IMPLEMENTING EFFICIENT MARKET STRUCTURE*

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Abstract

This article studies the design of optimal mechanisms to regulate entry in natural oligopoly markets, assuming the regulator is unable to control the behavior of firms once they are in the market. We adapt the Clarke–Groves mechanism, characterize the optimal mechanism that maximizes the weighted sum of expected social surplus and expected tax revenue, and show that these mechanisms generally avoid budget deficits and prevent excessive entry.

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1 Introduction

In recent years public policy has been engaged in redesigning markets on a massive scale. In Eastern Europe, many formerly state owned enterprises were privatized after the breakdown of formerly communist regimes, and further privatization programs are on the way. And in Western economies, privatization and deregulation were launched in many tightly regulated industries, ranging from public utilities to telecommunications. These policies have by and large been successful in building private markets and raising revenue. However, they have often failed to pay sufficient attention to the market structure implied by particular privatization and deregulation schemes.

The recent allocation of government franchises for operating wireless telecommunication through spectrum auctions is a case in point. These auctions raised an enormous amount of revenue, which earned them high praise both in the profession and the general public, but they may not have created the best market structure.[1]

Usually an auction is said to be efficient if the objects are allocated to the bidders who value them most. However, in the case of awarding spectrum rights, this principle does not apply without qualification. For example, if bidders were allowed to get all spectrum rights, the winner of the auction would typically monopolize the market. Awarding a monopoly may raise the highest revenue for the auctioneer, but typically at a loss in social welfare.

The recent spectrum auctions did not completely ignore market structure. Indeed, in the U.S. the market was broken down into many regional submarkets which, in the case of mobile phone services, had to be supplied by two providers, and various affirmative action schemes were employed to give preferential treatment to minority operated firms. However, in other countries, nationwide spectrum rights were sometimes auctioned strictly to the highest bidder. For example, in Germany the regulator recently auctioned ten twin–paired radio frequencies for mobile telecommunications in the 1800 MHz range in this way. All four existing providers did participate in the auction, but in the end all frequencies were awarded to the two major providers (see Cane (1999)): Mannes-

[1]Market structure is, of course, also a concern in private industry. A case in point is the relationship between the Holland Sweetener Company that challenged the monopolistic position of Monsanto, the producer of NutraSweet, and the major buyers of such sweeteners such as Coke and Pepsi. The latter wished to continue buying NutraSweet, but also had a vested interest to keep the new competitor alive and well in order to restrain Monsanto's monopoly.

mann Mobilfunk, who won the larger share, and DeTeMobil (Deutsche Telekom).

If the concern for market structure is taken serious at all by public policy, it is usually accounted for by imposing restrictions on the (minimum) number of suppliers who must serve the market. However, the issue is usually complicated by the fact that the regulator has incomplete information about relevant market characteristics, and cannot know how many firms should serve a particular market unless firms reveal their individual characteristics. Therefore, the design of franchising or privatization schemes usually cannot be separated from the design of mechanisms that detect and implement the right market structure.

The purpose of the present paper is to design optimal mechanisms to implement the optimal market structure under conditions of incomplete information, when the regulator cannot know which and how many firms should participate in the market, unless he induces firms to reveal their relevant private information. In particular, we

- ▷ characterize the optimal Groves mechanism that yields the highest tax revenue in the class of mechanisms that implement efficiency;
- ▷ solve the optimal mechanism that maximizes a weighted sum of tax revenue and social surplus, which is relevant if general taxation is subject to a deadweight loss;
- ▷ show that free entry would lead to inefficiency;
- ▷ show that optimal mechanisms are generally deficit free.

There is a small literature on the design of market structure, especially in the context of procurement. This literature usually analyzes particular mechanisms but does not consider optimal mechanisms. To our knowledge there are three exceptions: Dana and Spier (1994), McGuire and Riordan (1995), and Auriol and Laffont (1992). However, all three papers restrict the analysis to two potential suppliers so that the choice is between duopoly and monopoly.

Dana and Spier (1994) analyze the optimal market structure when tax revenue and efficiency matter. Their model is the most closely related to our paper, since they also assume that firms are not regulated once they are in the market. Similarly, McGuire and Riordan (1995) analyze the market structure in the particular context of two firms that produce differentiated products. The main difference between these and our paper is that we do not restrict the number of firms to just two firms.

In turn, Auriol and Laffont (1992) consider a framework where firms are regulated once they are in the market. Firms' marginal costs are private information whereas fixed costs are common knowledge. They show that monopoly is favored if market structure is chosen after marginal costs have been revealed, whereas the government prefers duopoly if the market structure is chosen ex ante. In contrast to their model, we assume that the government cannot or does not wish to regulate firms once they are in the market. Moreover, in our model, fixed costs are private information, which is plausible when firms' marginal costs are relatively insignificant, as in the telecommunications industry.

An important question is whether optimal mechanisms are deficit free. Interestingly, this question bears some relationship to the literature concerning excessive entry in oligopoly markets (see Mankiw and Whinston (1986) and Suzumura (1995)). As it turns out, optimal mechanisms are deficit free if and only if excessive entry occurs in a hypothetical free entry game. Adapting a well–known excessive entry result by Mankiw and Whinston, we conclude that optimal mechanisms are deficit free if the integer constraint on the number of firms does not bind.

The plan of the paper is as follows. *Section 2* introduces the model. *Section 3* explains some basic properties of feasible direct revelation mechanisms that are crucial for the design of optimal mechanisms. *Section 4* analyzes the implementation of efficient market structure in dominant strategies and characterizes the tax revenue maximizing Groves mechanism. *Section 5* solves the optimal mechanism when tax revenue matters more than social surplus, due to the deadweight loss of general taxation. *Section 6* shows why free entry cannot generally implement the efficient market structure, which explains why entry regulation is desirable in natural oligopoly, and *Section 7* shows that the optimal mechanism is generally deficit free. The paper closes in *Section 8* with a discussion.

