# `drdid` and `csdid`:

## Doubly robust DID with multiple time periods

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# Special Thanks


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# Introduction

- DID is one of the most popular methods of applied researchers aiming to analyze Causal Effects.
  - The canonical DID (2x2) compares the changes in the outcome of treated units with changes observed among non-treated/control units.
  - Under the Parallel Trends Assumption (PTA), differences in those changes (DiD) identify the Average Treatment Effects of the treated units (ATT).
- Empirical research typically deviates from the Canonical design because:
  - Researchers have access to many periods. $(T > 2)$
  - Treatment may occur at different points in time. $(G > 1)$
  - Groups may be different in terms of observed characteristics. $(X's)$

# Introduction

- Simple Solution: Identify the ATT ($\theta$) by
  - Adding fixed effects for individuals or cohorts.
  - Adding time fixed effects.
  - Adding controls

$$y = \alpha_i + \delta_t + \beta X + \theta^{twfe} * (Eff\_Tr) + u$$

- Recent research (de Chaisemartin and D'Haultoeuille, 2020; Goodman-Bacon, 2021; Borusyak and Jaravel, 2017) has shown that this simple generalization (A.K.A. TWFE) may not be adequate to identify an ATT when effects are heterogeneous.

- While others (Abadie, 2005; Heckman et al 1997; Imbens and Wooldrige, 2009; Sant'Anna and Zhao, 2020) have proposed many alternatives to better account for differences in controls.

# Introduction

With this framework, today I'll focus on the following topics:

- The identification of the ATT using canonical DID designs, the role of covariates (based on Sant'Anna and Zhao (2020) - SZ ), and its implementation via `drdid`.
- The problems related with the simple TWFE generalization of the DID estimators. (based on Goodman-Bacon (2021))
- The estimation of the DID effects with Multiple Periods (based on Callaway and Sant'Anna (2021)-CS), and its implementation via `csdid`.

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# Canonical 2x2 DID

Setup

- Assume that we have access to a panel data where all units are followed over two periods: $t = 0, 1$.
- All units fall within two groups: Treated or untreated/control, status which cannot be changed ($D_i$).

Under full heterogeneity, potential and observed outcomes for a unit $i$ at time $t$ can be written as:

$$y_{i,t}(W) = \mu_i + \gamma_i t + \theta_i W \times t$$

$$y_{i,t} = D_i y_{i,t}(1) + (1 - D_i) y_{i,t}(0)$$

$$y_{i,0} = y_{i,0}(1) = y_{i,0}(0)$$

# Canonical 2x2 DID

In this framework, the treatment effect for unit $i$ at $t = 1$ is the difference between both Potential Outcomes:

$$\theta_i = y_{i,1}(1) - y_{i,1}(0)$$

And the ATT is:

$$\begin{aligned} ATT = E(\theta_i | D_i = 1) &= E_1(\theta_i) \\ &= E_1(y_{i,1}) - E_1(y_{i,1}(0)) \\ &= E_1(y_{i,1}) - (E_1(y_{i,0}) + E_1(\gamma_i)) \end{aligned}$$

If we can estimate $E_1(\gamma_i)$, we can estimate the ATT.

# Canonical 2x2 DID

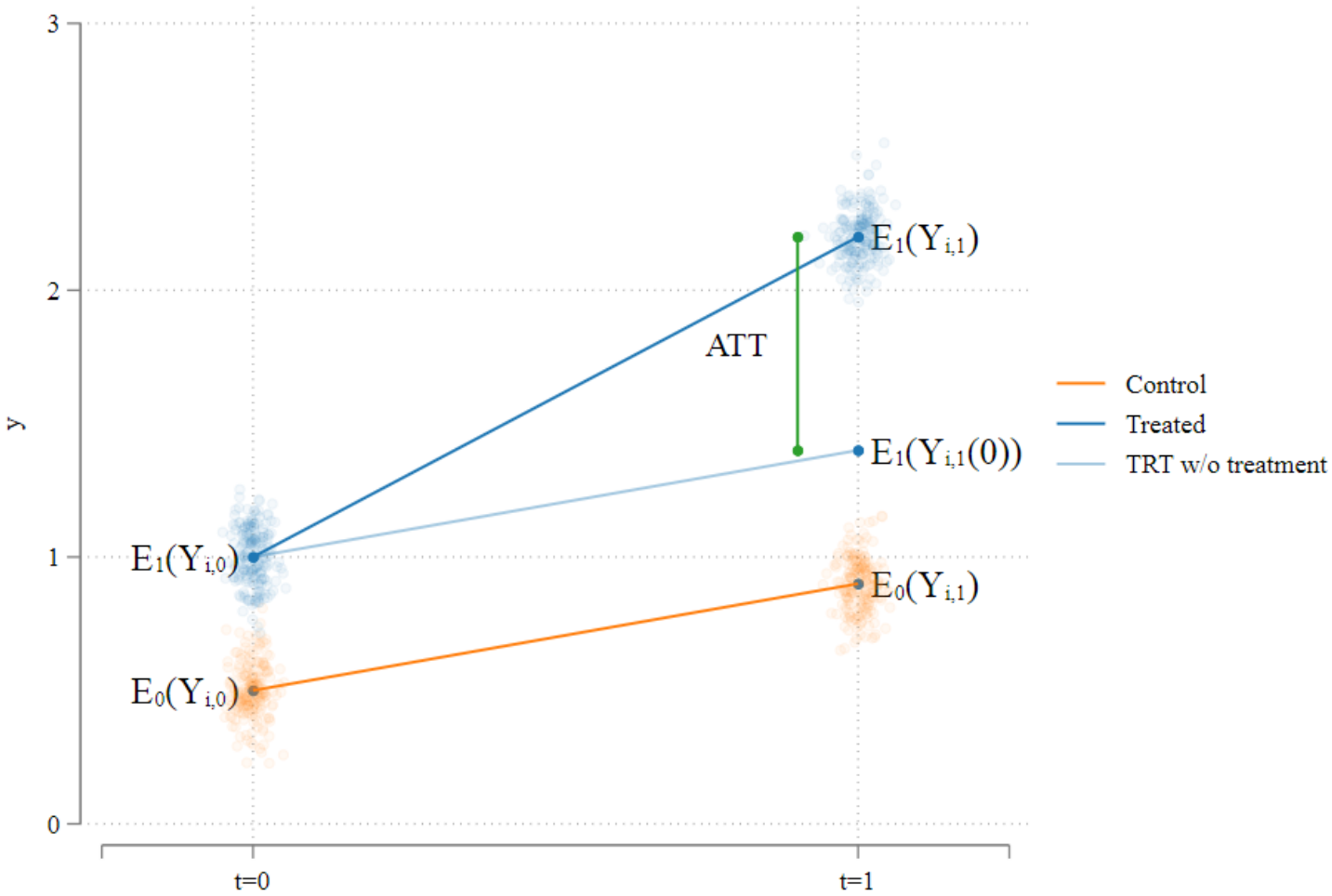
Thus, we rely on the Parallel trends Assumption (PTA)

$$E_1(\gamma_i) = E_0(\gamma_i)$$
$$E_1(y_{i,1}(0) - y_{i,0}) = E_0(y_{i,1} - y_{i,0})$$

This assumes that the growth experienced by the control group should be the same as the growth of the treated group in absence of the treatment.

$$ATT = [E_1(y_{i,1} - y_{i,0})] - [E_0(y_{i,1} - y_{i,0})]$$

# Canonical 2x2 DID



# 2x2 DID with Covariates

Simple 2x2 DID identifies the ATT under Unconditional PTA.
This may be appropriate if:

- The treated and control groups have similar characteristics; or if
- The change in outcome $\gamma_i$ does not depend on characteristics.

