Allocating and Funding Universal Service Obligations in a Competitive Network Market

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Abstract

We examine, in a network market open to competition, various mechanisms of allocating and funding "universal service obligations" among agents (rival operators and consumers). The obligations we consider are geographic ubiquity and non discrimination. We analyze from efficiency and equity point of view the respective advantages of a "restricted-entry" system (where the entrant is not allowed to serve high cost consumers) and the "pay or play" system at work for instance in Australia. We show that the pay or play regulation always dominates the restricted-entry regulation under ubiquity constraint alone. This result no longer holds when the regulator imposes also the non discrimination constraint.

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1 Introduction

The transition towards a more competitive regime on the markets of public utilities rises a number of new questions. In network utilities like telecommunications, electricity, gaz, the regulator often values an "equal access" of all consumers to the service at an "affordable tariff"[1]. Whereas networks were previously operated by monopolies who were in charge with these universal service obligations (USOs), and used cross-subsidies between "profitable" and "unprofitable" users, the arrival of new entrants on markets that are now open to competition induces cream skimming on profitable segments of the market (see Laffont and Tirole (1998)), and makes previous monopolies unable to finance these obligations through cross-subsidies[2]. Moreover, competition leads to outcomes that are not necessarily desirable from the regulator's point of view. Without regulatory constraints, some users would then be excluded of the market, and users with different consumption or costs characteristics would face different tariffs. If the regulator values equality with respect to tariffs and/or access of all the users to the market, he must then impose "universal service obligations" (USOs). In practice, those obligations include two types of components. The "social" component includes the fact that light users should be offered tariffs similar to heavy users, or low tariffs for targeted consumers (see Gasmi, Laffont, Sharkey (1999)). The "geographical" component can be divided into two obligations: ubiquity and non discrimination. The "ubiquity constraint" states that all consumers should be connected to a network, whatever their location. The "non discrimination" constraint states that the same tariff should be proposed to all those consumers, whatever their location or their connection cost. These constraints may be imposed together or independently[3]. Those constraints are particularly relevant in developed countries for promoting access of a large number of consumers to new technologies, like Internet. In developing countries, and particularly in case of privatization, those constraints are crucial to ensure a large development of networks.

The universal service obligations raise two series of questions. First, which USO's should be imposed to whom (the allocation problem) and second, who should pay for the USO's (the funding problem). The combination of various solutions to the first and to the second question define different regulatory mechanisms that have different implications in terms of global and/or partial surplus. Compared to an unconstrained competition between network providers, universal service obligations induce distortions on the competitive entry process and on the equilibrium market structure. Those distortion generate both social benefits and social costs. The question of how to share these costs and benefits becomes one of the

[1] See e.g., Federal Communications Commission, "In the matter of Federal-Sate Joint Board on Universal Service", CC Docket, n 96-45, Nov. 8, 1996.

[2] For a complete theory of competition in the sectors of public utilities, see Laffont and Tirole (1993).

[3] For instance in France, the provision of gas is not submitted to the ubiquity constraint: the operator Gaz De France may choose not to serve a given area. But when it serves it, it is submitted to a non discrimination constraint, that is, it must offer the same tariff (or the same menu) to any consumer of the areas that are served.

main questions for regulators and has received various answers in different countries.

The main tasks for regulators consist in determining optimal rules for allocating and funding those USOs. On the allocation side, two alternative approachs exist, depending on who between the incumbent and its competitors incur USOs. In most countries, following openess to competition, only the incumbent is in charge of USOs. In certain cases, productive efficiency would require that competitors incur some of the USOs. For instance, USOs can be auctionned (see Milgrom (1996), Weller (1998))[4]. On the funding side, three methods exist. Most systems share the property that they have to be self funded. The cost of the USOs is then financed through taxes, levied on all units of goods. An alternative method consists in financing USOs through lump sum transfers. Finally, probably the less distorting scheme consists in funding USOs through cross-subsidies.

In this paper, we focus mainly on two allocation mechanisms and funding systems. The "restricted-entry" rule gives to the incumbent the obligation of serving non profitable users at the same tariff as profitable ones, whereas the pay or play regulation[5] allows the entrant to serve the market. On the funding side, both rules allow for cross-subsidies and taxation. We compare them from a global and partial surplus point of view. In the last section of the paper, we briefly discuss other possibilities, like auctions on the allocations side and lump sum transfers on the funding side.

We examine various procedures of tax determination. Among others, we analyze the case where the tax is designed to be "competitively neutral", that is, the incumbent must obtain through taxes, if it is possible, the same amount of profits than under a competitive regime. This view corresponds to that developped in the US in the Telecommunication Act of 1996[6]. However, it differs greatly from a determination of taxes through a first best principle, and thus induces social losses. An important remark is thus that the "cost of universal service" is conditionnal to the way it is funded. As pointed out by Panzar (1999), '*any USO costing exercise must begin with a careful specification of an unsubsidized market scenario that would prevail in the absence of the USO*'. Many proposed measures of USO costs (in particular the accountable approach of '*net avoidable costs*'[7]) ignore this basic requirement. In this paper, we define precisely the benchmark situation, where no USO is imposed, and then we carefully list and evaluate the economic distortions due to the presence of the tax. In our framework, the market structure as well as the cost of the USO's are endogeneous, since they depend on the regulation rule and the tax determination procedure.

We build a model where two operators compete for a market of a network good. Consumers are heterogenous with respect to their connection costs. The regulator imposes different combinations of USOs, and funds them through taxes. The incumbent firm is

[4]For instance, USOs are auctionned for the postal service in Germany or for telecommunications in the US (and more recently in Germany and Austria).

[5]The pay or play regualtion is applied in Australia and has been discussed in UK in the telecommunication sector.

[6]See Pub. L. No. 104-104, 110 Stat. 56 (1996) (1996 Act), 47 U.S.C. 254(b).

[7]See for instance WIK (1997).

in charge with the USOs. We then compare two regulatory mechanisms. In the "pay or play" system, the entrant may choose to serve the non profitable users, whereas in the "restricted-entry" system, he is not allowed to. We examine the equilibria of the corresponding games, and compare them in terms of social welfare and in terms of transfers between agents. In a last section, we compare the funding of USOs through taxation with the case lump sum transfer scheme. Finally, we compare these two regulatory mechanisms with a second price auction on the market of non profitable users.

2 Model and notations

2.1 The framework

We consider the following situation. Two firms, denoted I and E, compete for the market of a network good. This network has a character of public utility, that is, the public authority may pursue social objectives such as "equal access" of all the consumers to the service provided by the network. Typically, this situation reflects liberalization of a network market, such as electricity, gaz, telecommunications or airlines, where an incumbent firm (I) faces a entrant (E).

There are two types of users denoted by $\mu = \underline{\mu}, \overline{\mu}$. The type of a user corresponds to its location across the geographical space: the value $\mu = \underline{\mu}$ (resp. $\mu = \overline{\mu}$) denotes a location where the cost of connection to the network is low (resp. high). For instance $\underline{\mu}$ is a measure of the cost of connection in high density areas (urban areas), whereas $\overline{\mu}$ corresponds to low density areas (rural areas). The proportion of consumers of type $\overline{\mu}$ is $\overline{\alpha}$ and that of consumers $\underline{\mu}$ is $\underline{\alpha}$.

A consumer who buys q units of the good and who is charged a tariff $T(q)$[8] receives a net surplus equal to: $u = w(q) - T(q)$, where $w(q)$ is supposed to be increasing and concave. Here, the only element of heterogeneity concerns the connection costs. The demand addressed to firm K by a consumer facing the tariff $T(q)$ is given by $w'(q) = T'(q)$. One should note that the demand of a particular consumer is independant of its connection cost.

Firms are endowed with the following technologies. Firm $K = I, E$ incurs a cost $C_K(q, \mu)$, when providing q units to consumer $\mu = \underline{\mu}, \bar{\mu}$. We assume that

$$C_K(q, \underline{\mu}) < C_K(q, \bar{\mu})$$

for all q and for both firms $K = I, E$. If the consumers $\underline{\mu}$ and $\bar{\mu}$ are respectively urban and rural users, this assumption expresses the fact that connecting rural users to the network is more costly to any firm than connecting urban users, whatever the level of their consumption[9]. The profit of firm K is then $\pi_K(q, \mu) = T_K(q, \mu) - C_K(q, \mu)$.

[8]Under the assumption of homogeneous preferences, a two-part tariff allows to achieve the same allocation than more general non linear tariffs. In a more general model, consumers could be heterogenous in their tastes for the good.

[9]In order to focus on questions related to universal service obligations, we do not consider any problem of

Remark 1 *It is equivalent to work with tariff variables $T(\mu, q)$ or with variables u and q, where u denotes the utility level offered to a consumer. We deal thereafter with (u, q). We denote by $\underline{u}_K$ (respectively $\overline{u}_K$) the utility offered by firm K to consumers of type $\underline{\mu}$ (respectively $\overline{\mu}$).*

Remark 2 *The surplus derived from the relationship between a consumer of type μ and firm K is $S_K(\mu, q) = w(q) - T_K(q, \mu) + \pi_K(\mu, q) = w(q) - C_K(q, \mu)$.*

The first best quantity is defined by $q_K^{FB} = \text{argmax}_{q \geq 0} S_K(\mu, q)$. The first best surplus is $S_K(\mu) = w(q^{FB}) - C_K(q^{FB}, \mu)$ if $q_K^{FB} > 0$. This value $S_K(\mu)$ is the value to be shared between the firm and the consumer, the share of the consumer being u_K. For simplicity, we note $\bar{S}_K = S_K(\overline{\mu})$ and $\underline{S}_K = S_K(\underline{\mu})$.

