Uniform Nonparametric Inference for Time Series using Stata

Jia Li, Zhipeng Liao, and Mengsi Gao

Singapore Management University

OLS and two popular Stata commands

Applied researchers are often interested in a linear regression

$$Y_t = a + bX_t + \epsilon_t.$$

Stata offers two convenient commands

Cross-sectional regression

```
reg y x, robust
```

Time-series regression

```
newey y x, lag(5)
```

Nonparametric regressions for time series data

We are interested in a nonparametric regression

$$Y_t = g(X_t) + \epsilon_t,$$

where $g(\cdot)$ is the conditional expectation function

$$g(x) = \mathbb{E}[Y_t | X_t = x], \quad x \in \mathcal{X}.$$

Nonparametric series regression:

$$Y_t = a + b_1 X_t + b_2 X_t^2 + b_3 X_t^3 + \cdots .$$

Our new `tssreg` command

```
tssreg y x
tssreg y x, lag(5)
tssreg y x, lag(5) plot
```

Default output: "Functional t-test"

```
. use "data.dta", clear
. tsset timevar
        time variable:  timevar, 1 to 215
                delta:  1 unit
. tssreg y x
```

Transformation:	sup-t	5% critical value	P>|t|
Rank	11.1841	2.8545	0.000

- $H_0 : g(\cdot) = 0$.
- The sup-t statistic: $\sup_{x \in \mathcal{X}} \frac{|\hat{g}(x)|}{\hat{\sigma}(x)}$
- The critical value is obtained by simulating Gaussian processes, which may vary slightly due to random draws.

