HOW ROBUST ARE NOMINAL WAGE RIGIDITIES?

by

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Abstract: Several studies indicate that firms are reluctant to cut nominal wages during periods of relatively high nominal per capita GDP growth. It has been argued, however, that in an environment with a low nominal per capita GDP growth, i.e., when nominal wage cuts become customary, firms would no longer hesitate to cut nominal pay. To examine this argument we use data from Switzerland where nominal GDP growth has been very low between 1991 and 1997. *It turns out that the rigidity of nominal wages is a robust phenomenon that does not vanish but even increases as inflation decreases. Nominal wage rigidity constitutes a considerable obstacle to real wage adjustments.* Our estimates indicate that wage rigidity is almost complete for full-time workers who stay with the same employer, but we find little evidence of nominal rigidities for workers who switch employers. We also find evidence that, in the absence of downward nominal rigidity, real wages would indeed be quite responsive to unemployment.

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1. Introduction

One of the core questions of modern macroeconomics is whether and through which channels nominal demand shocks affect output and employment. In this context, the inertia of nominal wages plays a key role. The extent and the nature of downward nominal wage rigidity are likely to have strong implications for the real effects of nominal demand disturbances. In recent years there has been a renewed interest in the question of nominal wage rigidity (e.g., Akerlof, Dickens, and Perry, 1996; Card and Hyslop 1996, 1996; Kahn 1996; Lebow, Stockton, and Washer, 1995; McLaughlin, 1994; Altonji and Devereux, 1999; Smith, 1998), and, in particular, whether there are forces that prevent nominal wage cuts.

Below, we will argue that there are serious reasons – the presence of efficient nominal wage contracts, the existence of nominal loss aversion and nominal fairness standards – that may prevent firms from implementing nominal pay cuts. Most of the above cited evidence indicates that nominal pay cuts are indeed relatively infrequent. In this sense, the evidence is consistent with the assertion of nominal wage rigidity. A major counter-argument is, however, that the infrequency of nominal pay cuts is due to the fact that nominal per capita GDP growth and, hence, average nominal wages were rapidly rising during the periods considered by these authors (Gordon, 1996; Mankiw, 1996). At the same time, a considerable fraction of workers receives small real wage cuts. If, instead, nominal per capita GDP growth were low for a number of consecutive years, nominal pay cuts would be much more frequent and would, hence, no longer be viewed as something special. After all, individuals were prepared to take real wage cuts in previous years. As a consequence, fairness standards and the reference points used to measure losses would adjust so that firms would no longer be reluctant to cut nominal pay.\footnote{This argument is neatly summarised by Gordon (1996, p. 62): "The … attempt to reason from evidence on nominal wage rigidity in an environment of rapid \textit{positive} average nominal wage change to a hypothetical situation of \textit{zero} average nominal wage change is subject to the Lucas critique. If the macroeconomic environment were different, microeconomic behavior would be different. Nominal wage reductions would no longer be seen as unusual if the average nominal wage was not growing. Workers would not see them as unfair, and firms would not shy away from imposing them." See also the recent edition of \textit{The Economist} (1999), who makes the same point.}

This paper examines whether nominal wage rigidities disappear in an environment of very low nominal per capita GDP growth. For our purposes the data sets used in the above-
mentioned papers have the disadvantage that nominal GDP growth per capita was substantial. In this paper we take advantage of the low inflation during a period of almost zero real GDP growth in Switzerland between 1991 and 1997. Switzerland experienced inflation and growth rates close to zero in several consecutive years. Note that both low inflation and low real GDP growth provide an ideal environment for the study of the above counter-argument. Low real GDP growth implies that average real wage growth is moderate. Inflation rates that are close to zero mean that real wage growth frequently would have to take the form of nominal wage cuts. Thus, if fairness standards and reference points indeed adjust to this environment, we should find no, or at least evidence of fading, nominal wage rigidity.

Previous examinations of nominal wage rigidity are based on survey data. Survey data contain measurement error which makes the detection of nominal rigidity difficult. Some authors (e.g., Akerlof, Dickens and Perry, 1996) have argued that measurement error hides nominal rigidity while others (e.g., Smith 1999) claim that measurement error gives rise to spurious nominal inertia. In this paper we tackle the issue of measurement error in two ways. First, we are in the fortunate position to use two independent data sources that provide information about individual wages between 1991 and 1997. One of these data sources is a random sample from the Swiss Social Insurance File (SIF), which records all financial transactions between workers and their firms. The SIF contains accurate earnings information. Second, our econometric analyses take into account the possibility of measurement error explicitly. Based on the method developed by Altonji and Devereux (1999) we estimate the amount of measurement error in a parametric way. This allows us to separate the effects of nominal wage rigidity from the other determinants of real wage changes and from measurement error. The method of Altonji and Devereux relies on the intuition that there may be obstacles to nominal wage cuts that prevent firms from implementing all desired wage cuts. In particular, if the desired wage cut, that is defined as the wage cut that would obtain in the absence of downward rigidity, is sufficiently small firms will not implement the cut. We motivate the use of this method with evidence from personnel records of two large firms in Switzerland. Wage cuts are very rare in these records and there is no tendency towards more wage cuts.

Our results indicate that this is also the case in the two representative data sources. More importantly, the reluctance to cut nominal wages is a robust phenomenon: The fall in inflation from roughly 5 percent in 1991 to zero in 1997 is accompanied with a considerable rise in downward wage rigidity. According to the conservative estimates from our SIF sample the fraction of workers who did not experience nominal cuts due to downwardly rigid wages rose
from 25 percent in 1991 to roughly 60 percent in 1997. For this group of workers the presence of nominal rigidity prevented real wage cuts of 6.3 percent in 1991 and of 13.3 percent in 1997. Thus, if anything, nominal inertia became more important as inflation decreased and that the Swiss experience casts doubt on the conjectures of Gordon and Mankiw (1996). We believe that this result is important for a further reason. The Swiss labor market is one of the least regulated and least unionized labor markets in Europe. In Switzerland employers are allowed to renegotiate a contract, e.g. demand a wage cut, with explicit threat of dismissal. Unless the worker agrees, the employment relation ends. But this implies that wage rigidities due to the presence of efficient nominal contracts cannot be important (Malcomson, 1997).

The strength and persistence of downward wage rigidity are surprising and point to the importance of the underlying behavioral forces. According to our most conservative estimate, the desired wage cut must exceed 15 percent to induce firms to actually cut the workers’ nominal pay. If the desired wage cut is less, workers receive wage freezes instead. Our least conservative estimate indicates, however, that the threshold value for cutting nominal pay is 36 percent. Moreover, we find significant differences between full-time and part-time employees, and little downward rigidity for workers who switch employers. Nominal rigidity is more pronounced for full-time workers: At most 3.3 percent receive wage cuts and between 45 and 60 percent of the full-time workers receive wage freezes instead of cuts. In contrast, part-time workers’ wages are more flexible and at least 11 percent receive wage cuts. The sizeable differences between job stayers and job movers and between part-time workers and full-time workers lend further credibility to our results because, as we argue in more detail below, the behavioral forces that give rise to nominal rigidity are likely to be most important for full-time stayers.

We show that in the absence of downward rigidities, real wages would indeed be quite flexible in their response to unemployment. High unemployment growth reduces wage growth substantially. A one percent growth in the regional unemployment rate lowers wage growth by almost one percent, provided that it doesn’t entail a nominal wage cut.

Since downward wage rigidity has quite large an impact on real wages and the effect is not fading at lower rates of inflation, our findings have potentially important implications for monetary policy. The estimates are consistent with the view that low inflation imposes permanently higher unit labor costs, as less real wage cuts occur, and that full-time incumbents are disproportionately affected. This may lead to permanently lower overall employment and distortions between full-time and part-time jobs.
The remainder of the paper is structured as follows: Section 2 discusses potential reasons for the downward rigidity of nominal wages. Section 3 reviews previous studies that used survey data similar to our second data source and highlights the advantages of our data sets. Section 4 provides a description of the empirical pattern of wage changes in Switzerland between 1991 and 1997 and Section 5 presents the empirical model of wage changes. Section 6 presents the results and Section 7 concludes the paper.

2. Reasons for Nominal Wage Rigidity

In a frictionless economy with zero costs of replacing workers and fully optimizing agents nominal wage rigidity does not occur. In this economy employers can always credibly threaten to replace incumbents who are unwilling to accept nominal pay cuts. However, in the presence of legal or economic frictions that cause positive replacement costs there are at least three reasons why nominal wage cuts cannot be enforced by the employers or why employers are unwilling to enforce them:

(i) Employers and workers may have implemented nominal wage contracts that can only be changed by mutual consent.

(ii) Nominal pay cuts are likely to be experienced as particularly painful for reasons of nominal loss aversion.

(iii) Nominal pay cuts are likely to violate standards of fairness and are thus interpreted as an insult by the employees.

