Conditional Chi-squared Tests for Moment Inequality Models and Their STATA Implementation

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STATA Symposium

Introduction

- This talk will cover two tests for moment inequality models.
  - The first one is the Conditional Chi-squared (CC) Test for full-vector inference. This part will be based on:

    Gregory Cox and Xiaoxia Shi, "Simple Adaptive Size-Exact Testing for Full-vector and Subvector Inference in Moment Inequality Models," *the Review of Economic Studies*, forthcoming.
  - The second one is the subvector CC test for sub-vector inference. This part will be based on the above paper as well as work-in-progress by the same authors.
- Both tests are based on fully analytical critical values (chi-squared), and uses no user-chosen tuning parameter.
- Both are designed when the number of moment inequalities are finite.

Introduction

- STATA commands that implement the two tests are in progress.
- The MATA implementation of the tests have been completed and can replicate Matlab results.
    - that is, despite the curious absence of a build-in MATA function for quadratic programming with inequality constraints (QPIC).
    - A QPIC function is essential to our implementation – my RA wrote his own!
    - Please note his name: **Guangyao Zhou**, who is applying to your PhD programs this Fall.
- We are (he is) working on packaging it into a STATA command.

Moment Inequality Models

- A moment inequality model:

$$\mathbb{E}[m(W_i, \theta_0)] \geq 0,$$

  where m is the vector of moment functions known up to the unknown θ_0, and W_i is the vector of observables.

- Used for inference in many areas of economics as a solution to missing data, multiple equilibria, big unsolvable games, etc.

Example 1: Interval Outcome Regression

Y^* : a latent outcome; X : explanatory variables

Y_L, Y_U: observed lower and upper bounds variables, $Y_L \leq Y^* \leq Y^U$, a.s.

Model: $\mathbb{E}[Y^* - X'\beta|X] = 0$.

Without further information/restriction, this implies that

$$\mathbb{E}[Y_U - X'\beta|X] \geq 0$$
$$\mathbb{E}[X'\beta - Y_L|X] \geq 0.$$

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$$\mathbb{E}[X'\beta - Y_L|X] \geq 0.$$

A special case: interval due to selection:

$$Y_L = S * Y^* + (1 - S)y_\ell, \quad Y_U = S * Y^* + (1 - S)y_u.$$

— S: a binary selection variable; y_ℓ, y_u bounds on support of Y^*.

Ref. Manski and Tamer (2002), Blundell Gosling Ichimura, and Meghir (2007), Kreider and Pepper (2007), Kreider, Pepper, Gundersen, and Jolliffe (2012), among others

Example 2: Multiple Equilibria

Consider a two by two game, where the net profit of player j is

$$\pi_j = Y_j * [X_j'\beta + \delta_j * Y_{-j} - \varepsilon_j], \text{ where } j = 1, 2,$$

Y_j is a binary action (say entry), Y_{-j}: the action of the other player, and $(\varepsilon_1, \varepsilon_2) \sim f(\sigma; X_1, X_2)$. Let $\delta_1, \delta_2 < 0$.

This model can predict the probabilities of $(0,0)$, $(1,1)$, $(0,1)$ and $(1,0)$ as unique equilbria, and that of multiple equilibria $\{(0,1),(1,0)\}$. Then,

$$\begin{aligned}
&\mathbb{E}[(Y_1, Y_2) = (0,0)|X] = p_{00}(\beta, \delta, \sigma; X) \\
&\mathbb{E}[(Y_1, Y_2) = (1,1)|X] = p_{11}(\beta, \delta, \sigma; X) \\
&\mathbb{E}[(Y_1, Y_2) = (1,0)|X] \leq p_{10}(\beta, \delta, \sigma; X) + p_{\{10,01\}}(\beta, \delta, \sigma; X) \\
&\mathbb{E}[(Y_1, Y_2) = (0,1)|X] \leq p_{01}(\beta, \delta, \sigma; X) + p_{\{10,01\}}(\beta, \delta, \sigma; X)
\end{aligned}$$

Ref. Ciliberto and Tamer (2009), Kawai and Watanabe (2013), Magnolfi and Roncoroni (2016), Sheng (2016), He (2017), Fack, Grenet, and He (2019) among others

Example 3: Revealed Preference Approach to Complex Games

Difficuties in solving complex games can hinder empirical work. But often, it is much easier to write down revealed-preference inequalities under relatively weak assumptions.

