A Multivariate GARCH Model with Time-Varying Correlations

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Abstract: In this paper we propose a new multivariate GARCH model with time-varying correlations. We adopt the vech representation based on the conditional variances and the conditional correlations. While each conditional-variance term is assumed to follow a univariate GARCH formulation, the conditional-correlation matrix is postulated to follow an autoregressive moving average type of analogue. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we manage to construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimisation. Thus, our new model retains the intuition and interpretation of the univariate GARCH model and yet satisfies the positive-definite condition as found in the constant-correlation and BEKK models. We report some Monte Carlo results on the finite-sample distributions of the QMLE of the varying-correlation MGARCH model. The new model is applied to some real data sets. It is found that extending the constant-correlation model to allow for time-varying correlations provides some interesting time histories that are not available in a constant-correlation model.

Key Words: BEKK model, constant correlation, Monte Carlo method, multivariate GARCH model, quasi maximum likelihood estimate, varying correlation

JEL Classification: C12

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1 Introduction

Following the success of the autoregressive conditional heteroscedasticity (ARCH) model and the generalized ARCH (GARCH) model in describing the time-varying variances of economic data in the univariate case many researchers have extended these models to the multivariate dimension. Applications of the multivariate GARCH (MGARCH) models to financial data have been particularly popular. For example, Bollerslev (1990) studied the changing variance structure of the exchange rate regime in the European Monetary System assuming the correlations to be time invariant. Kroner and Claessens (1991) applied the models to calculate the optimal debt portfolio in multiple currencies. Lien and Luo (1994) evaluated the multiperiod hedge ratios of currency futures in a MGARCH framework. Karolyi (1995) examined the international transmission of stock returns and volatility using different versions of MGARCH models. Baillie and Myers (1991) estimated the optimal hedge ratios for commodity futures and argued that these ratios are nonstationary. Gourieroux (1997, Chapter 6) presented a survey of several versions of MGARCH models. See also Bollerslev, Chou and Kroner (1992) and Bera and Higgins (1993) for surveys on the methodology and applications of GARCH and MGARCH models.

Bollerslev, Engle and Wooldridge (1988) provided the basic framework for a MGARCH model. They extended the GARCH representation in the univariate case to the vectorized conditional-variance matrix. Their specification follows the traditional autoregressive moving average time series analogue. While this so-called vech representation is very general, it involves a large number of parameters. Empirical applications require further restrictions and simplifications. A popular member of the vech-representation family is the diagonal form. Under the diagonal form, each variance-covariance term is postulated to follow a GARCH-type equation with the lagged variance-covariance term and the product of the corresponding lagged residuals as the right-hand-side variables in the conditional-(co)variance equation.

It is often di±cult to verify the condition that the conditional-variance matrix of an estimated MGARCH model is positive de¯nite.[1] Furthermore, such conditions are often very di±cult to impose during the optimisation of the log-likelihood function. Bollerslev (1990) suggested the constant-correlation MGARCH (CC-MGARCH) model that can overcome these di±culties. He pointed out that under the assumption of constant correlations, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix. As the sample correlation matrix is always positive de¯nite, the optimisation will not fail as long as the conditional variances are positive. In addition, when the correlation matrix is concentrated out of the log-likelihood function further simpli¯cation is achieved in the optimisation.

Due to its computational simplicity, the CC-MGARCH model is very popular among empirical researchers. However, while the constant-correlation assumption provides a convenient MGARCH model for estimation, some studies found that this assumption is not supported by some ¯nancial data. Bera and Kim (1996) and Tse (1998) found that the stock returns across di®erent national markets exhibit time-varying correlations. Thus, there is a need to extend the MGARCH models to incorporate time-varying correlations and yet retain the appealing feature of satisfying the positive-de¯nite condition during the optimisation.

Engle and Kroner (1995) proposed a class of MGARCH model called the BEKK (named after Baba, Engle, Kraft and Kroner) model. The motivation is to ensure the condition of a positive de¯nite conditional-variance matrix in the process of optimisation. Engle and Kroner provided some theoretical analysis of the BEKK model and related it to the vech-representation form. Another approach examines the conditional variance as a factor model. The works by Diebold and Nerlove (1989), Engel and Rodrigues (1989)

[1]Engle, Granger and Kraft (1984) presented the necessary conditions for the conditional-variance matrix to be positive de¯nite in a bivariate ARCH model. Extensions of these results to more general models are, however, intractable.

and Engle, Ng and Rothschild (1990) were along this line. One disadvantage of the BEKK and factor models is that the parameters cannot be easily interpreted, and their net e®ects on the future variances and covariances are not readily seen. Bera, Garcia and Roh (1997) reported that the BEKK model does not perform well in the estimation of the optimal hedge ratios. Lien, Tse and Tsui (1998) reported di±culties in getting convergence when using the BEKK model to estimate the conditional-variance structure of spot and futures prices.

In this paper we propose a new MGARCH model with time-varying correlations. Basically we adopt the vech representation. The variables of interest are, however, the conditional variances and conditional correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analogue to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we manage to construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive de¯nite during the optimisation. Thus, our new model retains the intuition and interpretation of the univariate GARCH model and yet satis¯es the positive-de¯nite condition as found in the constant-correlation and BEKK models.

The plan of the rest of the paper is as follows. In Section 2 we describe the construction of the varying-correlation MGARCH model. As in other MGARCH models, the new model can be estimated using a quasi-MLE (QMLE) approach. Some Monte Carlo results on the ¯nite-sample distributions of the QMLE of the varying-correlation MGARCH model are reported in Section 3. Section 4 describes some illustrative examples of the new model using some real data sets. These are the exchange rate data, interest rate data and stock price data. The new model is compared against the CC-MGARCH model. It is found that extending the constant-correlation model to allow for time varying correlations provides some interesting empirical results. The estimated conditional-correlation path provides an interesting time history that would be lost in a constant-correlation model. Finally, we give some concluding remarks in Section 5.

2 A Varying-Correlation MGARCH Model

Consider a multivariate time series of observations $\{y_t\}$, $t = 1, \ldots, T$, with K elements each, so that $y_t = (y_{1t}, \ldots, y_{Kt})'$. We assume that the observations are of zero (or known) mean. This assumption simplifies the discussions without straining the notations.[2]

The conditional variance of y_t is assumed to follow the time-varying structure given by

$$\mathrm{Var}(y_t \mid \Psi_{t-1}) = \Omega_t, \qquad (1)$$

where Ψ_t is the information set at time t. We denote the variance elements of Ω_t by σ^2_{it}, for $i = 1, \ldots, K$, and the covariance elements by σ_{ijt}, where $1 \leq i < j \leq K$. Denoting D_t as the $K \times K$ diagonal matrix with the ith diagonal element being σ_{it}, we let $\varepsilon_t = D_t^{-1} y_t$. Thus, ε_t is the standardised residual and is assumed to be serially independently distributed with mean zero and variance matrix $\Gamma_t = \{\rho_{ijt}\}$. Of course, Γ_t is also the correlation matrix of y_t and $\Omega_t = D_t \Gamma_t D_t$.