2 The model

Consider a natural oligopoly market with a large number of potential firms. All firms have the same variable costs, but different fixed costs. The regulator has decided to award a limited number of unrestricted licenses to operate in this market. The regulator's problem is to determine the optimal number of licenses to be issued if efficiency or tax revenue matters, and to assign them to particular firms. This problem is complicated by the fact that fixed costs are firms' private information.

After licenses have been awarded, the licensees play some market game that gives rise to identical equilibrium profits π (before deducting fixed costs) and aggregate consumer surplus C, that are a function of the number of licenses n. These reduced form payoff functions have the following properties.

Assumption 1 *Aggregate producer surplus* $\Pi(n) := n\pi(n)$ *is decreasing:* $\Pi(n+1) < \Pi(n)$, *aggregate consumer surplus is increasing:* $C(n+1) > C(n)$, *and aggregate social surplus* $S(n) := \Pi(n) + C(n)$ *is increasing:* $S(n+1) > S(n)$, *but at a decreasing rate:* $\Delta S(n) > \Delta S(n+1)$, $\Delta S(n) := S(n) - S(n-1)$. *No payoffs are generated if no licenses are issued:* $\Pi(0) = C(0) = 0$.

From the regulator's perspective, firms' privately known fixed costs are independent random variables $\hat{\theta} := (\hat{\theta}_1, \ldots, \hat{\theta}_N)$, drawn from a distribution $G(x_1, \ldots, x_N) = \prod_{i=1}^{N} G_k(x_k)$, with support $\Theta := \prod_{i=1}^{N}[\underline{\theta}_i, \bar{\theta}_i]$, and with positive densities $g_k(x_k) := G'_k(x_k)$.

A market game that gives rise to the properties stated in Assumption 1 is the standard Cournot game, provided that game has a unique equilibrium in pure strategies. In the present framework, a frequently employed sufficient condition for existence and uniqueness is (see Selten (1970) and Szidarovszky and Yakowitz (1977))[2]

Assumption 2 *Market demand* $P(Q)$ *has a finite satiation point* $\bar{Q}$, *i.e.* $P(Q) = 0, \forall Q \geq \bar{Q}$, *is twice continuously differentiable with* $P(Q) > 0$, *and* $P'(Q) < 0$, $\forall Q \in [0, \bar{Q})$, *and satisfies the condition:* $P''(Q)Q_i + P'(Q) < 0$, $\forall q_i > 0, Q \in [0, \bar{Q})]$.

Finally, we assume:

Assumption 3 *The least cost monopoly profit is nonnegative, which requires* $\pi(1) \geq \min\{\bar{\theta}_1, \ldots, \bar{\theta}_N\}$, *and* $\max\{\underline{\theta}_1, \ldots, \underline{\theta}_N\} \leq \Delta S(N)$, *which implies that each conceivable market size can be efficient.*

3 Basic properties of feasible mechanisms

We begin with some basic properties of feasible regulatory mechanisms. By the Revelation Principle (Myerson (1979)) attention can be restricted

[2] Of course, the market game may differ from the Cournot game; however, without specifying the game one cannot know which assumptions are required for uniqueness of the solution of that game.

to *direct revelation mechanisms*, where the regulator asks firms to independently announce their fixed costs, and then determines who gets a license and how much each firm has to pay.

Direct revelation mechanisms are described by two outcome functions (p, t) of the form $p : \Theta^N \to [0,1]^N$ and $t : \Theta^N \to \mathbb{R}^N$. For each vector of announced fixed costs $\phi \in \Theta^N$, the allocation rule $p_k(\phi)$ is the probability that firm k gets a license, and the payment rule $t_k(\phi)$ is firm k's payment to the regulator. Note, a firm may have to pay something even if it is not awarded a license.

We denote by $n_p(\phi) := \sum p_k(\phi)$ the number of firms that obtain a license if the mechanism (p, t) is used, and define the expected payment, probability of being in the market and profit in this case of firm k when it announces fixed costs ϕ_k as

$$\bar{t}_k(\phi_k) := E_{\hat{\theta}_{-k}}\left[t_k(\phi_k, \hat{\theta}_{-k})\right], \tag{1}$$

$$\bar{p}_k(\phi_k) := E_{\hat{\theta}_{-k}}\left[p_k(\phi_k, \hat{\theta}_{-k})\right], \tag{2}$$

$$\bar{\pi}_k(\phi_k) := E_{\hat{\theta}_{-k}}\left[\pi(n_p(\phi_k, \hat{\theta}_{-k}))p_k(\phi_k, \hat{\theta}_{-k})\right], \tag{3}$$

Expectation is taken over the fixed costs $\hat{\theta}_{-k}$ of all firms except k. Note that $\bar{\pi}$ does not depend on the payment rule t. Moreover, let

$$U_k(\phi_k \mid \theta_k) := \bar{\pi}_k(\phi_k) - \theta_k \bar{p}_k(\phi_k) - \bar{t}_k(\phi_k) \tag{4}$$

denote the expected payoff of firm k if it announces ϕ_k while its true fixed cost is θ_k. Finally, we define

$$\bar{U}_k(\theta_k) := U_k(\theta_k \mid \theta_k). \tag{5}$$

Using this notation, a direct revelation mechanism (p, t) is *incentive compatible* if $\bar{U}_k(\theta_k) \geq U_k(\phi_k, \theta_k)$, for all ϕ_k and k, it satisfies the *interim participation constraint* if $\bar{U}_k(\theta_k) \geq 0$, for all θ_k and k, and it is called *feasible* if it is both incentive compatible and satisfies the interim participation constraint.