Let $\pi_j(d_j, d_{-j}, X; \theta)$ be the payoff function of player j, when her action is d_j and her fellow players' actions are d_{-j}. Let the observed equilibrium action profile be $(\mathbf{d}_j)_{j\in\mathcal{J}}$.

By revealed preference, we have

$$\mathbb{E}[\pi_j(\mathbf{d}_j, \mathbf{d}_{-j}, X; \theta) - \pi_j(d_j, \mathbf{d}_{-j}, X; \theta)|\mathcal{I}_j] \geq 0,$$

for some $d_j \neq \mathbf{d}_j$, and appropriate information set $\mathcal{I}_j$. Appropriate structural assumptions can replace $\mathcal{I}_j$ by a vector of observables.

Holmes (2011),Ho, Ho, and Mortimer (2012), Ho and Pakes (2014), Pakes, Porter, Ho, and Ishii (2015), Ho and Rosen (2017), Morales, Sheu, Zahler (2019) among others

Inference by Test Inversion

- A moment inequality model typically does not point identify the parameter. Inference for parameter is typically done by test inversion.
- Test Inversion: test $H_0 : \theta_0 = \theta$ for all $\theta \in \Theta$, and collect all those that are not rejected to form a confidence region for θ_0.
- Let $T_n(\theta)$ be a test statistic and $cv(\theta, 1-\alpha)$ be a critical value. The confidence region is of the form

$$CS_n(1-\alpha) = \{\theta \in \Theta : T_n(\theta) \leq cv(\theta, 1-\alpha)\}.$$

Practical Challenges

$$CS_n(1-\alpha) = \{\theta \in \Theta : T_n(\theta) \le cv(\theta, 1-\alpha)\}.$$

- Need to compute $T_n(\theta)$ and $cv(\theta, 1-\alpha)$ for a large number of θ values, and that number increases *exponentially* with d_θ – the dimension of θ.
  - State of the art tests in the literature have simulated $cv(\theta, 1-\alpha)$, nontrivial to compute.
  - They also require tuning parameters that are used to determine which inequalities are binding.

Practical Challenges

$$CS_n(1-\alpha) = \{\theta \in \Theta : T_n(\theta) \leq cv(\theta, 1-\alpha)\}.$$

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  - State of the art tests in the literature have simulated $cv(\theta, 1-\alpha)$, nontrivial to compute.
  - They also require tuning parameters that are used to determine which inequalities are binding.
- No easy way to focus on a subvector of θ.
  - Computational: have to search in the space of the full vector, even if many of the parameters are nuisance parameters.
  - Theoretical: projection of the joint confidence set often conservative.

The Conditional Chi-Squared Test

The Conditional Chi-squared Test

- Compare the likelihood ratio statistic to a chi-squared critical value
- Degree of freedom is the number of active inequalities

Active Inequalities

- Hold with equality at the restricted estimator for the moments
- Sample counterpart of binding inequalities

Desirable Properties

- Simple
  - No tuning parameters
  - No simulation. **Computationally very fast**
- Adapts to slack inequalities (without a deliberate moment selection step)
- Exact size in the normal model with known variance, and asymptotically uniformly correct size with asymptotically normal moments
- Competitive size/power to alternatives
- **Easy subvector inference** (if model is linear in nuisance parameters)

Literature Review

- Full-vector Inference: Chernozhukov, Hong and Tamer (2007), Romano and Shaikh (2008), Andrews and Guggenberger (2009), Andrews and Soares (2010), Bugni (2010), Canay (2010), Stoye (2009), Andrews Barwick (2012), Romano, Shaikh and Wolf (2014). Review papers: Canay and Shaikh (2017) and Molinari (2020) .
- Subvector Inference: Kaido and Santos (2014), Kaido, Molinari, and Stoye (2019), Bugni, Canay, and Shi (2017), Chen, Christensen, and Tamer (2018), Gafarov (2019), Cho and Russell (2020), Andrews, Roth, and Pakes (2020).