To specify the conditional variance of y_t, we adopt the vech-diagonal formulation as initiated by Bollerslev, Engle and Wooldridge (1988). Thus, each conditional-variance term follows a univariate GARCH(p, q) model given by the following equation:

$$\sigma^2_{it} = \omega_i + \sum_{h=1}^{p} \alpha_{ih} \sigma^2_{i,t-h} + \sum_{h=1}^{q} \beta_{ih} y^2_{i,t-h}; \qquad i = 1, \cdots, K; \qquad (2)$$

where ω_i, α_{ih} and β_{ih} are nonnegative, and $\sum_{h=1}^{p} \alpha_{ih} + \sum_{h=1}^{q} \beta_{ih} < 1$, for $i = 1, \ldots, K$. Note that we may allow (p, q) to vary with i so that (p, q) should be regarded as the generic

[2] Additional parameters would be required to represent the conditional-mean equation in the complete model if the mean is unknown. Under certain conditions, the MLE of the parameters in the conditional-mean equation is asymptotically uncorrelated with the MLE of the parameters of the conditional-variance equation. Under such circumstances, we may treat y_t as pre-filtered observations (see Bera and Higgins (1993) for further discussions). Otherwise, the parameter vector has to be augmented to take account of the unknown mean.

order of the univariate GARCH process. Researchers adopting the vech-diagonal form typically assume that the above equation also applies to the conditional-covariance terms in which σ_{it}^2 is replaced by σ_{ijt} and y_{it}^2 is replaced by $y_{it}y_{jt}$ for $1 \le i < j \le K$. We shall, however, deviate from this approach. Specifically, we shall focus on the conditional-correlation matrix and adopt an autoregressive moving average analogue on this matrix. Thus, we assume that the time-varying conditional-correlation matrix Γ_t is generated from the following recursion:

$$\Gamma_t = (1 - \theta_1 - \theta_2)\Gamma + \theta_1 \Gamma_{t-1} + \theta_2 \Psi_{t-1}, \qquad (3)$$

where $\Gamma = \{\rho_{ij}\}$ is a (time-invariant) $K \times K$ positive definite parameter matrix with unit diagonal elements and Ψ_{t-1} is a $K \times K$ matrix whose elements are functions of the lagged observations of y_t.[3] The functional form of Ψ_{t-1} will be specified below. The parameters θ_1 and θ_2 are assumed to be nonnegative with the additional constraint that $\theta_1 + \theta_2 \le 1$. Thus, Γ_t is a weighted average of Γ, Γ_{t-1} and Ψ_{t-1}. Hence, if Ψ_{t-1} is a well-defined correlation matrix (i.e., positive definite with unit diagonal elements), Γ_t will also be a well-defined correlation matrix.[4]

It can be observed that Ψ_{t-1} is analogous to $y_{i,t-1}^2$ in the univariate GARCH(1, 1) model. However, as Γ_t is a standardised measure, we also require Ψ_{t-1} to depend on the (lagged) standardised residuals ε_t. Denoting $\Psi_t = \{\psi_{ijt}\}$, we propose to consider the following specification for Ψ_{t-1}:

$$\psi_{ij,t-1} = \frac{\sum_{h=1}^{M} \varepsilon_{i,t-h}\varepsilon_{j,t-h}}{\sqrt{\left(\sum_{h=1}^{M} \varepsilon_{i,t-h}^2\right)\left(\sum_{h=1}^{M} \varepsilon_{j,t-h}^2\right)}}, \qquad 1 \le i < j \le K. \qquad (4)$$

Thus, Ψ_{t-1} is the sample correlation matrix of $\{\varepsilon_{t-1}, \cdots, \varepsilon_{t-M}\}$. We define E_{t-1} as the $K \times M$ matrix given by $E_{t-1} = (\varepsilon_{t-1}, \cdots, \varepsilon_{t-M})$. If B_{t-1} is the $K \times K$ diagonal matrix

[3] For the sake of simplicity and at the risk of being not thorough, we shall describe a correlation matrix as being positive definite. It is not difficult to see that for some statements made in this section, the term "positive definite" should, strictly speaking, be replaced by the term "positive semi-definite".

[4] This statement is subject to the condition that the recursion starts with a well-defined correlation matrix Γ_0. Under such conditions, the diagonal elements of Γ_t are unity and Γ_t remains positive definite.

with the ith diagonal element being $(\sum_{h=1}^{M} \epsilon^2_{i,t-h})^{1/2}$ for $i = 1, \ldots, K$, then we have

$$\Psi_{t-1} = B_{t-1}^{-1} E_{t-1} E_{t-1}' B_{t-1}^{-1}. \qquad (5)$$

Note that when $M = 1$, Ψ_{t-1} is identically equal to the matrix of unity. Updating the conditional-correlation matrix with respect to the matrix of unity is of course not meaningful. Thus, taking first-order lag for the formulation of Ψ_{t-1} is not sufficient. Indeed, $M \geq K$ is a necessary condition for Ψ_{t-1} to be positive definite. When positive-definiteness is satisfied, Ψ_{t-1} is a well-defined correlation matrix. Thus, the condition $M \geq K$ will be imposed subsequently.

Equation (3) is analogous to the univariate GARCH equation, with the additional restriction that the sum of the coefficients is equal to 1. Indeed, Γ_t involves updating the conditional-correlation matrix with respect to the latest conditional-correlation matrix Γ_{t-1} and a sample estimate of the conditional-correlation matrix based on the recent M standardised residuals. We shall call the model specified by (2), (3) and (5) the varying-correlation MGARCH (VC-MGARCH) model.

Assuming normality, $y_t \sim N(0, D_t \Gamma_t D_t)$, so that the log-likelihood ℓ_t of the observation y_t is given by:

$$\ell_t = -\frac{1}{2} \ln|D_t \Gamma_t D_t| - \frac{1}{2} y_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} y_t \qquad (6)$$

$$= -\frac{1}{2} \ln|\Gamma_t| - \frac{1}{2} \sum_{i=1}^{K} \ln \sigma_{it}^2 - \frac{1}{2} y_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} y_t, \qquad (7)$$

from which we can obtain the log-likelihood function of the sample as $\ell = \sum_{t=1}^{T} \ell_t$. Hence, the log-likelihood function is conditional on the values Γ_0, Ψ_0 and y_0 being fixed. These assumptions have no effects on the asymptotic distribution of the QMLE. Denoting $\theta = (\omega_1, \alpha_{11}, \ldots, \alpha_{1p}, \beta_{11}, \ldots, \beta_{1q}, \omega_2, \ldots, \beta_{Kq}, \rho_{12}, \ldots, \rho_{K-1,K}, \theta_1, \theta_2)$ as the parameter vector of the model, the QMLE of θ is obtained by maximising ℓ with respect to θ. We shall denote this value by $\hat{\theta}$.

For parameter parsimony, (p, q) is usually taken to be of low order. For $p = q = 1$, the total number of parameters in the VC-MGARCH model is $3K + K(K+1)/2 + 2$. In comparison, an unrestricted BEKK model with order 1 for both the lagged conditional-covariance matrix term and the outer product of the lagged residuals term has $K(K+1)/2 + 2K^2$ parameters. For example, for $K = 2$, 3 and 4, the number of parameters in the VC-MGARCH model is 9, 14 and 20, respectively, while that for the BEKK model is 11, 24 and 42, respectively. The number of parameters in the VC-MGARCH model always exceeds that of the constant-correlation model by 2, due to the parameters θ_1 and θ_2. Indeed the CC-MGARCH model is nested within the VC-MGARCH model by imposing the restrictions $\theta_1 = \theta_2 = 0$.

The conditions $0 \leq \theta_1, \theta_2 \leq 1$ and $\theta_1 + \theta_2 \leq 1$ pose some problems in the optimisation. One way to get around this difficulty is through transformation. For example, we may define $\theta_i = \lambda_i^2/(1 + \lambda_1^2 + \lambda_2^2)$ for $i = 1, 2$, where λ_1 and λ_2 are unrestricted parameters. The log-likelihood function may be initially optimised with respect to λ_1, λ_2 and other parameters of interest. The optimisation is then shifted to the original vector θ when convergence with respect to λ_1, λ_2 and other parameters has been achieved. This technique is used in the computations reported in this paper.

3 Some Monte Carlo Results

Although the GARCH type of models have been applied extensively in the literature, little has been known about the theoretical asymptotic distribution of the QMLE of these models. Consistency and asymptotic normality have often been assumed. The works of Weiss (1986) and Lumsdaine (1996) represent few of the studies on the asymptotic distribution of the QMLE in the univariate case. For MGARCH models, theoretical results are even more scanty.