In the past an important objection against nominal wage contracts has been that they are inefficient. However, as has been shown by MacLeod and Malcomson (1993) and, more recently, by Holden (1999) nominal wage contracts that can only be changed by mutual consent provide efficient protection of general and specific investments in human capital. Thus, it may well be in the interest of employers and workers to implement such contracts. The important implication of these contracts is that the nominal wage will change only if the outside option of either the employer or the worker becomes binding at the prevailing wage. However, due to the presence of specific human capital, the outside options will frequently

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2 The arguments in MacLeod and Malcomson (1993) and Holden (1999) apply to non-unionized economies. Holden (1994) shows that in a unionized economy, in which production continues under the terms of the old contract while the parties are bargaining over a new contract, nominal rigidity arises.
not be binding so that sufficiently small changes in the outside options do not affect the nominal wage. Decreases in the value of the marginal product of the worker, in particular, may not cause nominal wage decreases. These contracts require if a contract is renegotiated and one party does agree to the proposed change in terms, the old contract prevails. Switzerland is a notable exception among European countries, because Swiss law allows to change the terms of a contract under the explicit threat of dismissal otherwise. As discussed in Malcomson (1997), nominal contracts of this kind cannot be sustained in this setting.

Absent this explanation, there are at least two other explanations why wage cuts are rarely observed. The notion of loss aversion as developed by Kahneman and Tversky (1979) is based on the idea that losses and gains relative to a neutral reference point are the "carrier of subjective value" that motivates behavior. Loss aversion means that losses are psychologically more salient than gains of the same absolute size, i.e. they are experienced as particularly painful and are thus likely to trigger different behaviors than, e.g. a reduction in gains. There is ample evidence from questionnaire studies and many experimental examinations indicating the behavioral relevance of loss aversion (e.g., Tversky and Kahneman, 1991). Many people are willing to take more risks in the domain of losses (Kahneman and Tversky, 1979) and their financial decisions seem to be affected by loss aversion (Thaler and Tversky, 1996; Benartzi and Thaler 1995). Moreover, they make systematically sub-optimal decisions to avoid losses in intertemporal choice situations (Fehr and Zych, 1997), their ability to coordinate on Pareto-superior equilibria is systematically affected by the desire to avoid losses3 (Cachon and Camerer, 1996) and price formation in competitive experimental markets also seems to be significantly influenced by loss aversion (Myagkoff and Plott, 1997).

In a recent paper, Genesove and Mayer (1998) provide strong evidence for nominal loss aversion in housing markets. They show that in a given market situation, i.e., for a given expected selling price, those sellers who bought their house at a higher nominal purchase price than the prevailing expected selling price ask for substantially higher selling prices than those sellers who bought their house below the prevailing expected selling price. Moreover, it turns out that sellers who face nominal losses relative to the original purchase price do in fact sell

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3 Cachon and Camerer show that in coordination games with multiple Pareto-ranked equilibria, the subtraction of a constant number from all payoffs, so that Pareto-inferior equilibria involve losses, enables agents to avoid the play of inferior equilibria. Note that the subtraction of a constant from all payoffs leaves the game strategically unchanged. The authors show, however, that the play of inferior equilibria cannot be avoided when these equilibria do not involve losses.
their houses at higher prices. Interestingly, Genesove and Mayer also find that nominal loss aversion is not only exhibited by owner-occupants but also by professional investors in the housing market.

In our view there is no reason why workers should exhibit less nominal loss aversion than owner-occupants or investors in the housing market. From a psychological viewpoint, it seems rather likely that nominal pay cuts are interpreted as losses. The pain inflicted by these losses may well trigger resentments that induce workers to take actions (e.g. shirking or quitting) that are costly to the firm. This, in turn, may prevent firms from implementing nominal pay reductions.

Survey and experimental studies indicate that fairness standards are relevant for the behavior of employees and employers (e.g. Bewley, 1995, 1999; Blinder and Choi, 1990, Campbell and Kamlni, 1997; Fehr and Falk, 1999; Fehr, Kirchsteiger, and Riedl, 1993). Virtually all of these studies suggest that violations of fairness standards have negative effects on work morale and that firms shy away from violating these standards. There is also evidence that judgments of fairness and job satisfaction are affected by nominal pay (Kahneman, Knetsch and Thaler, 1986; Shafir, Diamond, and Tversky, 1997). This is neatly expressed by the president of a large division (32,000 employees) of an insurance company, interviewed by Bewley (1999):

"Real pay cuts (through inflation) are easier than nominal ones. Inflation is gradual. Real pay cuts give people more time to adjust than a sudden 10% cut in pay. ... Nominal pay cuts are an insult, even if everybody is cut."

Bewley collected data from 236 managers and human resource officers regarding the consequences of pay cuts. He draws a consistent picture of compensation officers unwilling to cut nominal wages: 69% agreed that nominal pay cuts hurt morale, 42% feared a direct effect on productivity, 41% asserted that it increased turnover, and some 10% feared more drastic retaliation from employees such as theft or even sabotage.

In our view the employees’ resentment in response to the experience of a nominal loss and the violation of fairness standards caused by the nominal pay reductions represent plausible forces inhibiting such cuts. However, in this context one has to take into account that both the definition and, hence, the experience of a loss and the definition of what is fair may shift in response to changes in the economic environment. Put differently: In a situation of very low average growth of nominal wages, pay cuts may become customary so that they are no longer perceived as losses or as a violation of a fairness standard. Therefore, loss aversion and
fairness considerations may no longer inhibit nominal pay cuts. This claim can only be examined with data from periods of low average nominal wage growth and that is why the Swiss experience since 1991 is of general interest.

Before we proceed further we would like to point out that the behavioral forces contributing to nominal wage rigidity are likely to be different for different categories of workers. For example, if it is indeed the case that nominal wage cuts hurt work morale it may be better for firms to fire a worker than to cut her nominal wage. It seems very likely that the new employer of the fired worker is less constrained by the impact of historically evolving fairness standards or by the worker's employment history. Therefore, it seems much easier to impose pay cuts on job movers than on job stayers. A similar argument can be made with regard to full-time and part-time workers. It seems likely that full-time workers have more specific human capital so that nominal wage contracts are probably more important for them than for part-time workers. In addition, for a firm the loyalty and work morale of full-time workers is, of course more important than the loyalty and work morale of part-time workers. Moreover, the relevance of fairness standards is likely to be more important for workers with a greater attachment to the firm. Therefore, firms are likely to be more reluctant to cut the nominal wages of full-time workers. Thus, if the behavioral forces discussed above are relevant we should observe that nominal inertia differs across these categories of workers.

3. Previous Studies

In this section, we shortly review five recent studies of wage rigidities that use data of individual wage changes from panel surveys or directly ask individuals to state the wage changes. McLaughlin (1994) presents an early study of wage rigidities in the U.S. He finds substantial variability in real wage changes, and concludes that wage changes are not skewed away from wage cuts. In his analysis, the frequency of wage cuts is unaffected by inflation. 4 Yet, other authors’ results deviate considerably from McLaughlin's study. Kahn (1997) mentions that the skewness statistics are dominated by observations far away from the median and therefore susceptible to outlier influences. She shows that in a virtually identical data set as McLaughlin's a given real wage change is less likely if it entails a nominal pay cut. She

4 Smith (1999) examines the British experience and argues that most of the observed wage rigidity is essentially an artifact of surveys and in fact produced by a very persistent form of measurement error.
finds strong evidence of nominal wage rigidities at the micro level. She does not address the issue how nominal rigidities affect real wage flexibility.

Card and Hyslop (1996) use a different technique to uncover nominal rigidities. They construct a counterfactual distribution of wage changes, i.e., a distribution of wage changes in the absence of nominal inertia. They find that, in the range of negative nominal wage changes, the difference between the counterfactual and the actual density of wage changes becomes larger as inflation declines. Hence, more individuals are affected by nominal rigidities at lower levels of inflation. Card and Hyslop also test whether low inflation leads to slower adjustment of real wages, but find no evidence for this.

Two studies have dealt explicitly with the fact that survey data may be error-ridden. Akerlof, Dickens, and Perry (1996) conduct a telephone survey and directly ask individuals to state their wage changes. They then pollute the data with a random distribution that closely resembles the distribution of measurement error from validation studies. The distribution they obtain shares many features of the distribution of wage changes obtained from labor force survey. They conclude that one cannot reject the hypothesis that the true distribution of wage changes exhibits total wage rigidity, but that this is masked by measurement error.

Altonji and Devereux (1999) develop an interesting model that explicitly allows for measurement error in a parametric way. We will give a detailed description of this model in section 5. They find that once one eliminates measurement error from the data, wages are far from flexible. In some specifications, they cannot reject the hypothesis that wages are perfectly rigid and there are no true wage cuts.

All of the above studies have in common that nominal per capita GDP growth was substantial over the period considered. The median growth rate of nominal GDP per capita is, for example, never below 6.4 percent in these studies (see Table 1). As pointed out in the introduction, it is therefore difficult to draw inferences from these studies about the behavior of individuals in an environment of low nominal growth. To examine whether nominal inertia vanishes or is reduced in an environment of low nominal GDP growth one needs a data set that is based on a prolonged period of low nominal growth. This is so because one cannot expect that nominal reference points and fairness standards adjust instantaneously to a new environment.

