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WHY IS THERE MONEY?
CONVERGENCE TO A MONETARY EQUILIBRIUM IN A GENERAL EQUILIBRIUM MODEL WITH TRANSACTION COSTS

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Why is there Money? Convergence to a Monetary Equilibrium in a General Equilibrium Model with Transaction Costs

(Preliminary)

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Abstract

This paper presents a class of examples where a nonmonetary economy converges in a tatonnement process to a monetary equilibrium. Exchange takes place in organized markets characterized by an array of trading posts where each pair of goods may be traded for one another. A barter equilibrium with m commodities is characterized by m(m-1)/2 commodity pair trading posts, most of which host active trade. A monetary equilibrium with unique money is characterized by active trade concentrated on m-1 posts, those trading in 'money' versus the m-1 nonmonetary commodities. There are two distinct sources of monetization: absence of double coincidence of wants and scale economies in transaction costs. As households discover that some pairwise markets (those dealing in the 'natural' money or those with high trading volumes) have lower transaction costs, they restructure their trades to take advantage of the low cost. Use of media of exchange arises endogenously from their low transaction cost. Uniqueness of the medium of exchange in equilibrium results from scale economies in the transaction technology.

I. Introduction: Formalizing Menger's 'Origin of Money'[1]

Exchange, like production and consumption, is a fundamental economic activity. In practice, trade is overwhelmingly monetary; the monetization of trade is a universal common denominator of exchange. The monetary instrument varies: gold, government fiat money, private issue banknotes, cattle, cowrie shells, cigarettes. *Money*, like *language* and the *wheel*, is one of the fundamental discoveries of civilization. Nevertheless, centuries of acculturation should not blind economists to the counterintuitive quality of monetary exchange. Monetary trade involves one party to a trade giving up something desirable (labor, his production, a previous acquisition) for something useless (a fiduciary token or a commonly traded commodity for which he has no immediate use) in the hope of advantageously retrading this latest acquisition. An essential issue at the foundations of monetary theory is to articulate the elementary economic conditions that allow this paradox to be sustained as a market equilibrium.

Over a century ago, Carl Menger presented precisely this challenge to monetary theory and proposed an outline of its solution, a theory of market liquidity and of the economic history of money, Menger (1892):

> It is obvious ... that a commodity should be given up by its owner ...for another more useful to him. But that every[one] ... should be ready to exchange his goods for little metal disks apparently useless as such ... or for documents representing [them] ...is...mysterious....
>
> why...is...economic man ...ready to accept a certain kind of commodity, *even if he does not need it*, ... in exchange for all the goods he has brought to market[?]
>
> The problem ... consists in giving an explanation of a general, homogeneous, course of action ...which ... makes for the common interest, and yet which seems to conflict with the ... interests of contracting individuals.

Menger is asking here why monetary trade is an equilibrium, the outcome of the interaction of agents' optimizing decisions.[2] Further, why is the function of mediation in trade concentrated on a

[1] This paper has benefited from seminars and helpful comments at the University of California - Santa Barbara, University of California - San Diego, NSF-NBER Conference on General Equilibrium Theory at Purdue University, Society for the Advancement of Behavioral Economics meeting at San Diego State University, Econometric Society meeting at the University of Wisconsin - Madison, SITE meeting at Stanford University, Federal Reserve Bank of Kansas City, Federal Reserve Bank of Minneapolis, Midwest Economic Theory Conference at the University of Illinois - Urbana Champaign, and from comments of Meenakshi Rajeev. Remaining errors are the author's.

[2] It is useful to distinguish the notion of money as a common medium of exchange from money as a government fiat instrument ("little metal disks apparently useless as such ... or ... documents representing [them]"). This paper will develop the foundations of a common medium of exchange. Fiat money, an easily recognized portable divisible fiduciary instrument with low transaction and inventory costs is an ideal medium of exchange if it has positive equilibrium value. Why it should have positive value is treated in Li and Wright(1998), Lerner(1947), Smith(1776), and Starr(1974). Adam Smith comments "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money…"

unique (or small number of) medium(a) of exchange? It is not a sufficient answer to cite the inconvenience of barter. Inconvenience of barter is the reason why monetization of trade is efficient but it does not explain why monetary trade is a market equilibrium. No agent can choose individually to monetize; monetization is the common outcome of the equilibrium of the trading process.

Menger's proposed solution to this puzzle focused on the liquidity of trading opportunities. "[Call] goods ... *more or less saleable*, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution." That is, a good is very saleable (liquid) if the price at which a household can sell it (the bid price) is very near the price at which it can buy (the ask price). "Men ... exchange goods ... for other goods ... more saleable....[which] become *generally* acceptable media of exchange [emphasis in original]." Hence, Menger suggests that liquid goods, those with narrow spreads between bid and ask prices, become principal media of exchange, money: Liquidity creates monetization. This is the insight that is formalized in the examples below.[3]

Our model starts out from a visualization due to Walras (1874). He describes the setting of trade in a market equilibrium as a complex of trading posts where goods trade pairwise against one another.

> In order to fix our ideas, we shall imagine that the place which serves as a market for the exchange of all the commodities ... for one another is divided into as many sectors as there are pairs of commodities exchanged. We should then have m(m-1)/2 special markets each identified by a signboard indicating the names of the two commodities exchanged there as well as their prices or rates of exchange...

Thus, if there are m goods, Walras envisions a large number, m(m-1)/2, of active trading posts. Buyers of good i for good j and buyers of good j for i go to the {i,j} trading post.

This picture is in contrast to the practice in actual economies. In a monetary economy, there are no active trading arrangements for most goods directly for one another. Almost all trade is of goods for money, a single distinguished commodity that enters into almost all trades. "Money buys goods. Goods buy money. Goods do not buy goods," Clower (1967). In a monetary economy, most of the m(m-1)/2 trading posts Walras posits will be inactive. Active trade will be concentrated on a narrow band of m-1 posts, those trading in 'money' versus the m-1 other nonmonetary commodities. In a monetary economy, households with supplies of good i and demands for good j trade i for j by first trading i for money and then money for j. They do not trade i for j directly.

How does this concentration of trade on a single intermediary good come about? Prof. Tobin (1980) emphasizes scale economy and a positive external effect:

> The use of a particular language or a particular money by one individual increases its value to other actual or potential users. Increasing returns to scale, in this sense, limits the number of languages or moneys in a society and indeed explains the tendency for one basic language or money to monopolize the field.

Tobin is arguing that there is a network externality; increasing activity in one market using a given medium of exchange enhances the effectiveness of all markets using that medium

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[3] It is not clear whether Menger regards liquidity as an inherent quality of a commodity or as endogenously determined in the market. In the examples below both sources of liquidity arise --- some goods have naturally lower transaction costs than others *ceteris paribus*, but the bid-ask spread is the market price of liquidity, endogenously determined in equilibrium.

so that the economy converges on a single common 'money'. This argument is formalized in section IV.

Einzig (1966, p. 345), writing from an anthropologic perspective suggests "Money tends to develop automatically out of barter, through the fact that favourite means of barter are apt to arise ... object[s] ... widely accepted for direct consumption."   Einzig assumes that liquid goods are likely to be those goods with high trading volumes, an observation consistent with Tobin's emphasis on scale economy in the transactions process.

Two distinct bases for monetization of trade naturally arise: absence of double coincidence of wants and scale economies in transaction cost. Absence of double coincidence of wants in a barter economy means that each good is traded more than once in going from endowment to consumption incurring transaction costs at each step. This arrangement favors the development of specialized media of exchange, low transaction cost goods carrying purchasing power from one market to the next.   Thus, in the absence of double coincidence of wants, a small number of specialized low transaction cost goods become 'money'. Scale economies in transaction costs are a separate source of specialization in the transaction function. Both with and without double coincidence of wants, scale economies in transaction cost promote uniqueness of the medium of exchange, a unique 'money'.

The research agenda for this paper is then to formulate a class of examples where the monetization of trade (specialization on a unique [or a few] medium[a] of exchange) is an outcome of market equilibrium. The modeling specification should be parsimonious. The asymmetric function of money should be endogenously determined from more fundamental assumptions; no distinctive role for money should be assumed directly.

The model portrays a trading post as a firm. The decision to operate a trading post depends on the post's ability to cover costs[4]. To emphasize the pairwise character of trade, the model posits budget constraints enforced at each transaction separately: the value of each household's sales to a trading post must equal the value of its purchases from the post. The multiplicity of budget constraints contrasts with the single budget constraint facing a household in a Walrasian model. The multiple budget constraints create the function of a carrier of value between trading posts and hence for a medium of exchange.

