A Note on Environmental Decisions under Uncertainty – The Impact of the Choice of the Welfare Measure*

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Abstract. The paper addresses the question, how environmental policy decisions under uncertainty depend on the underlying welfare concept. We study three different welfare measures: The first is directly based on the ex ante (expected) utility of a representative consumer whereas the second and the third are based on a valuation of policy changes compared to the status quo. Here the second criterion is based on the ex ante, the third on the ex post willingness-to-pay for policy changes with respect to the status quo. We show that decisions based on these measures coincide if and only if a risk neutral expected utility maximization is applied. Differences between the decisions are analyzed for both, risk averse expected utility maximization and the MaxiMin criterion. For risk-averse decision makers differences between the first and the second concept arise if the absolute risk-aversion of the decision maker is not constant in income. For risk aversion and the MaxiMin criterion, emission levels based on an optimization of ex post utility changes, e.g. abatement costs, exceed those based on the first or second concept if a reduction of emissions is socially optimal. Implications for applying the concepts of abatement costs and benefits from abatement are discussed.
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1 Introduction

Decisions are often to be made under uncertainty. The consequences of a decision may depend on a state of nature whose realization is not known at the time the decision is made. In particular the impact of environmental policy decisions is not perfectly known in most cases because the affected ecosystems are too complex. In order to make decisions in the context of risk or uncertainty usually the concept of expected utility maximization is applied. To do this it is necessary, however, that probabilities can be assigned to the states of nature. For the case that no such reliable probability distribution exists, the MaxiMin criterion could be employed as an alternative, i.e. the decision could be based on a valuation of the worst conceivable scenario. Not surprisingly, optimal decisions depend crucially on the criterion taken.

To determine expected utility and for the valuation of the worst case scenario one has to specify the welfare concept. Let us consider the problem of a decision on how much emissions of a pollutant should be abated. In partial analysis two different concepts can be found in the literature: In some models one tries to maximize expected utility which is often specified as expected benefits from emissions less the environmental damage from those emissions. In other models the expected increase of utility compared to a reference scenario is maximized. This is often done by optimizing benefits from reducing emissions, i.e. avoided damage, minus the abatement costs. The abatement costs are defined as the loss of benefits from emissions due to a reduction of emissions. This second concept depends on a reference level, referred to as the status quo emission level. Both concepts have been applied to environmental decisions under uncertainty. For example, Ulph and Ulph [1997] employ the first concept, whereas Welsch [1995] uses the second one. In both papers the authors assume that the direct economic benefits and costs of the decisions are certain, whereas the environmental consequences depend on a state of nature that is unknown at the time the decision is made. Since the two papers answer different questions, the consequences of the choice of the welfare concept cannot be compared directly. But, would we have to expect differences in results that arise only because the underlying welfare concepts differ? Are there reasons to take the first approach in some models while in others the second concept is applied?

In order to answer these questions, we will first motivate the two welfare concepts plus a third one by applying decision criteria from the literature on the valuation of discrete projects, i.e. decisions of "yes-or-no"-type. These criteria mostly refer to a measure of the compensating variation. This willingness-to-pay can be determined ex ante, i.e. before the uncertainty is resolved, or ex post, i.e. after the revelation of the true state of nature.[1] We then show that the three welfare measures coincide with respect to the derived "optimal" decisions, if and only if a risk neutral expected utility maximization is applied. For all other decision criteria optimal emission levels may vary due to the choice of the welfare functional. Thus, a thorough assessment of which welfare measure is the appropriate one is necessary before modeling questions in this context.

In this paper we analyze the impact of the choice of the welfare measure for risk-averse expected utility maximization and its extreme case, the MaxiMin criterion. In particular we show that if a reduction of emissions is socially desirable, the optimal emission levels based on a ex ante compensation measure are lower than those based on the ex post measure, i.e. on the change in utility of a representative consumer. This result holds true for non-risk-neutral expected utility maximization as well as for the MaxiMin criterion.

The paper is organized as follows. In section 2 we present our model. In section 3 we introduce and motivate the three different welfare concepts. Section 4 shows that in general equivalence of welfare measures is tantamount to a risk neutral decision criterion. In section 5 decisions based on a risk averse expected utility maximization are studied, in section 6 we look at the MaxiMin criterion. Section 7 concludes.