Outline

1. Conditional Chi-Squared Test
2. Subvector Inference
3. Simulations
   - Full-Vector Test
   - Subvector Test

Moment Inequality Models

$$A\mathbb{E}_F\bar{m}(\theta_0) \leq b$$

$$\bar{m}(\theta_0) = \frac{1}{n}\sum_{i=1}^{n} m(W_i, \theta_0)$$

- W_i is a vector of observables with distribution F
- $m(\cdot, \theta_0)$ is a known d_m-vector of moment functions
- θ are unknown parameters
- A is a $d_A \times d_m$ known matrix: $d_A \geq d_m$, often $d_A > d_m$.
- b is a known d_A-vector

Full-Vector Inference

- The inequalities may not point identify θ_0, and thus typically does not allow a standard point estimator.
- Confidence sets can be constructed for θ_0 by inverting the test for

$$H_0 : \theta_0 = \theta$$

or

$$H_0 : A\mathbb{E}_F \bar{m}(\theta) \leq b$$

for all $\theta \in \Theta$.

A Conditional Chi-Squared (CC) Test

- Let $\widehat{\Sigma}(\theta)$ denote an estimator of the variance of $\sqrt{n}\bar{m}(\theta)$.
- Consider the QLR statistic:

$$T(\theta) = \min_{\mu: A\mu \leq b} n(\bar{m}(\theta) - \mu)' \widehat{\Sigma}^{-1}(\theta)(\bar{m}(\theta) - \mu).$$

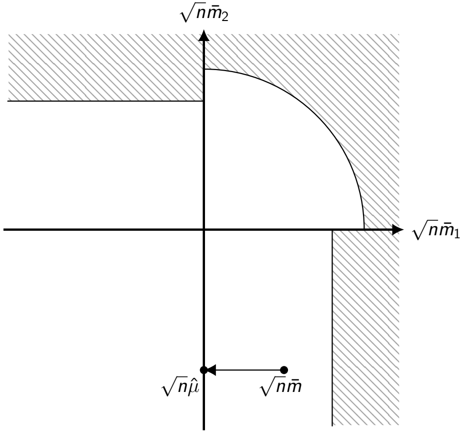
- Let $\hat{\mu}$ denote the solution to the above minimization problem.
- Let $\widehat{J}$ denote the collection of j's for which $a_j' \hat{\mu} = b_j$.
  - a_j' denotes the jth row of A and b_j denotes the jth element of b
- Let $\hat{r}$ denote the rank of $A_{\widehat{J}}$, the submatrix of A formed by rows with indices in $\widehat{J}$.
- Reject if

$$T(\theta) > \chi^2_{\hat{r}, 1-\alpha}.$$

A Simple Example

$$H_0 : \mathbb{E}\bar{m}_1 \le 0 \qquad \mathbb{E}\bar{m}_2 \le 0$$

$$\widehat{\Sigma}(\theta) = Var(\sqrt{n}\bar{m}(\theta)) = I.$$



Properties

- Cox and Shi (2022, RESTUD) show that the CC test has correct level in finite sample, if the sample moments are normal and the variance matrix is known.
- The CC test has correct uniform asymptotic level, if the sample moments are asymptotically normal and the variance matrix is estimated consistently.
- The CC test can be slightly conservative, if the model contains no equality constraint. A simple data-dependent refinement can be done to make it size correct, but in practice, it works fine without the refinement.
- The test has the adaptive property: inequalities very slack are automatically disregarded by the test.

STATA Implementation of the CC Test

- We are considering the syntax:

  CCTest **varlist** [if] [in] [, A, b, refined]

  - **varlist** contains the variables that are the elements of $m(W_i, \theta)$ (at a given θ) value.