Adding `plot` option

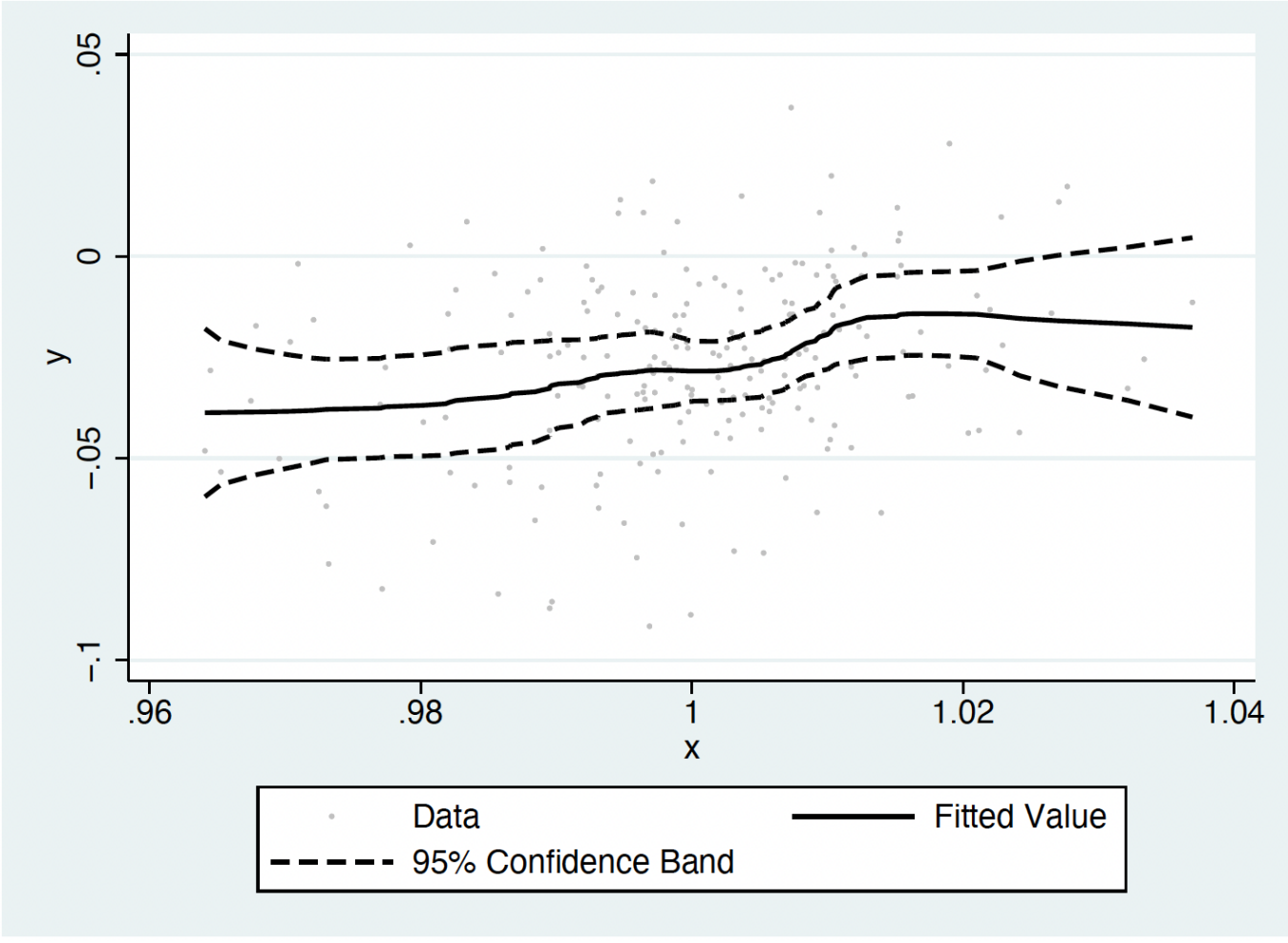


Figure 1: Nonparametric estimate and **uniform** confidence band.

Uniform confidence band

The **uniform** confidence band covers the entire function with 95% probability in large samples, i.e.,

$$\mathbb{P}(L(x) \leq g(x) \leq U(x) \text{ for all } x \in \mathcal{X}) \approx 95\%.$$

See Li and Liao (2020, Journal of Econometrics).

Typical nonparametric procedure only gives a **pointwise** confidence band: for **a given** x,

$$\mathbb{P}(L(x) \leq g(x) \leq U(x)) \approx 95\%.$$

One could not make inferential claims on the whole function with only pointwise inference.

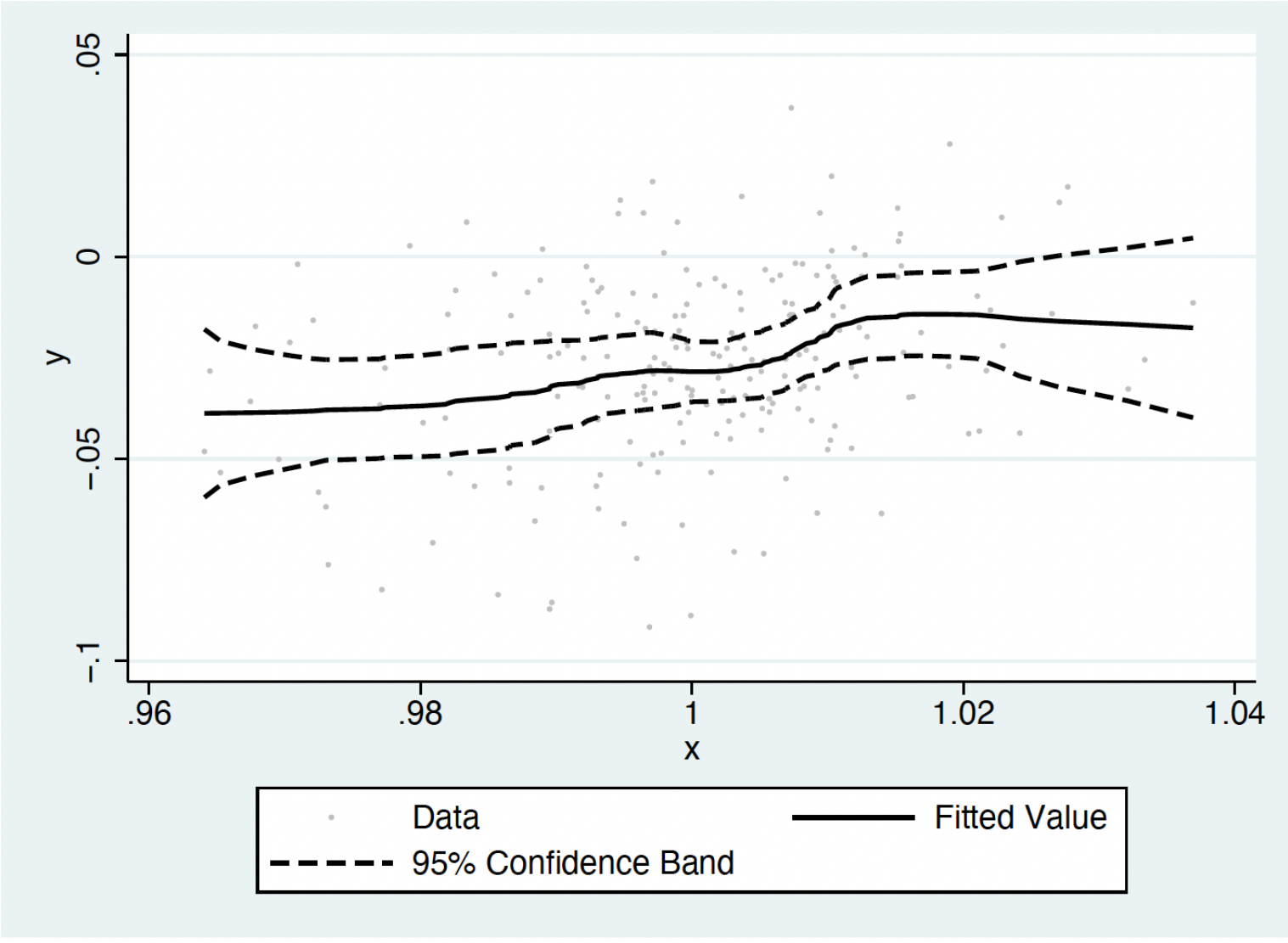
How is the nonparametric series estimation implemented?