THEOREM 1 (REVENUE–EQUIVALENCE) *A feasible direct revelation mechanism (p, t) gives rise to the following payoffs $\bar{U}_k(\theta_k)$ and expected tax* *revenue T:*

$$\bar{U}_k(\theta_k) = \bar{U}_k(\bar{\theta}_k) + \int_{\theta_k}^{\bar{\theta}_k} \bar{p}_k(x)dx \tag{6}$$

$$\bar{U}_k(\bar{\theta}_k) \geq 0 \tag{7}$$

$$T = \sum_{k=1}^{N} \Big(E\Big[\Big(\bar{\pi}_k(\hat{\theta}_k) - \hat{\theta}_k \bar{p}_k(\hat{\theta}_k) \Big) - \int_{\hat{\theta}_k}^{\bar{\theta}_k} \bar{p}_k(x)dx \Big] - \bar{U}_k(\bar{\theta}_k) \Big). \tag{8}$$

In particular, all such mechanisms that have the same allocation rule p and reservation utilities $\bar{U}_k(\bar{\theta}_k)$ also give rise to the same payoffs and expected tax revenue.

Proof In a feasible direct revelation mechanism truth–telling is a best reply to truth-telling. Hence,[3]

$$\frac{\partial U}{\partial \phi_k} (\phi_k \mid \theta_k) \big|_{\phi_k=\theta_k} = 0.$$

Using the envelope theorem, one obtains:

$$\bar{U}_k'(\theta_k) = \frac{\partial}{\partial \theta_k} U(\phi_k \mid \theta_k) \big|_{\phi_k=\theta_k} = -\bar{p}_k(\theta_k). \tag{9}$$

Integration gives (6), and hence $\bar{U}_k(\theta_k) \geq 0$ iff $\bar{U}_k(\bar{\theta}_k) \geq 0$, for all θ_k, by the monotonicity property (9). Using $\bar{U}_k(\theta_k) = (\bar{\pi}(\theta_k) - \theta_k) - \bar{t}_k(\theta_k)$ gives $\bar{t}_k(\theta_k)$, and (8) follows from the fact that $T = \sum E[\bar{t}_k(\hat{\theta}_k)]$. Since $\bar{\pi}_k$ does not depend on t, we conclude that, for a given allocation rule, all feasible direct revelation mechanisms give rise to the same payoffs and expected tax revenues, unless firms' reservation utilities differ. □

4 The optimal Groves mechanism

The *efficient* market structure which maximizes social welfare can be implemented in dominant strategies by a Groves mechanism. In this section we characterize the efficient market structure and derive that particular Groves mechanism which maximizes the expected tax revenue

[3] $\bar{U}_k$ is piecewise continuously differentiable if the density g is smooth. If differentiability is not satisfied, the proof is a bit more elaborate along the lines of the proof of Myerson (1981).

in the class of Bayesian mechanisms that implement the efficient market structure.

Note that implementation in Bayesian Nash equilibrium strategies is a much weaker requirement than implementation in dominant strategies. It may be somewhat surprising that a simple dominant strategy mechanism maximizes tax revenue in the larger class of mechanisms that implement the efficient allocation as a Bayesian Nash equilibrium.[4]

An allocation rule p generates the social welfare

$$W(p,\theta) = \sum_{k=1}^{N} \Big(\pi\left(n_p(\theta)\right) - \theta_k\Big) p_k(\theta) + C\left(n_p(\theta)\right) .$$

Call an allocation rule *monotone* if it assigns entry rights only to the firms with the lowest fixed cost, i.e. if $p_k(\theta) = 1$ and $\theta_l < \theta_k$ implies $p_l(\theta) = 1$. Clearly, the efficient rule must be monotone.

For monotone allocation rules social welfare can be written as

$$W(p,\theta) = \sum_{k=1}^{n_p(\theta)} \Big(\pi\left(n_p(\theta)\right) - f_k\Big) + C\left(n_p(\theta)\right) = S\left(n_p(\theta)\right) - \sum_{j=1}^{n_p(\theta)} f_j,$$

where f denotes the order statistic of θ. It remains to determine the optimal market size. For this purpose, rewrite welfare as

$$W(p,\theta) = \sum_{k=1}^{n_p(\theta)} \left[\Delta S(j) - f_j\right].$$

From this representation, we conclude that the j–th best firm should enter the market if and only if $f_j \le \Delta S(j)$. Therefore, the efficient market size is

$$n^\star(\theta) := \max\left\{k : f_k \le \Delta S(k)\right\} \tag{10}$$

The efficient allocation rule is

$$p_k^\star(\theta) = \begin{cases} 1 & \text{if } \theta_k \le \Delta S\left(n^\star(\theta)\right) \\ 0 & \text{else}. \end{cases} \tag{11}$$

Note that $n_{p^\star} = n^\star$.

As is well–known, it is possible to implement the efficient allocation rule in dominant strategies. The revenue maximizing mechanism in this class of mechanisms is characterized as follows.

[4]On the optimality of the Clarke–Groves mechanism in several other applications see Krishna and Perry (1998) and Schweizer (1999).

Theorem 2 (Optimal Groves Mechanism) *The mechanism $(p^\star, t^\star)$ with allocation rule $p^\star$ given by (11) and the payment rule*

$$t_k^\star(\theta) = \left(\pi\left(n^\star(\theta)\right) - \min\left\{\bar{\theta}_k, \Delta S\left(n^\star(\theta)\right), f_{n^\star(\theta)+1}\right\}\right) p_k^\star(\theta) \tag{12}$$

is a Groves mechanism.[5] *Moreover, it is the mechanism that maximizes tax revenue among all feasible mechanisms which implement the efficient market structure $p^\star$.*

Proof A Groves mechanism which implements $p^\star$ has payments of the form

$$t_k(\theta) = h_k(\theta_{-k}) - \Big[\sum_{j \neq k}\left(\pi\left(n^\star(\theta)\right) - \theta_j\right) p_j^\star(\theta) + C\left(n^\star(\theta)\right)\Big].$$

Let

$$h_k^\star(\theta_{-k}) = W\left(p^\star, (\bar{\theta}_k, \theta_{-k})\right)$$

be the social welfare which is generated if firm k has the highest fixed cost $\theta_k = \bar{\theta}_k$. We now show that

$$t_k^\star(\theta) = h_k^\star(\theta_{-k}) - \Big[\sum_{j \neq k}\left(\pi\left(n^\star(\theta)\right) - \theta_j\right) p_j^\star(\theta) + C\left(n^\star(\theta)\right)\Big]$$

can be simplified to (12).