- The MATA code computes $\bar{m}_n(\theta)$, the sample variance matrix, and implements a QPIC algorithm to compute the test statistic.

- The QPIC algorithm reports the identity of the active inequalities, which then yields $\hat{r}$. That is used to obtain the $\chi^2_{\hat{r}}$ critical value.

- The QPIC that Guangyao implements is an active-set algorithm.

Subvector Inference Problem

$$B\mathbb{E}_F\left[m(W_i,\theta_0)+C(W_i,\theta_0)\delta_0\right]\leq d(\theta_0).$$

- δ is a vector of nuisance parameters entering linearly
- B is a $d_B \times d_m$ known matrix
- $m(W_i, \theta_0)$ is a d_m-vector-valued function, known up to the unknown θ_0,
- $C(W_i, \theta_0)$ is a $d_m \times d_\delta$ matrix valued function, known up to the unknown θ_0,
- $d(\theta_0)$ is a d_B-vector, known up to the unknown θ_0

Subvector Inference Problem

- Hypothesis of interest:

$$H_0 : \exists\ \delta\ s.t.\ B\mathbb{E}_F\left[m(W_i,\theta) + C(W_i,\theta)\delta\right] \leq d(\theta)$$

- The collection of θ's at which this H_0 is not rejected forms a confidence set for θ_0.

Subvector Test

- Let $\bar{m}(\theta) = n^{-1}\sum_{i=1}^{n} m(W_i, \theta)$, and $\overline{C}(\theta) = n^{-1}\sum_{i=1}^{n} C(W_i, \theta)$.
- Let $\widehat{\Sigma}(\theta)$ be an appropriate variance-estimator (more on this later)
- Let $T_n(\theta) = \min_{\mu,\delta: B(\mu+\overline{C}(\theta)\delta)\leq d(\theta)} n(\bar{m}(\theta) - \mu)'\widehat{\Sigma}(\theta)^{-1}(\bar{m}(\theta) - \mu)$
- Let $(\hat{\mu}, \hat{\delta})$ be the solution to this minimization problem.
- Let $\hat{J}$ be the indices corresponding to the active inequalities among $B(\hat{\mu} + \overline{C}(\theta)\hat{\delta}) \leq d(\theta)$. And let $B_{\hat{J}}$ be the submatrix of B formed by the rows of B corresponding to indices in $\hat{J}$.
- Let $P_{B_{\hat{J}}\overline{C}(\theta)}$ denote the projection matrix onto the space spanned by the columns of $B_{\hat{J}}\overline{C}(\theta)$.

Subvector Test

- Let $\hat{r} = \text{rank}((I - P_{B_{\hat{J}}\overline{C}(\theta)})B_{\hat{J}})$. And let $cv(\theta, 1-\alpha) = F^{-1}_{\chi^2_{\hat{r}}}(1-\alpha)$.
- The subvector CC test is simply

$$\varphi_n(\alpha) = 1\{T_n(\theta) > cv(\theta, 1-\alpha)\}.$$

The Variance-Matrix

$$B\mathbb{E}_F[m(W_i,\theta_0) + C(W_i,\theta_0)\delta_0] \leq d(\theta_0)$$

- There are two special cases. The appropriate $\widehat{\Sigma}(\theta)$ for the special cases differ from each other and differ from the general case.
  - Special Case 1: $C(W_i,\theta)$ does not depend on W_i (and thus is known at any given θ and non-random). In this case, $\widehat{\Sigma}(\theta)$ is the sample variance matrix of $m(W_i,\theta)$.
  - Special Case 2: $C(W_i,\theta)$ is in fact $C(Z_i,\theta)$ for a subset of variables in W_i, denoted Z_i, and $B\mathbb{E}_F[m(W_i,\theta_0) + C(Z_i,\theta_0)\delta_0|Z_i] \leq d(\theta_0)$ is known to hold. In this case, $\widehat{\Sigma}(\theta)$ should estimate the *conditional* variance matrix of $\sqrt{n}\bar{m}(\theta)$ given $\{Z_i\}_{i=1}^n$.