Fact

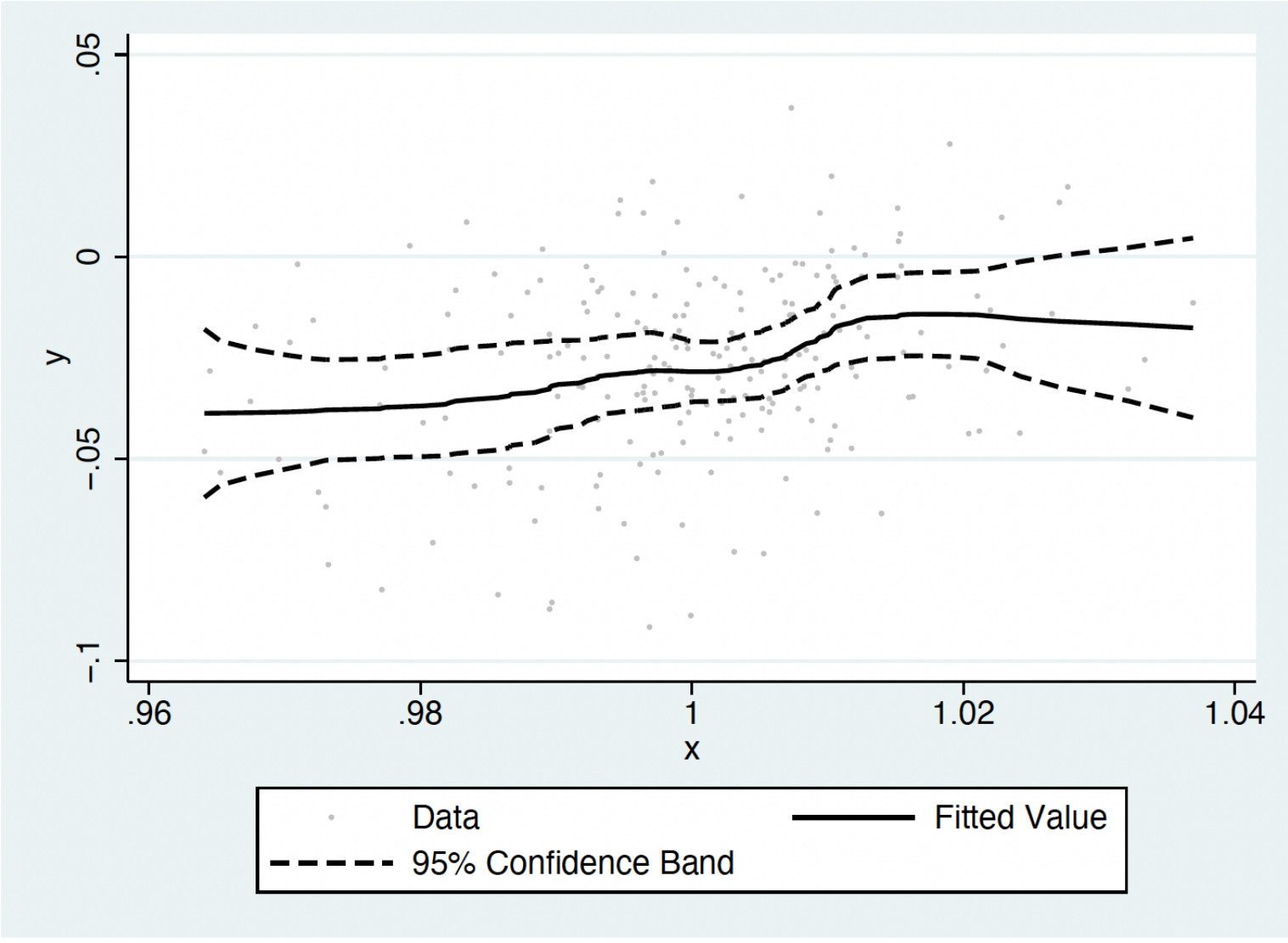
Legendre polynomials are orthogonal on $[-1, 1]$.