When monetization takes place, households supplying good i and demanding good j are induced to trade in a monetary fashion, first trading i for 'money' and then 'money' for j, by discovering that the transaction costs are lower in this indirect trade than in direct trade of i for j. Starting from a barter array consisting (as Walras posits) of $m(m-1)/2$ active trading posts, the allocation evolves through price and quantity adjustments to a monetary array where only one [or several] subset[s] of $m-1$ trading posts are active. The impetus for the concentration of the trading function in a few trading posts (those specializing in trade that includes the commodity[ies] that is [are] endogenously designated as 'money') in the monetary equilibrium

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[4] The practice of representing transaction costs in a trading firm or an economy-wide transaction technology (rather than modeling them at the level of the individual transactor) embodies both the notion that there are businesses specializing in the transaction function (retailers, wholesalers, etc.) and a convenient abbreviation. Rather than depict the transaction costs incurred at the level of the individual transactor separately, those costs are thought to be bundled into the costs of the transacting firm and priced in the difference between buying and selling prices of goods. See for example Foley (1970) and Hahn (1971).

comes from pricing the scale economies in transaction technology or from pricing inherently low transaction cost.

The class of examples presented in section III focuses on demand systems where double coincidence of wants is absent. They posit one [a few] good[s] with a more efficient transaction technology --- lower transaction costs than other goods at every trading volume. A more efficiently transacted good is a 'natural' money. The existence of 'natural' moneys is sufficient to concentrate trade in those goods but scale economies in the transaction technology are essential to uniqueness of 'money'. The range of possibilities can be summarized in a table:

Equilibrium Monetary Structure

Demand Structure	*Linear Transaction Technology*	*Increasing Returns Transaction Technology*
Absence of Double Coincidence of Wants	Monetary Equilibrium where low transaction cost instrument becomes 'money'; Possibly multiple 'moneys'	Monetary Equilibrium with Unique 'Money'
Full Double Coincidence of Wants	Nonmonetary equilibrium	Monetary Equilibrium with Unique 'Money'

(Column heading spanning the last two columns: **Returns to Scale in Transaction Technology**)

Though there are borderline cases, the main outlines are above: Monetization of trade and uniqueness of the medium of exchange depend both on the structure of demand and on the returns to scale in the transaction technology. Scale economies in transaction technology lead to uniqueness of the monetary instrument in equilibrium. The absence of double coincidence of wants leads to an equilibrium where a low transaction cost instrument is 'money.' If transaction costs are linear and there are several equally low transaction cost instruments, 'money' may not be unique. With full double coincidence of wants and linear transaction technology, equilibrium will be nonmonetary.

To treat scale economies at the level of the trading post, this paper will consider nonconvex transaction technologies and the resultant nonconvex transaction cost functions. Competitive equilibria are hence unlikely to exist and this paper concentrates on average cost pricing equilibria of monopolistic (monopolistically competitive) trading posts.

Section IV starts with an economy of diverse endowments and demands but with a double coincidence of wants. The class of examples developed there starts as Einzig suggests with goods most "widely accepted for direct consumption." With scale economies in the transaction technology, these high volume goods will also be those with the lowest unit transaction cost. Thus they are, in Menger's view, the most saleable, and excellent candidates for "*generally* acceptable media of exchange." As they are so adopted by some households, their trading volumes increase, reducing their average transaction costs, and making them more saleable still. This process converges to an equilibrium with a unique medium of exchange. Liquidity of pairwise markets is the essential point here; low combined transaction cost for the medium of exchange and the goods for which it is traded. As households discover that some pairwise markets (those with high trading volumes) have lower transaction costs, they rearrange their trades to take advantage of the low cost. That leads to even higher trading volumes and even lower costs at the most active trading posts[5]. The process converges to an equilibrium where only the high volume trading posts dealing in a single intermediary good ('money') are in use. Under nonconvex transaction costs, this implies a cost saving, since only m-1 trading posts need to operate, incurring significantly lower costs than m(m-1)/2 posts. Scale economies make it cost-saving to concentrate transactions in a few trading posts and a unique 'money'.

A bibliography of the issues involved in this inquiry appears in Ostroy and Starr (1990). In addition, note particularly Banerjee and Maskin (1996), Iwai (1995), Kiyotaki and Wright (1989), Ostroy and Starr (1974). The treatment of transaction costs in this paper resembles the general equilibrium models with transaction cost developed in Foley (1970), Hahn (1971), and Starrett (1973). The structure of bilateral trade here however is more detailed, with a budget constraint enforced at each trading post separately, so that their results do not immediately translate to the present setting.

This paper displays results similar to those of Iwai (1995) and Kiyotaki and Wright (1989), where traders are induced to concentrate their transactions on high volume markets by the reduced waiting time for a matching trade made possible by high volume. Indivisibility of traders --- a scale economy --- accounts for this effect. This paper differs in explicitly emphasizing transaction cost and portraying a dynamic adjustment process that leads from barter to a monetary equilibrium.

In most of the literature, the locus of economic interaction is the pair of traders (households) coming together to trade. Exchange takes place without the structure of a specialized transaction function (exchanges, retailers, wholesalers, etc.). The present paper and Starr and Stinchcombe (1999, 1998) emphasize an explicit resource using specialized exchange activity, the trading post for trade of a pair of commodities against one another, as the elementary unit of economic interaction. The market-maker there posts prices with a bid-ask spread to cover costs. It is there that supplies and demands are presented and matched. Starr and Stinchcombe (1999) characterizes monetary trade as the cost minimizing outcome of a centralized programming problem with a nonconvex transaction cost structure. This paper and Starr and Stinchcombe (1998) decentralize the problem, characterizing the monetization of trade as a market equilibrium.

II. An Economy with Pairwise Trade and Transaction Costs: An Average Cost Pricing Equilibrium

This section describes a population of households and trading firms, an average cost pricing equilibrium concept, and a tatonnement process to approach equilibrium. The distinctive features of the model are (i) transactions exchange pairs of goods, (ii) budget constraints are enforced at each transaction separately, generating a role for a carrier of value between transactions (a medium of exchange), and (iii) transaction costs may be linear or include scale economies.

Let N be the number of commodities. A typical household h, has endowment $r^h \in R^N_+$. r^h_n is h's endowment of good n. It is convenient to specify examples where a typical household has easily identified goods to be supplied and demanded. A typical household may be denoted

5 Hahn (1997) describes this situation as "If the number who can gain from trade is ... sufficiently [large] ... , the Pareto improving trade will take place. There is thus an externality induced by set-up costs."

h=[m,n] where m and n are integers between 1 and N-1 (inclusive). m denotes the good with which h is endowed. n denotes the good he prefers; [m,n]'s utility function can then be taken to be $u^{[m,n]}(x) = \Sigma_{i\neq n} x_i + 3x_n$.

Specify a nonconvex transactions technology for pairwise goods markets so that all transaction costs accrue in good N. Firm {i,j} is the market maker in trade between goods i and j. The typical transactions of firm {i,j} will consist of purchases of $y^B \in R^N_+$ and sales of $y^S \in R^N_+$. The subscript n denotes the n^{th} co-ordinate.

$$Y^{\{i,j\}} = \{ (y^B , y^S) \mid y^S_n = y^B_n = 0 \text{ for } n \neq i,j,N \; ; 0 \leq y^S_n \leq y^B_n \text{ for } n=i,j; \; y^S_N=0; \; y^B_N \geq \min[\delta^i y^B_i, \gamma^i] + \min[\delta^j y^B_j, \gamma^j] \}$$

where $\delta^i, \delta^j, \gamma^i, \gamma^j > 0$. In words, the transaction technology looks like this: The firm {i,j} makes a market in goods i and j, buying each good in order to resell it. It incurs transaction costs in good N. These costs vary directly (in proportions δ^i, δ^j) with volume of trade at low volume and then hit a ceiling after which they do not increase with trading volume. This specification is sufficiently general that it includes both linear and diminishing marginal cost cases. For γ^i, γ^j sufficiently large, costs are linear in the relevant range. For γ^i, γ^j sufficiently small there is a scale economy in the relevant range of usage; the transaction technology is nonconvex, displaying diminishing marginal costs. The transaction cost structure is separable in the two principal traded goods. The firm {i,j} buys good N to cover the transaction costs it incurs, paying for N in goods i and j.[6]

Good N is specialized as the input to the transactions process. It is convenient to arrange a special segment of the households, denoted H^2, to provide good N. For simplicity we will suppose that there is just one representative $h \in H^2$. He has an endowment of good N only, $r^h_N > \sum_{i=1}^{N-1} (N-2)\gamma^i$, and utility function $u^h(x) = \sum_{i=1}^{N} x_i$.