We shall maintain the following assumption throughout[10]:

$$\max(\bar{S}_I, \bar{S}_E) < 0 < \min(\underline{S}_I, \underline{S}_E), \tag{1}$$

which means that high cost consumers are not profitable and would not be served by any firm without universal service obligations.

2.2 Benchmark: The competitive case

As a benchmark, we study the case where no firm is submitted to any USO constraint, that is, each firm K is able to offer a perfectly discriminatory tariff to any consumer. Because of our assumption (1), firms then compete on user $\underline{\mu}$ only. The profit of firm E is given by:

$$\pi_E(\underline{u}_E, q_E) = \begin{cases} \alpha(\underline{S}_E(q) - \underline{u}_E) & \text{if } \underline{u}_E > \underline{u}_I \\ 0 & \text{if } \underline{u}_E < \underline{u}_I. \end{cases}$$

The profit of firm I has a symmetric expression. It is clear that we have $q_K = q_K^{FB}$ when the consumer is served by firm K. Therefore in what follows, we will call "strategies" the choices of the variable $\underline{u}_K$ by firm $K = I, E$.

Each firm reacts to the strategy of its rival by choosing the share of the surplus it leaves to the consumer. We can then compute the best reply function of firm E in response to the strategy $\underline{u}_I$ of firm I.

Facing $\underline{u}_I$, firm E can offer $\underline{u}_I + \varepsilon$ if $\pi_E = \underline{S}_E - \underline{u}_I - \varepsilon$ remains positive, that is, as long as $\underline{u}_I < \underline{S}_E$. If $\underline{u}_I > \underline{S}_E$, firm E has no incentive to be active. Thus the best reply function of E to $\underline{u}_I$ is given by:

$$\underline{u}_E(\underline{u}_I) = \begin{cases} \underline{u}_I & \text{if } \underline{u}_I < \underline{S}_E \\ \text{any value in } [0, \underline{u}_I[& \text{if } \underline{u}_I > \underline{S}_E. \end{cases}$$

interconnection between both networks, which amounts to assume that the interconnection charge between networks is zero. For more details on the interconnection problems, see Armstrong, Doyle and Vickers (1996).

[10] except in section A.6, where we relax this assumption, in order to study the impact of imposing a non discrimination constraint alone.

In what follows, we will use as a benchmark the sequential game in which firm I is the leader. This representation of the competition process suits better to the case where a (dominant) incumbent faces a new entrant. Finally, as we will see in the next section, the allocation of universal service obligations gives to the incumbent a first mover advantage.

The sequential game (where firm I acts as a leader) may lead to a continuum of subgame perfect equilibria (SPE). When $0 < \underline{\mathrm{S}}_I < \underline{\mathrm{S}}_E$, any value $u_I \in [0, \underline{\mathrm{S}}_E]$ leads to the same profit for the firm I, namely $\Pi_I = 0$ since E serves the consumer).

Lemma 1 *The subgame perfect equilibria of the game where firm I acts as a leader are defined as follows:*

- *if $0 < \underline{S}_E < \underline{S}_I$, there is a unique SPE: firm I serves the consumer, offers him $\underline{u}_I = \underline{S}_E$ and $\underline{q}_I = q_I^{\mathrm{FB}}$, and makes profit $\pi_I = \underline{S}_I - \underline{S}_E$;*
- *if $0 < \underline{S}_I < \underline{S}_E$, there is a continuum of SPE, parametrized by $\underline{u}_E \in [0, \underline{S}_E]$: firm E serves the consumer, who obtains any value $\underline{u}_E \in [0, S_E]$. Firm E gets $\Pi_E = \underline{S}_E - \underline{u}_E$ and firm I stays out of the market.*

2.3 Taxes and distortions

In what follows, we will often consider the case where firm E pays a tax t on each unit sold to consumer $\underline{\mu}$. We now state a few useful results concerning the surplus of the relationship between firm E and the consumer $\underline{\mu}$, in presence of the tax t. In the sequel, the presence of a tilda indicates the dependance of the variable with regard to the tax rate t.

Let $T(q)$ be the tariff posted by the firm and q be the quantity of good purchased by the consumer $\underline{\mu}$. The profit of the firm writes:

$$\widetilde{\pi}_E = T(q) - C_E(q, \underline{\mu}) - tq,$$

where t is the tax level. We denote $\tilde{S}_E$ the surplus from the relationship between E and $\underline{\mu}$

$$\tilde{S}_E(t, q) = \widetilde{\pi}_E + w(q) - T(q) = w(q) - C_E(q, \underline{\mu}) - tq.$$

Let $\tilde{S}_E(t)$ be the maximum value over $q \geq 0$ of the surplus:

$$\tilde{S}_E(t) = \max_{q \geq 0} \{w(q) - C_E(q, \underline{\mu}) - tq\}.$$

Let $\tilde{q}_E$ be the value of q for which the maximum is attained. The following lemma is proved in the appendix (section A.1).

Lemma 2 *The following properties are true:*

1. *The total surplus function $\tilde{S}_E$ is convex;*
2. *The functions $\tilde{S}_E$ and $\tilde{S}_E + t\tilde{q}_E$ are nonincreasing with respect to t;*

3. There exists a tax level t^e such that for $t \geq t^e$, firm E can extract no surplus from consumers $\underline{\mu}$: $\tilde{S}_E(t^e) = 0$.

The sum $\tilde{S}_E(t) + t\tilde{q}_E(t)$ represents the total amount that can be extracted from the relation between E and the consumer $\underline{\mu}$, that is, the whole amount of what the firm E and the consumer $\underline{\mu}$ receive on the one hand, $\tilde{S}_E(t)$, and the amount of tax collected on the other hand $t\tilde{q}_E$. The higher the tax, the lower this total amount. This partial effect should lead the regulator to choose not too high taxes.

Lemma 2 shows that the entrant is only active on the market when the tax level is not too high ($t < t^e$). Very high level of taxes eject firm E from the competition with firm I.

2.4 Definitions of and interactions between USOs

The Universal Service Obligations consist in two different constraints imposed by the regulator to the incumbent I:

- Ubiquity constraint (subscript U will identify thereafter the variables under this constraint): Each consumer should have access to the good, that is, each consumer must face a tariff such that he obtains a nonnegative level of utility;
- Non discrimination constraint (ND): If the incumbent chooses to serve both types of consumers, it must offer the same tariff to both. The incumbent may prefer to serve only one type;
- Ubiquity and non discrimination constraints (UND): the incumbent must offer to all consumers the same tariff and this tariff must be such that all the consumers have a nonnegative utility level.

Recall that we model the competition between firms I and E by a sequential game, where firm I is the leader: whatever the regulation framework, firm I first announces a pair $(\underline{u}_I, \bar{u}_I)$. The USO's may thus be represented by constraints on the space of strategies of the incumbent:

$$\begin{array}{ll} \text{USO=U}: & \underline{u}_I \geq 0, \bar{u}_I \geq 0 \\ \text{USO=ND}: & \text{if } \underline{u}_I \geq 0, \bar{u}_I \geq 0 \text{ then } \underline{u}_I = \bar{u}_I \\ \text{USO=UND}: & \underline{u}_I = \bar{u}_I \geq 0. \end{array}$$

Note, however, that these restrictions on I's strategies imply (at equilibrium) restrictions on E's strategies. Suppose for example that under USO=UND, firm E serves all the consumers. Then, it is clear that the consumers $\underline{\mu}$ and $\bar{\mu}$ get the same level of utility at equilibrium ($\underline{u}_E = \bar{u}_E$).

Imposing ND alone may lead to some paradoxical effects. It may happen that with no constraint, the incumbent would have served both types of consumers, but submitted to the ND constraint, it stops serving type $\overline{\mu}$ consumers. Therefore, a constraint U may be required together with ND in order to make ND effective (see section A.6 for more details). In this paper, we focus on constraints U and UND.

2.5 Regulatory rules

The purpose of the paper is to examine the consequences on consumer's surplus, on profits and on global surplus, of different ways of allocating and funding the USOs (U or UND). We consider essentially two regulatory frameworks, denoted by "restricted-entry regulation" and "pay or play regulation".

Whatever the regulatory mechanism, the timing of the game is the following. The regulator first announces the level of the tax t paid on each unit of the good. It also announces that at the end of the game the amount of tax collected will be allocated to the incumbent, if it serves the high cost consumer [11]. Then firms compete, that is, they choose the share of the surplus they will leave to consumers, the tax rate being taken as exogenous.

2.5.1 The various regimes

As we will see further, the regulatory rules may lead at equilibrium to one of the four following situations:

- Firm I serves both types of consumers, and cross-subsidizes between them. We call this regime "I cross-subsidizes" (ICS). No tax is raised nor perceived;
- Firm E serves both types of consumers, and cross-subsidizes between them; we call this regime "E cross-subsidizes" (ECS). No tax is raised nor perceived;
- Firm I serves the high costs consumers, E serves the low cost consumers and pays a tax; we call this situation the "taxation regime" (TR);
- High costs consumers are excluded from the consumption of the good, low cost consumers being served by either of the two firms; we call this regime "exclusion regime" (ER).

2.5.2 'Restricted-entry' regulation versus 'pay or play' regulation

Under the 'restricted-entry' regulation, firm E is not allowed to serve high cost consumers $\bar{\mu}$. Under USO=U or UND, these consumers are served by the incumbent. Two regimes may occur: ICS and TR.

