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Discrete-time multistate regression models in Stata: The `dtms` module

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Outline

- Multistate methodology
- The **`dtms`** package
  - Example
  - Package highlights / contributions
  - Results organization and storage: Tree management
  - Analytical contributions
  - Computational aspects
  - Package shortcomings / To be implemented

Discrete-Time Multistate Estimation

- Discrete set of states, e.g., employment status, health status
- Discrete-time
  - Evenly spaced (intermittently observed) data
- Under Markov assumption, use
  - **(1)** regression models to
  - **(2)** predict transition probabilities, then
  - **(3)** apply matrix formulas to get stats like:
    - state and life expectancies (LEXP)
    - lifetime risk (LRSK), mean age at first entry (MAFN)
    - other newly introduced stats, like state at absorption (STAB) and mean duration of episodes (MDUR)

Continuous v. Discrete

- Data dependent choice
- Choice also depends on research question
- Computational advantage depends on application
- Ease of application
- Software exists mostly for continuous time

Package Installation

```
. net install dtms, from(https://user.demogr.mpg.de/schneider/stata)
```

- At some point, installation will be via SSC.
- Requires Stata version 16.1 or higher.
- Announcement / thread on Stata Forum:
  https://www.statalist.org/forums/forum/general-stata-discussion/general/1690703-dtms-new-stata-command-for-discrete-time-multistate-model-estimation

Example

- All example statements to follow are listed in the addendum slides.
- `dtms` estimation proceeds in sequential steps:

(2) regression (`mlogit`)

(3) predict transition probabilities

(4) calculate various results from them

Very first step: **(1)** Model setup, so there are a total of four steps

Example

```
.     dtms exampledata 12
(dtms example data set loaded)
(fictitious single-year data (II) on cognitive impairment (age centered at 50))
.     describe

Contains data
 Observations:        28,654                  fictitious single-year data (II) on cognitive impairment
                                                (age centered at 50)
    Variables:             8                  10 Jun 2023 11:01
                                              (_dta has notes)
---------------------------------------------------------------------------------------------------
Variable      Storage   Display    Value
    name         type    format    label      Variable label
---------------------------------------------------------------------------------------------------
id              int     %12.0g                Subject ID
n               float   %9.0g                 Subject obs number
year            int     %9.0g                 Survey year
age             float   %9.0g                 Exact age at interview (years, centered at 50)
cog3            byte    %9.0g      COG3       Cognitive impairment, 3-cat
sex             byte    %8.0g      SEX        Sex
educ            byte    %9.0g      EDUC       Education level, 3-cat
numdrinks       byte    %9.0g                 # of alc drinks / week
---------------------------------------------------------------------------------------------------
Sorted by: id  year
```

Example

```
. table cog3
```

	Frequency
Cognitive impairment, 3-cat	
1.none	21,534
2.mild	4,729
3.severe	1,181
4.dead	1,210
Total	28,654

```
. xttrans cog3
```

Cognitive impairment, 3-cat	Cognitive impairment, 3-cat				
	1	2	3	4	Total
1	90.66	6.84	0.04	2.46	100.00
2	20.82	60.30	9.22	9.66	100.00
3	0.61	20.52	56.77	22.10	100.00
Total	74.77	16.63	4.06	4.54	100.00

Example

```
. dtms clear
. dtms setup (impex) , states(1 none 2 mild 3 sevr 4 dead) abs(1) ages(50/110)
. dtms dir
  (S) impex : (no label) | tra IDs: 1 2 3 | abs IDs: 4 | 61 ages: 50-110
.
. mlogit cog3 iL.cog3 c.age c.age#c.age sex##educ c.numdrinks , cluster(id) rrr
      (output omitted)

. dtms estimate frome (impex mlog) , agevar(age, center(50)) orgnvar(L.cog3)
. dtms dir
  (S) impex : (no label) | tra IDs: 1 2 3 | abs IDs: 4 | 61 ages: 50-110
    (E) mlog : (no label) | cmdline: mlogit cog3 iL.cog3 c.age c.age#c.age se..

. dtms transprob atmeans (impex mlog allmeans)
. dtms dir
  (S) impex : (no label) | tra IDs: 1 2 3 | abs IDs: 4 | 61 ages: 50-110
    (E) mlog : (no label) | cmdline: mlogit cog3 iL.cog3 c.age c.age#c.age se..
      (T) allmeans : (no label) | dtms trans atmeans: L.cog3=(1 2 3) age=(1 2..
```

Example

```
. dtms proportion fixed (impex pfixed) , values(0.88 0.10 0.02)
. dtms result lexp (impex mlog allmeans lexp) , initprop(pfixed) format(%7.1f)

  Life expectancy


Remaining life expectancy

                 |  init
                 |     none       mild       sevr      total
-----------------+--------------------------------------------
state            |
            none |     24.5       21.3       14.3       24.0
            mild |      3.8        4.9        3.7        3.9
            sevr |      0.8        0.9        2.1        0.8
           total |     29.1       27.1       20.2       28.7
. dtms dir
  (S) impex : (no label) | tra IDs: 1 2 3 | abs IDs: 4 | 61 ages: 50-110
    (P) pfixed : (no label) | fixed:  0.880 0.100 0.020
    (E) mlog : (no label) | cmdline: mlogit cog3 iL.cog3 c.age c.age#c.age se..
      (T) allmeans : (no label) | dtms trans atmeans: L.cog3=(1 2 3) age=(1 2..
        (R) lexp : (no label) | prop: pfixed | timing: mid | calc: analytic |..
```

Simplification: a single fixed initial proportion is assumed for all groups in the example

Example

```
. dtms transprob atmeans (impex mlog edlow)  , at(educ=1)
. dtms transprob atmeans (impex mlog edhigh) , at(educ=3)
. dtms result lexp (impex mlog edlow  lexp) , initprop(pfixed) format(%7.1f)
. dtms result lexp (impex mlog edhigh lexp) , initprop(pfixed) format(%7.1f)
```

Remaining life expectancy

	init none	mild	sevr	total
state				
none	**18.5**	**14.7**	**9.8**	**17.9**
mild	**5.2**	**6.4**	**4.8**	**5.3**
sevr	**1.2**	**1.4**	**2.6**	**1.2**
total	**24.9**	**22.5**	**17.2**	**24.5**

Remaining life expectancy

	init none	mild	sevr	total
state				
none	**28.2**	**25.3**	**15.7**	**27.7**
mild	**2.3**	**3.2**	**2.3**	**2.4**
sevr	**0.3**	**0.4**	**1.6**	**0.4**
total	**30.8**	**28.9**	**19.5**	**30.4**

Example

```
. dtms result stab (impex mlog edlow  stab) , initpr(pfixed) form(%7.2f)
. dtms result stab (impex mlog edhigh stab) , initpr(pfixed) form(%7.2f) post nopv
```

State distribution before absorption

	init none	mild	sevr	total
state				
none	0.38	0.32	0.23	0.37
mild	0.40	0.44	0.32	0.40
sevr	0.22	0.23	0.46	0.23
total	1.00	1.00	1.00	1.00

State distribution before absorption

	init none	mild	sevr	total
state				
none	0.60	0.55	0.36	0.59
mild	0.29	0.33	0.21	0.30
sevr	0.11	0.12	0.43	0.11
total	1.00	1.00	1.00	1.00

State distribution before absorption:

	Coefficient	Std. err.	[95% conf.	interval]
none				
none	0.598	0.027	0.545	0.650
mild	0.295	0.023	0.250	0.340
sevr	0.107	0.019	0.070	0.144
mild				
none	0.552	0.026	0.500	0.604
mild	0.332	0.023	0.286	0.378
sevr	0.116	0.020	0.077	0.156
sevr				
none	0.358	0.033	0.293	0.422
mild	0.212	0.024	0.165	0.258
sevr	0.431	0.048	0.336	0.525
cond				
none	1.000	0.000	1.000	1.000
mild	1.000	0.000	1.000	1.000
sevr	1.000	0.000	1.000	1.000
state				
none	0.588	0.026	0.537	0.640
mild	0.297	0.023	0.253	0.341
sevr	0.115	0.019	0.078	0.151
total				
total	1.000	0.000	1.000	1.000

