Influence Analysis with Panel Data using Stata

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Motivation

- Short panel data sets (small N but $N \gg T$) are common in many fields of Economics, e.g.
  - Macro-level panel data
  - Experimental panel data
- Observational data may contain "anomalous" observations (Rousseeuw and Van Zomeren, 1990; Silva, 2001)
  - Vertical outliers (VO), good leverage (GL) points, bad leverage (BL) points ▸ Example ▸ DGP
- Large influence on the Least Squares (LS) estimates
  $\Longrightarrow$ Biased regression coefficients or standard errors (Donald and Maddala, 1993; Bramati and Croux, 2007; Verardi and Croux, 2009)

Motivation

- **Diagnostic plots** (leverage-vs-residual plots)
  - for cross-sectional data: `lvr2plot/lvr2plot2`
  - Less handy for panel data
- **Measures of influence** (Cook (1979)'s distance)
  - for cross-sectional data: `predict c, cooksd`
  - for panel data: `jackknife2, cooksd(`*newvar*`)` `bpd(`*newvar*`) :` *command*
  - These metrics may fail to flag multiple atypical cases (Atkinson and Mulira, 1993; Chatterjee and Hadi, 1988; Rousseeuw and Van Zomeren, 1990) unlike *pair-wise measures* (Lawrance, 1995)

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In this presentation

- I present a method to
  1. Detect and identify the type of anomalous unit
  2. Show how these affect the LS estimates, and the influence of other units
- I follow a *unit-wise* approach (full history of a unit)
- I propose two commands in Stata
  - `xtlvr2plot` – Leverage-vs-residual plot for panel data
  - `xtinfluence` – Influence analysis with panel data

Model and estimators

Static linear panel regression model with fixed effects

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + \alpha_i + u_{it}$$

Model after the *within-group* (WG) transformation

$$\widetilde{y}_{it} = \widetilde{\mathbf{x}}_{it}'\boldsymbol{\beta} + \widetilde{u}_{it}$$

where $\widetilde{y}_{it} = y_{it} - T^{-1}\sum_t y_{it}$, etc., and $\boldsymbol{\beta}$ is a vector of parameters.

The WG Estimator

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{\mathbf{x}}_{it}\widetilde{\mathbf{x}}_{it}'\right)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{\mathbf{x}}_{it}\widetilde{y}_{it}$$

Residual and Leverage

- The **average normalised residual** squared

$$\widehat{u}_i^* = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\widehat{u}_{it}}{\sqrt{\sum_i \widehat{u}_{it}^2}}\right)^2$$

  where $\widehat{u}_{it} = \widetilde{y}_{it} - \widetilde{\mathbf{x}}_{it}'\widehat{\boldsymbol{\beta}}$ are LS Residuals.

  Cut-off value: $c_{\widehat{u}_i^*} = \frac{2}{NT}$

- The **average individual leverage** of unit i at time t is

$$\overline{h}_i = \frac{1}{T}\sum_{t=1}^{T} h_{ii,tt}$$

  where $h_{ii,tt} = \widetilde{\mathbf{x}}_{it}'(\widetilde{\mathbf{X}}'\widetilde{\mathbf{X}})^{-1}\widetilde{\mathbf{x}}_{it}$, and $h_{ii,ts} = \widetilde{\mathbf{x}}_{it}'(\widetilde{\mathbf{X}}'\widetilde{\mathbf{X}})^{-1}\widetilde{\mathbf{x}}_{is}$ for $t, s = 1, \ldots, T$.

  Cut-off value: $c_{\overline{h}_i} = \frac{2(K+1)}{NT}$

xtlvr2plot: Syntax

`xtlvr2plot` – Leverage-versus-normalised residual squared plot for panel data.