Households formulate their trading plans deciding how much of each good to trade in each pairwise goods market (i.e. with each pairwise market maker). A typical firm (market maker) {i,j} is denoted by the pair of goods in which it makes a market. A typical household h=[m,n] is denoted by the pair of goods it will typically seek to exchange (m for n). This leads to the rather messy notation

$b^{[m,n]\{i,j\}}_{\ell}$ = planned purchase of good ℓ by household [m,n] on market {i,j}

$s^{[m,n]\{i,j\}}_{\ell}$ = planned sale of good ℓ by household [m,n] on market {i,j}

The trading firm {i,j} posts buying (bid) prices for goods i and j. The price of i is in units of j. The price of j is in units of i. Hence the selling (ask) price of j is the inverse of the bid price of i (and vice versa). Market conditions facing the typical household are characterized by the buying (bid) prices for goods traded in {i,j} (bid prices for purchases by the trading firm of i and j ---- and the implied selling (ask) prices of the counterpart good), and prices for inputs to the transaction technology. $q^{\{i,j\}}_i$ and $q^{\{i,j\}}_j$ are the buying (bid) prices of goods i and j at {i,j} --- the price at which the holder of i or j can sell for the other good. The price of good i is expressed in units of good j; the price of good j is expressed in units of good i. Hence $(q^{\{i,j\}}_i)^{-1}$ and $(q^{\{i,j\}}_j)^{-1}$ are the ask prices of j and i respectively. The trading post {i,j} covers its costs by the difference between the bid and ask prices of i and j, that is by the spread $(q^{\{i,j\}}_j)^{-1} - q^{\{i,j\}}_i$ and the spread $(q^{\{i,j\}}_i)^{-1} - q^{\{i,j\}}_j$.

[6] The transaction technology posited here supposes that all trading posts for good j have the same transaction technology for j. This is in contrast to Banerjee and Maskin (1996) where various traders have differing transaction costs (difficulty in assessing product quality) for the same good.

For example, on the apple-orange market, we might have q_{orange}=4 apples, $q_{apple}=({}^1/_6)$orange. Then the ask price of orange is $(q_{apple})^{-1}$=6 apples, and the ask price of apple is $(q_{orange})^{-1}=({}^1/_4)$ orange. The bid-ask spread on orange=2 apples; the bid-ask spread on apple=$({}^1/_{12})$ orange.

$q^{\{i,j\}}_{(i)N}$ is {i,j}'s buying price of good N in units of i; $q^{\{i,j\}}_{(j)N}$ is {i,j}'s buying price of good N in units of j. In the present simplified model, $q^{\{i,j\}}_{(i)N}$ and $q^{\{i,j\}}_{(j)N}$ are necessarily unity in equilibrium (when there is active trade).

Given $q^{\{i,j\}}_i$, $q^{\{i,j\}}_j$, for all {i,j} household h then forms its buying and selling plans. Household h faces the following constraints on its transaction plans:

(T.i) $b^{h\{i,j\}}_n > 0$, only if n=i,j; $s^{h\{i,j\}}_n > 0$, only if n=i,j .

(T.ii) $b^{h\{i,j\}}_i \leq q^{\{i,j\}}_j \cdot s^{h\{i,j\}}_j$, $b^{h\{i,j\}}_j \leq q^{\{i,j\}}_i \cdot s^{h\{i,j\}}_i$ for each {i,j}.

(T.iii) $x^h_n = r^h_n + \Sigma_{\{i,j\}} b^{h\{i,j\}}_n - \Sigma_{\{i,j\}} s^{h\{i,j\}}_n \geq 0,\ 1 \leq n \leq N$.

For $h \in H^2$, the constraints are similar:

(T.i) $b^{h\{i,j\}}_n > 0$, only if n=i,j; $s^{h\{i,j\}}_n > 0$, only if n=i,j,N.

(T.ii') Decompose $s^{h\{i,j\}}_N$ into nonnegative elements $s^{h\{i,j\}}_{(i)N}$ and $s^{h\{i,j\}}_{(j)N}$, so that $s^{h\{i,j\}}_{(i)N}+s^{h\{i,j\}}_{(j)N}=s^{h\{i,j\}}_N$, then we have $b^{h\{i,j\}}_i \leq q^{\{i,j\}}_{(i)N} \cdot s^{h\{i,j\}}_{(i)N}$, and $b^{h\{i,j\}}_j \leq q^{\{i,j\}}_{(j)N} \cdot s^{h\{i,j\}}_{(j)N}$, for each {i,j}.

(T.iii) $x^h_n = r^h_n + \Sigma_{\{i,j\}} b^{h\{i,j\}}_n - \Sigma_{\{i,j\}} s^{h\{i,j\}}_n \geq 0,\ 1 \leq n \leq N$.

Note that condition (T.ii) defines a budget balance requirement at the transaction level, implying the pairwise character of trade. (T.ii) is the source of the demand for a medium of exchange. Since the budget constraint applies to each pairwise transaction separately, there is a demand for a carrier of value to move purchasing power between successive transactions. Household h's behavior is described then as follows. h faces the array of bid prices $q^{\{i,j\}}_i$, $q^{\{i,j\}}_j$, and chooses $s^{h\{i,j\}}_n$ and $b^{h\{i,j\}}_n$, n= i, j, N(for h in H^2), to maximize $u^h(x^h)$ subject to (T.i), (T.ii), (T.iii). That is, h chooses which firms to transact with --- and hence which pairwise markets to transact in --- and a transaction plan to optimize utility, subject to a multiplicity of pairwise budget constraints.

The firm {i,j} covers its transaction costs from the bid-ask spread. The firms in this economy have nonconvex transaction technologies, generating a natural monopoly in each pairwise goods market. A competitive equilibrium is not an appropriate solution concept. The equilibrium notion I will use is an average cost pricing equilibrium resulting in zero profits for the typical trading firm (this has the additional technical benefit that trading firms make no net profit, so no account need be taken of their distribution to shareholders). The rationale for this choice of equilibrium concept is possible entry (by other similar firms) if any economic rent is actually earned. The presence of potential entrants and their actions is not explicitly modeled. There is a unique firm making a market in goods i and j, denoted indiscriminately {i,j} = {j,i}.

An average cost pricing equilibrium[7] consists of $q^{o\{i,j\}}_n$, $1\leq n\leq N$, so that :

- For each household h, there is a utility optimizing plan $b^{oh\{i,j\}}_n$, $s^{oh\{i,j\}}_n$, (subject to T.i, T.ii [or T.ii' for $h\in H^2$], T.iii) so that $\Sigma_h b^{oh\{i,j\}}_n = y^{o\{i,j\}S}_n$, $\Sigma_h s^{oh\{i,j\}}_n = y^{o\{i,j\}B}_n$, for each {i,j}, each n, where
- For each {i,j} there is $(y^{o\{i,j\}B}, y^{o\{i,j\}S}) \in Y^{\{i,j\}}$, further
- For $i,j\neq N$, $y^{o\{i,j\}B}_i = \sum_{h\notin H^2} s^{oh\{i,j\}}_i$, $y^{o\{i,j\}B}_j = \sum_{h\notin H^2} s^{oh\{i,j\}}_j$
- $y^{o\{i,j\}B}_N$ can be divided into two parts, $y^{o\{i,j\}B}_{(i)N} \geq 0$, $y^{o\{i,j\}B}_{(j)N} \geq 0$, so that $y^{o\{i,j\}B}_{(i)N} + y^{o\{i,j\}B}_{(j)N} = y^{o\{i,j\}B}_N$.
- $q^{o\{i,j\}}_N y^{o\{i,j\}B}_{(i)N} = y^{o\{i,j\}B}_i - q^{o\{i,j\}}_j y^{o\{i,j\}B}_j$. $q^{o\{i,j\}}_N y^{o\{i,j\}B}_{(j)N} = y^{o\{i,j\}B}_j - q^{o\{i,j\}}_i \cdot y^{o\{i,j\}B}_i$.

An equilibrium is said to be monetary with a unique money, μ, if --- for all households --- good μ is the only good that households both buy and sell. There are examples of monetary equilibria where there are several media of exchange. An equilibrium will be said to be monetary with multiple moneys, $\mu^1, \mu^2, ...$, if --- for all households --- $\mu^1, \mu^2, ...$ are the only goods that households both buy and sell.

Particularly in the case of scale economies in the transactions technology there is a strong tendency to multiple equilibria. This creates an interest in determining which of the several equilibria the economy will actually select. One solution to this problem is to posit an adjustment process to equilibrium that makes the choice. Hence we use the following

Tatonnement adjustment process for average cost pricing equilibrium:

Prices will be adjusted by an average cost pricing auctioneer. Specify the following adjustment process for prices.

STEP 0: The starting point is somewhat arbitrary. In each pairwise market the bid-ask spread is set to equal average costs at low trading volume.

CYCLE 1

STEP 1: Households compute their desired trades at the posted prices and report them for each pairwise market.

STEP 2: Average costs (and average cost prices) are computed for each pairwise market based on the outcome of STEP 1. Prices are adjusted upward for goods in excess demand at a trading post, downward for goods in excess supply, with the bid-ask spead adjusted to average cost. A market's (market making firm's) nonzero prices are specified only for those goods where the firm has the technical capability of being active in the market; other prices are unspecified, indicating no available trade.