In the univariate case, Engle, Hendry and Trumble (1985), Bollerslev and Wooldridge (1992) and Lumsdaine (1995) examined the small-sample properties of the QMLE of the

ARCH and GARCH models. In this section we report some results on the small-sample properties of the QMLE of the VC-MGARCH model based on a small-scale Monte Carlo experiment. It is not our intention to provide a comprehensive Monte Carlo study of the QMLE. We shall focus our interest on the small-sample bias and mean squared error only. The reliability of the inference concerning the model parameters will not be examined. Our results, however, will provide some preliminary evidence with respect to the small-sample properties of the QMLE of the VC-MGARCH model.

We consider bivariate VC-MGARCH models in which the conditional-variance equations are given by:

$$\sigma_{it}^2 = \omega_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i y_{i,t-1}^2; \qquad i = 1, 2; \tag{8}$$

and the conditional correlation coefficient is given by:

$$\rho_t = (1 - \theta_1 - \theta_2)\rho + \theta_1 \rho_{t-1} + \theta_2 \psi_{t-1}; \tag{9}$$

where ψ_{t-1} is given by:

$$\psi_{t-1} = \frac{\sum_{h=1}^{2} \epsilon_{1,t-h}\epsilon_{2,t-h}}{\sqrt{\left(\sum_{h=1}^{2} \epsilon_{1,t-h}^2\right)\left(\sum_{h=1}^{2} \epsilon_{2,t-h}^2\right)}}: \tag{10}$$

with $\epsilon_{it} = y_{it}/\sigma_{it}$ for $i = 1, 2$.[5]

We consider four experimental setups. The true parameter values of the data generating processes of these experiments, labelled E1 through E4, are given in Tables 1.1 and 1.2. Observations $\{y_t\}$ are generated from these models assuming the errors are normally distributed. We consider $T = 500$; 1000 and 1500. The QMLE are calculated for each generated sample. Using Monte Carlo samples of 1000 runs, we estimate the bias and mean squared error (MSE) of the QMLE.

E1 and E2 represent models with higher volatility persistence (as measured by $\alpha_i + \beta_i$), while E3 and E4 represent models with lower volatility persistence. The selected

[5] All computations reported in this paper assume $M = K$ in the definition of Ψ_t.

values of ½ in the experiments vary from 0.2 to 0.7. It can be seen from Tables 1.1 and 1.2 that the biases of the QMLE are generally quite small. The bias decreases with the sample size, although in some cases not steadily. Likewise, the same is true for the MSE. Overall, the Monte Carlo results suggest that the QMLE is likely to be consistent. For the sample sizes and models considered, the bias and MSE appear to be small.

In the next section, we report the empirical results of applying the VC-MGARCH model to some real data sets.

4 Some Illustrative Examples

We examine three sets of ¯nancial data. These are the exchange rate data, interest rate data and stock price data, denoted by DS1, DS2 and DS3, respectively. DS1 consists of two exchange rate (versus US dollar) series, namely, the British Pound (B) and the Deutschmark (D). These series represent daily observations from January 1990 to June 1998, with 2137 observations. DS2 covers three series of bond yield data, consisting of the yields of the 3-month Treasury Bill (M), the 1-year Treasury Note (O) and the 10-year Treasury Bond (T) in the US. The observations are weekly (Wednesday) ¯gures from January 1982 through April 1998, totalling 850 observations. DS3 covers the stock price indices of three national markets. These are the markets of Hong Kong (H), Japan (J) and Singapore (S), as measured by the Hang Seng Index, the Nikkei Stock Average 225 and the SES-ALL Index, respectively. We sample the data from every ¯fth trading day for the period of January 1990 through December 1997, with 340 observations. This sampling method alleviates the problem of nonsynchronous trading days for the three markets. DS1 and DS2 were downloaded from the website of the Federal Reserve Bank of Chicago. DS3 was compiled from various issues of the Stock Exchange of Singapore Journal.

Figures 1 through 3 present the plots of the eight series in the three data sets. For

Figure 3, the values of the Hang Seng Index and the Nikkei Stock Average 225 have been scaled down by a factor of one-tenth for the purpose of presentation. We can see that the bond yields moved closely together. In contrast, the stock market indices exhibited periods of divergence. For example, in the early 1990s, the Japan market experienced prolonged period of downturn, while the Hong Kong and Singapore markets were gradually moving upwards. Table 2 provides a summary of the descriptive statistics of the data. The summary statistics refer to those of the differences of the logarithmic series (expressed in percentage). It can be seen that all differenced logarithmic series exhibit negative skewness and excess kurtosis (compared to the normal distribution) in the unconditional distribution. While the exchange rate data demonstrate no evidence of serial correlation, both the interest rate data and the stock return data have significant serial correlation as suggested by the Q_1 statistics. The Q_2 statistics show that there is serial correlation in the conditional variance of all data sets and GARCH type of modelling may be required. In the subsequent analysis, we apply autoregressive filters to the differenced logarithmic series and model the filtered residuals using MGARCH models. The autoregressive filters are estimated using ordinary least squares (OLS).

We fit the CC-MGARCH model to all data sets using Bollerslev's (1990) algorithm. For DS2 and DS3 we consider trivariate model as well as bivariate (pairwise) models. The results are summarised in Table 3. It can be observed that the estimates of α, β and ρ are all statistically significant at the 5 percent level for DS1 and DS2. For DS3, which has the smallest sample size, some estimates of α and β are not statistically significant, while all estimates of ρ are significant. The exchange rate data have the highest intensity of persistence in volatility according to the estimates of $\alpha + \beta$. Rather interestingly, the estimates of $\alpha + \beta$ are quite robust for each series, regardless of what other series are included in the CC-MGARCH (whether bivariate or trivariate) system. For example, the estimates of $\alpha + \beta$ for the returns in the Japanese market (J) vary from 0.9085 to 0.9194 for three different systems.

With respect to the correlation coefficients, the returns across different national markets have the lowest correlation. In no case is the estimated correlation coefficient higher than 0.57. For the interest rate data, the correlation between the medium-term (O) and long-term (T) rates has the highest value. As expected, the correlation between the short-term (M) and long-term (T) rates has the lowest value. Again, it is quite remarkable that the estimated correlation coefficient between each pair of series is quite robust regardless of whether it is an estimate from a bivariate or trivariate system.

Table 4 summarises the estimation results of the VC-MGARCH models for the three data sets. It can be seen that the intensity of the volatility persistence has increased compared to the CC-MGARCH models. For example, for data sets DS2 and DS3, 7 out of 9 estimates of $\alpha + \beta$ are larger than the corresponding estimates in the constant-correlation models. For DS1 and DS2, the estimates of ρ in the varying-correlation models are larger than the corresponding estimates of ρ in the constant-correlation model. Also, for these two data sets the estimates of ρ are quite stable irrespective of the system in which this parameter is estimated. For DS3, however, the estimates of ρ are no longer stable with respect to the system in which it is estimated. For example, $\hat{\rho}_{HS}$ is 0.7314 in the bivariate system of (H, S), but is equal to 0.6207 in the trivariate system of (H, J, S). Also, while $\hat{\rho}_{JS}$ is 0.5691 in the system (J, S), it drops to 0.3333 in the system (H, J, S).

It can be seen that most estimates of θ_1 and θ_2 are statistically significant at the 5 percent level. The only exceptions are the estimates of θ_2 in the system (M, T) in DS2 and in the system (H, J) in DS3. As the CC-MGARCH model is nested within the VC-MGARCH model, ignoring the extension would induce model misspecification. We now proceed to examine the model diagnostics for the constant-correlation and varying-correlation models.