SZ assumes that the PTA holds only when considering groups with the same characteristics (Conditional PTA):

$$E_1(\gamma_i|X) = E_0(\gamma_i|X)$$
$$E_1(y_{i,1}(0) - y_{i,0}|X) = E_0(y_{i,1} - y_{i,0}|X) = \gamma(x)$$

Thus, the ATT estimation should adjust for differences in characteristics.

$$ATT = [E_1(y_{i,1} - y_{i,0})] - [E_1((E_0(y_{i,1} - y_{i,0}|X))]$$

# 2x2 DID with Covariates

What to kind of controls can be used?

**Panel**

- They should not be unique to treatment or control group
- They should capture pre-treatment characteristics
  (only pre-treatment values are used)
- Time varying variables are allowed only if the changes are strictly exogenous. (bad control risk)

**Repeated Cross-section**

- Time varying controls are possible but SZ assumes stationary in the covariates (Abadie, 2005).
- They should not change much across time
- The changes should be strictly exogenous.
- Post-treatment units should represent their pre-treatment counterparts.

# 2x2 DID with Covariates: How?

How are ATT's Estimated?

SZ discusses the properties of three types of estimators, which accommodate to panel and repeated cross-sectional data.

- Outcome regression
- Reweighting approach (IPW and IPT)
- Doubly Robust Estimators: Combining OR and RW

For simplicity, I will focus on panel estimators.

# 2x2 DID with Covariates

$$ATT = E(\Delta y_i | D_i = 1) - \hat{E}(\gamma_i | D_i = 1)$$

**Outcome Regression**

$$S1 : \gamma_i = \Delta y_i = \gamma(x) + v_i \quad \forall i | D_i = 0$$
$$S2 : E(\gamma_i | x) = \hat{\gamma}(x)$$
$$S3 : \widehat{ATT}_{OR} = E(\Delta y_i | D_i = 1) - E(\hat{\gamma}(x) | D_i = 1)$$

Depends strongly on the correct specification for the outcome change.

# 2x2 DID with Covariates

$$ATT = E(\Delta y_i | D_i = 1) - \hat{E}(\gamma_i | D_i = 1)$$

**Re-weighted Approach**

$$S1 : P(D_i = 1|X) = F(X) \rightarrow \hat{\pi}(X)$$

$$S2 : \omega(x) = \frac{\hat{\pi}(X)}{1 - \hat{\pi}(X)}$$

$$S3 : \widehat{ATT}^{1}_{ipw} = E(\Delta y_i | D_i = 1) - \frac{E(\omega(x)\Delta y_i | D_i = 0)}{E(\omega(x)|D_i = 0)}$$

$$S3 : \widehat{ATT}^{2}_{ipw} = E(\Delta y_i | D_i = 1) - \frac{E(\omega(x)\Delta y_i | D_i = 0)}{E(D_i)/(1 - E(D_i))}$$

Depends strongly on the correct specification of the propensity score.

# 2x2 DID with Covariates

$$ATT = E(\Delta y_i | D_i = 1) - \hat{E}(\gamma_i | D_i = 1)$$

**Doubly Robust OPT 1**

$$S1 : P(D_i = 1|X) = \hat{\pi}(X) \rightarrow \omega(x) = \frac{\hat{\pi}(X)}{1 - \hat{\pi}(X)}$$

$$S2 : \gamma_i = \Delta y_i = \gamma_\omega(x) + v_i \quad \forall i | D_i = 0 \quad weighted \quad by \quad \omega(x)$$

$$S3 : \widehat{ATT}^1_{DR} = E(\Delta y_i | D_i = 1) - E(\hat{\gamma}_\omega(x) | D_i = 1)$$

Both Doubly Robust estimators are consistent if either the outcome model or the propensity score is correctly specified

# 2x2 DID with Covariates

$$ATT = E(\Delta y_i | D_i = 1) - \hat{E}(\gamma_i | D_i = 1)$$

**Doubly Robust OPT 2**

$$S1 : P(D_i = 1|X) = \hat{\pi}(X) \rightarrow \omega(x) = \frac{\hat{\pi}(X)}{1 - \hat{\pi}(X)}$$

$$S2 : \gamma_i = \Delta y_i = \gamma(x) + v_i \quad \forall i | D_i = 0$$

$$S3 : \widehat{ATT}^2_{DR} = E(\Delta y_i | D_i = 1)$$

$$- E(\hat{\gamma}(x)|D_i = 1) - \frac{E\Big(\omega(x)(\Delta y_i - \hat{\gamma}(x))|D_i = 0\Big)}{E\Big(\omega(x)|D_i = 0\Big)}$$

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# drdid: Doubly Robust DID estimator

```
# Command is available from ssc
ssc install drdid, replace
# General Syntax
drdid y x1 x2 ... [if/in] [w=.], [ivar(pid)]  time(tmt) treatment(trt) //
        [method] [SE CI options]
# ivar -> Panel, Otherwise RC. tmt and trt Should be Binary
# "Method" Options:
drimp [rc1]   : ATT_DR  OPT 1: p(trt=1|X)~IPT
dripw [rc1]   : ATT_DR  OPT 2: p(trt=1|X)~Logit
stipw         : ATT_IPW OPT 1
ipw           : ATT_IPW OPT 2
reg           : ATT_OR
all           : Estimates all options
```

# drdid: Doubly Robust DID estimator

```
# "SE/CI" options: Default Asymptotic Standard Errors
cluster(cvar)    : Clustered Standard errors
wboot[(opt)]     : Wildbootstrap:
   reps()        : Repetitions, Default 999
   wbtype()      : WBtype: mammen / rademacher
rseed()          : Seed for replication
gmm              : Estimation via gmm.
level()          : Level of Confidence. Default 95
# Other options
noisily          : Shows all intermediate steps

## Additional utilities
# Predicts IPWeights or Pscores
drdid_predict {newvar}, [weight pscore]
# Displays Intermediate Steps
drdid_display, bmatrix(name) vmatrix(name)
```