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[7] Meenakshi Rajeev reminds me that a reasonable additional constraint on equilibrium is that there should be nonnegative stocks of each good at it's trading post throughout the execution of trade. Formalizing this notion requires adding a time dimension within the trading process. An alternative interpretation is that the scale economy in transaction cost reflects the cost of acquiring an inventory for each active trading post to maintain nonnegative stocks throughout. The linear cost case can be interpreted that traders accept warehouse receipts for desired goods and costlessly revisit the trading post to acquire needed stocks that have accumulated there. See Rajeev (forthcoming).

CYCLE 2
Repeat STEP 1 (at the new posted prices) and STEP 2.
CYCLE 3, CYCLE 4, ... repeat until the process converges.

III. Absence of Double Coincidence of Wants: A Class of Examples where Money is the Low Transaction Cost Instrument

Jevons (1893) reminds us that monetization of trade follows in part from the absence of a double coincidence of wants. In the present model, that logic is particularly powerful. Absence of coincidence of wants means that the typical endowment will be traded more than once in moving from endowment to consumption. Barter trade successfully rearranging the allocation to an equilibrium will transact an endowment first at the trading post where it is supplied and again at a distinct post where it is demanded. Hence the alternative of monetary trade (substituting for the retrade of nonmonetary goods) can be undertaken without increasing total trading volume or transaction cost, even without scale economies.

Generations of economists have noted that some goods are more suitable than others as media of exchange[8]. Some of the properties of money --- general acceptability and price predictability, for example --- are conferred as part of the monetary equilibrium. Others are the indigenous property of the commodity: durability, portability, cognizibility, divisibility. The transaction technology $Y^{\{i,j\}}$ is sufficiently flexible to distinguish transaction costs differing among commodities, for example to distinguish the transactions costs of mint-standardized gold medallions from those of fresh fish.

We now formalize the classic notion of the absence of double coincidence of wants. Let N be an integer, $N \geq 8$. A bit of additional notation is helpful to characterize permutations of the N commodities. Let

$$m \oplus i = \begin{cases} m+i & \text{if } m+i < N \\ m+i+1-N & \text{if } m+i \geq N \end{cases}$$

That is, $m\oplus i$ denotes m+i mod N with a jump at N (since good N is used primarily as an input to the transaction process). Recall that [m,n] denotes a household endowed with good m, strongly preferring good n. Using the notation above, let $H^3 = \{[m,m\oplus i] \mid m=1,2, ..., N-1; i=1,2; r^{[m,m\oplus i]}_m=A, \text{ for all } m\}$. H^3 characterizes a population of 2(N-1) households with the same size of initial endowment, so that no pair of them have reciprocal matching endowments and preferences but so that their endowments in aggregate can be reallocated to make each one significantly better off (roughly by arranging the households clockwise in a circle ordered by endowment good and having each household [m,m⊕i] send his endowment i places counterclockwise). Recall the transactions technology introduced in the previous section:

$$Y^{\{i,j\}} = \{ (y^B, y^S) \mid y^S_n = y^B_n = 0 \text{ for } n \neq i,j,N ;\ 0 \leq y^S_n \leq y^B_n \text{ for } n=i,j;\ y^S_N=0;$$
$$y^B_N \geq \min[\delta^i y^B_i, \gamma^i] + \min[\delta^j y^B_j, \gamma^j] \}$$

where $\delta^i, \delta^j, \gamma^i, \gamma^j > 0$. Let $0 < \delta^i, \delta^j < {}^1/_3$. We will use H^3 and this transactions technology to present a class of examples demonstrating

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[8] I am indebted to several colleagues --- including Henning Bohn, Harold Cole, James Hamilton, Harry Markowitz, Chris Phelan and Bruce Smith --- for reminding me how foolish it is to ignore this point.

(i) Absent double coincidence of wants and under uniform linear (constant marginal) transaction costs with a natural money (lowest transaction cost instrument) there is a monetary equilibrium. If there is only a single lowest transaction cost instrument then the 'money' is unique. If there are several equally low cost natural moneys then the monetary instrument need not be unique (Examples III.1 and III.2).

(ii) Absent double coincidence of wants, if there are scale economies in transaction costs, then a tatonnement process can lead to a unique medium of exchange in equilibrium. Uniqueness of the medium of exchange can occur even when there are several natural moneys that can act as media of exchange with equivalent low cost transaction technologies (Example III.3).

(iii) Absent double coincidence of wants and without a 'natural' money or scale economies in transaction cost, there are both barter and monetary equilibria with identical transaction costs and welfare (Examples III.4, III.5).

<u>Example III.1</u>: Let the population of households be $H^2 \cup H^3$. Let $0<\delta^i<{}^1/_3$ for all i, and $0<\delta^1<\delta^i$, i=2, 3, ... N-1, and let $\gamma^i >$ NA all i. Transaction costs are constant and non-trivial for all goods but 1; scale economies are not evident in the relevant range. Then the tatonnement process converges to a monetary equilibrium with 1 as money.

Demonstrating Example III.1:

STEP 0: For all $1\leq i,j\leq N-1$, $i\neq j$, $q^{\{i,j\}}_{(i)N} = q^{\{i,j\}}_{(j)N} = 1$, $q^{\{i,j\}}_i=(1-\delta^i)$, $q^{\{i,j\}}_j=(1-\delta^j)$.

CYCLE 1, STEP 1: For each $[m,n] \in H^3$, set $s^{[m,n]\{m,n\}}_m=A$, $b^{[m,n]\{m,n\}}_n=q^{\{m,n\}}_m A$.

STEP 2: Note that the allocation in STEP 1 is not market clearing. For each {i,j}, $i\neq j$, $j=1, i\oplus 1, i\oplus 2$, set $q^{\{i,j\}}_i = \dfrac{1-\delta^i}{1+\delta^j}$, $q^{\{i,j\}}_j=1$, $q^{\{i,j\}}_{(i)N}=1$. Note that $q^{\{i,1\}}_i > q^{\{i,j\}}_i$ for $j\neq 1$.[9]

CYCLE 2, STEP 1: For each $[m,n] \in H^3$, set $s^{[m,n]\{m,1\}}_m=A$, $b^{[m,n]\{m,1\}}_1=q^{\{m,1\}}_m A$ $=s^{[m,n]\{n,1\}}_1=b^{[m,n]\{n,1\}}_n$. This allocation is market clearing.

STEP 2: Repeat CYCLE 1, STEP 2.

CYCLE 3, STEP 1: Repeat CYCLE 2, STEP 1.

CONVERGENCE.

In Example III.1 the first round of planned trades is out of equilibrium. Each household proposes its desired net trade to the trading post specializing in that pair of goods, but this is not market clearing in the absence of double coincidence of wants. Trading posts reprice their goods to reflect the disequilibrium: trading posts' bid prices of households' desired supplies are marked down to reflect the transaction cost of acquiring the desired match. The bid price of m at each {m,m⊕i} post includes a discount for both the transaction cost of m and for the countertrade m⊕i. Similarly at the {m,1} post there is discounting of the bid price of m for transaction costs on both m and 1. But this leads to pricing trade at {m,1}, m=2,3,...,N-1, the posts of the low transaction cost good 1, more attractively than at those of high transaction cost goods. Households respond to this pricing by rearranging their planned trades to good 1's trading posts. Household [m,m⊕i] sells m for 1 to {m,1} at a bid price discounted for the transaction costs of m and 1 . He then trades 1 at par for m⊕i at {1,m⊕i}. All trade goes through good 1's trading posts. Good 1 has become 'money.'

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9 The tatonnement process requires this price adjustment for {i, i⊕1} and {i, i⊕2}. It is convenient to extend it to all {i,1}.

Example III.2: Let the population of households be $H^2 \cup H^3$. Let $0<\delta^1 = \delta^2 = \delta^3 < \delta^i < {}^1/_3$, i=4,5,...N-1, and let $\gamma^i > NA$ all i. Then there is a continuum of equilibria with 1,2,3 acting as 'money' in proportions from 0% to 100%. Consumptions and utilities of all households are the same as in the equilibrium of Example III.1.

Demonstrating Example III.2: Choose $\alpha^1, \alpha^2, \alpha^3 \geq 0$; $\alpha^1+\alpha^2+\alpha^3 = 1$. For each {i,j}, i≠j, j=1,2,3,i⊕1,i⊕2, set $q^{\{i,j\}}{}_i=\frac{1-\delta^i}{1+\delta^j}$, $q^{\{i,j\}}{}_j=1$, $q^{\{i,j\}}{}_{(i)N}=1$. Note that $q^{\{i,\ell\}}{}_i > q^{\{i,j\}}{}_i$ for ℓ=1,2,3; j≠1,2,3. For each $[m,n] \in H^3$, each ℓ = 1,2,3, set $s^{[m,n]\{m,\ell\}}{}_m=\alpha^\ell A$, and set $b^{[m,n]\{m,\ell\}}{}_\ell=q^{\{m,\ell\}}{}_m\alpha^\ell A=s^{[m,n]\{n,\ell\}}{}_\ell=b^{[m,n]\{n,\ell\}}{}_n$. This allocation is market clearing.

