Structural Change and Long Memory in Volatility: New Evidence from Daily Exchange Rates*

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Abstract

Using a Markov-Switching Fractionally Integrated GARCH model of a daily exchange rate (DEM-USD), we provide evidence in favour of a strong interaction between structural change and long memory in the variance. It is however found that these features are "imperfect substitutes" in the sense that both features are required to capture all of the observed persistence in the volatility.

Keywords: Structural Change; FIGARCH; Persistence in Volatility; Daily Exchange Rate Returns

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1 Introduction

Recent developments in times series econometrics have been concerned with the interaction between structural change and long range dependence in the two first moments of financial series. Diebold and Inoue (1999) show that, under some conditions, stochastic regime switching is observationnally equivalent to long memory, asymptotically and in finite samples. Granger and Hyung (1999) emphasize a positive relationship between occasional structural breaks and the fractional degree of integration. Mikosch and Stărică (1999) argue that structural change in asset returns could be responsible for the long memory in the volatility. Furthermore, empirical analysis, mostly based on the stock exchange returns, document on the much lower estimated persistence obtained when accounting for structural change, either in the conditional mean or in the conditional variance. Granger and Hyung (1999) show that there is much less evidence of long memory in the absolute and squared S&P 500 daily returns when adjusting for breaks. On 30 stocks and 2 indexes, Kim and Kon (1999) also show that the persistence captured in a Garch model is strongly reduced when including previously identified structural shifts in the variance.

In this paper, we provide further evidence of the strong interaction between structural change and long memory in the field of exchange rate volatility. The choice of this field is quite important since previous analysis (Baillie, Bollerslev and Mikkelsen 1996, Bollerslev and Mikkelsen 1996, Tse 1998, Beine, Laurent and Lecourt 1999) have used exchange rate returns to provide strong evidence in favour of the FIGARCH model. Simultaneously, some evidence in favour of structural change in the conditional variance of exchange rate returns has been advanced by Bollen, Gray and Whaley (2000) through a Markov-Switching analysis. To allow both for structural change and long memory, we estimate a new model, the Markov-Switching FIGARCH model that jointly integrates both features.[1] This contrasts with previously mentioned empirical contributions which rely on a two-step procedure, i.e. identification of the structural breaks followed by the estimation of break-filtered models.

[1] A similar approach including both FIGARCH and regime switching features has simultaneously and independently been proposed by Hsieh, Chung and Lin (1999). Comparing their analysis with ours, one may identify several differences. First, their analysis is applied on stock returns (S&P 500). Secondly, on the methodological side, only the scale parameter is made dependent on the regime. Finally and maybe more importantly, the main conclusions are quite different: while we conclude in (low) complementarity between structural change and long memory (see below), their analysis emphasize the spurious feature of long memory.

Basically, the advantages of our approach are twofold. The first one is of course related to efficiency in the estimation of the final model parameters. A one-step estimation allows to control for the interaction between parameters thought to capture long memory and those related to structural change. The second one is related to the data generating process that the researcher has in mind. Former studies assume that structural change occur and is correctlty detected and then, investigates the impact of its presence on the estimated persistence of shocks. Nevertheless, as Granger and Hyung (1999) show, if the true process is I(d), spurious breaks will be detected.[2] By allowing for long memory and structural changes, our integrated approach is less subject to these biases.

2 The Single-Regime FIGARCH model

We focus on daily nominal exchange rate returns (denoted r_t) of the Deutsche Mark (DEM) against the US Dollar (USD) obtained from the Federal Reserve Bank. Our estimation period ranges from January 1 1980 to December 31 1998 and involves 4739 data points. As a starting point, one can estimate the FIGARCH (p, d, q) model introduced by Baillie, Bollerslev and Mikkelsen (1996) that allows for long run dependence in the conditional variance:

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \mid \Omega_t \sim N(0, \sigma_t^2) \tag{1}$$

$$\sigma_t^2 = \omega + \beta(L)\sigma_t^2 + \left[1 - \beta(L) - \phi(L)\,(1 - L)^d\right]\epsilon_t^2 \tag{2}$$

where μ is the mean of the process and Ω_t is the information set at time t. $\beta(L) = \beta_1 L + ... + \beta_p L^p$ and $\phi(L) = 1 - \phi_1 L - ... - \phi_q L^q$ are the lag polynomials of respective orders p and q of which all roots lie outside the unit circle. μ, ω, β's, ϕ's and d are parameters to be estimated with d being