In this study we take advantage of the long recession with low inflation in Switzerland over the years 1991 to 1997. Table 1 shows that during this time period the median nominal per capita GDP growth is much lower (1.7 percent) than the median growth in the US during the
time periods covered by the above cited studies. In addition, Table 1 reports the maximum number of consecutive years during which nominal per capita GDP growth is below 4 percent and below 2 percent. For the time periods covered by the above studies it occasionally happened that in a single year nominal per capita GDP growth was below 4 percent. However, nominal growth rates below 4 percent never occurred in two or more consecutive years. In contrast, in Switzerland nominal growth per capita was below 4 percent during the period considered. Moreover, in three consecutive years nominal growth was even below 2 percent.

4. The Pattern of Nominal Wage Changes

We use two different data sets, each with its own features, to evaluate the extent of nominal wage rigidities: The Swiss Labor Force Survey (SLFS) 1991 – 1998 allows us to calculate changes in hourly wages for non-self employed individuals. In total, the SLFS provides 27,238 observations of wage changes, out of which 24,567 observations come from workers who stayed with the same employer\(^5\) over at least two consecutive years. The Swiss Labor Force Survey is useful for two reasons. It provides extensive information on worker characteristics. The characteristics include the usual determinants of wage growth such as tenure, labor market experience, education levels, gender, age and nationality. Second, it restores comparability with studies that used similar data sets.

The second source of data is a large random sample from the Social Insurance Files (SIF). All financial transactions between firms and workers are recorded in the Social Insurance Files. The sample covers essentially the same period of time\(^6\). We obtain a sample of 147,439 observations of wage changes over the time span. This sample has three advantages. First, measurement error is not an issue. The earnings information obtained from the SIF is accurate. Second, the sample is comfortably large. Third, since the SIF data covers the same period of time as the SLFS, we can replicate the empirical analysis we conduct with the SLFS and check whether analyses that used survey data overstated the extent of nominal wage rigidities. We should also mention that the SIF data has three problems. First, it is impossible

\(^5\) In slight abuse of terminology, we will refer to these as 'job stayers'. We are aware that this term more accurately describes employees who stay on the same job assignment with the same employer.

\(^6\) The Social Security Files are December to December data, while the SLFS is conducted in May. Henceforth, referring to wage changes in e.g. 1993, we mean wage changes between May 1993 and May 1994 for the SLFS and wage changes between December 1992 and December 1993 for the SSF.
to identify job stayers with certainty. We only include workers who were insured by the same local social insurance agency in two consecutive years since these are most likely to be job stayers. However, if an individual changes the employer, but both employers are associated with the same local agency, the individual may still be included in our sample. Thus, we may wrongly include job movers in our SIF sample, which could understate the true degree of nominal wage rigidity. Second, we have no information on hours worked and salary components. Hence, when referring to wage changes in the sample, we should bear in mind that we only observe changes in earnings per year. For instance, temporary variations in hours, which arise, e.g., through different overtime in two years, look like a ‘wage change’ in our sample. As we will illustrate below, this can generate a substantial number of false negatives in a sample with low average nominal income growth. Third, the worker characteristics we observe are not the same as in the SLFS. They include age, nationality, gender, details on the agency that recorded the payment and the period of time to which it applies.

Figure 1a summarizes the distribution of nominal wage changes of job stayers in Switzerland between 1991 and 1997. Consider first the figure on the left which displays the histogram obtained from the SLFS. It exhibits common characteristics for this type of histograms:

1. There is a spike at zero: The largest bin is the one containing no and small, but positive nominal wage changes (between zero and 2 percent).\(^7\)

2. There is an asymmetry in the distribution of wage changes. Negative wage changes are observed much less frequently than positive wage changes.

Compare this to the right panel of Figure 1a, which is based on the SIF data using identical bins. Two features deserve to be mentioned here: First, the SIF distribution is more centered on zero than the SLFS distribution. For instance, 45% of all observations in the SLFS are between zero and 10 percent, while the corresponding figure is 59% in the SIF. Second, the asymmetry is more pronounced in the SIF sample. The pile-up of observations just above zero is much heavier and the discontinuity around zero obvious. One possible explanation for the lack of centrality and asymmetry in the SLFS is the presence of measurement error.

Table 2 provides additional information on wage changes in our two data sources together with the inflation rate and real per capita GDP growth. The table shows that the sharp

\(^7\) For the exact fraction of zero wage changes see Table 2.
decrease in the rate of inflation at the beginning of the period considered is associated with more observed wage cuts in the labor force survey and more zero wage changes. However, the fraction of observed wage cuts is almost always smaller in the SIF sample and the same holds for the fraction of zero wage changes. This again suggests that measurement error is important in the labor force survey: Imagine that the distribution of true wage changes has no, or only few, negative entries. Assume further that measurement error is important. Then, as the distribution moves closer to zero over time, measurement error creates a larger number of negative observations. Therefore, we measure more wage cuts in the SLFS sample. Note that the fact that we cannot control for hours variation in the SIF sample only strengthens this argument because it is likely to produce false negatives in this sample, a point to which we return below.

Figure 2 shows the evolution of the distribution of wage changes over time, using the SIF sample. The sequence of distributions conveys the impression that the decline in inflation is associated with a rise in downwards rigidity. Consider, first, the three panels for 1991, 1992, and 1993. In these years the distribution is (locally) symmetric around its median. The bins to the left and the right of the median are of similar size. Compare this to the distribution of wage changes in the low inflation years 1995 to 1997, where the median is much closer to zero. Now compare the bins to the left and right. The distribution is squished at zero and exhibits a pronounced asymmetry around it. However, there is only a relatively small increase in the frequency of negative wage changes, while the cluster to the right of zero is getting much larger.

To summarize the above discussion, we claim the following:

1. Measurement error in the SLFS is important and contributes to understating the true extent of downward nominal rigidity.

2. Although we observe many earnings decreases in the SIF, these most likely reflect temporary variations in hours and not true wage cuts

The validity of these claims is essential to the empirical approach that we discuss below. We obtained personnel records from two large firms in Switzerland to corroborate further evidence for the two claims. Firm A is a large firm in the service industry with approximately 10,000 employees. The personnel records go from 1993 to 1999. Average wage growth in Firm A was 3.8% (standard deviation: 5.3 percent). Firm B is a medium size firm that is in the service industry and has a (declining) branch in manufacturing. The records start in 1984 and
end in 1999. Employment per year drops from about 2000 in 1990 to 1000 in 1998, from where it started to rise again\(^8\). Wages grew on average by 5.7% (standard deviation: 5%).

Figure 1b displays the distribution of wage changes in the two firms, where we chose the X-scale to preserve comparability with figure 1a. There are two striking features in figure 1b. First and foremost, there are very few wage cuts. In Firm A (N=35,779), only 1.7% of all observations are wage cuts. In Firm B (N=20,236), the fraction is even lower (.4%). Both distributions exhibit a discontinuity at zero that could hardly be more pronounced and need not be discussed any further. Second, the distribution of wage changes in both firms is much more centered than those in the companion figure 1a. While in both firms, 89% of all observations are between zero and 10 percent, the same statistic is only 59% for the SIF and 45% for the SLFS. Both facts are consistent with the first claim. Adding a (symmetric) random disturbance calibrated to match the distribution of measurement error, as e.g. in Bound and Krueger (1991), to any of the two histograms would create false negatives and make the distribution less centered. In short, it would produce a figure much like the histogram of wage changes from the SLFS.

We provide evidence for the second claim in figure 3. The first panel reproduces the distribution of wage changes in Firm B, but only for the period 1993 to 1998. We constrain the sample, because information on overtime payments are only available over this period of time. In the second panel, we intentionally inject these overtime payments into the salaries to calculate 'polluted' wage changes as we would observe them in the SIF, other things being equal. The distribution of 'wage' changes in the second panel now contains a sizeable fraction of negatives (8%) and is less centered than the true distribution around zero. The fraction of observations between zero and 10 percent declines to 77%. While this does not replicate the moments of the SIF sample perfectly, it goes long way and presents evidence for the second claim. Keep in mind that average salary growth is high in firm B, hence any random disturbance that we add would generate more negatives in a low-growth firm.

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\(^8\) The reason is that Firm B closed its manufacturing plants, which was accompanied with a large employment decrease, many of which were dismissals.
5. An Empirical Model of Wage Changes

The previous discussion added to the plausibility of our two claims and the subsequent treatment of these features. Thus, measurement error can generate wage decreases in the SLFS data. Likewise, unobserved variation in working hours could be the source of the increased frequency of negative earnings changes in our SIF data. However, the raw observations in Table 2 could also represent true wage cuts per working time unit. Therefore, we need an econometric model that allows explicitly for the presence of measurement error so that one can separate true wage changes from wage changes that merely reflect measurement error. The method developed by Altonji and Devereux (1999) delivers exactly this product.