Under the 'pay or play' regulation (POP), firm E is allowed to serve consumer $\bar{\mu}$. When E chooses to serve $\bar{\mu}$, it does not pay nor receive any tax. Therefore, firms' payoffs are not symmetric[12]. Under the 'play or pay' regulation, the identity of the firm who serves $\bar{\mu}$ is endogeneously determined. Three regimes may occur: ICS, TR, and ECS.

[11] If the incumbent serves both types of consumers, it pays the tax but then it also receives the amount of the tax; it is thus neutral for its profit.

[12] By contrast, auction mechanisms generally lead to symmetric payoffs (see for example Anton and alii [1]).

Under the restricted-entry regulation , firm E's strategy reduces to the choice of $\underline{u}_E$. Under the POP regulation, firm E may in addition choose to serve $\overline{\mu}$ consumers and thus also chooses $\bar{u}_E$.

2.5.3 The choice of the tax level

The choice of the tax level depends on the objectives pursued by the regulator. We focus here on three particular procedures: the first best tax, the balanced-budget tax and the competitively neutral tax.

The *first best tax* is the level of tax that maximizes the social welfare. However, this first best level may lead to a loss in profit for the incumbent.

Therefore, we introduce a second-best tax, namely the *balanced-budget tax*, defined by the maximization of the welfare, under the constraint:

$$\widetilde{\pi}_I^{USO} = 0.$$

We also analyse the "*competitively neutral*" tax, where firm I is compensated and receives, when it is possible, the profit of the benchmark case. Let $\widetilde{\pi}_K^{USO}$ be the profit of firm K at the equilibrium of this competitive process with tax, when the set of obligations is USO=U or UND. Under competitive neutrality, the tax rate t^{USO} is determined by the equality

$$\widetilde{\pi}_I^{USO} = \pi_I \qquad \text{("competitive neutrality condition")}$$

The idea of competitive neutrality is that the funding and the presence of USOs are designed not to affect the profit of firm I with respect to the unconstrained situation (benchmark)[13].

Budget balance and competitive neutrality may or may not coincide, according on what happens in the benchmark case.

2.6 Welfare criteria and productive efficiency

Generally, USOs are imposed in a purpose of enhancing the situation of unfavored consumers (by the constraint U) or to achieve equity objectives (by the constraint ND). In our model, "unfavored" consumers correspond to high connection cost consumers (the $\overline{\mu}$ type). We assume that the public authority values by itself the access to the network of all consumers and denote by k the valuation of the access by the regulator. The welfares in the different regimes are

$W_{II} = \underline{\alpha}\underline{S}_I + \overline{\alpha}\overline{S}_I + k$	in the ICS regime
$W_{EE} = \underline{\alpha}\underline{S}_E + \overline{\alpha}\overline{S}_E + k$	in the ECS regime
$\widetilde{W}_{EI} = \underline{\alpha}(\underline{\widetilde{S}}_E + t\underline{\widetilde{q}}_E) + \overline{\alpha}\overline{S}_I + k$	in the taxation regime
$W_{EI} = \underline{\alpha}\underline{S}_E + \overline{\alpha}\overline{S}_I + k$	in the taxation regime with $t = 0$
$W_{K0} = \underline{\alpha}\underline{S}_K + \underline{\alpha}k$	in the exclusion regime (firm K serves $\underline{\mu}$).

[13]Firstly, remark that the competitive neutrality tax depends on the endogeneous market structure when USOs are imposed. Secondly, this criteria compensates perfectly the firm who incurs USOs but not its competitors.

In the benchmark case analysed above, and under our assuption $\overline{S}_K < 0$, the welfare is thus W_{I0} or W_{E0} according to which firm is the most efficient on the market of low costs consumers. The following remark follows readily from Lemma 2.

Remark 3 *The welfare in the taxation regime $\widetilde{W}_{EI}$ is nonincreasing with respect to the tax level t.*

Let W be the value of the global welfare at the competitive equilibrium, and $\widetilde{W}^{USO}(t)$ the value of the global welfare at equilibrium with USO and tax. Then the difference $\Delta W^{USO} = \widetilde{W}^{USO}(t) - W$, that may be positive or negative, measures the variation of welfare due to the USO constraint. The variations of the global welfare may be decomposed into individual contributions, which allows to evaluate the transfers associated with a particular regime. Both constraints also induce welfare losses. According to the way USOs are attributed and funded, these losses may be incurred by firms and/or by $\underline{\mu}$ type consumers. As a whole, the global welfare may increase or decrease, according to which partial effect dominates. Moreover, the two systems that we examine may lead to opposite conclusions. An interesting question is to compare the welfares under the restricted-entry and the pay or play regulation.

3 The restricted-entry regulation: Firm I serves high cost consumers

Recall that under the restricted-entry rule, the USOs are imposed to the incumbent (firm I) and that the entrant (firm E) cannot choose to serve the consumers relevant for the USO instead of paying the tax. We consider successively the cases where the USO is restricted to U only, and defined by UND. We derive first the equilibria of the game, identify thereafter the possible inefficiencies generated by the situation and proceed finally with the welfare and redistribution analysis.

3.1 Equilibria of the competition process under USOs

The constraints U and UND that the regulator can impose on firm I have different consequences on the equilibrium configurations and on their welfare implications. We examine successively these various constraints. Recall that we restrict ourselves to the case where $\bar{S}_I < 0$ and $\bar{S}_E < 0$.

3.1.1 Equilibria under ubiquity constraint alone

As explained in the introduction, the competition process (for a given tax level t) is modelled by a sequential game, where the incumbent acts as a leader: firm I announces two values $\underline{u}_I$ and $\bar{u}_I$ such that: $\underline{u}_I \geq 0$ and $\bar{u}_I \geq 0$ (ubiquity constraint). Since firm E is not allowed to serve consumer $\bar{\mu}$, we have clearly: $\bar{u}_I = 0$. Firm E's profit function is

$$\widetilde{\pi}_E = \begin{cases} 0 & \text{in the ICS regime} \\ \underline{\alpha}(\tilde{S}_E - \underline{u}_I) & \text{in the taxation regime.} \end{cases}$$

Firm E chooses to serve $\underline{\mu}$ if and only if $\underline{u}_I < \tilde{S}_E$. Therefore the announcement of $\underline{u}_I$ determines the regime that will prevail: $\underline{u}_I < \tilde{S}_E$ leads to the taxation regime, $\underline{u}_I > \tilde{S}_E$ leads to the ICS regime. Firm I's profit function is then given by

$$\widetilde{\pi}_I = \begin{cases} \underline{\alpha}(\underline{\mathrm{S}}_I - \underline{u}_I) + \bar{\alpha}\bar{S}_I & \text{if } \underline{u}_I > \tilde{S}_E \quad \text{ICS regime} \\ \underline{\alpha}t\tilde{q}_E + \bar{\alpha}\bar{S}_I & \text{if } \underline{u}_I < \tilde{S}_E \quad \text{taxation regime} \end{cases}$$

Then the strategy of the incumbent reduces to the choice of the regime: In the taxation regime, firm I's profit does not depend on $0 \leq \underline{u}_I < \tilde{S}_E$ (which leads to a multiplicity of equilibria); in the ICS regime, firm I's optimal announcement is of course $\underline{u}_I = \tilde{S}_E$. Then firm I has to compare the corresponding values of its profit. We have:

$$\widetilde{\pi}_I = \max\left(\underline{\alpha}(\underline{\mathrm{S}}_I - \tilde{S}_E) + \bar{\alpha}\bar{S}_I, \underline{\alpha}t\tilde{q}_E + \bar{\alpha}\bar{S}_I\right).$$

In all the sequel, we will express the profits of firm I in the different regimes in terms of the corresponding welfares. We can rewrite $\widetilde{\pi}_I$ as

$$\widetilde{\pi}_I = \max\left(W_{II} - k - \underline{\alpha}\tilde{S}_E, \widetilde{W}_{EI} - k - \underline{\alpha}\tilde{S}_E\right).$$

Up to a constant $(\underline{\alpha}\tilde{S}_E + k)$, the profits of I in the regimes ICS and TR coincide with the corresponding welfares W_{II} and $\widetilde{W}_{EI}$. We introduce the following convention to express the fact that firm I's compares the quantities W_{II} and $\widetilde{W}_{EI}$ when choosing the regime:

$$\widetilde{\Pi}_I = \max(W_{II}, \widetilde{W}_{EI}).$$

The notation $\widetilde{\Pi}_I$ denotes the profit of I (in presence of the tax t) in the regimes ICS and TR up to a constant. This leads to the following proposition.

Proposition 1 (Restricted-entry regulation, USO=U) *For any level of the tax, the perfect subgame equilibria lead to the regime associated to the highest value of the welfare: Firm I chooses the taxation regime when $\widetilde{W}_{EI} > W_{II}$ and it chooses to cross-subsidize when $W_{II} > \widetilde{W}_{EI}$.*

It is interesting to note that since the regulator can transfer its objective to the incumbent, the restricted-entry regulation avoids incentives problems that could appear due to informational asymmetries between parties: even if the regulator observes neither the costs of the firms nor the consumers' characteristics, delegation to the incumbent of the ubiquity constraint solves the adverse selection problems associated with unobservability of the firms' and the consumers' characteristics[14].

[14]However, second price auctions would also solve such informational problems.

By Lemma 2, the fonction $\widetilde{W}_{EI}$ decreases with t. Then if $W_{EI} = \widetilde{W}_{EI}(0) < W_{II}$, we have: $\widetilde{W}_{EI}(t) < W_{II}$ for all $t \geq 0$ and the ICS regime occurs for all tax level. If $W_{EI} \geq W_{II}$, there exists a tax level τ^U such that

$$\widetilde{W}_{EI}(\tau^{\mathrm{U}}) = W_{II}.$$

The taxation regime occurs for $t \leq \tau^{\mathrm{U}}$ and the ICS regime occurs for $t > \tau^{\mathrm{U}}$. It is intuitive that when the tax rate is high, the entrant prefers to stay out of the market. A given level of tax generates a given level of the sum $\tilde{S}_E + t\tilde{q}_E$, which measures the "efficiency" of the relationship between E and the $\underline{\mu}$ consumer, conditionnally to t. Recall that this quantity is a decreasing function of t.