[2] By contrast, when break dates are supposed to be known, dummy variables can be directly included and Likelihood Ratio Tests (LRT) conducted to assess the occurrence of structural change. See Beine and Laurent (2000) for illustration.

the fractional integration parameter[3] and finally, L is the lag operator[4]. Maximum likelihood estimation results of the FIGARCH $(1, d, 0)$ are reported in column 2 of Table 1. As found by Baillie *et al.* (1996), and Tse (1998), estimates of d suggest that the stable GARCH $(d = 0)$ or the integrated GARCH $(d = 1)$ models are rejected in favour of a long memory in the conditional variance.

Table 1: FIGARCH and MS-(FI)GARCH models for the DEM-USD exchange rate

	FIGARCH(1,d,0)	SW-GARCH(1,1)	SW-FIGARCH(1,d,0)
$\mu \mid \mu_1$	-0.0061	0.0303	0.0571
	(0.0092)	(0.0298)	(0.0400)
μ_2	-	-0.0148	-0.0193
		(0.0123)	(0.0106)
$\omega \mid \omega_1$	0.0724	0.8794	1.0150
	(0.0135)	(0.0702)	(0.0940)
ω_2	-	0.1772	0.0770
		(0.0415)	(0.0255)
$\beta_1 \mid \beta_{1,2}$	0.2060	0.2280	0.1109
	(0.0501)	(0.1315)	(0.0342)
$\phi_{1,2}$	-	-0.0069	-
		(0.0193)	
$d \mid d_2$	0.2754	-	0.0958
	(0.0360)		(0.0204)
p_{11}	-	0.9617	0.9227
		(0.0152)	(0.0326)
q_{22}	-	0.9794	0.9747
		(0.0084)	(0.0104)
AIC	2.0114	1.9890	1.9820
SBIC	2.0169	2.0003	1.9930
$Q^2(20)$	26.4796	19.6910	22.9882
$Q^2(50)$	58.6737	55.2263	59.1267
LogL	-4762.0911	-4705.0643	-4688.5829

Note: asymptotic standard errors are in parentheses.
AIC = Akaike information criterion.
SBIC = Schwarz Baysian information criterion.
$Q^2(.)$ = Ljung-Box statistic on squared residuals.
LogL = Log-Likelihood.
Source: Federal Reserve Statistical Release H.10.

[3] We follow Baillie *et al* (1996) and truncate the infinite Taylor approximation of $(1 - L)^d$ at a number of lags equal to 1000. Chung (1999) proposes an alternative specification of the FIGARCH due to the strong relationship between ω and the truncation order. We do not tackle this issue in this paper because the parameter of interest is d, which is not affected by this choice.

[4] In the case of a FIGARCH$(1, d, 0)$, $\phi_1 = \phi_2 = \cdots = \phi_q = 0$.

3 The Markov-Switching FIGARCH model

Let us now focus on structural change through a Markov-Switching process. Although these processes are not the only ones allowing for structural change, Markov-Switching processes (MS) have been specifically investigated by some of the above mentionned theoretical contributions. The new model to be estimated becomes a MS-FIGARCH model in which the mean and variance parameters are made dependent upon a latent state variable s_t $(s_t = 1, 2)$:

$$r_t = \mu_{s_t} + \epsilon_t, \qquad \epsilon_t \mid \Omega_t \sim N(0, \sigma^2_{s_t}) \tag{3}$$

$$\sigma^2_{s_t} = \omega_{s_t} + \beta_{1,s_t}\tilde{\sigma}^2_{t-1} + \left[1 - \beta_{1,s_t}L - (1-\phi_{1,s_t}L)(1-L)^{d,s_t}\right]\tilde{\epsilon}^2_t \tag{4}$$