The logic of Example III.2 is merely the multi-money version of III.1. The first round of planned trades is out of equilibrium. Each household proposes its desired net trade to the trading post specializing in that pair of goods, but this is not market clearing in the absence of double coincidence of wants. Trading posts reprice their goods to reflect the disequilibrium: trading posts' bid prices of households' desired supplies are marked down to reflect the transaction cost of acquiring the desired match. But this leads to pricing trade at the posts of the low transaction cost goods, 1,2,3 , more attractively than those of high transaction cost goods. Households respond to this pricing by rearranging their planned trades to goods 1, 2, and 3's trading posts. All trade then goes through good 1, 2, 3's trading posts. Goods 1, 2, 3 have become common media of exchange. They can be used however in any proportionate combination from 0% to 100% since absent economies of scale there is no reason further to specialize.

Example III.3: Let H^4 = {[1, n], [n, 1]| n=2,3,...,N-1; household endowments =A>0}. Let the population of households be $H^2 \cup H^3 \cup H^4$. Let $0<\delta^1 = \delta^2 = \delta^3 < {}^1/_3 < \delta^4 = \delta^5 = ...= \delta^{N-1}$. For all i let $\gamma^i = ({}^1/_2)\delta^i A$. That is, goods 1,2,3 have low transaction costs and all goods' scale economies enter at a low level of activity. Then the tatonnement process described above leads to a monetary equilibrium with 1 as 'money'.

Demonstrating Example III.3:

STEP 0: For all 1≤i,j≤N-1, i≠j, $q^{\{i,j\}}{}_{(i)N} = q^{\{i,j\}}{}_{(j)N} = 1$, $q^{\{i,j\}}{}_i=(1-\delta^i)$, $q^{\{i,j\}}{}_j=(1-\delta^j)$.

CYCLE 1, STEP 1: For each $[m,n] \in H^3 \cup H^4$, set $s^{[m,n]\{m,n\}}{}_m=A$, $b^{[m,n]\{m,n\}}{}_n=q^{\{m,n\}}{}_m A$.

STEP 2: Note that the allocation in STEP 1 is not market clearing. For each {i,j}, i,j≠1, i=1,2,...,N-1, j=2,3,i⊕k, k=1,2, set $q^{\{i,j\}}{}_i=\frac{1-(\delta^i/2)}{1+\delta^j}$, $q^{\{i,j\}}{}_j=1$, $q^{\{i,j\}}{}_{(i)N}=1$. For j≠N-2,N-1,1,2,3, $q^{\{1,j\}}{}_j= \frac{1-(\delta^j/2)}{1+(\delta^1/2)}$, $q^{\{1,j\}}{}_1=1$; $q^{\{1,j\}}{}_j$ for j=N-2,N-1,1,2,3, is slightly higher.

CYCLE 2, STEP 1: For each $[m,n] \in H^3 \cup H^4$, set $s^{[m,n]\{m,1\}}{}_m=A$, $b^{[m,n]\{m,1\}}{}_1=q^{\{m,1\}}{}_m A$ $=s^{[m,n]\{n,1\}}{}_1=b^{[m,n]\{n,1\}}{}_n$. This allocation is market clearing.

STEP 2: For each {i,j}, i,j≠1, i=1,2,...,N-1, j=i⊕k, k=1,2, set $q^{\{i,j\}}{}_i= \frac{1-\delta^i}{1+\delta^j}$, $q^{\{i,j\}}{}_j=1$, $q^{\{i,j\}}{}_{(i)N}=1$. $q^{\{1,i\}}{}_i=\frac{1-(\delta^j/6)}{1+(\delta^1/6)}$, $q^{\{1,i\}}{}_1=1$.

CYCLE 3, STEP 1: Repeat Cycle 2, Step 1.

CYCLE 3, STEP 2: Repeat Cycle 2, Step 2.

CONVERGENCE

What's happening in Example III.3? The rabbit goes into the hat when H^4 is introduced. The economy starts out with a large volume of trade going through good 1. The scale economy in transaction costs means that the average transaction cost on the typical market including good 1, {1,n} is relatively low. At Cycle 2, Step 1, all agents not currently participating in markets for good 1 recognize that at prevailing transaction costs, it is advantageous to reallocate trades from direct trading markets {i,j} to indirect trade through good 1 as medium of exchange trading on {1,i}, {1,j}. Identical transaction costs can be achieved through good 2 or 3 as the medium of exchange. However, the starting position where markets including good 1, {1,i},{1,j}, have already achieved significant scale economies leads the adjustment process to concentrate on good 1 as the unique medium of exchange.[10]

Example III.4: Let the population of households be $H^2 \cup H^3 \cup \{0\}$. The household designated 0 is a dummy arbitrageur, with neither positive endowment nor tastes. Let $0<\delta^i<{}^1/_3$ be the same value for all i = 1, 2, 3, ... N-1, and let γ^i > NA all i. That is, transaction costs are constant, uniform, and non-trivial for all goods; scale economies are not evident in the relevant range. Then the economy has a nonmonetary equilibrium.

Demonstrating Example III.4: For each {i,j}, i=1,2,...,N-1, j=i⊕k, k=1,2, set $q^{\{i,j\}}_i=\frac{1-\delta^i}{1+\delta^j}$, $q^{\{i,j\}}_j=1$, $q^{\{i,j\}}_{(i)N}=1$. Then for each $[m,n] \in H^3$, set $s^{[m,n]\{m,n\}}_m=A$, $b^{[m,n]\{m,n\}}_n=q^{\{m,n\}}_m A$. For each {i,j}, i=1,2,...,N-1, j=i⊕k, k=1,2, set $s^{0\{i,j\}}_j=\frac{1-\delta^i}{1+\delta^j}A$, $b^{0\{i,j\}}_i=\frac{1-\delta^i}{1+\delta^j}A$. Trading post {i,j} has a net return from the trades above of i in the amount $A(1-\frac{1-\delta^i}{1+\delta^j})$ which is sufficient to acquire the same volume of good N from H^2 and cover transaction costs. This array of prices and allocation is an average cost pricing (and competitive) equilibrium.

Example III.5: Let the population of households be $H^2 \cup H^3$. Under the same parameter values as in Example III.4, for any good μ = 1,2,...N-1, there is a monetary equilibrium with μ acting as money. Consumptions and utilities of all households are the same as in Example III.4.

Demonstrating Example III.5: For each {μ,j}, j=1,2,...,N-1, $j\neq\mu$, set $q^{\{\mu,j\}}_\mu=1$, $q^{\{\mu,j\}}_j=\frac{1-\delta^j}{1+\delta^\mu}$. For each $[m,n]\in H^3$, let $s^{[m,n]\{\mu,m\}}_m=A$, $b^{[m,n]\{\mu,m\}}_\mu= q^{\{\mu,m\}}_m A$, $s^{[m,n]\{\mu,n\}}_\mu=q^{\{\mu,m\}}_m A=b^{[m,n]\{\mu,n\}}_n$.

Example III.5 represents a case where it is pointless to use money, but where a monetary allocation is nevertheless possible. Absent a natural money or scale economies in transaction costs there is no price system inducement to monetize or to concentrate on a small number of media of exchange. Money is merely performing --- at no cost saving --- the arbitrage function a middleman trader can perform in Example III.4.

IV. Scale Economy in Transaction Costs: A Class of Examples where Liquidity Comes from Common Usage

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10 It is tempting to seek a more subtle example where the initial position is not decisively dominated by good 1 and where the adjustment takes place more gradually. I have not succeeded in formulating that treatment for the absence of double coincidence of wants case.

This section uses a class of examples to illustrate convergence from barter to a monetary average cost pricing equilibrium in a pure exchange economy with full double coincidence of wants, with pairwise goods markets, and nonconvex transaction technology. Define a household population H^1 as follows: Let N be an even integer $N \geq 8$. Let $H^1 = \{ [m,n] \mid 1 \leq m,n \leq N-1, m \neq n; r^{[m,n]}_m = A > 0$, except $r^{[m,1]}_m = 2A = r^{[1,m]}$ for $m \neq N/2$, $r^{[N/2,1]}_{N/2} = 3A = r^{[1, N/2]}_1 \}$.

<u>Example IV</u>: Let the population be $H^1 \cup H^2$, let $\delta^i = {}^1/_2$, $\gamma^i = (.6)A$, all i. Then the tatonnement process converges to a monetary equilibrium where 1 is the unique money.