# Example

```
use https://friosavila.github.io/playingwithstata/drdid/lalonde.dta, clear

. drdid re age educ black married nodegree hisp re74 if treated==0 | sample==2,
ivar(id) time(year) tr(experimental) dripw

Doubly robust difference-in-differences                  Number of obs = 32,834
Outcome model  : least squares
Treatment model: inverse probability
------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ATET         |
experimental |
   (1 vs 0)  |  -871.3271   396.0211    -2.20   0.028    -1647.514   -95.14007
------------------------------------------------------------------------------
```

# Example

```
drdid re age educ black married nodegree hisp re74 if
treated==0 | sample==2, cluster(id) time(year) tr(experimental) drimp
# output Omitted
drdid_predict wgt, weight

tabstat age educ black married nodegree hisp re74  if
treated==0 | sample==2 [w=wgt], by(experimental)

exper |       age      educ     black   married  nodegree      hisp      re74
------+------------------------------------------------------------------------
    0 |  24.44706  10.18824        .8  .1576471  .8141176  .1129412  3672.485
    1 |  24.44706  10.18824        .8  .1576471  .8141176  .1129412  3672.485
------+------------------------------------------------------------------------
Total |  24.44706  10.18824        .8  .1576471  .8141176  .1129412  3672.485
-------------------------------------------------------------------------------
```

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# GxT and the TWFE

As mentioned before, empirical research usually differs from the canonical approach: Many periods (T) and treatment at different times (G).

The common approach is to estimate this model using a TWFE:

$$y_{it} = \alpha_i + \gamma_t + \theta^{TWFE} * p_{it} + u_{it}$$

$$p_{it} = 1 \quad if \ already \ treated$$

This would capture the ATT only if the effect is homogeneous across units.

# Limitations of TWFE

There are two ways of understanding the limitations of the simple TWFE estimator.

**Negative Weights** Assume a balanced panel and no controls:

$$\hat{\theta}^{TWFE} = \frac{\sum \tilde{P}_{it} y_{it}}{\sum \tilde{P}_{it}^2} \quad ; \tilde{P}_{it} = P_{it} + \bar{P} - (\bar{P}_i + \bar{P}_t)$$

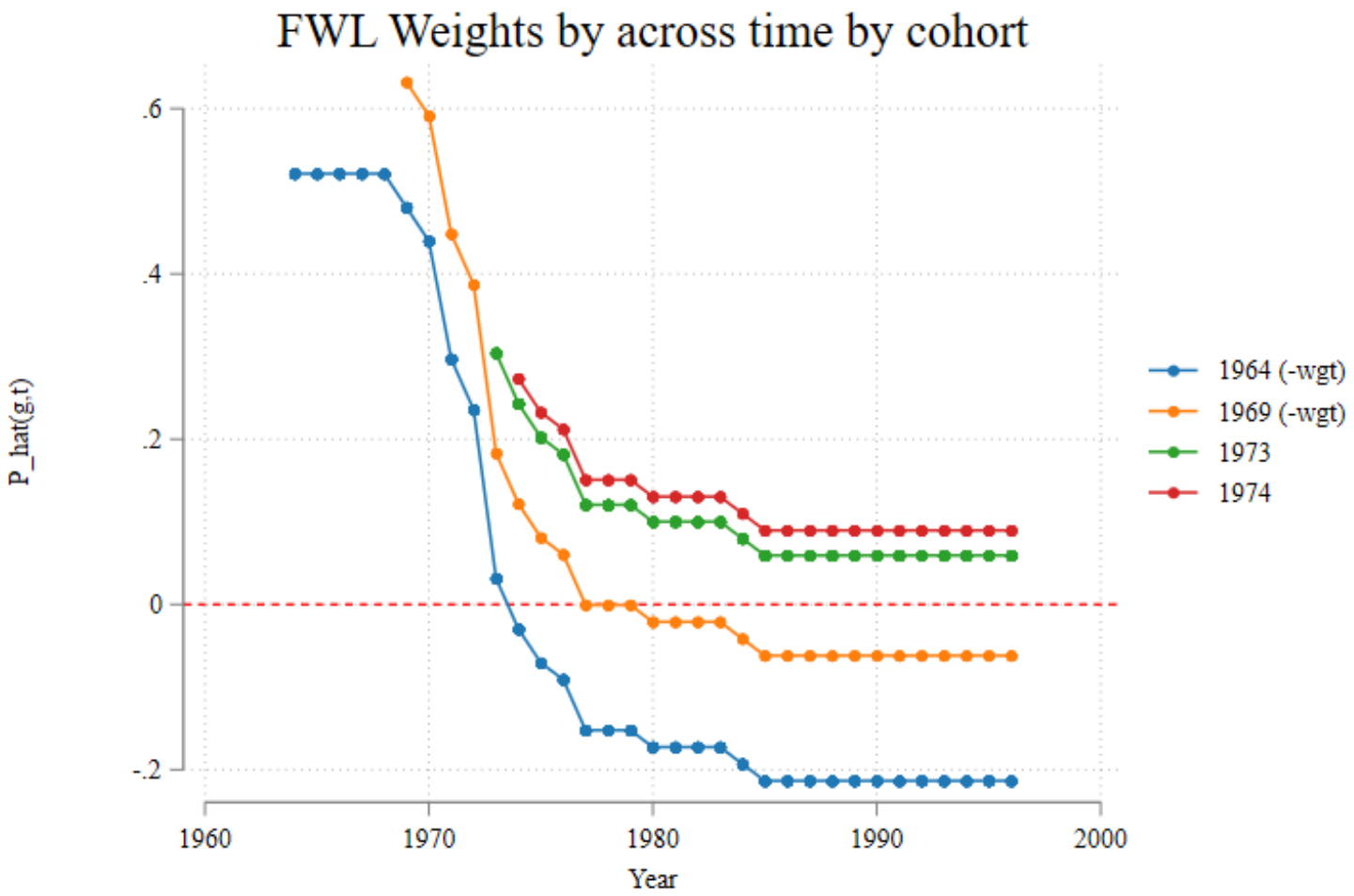
Treated units $P_{it} = 1$ should always receive a positive weight.
however, $\tilde{P}_{it}$ could be negative because:

- $\bar{P}_t$ is larger at later periods
- $\bar{P}_i$ is larger for units treated earlier

Thus, $\tilde{P}_{it}$ could be negative for units treated earlier, but seen at later periods.