Table 5 summarises a battery of diagnostic tests for the fitted MGARCH models. $Q_i(20)$ tests for the autocorrelation in the standardised residuals of series i. It is the

Box-Pierce statistic based on the autocorrelation coefficients of $\hat{\epsilon}_{it}$ up to order 20. Likewise, $Q_{ij}(20)$ is the Box-Pierce statistic based on the autocorrelation coefficients of $\hat{\epsilon}_{it}\hat{\epsilon}_{jt} - \delta_{ijt}$, where $\delta_{ijt} \equiv 1$ for $i = j$, and $\delta_{ijt} = \hat{\rho}_{ijt}$ for $i \neq j$, up to order 20. We would expect $\hat{\epsilon}_{it}\hat{\epsilon}_{jt} - \delta_{ijt}$ to be approximately serially uncorrelated if the conditional variance-covariance equations are correct. Thus, $Q_{ij}(20)$ can be used to test for inadequacy of the conditional variance-covariance structure. Although the Q statistics are not asymptotically distributed as χ^2 (see Li and Mak (1994) and Ling and Li (1997)), the χ^2 approximation has nonetheless been used as a rule of thumb for the asymptotic distribution (see, e.g., Bollerslev (1990) Footnote 7). LM_C is the Lagrange multiplier test for the assumption of constant correlations in a MGARCH model suggested by Tse (1998). It is asymptotically distributed as a χ^2_R, where $R = K(K - 1)/2$, under the null. In Panel B of the table we also present the likelihood ratio statistic LR, which tests for the restriction $\theta_1 = \theta_2 = 0$.

For the CC-MGARCH model of DS1, although LM_C cannot detect any violation of the constant-correlation assumption, Q_{12} suggests the contrary. Otherwise, all other diagnostics do not detect any model misspecification. On the other hand, the Q_{12} statistic of the VC-MGARCH model is insignificant at the 5 percent level. As expected, the LR statistic is highly significant. Overall, the results suggest the superiority of the VC-MGARCH model over the CC-MGARCH model.

For DS2, the CC-MGARCH model passes most of the diagnostics, except for the LM_C statistic of the (M, T) system. For the VC-MGARCH model, all diagnostics cannot detect any misspecifications. Although the LR statistics are significant at the 5 percent level, a comparison of Tables 3 and 4 shows that the estimates of the ω, α and β do not differ much over the two models. Thus, for the interest rate system, not much is lost by imposing constant correlations.

For the stock price data, significant time-varying correlation is detected by the LM_C statistic in the bivariate system (H, S) as well as the trivariate system. Likewise, the

LR statistics in these two cases reject the joint hypothesis of $\theta_1 = \theta_2 = 0$. Otherwise, there is no indication of time-varying correlations in the bivariate systems involving Japan.[6] Both the CC-MGARCH and VC-MGARCH models pass the various Q_i and Q_{ij} diagnostics.

Table 6 reports the summary statistics of the in-sample conditional variances, co-variances and correlations of the VC-MGARCH(1, 1) models. It can be seen that the sample means of the conditional correlations are remarkably close to the QMLE of the correlation coefficients in Table 3. Nonetheless, the range of the conditional correlations can be quite large. For example, for DS1 the range of $\{\rho_{BDt}\}$ is 0.5884, with a mean of 0.7001. For the interest rate data, the ranges of the conditional correlations appear to be smaller. In the trivariate system, the minimum is 0.1777 (for $\{\rho_{OTt}\}$) and the maximum is 0.2856 (for $\{\rho_{MTt}\}$). For the data set DS3, we can see that Hong Kong is the most volatile market, followed by Japan and then Singapore. While the Hong Kong and Singapore markets exhibit higher co-movements, the Japan market appears to have low correlations with Hong Kong and Singapore.

To obtain a clearer picture of the time history of the conditional correlations, we plot the time paths of the conditional correlations based on the bivariate VC-MGARCH(1, 1) models. The plots are presented in Figures 4 through 10, in which both the conditional correlations and the QMLE of the CC-MGARCH(1, 1) models (given by the dotted lines) are provided.

From Figure 4 we can see that the conditional correlations of the British Pound and the Deutschmark were quite unstable. Broadly speaking, in the earlier periods of the sample, the conditional correlations were mostly above the estimated value of 0.7168 obtained from the constant-correlation model. After October 1995, however, the conditional correlations dropped below this value. During this period, the British Pound

[6] We must note, however, that the lack of statistical significance could be due to the small sample size in this data set. Furthermore, the correlations between the Japan market and the Hong Kong and Singapore markets are quite low (see Table 6 below).

was experiencing an upward drift while the Deutschmark was following a downward trend. This time history of the conditional correlations of the two currencies cannot be extracted from the constant-correlation model.

From Figures 5 through 7, we can observe that the conditional correlations of the three bond yields °uctuate quite randomly around the values estimated from the constant-correlation model. This is particularly obvious for the correlations between the 1-year and 10-year yields (i.e., the system (O, T)). In comparison, the conditional correlations between the 3-month and 1-year rates show slight tendency of dropping in the later period of the sample.

Turning to Figures 8 and 9, we can see that the path of the conditional correlations between Japan and Singapore appears to be di®erent from the other combinations. These two markets experienced drop in correlations in the later part of the sample period. Except for a brief period in 1996, the conditional correlations after May 1992 were below the average of 0.3149 estimated from the constant-correlation model. The conditional correlations between the Hong Kong and Singapore markets experienced the most volatile °uctuations. The °uctuations occurred throughout the sample period and are strong indications of the unstable relationship between the two markets.

We shall end this section by stating that it is not our intention to claim that the VC-MGARCH models as presented represent the best MGARCH models for the data. Other MGARCH models could also provide the conditional-correlation structure as presented here. The VC-MGARCH model, however, does provide a viable alternative that is relatively easy to estimate.

5 Conclusions

In this paper we propose a new MGARCH model with time-varying correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analogue to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we manage to construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimisation.

We report some Monte Carlo results on the finite-sample distributions of the QMLE of the varying-correlation MGARCH model. It is found that the bias and MSE of the QMLE are small for sample sizes of 500 or above. The new model is applied to three real data sets, namely, exchange rate data, interest rate data and stock price data. The new model is found to pass the model diagnostics satisfactorily, while the constant-correlation MGARCH model is found to be inadequate in some cases. Extending the constant-correlation model to allow for time-varying correlations provides some interesting empirical results. In particular, the estimated conditional-correlation path provides an interesting time history that would not be available in a constant-correlation model. It is hoped that the varying-correlation MGARCH model would provide a useful alternative for modelling multivariate conditional heteroscedasticity in empirical applications.

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Table 1.1: Estimated Bias and MSE of the QMLE of Bivariate VC-MGARCH(1, 1) Models

Parameters	Experiment E1				Experiment E2			
	True Value	Sample Size	Bias	MSE	True Value	Sample Size	Bias	MSE
ω_1	0.4	500	0.0907	0.0687	0.4	500	0.1166	0.0993
		1000	0.0363	0.0194		1000	0.0487	0.0273
		1500	0.0266	0.0116		1500	0.0328	0.0157
α_1	0.8	500	−0.0135	0.0033	0.8	500	−0.0183	0.0043
		1000	−0.0056	0.0012		1000	−0.0070	0.0016
		1500	−0.0046	0.0008		1500	−0.0050	0.0010
β_1	0.15	500	−0.0007	0.0013	0.15	500	0.0005	0.0017
		1000	−0.0005	0.0006		1000	−0.0010	0.0008
		1500	0.0005	0.0004		1500	−0.0004	0.0005
ω_2	0.2	500	0.0313	0.0095	0.2	500	0.0364	0.0118
		1000	0.0132	0.0031		1000	0.0123	0.0040
		1500	0.0076	0.0017		1500	0.0089	0.0024
α_2	0.7	500	−0.0170	0.0062	0.7	500	−0.0230	0.0079
		1000	−0.0094	0.0023		1000	−0.0075	0.0031
		1500	−0.0043	0.0015		1500	−0.0047	0.0018
β_2	0.2	500	−0.0018	0.0023	0.2	500	0.0011	0.0030
		1000	0.0013	0.0010		1000	−0.0003	0.0013
		1500	−0.0005	0.0008		1500	−0.0005	0.0009
ρ	0.7	500	−0.0011	0.0028	0.2	500	−0.0008	0.0077
		1000	−0.0227	0.0084		1000	−0.0012	0.0034
		1500	0.0010	0.0009		1500	0.0001	0.0022
θ_1	0.8	500	−0.0018	0.0014	0.8	500	−0.0358	0.0181
		1000	−0.0090	0.0023		1000	−0.0194	0.0065
		1500	0.0011	0.0004		1500	−0.0111	0.0029
θ_2	0.1	500	−0.0006	0.0008	0.1	500	0.0043	0.0016
		1000	−0.0064	0.0014		1000	0.0023	0.0008
		1500	0.0005	0.0003		1500	0.0011	0.0004

Notes: See equations (8), (9) and (10) for the data generating process.