Example

```
. dtms dir
  (S) impex : (no label) | tra IDs: 1 2 3 | abs IDs: 4 | 61 ages: 50-110
    (P) pfixed : (no label) | fixed:  0.880 0.100 0.020
    (E) mlog : (no label) | cmdline: mlogit cog3 iL.cog3 c.age c.age#c.age se..
      (T) allmeans : (no label) | dtms trans atmeans: L.cog3=(1 2 3) age=(1 2..
        (R) lexp : (no label) | prop: pfixed | timing: mid | calc: analytic |..
      (T) edlow : (no label) | dtms trans atmeans: L.cog3=(1 2 3) age=(1 2 3 ..
        (R) lexp : (no label) | prop: pfixed | timing: mid | calc: analytic |..
        (R) stab : (no label) | prop: pfixed | timing:  | calc: analytic | ST..
      (T) edhigh : (no label) | dtms trans atmeans: L.cog3=(1 2 3) age=(1 2 3..
        (R) lexp : (no label) | prop: pfixed | timing: mid | calc: analytic |..
        (R) stab : (no label) | prop: pfixed | timing:  | calc: analytic | ST..
```

```
. dtms file save using temptree.dtms , replace
```

Example

```
. dtms combine (impex mlog edhigh lexp) , with(edlow lexp) post(state total)

  2 combined dtms results
  result type(s) : lexp
  model name     : impex mlog
  result names   : 1: edhigh lexp
                   2: edlow lexp
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
1_lexp_state						
none	27.654	0.755	36.644	0.000	26.175	29.133
mild	2.388	0.271	8.810	0.000	1.857	2.919
sevr	0.370	0.100	3.693	0.000	0.174	0.566
1_lexp_total						
total	30.412	0.737	41.285	0.000	28.968	31.855
2_lexp_state						
none	17.940	0.643	27.901	0.000	16.680	19.201
mild	5.292	0.367	14.433	0.000	4.574	6.011
sevr	1.243	0.154	8.048	0.000	0.940	1.545
2_lexp_total						
total	24.476	0.676	36.193	0.000	23.150	25.801

Example

```
. dtms combine (impex mlog edhigh lexp) , with(edlow lexp) post(state total) diff

  2 combined dtms results
  result type(s) : lexp difference
  model name     : impex mlog
  result names   : 1: edhigh lexp
                   2: edlow lexp
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
lexp_state						
none	9.714	0.923	10.529	0.000	7.906	11.522
mild	-2.905	0.436	-6.662	0.000	-3.759	-2.050
sevr	-0.873	0.182	-4.805	0.000	-1.229	-0.517
lexp_total						
total	5.936	0.881	6.738	0.000	4.209	7.663

Package Highlights / Contributions

- A **framework for organizing** and saving results
- Up to 9 states
- Automated estimation of transition probabilities
- **14+ different outcome statistics**
- **Asymptotic covariance matrices**
- **Partial age ranges**
- **Group comparisons**
  linear and nonlinear hypothesis testing on any number of and on any type of results
- **Speed** (interactive time range)
- **Markov chains with rewards**
  general implementation; includes the possibility of user-defined rewards
- Works with `svy` estimation
- Generation of data sets with simulated trajectories
- Simulation-based results as alternative to analytical ones

All help files and/or subcommands of the **dtms** package:

package and conceptual overview (help dtms)

managing the dtms tree or its elements (help dtms tree)

dtms dir	list a dtms tree
dtms describe	describe a dtms tree element
dtms label	label a dtms tree element
dtms rename	rename a dtms tree element
dtms drop	drop a dtms tree element
dtms usedby	list dtms tree elements that use a particular side tree element
dtms file	save and load setups and all of their downstream elements
dtms settings	query and modify global dtms settings
dtms clear	delete the entire dtms tree and all global dtms settings

adding elements to the dtms tree (help dtms add)

dtms setup	add basic model setup
dtms proportion	add initial proportion
dtms rewards	add non-standard transition timing specification
dtms estimate	add model regression estimate
dtms transprob	add transition probabillities
dtms simdata	add simulated trajectories

calculate results (help dtms result)

dtms result	calculate and add one of the 14+ different outcomes to the dtms tree

extract information from the dtms tree (help dtms extract)

dtms erestore	restore e()-results that are held in some dtms tree elements
dtms combine	post combined estimates of two result tree elements to e()
dtms getmatrix	query various kinds of matrices related to dtms calculations
dtms matbrowse	browse Stata matrix in Stata's data browser

extensive examples and helper commands for executing them (help dtms examples)

dtms exampledata	load example data sets
dtms exampletree	load example tree elements into the dtms tree

Tree Management

<u>The dtms tree</u>

The dtms tree is structured as:

```
                                              dtms
                                                 |
                            ------------------------ [...]
                            |                    |            LEVEL 1
           --------------- S1        [...] --- S2               MAIN : S-ETUP
          /               / |                / |
         /         -------  |            [...]   |
        /         /         |                    |
       /         /    ---------- [...]     --------- [...]    - - - - - - - - -
   P1--      W1--     |     |              |     |            LEVEL 2
   P2/|      W1/|    E1    E2             E1   E2               SIDE : P-ROPORTION
[...]/    [...]/      |     |              |     |                     RE-W-ARDS
                    [...] [...]            |  [...]             MAIN : E-STIMATE
                                           |
                                     ------------ [...]       - - - - - - - - -
                                     |     |                  LEVEL 3
                              ----- T1    T2                    MAIN : T-RANSPROB
                             /       |     |
                            /        |   [...]
                           /         |
                          /    ----------- [...]              - - - - - - - - -
                      I1--     |     |                        LEVEL 4
                      I2/|    R1    R2                          SIDE : S-I-MDATA
                   [...]/                                       MAIN : R-ESULT
```

where the four main tree elements are:

S# = (Model) Setup number #
E# = Estimate number # of the setup it belongs to
T# = Transition probability ("transprob") number # of the estimate it belongs to
R# = Result number # of the transprob it belongs to

plus three helper (side tree) elements:

P# = (Initial) Proportion number # of the setup it belongs to
W# = (Time) Rewards number # of the setup they belong to
I# = Simulation data number # of the transprob it belongs to

Tree Management

```
                                              dtms
                                                |
                            ------------------------ [...]
                            |                   |             LEVEL 1
           --------------- S1        [...] --- S2               MAIN : S-ETUP
          /               / |                /  |
         /         -------  |            [...]  |
        /         /         |                   |
       /         /    ---------- [...]     --------- [...]    - - - - - - - - -
   P1--      W1--     |     |              |    |             LEVEL 2
   P2/|      W1/|    E1    E2             E1   E2               SIDE : P-ROPORTION
[...]/    [...]/      |     |              |    |                      RE-W-ARDS
                    [...] [...]            |  [...]             MAIN : E-STIMATE
                                           |
                                     ------------ [...]       - - - - - - - - -
                                     |     |                  LEVEL 3
                              ----- T1    T2                    MAIN : T-RANSPROB
                             /       |     |
                            /        |   [...]
                           /         |
                          /    ----------- [...]              - - - - - - - - -
                      I1--     |     |                        LEVEL 4
                      I2/|    R1    R2                          SIDE : S-I-MDATA
                   [...]/                                       MAIN : R-ESULT
```

Analytical Contributions

- The material that follows will soon be available in two working papers: Schneider (2023) and Schneider and Myrskylä (2023).
- Large sections of the mathematical derivations are already in the methods doc included in the `dtms` package.