The following result gives the consumers' surpluses in the different regimes.

Corollary 1 (SPE in the restricted-entry game, USO=U) *In both regimes, the consumer $\bar{\mu}$ gets $\bar{u} = 0$. The utility of consumer $\underline{\mu}$ depends on the regime:*

$$\underline{u} = \begin{cases} \underline{u}_I = \tilde{S}_E & \textit{in the ICS regime} \\ \underline{u}_E \in [0, \tilde{S}_E] & \textit{in the taxation regime.} \end{cases}$$

The cross-subsidy regime corresponds to a situation where I is relatively more efficient on the market of low cost consumers than E when the latter is submitted to the tax. In this case, I announces for both types of consumers values that E cannot match because the tax is too high (and thus the quantity $\tilde{S}_E + t\tilde{q}_E$ is too small). Therefore, I serves both types of consumers and cross-subsidizes between them (that is, I finances the deficit created by the constraint U on high costs consumers by a transfer levied on low cost consumers).

In the taxation regime, by contrast, the sum of the surplus generated by the relationship between E and $\underline{\mu}$ and of the amount of taxes collected is sufficiently high to allow both firms to be active, each of them serving a different segment of the market. In this regime, there are multiple equilibria[15] which can only be selected by an additionnal criterion: a criterion of dominance for firms would imply $\underline{u} = 0$, whereas if there would exist a second entrant, competition on the $\underline{\mu}$ would lead to $\underline{u} = \tilde{S}_E$. An important consequence of this result is the following.

Under ubiquity, a criterion of 'pure' productive efficiency would consist in the comparison of W_{II} an W_{EI}. By Proposition 1, we know that firm I compares W_{II} and $\widetilde{W}_{EI}$. Since $\widetilde{W}_{EI} < W_{EI}$, it follows that the ICS regime is "too often" implemented with regard to 'pure' productive efficiency.

However, a reasonable criterion of productive efficiency should take into account the fact that ubiquity is funded through taxes. If we define productive efficiency as the comparison between W_{II} and $\widetilde{W}_{EI}$, we see that the delegation of the regulator's objective function to firm I allows to choose the socially preferred regime.

[15]If we had used a simultaneous game instead of a sequential game, the equilibrium would have been unique. Note, however, that a Nash equilibrium may fail to exist when the regulator imposes also the non discrimination constraint.

In addition, a tax level equal to 0 allows to achieve pure productive efficiency. A zero tax, however, may fulfill neither of the tax constraints (budget balance of the incumbent or competitive neutrality).

3.1.2 Equilibria of the restricted-entry game with non discrimination constraint

The regulator imposes that each consumer receives the same nonnegative surplus. This constraint is implemented by restricting the space of strategies of the incumbent ($\underline{u}_I = \bar{u}_I \geq 0$). The proof of the following can be found in appendix (section A.2).

Proposition 2 (Restricted-entry regulation, USO=UND) *Firm I's profit is given, up to a constant, by*

$$\widetilde{\Pi}_I \equiv \max(W_{II} - \bar{\alpha}\tilde{S}_E, \widetilde{W}_{EI}).$$

By contrast with the case USO=U (see Proposition 1), the profit of the incumbent does not coincide with the welfares in the regimes ICS and TR. Under UND, the choice of the incumbent differs from that of the regulator (for any given tax level t).

By Lemma 2, the function $\widetilde{W}_{EI} + \bar{\alpha}\tilde{S}_E$ is decreasing with respect to t. Let τ^{UND} be the value of the tax for which

$$\widetilde{W}_{EI}(\tau^{\mathrm{UND}}) = W_{II} - \overline{\alpha}\tilde{S}_E(\tau^{\mathrm{UND}}).$$

It is worth noting that $\tau^{\mathrm{UND}} \geq \tau^{\mathrm{U}}$. We have the following

Corollary 2 (SPE in the restricted-entry game, USO=UND) *The ICS regime occurs if and only if*

$$\widetilde{W}_{EI} \leq W_{II} - \bar{\alpha}\tilde{S}_E \quad \textit{or, equivalently} \quad t \geq \tau^{\mathrm{UND}}.$$

The taxation regime can occur only if $\underline{S}_E > \underline{\alpha S}_I$. In that case, it appears when $0 \leq t < \tau^{\mathrm{UND}}$. Consumers' surplus depends on the regime:

- *cross-subsidy regime (ICS): $\underline{u}_I = \overline{u}_I = u_I = \tilde{S}_E$;*
- *Taxation regime (TR): $\underline{u} = \overline{u} = 0$.*

Recall that we have identified a first distortion due to the tax: Under USO=U, firm I compares W_{II} and $\widetilde{W}_{EI}$, instead of W_{II} and W_{EI}. The tax distortion $W_{EI} - \widetilde{W}_{EI}$ implies that the ICS regime is too often implemented.

Proposition 2 brings out an additional distortion under USO=UND, coming from the fact that under UND firm I has to leave $\overline{\alpha}\tilde{S}_E$ to the $\overline{\mu}$ consumers. The distortion $\bar{\alpha}\tilde{S}_E$ has the opposite impact: compared to productive efficiency, firm I chooses for too large a set of parameters to remain in the taxation regime, instead of choosing the ICS regime

($\tau^{\mathrm{UND}} \geq \tau^{\mathrm{U}}$). The ND constraint softens the competition: since the incumbent has to leave the same utility to both consumers, it is less agressive on the low cost consumers' market[16]. By contrast, the taxation regime allows the incumbent to leave a zero utility to high costs consumers.

The distortion due to ND appears particularly clearly in the following case. Suppose that firm I is more efficient on both markets of consumers $\overline{\mu}$ and $\underline{\mu}$ ($0 < \underline{S}_E < \underline{S}_I$ and $\overline{S}_E < \overline{S}_I < 0$, so that $W_{II} > W_{EI}$). When ubiquity (U) alone is at work, firm I serves $\overline{\mu}$ and $\underline{\mu}$ (cross-subsidy regime). Under UND, however, firm E serves $\underline{\mu}$ if and only if the condition

$$W_{EI} < W_{II} < \widetilde{W}_{EI} + \bar{\alpha}\tilde{S}_E$$

is satisfied. This condition expresses the fact that the ND distortion ($\bar{\alpha}\tilde{S}_E$) dominates the tax distortion ($W_{EI} - \widetilde{W}_{EI}$). In that case, firm E is active on the market whereas I is more efficient.

We now can analyse the welfare and redistributive implications of imposing USOs depending on their funding is realized through cross-subsidies or through taxation.

3.2 Welfare and redistribution analysis

Consider first the case where U is imposed alone. The following proposition, which follows directly from Proposition 1, states the implications of U in terms of welfare.

Lemma 3 (Restricted-entry regulation, USO=U, welfare analysis) *The welfare under the restricted-entry regulation and with the ubiquity constraint is given by*

$$W_{re}^{U}(t) = \max(\widetilde{W}_{EI}(t), W_{II}).$$

The function W_{re}^{U} is nonincreasing and continuous with respect to the tax level t.

More precisely, it is strictly decreasing in the taxation regime and then constant in the cross-subsidy regime.

Consider now the case where both constraints are imposed together (UND). We have the following lemma.

Lemma 4 (Restricted-entry regulation, USO=UND, welfare analysis) *The welfare under the restricted-entry regulation with USOs U and ND is given by*

$$W_{re}^{UND}(t) = \begin{cases} \widetilde{W}_{EI} & \text{if } \widetilde{W}_{EI} \geq W_{II} - \bar{\alpha}\tilde{S}_E \\ W_{II} & \text{if } \widetilde{W}_{EI} \leq W_{II} - \bar{\alpha}\tilde{S}_E. \end{cases}$$

The function W_{re}^{UND} is non increasing with respect to t and discontinuous at $t = \tau^{\mathrm{UND}}$.

[16] This result is also present in Anton and alii [1]

A consequence of this proposition is that the tax level which maximizes the welfare (that is, the first best tax), in the case where the taxation regime exists, i.e. when $W_{EI} > W_{II}$, is $t^{FB} = 0$. In the cross-subsidy regime, the welfare is constant with respect to the tax rate. However, for this level t^{FB} of the tax rate, firm I incurs a deficit. Therefore it is interesting to examine what happens for the tax level t^{BB} that guarantees the budget balance of firm I: $\widetilde{\pi}_I(t^{\mathrm{BB}}) = 0$. This is done in the following proposition.

Proposition 3 *Assume $\overline{S}_E < 0, \overline{S}_I < 0$ and $\underline{S}_I < \underline{S}_E$ or $W_{II} < W_{EI}$. The profit of the incumbent in the benchmark case is $\pi_I = 0$. Then, whatever the USOs at work:*

- *Competitive neutrality and budget balance are equivalent.*
- *The taxation regime exists for small values of t and the first best tax is $t^{FB} = 0$. This first best tax implies a deficit for the incumbent: $\widetilde{\pi}_I(0) = \overline{\alpha}\overline{S}_I < 0$.*
- *If $\underline{\alpha}\underline{S}_I + \overline{\alpha}\overline{S}_I = \widetilde{\pi}_I(t^e) \geq 0$, that is, when the monopoly of I is viable, then there exists at least a competitively neutral tax.*

When $t = t^{\mathrm{BB}}$ the welfare is $\widetilde{W}_{EI} = \underline{\alpha}\left(\tilde{S}_E + t^{\mathrm{BB}}\tilde{q}_E\right) + \overline{\alpha}\overline{S}_I + k$ in the taxation regime, whereas in the case where I cross-subsidizes, it is W_{II}. In the benchmark case, it is equal to $W_{E0} = \underline{\alpha}\underline{S}_E + \underline{\alpha}k$. Clearly the comparison with the benchmark depends on the value of k.