Table 1.2: Estimated Bias and MSE of the QMLE of Bivariate VC-MGARCH(1, 1) Models

Parameters	Experiment E3				Experiment E4			
	True Value	Sample Size	Bias	MSE	True Value	Sample Size	Bias	MSE
ω_1	0.4	500	0.0280	0.0177	0.4	500	0.0315	0.0188
		1000	0.0153	0.0077		1000	0.0114	0.0088
		1500	0.0056	0.0044		1500	0.0051	0.0052
α_1	0.5	500	−0.0184	0.0104	0.5	500	−0.0181	0.0109
		1000	−0.0092	0.0045		1000	−0.0067	0.0051
		1500	0.0028	0.0027		1500	−0.0025	0.0031
β_1	0.3	500	0.0005	0.0041	0.3	500	−0.0032	0.0042
		1000	0.0021	0.0020		1000	−0.0019	0.0021
		1500	−0.0010	0.0015		1500	−0.0022	0.0015
ω_2	0.2	500	0.0177	0.0064	0.2	500	0.0219	0.0081
		1000	0.0092	0.0031		1000	0.0109	0.0032
		1500	0.0072	0.0021		1500	0.0089	0.0024
α_2	0.4	500	−0.0409	0.0335	0.5	500	−0.0352	0.0268
		1000	−0.0166	0.0162		1000	−0.0188	0.0110
		1500	−0.0153	0.0118		1500	−0.0089	0.0074
β_2	0.2	500	0.0011	0.0038	0.2	500	−0.0008	0.0034
		1000	−0.0007	0.0018		1000	0.0016	0.0017
		1500	0.0005	0.0013		1500	0.0021	0.0013
ρ	0.5	500	−0.0093	0.0079	0.2	500	0.0002	0.0139
		1000	0.0012	0.0037		1000	0.0007	0.0068
		1500	−0.0010	0.0024		1500	0.0001	0.0041
θ_1	0.7	500	−0.0136	0.0052	0.6	500	−0.0137	0.0055
		1000	−0.0054	0.0022		1000	−0.0035	0.0023
		1500	−0.0031	0.0014		1500	−0.0058	0.0016
θ_2	0.2	500	0.0022	0.0016	0.3	500	0.0035	0.0023
		1000	0.0004	0.0008		1000	−0.0009	0.0010
		1500	0.0003	0.0005		1500	0.0019	0.0007

Notes: See equations (8), (9) and (10) for the data generating process.

Table 2: Summary Statistics of the Di®erenced Logarithmic Series of Various Data Sets

Variable (Code)	Mean	Std Dev	Minimum	Maximum	Std Skewness	Std Kurtosis	$Q_1(20)$	$Q_2(20)$	Num of Obs
Panel A: Forex Market Data (DS1)									
British Pound (B)	0.0008	0.6498	{4.3306	3.2467	{28.6798	125.0248	18.4091	422.5185	2137
Deutschmark (D)	{0.0025	0.6919	{3.4671	3.3440	{28.2742	95.0182	12.1631	211.7246	2137
Panel B: Bond Market Data (DS2)									
3-Month T-Bill (M)	{0.1019	2.3643	{21.2535	10.4069	{363.1040	228.7802	65.0449	106.4135	850
1-Year T-Note (O)	{0.1104	2.1071	{14.3009	6.5305	{138.6335	108.3323	104.6519	91.8969	850
10-Year T-Bond (T)	{0.1102	1.6528	{7.7080	5.5091	{27.0643	48.6885	72.0995	65.6332	850
Panel C: Stock Market Data (DS3)									
Hong Kong (H)	0.5218	3.3224	{10.3635	12.0917	{31.4482	26.3981	38.0765	28.1933	340
Japan (J)	{0.2015	3.2499	{10.2969	11.5091	{13.3198	30.9661	24.8226	34.2806	340
Singapore (S)	0.0306	2.3391	{13.5471	6.9306	{96.0536	55.8701	43.5073	87.6590	340

Notes: $Q_1(20)$ is the Box-Pierce portmanteau statistic of the di®erenced logarithmic series based on the autocorrelation coe±cients up to order 20. Similarly, $Q_2(20)$ is the portmanteau statistic of the squared di®erenced logarithmic series.

Table 3: Estimation Results of CC-MGARCH(1, 1) Models

Data	K	Variable	!	®	¯	Correlations
DS1	2	B	0.0117	0.9221	0.0464	½BD = 0.7168
			(0.0030)	(0.0145)	(0.0081)	(0.0107)
		D	0.0162	0.9274	0.0362	
			(0.0042)	(0.0141)	(0.0067)	
DS2	2	M	0.6598	0.6259	0.2249	½MO = 0.7623
			(0.2739)	(0.1127)	(0.0828)	(0.0217)
		O	0.3558	0.8129	0.0961	
			(0.0913)	(0.0270)	(0.0255)	
	2	M	0.5959	0.5883	0.3033	½MT = 0.5232
			(0.2888)	(0.1454)	(0.1353)	(0.0322)
		T	0.2605	0.8384	0.0605	
			(0.0720)	(0.0288)	(0.0205)	
	2	O	0.2241	0.8284	0.1215	½OT = 0.8249
			(0.0781)	(0.0392)	(0.0377)	(0.0133)
		T	0.2116	0.8591	0.0589	
			(0.0576)	(0.0244)	(0.0154)	
	3	M	0.6020	0.6568	0.2050	½MO = 0.7627
			(0.2908)	(0.1224)	(0.0841)	(0.0225)
		O	0.3440	0.8152	0.0971	½MT = 0.5264
			(0.0871)	(0.0301)	(0.0237)	(0.0340)
		T	0.2278	0.8603	0.0505	½OT = 0.8246
			(0.0663)	(0.0285)	(0.0139)	(0.0135)
DS3	2	H	0.6250	0.8721	0.0999	½HJ = 0.2073
			(0.4604)	(0.0471)	(0.0507)	(0.0600)
		J	0.9569	0.8079	0.1006	
			(0.6010)	(0.0774)	(0.0481)	
	2	H	1.0243	0.8499	0.0918	½HS = 0.5645
			(0.5177)	(0.0414)	(0.0404)	(0.0519)
		S	0.6106	0.8186	0.0829	
			(2.1266)	(0.4753)	(0.1192)	
	2	J	0.8439	0.8233	0.0961	½JS = 0.3149
			(0.5115)	(0.0625)	(0.0406)	(0.0572)
		S	0.5639	0.8156	0.0966	
			(0.6395)	(0.1408)	(0.0545)	
	3	H	1.0140	0.8522	0.0900	½HJ = 0.2146
			(0.5099)	(0.0389)	(0.0403)	(0.0607)
		J	0.8652	0.8200	0.0969	½HS = 0.5663
			(0.5460)	(0.0686)	(0.0425)	(0.0524)
		S	0.7917	0.7867	0.0825	½JS = 0.3220
			(1.9858)	(0.4195)	(0.0863)	(0.0600)

Notes: The ¯gures in parentheses are standard errors.