Analytical Contributions: Preliminaries

Markov transition matrix:

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{U}_2 & \mathbf{0} & & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{U}_3 & & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & & \ddots & & & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{U}_{\bar{a}-1} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{d}_2 & \boldsymbol{d}_3 & \cdots & \boldsymbol{d}_{\bar{a}-1} & \boldsymbol{d}_{\bar{a}} & 1 \end{bmatrix}$$

Submatrices have trans. prob. at age a:

$$U_a = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1\bar{s}} \\ u_{21} & u_{22} & & u_{2\bar{s}} \\ \vdots & & \ddots & \vdots \\ u_{\bar{s}1} & u_{\bar{s}2} & \cdots & u_{\bar{s}\bar{s}} \end{bmatrix}$$

Sometimes only the transient states are needed:

$$\boldsymbol{U} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \boldsymbol{U}_2 & \mathbf{0} & & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{U}_3 & & \mathbf{0} & \mathbf{0} \\ \vdots & & \ddots & & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{U}_{\bar{a}-1} & \mathbf{0} \end{bmatrix}$$

The **fundamental matrix** has elements for LEXP: $\boldsymbol{F} = (\boldsymbol{I} - \boldsymbol{U})^{-1}$

Analytical Contributions: Asymptotics, LEXP

The first column of the fundamental matrix contains all the elements of state expectancies:

$$\boldsymbol{F} = (\boldsymbol{I} - \boldsymbol{U})^{-1} = \sum_{a=0}^{\bar{a}-2} \boldsymbol{U}^{a} = \begin{bmatrix} \boldsymbol{I} & | & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{U}_2 & | & \boldsymbol{I} & & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{U}_3 \cdot \boldsymbol{U}_2 & | & \boldsymbol{U}_3 & & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & | & & \ddots & & \vdots \\ \boldsymbol{U}_{\bar{a}-1} \cdot \ldots \cdot \boldsymbol{U}_2 & | & \boldsymbol{U}_{\bar{a}-1} \cdot \ldots \cdot \boldsymbol{U}_3 & \cdots & \boldsymbol{U}_{\bar{a}-1} & \boldsymbol{I} \end{bmatrix}$$

$$\boldsymbol{F}_1 = \left| \boldsymbol{U}_2 \quad \boldsymbol{U}_3 \boldsymbol{U}_2 \quad \cdots \quad \prod_{a=\bar{a}_{-1}}^{2} \boldsymbol{U}_a \right| = [\boldsymbol{f}_2 \quad \boldsymbol{f}_3 \quad \cdots \quad \boldsymbol{f}_{\bar{a}_{-1}}]$$

Let $\widetilde{\boldsymbol{U}} = [\boldsymbol{U}_2 \quad \ldots \quad \boldsymbol{U}_{\bar{a}-1}]$, then $\boldsymbol{V}^{tr} = \mathrm{cov}[\mathrm{vec}[\widetilde{\boldsymbol{U}}]]$.

Use the delta method to compute the covariance matrix:

$$\boldsymbol{V}^{F_1} = \boldsymbol{G}^{F_1} \boldsymbol{V}^{tr} \boldsymbol{G}^{F_1\prime} \qquad \boldsymbol{G}^{F_1} = \frac{\partial \,\mathrm{vec}\, \boldsymbol{V}^{F_1}}{\partial \mathrm{vec} \widetilde{\boldsymbol{U}'}}$$

Analytical Contributions: Asymptotics, LEXP

To summarize: Life expectancy covariance matrix:

$$\boldsymbol{V}^{tr} = \boldsymbol{G}^{tr}\boldsymbol{V}^{ml}\boldsymbol{G}^{tr\prime}$$

$$\boldsymbol{V}^{F_1} = \boldsymbol{G}^{F_1}\boldsymbol{V}^{tr}\boldsymbol{G}^{F_1\prime}$$

$$\boldsymbol{V}^{E} = (\boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1})\boldsymbol{V}^{F_1}(\boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1})'$$

Calculations repeatedly use the **delta method**.

Analytical Contributions: Asymptotics, LEXP

$$\boldsymbol{V}^{F_1} = \boldsymbol{G}^{F_1}\boldsymbol{V}^{tr}\boldsymbol{G}^{F_1\prime}$$

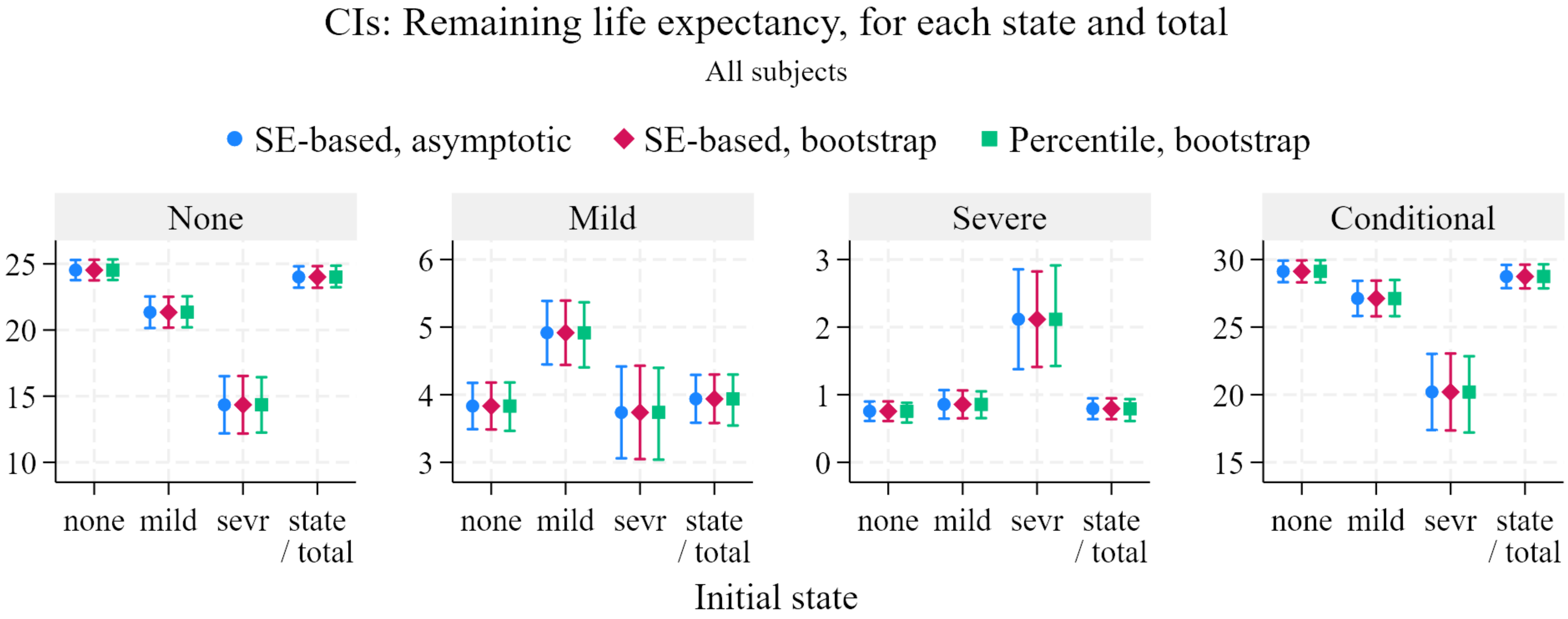
$$\frac{\partial\,\mathrm{vec}(\boldsymbol{f}_i)}{\partial\,\mathrm{vec}(\boldsymbol{U}_j)'} = \frac{\partial\,\mathrm{vec}\left(\prod_{a=i}^{j+1}\boldsymbol{U}_a \times \boldsymbol{U}_j \times \prod_{a=j-1}^{2}\boldsymbol{U}_a\right)}{\partial\,\mathrm{vec}(\boldsymbol{U}_j)'}$$