If $p_k^\star(\theta) = 0$, then also $p^\star(\bar{\theta}_k, \theta_{-k}) = 0$. Thus, we have $W(p^\star, \theta) = W(p^\star, (\bar{\theta}_k, \theta_{-k}))$, and it follows that $t_k^\star(\theta) = 0$. If $p_k^\star(\theta) = 1$, we distinguish the following cases:

1) $\bar{\theta}_k = \min\left\{\bar{\theta}_k, \Delta S\left(n^\star(\theta)\right), f_{n^\star(\theta)+1}\right\}$:

Then firm k stays in the market with fixed costs $\bar{\theta}_k$. Thus, $p^\star(\bar{\theta}_k, \theta_{-k}) = p^\star(\theta)$, $n^\star(\bar{\theta}_k, \theta_{-k}) = n^\star(\theta)$, and therefore

$$h_k^\star(\theta_{-k}) = \sum_{j \neq k}\left(\pi\left(n^\star(\theta)\right) - \theta_j\right) p^\star(\theta) + \pi(n^\star(\theta)) - \bar{\theta}_k + C\left(n^\star(\theta)\right),$$

which yields

$$t_k^\star(\theta) = \pi\left(n^\star(\theta)\right) - \bar{\theta}_k\,.$$

2) $f_{n^\star(\theta)+1} = \min\left\{\bar{\theta}_k, \Delta S\left(n^\star(\theta)\right), f_{n^\star(\theta)+1}\right\}$:

[5] Recall that f is the order statistic of θ.

In this case, the firm with fixed costs $f_{n^\star(\theta)+1}$ replaces firm k. Therefore, $n^\star(\bar\theta_k, \theta_{-k}) = n^\star(\theta)$, $p^\star(\bar\theta_k, \theta_{-k}) \neq p^\star(\theta)$, and $p_i^\star(\bar\theta_k, \theta_{-k}) = 1$ for the firm with the $n^\star(\theta) + 1$-highest fixed costs. Thus,

$$h_k^\star(\theta_{-k}) = \sum_j \Big(\pi\left(n^\star(\theta)\right) - \theta_j\Big) p_j^\star(\bar\theta_k, \theta_{-k}) + C\left(n^\star(\theta)\right),$$

which yields

$$t_k^\star(\theta) = \pi\left(n^\star(\theta)\right) - f_{n^\star(\theta)+1}\,.$$

3) $\Delta S\left(n^\star(\theta)\right) = \min\left\{\bar\theta_k, \Delta S\left(n^\star(\theta)\right), f_{n^\star(\theta)+1}\right\}$:
Here, firm k drops out of the market and no other firm takes its place. Therefore, $n^\star(\bar\theta_k, \theta_{-k}) = n^\star(\theta) - 1$ and the kth element of $p^\star$ becomes zero. We get

$$\begin{aligned} h_k^\star(\theta_{-k}) &= \sum_j \Big(\pi\left(n^\star(\theta) - 1\right) - \theta_j\Big) p^\star(\bar\theta_k, \theta_{-k}) + C\left(n^\star(\theta) - 1\right) \\ &= \sum_{j \neq k} \Big(\pi\left(n^\star(\theta) - 1\right) - \theta_j\Big) p_j^\star(\theta) + C\left(n^\star(\theta) - 1\right) \end{aligned}$$

Thus,

$$\begin{aligned} t_k &= \sum_{j \neq k} \Big[\pi\left(n^\star(\theta) - 1\right) - \pi\left(n^\star(\theta)\right)\Big] p_j^*(\theta) - \Delta C\left(n^\star(\theta)\right) \\ &= \pi\left(n^\star(\theta)\right) - \Delta\Pi\left(n^\star(\theta)\right) - \Delta C\left(n^\star(\theta)\right) \\ &= \pi\left(n^\star(\theta)\right) - \Delta S\left(n^\star(\theta)\right). \end{aligned}$$

We now show that the expected utility of firm k is zero in the worst case, when its fixed cost is equal to $\bar\theta_k$. In this event it can only make a profit if it is in the market, that is if $\bar\theta_k \leq \Delta S(n^\star(\bar\theta_k, \theta_{-k}))$. But this entails $\bar\theta_k \leq f_{n^\star(\theta)+1}$ by definition of $p^\star$. Thus, firm k has to pay $\pi(n^\star(\bar\theta_k, \theta_{-k})) - \bar\theta_k$ in this case, which is exactly equal to its profit so that $\bar U_k(\bar\theta_k) = 0$.

As we know from the Revenue–Equivalence Theorem, in each feasible mechanism that implements $p^\star$ the regulator is only free to choose firms' reservation utilities $\bar U_k(\bar\theta_k)$. In view of the participation constraint, it is therefore optimal to set all of them equal to zero. This is exactly what the mechanism $(p^\star, t^\star)$ does. □

5 Optimal mechanisms if tax revenue matters

The government may care more for tax revenue than for consumer and producer surplus because the marginal cost of raising other taxes to fund government expenditures is greater than one, due to welfare distortions associated with general taxation. This suggests that the regulator maximizes a convex combination of expected tax revenue and social surplus: $\lambda T+(1-\lambda)E[W]$, or equivalently $(\mu-1)T+E[W]$, where $\mu := 1/(1-\lambda) \geq 1$ represents the marginal cost of general taxation.[6] Generally, a preference for tax revenue, $\mu > 1$, makes it optimal to deviate from the allocation rule that maximizes social welfare.