`xtlvr2plot` *depvar* [*indepvar*] [*if*] [*in*] [, *options*]

options

graph_opts	graph options allowed for `twoway scatter`
Generated variables	
`_lev`	average individual leverage
`_normres2`	average individual residual squared

xtlvr2plot: Example of code

```
** Use of the 'xtlvr2plot' command
xtset id t

xtlvr2plot y x,                                   ///
    mlabel(id)                                    ///
    xlabel(, format(%9.3fc))                      ///
    ylabel(, angle(h) format(%9.3fc))             ///
    title("Unit-wise Evaluation", size(medsmall)) ///
    saving("xtlvr2plot_example.gph", replace)
```

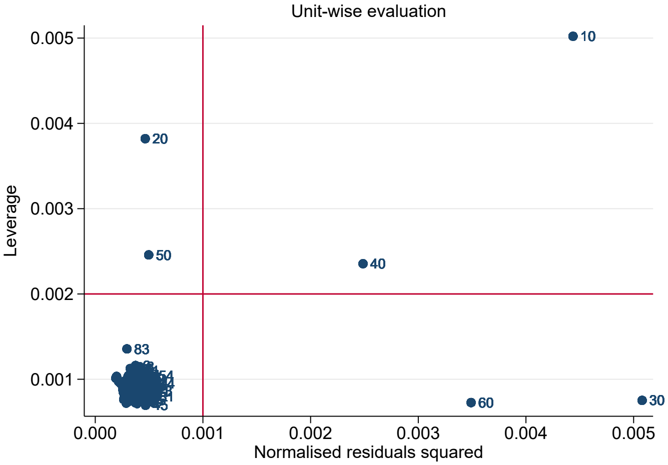
xtlvr2plot: Summary Table

```
** Summary table w/detected anomalous units
** generated by 'xtlvr2plot'
```

```
              Anomalous units
---------------------------------------------
x-cutoff =    0.001
y-cutoff =    0.002
---------------------------------------------
Good leverage units
 - Count : 2
 - List  : 20 50
Bad leverage units
 - Count : 2
 - List  : 10 40
Vertical outliers
 - Count : 2
 - List  : 30 60
---------------------------------------------
```

Note: Units 10 and 40 are set to be bad leverage units; units 20 and 50 good leverage units; units 30 and 60 vertical outliers.

xtlvr2plot



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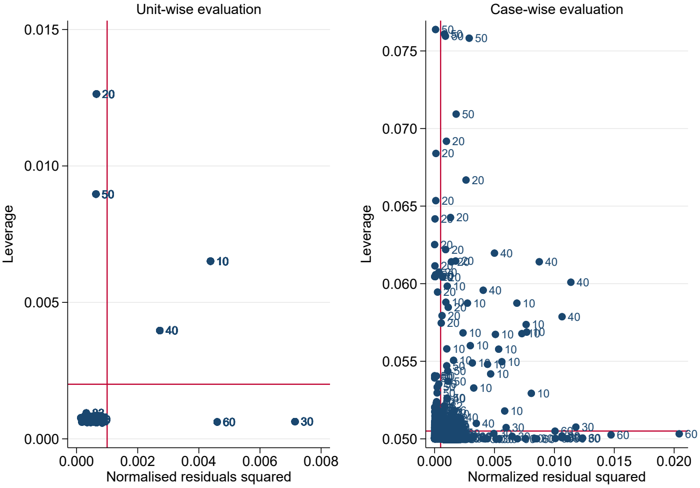
xtlvr2plot vs lvr2plot



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Influence analysis: Overview

- How anomalous units may affect the LS estimates

  1. Joint influence
  2. Joint effects
  3. Conditional influence
  4. Conditional effects

Influence analysis: Joint influence

- For $i \neq j$,

$$\mathrm{C}_{ij}(\widehat{\boldsymbol{\beta}}) = \big(\widehat{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}}_{(i,j)}\big)'\big(\widetilde{\mathbf{X}}'\widetilde{\mathbf{X}}\big)\big(\widehat{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}}_{(i,j)}\big)(s^2K)^{-1}$$

  where $\widehat{\boldsymbol{\beta}}_{(i,j)}$ is WG estimator w/t units i and j, s is RMSE, K is #covariates

  - Influence exerted by a pair (i,j) on LS estimates *jointly*
  - Comparison of LS estimates *with* and *without* the *pair*
  - $\mathrm{C}_{ij}(\widehat{\boldsymbol{\beta}}) = \mathrm{C}_{ji}(\widehat{\boldsymbol{\beta}})$