The last point of Proposition 3 follows from the continuity of the profit function $\widetilde{\pi}_I$ with respect to t. Since $\widetilde{\pi}_I(0) < 0$ and $\widetilde{\pi}_I(t^e) > 0$, there exists at least a solution to the equation $\widetilde{\pi}_I(t) = 0$.

Remark 4 *Suppose now $\underline{S}_I > \underline{S}_E$ (and still $\overline{S}_E < 0, \overline{S}_I < 0$).*

- *The profit of firm I in the benchmark case is: $\pi_I = \underline{\alpha}(\underline{S}_I - \underline{S}_E) > 0$. Therefore, competitive neutrality implies budget balance for the firm I.*
- *If $USO = U$, the cross-subsidy regime prevails for all t. The welfare does not depend on the tax level.*
- *If $USO = UND$, the taxation regime occurs when $t < \tau^{UND}$ if $\underline{S}_E > \underline{\alpha}\underline{S}_I$.*
- *In both cases, if $\underline{\alpha}\underline{S}_E + \overline{\alpha}\overline{S}_I \geq 0$, then there exists a competitively neutral tax.*

At this stage, we can discuss the social benefits and costs of the USO's.

Assume for instance that $\underline{S}_E < \underline{S}_I$ (and still $\overline{S}_I$, $\overline{S}_E < 0$). Then, in the benchmark case, firm I serves the low cost consumers and high costs consumers are not served; the welfare is then W_{I0}. If UND is imposed, then the welfare is given by the above proposition. In the taxation regime (see point 3 of the above remark), the variation of welfare with regard to the benchmark case can be written as:

$$\Delta W^{\mathrm{UND}} = \widetilde{W}_{EI} - W_{I0} = \bar{\alpha}k + \bar{\alpha}\bar{S}_I + \underline{\alpha}(\tilde{S}_E + t\tilde{q}_E - \underline{\mathrm{S}}_E) + \underline{\alpha}(\underline{\mathrm{S}}_E - \underline{\mathrm{S}}_I).$$

The first term, which is positive, represents the gain, in terms of social welfare, due to the connection of the high costs consumers. The other terms are negative. The term $\bar{\alpha}\bar{S}_I$ represents the cost associated with the connection of these consumers. The third term represents the loss in *allocative efficiency* (distortion of the surplus due to the taxation). The last term represents the *productive inefficiency*, due to the fact that the most efficient firm for serving these consumers, that is, firm I is replaced by firm E (less efficient).

The choice of a tax level has also important consequences in terms of redistribution. We now compare the benefits obtained by the various classes of consumers accross the various possible regimes.

Suppose first that UND is at work. If the tax rate t leads to the regime where I cross-subsidizes, then both types of consumers obtain a positive surplus $\underline{u}_I = \bar{u}_I = \tilde{S}_E > 0$, which depends on t. As Gasmi et alii (1999) pointed out, cross-subsidization corresponds to implicit taxation and has important redistributional effects.

In the taxation regime, where the tax is perceived, all consumers have $\underline{u}_E = \bar{u}_I = 0$. Therefore, the choice between cross-subsidies or taxation regime raises an equity issue, that has to be solved by political choices and not only through *ex-ante* criteria, such as competitive neutrality.

Suppose now USO=U. In the taxation regime, the sharing of the surplus between $\underline{\mu}$ consumers and E cannot be determined. However, they have $\underline{u}_I = \tilde{S}_E$ in the cross-subsidies regime and $\underline{u}_E \leq \tilde{S}_E$ in the taxation regime. High costs consumers are indifferent, as whatever the regime, they have $\bar{u}_I = 0$. The ubiquity constraint generates no redistributional effect between consumers. However, both consumers are better in the ICS regime.

4 Pay or Play: the entrant may serve high cost consumers

In this section, we investigate the alternative allocation rule where the entrant may choose to serve the non profitable users instead of paying the tax. As in the previous section we consider separately the effects of U and of UND.

4.1 The ubiquity constraint under the pay or play rule

We first compute the equilibria of the pay or play game and thereafter examine its welfare properties.

4.1.1 Equilibria of the competition process

The first important result shows that the profit of firm I coincides again with that of the regulator and that the social objective can thus be decentralized. But compared to the restricted-entry regulation, the pay or play regulation allows to achieve another regime where firm E serves all the consumers (the ECS regime) and finances its deficit on high costs consumers through cross-subsidies. Let τ_{pop} be the value of the tax for which

$$\underline{\alpha}\left(\underline{S}_E - \tilde{S}_E\right) + \overline{\alpha}\overline{S}_E = 0$$

that is, $\underline{\alpha}\tilde{S}_E = W_{EE} - k$. If t is lower than this threshold, then E has no incentive to serve the non profitable users whereas if it is higher, E can profitably serve both classes by cross-subsidization.

The proposition and the corollary below are proved in appendix (section A.3).

Proposition 4 (POP regulation, USO=U) *Under the pay or play regulation, when only the ubiquity constraint is at work, the ECS regime may appear only for $t > \tau_{\text{pop}}$. The profit of the incumbent is given (up to a constant) by*

$$\widetilde{\Pi}_I = \begin{cases} \max(W_{II}, \widetilde{W}_{EI}) & \text{if } t \leq \tau_{\text{pop}} \\ \max(W_{II}, \widetilde{W}_{EI}, W_{EE}) & \text{if } t > \tau_{\text{pop}}. \end{cases}$$

Therefore the profit is identical, up to a constant, to the welfare in the corresponding regime.

As in the restricted-entry regulation, the choice of the regime by firm I is thus that of the regulator (for a given level of the tax) who can thus decentralize the social optimum.

However, the pay or play regulation now introduces, for some values of the tax rate, the possibility that firm E serves both classes of consumers, a situation which was not achievable under the restricted-entry regulation. The following lemma now gives the equilibria of the game.

Corollary 3 (POP, USO=U, Perfect equilibria) *At equilibrium of the pay or play regulation with ubiquity constraint, two cases may appear, according to the value of the tax.*

(i) When $t \leq \tau_{pop}$, the situation is identical to the restricted-entry regulation. The regime ICS and TR may occur. The welfare is given by

$$W^U_{pop} = W^U_{re} = \max(\widetilde{W}_{EI}, W_{II}).$$

The equilibria are the same as in the restricted-entry game:

- *if $\widetilde{W}_{EI} < W_{II}$: I cross-subsidizes (ICS), $\underline{u}_I = \tilde{S}_E, \overline{u}_I = 0$*
- *if $\widetilde{W}_{EI} > W_{II}$: Taxation regime (TR), $\underline{u}_E \in [0, \tilde{S}_E], \overline{u}_I = 0$.*

(ii) When $t > \tau_{pop}$, three regimes (ICS, TR, ECS) may occur. Then the welfare is given by

$$W^U_{pop} = \max(\widetilde{W}_{EI}, W_{II}, W_{EE}).$$

The equilibria are given by

- *if* $W^U_{pop} = W_{II}$, *I cross-subsidizes (ICS) and* $(\underline{u}_I, \overline{u}_I) \in D_{II}$.
- *if* $W^U_{pop} = \widetilde{W}_{EI}$, *the taxation regime occurs (TR), and* $(\underline{u}_I, \overline{u}_I) \in D_{EI}$.
- *if* $W^U_{pop} = W_{EE}$, *E cross-subsidizes (ECS) and* $(\underline{u}_I, \overline{u}_I) \in D_{EE}$ *where*

  $D_{EE} = \left\{(\underline{u}_I, \overline{u}_I) / \overline{\alpha u}_I \leq \underline{\alpha}(\underline{S}_E - \widetilde{S}_E) + \overline{\alpha}\overline{S}_E \text{ and } \underline{\alpha u}_I + \overline{\alpha u}_I \leq W_{EE} - k\right\}$

  $D_{II} = \left\{(\underline{u}_I, \overline{u}_I) / \underline{u}_I \geq \widetilde{S}_E \text{ and } \underline{\alpha u}_I + \overline{\alpha u}_I = W_{EE} - k\right\}$

  $D_{EI} = \left\{(\underline{u}_I, \overline{u}_I) / \overline{\alpha u}_I = \underline{\alpha}(\underline{S}_E - \widetilde{S}_E) + \overline{\alpha}\overline{S}_E \text{ and } \underline{u}_I \leq \widetilde{S}_E\right\}$.

The pay or play regulation has a number of welfare implications that are now examined more in detail.