To prepare the thus generalized optimal mechanism design problem we introduce the definitions of "virtual social surplus":

$$S_\lambda(n) := S(n) - \lambda C(n), \tag{13}$$

and "priority levels" (that name will become clear later on):

$$\gamma_k(\theta_k) := \theta_k + \lambda \frac{G_k(\theta_k)}{g_k(\theta_k)}. \tag{14}$$

Furthermore, we let $\zeta = (\zeta_1, \ldots, \zeta_N)$ be the order statistic of γ, where ζ_1 denotes the lowest and ζ_N the highest priority level.

In addition, we make the following assumption:

Assumption 4 *Priority levels $\gamma_k(\theta_k)$ are strict monotone increasing for all $k \in \{1, \ldots, N\}$. A sufficient condition is that hazard rates $g_k(\theta_k)/G_k(\theta_k)$ are strict monotone decreasing.*

Assumption 4 considerably simplifies the characterization of the optimal mechanism since it assures that the second–order condition of the incentive compatibility constraint is satisfied. However, it can be dispensed with by employing a convexification argument developed in Myerson (1981) and Baron and Myerson (1982).[7]

Lemma 1 *The following mechanism (p^λ, t^λ) maximizes the convex combination of expected tax revenue and social welfare, $L_\lambda := \lambda T + (1-\lambda)E[W]$*

[6] This objective function is frequently used in public economics, see for example Laffont and Tirole (1993) and Dana and Spier (1994).

[7] For a more accessible account of this procedure see also Landsberger and Tsirelson (1999)).

in the class of feasible direct revelation mechanisms.

$$p_k^\lambda(\theta) = \begin{cases} 1 & \gamma_k(\theta) \le \Delta S_\lambda(n_\lambda(\theta)) \\ 0 & \textit{otherwise}, \end{cases} \tag{15}$$

$$t_k^\lambda(\theta) = p_k^\lambda(\theta)\,(\pi(n_\lambda(\theta)) - \theta_k) - \int_{\theta_k}^{\bar\theta_k} p_k^\lambda(x, \theta_{-k})dx, \tag{16}$$

$$n_\lambda(\theta) := \max\{k : \zeta_k \le \Delta S_\lambda(k)\}. \tag{17}$$

PROOF The expected tax revenue of feasible direct revelation mechanisms has already been determined in (8). A necessary condition for the maximum of T is $\bar U_k(\bar\theta_k) = 0$, for all k. Therefore, the regulator's objective function, L_λ, can be written in the form:

$$\begin{aligned} L_\lambda =& \mathbf{E}\Big[\sum_{k=1}^{N} \lambda\Big((\pi(n_p(\hat\theta)) - \hat\theta_k)p_k(\hat\theta) - \int_{\hat\theta_k}^{\bar\theta_k} p_k^\lambda(x, \hat\theta_{-k})dx\Big) \\ & + (1-\lambda)\Big(\sum_{k=1}^{N}\Big(\pi(n_p(\hat\theta))p_k(\hat\theta) - \hat\theta_k p_k(\hat\theta)\Big) + C(n_p(\hat\theta))\Big)\Big]. \end{aligned} \tag{18}$$

Using Fubini's theorem, we obtain

$$\begin{aligned} \mathbf{E}\Big[\int_{\hat\theta_k}^{\bar\theta_k} \bar p_k(x)dx\Big] &= \int_{\underline\theta_k}^{\bar\theta_k}\int_{\Theta_{-k}} p_k(x, \hat\theta_{-k})g_{-k}(\hat\theta_{-k})d\hat\theta_{-k}G_k(x)dx \\ &= \int_{\Theta_{-k}} p_k(x, \hat\theta_{-k})G_k(\hat\theta_k)g_{-k}(\hat\theta_{-k})d\hat\theta_{-k} \\ &= \int_{\Theta} p_k(\hat\theta)\frac{G_k(\hat\theta_k)}{g_k(\hat\theta_k)}g(\hat\theta)d\hat\theta\,. \end{aligned}$$

Plugging this back into (18), it follows that L_λ is equal to

$$\int_{\Theta}\Big(\sum_{k=1}^{N} p_k(\hat\theta)\Big[\pi(n_p(\hat\theta)) - \Big(\hat\theta_k + \lambda\frac{G_k(\hat\theta_k)}{g_k(\hat\theta_k)}\Big)\Big] + (1-\lambda)C(n_p(\hat\theta))\Big)g(\hat\theta)d\hat\theta.$$

It is sufficient to maximize pointwise for every vector of types (θ). Hence, we focus on

$$\sum_{k=1}^{N} p_k(\hat\theta)\Big[\pi(n_p(\hat\theta)) - \Big(\hat\theta_k + \lambda\frac{G_k(\hat\theta_k)}{g_k(\hat\theta_k)}\Big)\Big] + (1-\lambda)C(n_p(\hat\theta))\,.$$

Obviously we want the firms with the lowest priority levels to enter the market. We can therefore restrict attention to allocation rules p which are monotone in the priority levels. For such mechanisms, the sum can be transformed into

$$\sum_{k=1}^{n_p(\theta)} \Big(\pi(n_p(\hat{\theta})) - \zeta_k\Big) + (1-\lambda)C(n_p(\hat{\theta})) = S_\lambda(n_p(\theta)) - \sum_{k=1}^{n_p(\theta)} \zeta_k$$

Thus, the regulator has to determine the size n_p which maximizes the difference between virtual social surplus and the sum of priority levels. Now, because of

$$S_\lambda(n_p(\theta)) - \sum_{k=1}^{n_p(\theta)} \zeta_k = \sum_{k=1}^{n_p} \Delta S_\lambda(n_p(\theta)) - \zeta_k\,,$$

the optimal number of firms in the market is given by

$$n_\lambda(\theta) := \max\{k : \zeta_k \leq \Delta S_\lambda(k)\}.$$

Thus, the allocation rule p^λ which maximizes the difference between virtual social surplus and the sum of priority levels satisfies

$$p_k^\lambda(\theta) = 1 \;\text{ if }\; \gamma_k(\theta_k) \leq \Delta S_\lambda(n_\lambda),$$

and $p_k^\lambda = 0$ otherwise.