- $\mathrm{C}_{ij}(\widehat{\boldsymbol{\beta}}) \sim \mathrm{F}(\nu_1, \nu_2)$
  where $\nu_1 = k + 1$ and $\nu_2 = NT - N - (k + 1)$

Influence analysis: Joint influence

- For $i = j$,

$$\mathrm{C}_{ii}(\widehat{\boldsymbol{\beta}}) = \big(\widehat{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}}_{(i)}\big)'\big(\widetilde{\mathbf{X}}'\widetilde{\mathbf{X}}\big)\big(\widehat{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}}_{(i)}\big)(s^2K)^{-1}$$

where $\widehat{\boldsymbol{\beta}}_{(i)}$ is WG estimator w/t unit i

  - i's influence on LS estimates (as in Belotti and Peracchi (2020))

- $\mathrm{C}_{ii}(\widehat{\boldsymbol{\beta}}) \sim \mathrm{F}(\nu_1, \nu_2)$
where $\nu_1 = k + 1$ and $\nu_2 = NT - N - (k + 1)$

Influence analysis: Joint effects

- For $i \neq j$,

$$\mathrm{K}_{j|i} = \frac{\mathrm{C}_{ij}(\widehat{\boldsymbol{\beta}})}{\mathrm{C}_{ii}(\widehat{\boldsymbol{\beta}})}$$

  - How much the pair is influential wrt i
  - For $i = j$, $\mathrm{K}_{j|i} = 1$

- For large values of $\mathrm{K}_{j|i}$
  - The most influential unit (j) *alters* the effect of the least (i)
  - j either enhances or reduces the effect of i on the LS estimates $\Rightarrow$ based on the conditional effect

Influence analysis: Joint effects

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Influence analysis: Conditional influence

For $i \neq j$,

$$\mathrm{C}_{i(j)}(\widehat{\boldsymbol{\beta}}) = \big(\widehat{\boldsymbol{\beta}}_{(i,j)} - \widehat{\boldsymbol{\beta}}_{(j)}\big)' \left(\sum_{\substack{i=1 \\ i \neq j}}^{N} \widetilde{\mathbf{X}}'_{i(j)} \widetilde{\mathbf{X}}_{i(j)} \right) \big(\widehat{\boldsymbol{\beta}}_{(i,j)} - \widehat{\boldsymbol{\beta}}_{(j)}\big) (s^2 K)^{-1}$$

- Influence exerted by i on LS estimates without j in the sample
- How the absence of j affects the influence i on LS estimates
- $\mathrm{C}_{i(j)}(\widehat{\boldsymbol{\beta}}) = 0$ for $i = j$
- $\mathrm{C}_{i(j)}(\widehat{\boldsymbol{\beta}}) \neq \mathrm{C}_{j(i)}(\widehat{\boldsymbol{\beta}})$
- $\mathrm{C}_{i(j)}(\widehat{\boldsymbol{\beta}}) \sim F(\nu_1, \nu_2)$

Influence analysis: Conditional effects

- For $i \neq j$

$$\mathrm{M}_{i(j)} = \frac{\mathrm{C}_{i(j)}(\widehat{\boldsymbol{\beta}})}{\mathrm{C}_{ii}(\widehat{\boldsymbol{\beta}})}$$

  - How influence of i changes before and after the deletion of j

- If $\mathrm{M}_{i(j)} \geq 1$
  - influence of i increases without j in the sample
  - j *masks* i

- If $\mathrm{M}_{i(j)} < 1$
  - influence of i decreases without j in the sample
  - j *boosts* i

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xtinfluence: Syntax

xtinfluence – Influence analysis for panel data displaying the measures and effects of unit j against unit i. The size of the symbols is proportional to the magnitude of the calculated measures.

xtinfluence *depvar* [*indepvar*] [*if*] [*in*] [, *options*]

options

figure(*graphtype*)	display diagnostic plots like *graphtype* allows for the choice between scatter plot or heat plot; default is scatter
graph_opts	graph options allowed for scatter and heatplot
saving(*filename*)	save .dta and .pdf file with the specified name and location