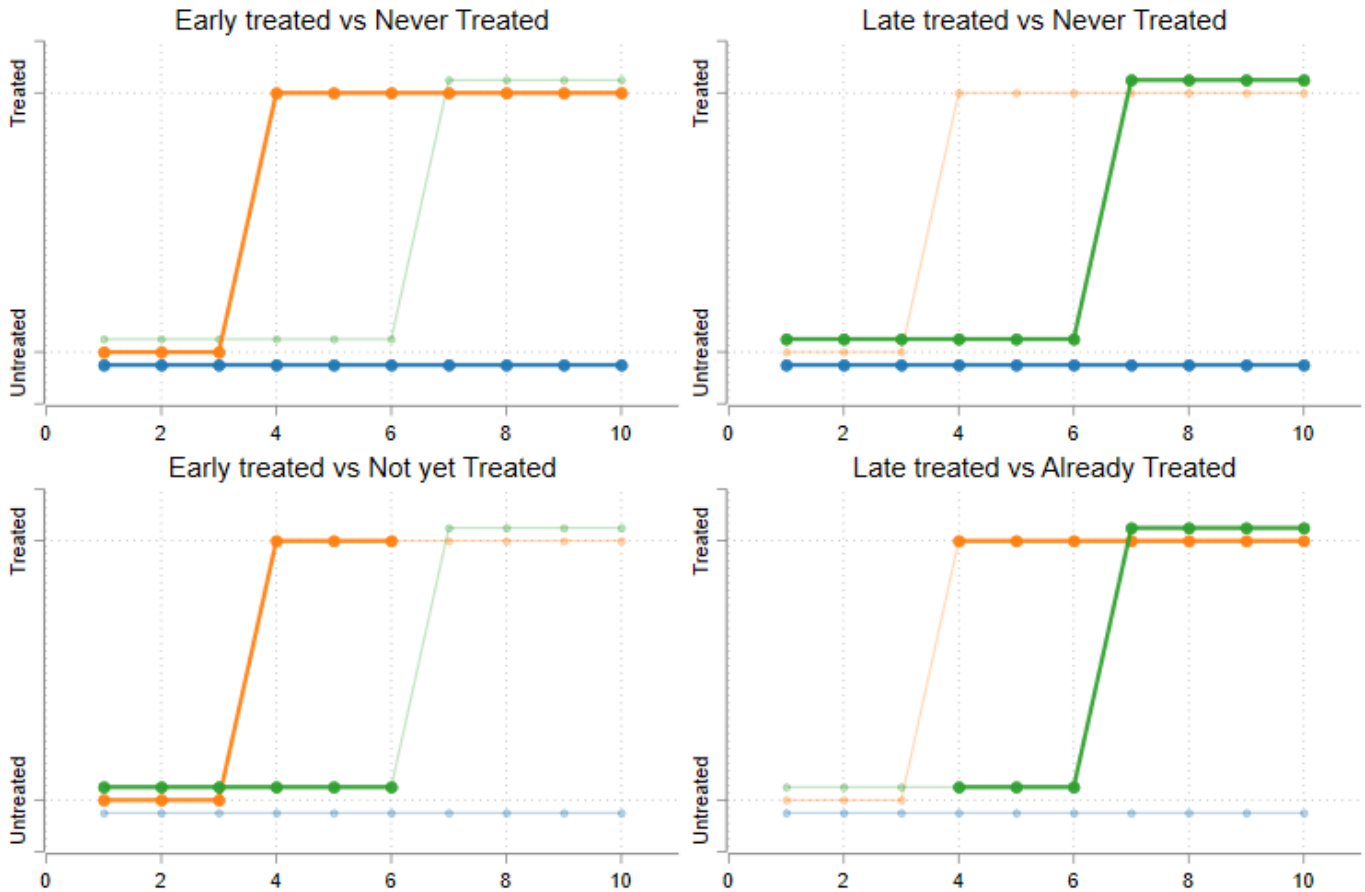
# Negative Weights

`Data:Bacon_example.dta`



# Good vs Bad Control groups

Goodman-Bacon (2021): TWFE estimator can be described as a weighted average of all 2x2 DID designs given the variation in the treatment status. But not all are good designs:



# Solution: a more flexible estimator

The "problem" with TWFE is how LR computes coefficients.

The alternative is to use a more flexible specification, or/and avoiding "bad" control groups.

There are many options, but two are the easiest to understand:
Opt1: Sun and Abraham (2021) and Wooldridge (2021)

$$y_{it} = \alpha_i + \gamma_t + \sum_{g=g_0}^{G} \sum_{s=g}^{T} \theta(g,s) * 1(G=g, t=s) + e$$

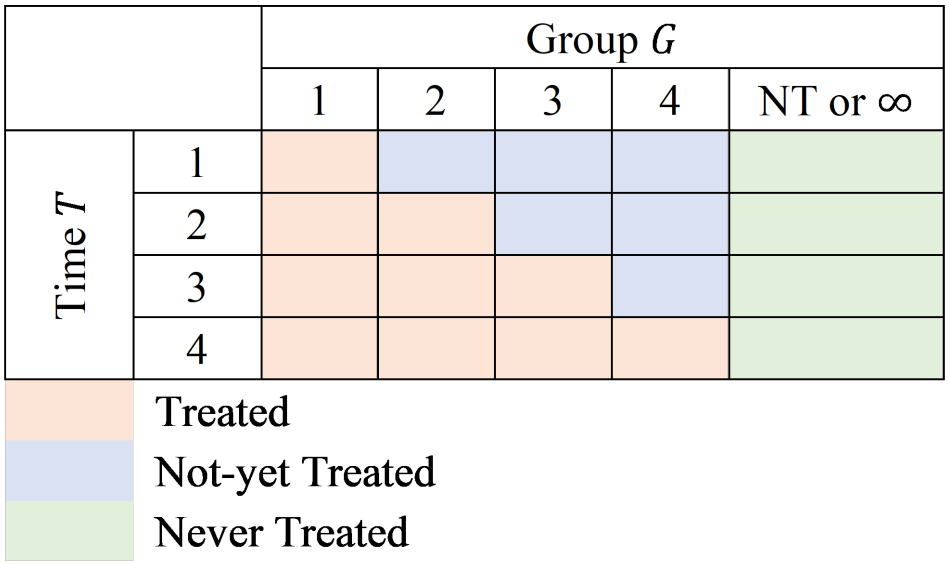
Opt2: Callaway and Sant'Anna (2021)

Breakdown a single GxT DID to multiple 2x2 DID.