The Variance-Matrix, the General Case

$$B\mathbb{E}_F[m(W_i,\theta_0)+C(W_i,\theta_0)\delta_0]\leq d(\theta_0)$$

- In the general case, $\widehat{\Sigma}(\theta)$ should be the sample variance matrix of

$$m(W_i,\theta)+C(W_i,\theta)\tilde{\delta},$$

  where $\tilde{\delta}$ is a preliminary estimator of a point in the identified set of δ_0.
- Let $\tilde{\mu}$ be the (unique) solution to a first-step QPIC problem:

$$\min_{\mu,\delta:B(\mu+\overline{C}(\theta)\delta)\leq d(\theta)} n\|\bar{m}-\mu\|^2$$

  And let $\tilde{\delta}$ be the (unique) solution to $\min_{\delta:B(\tilde{\mu}+\overline{C}(\theta)\delta)\leq d(\theta)}\|\delta\|^2$.

Stata Implementation of the Subvector CC Test

- We are considering the syntax (suppose that $d_m = M$):

  subCC_test (Z) (x1 x2 ... xM) y1 y2 ... yM [if] [in] [, B, d, C]

  - Z holds the conditioning variables, if there are any.
  - xj holds the variables that are the elements of the jth row of $C(W_i, \theta)$, if $C(W_i, \theta)$ depends on W_i.
  - yj is the variable that is the jth element of $m(W_i, \theta)$.
  - d is $d(\theta)$.
  - C is $C(W_i, \theta)$ when $C(W_i, \theta)$ does not depend on W_i (and thus is in fact $C(\theta)$).

Matlab Simulation I: Full Vector Test

- We follow Andrews and Barwick's (2012, AB) design, and consider p moment inequalities:

$$H_0 : \mathbb{E}W_j \leq 0 \text{ for } j = 1, \ldots, p,$$

where $(W_1, \ldots, W_p)'$ has mean $(\mu_1, \ldots, \mu_p)'$, unit variances, and correlation matrix Ω.

  - Consider $p = 2, 4, 10$ and $\Omega = \Omega_{\text{Neg}}, \Omega_{\text{zero}}, \Omega_{\text{pos}}$ from AB.

- We calculate
  - maximum null rejection probability (MNRP)
  - weighted average power (WAP)
  - size-corrected weighted average power (SCWAP)
  - time used for the calculation (Time)

Alternatives Considered

Andrews and Barwick (2012)

- Generalized moment selection to remove moments that are far from binding
  - Requires tuning parameter κ
- Bootstrap the distribution of an Adjusted QLR statistic
- Size-correct using a table for κ and size-correction constants
  - κ and the size-correction constants chosen to maximize weighted average power, computed via simulation by AB.

Romano, Shaikh and Wolf (2014)

- Construct a level β confidence set for slackness of the inequalities.
- Bootstrap the $\alpha - \beta$ quantile of the distribution of the QLR statistic (RSW1) and the MAX statistic (RSW2) at the least favorable point in the confidence set.