Saved data sets

*filename*_adj_mtx.dta	Automatically saves a data set with the influence measures and effects generated by the command

xtinfluence: Example

```
**Use of the 'xtinfluence' command
xtset id t

** Heat plot
xtinfluence y x, figure(heat)                                   ///
        keylabels(all, interval) color(RdBu, reverse)           ///
        lev(30) statistic(max)                                  ///
        xlabel(5(10)100, angle(h) labsize(small))               ///
        xmtick(##10) xmlabel(##2, angle(h))                     ///
        ylabel(5(10)100, angle(h))                              ///
        ymtick(##10) ymlabel(##2, angle(h))                     ///
        saving("xtinfluence_heat")

** Scatter plot
xtinfluence y x, figure(scatter)                                ///
        xlabel(5(10)100, angle(h) labsize(small))               ///
        xmtick(##10) xmlabel(##2, angle(h))                     ///
        ylabel(5(10)100, angle(h))                              ///
        ymtick(##10) ymlabel(##2, angle(h))                     ///
        saving("xtinfluence_scatter")
```

Influence analysis: Summary table

Variable	Obs	Mean	Std. dev.	Min	Max
C	10,000	.3811386	2.200585	2.35e-11	33.58732
K	10,000	16156.08	1242556	4.42e-08	1.23e+08
cC	10,000	.0038312	.0353837	0	.6169614
M	9,900	.0305928	.6922132	4.39e-06	65.47916

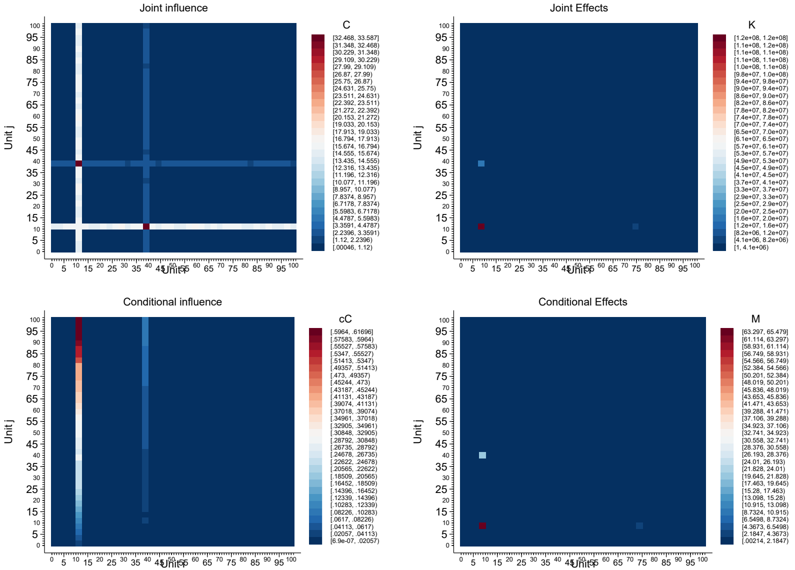
```
------------------------------------------------
             Influence analysis
------------------------------------------------
v1 = k+1 =  2
v2 = NT-N-k-1 = 1898
c1 = 4/N = .04
c2 = F(v1,v2,.5) =   0.6934
------------------------------------------------
Cii >= c1
 - Count : 8
 - List  : 8 10 20 34 40 43 50 65
Cii >= c2
 - Count : 2
 - List  : 10 40
i with K >= p99
 - Count : 30
 - List  : 3 4 6 9 11 13 14 19 24 27 47 49 55 57 62 64 67 68 69 71 72 74 76 77 79 84 86 89 93 95
j with K >= p99
 - Count :
 - List  :
i with M >= 1
 - Count : 2
 - List  : 9 74
j with M >= 1
 - Count : 2
 - List  : 10 40
------------------------------------------------
```