4.1.2 Welfare consequences of the ubiquity constraint: A comparison between the restricted-entry and the pay or play regulations

The virtue of the pay or play regulation, compared to the restricted-entry regulation, is that it allows, in situations where E is more efficient, to allocate the market of high costs consumers to firm E. It results that the pay or play regulation offers an additionnal possibility for the allocation of the users compared to the restricted-entry one. It thus enhances productive efficiency. Moreover, it is easy to check the following points:

- In the taxation regime, high costs consumers obtain $\overline{u} = 0$ in the restricted-entry regulation, and $\overline{u} = \frac{1}{\overline{\alpha}}\left[\underline{\alpha}(\underline{S}_E - \widetilde{S}_E) + \overline{\alpha}\overline{S}_E\right] > 0$ in the pay or play regulation. Therefore the high cost consumers prefer the pay or play regulation.
- In the regime where I cross-subsidizes, consumers $\overline{\mu}$ have $\overline{u} = 0$ in the restricted-entry regulation and a value $\overline{u} \in \left[0, W_{EE} - k - \underline{\alpha}\widetilde{S}_E\right]$ in the pay or play regulation; consumers $\underline{\mu}$ have $\underline{u} = \widetilde{S}_E$ under the restricted-entry regulation, whereas they obtain a value $\underline{u} \in \left[\widetilde{S}_E, W_{EE} - k\right]$ in the pay or play regulation. Thus both classes of consumers benefit from the pay or play regulation instead of the restricted-entry one.
- In the regime where E cross-subsidizes, there is a multiplicity of equilibria that share differently the surplus between consumers and firm E.

An important consequence of the first remark is that the pay or play regulation involves redistribution between consumers, although the ND constraint is not imposed (this redistribution does not appear in the restricted-entry regulation, under USO=U). These remarks lead to the following proposition.

Proposition 5 (USO=U, Welfare and utility comparison) *When the ubiquity constraint alone is at work, the welfare and the consumers'surplus are both always at least higher under the pay or play regulation than under the restricted-entry regulation.*

We have seen that the welfare configuration depends on the comparison between three terms: W_{II}, W_{EI}, W_{EE}. More precisely, at equilibrium, the welfare exhibits the following properties.

- If W_{II} is the highest, then the welfare is flat (constant with respect to t), the equilibrium configuration is that where I cross-subsidizes. Whatever the value of t, the profit of firm I is $\pi_I = W_{II} - W_{EE} > 0$. Every t is then a first best tax.
- If W_{EE} is the highest, then the first best is achieved for all $t > \tau_{\text{pop}}$. For such values of t, we have $\pi_I = 0$, and these tax rates verify the budget constraint and are competitively neutral.
- If W_{EI} is the highest, the first best is achieved for $t = 0$ and for that value, the profit of firm I is $\widetilde{\pi}_I(0) = \overline{\alpha}\overline{S}_I < 0$. The balanced-budget tax t^{BB} is thus a second best. Budget balance and competitive neutrality coincide, but this (common) level of the tax may lead to any of the possible regimes.

4.2 Ubiquity and non discrimination under pay or play regulation

We now examine the case where the incumbent is tied down to a non discrimination constraint. Firm E is allowed to serve the high costs consumers $\bar{\mu}$. Since we assume $\bar{S}_E < 0$, it is clear that firm E never serves consumer $\bar{\mu}$ only. Consider the case (ECS) where firm E serve both types of consumers. Recall that I must announce $\underline{u}_I = \bar{u}_I \geq 0$ because of the ubiquity and non discrimination constraints. It follows that, in the case (ECS)

$$\underline{u}_E = \bar{u}_E \geq 0.$$

In other words, firm E has to fulfill both constraints also (in case it serves $\underline{\mu}$ and $\bar{\mu}$). Therefore, in that case, there is no reason for E to pay a tax when it serves both markets and cross-subsidizes (ECS regime).

Recall that we have defined τ_{pop} by

$$\underline{\alpha}\mathrm{S}_E + \bar{\alpha}\bar{S}_E = \underline{\alpha}\tilde{S}_E.$$

We introduce another threshold $\bar{\tau}$, defined by

$$\underline{\alpha}\mathrm{S}_E + \bar{\alpha}\bar{S}_E = \tilde{S}_E.$$

Since $\tilde{S}_E$ is decreasing with respect to t, we have: $\tau_{\text{pop}} \leq \bar{\tau}$. As usual, the regimes at equilibrium follow immediately from the profit function of firm I. The proposition below (proved in appendix section A.4) gives the profit of the incumbent in the different regimes.

Proposition 6 (USO=UND, POP regulation, SPE) *Under the pay or play regulation and USO=UND, the profit function of firm I is given (up to a constant) by*

$$\widetilde{\Pi}_I = \begin{cases} \max(W_{II} - \bar{\alpha}\tilde{S}_E, \widetilde{W}_{EI}) & \text{if } t \le \tau_{\text{pop}} \\ \max(W_{II} - [\tilde{S}_E - (\underline{\alpha S}_E + \bar{\alpha}\bar{S}_E)], \widetilde{W}_{EI}, W_{EE}) & \text{if } \tau_{\text{pop}} \le t \le \bar{\tau} \\ \max(W_{II}, W_{EE}) & \text{if } \bar{\tau} \le t. \end{cases}$$

Three configurations may prevail at equilibrium, according to the level of the tax rate:

- when the tax rate is very low, it is never profitable for firm E to serve both markets. Therefore, two regimes may appear, ICS or TR, according to the value of the tax;
- for intermediate values of the tax rate, letting E cross-subsidize may appear at equilibrium, depending on the efficiency of both firms;
- when the tax rate is very high, the total amount generated by the taxation regime is so low that this regime disappears, and the equilibrium is either ICS or ECS, according to the respective efficiency of both firms.

Note that with regard to "pure" productive efficiency, the first two configurations involve a distortion, due to the non discrimination constraint. However, this distortion has not the same expression in both cases. More precisely, the distortion induced by the non discrimination constraint writes $\bar{\alpha}\tilde{S}_E$ for $t \le \tau_{\text{pop}}$ (as under the restricted-entry regulation), $\tilde{S}_E - (\underline{\alpha \mathrm{S}}_E + \bar{\alpha}\bar{S}_E)$ for $\tau_{\text{pop}} \le t \le \bar{\tau}$ and cancels for $t \ge \bar{\tau}$. It is easy to check that the distortion is continuous and decreasing with respect to the tax rate.

In the following proposition (see section A.4 for the proof), we compare the welfares under the restricted-entry and the POP regulation (for USO=UND).

Proposition 7 (USO=UND, Welfare comparison) *If $t \le \tau_{\text{pop}}$, the welfare is the same in the POP regulation as in the restricted-entry regulation. If $t > \tau_{\text{pop}}$, this is no longer the case. The welfare under the POP regulation may be strictly lower than under the restricted-entry regulation.*

(i) Suppose first

$$\begin{cases} \tau_{\text{pop}} \le t \le \bar{\tau} \text{ or, equivalently, } \underline{\alpha}\tilde{S}_E \le \underline{\alpha S}_E + \bar{\alpha}\bar{S}_E \le \tilde{S}_E \\ W_{EE} < W_{II} < W_{EE} + [\tilde{S}_E - (\underline{\alpha S}_E + \bar{\alpha}\bar{S}_E)] \\ \widetilde{W}_{EI} + \bar{\alpha}\tilde{S}_E < W_{II}. \end{cases} \quad (2)$$

Then under the restricted-entry regulation, the ICS regime prevails and the welfare is W_{II}, while the POP regulation leads to the ECS regime, where the welfare is $W_{EE} < W_{II}$.

(ii) Suppose now

$$\begin{cases} t > \bar{\tau} \text{ or, equivalently, } \tilde{S}_E \le \underline{\alpha S}_E + \bar{\alpha}\bar{S}_E \\ \widetilde{W}_{EI} > \max(W_{EE}, W_{II}). \end{cases}$$

Then under the restricted-entry regulation, the taxation regime prevails and the welfare is $\widetilde{W}_{EI}$, while the POP regime leads to the regimes ECS or ICS, and thus to a lower welfare.

Unlike what happens when the ubiquity constraint alone is at work (see Proposition 5), the welfare in the POP regulation may be strictly lower than in the restricted-entry regulation. The loss of welfare when it appears may come from the distortion $\tilde{S}_E - (\underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E)$ due to the ND constraint (case (i) above) or to the disappearance of the taxation regime in the POP regulation for high values of the tax level (case (ii) above).

According to Proposition 6, *ex-ante* criteria (competitive neutrality,...) may rule out some socially optimal regimes. The regulator should take this into account when determining the tax level and choosing the regulation rules.

5 Discussion

In this paper, we have analysed and compared the properties of the restricted-entry regulation and the pay or play regulation for allocating and funding universal service obligations in a network market open to new competitors. Of course, other mechanisms may be implemented to fill these universal service obligations both on the allocation and on the funding sides. We now briefly mention two alternative regulatory frameworks : lump sum transfers and second-price auctions.

5.1 Lump sum transfers

In this paper, we have concentrated here on the mechanisms that are financed through taxation. Losses associated with serving the high costs consumers could also be funded through lump sum transfers levied on tax payers.

Suppose for instance USO=U and a transfer T is used to fund the USO[17]. We briefly describe the situation in case the entrant is not allowed to serve high costs consumers. The profit of the incumbent is

$$\pi_I = \begin{cases} \underline{\alpha}(\underline{\mathrm{S}}_I - \underline{\mathrm{S}}_E) + \bar{\alpha}\bar{S}_I + T & \text{if I serves } \underline{\mu} \text{ and } \bar{\mu} \\ \bar{\alpha}\bar{S}_I + T & \text{if E serves } \underline{\mu} \text{ and I serves } \bar{\mu}. \end{cases}$$

The welfares in the different regimes are

$$\begin{array}{l} W_{II}^T = \underline{\alpha}\underline{\mathrm{S}}_I + \bar{\alpha}\bar{S}_I + k + T - (1+\lambda)T = W_{II} - \lambda T \\ W_{EI}^T = \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_I + k + T - (1+\lambda)T = W_{EI} - \lambda T, \end{array}$$

where λ denotes the cost of transfer of public funds (a transfer equal to T thus generates a dead weight loss equal to λT). The profit of the incumbent is given, up to a constant, by

$$\Pi_I = \max(W_{II}, W_{EI}).$$

Recall that with the unit tax, we had $\widetilde{\Pi}_I = \max(W_{II}, \widetilde{W_{EI}})$ (see Proposition 1). Funding the ubiquity constraint through lump sum transfers thus guarantees "pure" productive efficiency.