One can aggregate ATT's as needed

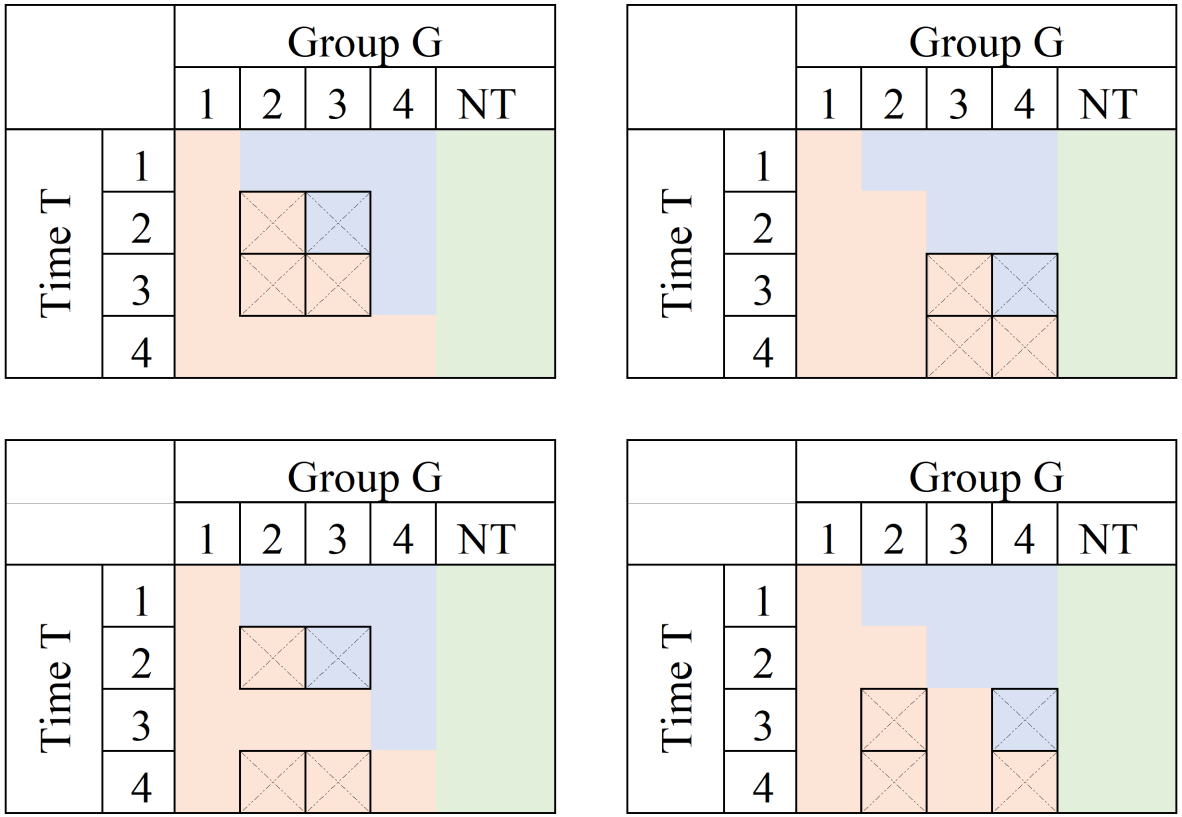
# DID with Multiple Time Periods

- CS breaks down the problem from one GxT DID to many 2x2 DID, using only good designs.
- Each design estimates a particular ATT(G,T).
- `drdid` can be applied on each design independently.