The general idea behind this method is the following: Due to the reasons outlined in section 2 nominal wage cuts are perceived as costly by the firms. Therefore, firms will not implement all desired wage cuts, i.e., there will be a difference between the desired or “notional” wage cuts and actually implemented wage cuts. However, the larger the desired wage cut the more likely it is that the benefits will outweigh the costs. Hence, there may exist a critical value \( \alpha \), below which the firm will be unwilling to cut the wage but above which the benefits outweigh the costs. Our main focus is on estimating this parameter. Nominal inertia may, however, not only prevent wage cuts. It may also reduce the wage cut relative to the desired cut. The model below also allows for this possibility. To achieve our goals, we need a complete model of wage changes that also incorporates the determinants of wages in the absence of rigidities and allows for unobserved variation in the data. We can then estimate all the parameters of interest jointly.

The general structure of the estimated model is as follows:

\[
\Delta y_{it} = \begin{cases} 
  x_{it} 'b + e_i + m_i & \text{if } x_{it} 'b + e_i \geq 0 \\
  m_i & \text{if } -\alpha \leq x_{it} 'b + e_i < 0 \\
  x_{it} 'b + \lambda + m_i & \text{if } x_{it} 'b + e_i < -\alpha 
\end{cases} \tag{1}
\]

where \( \Delta y_{it} \) is the observed log wage change of individual \( i \) in period \( t \), \( x_{it} 'b + e_i \) is the desired wage change that would be implemented in the absence of downwards nominal wage rigidity, \( x_{it} \) is a set of observable variables, \( e_i \) represents unobserved change in productivity, and \( m_i \) is unobserved variation in the data, which will be measurement error in the SLFS and temporary hours in the SIF. \( \lambda \) measures the extent to which wage cuts, if implemented, are constrained. As modeled above, \( \lambda \) is simply a constant reduction of the wage cut relative to
the desired cut. In the absence of nominal inertia and measurement error the determinants of wage changes are solely given by \( x_{it} \). In our empirical estimates below \( x_{it} \) contains variables like labor market experience, age, tenure and observable skills of worker \( i \). In addition, we included the firm size, the change in the regional unemployment rate, year dummies and a dummy that captures the foreigner status of \( i \). However, if nominal inertia and measurement error are potentially important, the empirical model must take this into account. Therefore, observed wage changes can fall into one of the following three regimes:

(i) If the desired nominal wage change is positive there are no forces that inhibit this wage change, i.e., we observe \( x_{it} \ 'b + e_i + m_a \) in the data and the likelihood of this occurring is

\[
f_{e+m}(\Delta y_a - x_{it} \ 'b \mid x_{it} \ 'b + e_a > 0)
\]

where \( f_{e+m}(\cdot) \) is the density of the sum of \( e \) and \( m \).

(ii) If \( x \ 'b + e \) falls between zero and \(-\alpha\), the firm will not cut the worker’s wage but give him a pay freeze instead. The observed ‘wage change’ is then entirely due to unobserved variation. Hence the likelihood of falling in this regime only depends on the distribution of \( m \) and is given by

\[
f_m(\Delta y_a - x_{it} \ 'b \mid -\alpha < x_{it} \ 'b + e_a < 0)
\]

(iii) If the desired wage cut is larger than \( \alpha \), the firm will implement the wage cut although the size of the cut may be smaller than desired. The conditional density for this event is

\[
f_{e+m}(\Delta y_a - x_{it} \ 'b - \lambda \mid x_{it} \ 'b + e_a < -\alpha)
\]

Since it cannot be observed which regime generated a particular observation, the likelihood of an observation sums up to

\[
l_a = f_{e+m}(\Delta y_{it} - x_{it} \ 'b \mid x_{it} \ 'b + e_{it} > 0) \cdot \Pr(x_{it} \ 'b + e_{it} > 0) \\
+ f_m(\Delta y_a - x_{it} \ 'b \mid -\alpha < x_{it} \ 'b + e_{it} < 0) \cdot \Pr(-\alpha < x_{it} \ 'b + e_{it} < 0) \\
+ f_{e+m}(\Delta y_{it} - x_{it} \ 'b - \lambda \mid x_{it} \ 'b + e_{it} < -\alpha) \cdot \Pr(x_{it} \ 'b + e_{it} < -\alpha)
\]

We assume that \( e \) and \( m \) are i.i.d. normal and estimate the parameters by maximum likelihood. In the appendix, where we derive (2) rigorously, we also show that in this case, the conditional densities involving \( f_{e+m}(\cdot) \) take a particularly simple form, whereas they are rather complex in the general case.
One feature of this approach is that it nests both extreme cases, i.e., the cases of perfect wage flexibility and of perfect wage rigidity. As $\alpha \to 0$ there is no wage rigidity. In this case the model collapses to a simple OLS regression of $\Delta y_a$ where only the sum of $e$ and $m$ is identified. If, at the other extreme, $\alpha \to \infty$, there are no true wage cuts and the third regime drops out. Hence, the model nests both extreme cases, and any intermediate one. We would like to emphasize that the above model does not assume that observed wage cuts merely represent measurement error (or variation in hours). Instead, it provides joint estimates of the threshold value $\alpha$ and the variance of the distribution of measurement errors. If we estimate an $\alpha$ that is close to zero most observed wage cuts represent true wage cuts. However, if we estimate large values of $\alpha$ many observed wage cuts will be lower than $\alpha$ and, hence, the extent of measurement error is bigger.

A second feature is that we can estimate the determinants of $\alpha$. Instead of imposing the restriction (as in equation (1)) that $\alpha$ is the same for all workers in all years we can allow for year-specific $\alpha$’s or for different $\alpha$’s for different groups of workers. In particular, by estimating year-specific $\alpha$’s we can observe whether $\alpha$ is lower in low-inflation years that provide direct evidence for the validity of the conjecture put forward by Gordon and Mankiw (1996). Finally, by allowing variations of $\alpha$ across different categories of workers we can also examine, e.g., whether $\alpha$ is different for full-time and part-time workers or for job stayers and job movers.

6. Results

In this section, we discuss the results obtained by estimating the previously outlined model. We first present the overall tests for the presence of downward nominal wage rigidity. We then evaluate the stability of these estimates as inflation gets very low. Finally, we assess the implications of the model for different types of workers and the extent to which downward wage rigidity prevents real wage cuts.

We estimate the basic model (1) in two versions. In the ‘continuous' model, we assume that the unobserved variation in the data is normally distributed. That is, everybody makes mistakes when reporting earnings to a labor force survey, and everybody's hours vary. In the ‘mixed’ model we allow for the case that, in every year, a fraction $p$ (that will be estimated) of all individuals has no unobserved variation, but that the rest of the sample draws a normally distributed error. This amounts to saying that in the SLFS, a fraction $p$ of all respondents
states the correct income, but the rest makes normally distributed errors. In analogy, in the SIF sample, a fraction \( p \) of all individuals has no variation in hours in that particular year.

The key difference between the continuous and the mixed model is the way the observations with \( \Delta y_i = 0 \) are treated. A realization of zero has probability zero in a normal distribution. Hence, the mixed model treats all observations with an observed wage freeze as unaffected by the distribution of \( m \) and, hence, does not use them to estimate its parameters.

The mixed model is appropriate for the SIF sample since we know from several other data sources (e.g. the SLFS) that many people did not work overtime during the previous year. Thus, for a fraction of the people in the SIF sample an observed earnings change of zero represents a true wage freeze. However, the mixed model is more problematic to use with the SLFS, if rounding by respondents causes many of the observations with a zero wage change, as claimed e.g. in Smith (1999). The mixed model will treat these observations as true wage freezes, whereas they could also be small wage increases which are subject to a rounding error. Hence, the mixed model will tend to overstate the extent of nominal rigidities if rounding is a serious problem. This is not the case in the continuous model, which treats all observations around zero alike. If rounding is symmetric, the continuous model will not be biased in any particular direction. Hence, by comparing the estimates across both methods and both data sets, we gain insights on how important rounding errors are and by how much they affect our conclusions.

### 6.1 Are Wages Flexible?

The basic results for both samples and models are displayed in Table 3. Consider first the estimates from the SLFS. We find strong evidence for nominal wage rigidities in both, the continuous and mixed model specifications. \( \alpha \) is large and estimated with a tiny standard error. In both models, the drop in productivity must be substantially larger than 20 percent in order to induce the firm to cut its employee’s wage.

The underlying wage growth function displays substantial wage flexibility if wage setting is not constrained by downward rigidity. We find a declining experience profile for wage growth. In order to avoid awkward polynomials we only include log experience to capture the curvature. The estimated coefficient is negative and highly significant. Increasing labor market experience from one to ten years decreases wage growth by 2.1 percent. We also find evidence for a wage curve, but one that allows for nominal rigidities. The estimated coefficient on the change in regional unemployment is also significant. It suggests that if
regional unemployment rises from two to four percent, as it frequently happened over the course of one year during our sample period, wage growth is reduced by 1.6 percent. Our result differs from the ‘classical’ wage curve found by Blanchflower and Oswald (1994) in an important respect. Our model suggests that unemployment has a strong effect on wages, but only in years where average wage growth is high enough. Unemployment growth reduces wage growth only if it doesn't require a nominal pay cut. Hence, wages would indeed be quite flexible in the absence of nominal wage rigidities.