Demonstrating Example IV: The endowment and tastes of the household side of the market looks like this. Good N is held by the household in H^2. His tastes are very simple: all goods are perfect substitutes. Households in H^1 have distinct preferences and endowments. Each is endowed with one good and strongly prefers another. Their tastes are uniformly distributed among goods 1 through N-1. For most pairs of goods m,n , the desired net trade is uniformly distributed as well; the desired trade between them is A. For pairs 1,n the desired trading volume is 2A except for the pair 1,N/2 where the desired volume is 3A. This structure of preferences and endowments creates a desire for relatively high trading volumes among households trading in good 1.

The scale economy in transactions costs begins to be apparent at trading volumes just slightly larger than the endowment of most households. The scale economy is manifest well within the desired trading volumes of households endowed with or desiring good 1. The tatonnement is illustrated in Figure 1. Each numbered node in the figure represents a commodity. The chord connecting nodes i and j represents an active market in the pair i,j. If there is no chord, there is no active market. A broken line chord represents a low volume (eventually high cost) market. A solid chord represents a moderate volume (eventually moderate cost) market. A heavy chord represents a high volume (low cost) market. The progression from barter to money is then the movement from a diffuse array of many active low volume markets to the concentration on a connected family of high volume (low cost) markets. The tatonnement proceeds as follows:

STEP 0: For all $1 \leq i,j \leq N-1$, $i \neq j$, $q^{\{i,j\}}_{(i)N} = q^{\{i,j\}}_{(j)N} = 1$, $q^{\{i,j\}}_i = q^{\{i,j\}}_j = {}^1/_2$.

CYCLE 1, STEP 1:

- For $[m,n] \in H^1$, $m \neq 1 \neq n$, $b^{[m,n]\{m,n\}}_n = ({}^1/_2)A = q^{\{m,n\}}_m A$, $s^{[m,n]\{m,n\}}_m = A$; all other purchases and sales are nil.
- For $[m,1] \in H^1$, $m \neq N/2$, $b^{[m,1]\{m,1\}}_1 = A = q^{\{m,1\}}_m 2A$, $s^{[m,1]\{m,1\}}_m = 2A$; all other purchases and sales are nil. For $[1,n] \in H^1$, $n \neq N/2$, $b^{[1,n]\{1,n\}}_n = A = q^{\{1,n\}}_1 2A$, $s^{[1,n]\{1,n\}}_1 = 2A$; all other purchases and sales are nil.
- For the two remaining elements of H^1, [1, N/2] and [N/2, 1], $b^{[1, N/2]\{N/2,1\}}_{N/2} = ({}^3/_2)A = q^{\{N/2,1\}}_1 3A$, $s^{[1, N/2]\{N/2,1\}}_1 = 3A$; $b^{[N/2,1]\{N/2,1\}}_1 = ({}^3/_2)A = q^{\{N/2,1\}}_{N/2} 3A$, $s^{[N/2,1]\{N/2,1\}}_{N/2} = 3A$; all other purchases and sales are nil.
- For $h \in H^2$, for $i \neq 1 \neq j$, $b^{h\{i,j\}}_i = b^{h\{i,j\}}_j = A/2$, $s^{h\{i,j\}}_N = A$; for i or j =1, $b^{h\{i,j\}}_i = b^{h\{i,j\}}_j = \gamma = (.6)A$, $s^{h\{i,j\}}_N = 2\gamma = (1.2)A$.

STEP 2:

- For $\{m,n\}$ where $m\neq 1\neq n$, $1=q^{\{m,n\}}_{(m)N}= q^{\{m,n\}}_{(n)N}$, $q^{\{m,n\}}_{m}=q^{\{m,n\}}_{n}=({}^1/_2)$.
- For $\{m,1\}$, $m\neq N/2$, $1=q^{\{m,1\}}_{(1)N}=q^{\{m,1\}}_{(m)N}$, $q^{\{m,1\}}_{m}=q^{\{m,1\}}_{1}=\dfrac{2A-\gamma}{2A}=.70$
- For $\{N/2,1\}$, $1=q^{\{N/2,1\}}_{(N/2)N}=q^{\{N/2,1\}}_{(1)N}$, $q^{\{N/2,1\}}_{N/2}=q^{\{N/2,1\}}_{1}=\dfrac{3A-\gamma}{3A}=.80$

At this stage we can see the initial effect of the scale economy. At STEP 0 prices started essentially equivalent in all pairwise markets. But the prices announced at the end of CYCLE 1 STEP 2 show that the bid prices of goods are much higher in the high volume markets; the bid-ask spread is lower there. The high volume markets are more liquid.

On entering CYCLE 2 STEP 1 households recalculate their desired trades. Those who have been trading on {N/2, 1} and on {m,1} find that trade on these markets has become even more attractive since the bid-ask spreads have narrowed. Those who had been trading on {N/2,m} face a quandary: goods N/2 and m are the goods that they want to trade, but trading indirectly through good 1 in {N/2, 1} and {m,1} may be a lower cost alternative. $q^{\{m,1\}}_{m}\cdot q^{\{N/2,1\}}_{1}=.56>.5=q^{\{m,N/2\}}_{m}$. Household [m,N/2] can get more N/2 for his m by trading indirectly through the markets with good 1, and household [N/2,m] can get more m for his N/2 by trading indirectly through the markets with good 1. They decide to trade through good 1. Good 1 is beginning to take on the character of money.

The transformation of good 1 into money is not complete however. Household [m,n] for $m\neq N/2\neq n$ considers but does not adopt indirect trade through good 1. He calculates $q^{\{m,1\}}_{m}\cdot q^{\{n,1\}}_{1}=.49<.5=q^{\{m,n\}}_{m}$. Household [m,n] still gets a better deal trading directly good m for n.

CYCLE 2, STEP 1:

- For $[m,n]\in H^1$, $m,n\neq N/2$, $m,n\neq 1$, $s^{[m,n]\{m,n\}}_{m}= A$, $b^{[m,n]\{m,n\}}_{n}= Aq^{\{m,n\}}_{m}$; all other purchases and sales are nil.

For [m,N/2], $m\neq 1$, $s^{[m,N/2]\{m,1\}}_{m}= A$, $b^{[m,N/2]\{m,1\}}_{1}= Aq^{\{m,1\}}_{m}$, $s^{[m,N/2]\{1,N/2\}}_{1}=Aq^{\{m,1\}}_{m}$, $b^{[m,N/2]\{1,N/2\}}_{1}= Aq^{\{m,1\}}_{m}q^{\{N/2,1\}}_{1}$; all other purchases and sales are nil.

For [N/2,n] , $n\neq 1$, $s^{[N/2,n]\{N/2,1\}}_{N/2}= A$, $b^{[N/2,n]\{N/2,1\}}_{1}= Aq^{\{N/2,1\}}_{N/2}$, $s^{[N/2,n]\{1,n\}}_{1}=Aq^{\{N/2,1\}}_{N/2}$, $b^{[N/2,n]\{1,n\}}_{n}= Aq^{\{N/2,1\}}_{N/2}q^{\{1,n\}}_{1}$; all other purchases and sales are nil.

For [m,1] , $m\neq N/2$, $s^{[m,1]\{m,1\}}_{m}=2A$, $b^{[m,1]\{m,1\}}_{1}= 2Aq^{\{m,1\}}_{m}$; all other purchases and sales are nil. For [1,n] , $n\neq N/2$, $s^{[1,n]\{1,n\}}_{1}=2A$, $b^{[1,n]\{1,n\}}_{n}= 2Aq^{\{n,1\}}_{1}$; all other purchases and sales are nil.

For [N/2,1], $s^{[N/2,1]\{N/2,1\}}_{N/2}= 3A$, $b^{[N/2,1]\{N/2,1\}}_{1}= 3Aq^{\{N/2,1\}}_{N/2}$. For [1, N/2], $s^{[1,N/2]\{N/2,1\}}_{1}= 3A$, $b^{[1,N/2]\{N/2,1\}}_{1}= 3Aq^{\{N/2,1\}}_{1}$.

- For $h\in H^2$, for each $\{1,j\}$, $b^{h\{1,j\}}_{j}=\gamma=s^{h\{1,j\}}_{N}$; for each $\{i,j\}$ so that $1\neq j\neq N/2\neq i\neq 1$, $b^{h\{i,j\}}_{j}=A/2=s^{h\{i,j\}}_{N}$; all other $b^{h\{i,j\}}_{j}$ and $s^{h\{i,j\}}_{j}$ are nil. In particular $b^{h\{i,N/2\}}_{i}$ and $s^{h\{i,N/2\}}_{N}$ are nil.