*filename*_adj_mtx.dta

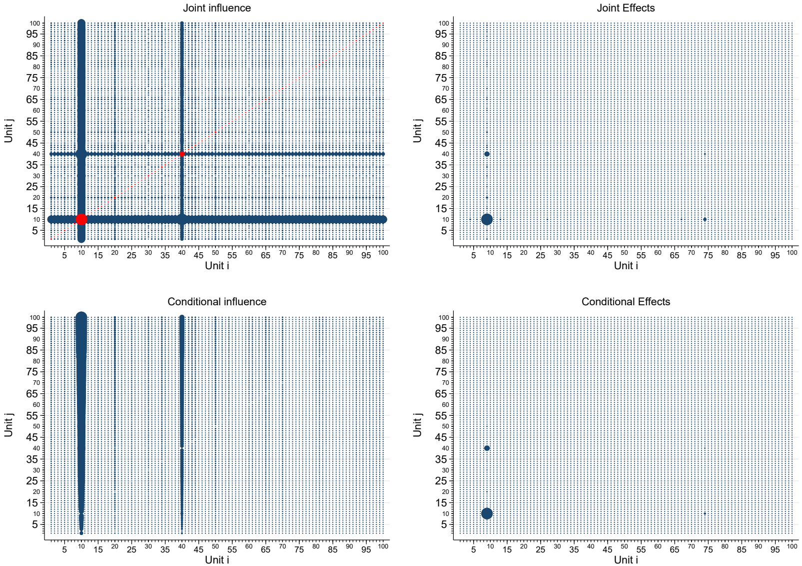
The saved data set resembles a directed and weighted adjacency list

	i	j	C	K	cC	M
1	1	1	.0318985	1	0	0
2	1	2	.0779802	2.444638	8.05e-06	.0002523
3	1	3	.0379366	1.189292	.000065	.0020391
4	1	4	.0812006	2.545595	.0000804	.0025191
5	1	5	.0384888	1.206603	.0000916	.0028703
6	1	6	.0619195	1.941144	.000091	.0028528
7	1	7	.0802803	2.516744	.0001116	.0034988
8	1	8	.0322271	1.010302	.0001236	.003874
9	1	9	.0102966	.3227937	.0001144	.0035852
10	1	10	34.86443	1092.981	.0001167	.0036569
11	1	11	.0380862	1.193983	.0001264	.0039615
12	1	12	.0524164	1.643225	.0001519	.0047621
13	1	13	.0510088	1.599099	.0001667	.005226
14	1	14	.0550416	1.725525	.0001834	.0057488
15	1	15	.0617752	1.936618	.0001679	.0052648
16	1	16	.0591808	1.855285	.000202	.0063336
17	1	17	.0512263	1.605917	.0001969	.0061739
18	1	18	.067513	2.116496	.0002049	.006424
19	1	19	.0904264	2.834818	.000237	.0074296
20	1	20	11.59427	363.474	.0005592	.0175295
21	1	21	.0564583	1.769938	.0002562	.0080332
22	1	22	.0020566	.0644732	.0002375	.0074454
23	1	23	.091529	2.869384	.0002585	.0081049
24	1	24	.026083	.8176892	.0002669	.0083674
25	1	25	.0945991	2.965631	.0003046	.0095503

Influence analysis: Heat plot



Influence analysis: Scatter plot



Conclusion

- The proposed STATA commands allow to
  1. Identify anomalous units and their type (unit-wise leverage-vs-residual plot)
  2. Investigate how anomalous units may affect the LS estimates (joint and conditional influence and effects)
- Once identified the type of anomaly in the sample
  1. Methods for measurement error if error in the data entry
  2. Robust estimation techniques if VO and BL (Bramati and Croux, 2007; Verardi and Croux, 2009; Aquaro and Čížek, 2013, 2014; Jiao, 2022)
  3. Jackknife-type standard errors if GL (MacKinnon and White, 1985; Davidson et al., 1993; MacKinnon, 2013; Belotti and Peracchi, 2020; Polselli, 2022)

Thank you for your attention!