$$\boldsymbol{G}^{F_1} = \begin{array}{c|cccccc} & \boldsymbol{U}_2 & \boldsymbol{U}_3 & \boldsymbol{U}_4 & \cdots & \boldsymbol{U}_{\bar{a}_{-1}} \\ \hline \boldsymbol{U}_2 & \boldsymbol{I}\otimes\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{U}_3\boldsymbol{U}_2 & \boldsymbol{I}\otimes\boldsymbol{U}_3 & \boldsymbol{U}_2'\otimes\boldsymbol{I} & \boldsymbol{0} & & \boldsymbol{0} \\ \boldsymbol{U}_4\boldsymbol{U}_3\boldsymbol{U}_2 & \boldsymbol{I}\otimes\boldsymbol{U}_4\boldsymbol{U}_3 & \boldsymbol{U}_2'\otimes\boldsymbol{U}_4 & (\boldsymbol{U}_3\boldsymbol{U}_2)'\otimes\boldsymbol{I} & & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{U}_{\bar{a}_{-1}}\ldots\boldsymbol{U}_2 & \boldsymbol{I}\otimes\boldsymbol{U}_{\bar{a}_{-1}}\ldots\boldsymbol{U}_3 & \boldsymbol{U}_2'\otimes\boldsymbol{U}_{\bar{a}_{-1}}\ldots\boldsymbol{U}_4 & (\boldsymbol{U}_3\boldsymbol{U}_2)'\otimes\boldsymbol{U}_{\bar{a}_{-1}}\ldots\boldsymbol{U}_5 & \cdots & (\boldsymbol{U}_{\bar{a}_{-2}}\ldots\boldsymbol{U}_2)\otimes\boldsymbol{I} \end{array}$$

$$\boldsymbol{V}^{E} = \left(\boldsymbol{I}_{\bar{s}^2}\otimes\boldsymbol{r}_{+1}\right)\boldsymbol{V}^{F_1}\left(\boldsymbol{I}_{\bar{s}^2}\otimes\boldsymbol{r}_{+1}\right)'$$

LEXP: Comparison Analytic v. Bootstrap CIs

```
. dtms erestore (impex mlog allmeans lexp) , replay
```



Analytical Contributions: Asymptotics, LRSK/MAFN

Output for lifetime risk / mean age at first entry:

```
. dtms result mafn (impex mlog allmeans mafn) , initst(1) intermed(2) target(3) post(lrsk mafn)
```

Mean age (at) first entry / lifetime risk:

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
lrsk						
lrsk	0.281	0.016	17.372	0.000	0.249	0.312
mafn						
mafn	76.385	0.671	113.776	0.000	75.069	77.701

Analytical Contributions: Asymptotics, LRSK/MAFN

Derivations similar to LEXP, but more complicated.

$$
\boldsymbol{G}_A^{F_{BA}} = \begin{array}{c|cccccc}
 & \boldsymbol{A}_2 & \boldsymbol{A}_3 & \boldsymbol{A}_4 & \cdots & \boldsymbol{A}_{\bar{a}-2} & \boldsymbol{A}_{\bar{a}-1} \\
\hline
\overline{\boldsymbol{B}}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\
\overline{\boldsymbol{B}}_3\boldsymbol{A}_2 & \boldsymbol{I}\otimes\overline{\boldsymbol{B}}_3 & \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0} & \boldsymbol{0} \\
\overline{\boldsymbol{B}}_4\boldsymbol{A}_3\boldsymbol{A}_2 & \boldsymbol{I}\otimes\overline{\boldsymbol{B}}_4\boldsymbol{A}_3 & \boldsymbol{A}_2{}'\otimes\overline{\boldsymbol{B}}_4 & \boldsymbol{0} & & \boldsymbol{0} & \boldsymbol{0} \\
\overline{\boldsymbol{B}}_4\boldsymbol{A}_4\boldsymbol{A}_3\boldsymbol{A}_2 & \boldsymbol{I}\otimes\overline{\boldsymbol{B}}_5\boldsymbol{A}_4\boldsymbol{A}_3 & \boldsymbol{A}_2{}'\otimes\overline{\boldsymbol{B}}_5\boldsymbol{A}_4 & (\boldsymbol{A}_3\boldsymbol{A}_2)'\otimes\overline{\boldsymbol{B}}_5 & & \boldsymbol{0} & \boldsymbol{0} \\
\vdots & \vdots & \vdots & \vdots & \ddots & & \vdots \\
\overline{\boldsymbol{B}}_{\bar{a}-1}\boldsymbol{A}_{\bar{a}-2}\ldots\boldsymbol{A}_2 & \boldsymbol{I}\otimes\overline{\boldsymbol{B}}_{\bar{a}-1}\boldsymbol{A}_{\bar{a}-2}\ldots\boldsymbol{A}_3 & \boldsymbol{A}_2{}'\otimes\overline{\boldsymbol{B}}_{\bar{a}-1}\boldsymbol{A}_{\bar{a}-2}\ldots\boldsymbol{A}_4 & (\boldsymbol{A}_3\boldsymbol{A}_2)'\otimes\overline{\boldsymbol{B}}_{\bar{a}-1}\boldsymbol{A}_{\bar{a}-2}\ldots\boldsymbol{A}_5 & \cdots & (\boldsymbol{A}_{\bar{a}-3}\ldots\boldsymbol{A}_2)'\otimes\overline{\boldsymbol{B}}_{\bar{a}-1} & \boldsymbol{0} \\
\overline{\boldsymbol{B}}_{\bar{a}}\boldsymbol{A}_{\bar{a}-1}\ldots\boldsymbol{A}_2 & \boldsymbol{I}\otimes\overline{\boldsymbol{B}}_{\bar{a}}\boldsymbol{A}_{\bar{a}-1}\ldots\boldsymbol{A}_3 & \boldsymbol{A}_2{}'\otimes\overline{\boldsymbol{B}}_{\bar{a}}\boldsymbol{A}_{\bar{a}-1}\ldots\boldsymbol{A}_4 & (\boldsymbol{A}_3\boldsymbol{A}_2)'\otimes\overline{\boldsymbol{B}}_{\bar{a}}\boldsymbol{A}_{\bar{a}-1}\ldots\boldsymbol{A}_5 & & (\boldsymbol{A}_{\bar{a}-3}\ldots\boldsymbol{A}_2)'\otimes\overline{\boldsymbol{B}}_{\bar{a}}\boldsymbol{A}_{\bar{a}-1} & (\boldsymbol{A}_{\bar{a}-2}\ldots\boldsymbol{A}_2)'\otimes\overline{\boldsymbol{B}}_{\bar{a}}
\end{array}
$$

$$
\boldsymbol{G}_B^{F_{BA}} = \begin{array}{c|cccccc}
 & \overline{\boldsymbol{B}}_2 & \overline{\boldsymbol{B}}_3 & \overline{\boldsymbol{B}}_4 & \ldots & \overline{\boldsymbol{B}}_{\bar{a}-1} & \overline{\boldsymbol{B}}_{\bar{a}} \\
\hline
\overline{\boldsymbol{B}}_2 & \boldsymbol{I}_{\bar{s}_A} & \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0} & \boldsymbol{0} \\
\overline{\boldsymbol{B}}_3\boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{A}_2{}' & \boldsymbol{0} & & \boldsymbol{0} & \boldsymbol{0} \\
\overline{\boldsymbol{B}}_4\boldsymbol{A}_3\boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{0} & (\boldsymbol{A}_3\boldsymbol{A}_2)' & & \boldsymbol{0} & \boldsymbol{0} \\
\vdots & \vdots & \vdots & \vdots & \ddots & & \vdots \\
\overline{\boldsymbol{B}}_{\bar{a}-1}\boldsymbol{A}_{\bar{a}-2}\ldots\boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & (\boldsymbol{A}_{\bar{a}-2}\ldots\boldsymbol{A}_2)' & \boldsymbol{0} \\
\overline{\boldsymbol{B}}_{\bar{a}}\boldsymbol{A}_{\bar{a}-1}\ldots\boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0} & (\boldsymbol{A}_{\bar{a}-1}\ldots\boldsymbol{A}_2)'
\end{array}
$$