STEP 2:

- For $\{m,n\}$ where $m\neq 1\neq n$, $1=q^{\{m,n\}}_{(m)N}= q^{\{m,n\}}_{(n)N}$, $q^{\{m,n\}}_{m}=q^{\{m,n\}}_{n}=({}^1/_2)$.
- For $\{m,1\}$, $m\neq N/2$, $1=q^{\{m,1\}}_{(1)N}=q^{\{m,1\}}_{(m)N}$, $q^{\{m,1\}}_{m}=q^{\{m,1\}}_{1}=\dfrac{3A-\gamma}{3A}=.80$
- For $\{N/2,1\}$, $1=q^{\{N/2,1\}}_{(N/2)N}=q^{\{N/2,1\}}_{(1)N}$, $q^{\{N/2,1\}}_{N/2}=q^{\{N/2,1\}}_{1}=\dfrac{NA+(N-3)Aq^{\{m,1\}}_{m}-\gamma}{NA+(N-3)Aq^{\{m,1\}}_{m}}>.95$

As CYCLE 2 STEP 1 is completed, trade has become partially monetized. All trade in good N/2 goes through good 1 as a medium of exchange. As STEP 2 is completed, prices reflect the higher trading volumes on markets including 1. Going into CYCLE 3 STEP 1, typical [m,n] for $1\neq m\neq N/2\neq n\neq 1$, can reconsider whether to trade in goods m and n directly or to trade through good 1 as a medium of exchange. In order to make that decision he compares $q^{\{m,n\}}_m$ to the product $q^{\{n,1\}}_1 \cdot q^{\{m,1\}}_m$. The former is the value of m in terms of n in direct trade, the latter through trade mediated by good 1. This is the same comparison [m,n] made at CYCLE 2 STEP 1, and decided to continue to trade directly. But at the new posted prices we have $.5 = q^{\{m,n\}}_m < .64 = q^{\{n,1\}}_1 \cdot q^{\{m,1\}}_m$. It is more advantageous to trade indirectly. The outcome of CYCLE 3 STEP 1 will be full monetization; all trade will go through good 1.

CYCLE 3, STEP 1:

- For $[m,n] \in H^1$, $m,n\neq 1$, $s^{[m,n]\{m,1\}}_m = A$, $b^{[m,n]\{m,1\}}_1 = Aq^{\{m,1\}}_m$, $s^{[m,n]\{1,n\}}_1 = Aq^{\{m,1\}}_m$, $b^{[m,n]\{1,n\}}_n = A(q^{\{m,1\}}_m q^{\{n,1\}}_1)$; all other purchases and sales are nil.
- For $[m,1] \in H^1$, $m\neq 1$, $s^{[m,1]\{m,1\}}_m = 2A$, $b^{[m,1]\{m,1\}}_1 = 2Aq^{\{m,1\}}_m$; all other purchases and sales are nil. For $[1,n] \in H^1$, $n\neq 1$, $s^{[1,n]\{1,n\}}_1 = 2A$, $b^{[1,n]\{1,n\}}_n = 2Aq^{\{1,n\}}_1$; all other purchases and sales are nil.

For [N/2,1], $s^{[N/2,1]\{N/2,1\}}_{N/2} = 3A$, $b^{[N/2,1]\{N/2,1\}}_1 = 3Aq^{\{N/2,1\}}_{N/2}$. For [1, N/2], $s^{[1,N/2]\{N/2,1\}}_1 = 3A$, $b^{[1,N/2]\{N/2,1\}}_1 = 3Aq^{\{N/2,1\}}_1$.

- For $h \in H^2$, for each {i,j} with $i\neq 1\neq j$, all transactions are nil. For {1,j}, $2\leq j \leq N-1$, $b^{h\{1,j\}}_j = \gamma = s^{h\{i,j\}}_N$.

STEP 2:

- For {m,n} where $m\neq 1\neq n$, $1 = q^{\{m,n\}}_{(m)N} = q^{\{m,n\}}_{(n)N}$, $q^{\{m,n\}}_m = q^{\{m,n\}}_n = (^1/_2)$.
- For {m,1}, $m\neq N/2$, $1 = q^{\{m,1\}}_{(1)N} = q^{\{m,1\}}_{(m)N}$, $q^{\{m,1\}}_m = q^{\{m,1\}}_1 = \dfrac{(N-1)A+(N-3)Aq^{\{m,1\}}_m - \gamma}{(N-1)A+(N-3)Aq^{\{m,1\}}_m} > .914$
- For {N/2,1} , $1 = q^{\{N/2,1\}}_{(N/2)N} = q^{\{N/2,1\}}_{(1)N}$, $q^{\{N/2,1\}}_{N/2} = q^{\{N/2,1\}}_1 = \dfrac{NA+(N-3)Aq^{\{m,1\}}_m - \gamma}{NA+(N-3)Aq^{\{m,1\}}_m} > .952$

CYCLE 4, STEP 1: Repeat Cycle 3, Step 1
STEP 2: Repeat Cycle 3, Step 2
CONVERGENCE.

What's happening in Example IV? Preferences and endowments are structured so that at roughly the same prices for all goods, there is a balance between supply and demand. Some pairs of goods are more actively traded than others. Good 1 has approximately twice as much active demand (and supply) than most other goods. Good N/2 has slightly more active trade than most other goods, and that active trade is concentrated in a supplier who demands good 1 and a demander endowed with good 1.

Here's how trade takes place. The starting point is a barter economy, the full array of (N-1)(N-2)/2 trading posts. For every pair of goods (i,j), where $1\leq i,j \leq N-1$, there is a post where that pair can be traded. The starting prices are chosen (somewhat arbitrarily) to cover average costs at low trading volume. The bid-ask spread is uniform across trading posts so trade at each post is as attractive as anywhere else. Then each household computes its demands and supplies at those prices. It figures out what it wants to buy and sell and to which trading posts it should go to implement the trades. Since all bid-ask spreads start out equal, each household just goes to the post that trades in the pair of goods that the household wants to exchange for one another; demanders of good j who are endowed with good i go to {i,j}. Because of the distribution of demands and supplies, there is twice the trading volume on posts {1,j} than on most {i,j} and three times as much on {1,N/2}.

Then the average cost pricing auctioneer responds to the planned transactions. He prices bid/ask spreads in all markets to cover the costs of the trade on them. Since there is a scale economy in the transactions technology, this leads to slightly narrower bid/ask spreads on the {1,j} markets and an even narrower spread on the {1, N/2} market. The auctioneer announces his prices.

Households respond to the new prices. Households who want to buy or sell good N/2 discover that the bid/ask spread on market {1, N/2} is lower than on any other market trading N/2. It makes sense to channel transactions through this low cost market, even if the household has to undertake additional transactions to do so. Ordinarily households [i,N/2] and [N/2,i] would have gone directly to the market {i,N/2} to do their trading. But the combined transaction costs on {i,1} and on {1,N/2} are lower than those on {i,N/2}. Households [i,N/2] and [N/2,i] find that they incur lower transaction costs by trading through good 1 as an intermediary. They exchange i for 1 and 1 for N/2 (or N/2 for 1 and 1 for i) rather than trade directly. The market makers on the many different {i,1} markets, $2 \leq i \leq N-1$, find their trading volumes increased as the [i,N/2] and [N/2,i] traders move their trades to {i,1} and {N/2,1}.

The average cost pricing auctioneer responds to the revised trading plans once again. Bid-ask spreads decline on {i,1}, $2 \leq i \leq N-1$. Now the bid-ask spreads on {i,1} are less than half those on {i,j} for $i \neq 1 \neq j$. The auctioneer announces his prices.

Households respond to the new prices. For all households [i,j], it is now less expensive to trade through good 1 as an intermediary than to trade directly i for j or j for i. All [i,j] now trade on {i,1} and {j,1}; none trade on {i,j}, for $i \neq 1 \neq j$. Trade is fully monetized with good 1 as the 'money.'

The average cost pricing auctioneer re-prices the markets. Inactive markets, {i,j} for $i \neq 1 \neq j$, necessarily continue to post their starting prices (which reflected anticipated low trading volume). The active markets {i,1} get posted prices reflecting their high trading volumes, with narrow bid-ask spreads.

Households review the newly posted prices. The narrow bid-ask spreads on the {i,1} markets reinforce the attractiveness of their previous plans, which called for trading through good 1 as an intermediary. They leave their monetary trading plans in force. At current prices, it is much more economical to trade i for j by first trading i for 1 and then 1 for j than to trade i for j directly. High trading volumes on the {i,1} and {j,1} markets ensure low transaction costs and keep them attractive. All trade takes place at {i,1}, i=2,3,4, ..., N-1. Good 1 has become the unique 'money'.

The economy of Example IV comes to precisely the opposite outcome if $\gamma^i > NA$ for all i. Then there are no transaction cost scale economies in the relevant range. The equilibrium will be nonmonetary direct barter trade.

V. Conclusion

Examples IV and III.3 demonstrate the following conception of the monetization of the transactions process. Scale economies in the transactions technology mean that high volume markets will be low average cost markets. The transition from barter to monetary exchange is the transition from a complex of many thin markets --- one for trade of each pair of goods for one another to an array of a smaller number of thick markets dealing in each good versus a unique common medium of exchange. This transition is resource saving if the scale economies in transactions technology are large enough.