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https://github.com/POLSEAN/Influence-Analysis

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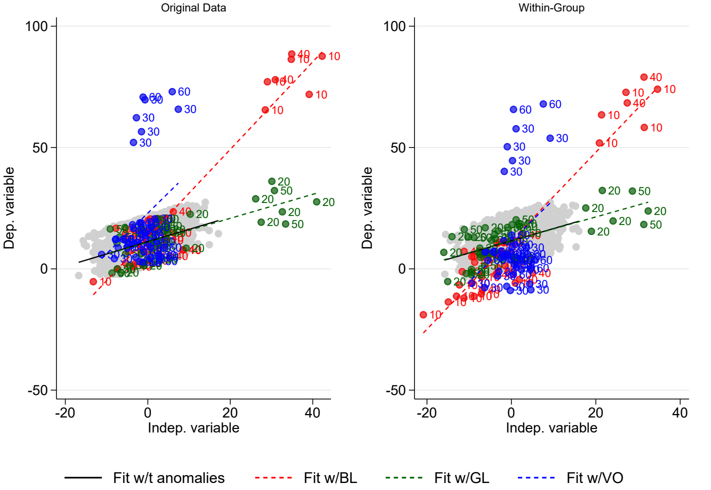
Scatter Plot DGP

Back



Note: Units 10 and 40 are bad leverage units; units 20 and 50 are good leverage units; units 30 and 60 are vertical outliers.

DGP ▸ Back

```
loc numobs 100
set obs 100
gen id = _n
expand 20

bys id: generate t = _n
bys id: gen z = rnormal(0,5)
**GL
bys id: replace z = z + rnormal(30,1) if id==20 & t<=5
bys id: replace z = z + rnormal(30,1) if id==50 & t<=2
**for BL
bys id: replace z = z + rnormal(30,1) if id==10 & t<=5
bys id: replace z = z + rnormal(30,1) if id==40 & t<=2
**line
bys id: gen a = runiform(0,20)
bys id: gen y = 1 + .5*z + a + runiform()
**BL
bys id: replace y = y + rnormal(50,1) if id==10 & t<=5
bys id: replace y = y + rnormal(50,1) if id==40 & t<=2
*VO
bys id: replace y = y + rnormal(50,1) if id==30 & t<=5
bys id: replace y = y + rnormal(50,1) if id==60 & t<=2
```

Example: Berka et al. (2018)

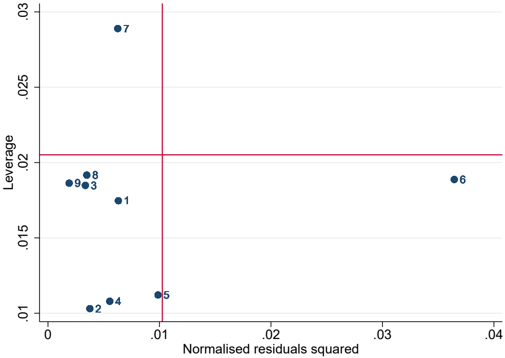
- They study relationship between real exchange rate and sectoral productivity in the Eurozone
- Regression model:

$$RER_{it} = \beta TFP_{it} + \mathbf{x}'_{it}\boldsymbol{\gamma} + \alpha_i + u_{it}$$

RER_{it}: real exchange rate in log
TFP_{it}: total factor productivity in log
$\mathbf{x}_{it}$: other controls
α_i: country fixed effects

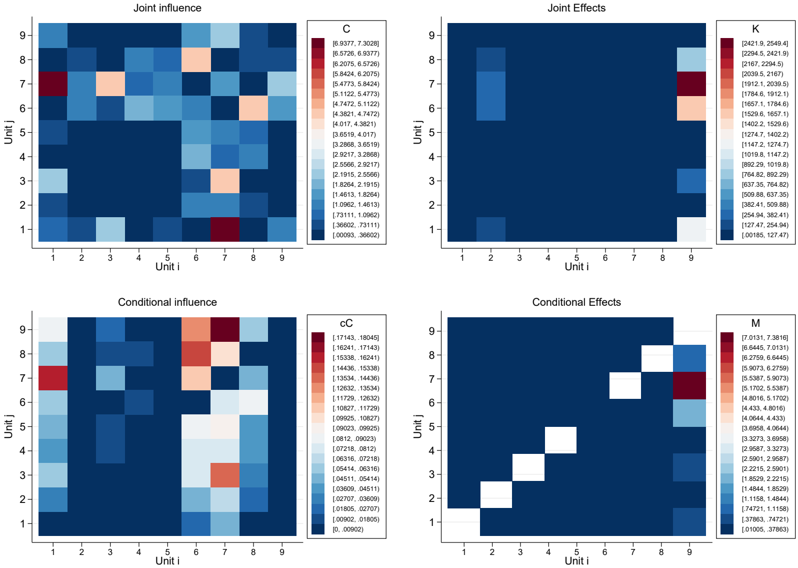
- Finding strong correlation between TFP and RER among high-income countries with floating nominal exchange rates
- Sample: 9 countries
- Time Period: 1995–2007
- Table 4, specification (2a)