[17]Remark that in such a case, the distortion on demand disappears.

Now suppose that the regulator chooses a second-best criterion and defines the transfer T^{BB} so as to compensate the incumbent if it is necessary

$$T^{\mathrm{BB}} = \begin{cases} \max(-\underline{\alpha}(\underline{\mathrm{S}}_I - \underline{\mathrm{S}}_E) - \bar{\alpha}\bar{S}_I, 0) & \text{when } W_{II} > W_{EI} \\ -\bar{\alpha}\bar{S}_I & \text{when } W_{EI} > W_{II}. \end{cases}$$

Note that $T^{\mathrm{BB}} \geq 0$. The comparison between the regulation modes depends on the value of $t^{\mathrm{BB}}, T^{\mathrm{BB}}$ and λ and is given in the following proposition.

Proposition 8 (USO=U, restricted-entry regulation, taxation v.s. transfers) *Funding the ubiquity constraint through taxation is better than through lump sum transfers if*

- $T^{\mathrm{BB}} > 0$, *when* $W_{II} > W_{EI}$;
- $\max(\widetilde{W}_{EI}(t^{\mathrm{BB}}), W_{II}) > W^T_{EI}(T^{\mathrm{BB}}) = W_{EI} - \lambda T^{\mathrm{BB}}$, *when* $W_{EI} > W_{II}$.

In the first case ($W_{II} > W_{EI}$), both taxation and lump sum transfers lead to the ICS regime. The welfare is W_{II} with the taxation rule and $W_{II} - \lambda T^{\mathrm{BB}} \leq W_{II}$ with the lump sum transfer. Funding USOs through taxation is thus strictly better than through lump sum transfers if $T^{\mathrm{BB}} > 0$, that is, if

$$\underline{\alpha}(\underline{\mathrm{S}}_I - \underline{\mathrm{S}}_E) + \bar{\alpha}\bar{S}_I < 0.$$

This condition expresses the fact that the incumbent makes losses when it serves both markets, facing the competitive pressure of the entrant. Moreover, the taxation rule provides the regulator with a tool to share the surplus between the incumbent's profit and the consumers' surplus: the tax rate influences the potential competitive pressure $\tilde{S}_E$ (and does not modify the welfare W_{II}). By contrast, with lump sum transfers, the regulator lacks such instruments to implement the sharing of the global surplus between agents.

Suppose now that productive efficiency requires that low costs consumers are served by the entrant ($W_{EI} > W_{II}$). We know that this market structure cannot be implemented by the tax if t^{BB} is too high, that is if $\widetilde{W}_{EI}(t^{\mathrm{BB}}) < W_{II}$. In that case, financing this USO through lump sum transfers is optimal if $W_{EI} - \lambda T^{\mathrm{BB}} > W_{II}$.

5.2 Second-price auctions

Concerning the allocation of the USOs, regulators often use second price auctions (see for example [1]): in such a mechanism, the ubiquity constraint may be sold to the competitors through an auction mechanism. Each competitor bids for a subsidy for serving the $\bar{\mu}$ consumers. The firm that requires the lowest subsidy wins the auctions (i.e. serves the market). Assume that this auction mechanism is financed through transfers. Then the government transfers to the winning firm the value required by the other firm to serve the high cost consumers.

Clearly, at equilibrium of this auction mechanism, each firm K requires a transfer equal to its loss $\bar{\alpha}\left|\overline{S}_K\right|$, the most efficient firm on the market of $\overline{\mu}$ consumers (i.e. the firm K for which $\overline{S}_K$ is maximum) wins the auction and receives $\left|\overline{\alpha}\overline{S}_{K'}\right|$ where K' is the other firm.

The comparison between auction and pay or play depends crucially on whether the auction mechanism allows cross-subsidization (no subsidy is perceived if the firm serves both consumers, see Anton et alii, 1999).

Assume that cross-subsidization is not allowed. Suppose for instance that the taxation regime prevails under the pay or play regulation. Then the welfare is $\widetilde{W}_{EI}$, whereas with a second price auction it is $W_{EI} - \lambda\overline{\alpha}\left|\overline{S}_E\right|$ where I is the winner of the auction. The comparison thus depends on the values of λ and of the taxes: the second price auction dominates the pay or play regulation if

$$\frac{W_{EI} - \widetilde{W}_{EI}}{\overline{\alpha}\left|\overline{S}_E\right|} > \lambda,$$

that is, if λ is not too high.

5.3 Concluding remarks

This paper is a first attempt to characterize the regulation systems from a welfare and a distributional point of view. However, we have only considered here the case of geographical differences between users. But users can also differ in their demand function, due either to various preferences for the network good or to the dispersion of their revenues. Light users (low income agents) can then be disadvantaged (or even excluded from the network) compared to heavy ones, because of the competition on the latter market. Another goal of the regulator could then be to protect the interests of light users. This requires to take into account a more complex structure of demand and another form of USO. We will examine this problem in a future paper.

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A Appendix

A.1 Proof of Lemma 2

The function $\tilde{S}_E$ is a supremum of affine functions, therefore it is a convex function. By the envelop theorem,we have

$$\frac{d\tilde{S}_E(t)}{dt} = -\widetilde{q}_E.$$

The function $\tilde{q}_E$ is therefore decreasing and we have

$$\frac{d}{dt}\left[\tilde{S}_E + t\tilde{q}_E\right] = t\frac{d\tilde{q}_E}{dt} \leq 0.$$

To see the last point, take $\bar{t} = w'(0) - c_E$, where $c_E > C'_E(q, \underline{\mu})$ for all q (the marginal cost $C'_E(q, \underline{\mu})$ is assumed to be bounded). Then the function

$$w(q) - C_E(q, \underline{\mu}) - \bar{t}q$$

is decreasing for $q \geq 0$. Therefore, it attains its maximum at $q = 0$ and $\tilde{S}_E(\bar{t}) = -C_E(0, \mu) \leq 0$. The continuity of $\tilde{S}_E$ gives the existence of t^e such that: $\tilde{S}_E(t^e) = 0$.

A.2 Proof of Proposition 2 and Lemma 2

We consider the restricted-entry regulation, with USO=UND. Recall that the incumbent announces one value $u_I \geq 0$. Since firm E is not allowed to serve $\bar{\mu}$, its profit function is

$$\widetilde{\pi}_E = \begin{cases} 0 & \text{ICS regime} \\ \tilde{S}_E - \underline{u}_I & \text{taxation regime.} \end{cases}$$

Firm E chooses to serve $\underline{\mu}$ if and only if $\underline{u}_I < \tilde{S}_E$. Therefore the announcement of $\underline{u}_I$ determines the regime that will prevail: $\underline{u}_I < \tilde{S}_E$ leads to the taxation regime, $\underline{u}_I > \tilde{S}_E$ leads to the ICS regime. Firm I's profit function is then given by

$$\widetilde{\pi}_I = \begin{cases} \underline{\alpha}\underline{\mathrm{S}}_I + \bar{\alpha}\bar{S}_I - u_I & \text{if } \underline{u}_I > \tilde{S}_E \quad \text{ICS regime} \\ \underline{\alpha}t\tilde{q}_E + \bar{\alpha}(\bar{S}_I - u_I) & \text{if } \underline{u}_I < \tilde{S}_E \quad \text{taxation regime} \end{cases}$$

Then the choice of the incumbent reduces to the choice of the regime: in the taxation regime, firm I's profit is maximum for $u_I = 0$ (unique equilibrium); in the ICS regime, firm I's optimal announcement is of course $\underline{u}_I = \tilde{S}_E$. Then firm I has to compare the corresponding values of its profit. We have:

$$\widetilde{\pi}_I = \max\left(\underline{\alpha}\underline{\mathrm{S}}_I + \bar{\alpha}\bar{S}_I - \tilde{S}_E, \underline{\alpha}t\tilde{q}_E + \bar{\alpha}\bar{S}_I\right).$$

The profit $\widetilde{\pi}_I$ can be written as

$$\widetilde{\pi}_I = \max\left(W_{II} - k - \underline{\alpha}\tilde{S}_E - \bar{\alpha}\tilde{S}_E, \widetilde{W}_{EI} - k - \underline{\alpha}\tilde{S}_E\right),$$

which gives Proposition 2 and Lemma 2.

A.3 Proof of Proposition 4 and Lemma 3

We consider the POP regulation, with USO=U. Recall that the incumbent announces two values $\underline{u}_I \geq 0$ and $\bar{u}_I \geq 0$. The profit of firm E is given by

$$\widetilde{\pi}_E = \begin{cases} 0 & \text{ICS regime} \\ \underline{\alpha}(\tilde{S}_E - \underline{u}_I) & \text{taxation regime} \\ \underline{\alpha}(\underline{\mathrm{S}}_E - \underline{u}_I) + \bar{\alpha}(\bar{S}_E - \bar{u}_I) & \text{ECS regime.} \end{cases}$$

Firm E prefers ECS to TR if and only if

$$\bar{\alpha}\bar{u}_I \leq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E.$$

The preceeding inequality is possible only if

$$\underline{\alpha}\tilde{S}_E \leq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E$$

or, equivalently, $t \geq \tau_{\text{pop}}$. Therefore, if $t < \tau_{\text{pop}}$, the ECS regime cannot occur and the POP regulation leads to the same equilibria as the restricted-entry regulation.