The extent of measurement error in our survey data is substantial although it is lower than expected. Our estimate of the standard deviation of the measurement error in the sample \( (\sigma_m) \) is roughly 7 percent. This is quite low compared to what validation studies of labor force surveys found (see, e.g., Angrist and Krueger (1999)). The standard errors obtained from validation studies are never below 10 percent, and sometimes considerably larger.

Notice also that the two different specifications of the measurement error do not alter the qualitative conclusions with respect to the extent of nominal wage rigidities. Previous studies (McLaughlin, 1994; Smith, 1999) have argued that the apparent rigidity of wages is largely due to rounding by individuals. We find little evidence for this claim. The estimated \( \alpha \) is 23.4 and 28.1 for the continuous and the mixed model, respectively. The difference is less than five percentage points. This is compatible with the view that rounding may have some effect but that this effect is small relative to the overall extent of nominal rigidity. Finally, we find evidence that if firms cut wages, they do not go all the way. The point estimate of \( \lambda \) suggests that if a wage cut occurs, it is lower than the actual productivity drop.

If we turn to the estimates obtained from the SIF sample, which are reported in the third and fourth column of table 3, we find again a large and highly significant \( \alpha \). The point estimates range from 0.15 for the continuous model to 0.2 for the mixed model where we included \( \lambda \). All models reject both, perfect flexibility and perfect rigidity. Notice also that the effects of unemployment on wages are very similar to the SFLS sample. Again, the point estimate of the coefficient on the (percentage point) change in regional unemployment is negative and highly significant. It implies that, in the absence of nominal wage rigidities, a rise in the unemployment rate from two to four percent would reduce wage growth by 1.8 percent. This supports our previous conclusion based on the survey data that nominal wages would be quite flexible and responsive to unemployment in the absence of nominal wage rigidities.

Unobserved variation in hours have a standard deviation of just a bit less than 4 percent, which seems plausible. As with the SLFS, the estimated \( \alpha \) is larger in the mixed model than
in the continuous model, but the difference is not large. This time, however, rounding error in the data is not an issue, and the estimates of the mixed model can be taken at face value.

At a first glance, the different estimates of $\alpha$’s across the two data sets seem to imply that there may be substantial differences in the estimated extent of nominal wage rigidity. This is not true, however, because the implied probability of a wage cut depends on the magnitude of $\alpha$ relative to the standard deviation of wage growth $\sigma$. The two data sets also produce fairly different estimates of $\sigma$. Since $\sigma$ is larger in the SFLS sample the higher threshold $\alpha$ need not imply that there will be much fewer true wage cuts in this sample. If we examine the frequency of true wage cuts that are implied by our estimates (see Table 3) we observe that these frequencies are quite similar across samples. While the SIF data predicts slightly more wage cuts, the difference to the SLFS is never more than 1.5 percentage points.

Taken together, Table 3 provides strong evidence for nominal wage rigidities over the whole sample period, using two independent data sources. It takes a substantial drop in productivity, around 20 percent, to induce firms to cut a worker’s nominal pay. These results closely parallel those obtained by Altonji & Devereux (1999) for the U.S.

### 6.2 Are Nominal Rigidities easily malleable?

Recall the conjecture put forward by Gordon (1996) that nominal wage rigidities will tend to vanish in an environment with low inflation and low GDP growth. The macroeconomic environment in Switzerland between 1991 and 1997 allows us to evaluate this claim explicitly. The most convenient way to do this is to estimate a year-specific $\alpha$ and to plot it together with the inflation rate over the years. This is done in Figure 4. It is obvious from both the SLFS and the SIF that $\alpha$ is not positively correlated with the inflation rate. In fact, the point estimates of $\alpha$ are increasing over time while inflation is decreasing.

Note that a weakening of nominal rigidities could take two forms. First, $\alpha$ could decline as employees’ fairness standards may adjust to the new nominal environment. Second, a somewhat weaker form of the conjecture predicts a higher dispersion among individuals, potentially leaving the average $\alpha$ unaffected. The idea here is that perhaps not all individuals

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9 The reason for this may be that our model does not pick up all the measurement error in the SFLS sample which could inflate the estimate of $\sigma$.

10 Recall that the SLFS is May-to-May data. Hence, we use May-to-May changes in the CPI measure inflation. Analogously, we use December-to-December CPI changes whenever we use the SSF data.
will adjust their reference points, but a significant fraction will. This would lead to different $\alpha$’s across individuals and would be reflected in a higher standard error in the estimated $\alpha$. A statistical test of the second conjecture is provided in Table 3. The estimates show that the increase over time is statistically significant. We can reject individual and joint zero restrictions on all year effects relative to the first year in the sample (1991). The estimates also show that the precision of the estimate is not declining. In both samples there is no increase in the standard errors.

A further measure of the extent of downward wage rigidity is the fraction of workers who actually do not receive wage cuts although – in the absence of nominal inertia – firms would cut their wages. Figure 5 plots the fraction of workers affected by nominal rigidity together with the estimated frequency of actual wage cuts. Irrespective of the data source, we get the same picture: There is essentially no or only a minor increase in the frequency of true wage cuts as inflation goes from five to zero percent and real GDP is roughly constant. Instead, the fraction of workers affected by nominal wage rigidities rises sharply in both samples. Thus, if anything, low inflation exacerbates the extent of downward rigidities.

6.3 Who is most affected?

As explained in section 2, there are reasons suggesting that wage rigidity is more important among job stayers than among job movers, and more important for full-time stayers compared to part-time stayers. In order to test for these differences, we estimate separate values of $\alpha$ for these groups of workers.

The results regarding the differences between full-time and part-time job stayers are displayed in Table 5. We find large differences between the two groups of employees. For part-time job stayers, the estimated $\alpha$ is between 20.5 and 23.1 percent, depending on the specification of measurement error. A comparison of the estimates for full-time and part-time stayers indicates that a much higher productivity drop is needed to induce firms to cut the wages of full-time stayers. The point estimate for full-time stayers is above 35 percent in both specifications, and the difference to the part-time stayers is significant. This difference is also reflected in the estimated frequency of wage cuts. Both models predict that wage cuts almost never occur for full-time job stayers. The frequency is below two percent in both specifications. We also see that wage cuts are more frequent for part-time job stayers: the point estimate is between 12 and 13 percent. Despite the higher frequency of wage cuts among part-time stayers this group is also strongly affected by downward rigidity. As Table 5 shows roughly 50 percent of the part-time stayers did not get wage cuts due to nominal rigidity.
The estimates in Table 5 are based on observations about job stayers only. In a further step we, therefore, add job movers to the sample to evaluate whether nominal rigidities are confined to job stayers only. Since we are also interested in the impact of different reasons for movements between jobs several waves of the SLFS cannot be used because the relevant information is only contained in the last three waves. Therefore our sample shrinks to 10,708 observations. Our estimates regarding the differences between movers and stayers are presented in Table 6.

Overall, job movers are much more likely to experience a wage cut than job stayers. While the threshold value for wage cuts is between 30 and 40 percent for full-time stayers it is slightly above 10 percent for job movers. Moreover, while at most 3.3 percent of the full-time stayers experience wage cuts the frequency of estimated wage cuts is much higher for job movers. Between 8 (in the mixed model) and 13 percent (in the continuous model) of those job movers who quit their job voluntarily experience wage cuts. Dismissed job movers have to accept wage cuts even more frequently. Between 16.6 and 18.1 percent of them experience wage cuts. It is also interesting to see that the wages of part-time stayers are cut much more frequently than the wages of full time stayers. Approximately between 11 and 14 percent of part-time stayers have to accept wage cuts. A final observation is that the difference between full-time and part-time workers, which is rather big for job stayers, disappears for job movers. For job movers it does not matter whether they work part-time or full-time because the fact that they move between jobs is already associated with much more flexible wages.

6.4 Effects on real wages

Finally, we are interested to what extent the presence of nominal inertia prevented real wage decreases of those workers who would have experienced wage cuts in the absence of forces inhibiting the cuts. The real wage impact of nominal rigidities is depicted in Figure 6. The figure shows that at the beginning of the time period considered, i.e. when inflation was roughly 4 percent nominal inertia prevented a real wage decrease of 6.3 percent among full-time stayers while in 1997 when inflation was zero the prevented real wage decrease of full-time stayers was pushed up to 13.3 percent. For part-time stayers we observe a similar time trend. Nominal inertia prevented a real wage cut of 4 percent in 1991 and of 9.5 percent in 1997. When discussing the aggregate consequences of nominal wage rigidity one has to combine these figures with the fraction of workers that is affected by nominal inertia. Recall from figure 5 that the fraction of workers that is affected by downward rigidity is also much
larger in 1997 than in 1991. For example, in the SLFS sample 45 percent of the full-time stayers were affected by nominal inertia in 1991 whereas in 1997 66 percent of full-time stayers would have experienced wage cuts in the absence of downward rigidity. Thus, the impact on the real wages of those affected by nominal inertia and the large increase in the fraction of affected workers suggests that fall in inflation prevented substantial real wages adjustments.