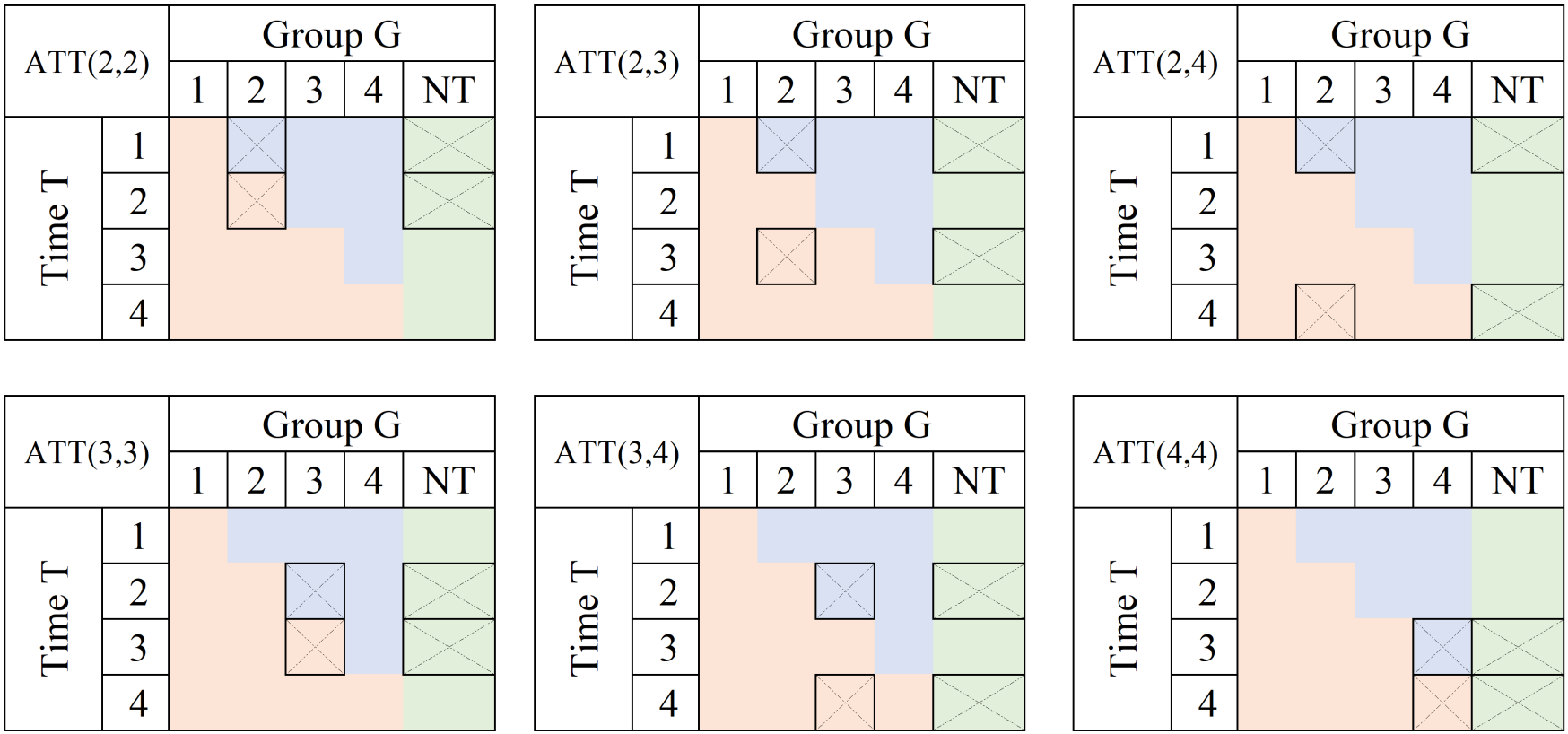
# DID with Multiple Time Periods

Figure: 2x2 to avoid



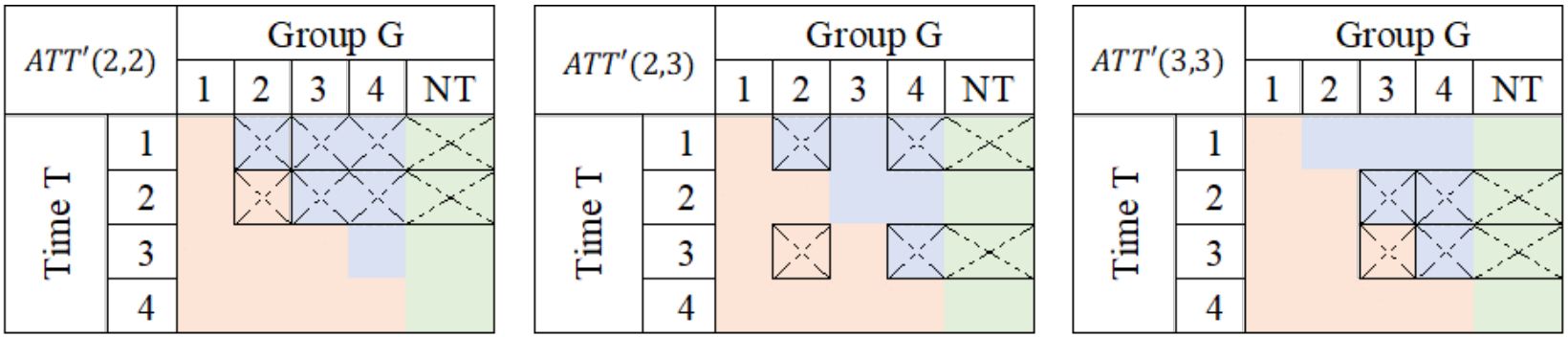
# DID with Multiple Time Periods

Figure: 2x2 Using Never Treated

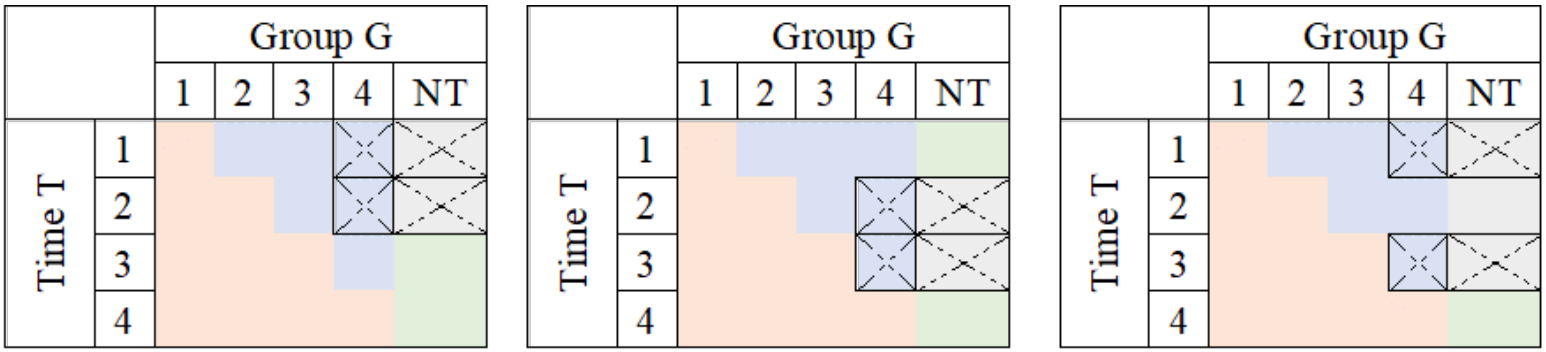


# DID with Multiple Time Periods

## 2x2 Using Not-yet Treated




## 2x2 Pre-treatment trends



# DID with Multiple Time Periods: ATT(g,t)

In CS, the main Building block is the ATT(g,t):

**ATT(g,t)**

The treatment effect for units treated at time $g$, measured at time $t$.

If $g \leq t$, it is useful to estimate treatment effects

$$ATT^{NT/NY}(g,t) = E(y_{i,t} - y_{i,g-1}|G_i = g) - E(y_{i,t} - y_{i,g-1}|G_i = NT \, or \, G_i > t)$$

If $g > t$, it is useful for pre-trend tests:

$$ATT^{S}(g,t) = E(y_{i,t} - y_{i,t-1}|G_i = g) - E(y_{i,t} - y_{i,t-1}|G_i = NT)$$

$$ATT^{L}(g,t) = E(y_{i,g-1} - y_{i,t-1}|G_i = g) - E(y_{i,g-1} - y_{i,t-1}|G_i = NT)$$

# Aggregating ATT(g,t)'s

CS identifies all feasible ATT(g,t)'s, but also suggests a series of possible aggregations.

General Structure:

$$ATT_{TYP} = \frac{\sum_{TYP} w_{g,t} ATT(g,t)}{\sum_{TYP} w_{g,t}}$$

where $w_{g,t}$ are based on the number of treated observations used in a particular ATT(g,t).

$$Simple : t \geq g$$
$$Group : t \geq g \& g = h$$
$$Calendar : t \geq g \& t = s$$
$$Event : t - g = e$$
$$CEvent : c_1 \leq t - g \leq c_2$$

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# csdid: DID with Multiple Time periods

```
# Command is available from ssc, requires drdid
ssc install csdid, replace
# General Syntax
csdid y x1 x2 ... [if/in] [w=.], [ivar(pid)]  time(T) gvar(G) //
        [method(@@@)] [long] [SE CI options]
# G=0 Never treated, and G subset of T (Not a 0-1 dummy)
# method(@@) options:
drimp [rc1]  (default) ; dripw [rc1]
stdipw ; ipw ; reg
# Pretreatment ATT
long          : Long Gaps for Pretreatment ATT(g,t)
# SE option
pointwise     : If "wboot" were requested, one can call for pointwise CI
                Default is uniform CI (valid for joint tests)
# One can save the RIF's into a file to save results.
saverif(file) [replace] : Useful for estimation of "wboot"
                          SE on other Aggregations
```

# csdid: DID with Multiple Time periods

```
# Post-Estimation:
after csdid    : estat        [agg pretrend], [option]
using rif_file: csdid_stats [agg ], [option SE options]
# Possible aggregations
attgt    , [post estore(@@) esave(@@) ]
simple   , [post estore(@@) esave(@@) ]
calendar, [post estore(@@) esave(@@) ]
group    , [post estore(@@) esave(@@) ]
event    , [window(%1 %2)] [post estore(@@) esave(@@) ]
cevent   ,  window(%1 %2)  [post estore(@@) esave(@@) ]

# Visualization (after csdid, estat or csdid_stat),
csdid_plot, [options]: Plots previously estimated ATTGT's
```