It remains to show that (p^λ, t^λ) is a feasible mechanism. Expected utility from this mechanism is given by

$$\bar{U}_k(\theta_k) = \int_{\theta_k}^{\bar{\theta}_k} p_k^\lambda(x, \theta_{-k})dx \geq 0\,.$$

Thus, the interim participation constraint is satisfied. By Assumption 4, the probability $\bar{p}_k^\lambda$ of being in the market is decreasing in θ_k. This is equivalent to incentive compatibility. □

From this result we can now derive a simple and intuitively appealing characterization of the optimal mechanism. For this purpose suppose fixed costs have been revealed to the regulator and consider a firm k that qualifies to be awarded a license, because its priority level is sufficiently low, according to the allocation rule (15). Then, the regulator can compute the supremum of fixed cost that this firm k could have had, given θ_{-k}, while still qualifying to get the license:

$$y_k^\lambda(\theta_{-k}) := \sup\Big\{x \in [\underline{\theta}_k, \bar{\theta}_k] : \gamma_k(x) \leq \min\{\zeta_{n_\lambda+1}, \Delta S_\lambda(n_\lambda)\}\Big\}.$$

This supremum of "winning" fixed costs plays a key role in the construction of the optimal payment rule, as follows.

Theorem 3 (Optimality if Tax Revenue Matters) *The optimal mechanism issues a license to the* $n_\lambda(\theta)$ *firms with the lowest priority levels. Payments are collected only from those firms that obtain a license, who then pay a transfer equal to the net profit they would have earned if they had the supremum of "winning" fixed costs:*

$$t_k(\theta) = \pi(n_\lambda(\theta)) - y_k^\lambda(\theta_{-k}). \tag{19}$$

Proof Employing the definition of $y_k^\lambda(\theta_{-k})$, the optimal allocation rule (15) is equivalent to

$$p_k^\lambda(\theta_k, \theta_{-k}) = \begin{cases} 1 & \text{if} \quad \theta_k \le y_k^\lambda(\theta_{-k}) \\ 0 & \text{otherwise.} \end{cases}$$

Integrating yields

$$\int_{\theta_k}^{\bar{\theta}_k} p_k^\lambda(x, \theta_{-k}) dx = \begin{cases} y_k^\lambda(\theta_{-k}) - \theta_k & \text{if} \quad \theta_k \le y_k^\lambda(\theta_{-k}) \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the optimal payment rule (16) can be written in the form:

$$t_k^\lambda(\theta) = \begin{cases} \pi(n_\lambda(\theta)) - y_k^\lambda(\theta_{-k}) & \text{if } \theta_k \le y_k^\lambda(\theta_{-k}) \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

□

Corollary 1 *If the optimal mechanism is adopted, the expected number of licensed firms,* n^λ*, is decreasing in* λ*. For* $\lambda = 0$ *we are back at implementing the welfare optimum, and for* $\lambda = 1$*, i.e. if tax revenue is the only thing that matters, the optimal mechanism implements monopoly.*

If firms' fixed costs are i.i.d. random variables, only the least cost firms get a license. However, if the $\hat{\theta}$'s differ, it is generally optimal to discriminate between firms, as follows.

Theorem 4 *Suppose hazard rates are monotone across firms. Then, the optimal mechanism exhibits "handicapping" of firms whose fixed costs are drawn from the more favorable distribution.*

PROOF Consider two firms, say firm 1 and firm 2, and assume firm 1 has a lower hazard rate, everywhere,

$$\forall x: \quad \frac{g_1}{G_1}(x) \leq \frac{g_2}{G_2}(x).$$

Then, for all y: $\int_y^{\bar{\theta}} g_1(x)/G_1(x)dx \leq \int_y^{\bar{\theta}} g_2(x)/G_2(x)dx$, which implies $\ln G_1(y) \geq \ln G_2(y)$, and hence $G_1(y) \geq G_2(y)$. In other words, $\hat{\theta}_1$ dominates $\hat{\theta}_2$ in the sense of first–order stochastic dominance. Since lower fixed costs are more favorable, we conclude that $\hat{\theta}_1$ is the more "favorable" random variable than $\hat{\theta}_2$. Using the definition of priority levels we conclude:

$$\forall x: \quad y_1(x) := x + \lambda \frac{G_2}{g_2}(x) \leq x + \lambda \frac{G_1}{g_1}(x) =: y_2(x).$$

Therefore, the allocation of licenses to the firms with the lowest priority levels, as advised by the optimal mechanism, handicaps those firms who draw their fixed cost from the more favorable distributions. □

We close with some examples to illustrate the gains from such discrimination. Suppose fixed costs are uniformly distributed on $[\underline{\theta}_i, \bar{\theta}_i]$. Then $g_i(\theta) = \frac{1}{\bar{\theta}_i - \underline{\theta}_i}$ and $G_i(\theta) = \frac{\theta_i - \underline{\theta}_i}{\bar{\theta}_i - \underline{\theta}_i}$. This yields priority levels

$$y_i(\theta_i) = \theta_i + \lambda(\theta_i - \underline{\theta}_i).$$

The markup of a firm's fixed costs is proportional to the extent it exceeds its minimal cost $\underline{\theta}_i$. In this sense, the optimal mechanism discriminates against "good" firms which increases their payments in case they win a license.