Table 4: Estimation Results of VC-MGARCH(1, 1) Models

Data	K	Variable	!	®	¯	$µ_1$	$µ_2$	Correlations
DS1	2	B	0.0059	0.9323	0.0516	0.9728	0.0187	½$_{BD}$ = 0.8987
			(0.0018)	(0.0110)	(0.0076)	(0.0037)	(0.0026)	(0.0192)
		D	0.0073	0.9415	0.0419			
			(0.0022)	(0.0095)	(0.0063)			
DS2	2	M	0.3935	0.7177	0.1993	0.8582	0.0502	½$_{MO}$ = 0.8217
			(0.2544)	(0.1280)	(0.0976)	(0.0696)	(0.0228)	(0.0267)
		O	0.2724	0.8184	0.1151			
			(0.0810)	(0.0286)	(0.0303)			
	2	M	0.5790	0.5968	0.3014	0.6564	0.0673	½$_{MT}$ = 0.5433
			(0.2787)	(0.1409)	(0.1331)	(0.0319)	(0.0399)	(0.0396)
		T	0.2596	0.8353	0.0627			
			(0.0730)	(0.0305)	(0.0211)			
	2	O	0.2214	0.8232	0.1295	0.4910	0.0474	½$_{OT}$ = 0.8345
			(0.0755)	(0.0378)	(0.0384)	(0.1293)	(0.0237)	(0.0142)
		T	0.2210	0.8534	0.0615			
			(0.0594)	(0.0246)	(0.0157)			
	3	M	0.3897	0.7295	0.1828	0.9389	0.0187	½$_{MO}$ = 0.7975
			(0.2770)	(0.1351)	(0.0941)	(0.0341)	(0.0095)	(0.0272)
		O	0.2657	0.8285	0.1035			½$_{MT}$ = 0.5580
			(0.0813)	(0.0307)	(0.0254)			(0.0453)
		T	0.2042	0.8691	0.0515			½$_{OT}$ = 0.8421
			(0.0658)	(0.0273)	(0.0134)			(0.0194)
DS3	2	H	0.6717	0.8703	0.0986	0.6545	0.0750	½$_{HJ}$ = 0.1968
			(0.4534)	(0.0450)	(0.0504)	(0.1172)	(0.0533)	(0.0734)
		J	0.8859	0.8131	0.1012			
			(0.5768)	(0.0790)	(0.0486)			
	2	H	0.8115	0.8719	0.0845	0.8398	0.0745	½$_{HS}$ = 0.7314
			(0.4599)	(0.0373)	(0.0335)	(0.0606)	(0.0272)	(0.1174)
		S	0.5684	0.8190	0.0945			
			(1.5522)	(0.3632)	(0.1169)			
	2	J	0.8021	0.8316	0.0900	0.9772	0.0197	½$_{JS}$ = 0.5691
			(0.4693)	(0.0573)	(0.0371)	(0.0092)	(0.0085)	(0.1326)
		S	0.5539	0.8181	0.0943			
			(0.6726)	(0.1505)	(0.0541)			
	3	H	0.9366	0.8646	0.0824	0.7343	0.0623	½$_{HJ}$ = 0.2048
			(0.4684)	(0.0368)	(0.0366)	(0.1851)	(0.0305)	(0.0779)
		J	0.8595	0.8157	0.1014			½$_{HS}$ = 0.6207
			(0.5063)	(0.0651)	(0.0415)			(0.0682)
		S	0.8180	0.7763	0.0922			½$_{JS}$ = 0.3333
			(1.3114)	(0.2764)	(0.0631)			(0.0696)

Notes: The ¯gures in parentheses are standard errors.

Table 5: Diagnostic Checks for the Constant-Correlation and Varying-Correlation Models

Tests	Forex Market (DS1)	Bond Market (DS2)				Stock Market (DS3)			
	(B, D)	(M, O)	(M, T)	(O, T)	(M, O, T)	(H, J)	(H, S)	(J, S)	(H, J, S)
Panel A: CC-MGARCH(1, 1) Model									
$Q_1(20)$	16.730	14.530	15.746	26.488	14.325	25.496	25.485	18.518	25.330
$Q_2(20)$	12.247	24.797	22.631	22.447	25.029	18.554	22.235	22.293	18.265
$Q_3(20)$					22.596				22.199
$Q_{11}(20)$	11.005	14.692	17.396	7.175	13.611	25.336	27.403	24.091	27.494
$Q_{22}(20)$	22.691	7.124	17.300	17.301	6.977	24.291	14.405	13.082	25.297
$Q_{33}(20)$					17.478				14.284
$Q_{12}(20)$	51.313	4.601	7.758	10.553	4.252	19.081	21.774	21.306	17.629
$Q_{13}(20)$					7.513				21.892
$Q_{23}(20)$					10.505				23.480
LMC	0.087	1.570	6.050	1.233	2.896	1.149	7.239	1.169	9.987
Panel B: VC-MGARCH(1, 1) Model									
$Q_1(20)$	16.934	14.970	15.742	26.711	14.724	25.481	25.196	18.501	25.095
$Q_2(20)$	9.745	25.898	22.625	22.466	25.771	18.534	22.285	22.278	18.247
$Q_3(20)$					22.596				22.200
$Q_{11}(20)$	7.022	12.469	17.192	7.406	11.840	25.639	27.110	23.923	27.680
$Q_{22}(20)$	14.752	6.970	17.252	17.246	6.728	24.322	13.403	13.235	25.457
$Q_{33}(20)$					17.436				13.473
$Q_{12}(20)$	27.998	3.664	7.655	10.626	3.593	19.309	21.292	21.923	18.041
$Q_{13}(20)$					7.001				21.972
$Q_{23}(20)$					10.333				22.320
LR	298.825	38.749	10.980	9.094	31.929	3.567	11.040	5.272	10.309

Notes: $Q_i(20)$ are the Box-Pierce portmanteau statistics based on the autocorrelation coe±cients of order up to 20 for the standardised residuals of variabe i. Similarly, $Q_{ij}(20)$ are the portmanteau statistics based on the autocorrelation coe±cients of order up to 20 for the products of the standardised residuals of variables i and j. The indices are according to the order of the coded variables in the parentheses (thus Q_{12} in the system (M, O, T) is Q_{MO}). LMC is the Lagrange multiplier test for constant correlations due to Tse (1998). LR is the likelihood ratio statistic for $H_0 : \mu_1 = \mu_2 = 0$.

Table 6: Summary Statistics of the Conditional Variance, Covariance and Correlation in the Estimated VC-MGARCH(1, 1) Models