MAFN: Comparison Analytic v. Bootstrap CIs

```
. dtms trans atmeans (impex mlog men) , at(sex=1)
. dtms trans atmeans (impex mlog wmn) , at(sex=2)
. qui dtms result mafn (impex mlog men mafn) , initst(1) intermed(2) target(3)
. qui dtms result mafn (impex mlog wmn mafn) , initst(1) intermed(2) target(3)
. qui dtms combine (impex mlog allmeans mafn) , with(men mafn) post(lrsk mafn)
. dtms combine e() , with(wmn mafn)
```

Analytical Contributions: Rewards

- The behind Markov chains with rewards: link probabilities of reaching certain states with transition probabilities out of that state, where each of these out-transitions is assigned a reward.
- LEXP is a special case.
- 12+ results in the package are based on the rewards method.
- Explaining rewards in more detail would need its own presentation.

```
epis  - number of episodes
stab  - state at absorption

mdur  - mean duration of episodes
maan  - mean age, all entries
maax  - mean age, all exits
maax# - mean age, all entries, detail for
        state encoded #
maan# - mean age, all exits, detail for state
        encoded #
maab  - mean age at absorption
mais  - mean age in state

ncnt  - entry count
xcnt  - exit count
ncnt# - entry count, detail for state #:
        breakdown w.r.t. exits to other
        states
xcnt# - exit count, detail for state #:
        breakdown w.r.t. entries from other
        states
```

Analytical Contributions: Asymptotics, Rewards

Derivations similar to LEXP, but more complicated.

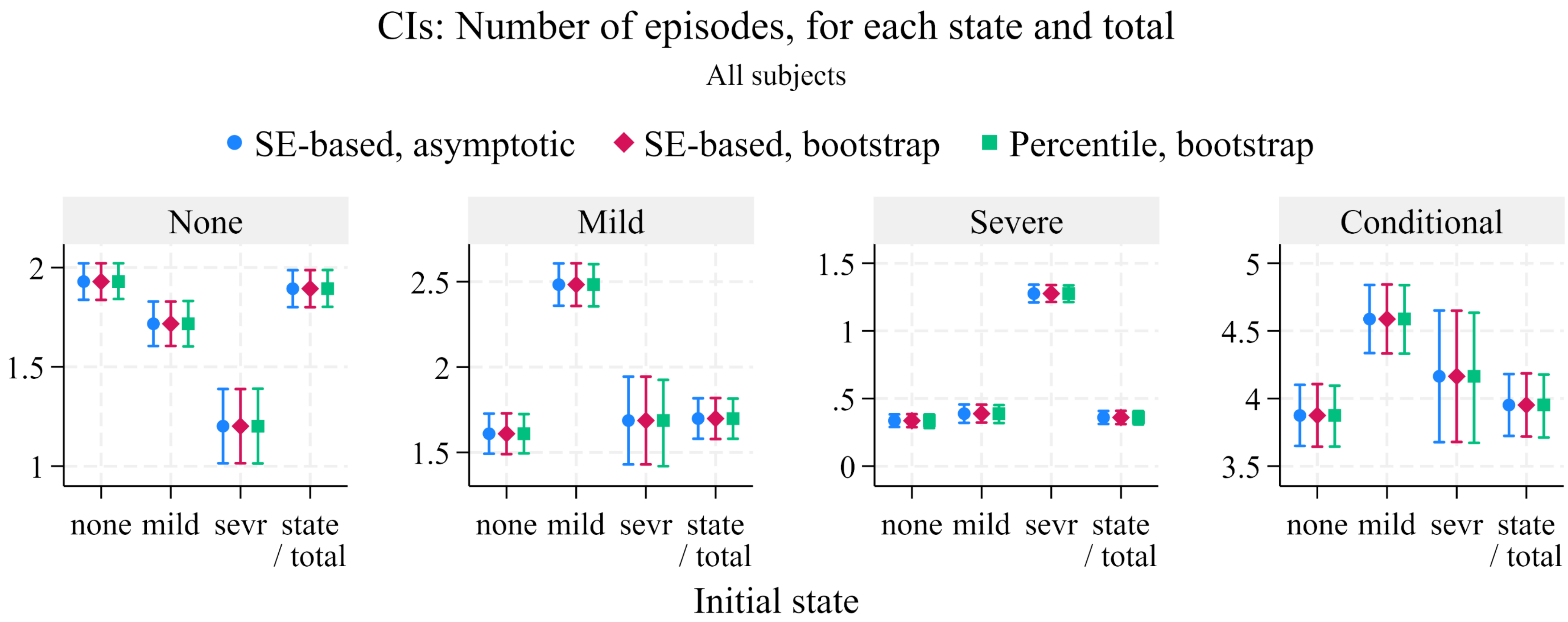
$$
\boldsymbol{G}^{F^S_R} = \begin{array}{c|cccccc}
 & \boldsymbol{U}_2 & \boldsymbol{U}_3 & \boldsymbol{U}_4 & \cdots & \boldsymbol{U}_{\bar{a}-1} \\
\hline
\overline{\boldsymbol{PR}}{}_2^s & [\boldsymbol{I} \otimes 1] \dfrac{\partial \operatorname{vec}(\overline{\boldsymbol{PR}}{}_2^s)}{\partial \operatorname{vec}(\boldsymbol{U}_2)'} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\
\overline{\boldsymbol{PR}}{}_3^s \boldsymbol{U}_2 & \boldsymbol{I} \otimes \overline{\boldsymbol{PR}}_3 & [\boldsymbol{U}_2{}' \otimes 1] \dfrac{\partial \operatorname{vec}(\overline{\boldsymbol{PR}}{}_3^s)}{\partial \operatorname{vec}(\boldsymbol{U}_3)'} & \boldsymbol{0} & & \boldsymbol{0} \\
\overline{\boldsymbol{PR}}{}_4^s \boldsymbol{U}_3 \boldsymbol{U}_2 & \boldsymbol{I} \otimes \overline{\boldsymbol{PR}}_4 \boldsymbol{U}_3 & \boldsymbol{U}_2{}' \otimes \overline{\boldsymbol{PR}}_4 & [(\boldsymbol{U}_3 \boldsymbol{U}_2)' \otimes 1] \dfrac{\partial \operatorname{vec}(\overline{\boldsymbol{PR}}{}_4^s)}{\partial \operatorname{vec}(\boldsymbol{U}_4)'} & & \boldsymbol{0} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\overline{\boldsymbol{PR}}{}_{\bar{a}-1}^s \boldsymbol{U}_{\bar{a}-2} \ldots \boldsymbol{U}_2 & \boldsymbol{I} \otimes \overline{\boldsymbol{PR}}_{\bar{a}-1} \ldots \boldsymbol{U}_3 & \boldsymbol{U}_2{}' \otimes \overline{\boldsymbol{PR}}_{\bar{a}-1} \ldots \boldsymbol{U}_4 & (\boldsymbol{U}_3 \boldsymbol{U}_2)' \otimes \overline{\boldsymbol{PR}}_{\bar{a}-1} \ldots \boldsymbol{U}_5 & & \left[\left(\boldsymbol{U}_{\bar{a}-2} \ldots \boldsymbol{U}_2\right)' \otimes 1\right] \dfrac{\partial \operatorname{vec}(\overline{\boldsymbol{PR}}{}_{\bar{a}-1}^s)}{\partial \operatorname{vec}\left(\boldsymbol{U}_{\bar{a}-1}\right)'} \\
\overline{\boldsymbol{PR}}{}_{\bar{a}}^s \boldsymbol{U}_{\bar{a}-1} \ldots \boldsymbol{U}_2 & \boldsymbol{I} \otimes \overline{\boldsymbol{PR}}_{\bar{a}} \ldots \boldsymbol{U}_3 & \boldsymbol{U}_2{}' \otimes \overline{\boldsymbol{PR}}_{\bar{a}} \ldots \boldsymbol{U}_4 & (\boldsymbol{U}_3 \boldsymbol{U}_2)' \otimes \overline{\boldsymbol{PR}}_{\bar{a}} \ldots \boldsymbol{U}_5 & \cdots & \left(\boldsymbol{U}_{\bar{a}-2} \ldots \boldsymbol{U}_2\right)' \otimes \overline{\boldsymbol{PR}}_{\bar{a}}
\end{array}
$$