Now let $N = 2$ and $\lambda = 1$ (only tax revenue matters). Moreover, let $\hat{\theta}_1$ be uniformly distributed on $[0, 1]$ and $\hat{\theta}_2$ uniformly distributed on $[1, 2]$, so that $\hat{\theta}_1$ is unambiguously the more favorably distributed random variable than $\hat{\theta}_2$. Priority levels are

$$y_1(\theta_1) = 2\theta_1, \quad y_2(\theta_2) = 2\theta_2 - 1. \tag{21}$$

And the tax revenue maximizing mechanism summarized in Theorem 3 deviates from the efficient selection of one firm by picking the inferior firm 2 whenever $\theta_2 < \theta_1 + 0.5$.

In order to gain some intuition why it is optimal to add a distortion in this manner, consider the impact of a distortion of efficiency on tax revenue that is due to awarding the license to the inferior firm 2 whenever

$\theta_2 < \theta_1 + \delta$, for $\delta > 0$. As one can easily confirm, if one starts from the efficient selection of one firm and raises δ just marginally, tax revenue jumps up in the event when firm 1 wins the license, which occurs almost with certainty, and declines when firm 2 wins (which almost never occurs).[8] Therefore, adding a small distortion, $\delta > 0$, unambiguously increases tax revenue.

In particular, tax revenue can be expressed as function R of the distortion parameter δ, as follows:

$$\begin{aligned} R(\delta) =& \pi(1) - \Big((1-\delta) + \int_1^{1+\delta}\int_0^{\theta_2-\delta}(\theta_2-\delta)d\theta_1 d\theta_2 \\ &+ \int_{1-\delta}^{1}\int_1^{\theta_1+\delta}(\theta_1+\delta)d\theta_2 d\theta_1\Big) \\ =& \pi(1) - \frac{4\delta^3 - 3\delta^2 + 6}{6}. \end{aligned}$$

It follows immediately that the tax revenue maximizing distortion is to set $\delta = 0.5$ (and the expected tax revenue is thus raised from $\pi(1) - 1$ to $\pi(1) - 23/24$), which is precisely the optimal mechanism summarized in Theorem 3 for $\lambda = 1$.

6 Free entry and inefficiency

A natural question is whether a mechanism is needed in the first place. Why not let free entry implement the efficient market size? Another important issue is whether the optimal mechanism, if it is adopted, contributes to a budget deficit. Ideally, the optimal mechanism should also be deficit free. As we show in this and the subsequent section, both issues are somewhat interrelated. In particular, free entry generally leads to an inefficient market size, and the optimal mechanism is generally deficit free.

To explain market size under free entry, consider a two–stage market game. After having observed their own fixed costs, firms simultaneously decide whether to enter into the market, and then, after entry decisions have been observed, play a Cournot–style market game. For simplicity,

[8] If firm 1 wins, one has either $\theta_2 > 1 + \delta$, in which case tax revenue is equal to $\pi(1) - 1$, or $\theta_2 < 1 + \delta$ and $\theta_1 \in (0, \theta_2 - \delta)$, in which case tax revenue is equal to $\pi(1) - (\theta_2 - \delta)$. Therefore, if firm 1 wins, tax revenue is always higher than in the efficient mechanism (where it is equal to $\pi(1) - 1$). However, if firm 2 wins, tax revenue is equal to $\pi(1) - (\theta_1 + \delta)$, which is evidently lower than $\pi(1) - 1$.

fixed costs are assumed to be i.i.d. random variables with the continuous probability distribution function $G : [0, \bar{\theta}] \to [0, 1]$, and $\bar{\theta} > \pi(N)$.[9]

In the Bayesian equilibrium of this game, firms enter the market iff their fixed costs are below some threshold level $c \in (0, \bar{\theta})$, which is uniquely determined by the condition of indifference between entry and non–entry:

$$\sum_{m=0}^{N-1} \pi(m+1) \binom{N-1}{m} G(c)^m (1 - G(c))^{N-1-m} - c = 0. \tag{22}$$

Thereby, the left-hand side is the equilibrium expected profit of the borderline firm with fixed cost equal to c if it enters, given that only the firms with fixed costs at or below c do enter.[10]

THEOREM 5 *Free entry generally gives rise to an inefficient market size.*

PROOF The equilibrium market size under free entry is equal to the efficient size n^* if

$$\Delta S(N^\star + 1) > c > \Delta S(n^\star).$$

The free entry equilibrium defines a deterministic threshold level c, implicitly defined in (22), whereas the efficient number of firms, n^*, and therefore $\Delta S(n^\star)$ is random, because it depends on the realization of fixed costs. Consequently, free entry generally fails to implement the efficient market size. □

7 Is the optimal mechanism deficit free?

We now show that the optimal mechanism is generally deficit free by using an excessive entry property of a hypothetical entry game. We proceed as follows: First, we introduce the hypothetical entry game and explain why it exhibits excessive entry. Second, we consider the optimal Groves mechanism analyzed in Section 4, which maximizes tax revenue in the class of mechanisms that implement the efficient market size. We show that this mechanism is deficit free. Third, we conclude that the optimal mechanism which maximizes the weighted sum of expected tax revenue and social surplus is also deficit free.

[9] If $\bar{\theta} \leq \pi(N)$ all firms would enter the market and no one would suffer a loss.

[10] For a proof of existence and uniqueness of the equilibrium threshold level c see Dixit and Shapiro (1986).

At the outset notice that the regulator would subsidize a firm that earns a negative profit in order to induce it to participate in the market game if this raises the social surplus by more than this firm's losses. Therefore, it is not obvious that the optimal mechanism should be deficit free.

Consider a hypothetical entry game which serves exclusively as a benchmark for our analysis. In this hypothetical game firms' fixed costs are common knowledge, and all least cost firms enter until the marginal firm earns zero profits and then play a Cournot–style market game. Integer constraints on the number of firms are ignored. This game gives rise to excessive entry, analogous to the well–known excessive entry property of symmetric Cournot market games discovered by Mankiw and Whinston (1986).