Example: Leverage-vs-residual plot ▸ Scatter



Note: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Ireland, 7-Italy, 8-Netherlands, 9-Spain.

Example: Network-like plots ▸ Summary



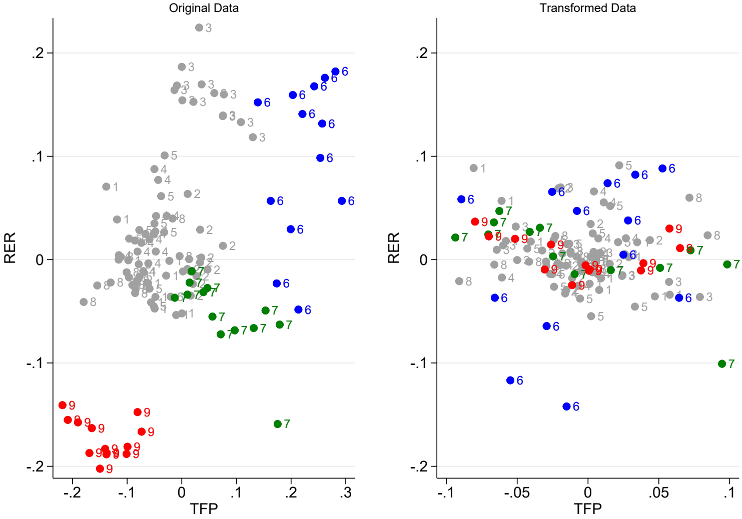
Example: Summary ▸ Back

Variable	Obs	Mean	Std. dev.	Min	Max
C	81	1.0233	1.472976	.0009253	7.30281
K	81	97.87085	368.2484	.0018538	2549.404
cC	81	.032125	.0439157	0	.1804506
M	72	.2303033	.8915019	.0046645	7.381636

```
-----------------------------------------------
              Influence analysis
-----------------------------------------------
v1 = k+1 =  2
v2 = NT-N-k-1 = 184
c1 = 4/N = .4444444444444444
c2 = F(v1,v2,.5) =   0.6958
-----------------------------------------------
Cii >= c1
 - Count : 4
 - List  : 1 6 7 8
Cii >= c2
 - Count : 3
 - List  : 1 6 7
i with K >= p99
 - Count : 1
 - List  : 9
j with K >= p99
 - Count :
 - List  :
i with M >= 1
 - Count : 1
 - List  : 9
j with M >= 1
 - Count : 2
 - List  : 6 7
-----------------------------------------------
```

Example: Scatter

Back



Note: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Ireland, 7-Italy, 8-Netherlands, 9-Spain.

Summary of Method

1. Identify anomalous units and their type with `xtlvr2plot`

2. Conduct the influence analysis with `xtinfluence`

   2.1 **Joint Influence Plot**
   - Identify units with high individual influence (main diagonal)
   - Identify pairs with high joint influence (off-diagonal)
   - Highly influential units swamp all other units

   2.2 **Joint Effect Plot**
   - Identify pairs with largest effect
   - j swamps the effect of i
   - j must be detected in (1) and (2.1)

   2.3 **Conditional Influence Plot**
   - Identify influential i conditional to removing j
   - Check if same units as (1) and (2.1)

   2.4 **Conditional Effect Plot**
   - Identify pairs with largest effect
   - j masks the effect of i
   - Compare identified pairs with (2.2)

3. Units detected in (1), (2.1) and (2.3) are anomalous; (2.2) and (2.4) explain how they affect the influence of other units and, hence, LS estimates