with

$$
\frac{\partial \operatorname{vec}(\overline{\boldsymbol{PR}}{}_a^s)}{\partial \operatorname{vec}(\boldsymbol{U}_a)'} = \frac{\partial \operatorname{vec}\left(\mathbf{1}_{\bar{s}} \cdot \left(\breve{\boldsymbol{P}}_a \odot \boldsymbol{R}_a^s\right)\right)}{\partial \operatorname{vec}(\boldsymbol{U}_a)'} = \left(\boldsymbol{I}_{\bar{s}} \otimes \mathbf{1}_{\bar{s}+1}\right) \cdot \boldsymbol{K}_{\bar{s}\bar{s}+1} \cdot \begin{bmatrix} \operatorname{diag}\left(\operatorname{vec}\left(\tilde{\boldsymbol{R}}_a^{s\prime}\right)\right) \\ -\left(\mathbf{1}_{\bar{s}} \otimes \operatorname{diag}(\tilde{\boldsymbol{r}}_a^s)\right) \end{bmatrix} \cdot \boldsymbol{K}_{\bar{s}\bar{s}}
$$

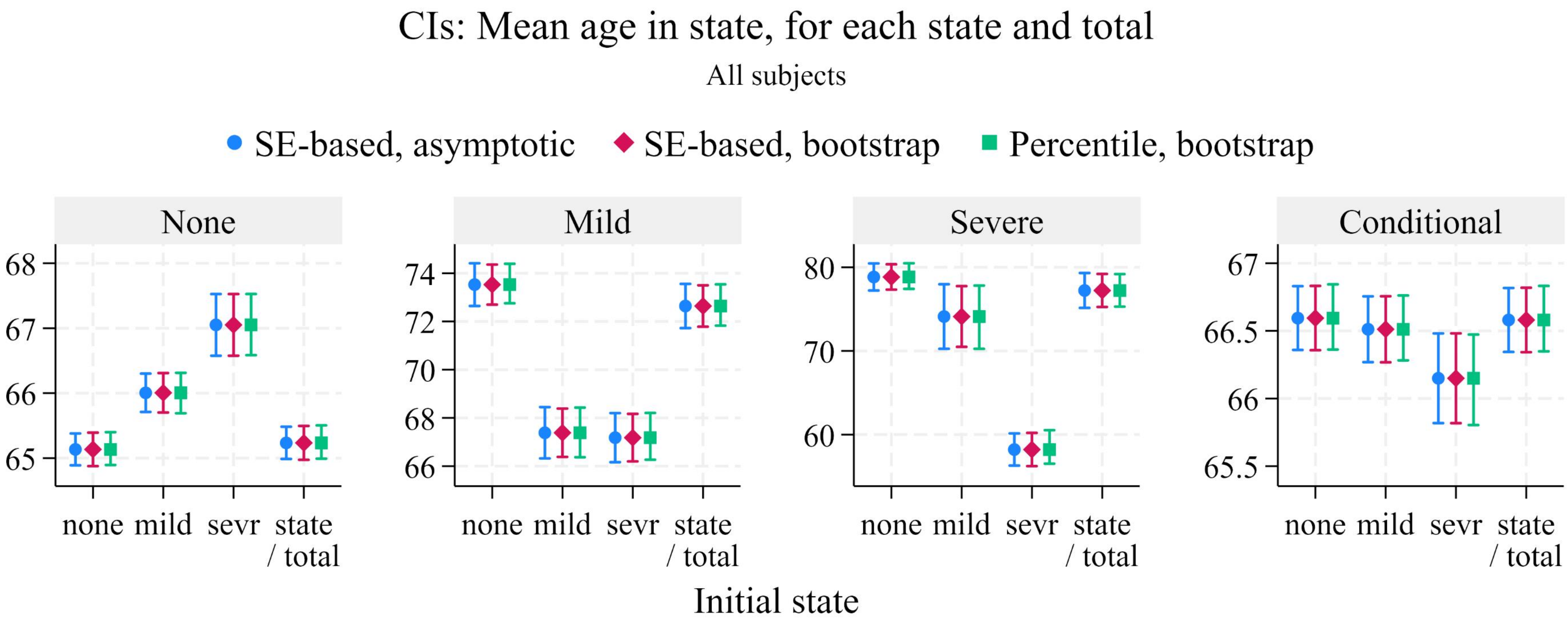
Rewards (EPIS): Comparison Analytic v. Bootstrap CIs

```
. dtms result epis (impex mlog allmeans epis) , initpr(pfixed) post
```



Rewards (MAIS): Comparison Analytic v. Bootstrap CIs

```
. dtms result mais (impex mlog allmeans mais) , initpr(pfixed) post
```



Analytical Contributions: Group Comparisons

Life expectancy covariance matrix, single group

$$\boldsymbol{V}^{tr} = \boldsymbol{G}^{tr}\boldsymbol{V}^{ml}\boldsymbol{G}^{tr\prime}$$
$$\boldsymbol{V}^{F_1} = \boldsymbol{G}^{F_1}\boldsymbol{V}^{tr}\boldsymbol{G}^{F_1\prime}$$
$$\boldsymbol{V}^{E} = (\boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1})\boldsymbol{V}^{F_1}(\boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1})'$$

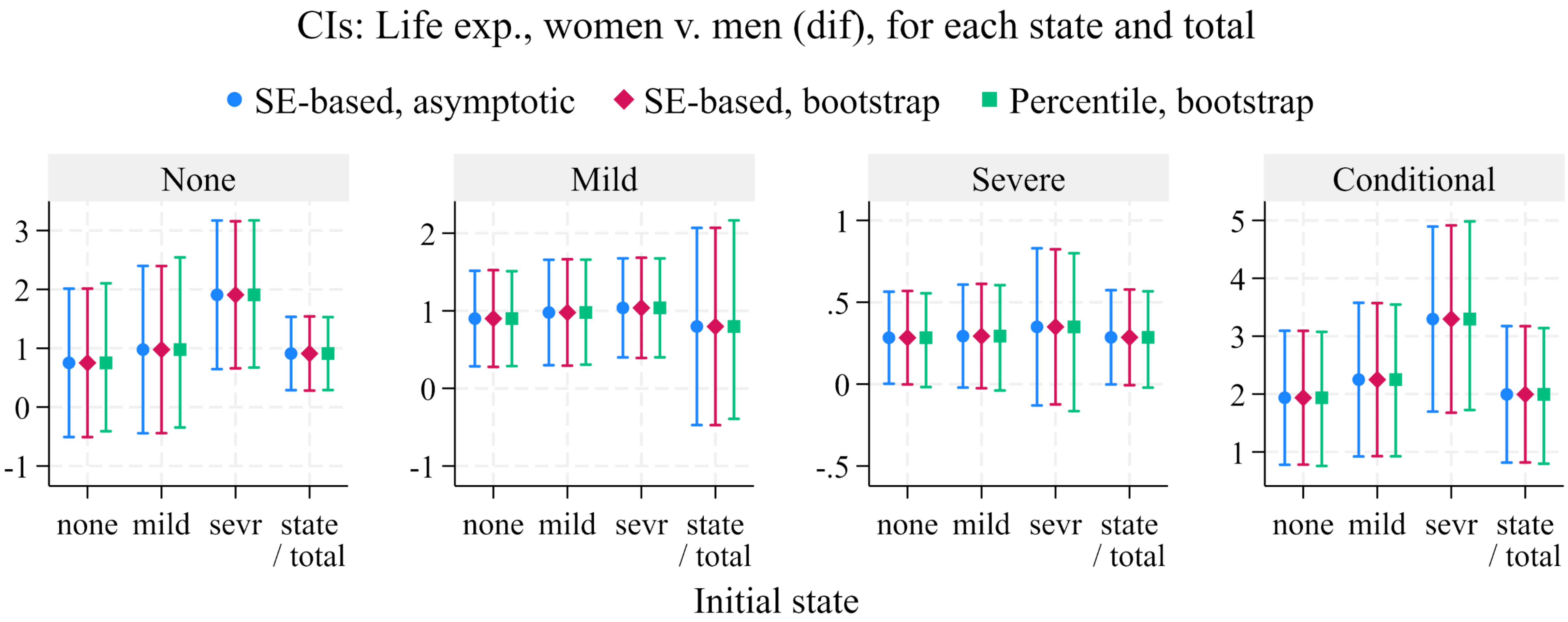
Life expectancy covariance matrix, two groups