LEMMA 2 *In the equilibrium of the hypothetical entry game, the number of entrants, $n^e(\theta)$, is greater than or equal to the welfare maximizing number of entrants, $n_0(\theta)$, for all θ.*

PROOF Ignoring integer constraints let $f(n)$ be a continuously differentiable function that denotes the n–th highest fixed costs, $q(n)$ each firm's equilibrium output, and $P(nq(n))$ the inverse market demand. Then, the social welfare generated by the n–firm hypothetical entry cum Cournot–style oligopoly game is equal to:

$$W(q(n), n) = \int_0^{nq(n)} P(y)dy - \int_0^{n} f(y)dy, \qquad (23)$$

and n^e is implicitly determined by the zero–profit condition:

$$\pi(n^e) - f(n^e) = 0. \qquad (24)$$

Differentiating (23) with respect to n yields, at $n = n^e$:

$$\begin{aligned} \left.\frac{dW}{dn}\right|_{n=n^e} &= P(n^e q(n^e)) \left(q(n^e) + n^e q'(n^e)\right) - f(n^e) \\ &= \pi(n^e) - f(n^e) + P(n^e q(n^e)) n^e q'(n^e) \\ &= P(n^e q(n^e)) n^e q'(n^e) \\ &< 0. \end{aligned} \qquad (25)$$

Hence, welfare can be increased by lowering n below n^e. □

THEOREM 6 *If integer constraints on the number of firms do not bind, the optimal mechanism is deficit free.*

PROOF The assertion follows easily for the optimal Groves mechanism that implements the efficient market size $n_0(\theta)$. Indeed, by definition of n_0 and n^e, combined with the excessive entry property $n^e(\theta) \geq n_0(\theta)$ one has, for all θ:

$$\begin{aligned}
\pi(n_0(\theta)) &\geq \pi(n^e(\theta)) \\
&\equiv f(n^e(\theta)) \\
&\geq f(n_0(\theta)) \\
&\equiv \frac{d}{dn}\int_0^{n_0(\theta)} f(x)dx \\
&\equiv \frac{d}{dn} S_0(n_0(\theta)).
\end{aligned}$$

Therefore, the regulator receives a nonnegative transfer from each firm that is awarded a license (others neither pay nor receive anything) equal to

$$t = \pi(n_0(\theta)) - \min\{\frac{d}{dn}S_0(n_0(\theta)), \bar{\theta}_i\} \geq 0.$$

Hence, the optimal Groves mechanism is deficit free.

Compared to the optimal Groves mechanism, which maximizes tax revenue in the class of mechanisms that implement the efficient market size, the optimal mechanism, analyzed in Section 5, gives more weight to tax revenue. Therefore, it yields at least as much tax revenue, and hence is also deficit free. □

Of course, it is not satisfactory to ignore the integer constraint on the number of firms, as we did so far, in this section. Indeed, if one takes this constraint into account, one can easily find examples, where the hypothetical entry game gives rise to insufficient entry. However, as Mankiw and Whinston (1986) have shown, $n^e \geq n^* - 1$, so that entry is never insufficient by more than one firm. Also, Perry (1984) performed numerical simulations assuming constant elasticity of demand curves and Cournot equilibrium, and showed that the integer constraint tends to matter only if the number of firms in the entry equilibrium tends to be in the order of one or two firms.

In addition, we report the following necessary and sufficient condition for the optimal mechanism to be deficit free, that does not ignore the integer constraint.

Theorem 7 *Taking the integer constraint into account, the optimal mechanism is always deficit free if and only if for all $n \geq 2$*

$$\Delta S(n) \leq \pi(n). \tag{26}$$

Proof Again, we first look at the efficient mechanism and then extend the result to the optimal mechanism.

Condition (26) entails $\min\{\bar{\theta}_k, \Delta S(n^\star(\theta)), f_{n^\star(\theta)+1}\} \leq \pi(n)$. This assures that the efficient mechanism does not subsidize any firm, which proves sufficiency.

It remains to be shown that (26) is also a necessary condition. For this purpose, suppose, per absurdum, that (26) is violated so that $\pi(n) < \Delta S(n)$ for some n. Now consider the event that $\pi(n) < f_n < \Delta S(n)$, for all $i = 1, \ldots, n$. Then, the efficient mechanism selects these n firms to participate in the market. Since participation is voluntary, this requires that they are subsidized. And the subsidy per active firm i is at least as high as

$$f_i - \pi(n) > 0, \quad \forall i = 1, \ldots, n.$$

Therefore, the efficient mechanism gives rise to a deficit.

The optimal mechanism gives more weight to tax revenue than the efficient mechanism. Therefore, the above condition also assures that the optimal mechanism is deficit free. □

8 Discussion

In the present paper we have analyzed the regulation of entry in a natural oligopoly market, in the tradition of the optimal mechanism design approach. The two key assumptions were that firms have private information about their fixed costs and that the regulator is unable to control the behavior of firms once they are in the market.

We addressed four issues: 1) the design of the optimal Groves mechanism that yields the highest tax revenue in the class of mechanisms that implement efficiency; 2) the design of the optimal mechanism that maximizes a weighted sum of tax revenue and social surplus, which is relevant if general taxation is subject to a deadweight loss; 3) a comparison of the optimal mechanism with the free entry market equilibrium, and 4) an assessment of the budgetary consequences of the optimal mechanism.

There are two main limitations of the present analysis. First, our analysis is restricted to an independent private values framework, where firms' fixed costs are independent random variables. In a framework of stochastic dependency, one would probably need other mechanisms, that supply firms with more information about each other, just like in an open ascending auction. The second major limitation has to do with the assumption that all potential firms have the same marginal cost, and differ only in their fixed costs. Ideally one would like to assume that firms have different fixed and marginal costs which is firms' private information. However, this gives rise to multi–dimensional mechanism design problems that are still not resolved with sufficient generality.

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