Table: normal distribution, known Ω, $n = 100$

	$k = 10$				$k = 4$				$k = 2$			
Test	MNRP	WAP	ScWAP	Time	MNRP	WAP	ScWAP	Time	MNRP	WAP	ScWAP	Time
					$\Omega = \Omega_{\text{Neg}}$							
RCC	.051	.61	.61	.003	.052	.62	.62	.003	.051	.62	.62	.003
CC	.051	.61	.60	.003	.049	.60	.61	.003	.046	.58	.60	.003
AB	.046	.53	.55	1.11	.051	.59	.59	1.07	.059	.65	.64	1.40
RSW1	.054	.58	.56	.551	.056	.60	.59	.538	.052	.64	.63	.701
RSW2	.050	.23	.23	.014	.052	.34	.34	.013	.052	.50	.49	.014
					$\Omega = \Omega_{\text{Zero}}$							
RCC	.052	.63	.63	.003	.052	.65	.65	.003	.051	.68	.68	.003
CC	.050	.62	.62	.003	.045	.62	.64	.003	.038	.61	.66	.004
AB	.043	.65	.66	1.08	.050	.67	.67	1.07	.056	.69	.67	1.39
RSW1	.053	.61	.60	.545	.056	.63	.62	.539	.052	.65	.65	.699
RSW2	.053	.54	.52	.014	.052	.62	.62	.014	.049	.66	.66	.014
					$\Omega = \Omega_{\text{Pos}}$							
RCC	.051	.76	.75	.003	.053	.75	.74	.003	.051	.72	.71	.003
CC	.038	.72	.76	.003	.033	.68	.74	.003	.032	.62	.69	.003
AB	.042	.78	.80	1.05	.051	.75	.75	1.03	.059	.72	.70	1.34
RSW1	.053	.77	.77	.547	.056	.73	.71	.534	.052	.67	.66	.700
RSW2	.052	.77	.77	.014	.052	.74	.74	.013	.049	.68	.69	.014

Note: CC, RCC, AB, RSW1 and RSW denote the conditional chi-squared test, the refined CC test, the adjusted quasi-likelihood ratio test with bootstrap critical value in AB, the two-step test in RSW based on the QLR statistic and that based on the Max statistic, respectively. MNRP, WAP, ScWAP and Time denote maximum null rejection probability, weighted average power, size-corrected WAP, and average computation time used in seconds in each Monte Carlo simulation. The AB test and the RSW tests use 1000 and 499 bootstrap draws respectively. The results for the CC, RCC, and RSW2 tests are based on 5000 simulations, while those for the AB and RSW1 tests are based on 2000 simulations for feasibility.

Table: normal distribution, estimated Ω, $n = 100$

Test	$k = 10$				$k = 4$				$k = 2$			
	MNRP	WAP	ScWAP	Time	MNRP	WAP	ScWAP	Time	MNRP	WAP	ScWAP	Time
$\Omega = \Omega_{\text{Neg}}$												
RCC	.074	.63	.54	.003	.058	.63	.61	.004	.053	.62	.61	.003
CC	.074	.63	.54	.003	.058	.61	.59	.004	.048	.59	.60	.003
AB	.046	.51	.53	1.23	.049	.58	.58	1.56	.056	.64	.63	1.13
RSW1	.054	.55	.53	.569	.053	.58	.58	.736	.050	.62	.62	.533
RSW2	.053	.24	.23	.026	.051	.34	.33	.026	.051	.49	.48	.023
$\Omega = \Omega_{\text{Zero}}$												
RCC	.069	.65	.59	.003	.053	.66	.65	.004	.051	.68	.68	.003
CC	.069	.64	.57	.003	.049	.63	.63	.004	.039	.61	.66	.003
AB	.043	.62	.64	1.23	.048	.66	.67	1.55	.053	.68	.67	1.14
RSW1	.052	.58	.58	.570	.053	.62	.61	.732	.050	.64	.64	.541
RSW2	.062	.54	.50	.026	.053	.61	.60	.026	.050	.64	.64	.024
$\Omega = \Omega_{\text{Pos}}$												
RCC	.056	.77	.75	.003	.054	.75	.74	.004	.051	.71	.71	.003
CC	.043	.73	.74	.003	.034	.68	.73	.004	.035	.63	.69	.003
AB	.044	.78	.79	1.19	.049	.74	.75	1.50	.055	.71	.70	1.07
RSW1	.053	.76	.75	.566	.053	.71	.71	.731	.052	.66	.66	.528
RSW2	.056	.76	.74	.026	.052	.73	.72	.027	.050	.67	.67	.023

Note: Same as Table 1.

Full Vector Simulation Takeaways

- With Ω known, RCC has exact size and CC is slightly conservative for small p.
- The performance of CC and RCC tests is competitive.
- Power curve calculations show that RCC and AB power functions cross: neither test dominates.
- Results are approximately the same with Ω unknown, and when the moments are non-normal.
- AB and RSW-QLR tests are much slower ($\approx 200\times$ or $\approx 400\times$); the RSW-MAX test is also slower (4 to $8\times$).