$$\boldsymbol{G}^{tr} = \begin{bmatrix} \boldsymbol{G}_1^{tr} \\ \boldsymbol{G}_2^{tr} \end{bmatrix}$$
$$\boldsymbol{G}^{F_1} = \begin{bmatrix} \boldsymbol{G}_1^{F_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_2^{F_1} \end{bmatrix}$$
$$\boldsymbol{V}^{E} = \begin{bmatrix} \boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1,1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1,2} \end{bmatrix} \boldsymbol{V}^{F_1} \begin{bmatrix} \boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1,1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{\bar{s}^2} \otimes \boldsymbol{r}_{+1,2} \end{bmatrix}'$$

The above generalizes to N groups in the obvious way.
The methods doc extends this to any type of result, and any mixture of results.

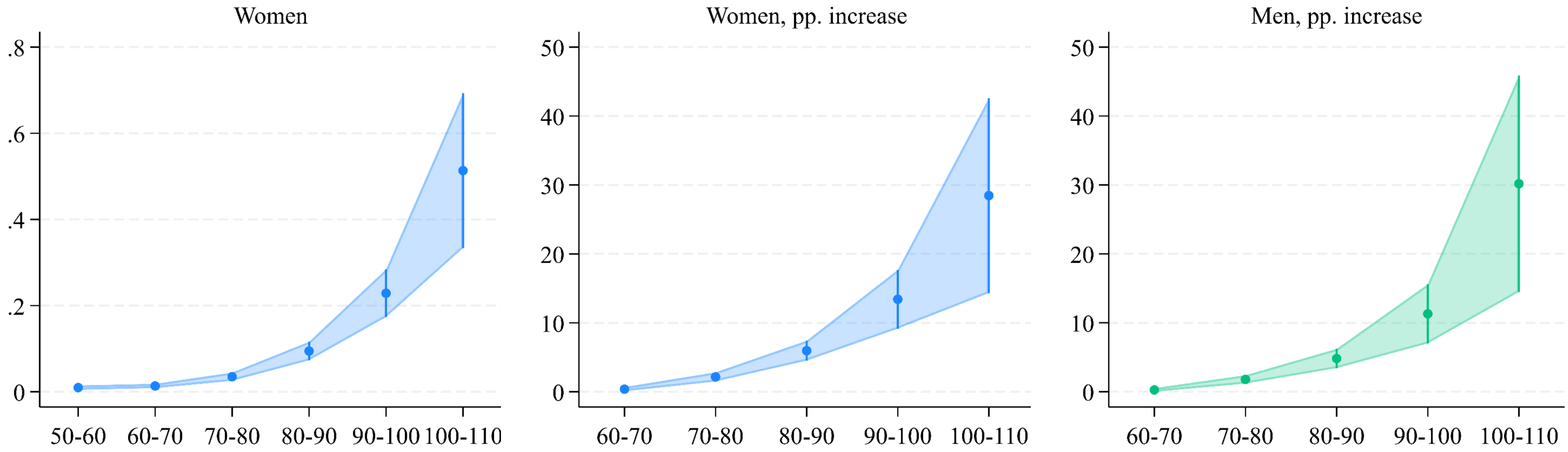
Analytical Contributions: Group Comparisons

```
. dtms result lexp (impex mlog wmn lexp) , initpr(pfixed)
. dtms result lexp (impex mlog men lexp) , initpr(pfixed)
. dtms combine (impex mlog wmn lexp) , with(men lexp) difference
```



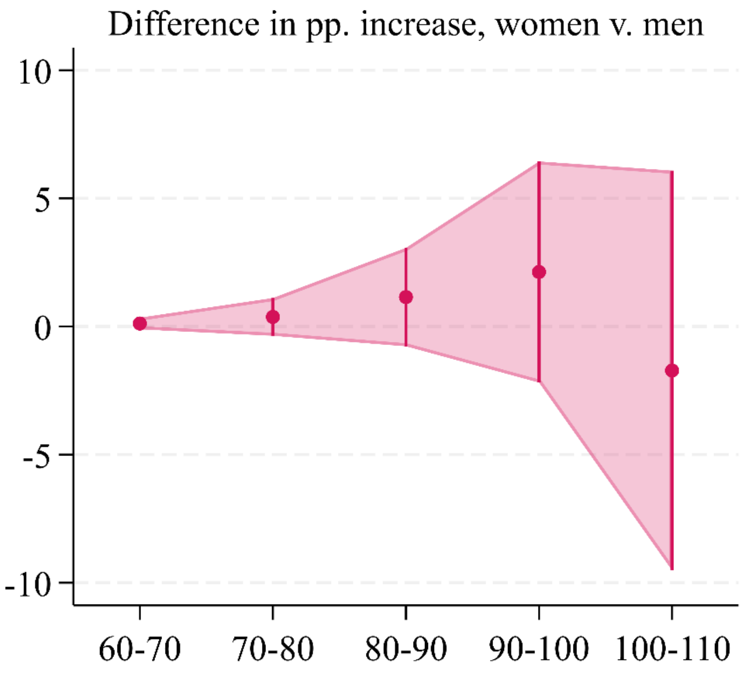
Analytical Contributions: Partial Age Ranges

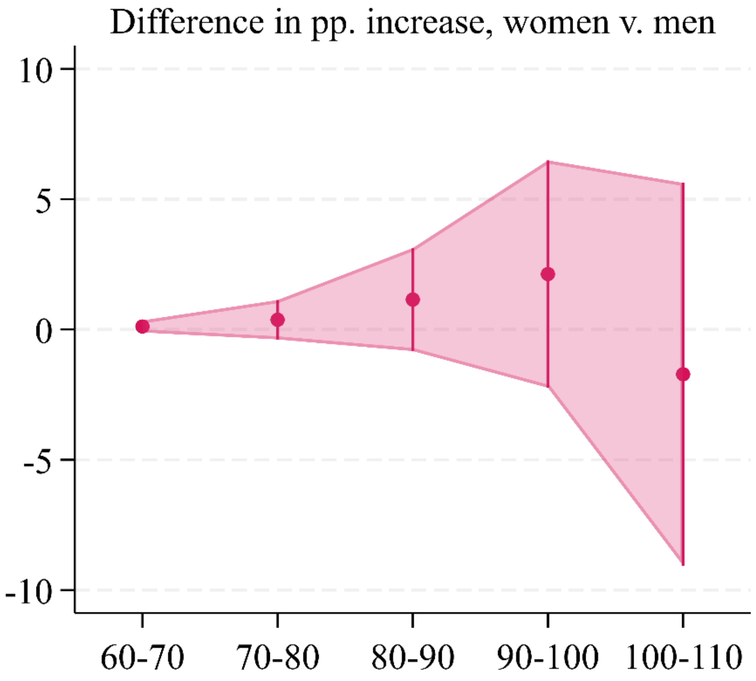
Illustrative example: Fraction of lifetime lived in severe impairment, by decade