- Rescale the X_t variable to the $[-1, 1]$ interval via a monotone transformation $X_t \mapsto \widetilde{X}_t$. The default is the rank transformation; more `method` options are available.
- Set up a series basis: Legendre polynomials of $\tilde{X}_t$. The default number of terms is $\mathtt{m} = 6$, corresponding to a 5th-order polynomial.
- Regress Y_t on the series basis, and get the nonparametric fit.
- The sampling error is captured by a Gaussian process. Critical values are obtained via simulation.

Use case I: Preliminary data/model exploration



Use case II: Conducting nonparametric inference



Use cases III: Specification test for conditional moments

Euler/Bellman equations in dynamic equilibrium models often imply conditional moment restrictions.

Example: Search and Matching Model

$$\mathbb{E}\left[Y_t | p_t\right] = 0, \quad \text{where}$$

$$Y_t \equiv p_{t+1} - \frac{\beta\theta_{t+1}c_{t+1}}{1-\beta} + \frac{(1-s)\,c_{t+1}}{(1-\beta)\,q\left(\theta_{t+1}\right)} - \frac{c_t}{(1-\beta)\,\delta q\left(\theta_t\right)} - z.$$

The parameters (β, q, δ, z) may be estimated or calibrated. The estimation/calibration error may be ignored because it will be dominated by the larger statistical error in the nonparametric test.

Stata implementation

```
tssreg y p, plot
```

Specification test for parametric models

- One may wonder whether a (non)linear specification is adequate.

$$Y_t = a + bX_t + e_t, \quad \text{or} \quad Y_t = f(X_t, \theta) + e_t.$$

- Estimate the model and then get the residual $\hat{e}_t$.
- Nonparametrically regress the residual on X_t, and check whether the nonparametric fit is statistically zero.

Stata implementation

```
tssreg e x, lag(5) plot
```

Related econometric methods to be coded

- `tssreg` is useful for testing **conditional moment equality** $\mathbb{E}[Y_t|X_t = x] = 0$ for all x.
- Li, Liao, and Quaedvlieg (2020, Review of Economic Studies) proposes a method for testing **conditional moment inequalities** $\mathbb{E}[Y_t|X_t = x] \geq 0$ for all x.
- Horvath, Li, Liao, and Patton (2021, Quantitative Economics) consider testing for **conditional quantile equalities** using a bootstrap.
- Li, Liao, and Zhou (2021, WP) extend the method to accommodate **panel** data with large T and possibly strong spatial correlation.