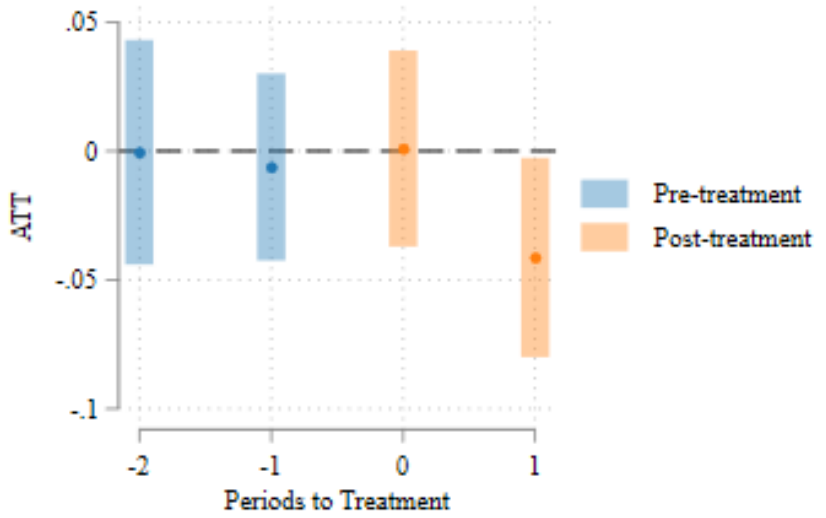
# csdid: Example

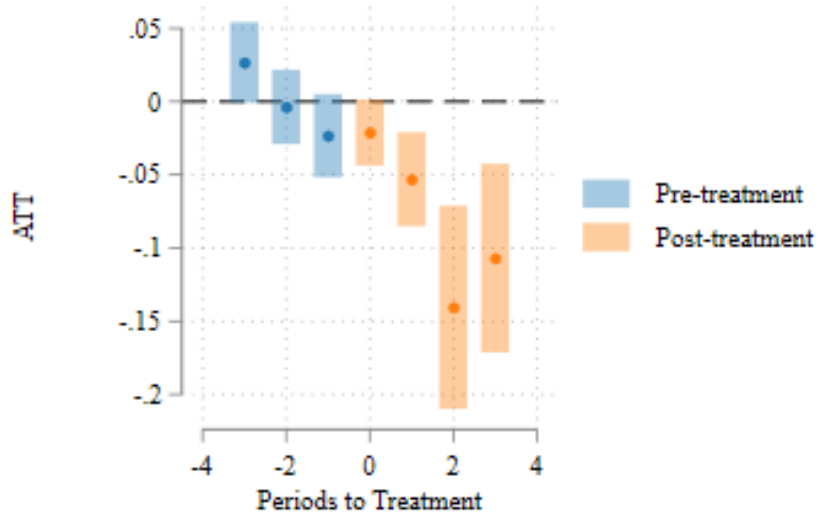
```
use https://friosavila.github.io/playingwithstata/drdid/mpdta.dta, clear
tab year first_treat
           |                first.treat
      year |         0      2004      2006      2007 |     Total
-----------+--------------------------------------------+----------
      2003 |       309        20        40       131 |       500
      2004 |       309        20        40       131 |       500
      2005 |       309        20        40       131 |       500
      2006 |       309        20        40       131 |       500
      2007 |       309        20        40       131 |       500
-----------+--------------------------------------------+----------
     Total |     1,545       100       200       655 |     2,500

# DID estimation, using Panel estimators, and DRIPW method.
# Asymptotic Standard errors
 csdid lemp lpop , ivar(countyreal) time(year) gvar(first_treat) method(dripw)
```

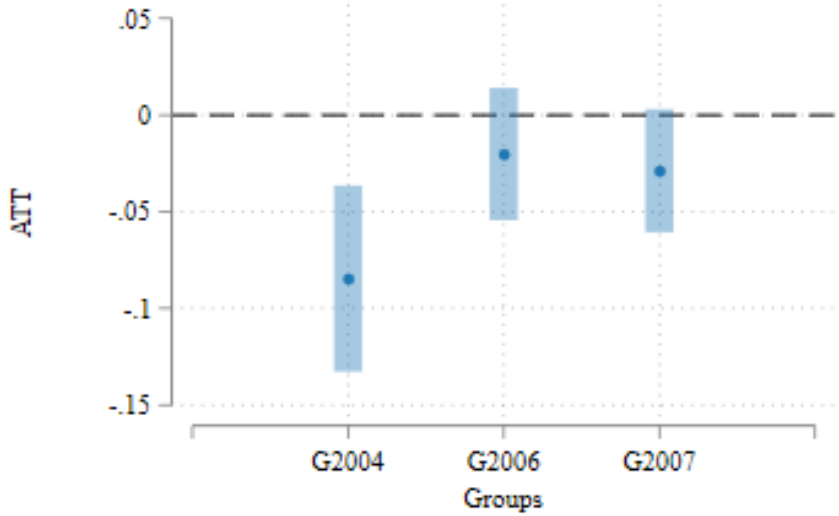
```
 csdid lemp lpop , ivar(countyreal) time(year) gvar(first_treat) method(dripw)
 saverif(rif)
File rif.dta will be used to save all RIFs
.......... # Each dot is a successful 2x2 estimation. X Otherwise
file rif.dta saved
Difference-in-difference with Multiple Time Periods # Type of Model
Outcome model  : least squares
Treatment model: inverse probability
------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
g2004        | # Indicates the Cohort
 t_2003_2004 |  -.0145297   .0221292    -0.66   0.511     -.057902    .0288427
 t_2003_2005 |  -.0764219   .0286713    -2.67   0.008    -.1326166   -.0202271
 t_2003_2006 |  -.1404483   .0353782    -3.97   0.000    -.2097882   -.0711084
 t_2003_2007 |  -.1069039   .0328865    -3.25   0.001    -.1713602   -.0424476
-------------+----------------------------------------------------------------
# t_#1_#2: Indicate which period was used as Pre (#1) and  Post (#2) Periods
# Ommited Output for g2006 & g2007

 Control: Never Treated
See Callaway and Sant'Anna (2020) for details
```