In section IV, with full double coincidence of wants, the emphasis is on scale economies alone; high volume imparts liquidity and monetization. The example shows that the transition progresses through individually rational decisions when prices reflect the scale economy and the initial condition includes one good (the latent 'money') with a relatively high transaction volume (hence low average transaction cost). Then, as Einzig notes, "favourite means of barter are apt to arise" and a barter economy thus converges incrementally to a monetary economy.

Conversely, section III emphasizes absence of double coincidence of wants. This setting necessarily generates higher trading volumes to fulfill budget balance (T.ii) and achieve an equilibrium allocation. A demand for carriers of purchasing power between trading posts endogenously arises and focuses on low transaction cost goods, the 'natural' moneys. Absent scale economy there is no impetus for uniqueness of 'money.'

Scale economy is not a necessary condition for uniqueness of the medium of exchange in equilibrium (Example III.1), but scale economy helps to ensure uniqueness. If there is a unique low transaction cost instrument in an economy with a linear transaction cost structure, that natural money may be the unique medium of exchange in equilibrium. If there are many equally low cost candidates for the medium of exchange, scale economy in transaction costs will allow one to be endogenously chosen as the unique medium of exchange. Inherent low cost and market determined high volume combine to yield unique monetization. Menger (1892) describes this transition:

> when any one has brought goods not highly saleable to market, the idea uppermost in his mind is to exchange them, not only for such as he happens to be in need of, but...for other goods...more saleable than his own...By...a mediate exchange, he gains the prospect of accomplishing his purpose more surely and economically than if he had confined himself to direct exchange...Men have been led...without convention, without legal compulsion,...to exchange...their wares...for other goods...more saleable...which ...have ...become generally acceptable media of exchange.

Thus, Menger argues that starting from a relatively primitive market setting, some goods will be more liquid than others. As they are adopted as media of exchange, markets for trade in them versus other goods become increasingly liquid. Eventually they become the common media of exchange in equilibrium. Examples III.3 and IV formalize this argument emphasizing that the increasing liquidity develops endogenously as a result of scale economy in the transaction process.

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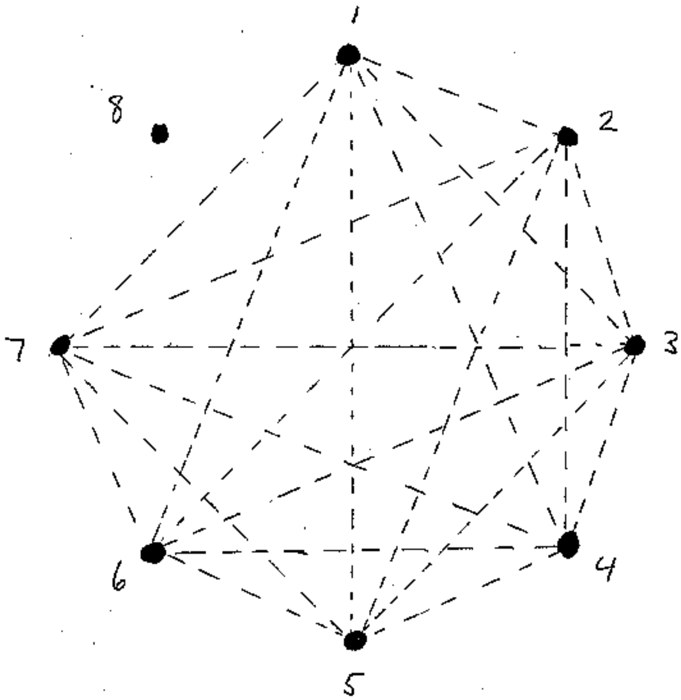
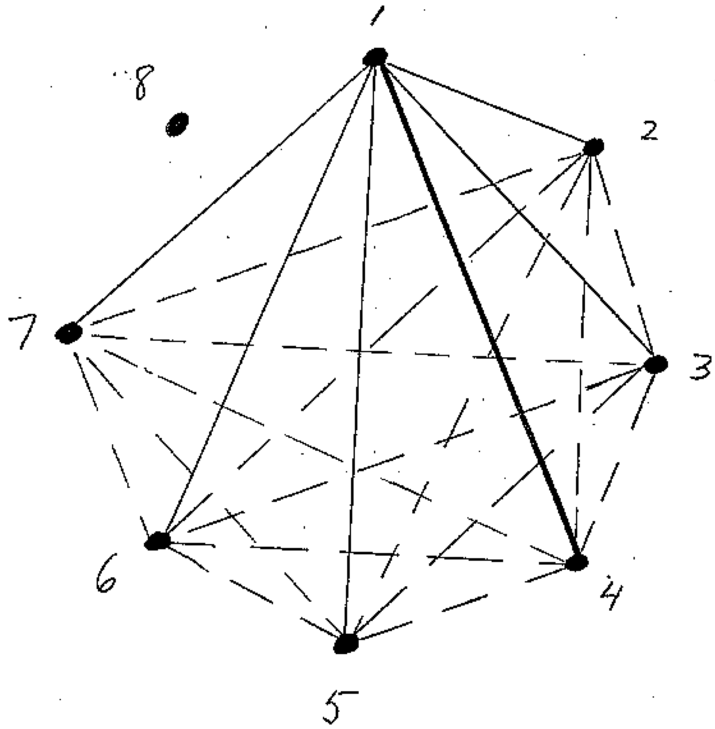
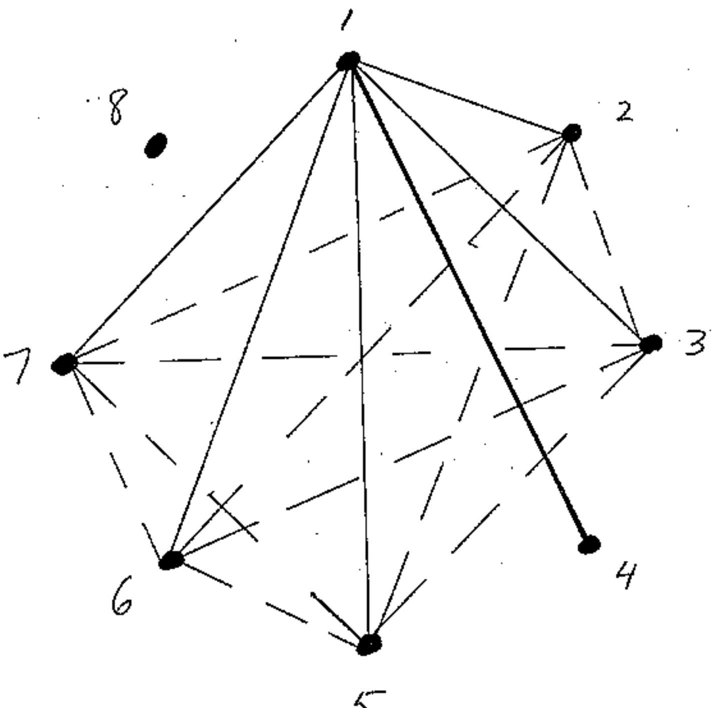


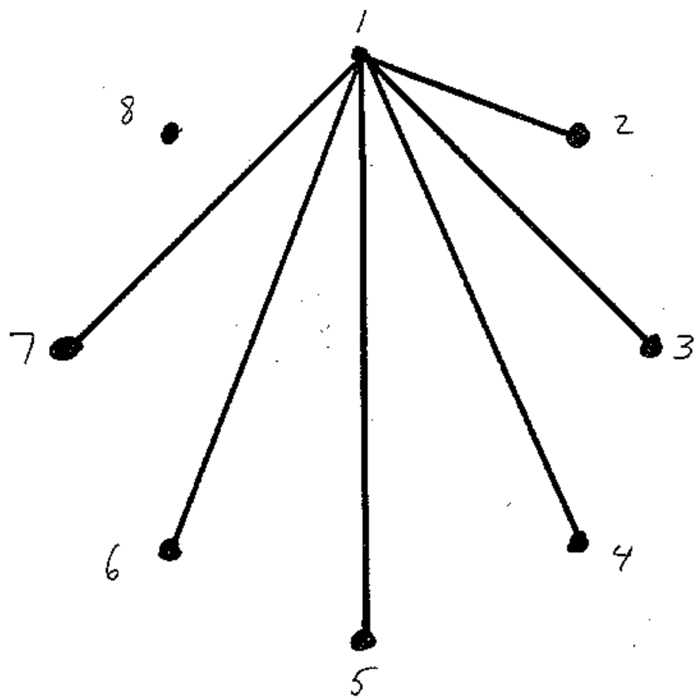
Barter Economy: Step 0



Barter Trade : Cycle 1



Partial Monetization: Cycle 2



Monetary Economy: Cycle 3

Heavy chord=High Volume Market
Solid chord=Moderate Volume Market
Broken chord=Low Volume Market
No chord=Inactive market

Figure 1