Data	System	Statistic	Mean	Std Dev	Minimum	Maximum
DS1	(B, D)	σ_B^2	0.4114	0.2660	0.1151	2.1116
		σ_D^2	0.4678	0.2033	0.1970	1.4227
		σ_{BD}	0.3176	0.2016	0.0616	1.3518
		ρ_{BD}	0.7001	0.1395	0.3487	0.9371
DS2	(M, O)	σ_M^2	4.9094	8.1492	1.5335	100.9584
		σ_O^2	4.0750	3.0793	1.7295	30.3173
		σ_{MO}	3.3570	3.9179	0.7924	44.2570
		ρ_{MO}	0.7519	0.0900	0.3748	0.8691
	(M, T)	σ_M^2	5.1534	10.0503	1.5062	150.1892
		σ_T^2	2.5526	0.6533	1.7611	7.2573
		σ_{MT}	1.7786	1.4943	0.5193	19.4827
		ρ_{MT}	0.5217	0.0753	0.2608	0.6277
	(O, T)	σ_O^2	4.2317	3.5258	1.5266	33.7753
		σ_T^2	2.5840	0.6858	1.7321	7.2102
		σ_{OT}	2.6669	1.3140	1.3243	13.2038
		ρ_{OT}	0.8257	0.0295	0.6876	0.8496
	(M, O, T)	σ_M^2	4.8061	7.6684	1.5843	92.8950
		σ_O^2	4.0039	2.8722	1.7825	27.6077
		σ_T^2	2.5698	0.6126	1.7926	6.4471
		σ_{MO}	3.2906	3.5850	1.1341	39.3440
		σ_{MT}	1.7629	1.2228	0.7532	12.9837
		σ_{OT}	2.5928	1.1185	1.4080	11.0433
		ρ_{MO}	0.7580	0.0584	0.5614	0.8324
		ρ_{MT}	0.5302	0.0574	0.3544	0.6400
		ρ_{OT}	0.8227	0.0299	0.6916	0.8693
DS3	(H, J)	σ_H^2	14.8791	7.2350	7.7632	111.6058
		σ_J^2	10.4010	3.7826	5.5870	26.3621
		σ_{HJ}	2.3826	1.4615	−0.5689	11.2181
		ρ_{HJ}	0.1961	0.0917	−0.0444	0.3586
	(H, S)	σ_H^2	14.7547	6.2244	8.5945	97.7936
		σ_S^2	6.0420	3.3804	3.5594	29.9887
		σ_{HS}	5.3986	2.9302	1.4648	33.8765
		ρ_{HS}	0.5667	0.1224	0.2199	0.7989
	(J, S)	σ_J^2	10.3513	3.5834	5.6473	25.0978
		σ_S^2	5.9269	3.3640	3.4586	29.8087
		σ_{JS}	2.4025	1.7890	0.5688	12.3114
		ρ_{JS}	0.2966	0.1217	0.0869	0.5540
	(H, J, S)	σ_H^2	14.7097	5.9743	8.9401	95.0937
		σ_J^2	10.4188	3.8265	5.5322	26.4870
		σ_S^2	5.9639	2.9457	3.9177	28.6671
		σ_{HJ}	2.4613	1.3580	−0.3170	10.3302
		σ_{HS}	5.3621	2.4003	2.3007	31.6027
		σ_{JS}	2.4581	1.3627	0.7467	11.0837
		ρ_{HJ}	0.2029	0.0856	−0.0270	0.3667
		ρ_{HS}	0.5732	0.0768	0.3544	0.6886
		ρ_{JS}	0.3115	0.0777	0.1219	0.4721

Notes: σ_i^2 and σ_{ij} are the conditional variance and covariance terms, respectively; ρ_{ij} is the conditional correlation.

Figure 1: Exchange Rates of British Pound and Deutschmark

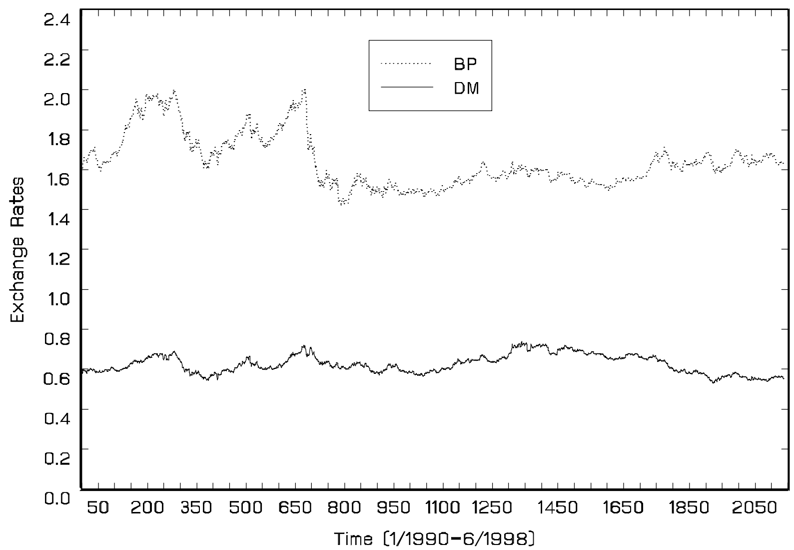

Figure 2: Yields of US Treasury Securities

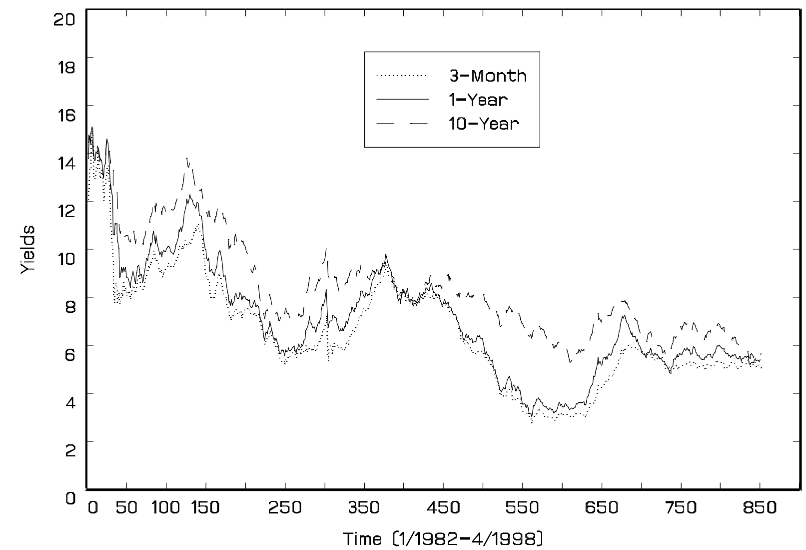

Figure 3: Stock Markets Indices

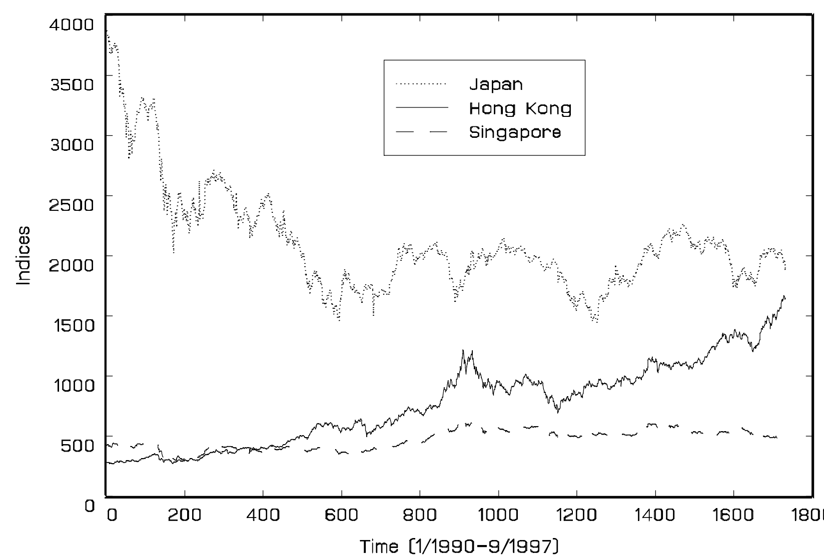

Figure 4: Conditional Correlation Coefficients of (B, D)

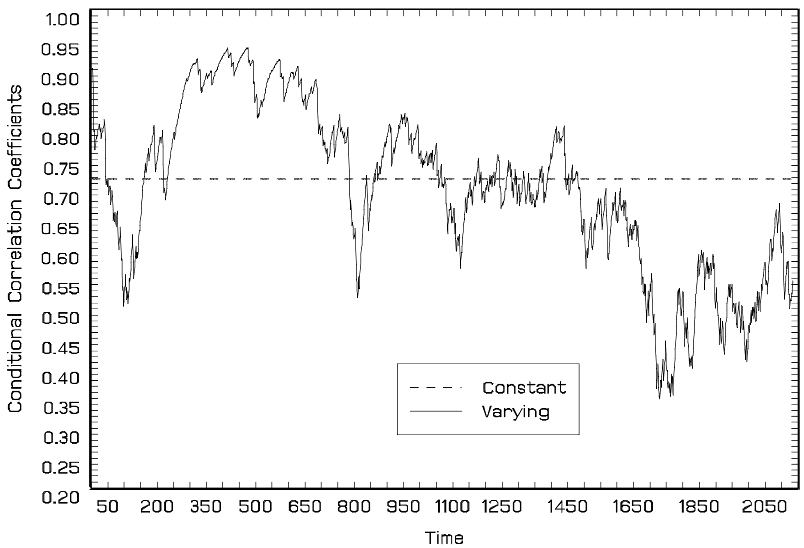

Figure 5: Conditional Correlation Coefficients of (M, O)