7. Concluding Remarks

The argument that nominal inertia vanishes during a relatively long period of low nominal per capita GDP growth is strongly rejected by our data. Our results do not lend support to the conjecture that the forces that contribute to nominal inertia are easily malleable by the macroeconomic environment. These results are based on two unique data sets that both cover the period between 1991 and 1997 in Switzerland, where inflation was below 1 percent during four of the seven years and real per capita GDP fluctuated between -.8 and +.5 during the first six years.

We use an approach that allows us to take unobserved variation in the data explicitly into account. Although observed wage cuts become more frequent in a low nominal growth environment, this is largely due to measurement error. We find no evidence that firms become less reluctant to cut nominal wages. According to our most conservative estimate, the required productivity drop is, on average, 15 percent to induce firms to cut nominal wages. For incumbent workers who work full-time the required productivity drop is even above 30 percent. If productivity falls by less firms, instead, prefer to leave nominal wages unchanged. If anything, the necessary productivity drop becomes larger if inflation decreases.

We find that the wages of full-time job stayers are much more rigid than those of part-time job stayers. In addition, we found little evidence for nominal rigidity among worker who switch employers. We also show that wages would indeed respond strongly to unemployment, if wage setting were not constrained by nominal rigidities. There is a negative and significant impact of changes in regional unemployment rates on wage growth. A percentage point increase in unemployment decreases incumbents’ wage growth by approximately 0.8 percent. This implies that real wages would exhibit substantial flexibility if only nominal rigidities were absent.
References


Fehr, Ernst and Peter K. Zych (1997), Intertemporal Choice under Habit Formation, forthcoming in: Handbook of Experimental Economic Results.


Smith, Jennifer (1999), Nominal Wage Rigidities: Evidence from the UK, unpublished manuscript, University of Warwick.


## Table 1: Nominal GDP per Capita Growth during the Sample Years

<table>
<thead>
<tr>
<th>Years Considered</th>
<th>Median</th>
<th>Number of Consecutive Years Below</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4 Percent</td>
</tr>
</tbody>
</table>

### A. Previous Studies (United States)

<table>
<thead>
<tr>
<th>Source</th>
<th>Years Considered</th>
<th>Median</th>
<th>4 Percent</th>
<th>2 Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card and Hyslop (1996)</td>
<td>1976 – 1991</td>
<td>7.19%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>McLaughlin (1994)</td>
<td>1976 – 1986</td>
<td>9.59%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Kahn (1997)</td>
<td>1971 – 1988</td>
<td>7.28%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Altonji and Devereux (1999)</td>
<td>1972 – 1992</td>
<td>6.92%</td>
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<td>0</td>
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</tbody>
</table>

### B. This Study (Switzerland)

<table>
<thead>
<tr>
<th>Years Considered</th>
<th>Median</th>
<th>Number of Consecutive Years Below</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991 – 1997</td>
<td>1.7%</td>
<td>All Years</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 Period</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YEAR</th>
<th>RATE OF INFLATION</th>
<th>PER CAPITA REAL GDP GROWTH</th>
<th>SOURCE: SWISS LABOR FORCE SURVEY</th>
<th>SOURCE: SOCIAL SECURITY FILES SAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>FRACTION WITH ZERO NOMINAL WAGE CHANGE</td>
<td>FRACTION WITH NOMINAL WAGE DECREASE</td>
</tr>
<tr>
<td>1991</td>
<td>4.7%</td>
<td>-0.8%</td>
<td>0.080</td>
<td>0.172</td>
</tr>
<tr>
<td>1992</td>
<td>3.7%</td>
<td>-0.1%</td>
<td>0.123</td>
<td>0.245</td>
</tr>
<tr>
<td>1993</td>
<td>1.1%</td>
<td>-0.5%</td>
<td>0.135</td>
<td>0.278</td>
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<tr>
<td>1994</td>
<td>1.6%</td>
<td>0.5%</td>
<td>0.075</td>
<td>0.262</td>
</tr>
<tr>
<td>1995</td>
<td>0.9%</td>
<td>0.6%</td>
<td>0.092</td>
<td>0.247</td>
</tr>
<tr>
<td>1996</td>
<td>0.6%</td>
<td>0%</td>
<td>0.205</td>
<td>0.321</td>
</tr>
<tr>
<td>1997</td>
<td>-0.02%</td>
<td>1.7%</td>
<td>0.217</td>
<td>0.279</td>
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## Table 3: Are Nominal Wage Rigidities Statistically Significant?

**ML Estimates**

<table>
<thead>
<tr>
<th></th>
<th>SWISS LABOR FORCE SURVEY</th>
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<th>SOCIAL SECURITY FILES</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>CONT. ERRORS</td>
<td>MIXED ERRORS</td>
<td>MIXED ERRORS</td>
<td>CONT. ERRORS</td>
</tr>
<tr>
<td><strong>Threshold Wage Cut $\alpha$</strong></td>
<td>.234** (.004)</td>
<td>.281** (.005)</td>
<td>.357** (.01)</td>
<td>.151** (.001)</td>
</tr>
<tr>
<td><strong>Wage Growth Function</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Experience</td>
<td>-.009** (.001)</td>
<td>-.012** (.002)</td>
<td>-.131** (.02)</td>
<td>-</td>
</tr>
<tr>
<td>Log Tenure</td>
<td>-.004** (.002)</td>
<td>-.005** (.002)</td>
<td>-.006** (.002)</td>
<td>-</td>
</tr>
<tr>
<td>Log Age</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-.067** (.004)</td>
</tr>
<tr>
<td>Change in regional Unemployment Rate</td>
<td>-.006* (.003)</td>
<td>-.007* (.003)</td>
<td>-.008* (.003)</td>
<td>-.007** (.001)</td>
</tr>
<tr>
<td>Foreigner (dummy variable)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-.05** (.015)</td>
</tr>
<tr>
<td>Foreigner*Log(Age)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.011** (.004)</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>.136</td>
<td>.135</td>
<td>.165</td>
<td>.099</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>.071</td>
<td>.075</td>
<td>.07</td>
<td>.039</td>
</tr>
<tr>
<td>$p$</td>
<td>-</td>
<td>.379</td>
<td>.363</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-</td>
<td>-</td>
<td>.12** (.01)</td>
<td>-</td>
</tr>
<tr>
<td><strong>Implied Frequency of Nominal Wage Cuts</strong></td>
<td>.062</td>
<td>.036</td>
<td>.045</td>
<td>.071</td>
</tr>
<tr>
<td>Fraction of Workers affected by Nominal Wage Rigidity</td>
<td>.503</td>
<td>.562</td>
<td>.623</td>
<td>.421</td>
</tr>
<tr>
<td>-----------------------------------------------------</td>
<td>------</td>
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<td>------</td>
<td>------</td>
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<tr>
<td>Year Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm-Size Effect</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>Number of Observations</td>
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<td>24,567</td>
<td>24,567</td>
<td>63,152</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>17,409</td>
<td>7,704</td>
<td>7,758</td>
<td>69,377</td>
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</table>


Notes: standard errors in parenthesis. *, ** denotes significance at the 5 percent and 1 percent level respectively
<table>
<thead>
<tr>
<th></th>
<th>Swiss Labor Force Survey</th>
<th>Social Security Files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mixed Model</td>
<td>Mixed Model</td>
</tr>
<tr>
<td><strong>Threshold Wage Cut</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Term (1991)</td>
<td>.285** (.01)</td>
<td>.145** (.004)</td>
</tr>
<tr>
<td><strong>Year Dummy Variables:</strong></td>
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<tr>
<td>Changes relative to 1991:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>.031* (.013)</td>
<td>.012** (.005)</td>
</tr>
<tr>
<td>1993</td>
<td>.058** (.013)</td>
<td>.047** (.005)</td>
</tr>
<tr>
<td>1994</td>
<td>.063** (.013)</td>
<td>.055** (.005)</td>
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<tr>
<td>1995</td>
<td>.044** (.014)</td>
<td>.057** (.005)</td>
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<tr>
<td>1996</td>
<td>.087** (.014)</td>
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<td>1997</td>
<td>.107** (.013)</td>
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<tr>
<td>$\sigma_e$</td>
<td>.161</td>
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<tr>
<td>$\sigma_m$</td>
<td>.069</td>
<td>.036</td>
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<tr>
<td>Number of Observations</td>
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<td>63,152</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>7,799</td>
<td>51,784</td>
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</table>

Notes: a. standard errors in parenthesis. *, ** denotes significance at the 5 percent and 1 percent level respectively.
b. Same wage growth function as in table 1. Both specifications include \( \lambda \).
### Table 5: Nominal Rigidities for Different Groups of Workers