This model is a natural extension of the Regime Switching GARCH model introduced by Gray (1996), in which d is set to 0 for both regimes.[5] By the way, all the parameters are allowed to switch, in contrast to Hamilton and Susmel (1994)'s Switching ARCH model and Hsieh, Chung and Lin (1999) (in which the markov process only governs a multiplicative parameter). To solve the problem of path dependence[6] induced by the GARCH process, Gray (1996) proposes to explain the conditional volatility of each regime $\left(\sigma^2_{s_t}\right)$ on past unconditional volatility $\left(\tilde{\sigma}^2_{t-1}\right)$ and unconditional squared residuals $\left(\tilde{\epsilon}^2_{t-1}\right)$, where[7]

$$\tilde{\epsilon}^2_{t-1} = \left\{r_t - \left[p_{1,t-1}\mu_1 + (1-p_{1,t-1})\mu_2\right]\right\}^2 \tag{5}$$

$$\tilde{\sigma}^2_{t-1} = p_{1,t-1}\left(\mu_1^2 + \sigma^2_{1,t-1}\right) + (1-p_{1,t-1})\left(\mu_2^2 + \sigma^2_{2,t-1}\right) - \left[p_{1,t-1}\mu_1^2 + (1-p_{1,t-1})\mu_2^2\right]^2 \tag{6}$$

$$p_{1,t} = \Pr\left(S_t = 1 \mid \Omega_{t-1}\right) \tag{7}$$

As the results will suggest, s_t will basically capture volatility regimes (high and low volatility);

[5] The MS-GARCH $(1,1)$ model is written as: $\sigma^2_{st} = \omega_{st} + \beta_{1,s_t}\tilde{\sigma}^2_{t-1} + \phi_{1,s_t}\tilde{\epsilon}^2_{t-1}$. For convenience, we use the same notation for the GARCH and the FIGARCH models.

[6] A regular extension of the SWARCH model introduced by Hamilton and Susmel (1994) to the GARCH and FIGARCH framework is not possible due to the time dependence of the conditional variance (because the GARCH model is an ARCH of infinite order). See Gray (1996, p. 34) for further details.

[7] The extension of this model to a three states Markov process is straightforward but is not investigated in this paper.

its dynamics is driven by a first order Markov Switching process with transition probabilities p_{ij}:

$$p_{ij} = \Pr ob(s_t = j \mid s_{t-1} = i) \tag{8}$$

These transition probabilities are constant in the sense that they depend only on the past state of the economy and can thus be collected in the following P matrix:[8]

$$P = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix} \tag{9}$$

This Markov-Switching model is estimated by the Expected Maximum Likelihood (EML) procedure (see for details Hamilton 1994 and Gray 1996). The estimation results are gathered in columns 3 and 4 of Table 1 (which reports only the finally retained models in order to save place). Column 3 reports the results of the MS-GARCH $(1,1)$ while the last column is related to the MS-FIGARCH $(1,d,0)$.

Some comments are in order.

First, our regimes are found to refer to volatility levels: the first regime $(s_t = 1)$ turns out to be the high volatility regime while the second one $(s_t = 2)$ refer to low volatility.[9] Notice that several specifications of the MS models have been estimated. Our retained specifications are a GARCH $(1,1)$ and a FIGARCH $(1,d,0)$ for the low volatility regime while a constant variance model turns out to capture the dynamics of the high volatility regime. Ljung-Box statistics on squared residuals clearly reject any remaining heteroskedasticity up to 50 lags.

Second, compared to the Single-Regime FIGARCH model, the MS models involve an important increase in the likelihood value. Of course, as reported by several authors (Hansen 1992, Garcia 1998), usual χ^2 based critical values cannot be formally used because of the non identification of

[8] The extension of this model to time-varying probabilities has been proposed by Diebold, Lee and Weinbach (1994) but is not explored in this paper.

[9] Indeed, both mean parameters turn out to be insignificant at usual confidence levels.

some parameters under the null hypothesis of no switching.[10] Nevertheless, given the very high values obtained for LR tests (LRT comparing the single-regime FIGARCH and MS-FIGARCH amounts to 147.02 for 3 additional parameters), we suspect some evidence in favour of Markov-Switching processes. Importantly, the information criteria (AIC and SBIC) clearly favour the MS models.

Finally, the MS-FIGARCH seems to perform better than the MS-GARCH, which is also confirmed by the usual LRT test and the information criteria.[11] Comparing the point estimate of d, our results show that allowing for structural change drastically reduces the estimated persistence of volatility shocks. In this sense, they illustrate the relevance of the theoretical results of Diebold and Inoue (1999) and Granger and Hyung (1999) to the modelling of daily exchange rates. Also, such a result is quite intuitive given the fact that the MS model accounts for the shocks persistence through the transition probabilities p_{11} and p_{22} that are found to be relatively high. Nevertheless, our estimation results suggest that as far as volatility is concerned, structural change and long memory are *imperfect substitutes*: under the low volatility regime, d remains significant at usual confidence levels and a FIGARCH $(1, d, 0)$ model is required to describe the process. This is consistent with the empirical evidence on stock returns volatility provided by Granger and Hyung (1999) and with Diebold and Inoue (1999) warnings about "*the temptation to jump to conclusions of structural change producing spurious inferences on long memory*".[12] In others terms, both features are necessary to capture the short run dynamics of exchange rate volatilty.