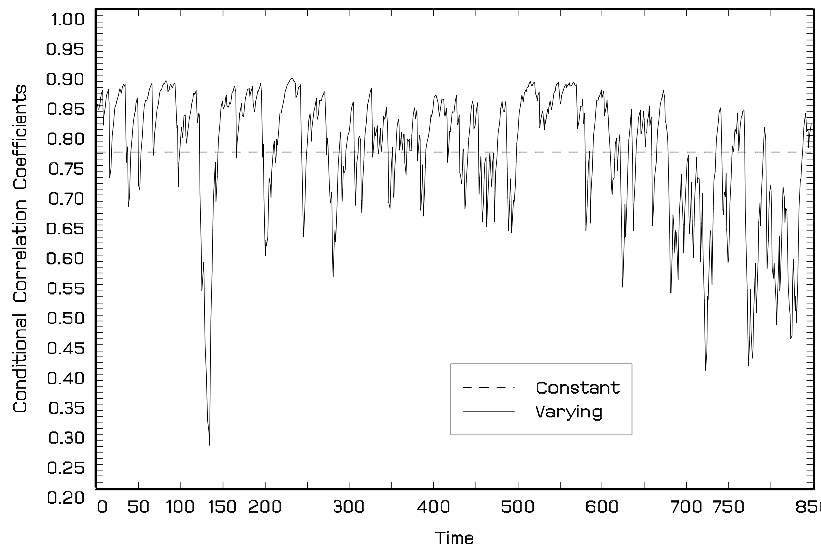

Figure 6: Conditional Correlation Coefficients of (M, T)

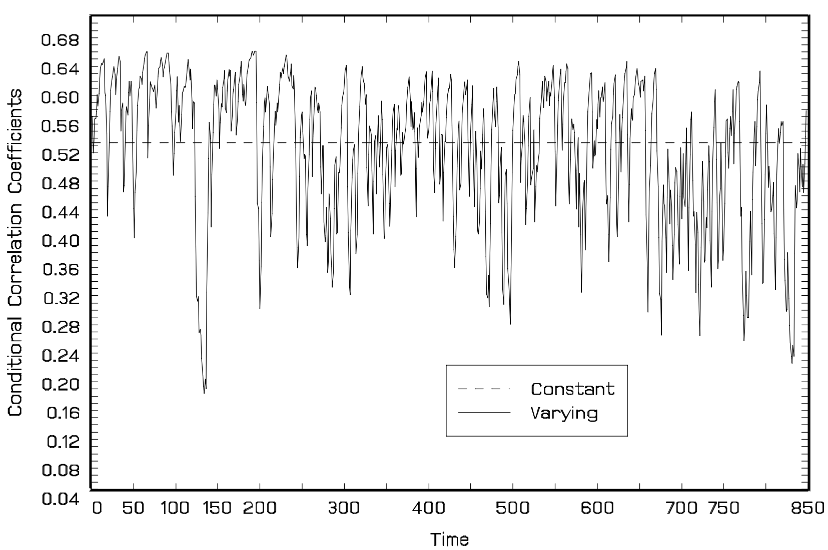

Figure 7: Conditional Correlation Coefficients of (O, T)

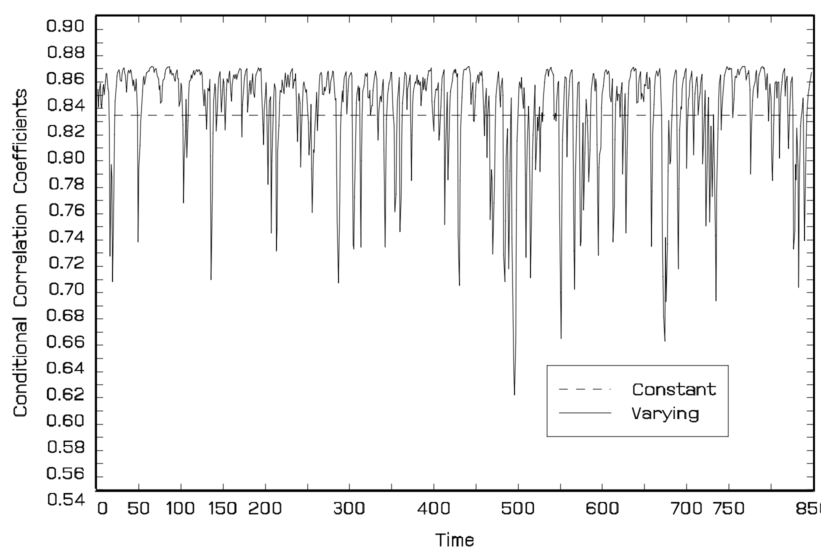

Figure 8: Conditional Correlation Coefficients of (H, S)

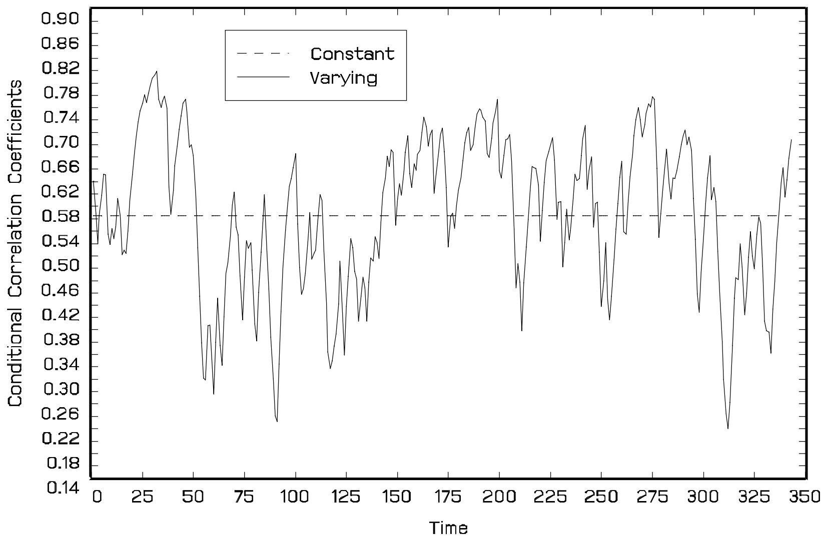

Figure 9: Conditional Correlation Coefficients of (H, J)

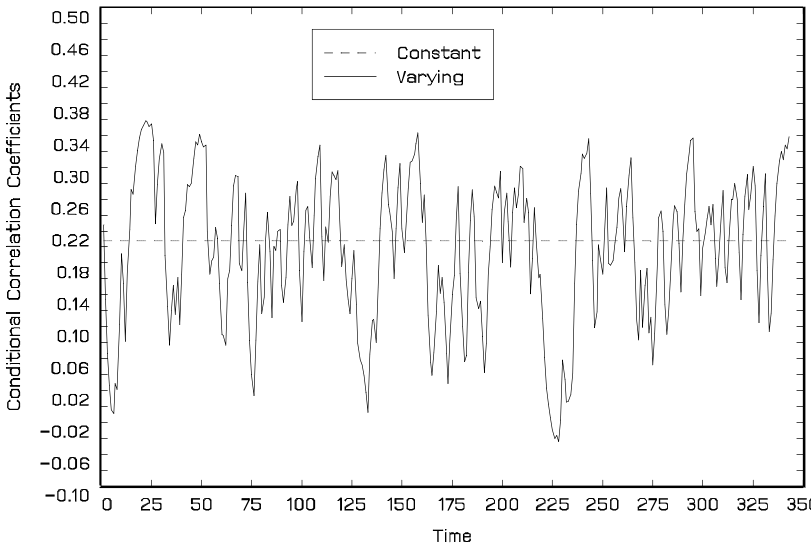

Figure 10: Conditional Correlation Coefficients of (J, S)