Matlab Simulation 2: Subvector Test

- We consider the logit version of the multinomial demand model in Gandhi, Lu, and Shi (2019).
- The model implies an IV regression model:

$$\mathbb{E}[Y^* - X'\theta_0 - Z_c'\delta_0 | Z_c, Z_e] = 0, \quad (1)$$

  where
  - Y^*: inverse demand of a product in a market
  - X: vector of endogenous product/market level variables (e.g. price)
  - Z_c: exogenous/control variables
  - Z_e: excluded instruments
- Y^* is unobserved because market shares are imprecisely measured.

Simulation 2: Subvector Test

- Using (imprecise) empirical market shares, we construct bounds for Y^*: Y_U and Y_L, which may *not* satisfy $Y^* \in [Y_L, Y_U]$, but satisfy

$$\mathbb{E}[Y_L|Z] \leq \mathbb{E}[Y^*|Z] \leq \mathbb{E}[Y_U|Z]. \tag{2}$$

- Thus we have the conditional moment inequalities

$$\begin{aligned} \mathbb{E}[Y_U - X'\theta_0|Z] - Z_c'\delta_0 &\geq 0 \\ -\mathbb{E}[Y_L - X'\theta_0|Z] + Z_c'\delta_0 &\geq 0. \end{aligned} \tag{3}$$

- Using a vector of functions of Z as instrumental functions $\mathcal{I}(Z)$, we get

$$E\left[\begin{pmatrix} \mathcal{I}(Z)(Y_U - X'\theta_0) \\ \mathcal{I}(Z)(-Y_L + X'\theta_0) \end{pmatrix} |Z\right] - \begin{pmatrix} \mathcal{I}(Z)Z_c' \\ \mathcal{I}(Z)Z_c' \end{pmatrix} \delta_0 \geq 0 \tag{4}$$

Subvector Test

- We consider a scalar X, a scalar Z_e, and a d_c-dimensional Z_c, where $d_c = 2, 3, 4$, which is also the number of nuisance parameters.
- We use an $\mathcal{I}(Z)$ that is $4, 8, 16$ dimensional for $d_c = 2, 3, 4$, giving us $8, 16, 32$ moment inequalities, respectively.
- We simulate a binary choice model with n i.i.d. markets.
- We set the DGP in such a way that d_c does not affect the ID set of θ_0 which can be numerically calculated.
- We use 5000 simulations to calculate the rejection rates in and outside of the ID set.
- We compare to the subvector test in Andrews, Roth, and Pakes (2021, ARP), which is proposed for the same model.

Confidence Interval

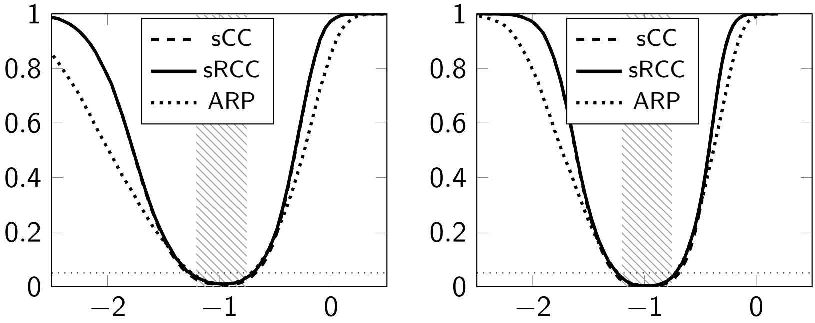
Table: Average Value, Length, and Computation Time (in seconds) of Confidence Intervals