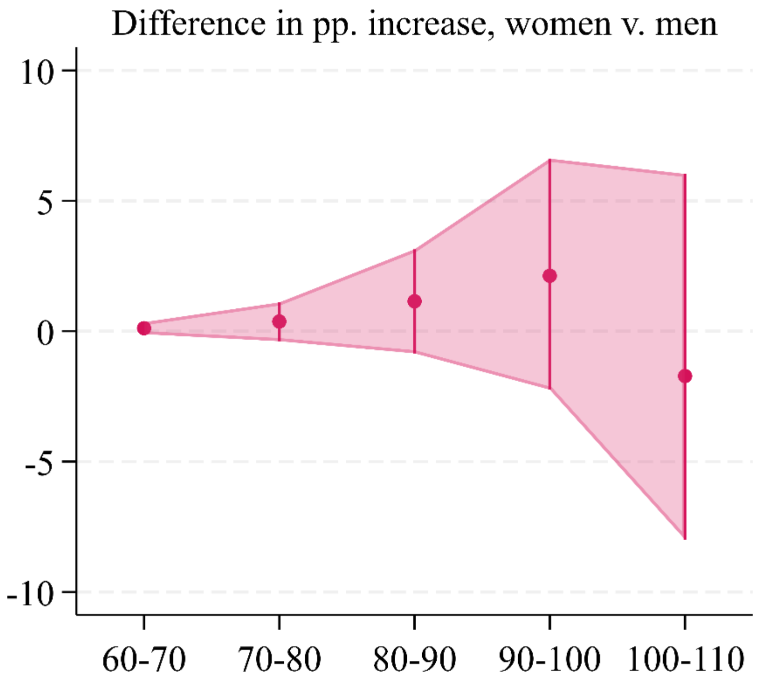


Analytical Contributions: Partial Age Ranges

Fraction of lifetime lived in severe impairment, by decade
SE-based asymptotic v. SE-based bootstrap v. percentile bootstrap







Computational Aspects

- Computational speed is one of the key contributions
  - What once took a day on 30 cores, now takes 5 minutes on a single core
  - Replacement of bootstrap by asymptotics
  - Calculation of covariance matrix of transition probabilities: code tailored to the purpose much faster than `margins`
- Mata object hierarchy underneath the dtms tree
- Self-threading code (Gould 2018)
- Memory consumption
  - Constant concern: Blows up with number of states and age classes
  - Intermediate results are not stored when a dtms tree is saved

Remaining Shortcomings / Future Improvements

- Asymptotic standard errors ignore the stochasticity of initial proportions
- Saved dtms trees incompatible across command versions
- Restrictions on state transitions not yet possible
  - disease progression models without recovery
  - relaxation of Markov assumption
- Functional form of age relatively restricted
- Import and processing of externally calculated covariance matrices for externally supplied transition probabilities not yet implemented
- Transition probabilities only calculated as conditional margins (this will not change)

Thank you

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References

Gould, William W. (2018). The Mata Book: A Book for Serious Programmers and Those Who Want to Be. Stata Press.

Lièvre, Agnès, Nicolas Brouard, and Christopher Heathcote. 2003. "The Estimation of Health Expectancies from Cross-Longitudinal Surveys." *Mathematical Population Studies* 10 (4): 211–48. Available at: https://doi.org/10.1080/713644739.

Lynch, Scott M. and J. Scott Brown (2005). "A New Approach to Estimating Life Tables with Covariates and Constructing Interval Estimates of Life Table Quantities." *Sociological Methodology*, 35(1): 189–237. Available at: https://doi.org/10.1111/j.0081-1750.2006.00168.x.

Schneider, Daniel C. (2023). "Inference for Discrete-Time Multistate Models: Asymptotic Covariance Matrices, Partial Age Ranges, and Group Comparisons." [working title] *MPDIR Working Paper*, forthcoming.

Schneider, Daniel C. and Mikko Myrskylä (2023). Extending Discrete-Time Multistate Models Using Markov Chains with Rewards: New Outcome Measures and Inference Results. [working title] *MPDIR Working Paper*, forthcoming.

Code from Slides

```
// introductory example
. dtms exampledata 12
. describe
. table cog3
. xttrans cog3

. dtms clear
. dtms setup (impex) , states(1 none 2 mild 3 sevr 4 dead) abs(1) ages(50/110)
. dtms dir

. mlogit cog3 iL.cog3 c.age c.age#c.age sex##educ c.numdrinks , cluster(id) rrr
. dtms estimate frome (impex mlog) , agevar(age, center(50)) orgnvar(L.cog3)
. dtms dir

. dtms transprob atmeans (impex mlog allmeans)
. dtms dir

. dtms proportion fixed (impex pfixed) , values(0.88 0.10 0.02)
. dtms result lexp (impex mlog allmeans lexp) , initprop(pfixed) format(%7.1f)
. dtms dir
```

Code from Slides

```
// introductory example (continued)
. dtms transprob atmeans (impex mlog edlow)  , at(educ=1)
. dtms transprob atmeans (impex mlog edhigh) , at(educ=3)
. dtms result lexp (impex mlog edlow  lexp) , initprop(pfixed) format(%7.1f)
. dtms result lexp (impex mlog edhigh lexp) , initprop(pfixed) format(%7.1f)

. dtms result stab (impex mlog edlow  stab) , initprop(pfixed) format(%7.2f)
. dtms result stab (impex mlog edhigh stab) , initprop(pfixed) format(%7.2f) post nopvalues

. dtms dir

* dtms file save using temptree.dtms , replace

. dtms combine (impex mlog edhigh lexp) , with(edlow lexp) post(state total)
. dtms combine (impex mlog edhigh lexp) , with(edlow lexp) post(state total) diff
```

Code from Slides

```
// Comparison Analytic v. Bootstrap CIs (boostrap code not included)
//      LEXP (life expectancy)
. dtms erestore (impex mlog allmeans lexp) , replay

//      MAFN (mean age at first entry)
. dtms result mafn (impex mlog allmeans mafn) , initst(1) intermed(2) target(3) post(lrsk mafn)

. dtms trans atmeans (impex mlog men) , at(sex=1)
. dtms trans atmeans (impex mlog wmn) , at(sex=2)
. qui dtms result mafn (impex mlog men mafn) , initst(1) intermed(2) target(3)
. qui dtms result mafn (impex mlog wmn mafn) , initst(1) intermed(2) target(3)
. qui dtms combine (impex mlog allmeans mafn) , with(men mafn) post(lrsk mafn)
. dtms combine e() , with(wmn mafn)
```

Code from Slides

```
// Comparison Analytic v. Bootstrap CIs (continued)
//      Rewards: EPIS (# of episodes)
. dtms result epis (impex mlog allmeans epis) , initpr(pfixed) post

//      Rewards: MAIS (mean age in state)
. dtms result mais (impex mlog allmeans mais) , initpr(pfixed) post

// Group Comparisons
dtms result lexp (impex mlog wmn lexp) , initpr(pfixed)
dtms result lexp (impex mlog men lexp) , initpr(pfixed)
dtms combine (impex mlog wmn lexp) , with(men lexp) difference

// Partial Age Ranges [code not shown]
```