4 Conclusion

This paper shows that the long memory documented in the volatility of exchange rates is drastically reduced when we to take into account the possible structural changes. However, unlike Hsieh, Chung and Lin (1999)'s investigation for stock returns, we find clear evidence of remaining long memory in the conditional second moment. It turns out that the exchange rate volatility dynamics is quite different from the stock returns volatility patterns.

[10] Garcia (1998) derived the distributions of critical values for some relatively simple models.

[11] Bollerslev and Mikkelsen (1996) show the usefulness of the AIC and SBIC to discriminate between the GARCH $(1, 1)$ and the FIGARCH $(1, d, 0)$.

[12] See Diebold and Inoue (1999) p.25.

References

[1] Baillie, R., Bollerslev, T., Mikkelsen, H., 1996, Fractionally Integrated Generalized Autoregressive Conditionnal Heteroskedasticity, *Journal of Econometrics*, 74, 3-30.

[2] Bollerslev, T., Mikkelsen, H-O., 1996, Modeling on Pricing Long Memory in Stock Market Volatility, *Journal of Econometrics*, 73, 151-184.

[3] Beine, M., Laurent, S., 2000, La persistance des chocs de volatilité sur le marché des changes s'est-elle modifiée depuis le début des années quatre-vingts ? Forthcoming in *La Revue Economique.*

[4] Beine, M., Laurent, S., Lecourt, C., 1999, Accounting for conditional leptokurtosis and closing days effects in FIGARCH models of daily exchange rates. Paper presented at the Sixth Workshop on Financial Modeling and Econometric Analysis, Lille III, January 1999.

[5] Bollen, P.B., Gray, S.F., Whaley, R.E., 2000, Regime-Switching in Foreign Exchange Rates: Evidence From Currency Option Prices, *Journal of Econometrics*, 94, 239-276.

[6] Chung, C.-F., 1999, Estimating the Fractionnaly Integrated GARCH Model, National Taiwan University working paper.

[7] Diebold, F.X., Inoue, A., 1999, Long Memory and Structural Change, Manuscript, Department of Finance, Stern School, NYU, May 1999.

[8] Diebold, F.X., Lee, J.H., Weinbach, G.C., 1994, Regime-switching with time-varying transition probabilities, in: C. Hargreaves, *Nonstationary time series analysis and cointegration*, (Oxford University Press), 283-302.

[9] Garcia, R., 1998, Asymptotic null distribution of the likelihood ratio test in Markow-Switching models, *International Economic Review*, 39(3), 763-788.

[10] Granger, C.W.J., Hyung, N., 1999, Occasional Structural Breaks and Long Memory, UCSD Discussion Paper 99-14, June 1999.

[11] Gray, S.F., 1996, Modeling the conditional distribution of interest rates as a regime-switching process, *Journal of Financial Economics,* 42, 27-62.

[12] Hamilton, J.D., 1994, Time series analysis (Princeton University Press, Princeton, N.J.).

[13] Hamilton, J.D. and R. Susmel, 1994, Autoregressive conditional heteroskedasticity and changes in regime, *Journal of Econometrics,* 64, 307-333.

[14] Hansen, B.E., 1992, The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP, *Journal of Applied Econometrics,* 7, S61-S82.

[15] Hsieh, C.-K., Chung, C.-F., Lin, C.-F. J., 1999, Long-Memory or Regime Changes: the Markov-Switching FIGARCH model, paper presented at the 1999 NBER/NSF Time Series Conference.

[16] Kim, D., Kon, S.J., 1999, Structural Change and Time Dependence in Models of Stock Returns, *Journal of Empirical Finance*, 6, 283-308.

[17] Mikosch, T., Stărică, C., 1999, Change of Structure in Financial Time series, Long Range Dependence and the Garch Model, Manuscript, Department of Statistics, University of Pennsylvania.

[18] Tse, Y.K., 1998, The Conditional Heteroskedasticity of the YEN-Dollar Exchange Rates, *Journal of Applied Econometrics*, 13, 1, 49-56.