For $t \geq \tau_{\text{pop}}$, the three regimes may appear, according to the value $\underline{u}_I, \bar{u}_I$:

$$\begin{aligned} \underline{\alpha}\underline{u}_I + \bar{\alpha}\bar{u}_I \geq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E \text{ and } \underline{u} \geq \tilde{S}_E &\implies \text{ICS} \\ \bar{\alpha}\bar{u}_I \geq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E \text{ and } \underline{u}_I \leq \tilde{S}_E &\implies \text{TR} \\ \bar{\alpha}\bar{u}_I \leq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E \text{ and } \underline{\alpha}\underline{u}_I + \bar{\alpha}\bar{u}_I \leq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E &\implies \text{ECS.} \end{aligned}$$

The profit function of the incumbent is

$$\widetilde{\pi}_I = \begin{cases} \underline{\alpha}(\underline{\mathrm{S}}_I - \underline{u}_I) + \bar{\alpha}(\bar{S}_I - \bar{u}_I) & \text{ICS regime} \\ \underline{\alpha}t\tilde{q}_E + \bar{\alpha}(\bar{S}_I - \bar{u}_I) & \text{taxation regime} \\ 0 & \text{ECS regime.} \end{cases}$$

In the ICS regime, it is optimal for the incumbent to announce $\underline{u}_I, \bar{u}_I$ such that $\underline{\alpha}\underline{u}_I + \bar{\alpha}\underline{u}_I = \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E$. In the taxation regime, it is optimal for I to announce $\bar{u}_I$ such that $\bar{\alpha}\bar{u}_I = \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E$. In the ECS regime, all the possible values of $(\underline{u}_I, \bar{u}_I)$ lead to the same profit, namely $\widetilde{\pi}_I = 0$. Finally firm I's profit is

$$\widetilde{\pi}_I = \max\left(\underline{\alpha}\underline{\mathrm{S}}_I + \bar{\alpha}\bar{S}_I - \underline{\alpha}\underline{\mathrm{S}}_E - \bar{\alpha}\bar{S}_E, \underline{\alpha}t\tilde{q}_E + \bar{\alpha}\bar{S}_I - [\underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E], 0\right),$$

which can be written as

$$\widetilde{\pi}_I = \max\left(W_{II} - W_{EE}, \widetilde{W}_{EI} - W_{EE}, W_{EE} - W_{EE}\right).$$

A.4 Proof of Proposition 6

We consider the POP regulation with USO=UND. Recall that the incumbent announces one value u_I. The profit of the other firm is

$$\widetilde{\pi}_E = \begin{cases} 0 & \text{ICS regime} \\ \underline{\alpha}(\tilde{S}_E - u_I) & \text{taxation regime} \\ \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - u_I & \text{ECS regime.} \end{cases}$$

Firm E prefers ECS to TR if and only if

$$\bar{\alpha}u_I \leq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E,$$

which is possible only if

$$\underline{\alpha}\tilde{S}_E \leq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E$$

or, equivalently, $t \geq \tau_{\text{pop}}$. Therefore, if $t < \tau_{\text{pop}}$, the ECS regime cannot occur and the POP regulation leads to the same equilibria as the restricted-entry regulation.

For $\tau_{\text{pop}} \leq t \leq \bar{\tau}$, we have:

$$\begin{aligned} 0 \leq u_I \leq [\underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E]/\bar{\alpha} &\implies \text{ECS} \\ (\underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E)/\bar{\alpha} \leq u_I \leq \tilde{S}_E &\implies \text{TR} \\ \tilde{S}_E \leq u_I &\implies \text{ICS} \end{aligned}$$

The profit function of the incumbent is

$$\widetilde{\pi}_I = \begin{cases} \underline{\alpha}\underline{\mathrm{S}}_I + \bar{\alpha}\bar{S}_I - u_I & \text{ICS regime} \\ \underline{\alpha}t\tilde{q}_E + \bar{\alpha}(\bar{S}_I - u_I) & \text{taxation regime} \\ 0 & \text{ECS regime.} \end{cases}$$

The optimal announcement leading to TR is of course $u_I = [\underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E]/\bar{\alpha}$ and the optimal announcement leading to ICS is $u_I = \tilde{S}_E$. This gives the profits

$$\widetilde{\pi}_I = \max\left(\underline{\alpha}\underline{\mathrm{S}}_I + \bar{\alpha}\bar{S}_I - \tilde{S}_E,\ \underline{\alpha}t\tilde{q}_E + \bar{\alpha}\underline{\mathrm{S}}_I - (\underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E - \underline{\alpha}\tilde{S}_E), 0\right),$$

which can be written

$$\widetilde{\pi}_I = \max\left(W_{II} - [\tilde{S}_E - \underline{\alpha}\underline{\mathrm{S}}_E - \bar{\alpha}\bar{S}_E] - W_{EE}, \widetilde{W}_{EI} - W_{EE}, W_{EE} - W_{EE}\right).$$

For $\bar{\tau} \leq t$, we have:

$$\begin{aligned} 0 \leq u_I \leq \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E &\implies \text{ECS} \\ \underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E \leq u_I &\implies \text{ICS.} \end{aligned}$$

The optimal announcement in ICS is of course $\underline{\alpha}\underline{\mathrm{S}}_E + \bar{\alpha}\bar{S}_E$, which gives the profit

$$\widetilde{\pi}_I = \max\left(\underline{\alpha}\underline{\mathrm{S}}_I + \bar{\alpha}\bar{S}_I - \underline{\alpha}\underline{\mathrm{S}}_E - \bar{\alpha}\bar{S}_E, 0\right),$$

which can be written

$$\widetilde{\pi}_I = \max\left(W_{II} - W_{EE}, W_{EE} - W_{EE}\right).$$

A.5 Proof of Proposition 7

(i) We first show that for each $(\underline{S}_E, \bar{S}_E)$ there exists $(\underline{S}_E, \bar{S}_I)$ such that the conditions (2) are satisfied. The third inequality of (2) is obtained by taking $\underline{S}_I$ large enough. Now we can choose $\bar{S}_I$ to ensure the second condition of (2).

Since $\widetilde{W}_{EI} < W_{II} - \bar{\alpha}\tilde{S}_E$, the restricted-entry regulation leads to ICS regime. By using assumptions (2), we obtain

$$W_{EE} > W_{II} - [\tilde{S}_E - (\underline{\alpha}\underline{S}_E + \bar{\alpha}\bar{S}_E)] > W_{II} - \bar{\alpha}\tilde{S}_E > \widetilde{W}_{EI}.$$

It follows that the POP regulation leads to ECS regime (see Proposition 6).

(ii) The proof is straightforward.

A.6 Imposing ND alone under the restricted-entry regulation

In this section, we assume: $\bar{S}_E < 0 < \bar{S}_I$. The incumbent can profitably serve the high cost consumers. We assume that the regulator imposes only the non discrimination constraint and that firm E cannot serve $\bar{\mu}$ (restricted-entry regulation). Firm E cannot be constrained by ND, since it never serves consumer $\bar{\mu}$. Firm I is compensated for the constraint ND by a tax t.

Lemma 5 (Perfect Equilibria in the restricted-entry game, USO=ND) *We make the following assumptions*

$$\begin{array}{l} \bar{S}_E < 0 < \bar{S}_I \\ \underline{\alpha S_I} > \bar{\alpha}\bar{S}_I + \underline{\alpha S_E} \\ \bar{S}_I < \tilde{S}_E. \end{array}$$

Then there exists a unique subgame perfect equilibrium in which firm I serves only $\underline{\mu}$ (giving them $\underline{u}_I = \tilde{S}_E$), $\bar{\mu}$ consumers being then excluded from the market.

Although I would serve both types of consumers in the benchmark case, imposing ND *alone* has the counterveiling effect of excluding high costs consumers from the market: firm I prefers to disconnect (or not to connect) these consumers rather than incurring losses due to cross-subsidization between both classes of consumers. This should lead the regulator to impose both constraints simultaneously.

Proof of Lemma 5

The possible regimes are ICS, TR and the exclusion regime, where $\underline{\mu}$ is served by I and $\bar{\mu}$ is not served. Firm E's profit function is

$$\widetilde{\pi}_E = \begin{cases} \underline{\alpha}(\tilde{S}_E - \underline{u}) & \text{taxation regime} \\ 0 & \text{ICS regime} \\ 0 & \text{exclusion regime (firm I),} \end{cases}$$

It follows that the profit of I is

$$\widetilde{\pi}_I = \begin{cases} \bar{\alpha}(\bar{S}_I - \bar{u}_I) + \underline{\alpha} t \tilde{q}_E & \text{taxation regime} \\ \underline{\alpha}\bar{S}_I + \bar{\alpha}\bar{S}_I - \underline{u} & \text{ICS regime} \\ \underline{\alpha}(\underline{\mathrm{S}}_I - \underline{u}) & \text{exclusion regime (firm I),} \end{cases}$$

The optimal announcement in the taxation regime and in the exclusion regime is of course $\tilde{S}_E$. Therefore firm I maximizes

$$\widetilde{\pi}_I = \max[\bar{\alpha}\bar{S}_I + \underline{\alpha} t \tilde{q}_E, \underline{\alpha}\bar{S}_I + \bar{\alpha}\bar{S}_I - \tilde{S}_E, \underline{\alpha}(\underline{\mathrm{S}}_I - \tilde{S}_E)].$$

The assumptions of the lemma ensure that the maximum is $\underline{\alpha}(\underline{\mathrm{S}}_I - \tilde{S}_E)$ (exclusion regime).