**ML estimates from Swiss Labor Force Survey**

<table>
<thead>
<tr>
<th></th>
<th>Mixed Model</th>
<th>Mixed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Threshold Wage Cut</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-Time Job Stayer</td>
<td>.36** (.006)</td>
<td>.391** (0.007)</td>
</tr>
<tr>
<td>Part-time Job Stayer</td>
<td>.205** (.005)</td>
<td>.231** (0.009)</td>
</tr>
<tr>
<td><strong>Frequency of Wage Cuts</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-Time Job Stayer</td>
<td>.014</td>
<td>.019</td>
</tr>
<tr>
<td>Part-time Job Stayer</td>
<td>.122</td>
<td>.134</td>
</tr>
<tr>
<td><strong>Fraction Affected by Nominal Wage Rigidities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-Time Job Stayer</td>
<td>.598</td>
<td>.634</td>
</tr>
<tr>
<td>Part-time Job Stayer</td>
<td>.475</td>
<td>.511</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>-</td>
<td>.04** (.001)</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>.142</td>
<td>0.145</td>
</tr>
<tr>
<td>( \sigma_m )</td>
<td>.074</td>
<td>0.075</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>24,567</td>
<td>24,567</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>7,989</td>
<td>8,020</td>
</tr>
</tbody>
</table>

**Notes:**

a. Standard errors in parenthesis. *, ** denotes significance at the 5 percent and 1 percent level respectively.

b. Same wage growth function as in table 1.
### Table 6: Nominal Rigidities for Movers and Stayers

ML estimates from Swiss Labor Force Survey

<table>
<thead>
<tr>
<th></th>
<th>Continuous model</th>
<th>Mixed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Threshold Wage Cut</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Job Stayers</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-Time</td>
<td>.309** (.009)</td>
<td>.413** (.011)</td>
</tr>
<tr>
<td>Part-time</td>
<td>.203** (.007)</td>
<td>.246** (.008)</td>
</tr>
<tr>
<td><em>Job Movers</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job quits</td>
<td>.119** (.026)</td>
<td>.138** (.027)</td>
</tr>
<tr>
<td>Dismissals</td>
<td>.103** (.026)</td>
<td>.106** (.026)</td>
</tr>
<tr>
<td>Other reasons</td>
<td>.13** (.027)</td>
<td>.13** (.03)</td>
</tr>
<tr>
<td>Full-time effect for</td>
<td>.019 (.021)</td>
<td>0.035 (.022)</td>
</tr>
<tr>
<td><em>Job Movers</em></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Frequency of Wage Cuts** |          |          |
| *Job Stayers*             |          |          |
| Full-time                 | .033     | .01      |
| Part-time                 | .139     | .114     |
| *Job Movers*              |          |          |
| Job quits                 | .129     | .081     |
| Dismissals                | .181     | .166     |
| Other reasons             | .144     | .134     |
| $\sigma_c$               | .141     | .144     |
| \( \sigma_m \) | .074 | .079 |
|\( \beta \) | - | .334 |
| Number of Observations | 10,708 | 10,708 |
| Log likelihood | 7,668 | 2,696 |

Notes:

a. standard errors in parenthesis. *, ** denotes significance at the 5 percent and 1 percent level respectively.

b. Same wage growth function as in table 1, but data is available only for 1995, 1996 and 1997.
Figure 1a: Four Views on Nominal Wage Changes

Switzerland 1990 - 1997

Source: Swiss Labor Force Survey

Source: Social Insurance Files
Figure 1b: Four Views on Nominal Wage Changes

Two Large Firms, Switzerland 1993/1984 - 1998

Source: Personnel Files, Firm A

Source: Personnel Files, Firm B
Figure 2: Distribution of Changes in Earnings by Year
Figure 3: The distribution of wage changes and overtime variations in True and 'Polluted' Wage Changes, Firm B, 1993 - 1998.
Figure 4: Are Nominal Rigidities easily malleable?
Figure 5: Extent of Nominal Wage Rigidities over time
Figure 6: By how much did Nominal Rigidities hold up real wages?
1. Appendix: The Likelihood Function

Observed wage changes are generated by the following process

\[
\Delta y_{it} = \begin{cases} 
  x'_{it} b + e_{it} + m_{it} & \text{if } x'_{it} b + e_{it} \geq 0 \\
  m_{it} & \text{if } -\alpha \leq x'_{it} b + e_{it} < 0 \\
  x'_{it} b + \lambda + e_{it} + m_{it} & \text{if } x'_{it} b + e_{it} < -\alpha
\end{cases}
\]

**Regime 1**

**Regime 2**

**Regime 3**

We assume that

\[
\begin{align*}
e_{it} & \sim N(0, \sigma_e) \\
m_{it} & \sim N(0, \sigma_m)
\end{align*}
\]

In order to derive the likelihood function, we note that

\[
F_{\Delta y}(q) = \Pr(\Delta y_{it} \leq q) = \\
\Pr(\Delta y_{it} \leq q, x'_{it} b + e_{it} \geq 0) \\
+ \Pr(\Delta y_{it} \leq q, -\alpha \leq x'_{it} b + e_{it} < 0) \\
+ \Pr(\Delta y_{it} \leq q, x'_{it} b + e_{it} < -\alpha)
\]

But since

\[
\Pr(\Delta y_{it} \leq q, x'_{it} b + e_{it} \geq 0) \\
= \Pr(\Delta y_{it} \leq q | x'_{it} b + e_{it} \geq 0) \cdot \Pr(x'_{it} b + e_{it} \geq 0)
\]

we can rewrite the cumulative distribution function of \( \Delta y \) as

\[
F_{\Delta y}(q) = F_{\Delta y}(q | x'_{it} b + e_{it} \geq 0) \cdot \Pr(x'_{it} b + e_{it} \geq 0) \\
+ F_{\Delta y}(q | -\alpha \leq x'_{it} b + e_{it} < 0) \cdot \Pr(-\alpha \leq x'_{it} b + e_{it} < 0) \\
+ F_{\Delta y}(q | x'_{it} b + e_{it} < -\alpha) \cdot \Pr(x'_{it} b + e_{it} < -\alpha)
\]

and the density is

\[
f_{\Delta y}(q) = f_{\Delta y}(q | x'_{it} b + e_{it} \geq 0) \cdot \Pr(x'_{it} b + e_{it} \geq 0) \\
+ f_{\Delta y}(q | -\alpha \leq x'_{it} b + e_{it} < 0) \cdot \Pr(-\alpha \leq x'_{it} b + e_{it} < 0) \\
+ f_{\Delta y}(q | x'_{it} b + e_{it} < -\alpha) \cdot \Pr(x'_{it} b + e_{it} < -\alpha)
\]

which is equation (2) in the text. Each part corresponds to one regime. We will now present the analytical expression for each part under the assumption that \( e \) and \( m \) are i.i.d. normal. The forms are derived in the next section.
Regime 1

The likelihood is given by

\[
f_{\Delta y} (\Delta y_{ht} | x_{it}' b + e_{it} \geq 0) \cdot \Pr(x_{it}' b + e_{it} \geq 0) \\
= f_{c+m} (\Delta y_{ht} - x_{it}' b | e_{it} \geq -x_{it}' b) \cdot \Pr(x_{it}' b + e_{it} \geq 0) \\
= \phi \left( \frac{\Delta y_{ht} - x_{it}' b}{\sqrt{\sigma^2_c + \sigma^2_m}} \right) \cdot \\
\left( 1 - \Phi \left( x_{it}' b s - \frac{(\Delta y_{ht} - x_{it}' b)}{s \sigma^2_m} \right) \right)
\]

where

\[
s = \frac{\sqrt{\sigma^2_c + \sigma^2_m}}{\sigma_c \sigma_m}
\]

and \(\phi(\cdot)\) and \(\Phi(\cdot)\) are the density and cumulative distribution function of the standard normal distribution respectively. The essential step consists in deriving the expression \(f_{c+m} (\Delta y_{ht} - x_{it}' b | e_{it} \geq -x_{it}' b) \cdot \Pr(x_{it}' b + e_{it} \geq 0)\), which we do in the next section.

Regime 2

The likelihood is

\[
f_{\Delta y} (\Delta y_{ht} | -\alpha \leq x_{it}' b + e_{it} < 0) \cdot \Pr (-\alpha \leq x_{it}' b + e_{it} < 0) \\
= f_{m} (\Delta y_{ht}) \cdot \Pr (-\alpha \leq x_{it}' b + e_{it} < 0) \\
= \frac{1}{\sigma_m} \phi \left( \frac{\Delta y_{ht}}{\sigma_m} \right) \left[ \Phi \left( -\frac{x_{it}' b}{\sigma_c} \right) - \Phi \left( \frac{-\alpha - x_{it}' b}{\sigma_c} \right) \right]
\]

since \(e\) and \(m\) are independent.