	$n = 500$			$n = 1000$		
	CI	Excess Length	Time	CI	Excess Length	Time
$d_c = 2$, 8 moment inequalities						
sCC	[-1.780, -.332]	1.00	0.1	[-1.615, -.433]	.736	0.1
ARP Hybrid	[-1.998, -.264]	1.29	111	[-1.736, -.395]	.895	109
$d_c = 3$, 16 moment inequalities						
sCC	[-1.852, -.293]	1.11	0.2	[-1.659, -.404]	.809	0.1
ARP Hybrid	[-2.219, -.123]	1.65	199	[-1.883, -.287]	1.15	120
$d_c = 4$, 32 moment inequalities						
sCC	[-1.921, -.254]	1.22	0.1	[-1.718, -.366]	.906	0.1
ARP Hybrid	[-2.596, -.011]	2.14	97	[-2.104, -.180]	1.48	145

Note: The identified set for θ_0 is $[-1.203, -.757]$. The computation times across different (n, d_c) cases are not comparable because they may have been performed by different computers on the computer cluster. The computation of different tests within each (n, d_c) case is always completed on the same computer. Thus the computation times across tests are comparable.

Power Curve: $d_c = 2$, and 8 moment inequalities

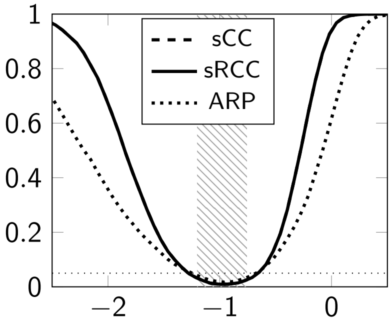
Figure: Rejection Rates of the sCC and sRCC Tests for $H_0 : \theta_0 = \theta$ at a Variety of θ Values at 2 Sample Sizes (nominal level = 5%)



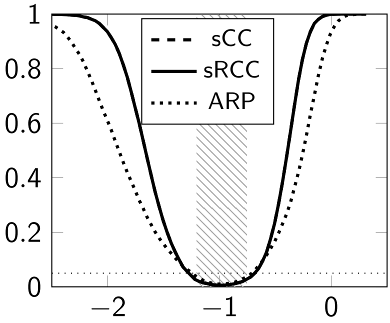
(a) $d_c = 2,\ n = 500$ (b) $d_c = 2,\ n = 1000$

Power Curve: $d_c = 3$ and 16 moment inequalities

Figure: Rejection Rates of the sCC and sRCC Tests for $H_0 : \theta_0 = \theta$ at a Variety of θ Values at 2 Sample Sizes (nominal level = 5%)



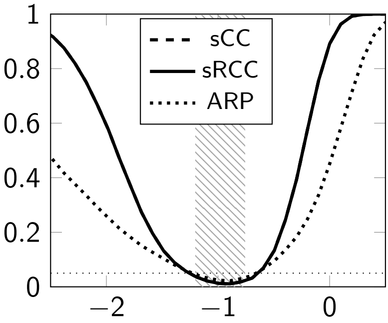
(c) $d_c = 3,\ n = 500$



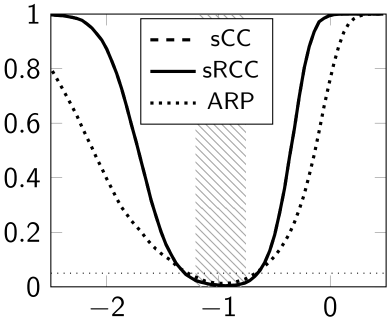
(d) $d_c = 3,\ n = 1000$

Power Curve: $d_c = 4$, and 32 moment inequalities

Figure: Rejection Rates of the sCC and sRCC Tests for $H_0 : \theta_0 = \theta$ at a Variety of θ Values at 2 Sample Sizes (nominal level = 5%)



(e) $d_c = 4, \ n = 500$



(f) $d_c = 4, \ n = 1000$

Summary

- Two extremely simple tests are presented for full-vector and for subvector inference in moment inequality models.
- The tests solve a QPIC problem to obtain the test statistics, and use chi-squared critical values with data dependent (but trivial to compute) degrees of freedom.
- Work-in-progress Stata commands are also presented.
- My RA **Guangyao Zhou** wrote a MATA function for QPIC problems that may be of independent interest. He is applying to PhD programs.
- Matlab simulations show good computational and power performance of the proposed tests.