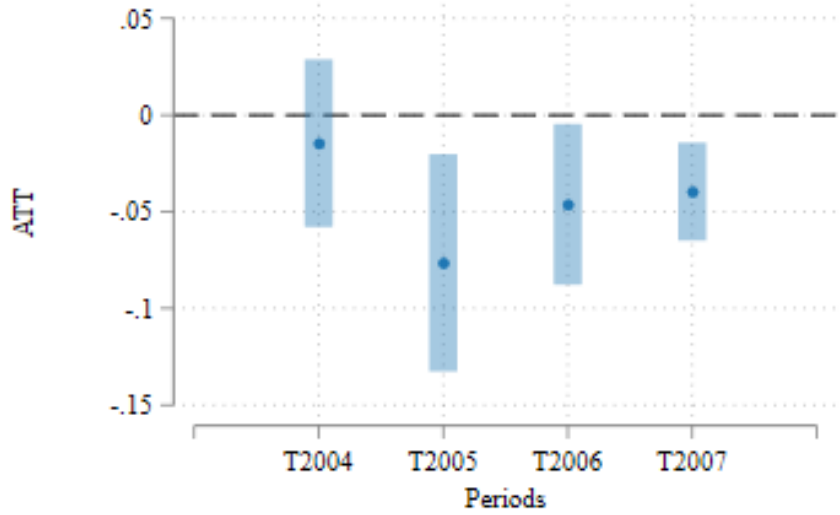
```
estat cevent, window(0 2)
ATT for events between 0 2 # Average of ATT from T+0 to T+2
Event Study:Aggregate effects
------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        ATTC |  -.0369435    .010938    -3.38   0.001    -.0583815   -.0155055
------------------------------------------------------------------------------

estat event
ATT by Periods Before and After treatment
Event Study:Dynamic effects
------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         Tm3 |   .0267278   .0140657     1.90   0.057    -.0008404     .054296
         Tm2 |  -.0036165   .0129283    -0.28   0.780    -.0289555    .0217226
         Tm1 |   -.023244   .0144851    -1.60   0.109    -.0516343    .0051463
         Tp0 |  -.0210604   .0114942    -1.83   0.067    -.0435886    .0014679
         Tp1 |  -.0530032   .0163465    -3.24   0.001    -.0850417   -.0209647
         Tp2 |  -.1404483   .0353782    -3.97   0.000    -.2097882   -.0711084
         Tp3 |  -.1069039   .0328865    -3.25   0.001    -.1713602   -.0424476
------------------------------------------------------------------------------
```

## csdid_plot


.05
0
-.05
-.1
ATT
-2
-1
0
1
Periods to Treatment
Pre-treatment
Post-treatment


.05
0
-.05
-.1
-.15
-.2
ATT
-4
-2
0
2
4
Periods to Treatment
Pre-treatment
Post-treatment


.05
0
-.05
-.1
-.15
ATT
G2004
G2006
G2007
Groups


.05
0
-.05
-.1
-.15
ATT
T2004
T2005
T2006
T2007
Periods

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# Conclusions

- The use of DID estimators of ATT's using DID has changed drastically over the last couple of years.
- Today I showed commands to implement two of these estimators:
- `drdid` implements Sant'Anna and Zhao (2020) estimator, which emphasizes the benefits of doubly robust DID estimators.
- `csdid` implements Callaway and Sant'Anna (2021), which proposes a strategy to identify and aggregate the treatment effects for GxT DID.

For other examples, discussions, and explanations check https://friosavila.github.io/playingwithstata/index.html

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# References

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# RC estimators

For next slides, I use the following notation:

$$E(y|D=d,T=t)=E_{d,t}(y)$$

where $y$ is the observed outcome for an unit.
$D=1$ if a unit is in the treated group.
$T=1$ if we are considering the Post-period.
Thus, $E_{1,0}(y)$ is the average observed outcome for the treated group, pre-treatment (T=0).
and the ATT estimator will be:

# RC Estimators: Regression Outcome

S1: Estimate 2 outcome models:

$$y = \theta_0(x) + e \; if \; D = 0 \; and \; T = 0$$

$$y = \theta_1(x) + e \; if \; D = 0 \; and \; T = 1$$

$$\hat{\gamma}(x) = \hat{\theta}_1(x) - \hat{\theta}_0(x)$$

S2: ATT:

$$ATT_{or} = E_{1,1}(y) - E_{1,0}(y) - E(\hat{\gamma}(x)|D = 1)$$

# RC Estimators: IPW1

S1: Estimate Pscore

$$P(D=1|X) = F(X) \rightarrow \hat{\pi}(x)$$

S2: Estimation of weights

$$\omega(x) = \frac{\hat{\pi}(x)}{1-\hat{\pi}(x)}$$

S3: ATT

$$ATT^1_{IPW} = E_{1,1}(y) - E_{1,0}(y) - \left( \frac{E_{0,1}(\omega(x)y)}{E_{0,1}(\omega(x))} - \frac{E_{0,0}(\omega(x)y)}{E_{0,0}(\omega(x))} \right)$$

# RC Estimators: IPW2

S1: Estimate Pscore $\hat{\pi}(x)$ and Weights $\omega(x)$
S2: Define $\pi = E(D)$ and $\tau = E(t)$
S3: Expected growth absent of treatment

$$\hat{E}(\gamma(x)|D=1) = \frac{E_{0,1}(\omega(x)y)}{\pi/(1-\pi)} \frac{E(t|D=0)}{\tau} - \frac{E_{0,0}(\omega(x)y)}{\pi/(1-\pi)} \frac{1-E(t|D=0)}{1-\tau}$$

S4: ATT

$$ATT^2_{IPW} = E_{1,1}(y)\frac{E(D\,t)}{\pi\tau} - E_{1,0}(y)\frac{E(D(1-t))}{\pi(1-\tau)} - E(\gamma(x)|D=1)$$

# RC Estimators: DR1

S1: Estimate Pscore $\pi(x)$ and weights $\omega(x)$
S2: Estimation of Outcome Regressions using WLS with $\omega(x)$ for the control group $(D = 0)$:

$$\theta^w_{0,1}(x)\, if\ t = 1\ and\ \theta^w_{0,0}(x)\, if\ t = 0$$

S3: Estimate ATT as:

$$ATT^1_{DR} = E_{1,1}(y - \hat{\theta}^w_{0,1}(x)) - E_{1,0}(y - \hat{\theta}^w_{0,0}(x)) -$$
$$\left( \frac{E_{0,1}(\omega(x)(y - \hat{\theta}^w_{0,1}(x))}{E_{0,1}(\omega(x))} - \frac{E_{0,0}(\omega(x)(y - \hat{\theta}^w_{0,0}(x))}{E_{0,0}(\omega(x))} \right)$$

# RC Estimators: DR2

S1: Estimate Pscore $\pi(x)$ and weights $\omega(x)$
S2: Estimation of Outcome Regressions using WLS with $\omega(x)$ for the control group ($D = 0$). And OR using OLS for the treated group ($D = 1$):

$$\theta^w_{0,1}(x); \theta^w_{0,0}(x)$$

$$\theta_{1,1}(x); \theta_{1,0}(x)$$

S3: Estimate ATT as

$$ATT^2_{DR} = E(\hat{\theta}_{1,1}(x) - \hat{\theta}_{1,0}(x) - (\hat{\theta}^w_{0,1}(x) - \hat{\theta}^w_{0,0}(x))|D = 1)$$
$$- \left( \frac{E_{0,1}(\omega(x)(y - \hat{\theta}^w_{0,1}(x))}{E_{0,1}(\omega(x))} - \frac{E_{0,0}(\omega(x)(y - \hat{\theta}^w_{0,0}(x))}{E_{0,0}(\omega(x))} \right)$$