Regime 3

Similarly to regime 1, the likelihood is

\[
f_{\Delta y} (\Delta y_{ht} | x_{it}' b + e_{it} \leq -\alpha) \cdot \Pr (x_{it}' b + e_{it} < -\alpha) \\
= f_{c+m} (\Delta y_{ht} - x_{it}' b - \lambda | e_{it} \leq -x_{it}' b - \alpha) \cdot \Pr (x_{it}' b + e_{it} < -\alpha) \\
= \phi \left( \frac{\Delta y_{ht} - x_{it}' b - \lambda}{\sqrt{\sigma^2_m + \sigma^2_c}} \right) \cdot \\
\Phi \left( -\frac{x_{it}' b - \alpha}{s} \right) s - \frac{(\Delta y_{ht} - x_{it}' b - \lambda)}{s \sigma^2_m}
\]
2. Deriving \( f_{x+y}(a|x \geq r) \cdot \Pr(x \geq r) \)

We are looking for the solution of \( f_{x+y}(a|x \geq r) \cdot \Pr(x \geq r) \), where \( X \) and \( Y \) are two independent normal random variables:

\[
y \sim N(\mu_y, \sigma_y) \\
x \sim N(0, \sigma_x)
\]

Remember that

\[
\Pr(x + y \leq a|x \geq r) \cdot \Pr(x \geq r) = \Pr(x + y \leq a, x \geq r)
\]

and that

\[
\Pr(x + y \leq a, x \geq r) = \int_{r}^{+\infty} \int_{-\infty}^{+\infty} f_y(y)f_x(x)dydx
\]

\[
= \int_{r}^{+\infty} \int_{-\infty}^{+\infty} f_y(u-x)f_x(x)dudx
\]

because \( X \) and \( Y \) are independent. Using the last two equations and taking the derivative with respect to \( a \), we obtain

\[
f_{x+y}(a|x \geq r) \cdot \Pr(x \geq r) = \frac{\partial \Pr(x + y \leq a|x \geq r)}{\partial a} \cdot \Pr(x \geq r)
\]

\[
= \frac{\partial \Pr(x + y \leq a, x \geq r)}{\partial a} = \int_{r}^{+\infty} f_y(a-x)f_x(x)dx
\]

All we need to do is evaluate the last integral. We first simplify the integrand as follows:

\[
f_y(a-x)f_x(x) = \frac{1}{2\pi\sigma_y\sigma_x} \exp \left[ -\frac{1}{2} \left( \frac{(a-x-\mu_y)^2}{\sigma_y^2} \right) - \frac{1}{2} \left( \frac{x^2}{\sigma_x^2} \right) \right]
\]

\[
= \frac{1}{2\pi\sigma_y\sigma_x} \exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2[(a-\mu_y)^2-2(a-\mu_y)x+x^2]+\sigma_yx^2}{(\sigma_y\sigma_x)^2} \right) \right]
\]

We separate terms involving \( x \) from others

\[
f_y(a-x)f_x(x) = \frac{1}{2\pi\sigma_y\sigma_x} \exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2(a-\mu_y)^2}{(\sigma_y\sigma_x)^2} + \frac{-2(a-\mu_y)x\sigma_x^2+x^2(\sigma_x^2+\sigma_y^2)}{(\sigma_y\sigma_x)^2} \right) \right]
\]
and define

\[ s = \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{\sigma_y \sigma_x}} \]

Substituting \( s \) into the last equation yields

\[ f_y(a-x)f_x(x) = \frac{1}{2\pi \sigma_y \sigma_x} \exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2(a - \mu_y)^2}{\sigma_y \sigma_x} - \frac{2(a - \mu_y)xs}{\sigma_y^2} + s^2 x^2 \right) \right] \]

which can also be written as

\[ f_y(a-x)f_x(x) = \frac{1}{2\pi \sigma_y \sigma_x} \exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2(a - \mu_y)^2}{\sigma_y \sigma_x} + \left( s - \frac{a - \mu_y}{\sigma_y^2} \right)^2 - \left( \frac{a - \mu_y}{\sigma_y^2} \right)^2 \right) \right] \]

The integral we have to solve now reads

\[ \int_r^{+\infty} f_y(a-x)f_x(x)dx = \frac{1}{2\pi \sigma_y \sigma_x} \exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2(a - \mu_y)^2}{\sigma_y \sigma_x} - \left( \frac{a - \mu_y}{\sigma_y^2} \right)^2 \right) \right] \int_r^{+\infty} \exp \left[ -\frac{1}{2} \left( s - \frac{a - \mu_y}{\sigma_y^2} \right)^2 \right] dx \]

Substitute

\[ u(x) = sx - \frac{(a - \mu_y)}{\sigma_y^2} \]

\[ \implies dx = \frac{1}{s} du \]

\[ \implies u(r) = rs - \frac{(a - \mu_y)}{\sigma_y^2} \]

and insert this into the last equation to obtain

\[ f_{x+y}(a|x \geq r) = \frac{1}{\sqrt{2\pi} \sigma_y \sigma_x s} \exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2(a - \mu_y)^2}{\sigma_y \sigma_x} - \left( \frac{a - \mu_y}{\sigma_y^2} \right)^2 \right) \right] \int_r^{+\infty} \exp \left[ -\frac{1}{2} u^2 \right] du \]
\[
\frac{1}{\sqrt{2\pi\sigma_y\sigma_x}} \exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2 (a - \mu_y)^2}{(\sigma_y\sigma_x)^2} - \left( \frac{(a - \mu_y)}{s\sigma_y^2} \right)^2 \right) \right] \cdot \left( 1 - \Phi \left( rs - \frac{(a - \mu_y)}{s\sigma_y^2} \right) \right),
\]

where \( \Phi(\cdot) \) is the cumulative distribution function of the standard normal distribution. The first term can be further simplified.

\[
\exp \left[ -\frac{1}{2} \left( \frac{\sigma_x^2 (a - \mu_y)^2}{(\sigma_y\sigma_x)^2} - \left( \frac{(a - \mu_y)}{s\sigma_y^2} \right)^2 \right) \right] = \exp \left[ -\frac{1}{2} (a - \mu_y)^2 \left( \frac{\sigma_x^2}{(\sigma_y\sigma_x)^2} - \frac{1}{s^2\sigma_y^4} \right) \right] = \exp \left[ -\frac{1}{2} (a - \mu_y)^2 \left( 1 - \frac{1}{s^2\sigma_y^4} \right) \right]
\]

Using the definition of \( s \), we get

\[
\exp \left[ -\frac{1}{2} \frac{(a - \mu_y)^2}{\sigma_y^2} \left( 1 - \frac{1}{s^2\sigma_y^4} \right) \right] = \exp \left[ -\frac{1}{2} \frac{(a - \mu_y)^2}{\sigma_y^2} \left( 1 - \frac{\sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right) \right] = \exp \left[ -\frac{1}{2} \frac{(a - \mu_y)^2}{\sqrt{\sigma_x^2 + \sigma_y^2}} \left( \frac{\sigma_y^2}{\sigma_x^2 + \sigma_y^2} \right) \right] = \exp \left[ -\frac{1}{2} \frac{(a - \mu_y)^2}{\sqrt{\sigma_x^2 + \sigma_y^2}} \right]
\]

and the whole expression is condensed to

\[
f_{x+y}(a|x \geq r) = \frac{1}{\sqrt{2\pi\sigma_y\sigma_x}} \exp \left[ -\frac{1}{2} \left( \frac{a - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \right)^2 \right] \cdot \left( 1 - \Phi \left( rs - \frac{(a - \mu_y)}{s\sigma_y^2} \right) \right),
\]

but since \( \sigma_y\sigma_x s = \sqrt{\sigma_x^2 + \sigma_y^2} \), we obtain the final result

\[
f_{x+y}(a|x \geq r) = \frac{1}{\sqrt{2\pi\sqrt{\sigma_x^2 + \sigma_y^2}}} \exp \left[ -\frac{1}{2} \left( \frac{a - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \right)^2 \right] \cdot \left( 1 - \Phi \left( rs - \frac{(a - \mu_y)}{s\sigma_y^2} \right) \right) = \frac{1}{\sqrt{\sigma_x^2 + \sigma_y^2}} \phi \left( \frac{a - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \right) \cdot \left( 1 - \Phi \left( rs - \frac{(a - \mu_y)}{s\sigma_y^2} \right) \right)
\]
where $\phi(\cdot)$ is the standard normal density function. The likelihood for regime 1 directly follows from this expression. It is now easy to derive the likelihood for regime 3. Since

$$P(x + y \leq a, x \geq r) = \int_{r}^{+\infty} \int_{-\infty}^{a-x} f_y(y) f_x(x) dy dx$$

it follows immediately that

$$P(x + y \leq a, x \leq r) = \int_{-\infty}^{r} \int_{-\infty}^{a-x} f_y(y) f_x(x) dy dx$$

and

$$f_{x+y}(a|x \leq r) \cdot \Pr(x \leq r) = \int_{-\infty}^{r} f_y(a-x) f_x(x) dx.$$ 

We can now reiterate the derivation of the likelihood function for regime 1. The only piece that is different are the integration boundaries.