2 The model

We study a simple static model in which the impact of a political decision on the utility of a representative consumer is uncertain. The consumer's utility can be measured in terms of a composite commodity (money), denoted by z. The gross ex post utility of the representative consumer depends on the government's decision on, say, an emission

[1] For a discussion of these measures the reader is referred to Bishop [1986] and Graham [1981].

level $e \in \mathbb{R}$. It is given by

$$u = U(e;s) ,$$

where $s = 1;\ldots;S$ represents the state of nature which is unknown at the time the decision is made. We assume that U is concave in e and twice differentiable, i.e.

$$U_{ee} := \frac{\partial^2 U}{\partial e^2} < 0 .$$

This specification of ex post utility covers situations where production or consumption of private goods results causes pollution. In case that the benefits from consuming that good (V) and the environmental damage caused by the emissions (D) can be monetarized separately, the utility function U could be written as

$$U(e;s) = V(e;s) - D(e;s) .$$

Such an approach was taken by **Ulph and Ulph** [1997]. In their model V is concave and does not depend on the state of nature s. D takes the special form $D(e;s) = sd(e)$, where d is a convex function.

We assume that the case where the government does not change its current policy corresponds to the status quo, which we denote by e^0. Further, there could be payments to or by the consumer in order to compensate for losses or gains due to the policy change. In this case, the net ex post utility z_s in state s may differ from the gross utility level $U(e;s)$. We assume that the consumer can rank different net ex post utility vectors, $(z_s)_s = (z_1;\ldots;z_S)$, by an ex ante utility function $W((z_s)_s)$. Realizing the impact of his decisions on the utility of the consumer a regulator has to choose the way, i.e. a welfare functional by which he ranks his decisions.

3 The different welfare measures

In this section we describe three different welfare measures which will be analyzed in the following sections.

First, if the regulator knew the valuation by the consumer he could just rank decisions in exactly the same way. This leads to a welfare function

$$W_1(e) = W((U(e;s))_s) .$$

However, in most cases such a procedure is not feasible due to informational restrictions. It may sometimes be easier for the regulator to assess the change in consumer's utility due to a certain political action. In the literature on the valuation of policy options, in those cases it is often referred to the contingent valuation method. This approach bases on comparing two policies by asking people for their willingness-to-pay for or their willingness-to-accept a policy change, respectively. In our model this would result in assessing a change from the status quo emission level $\bar{e}$ to a new level e.[2] This policy change, however, may have different impacts in different states of nature. Therefore, the ex ante willingness-to-pay (or compensating variation) generally differs from the state dependent ex post willingness-to-pay for a change of the emission level from e^0 to e. The ex ante compensating variation, $CV(e)$, in our model is implicitely determined by

$$W((U(e;s) - CV(e))_s) = W((U(e^0;s))_s) .^3$$

Based on this measure a policy change should be pursued if $CV(e) > 0$. Since when deciding about the emission level there is not only one but a continuum of possible emission levels, it may make sense to maximize the compensating variation. This leads to a second welfare functional

$$W_2(e) = CV(e) .$$

However, their is a discussion in the literature as to whether to aggregate ex ante or ex post willingness-to-pay.[4] In our model the ex post willingness-to-pay given a state of nature s is given by

$$CV(e;s) = U(e;s) - U(e^0;s) .$$

[2] See Hanley and Spash [1993] for an overview of the contingent valuation method.

[3] The measure of compensating variation often is referred to as option price, if the $W(\cdot)$ is a expected utility functional. Readers are referred to Ahlheim [1998], p 554.

[4] For a discussion see Graham [1981], Meier and Randall [1991], Ready [1995]. Ready gives normative arguments for the ex ante measure, whereas he points at difficulties to determine this measure.

These ex post improvements in consumers well-being could be aggregated by the regulator to a third welfare functional. Let us assume that the regulator assesses uncertain ex post values in exactly the same way as the consumer does. Then we obtain:

$$W_3(e) = W((CV(e;s))_s) .$$

Note that the welfare measures W_2 and W_3 depend on the status quo emission level e^0. The welfare measure w_1 is not affected by this reference scenario.

One could ask the question whether or not there is a natural welfare concept which should be applied to find an "optimal" decision. Given a normative point of view the first criterion seems to be the most plausible one since if the consumer could make the decision by himself, he would certainly maximize his ex ante expected utility. However, if the regulator has no complete information about the consumers' preferences and can only ask for changes in utility due to a certain decision, one of the last two measures has to be applied.

In the literature that analyzes "optimal" emission levels on a theoretical basis, also differing welfare concepts are employed. Let us consider the special case from the previous section where utility was given by the benefits from emissions through the consumption of a private good minus the environmental damage, $U(e;s) = V(e;s) - D(e;s)$. Ulph and Ulph [1997] directly take this utility as a welfare measure and thereby employ the first welfare concept. Other authors, as for example Welsch [1995], try to evaluate abatement programs by maximizing the benefits from the reduction of emissions minus the abatement costs. Benefits from the reduction thereby equal the abated environmental damage, $D(e^0;s) - D(e;s)$. Abatement costs are a measure for the loss in consumption compared to a reference scenario, i.e. $V(e^0;s) - V(e;s)$. Welsch therefore bases the analysis on the difference of utility levels, $U(e;s) - U(e^0;s)$. Hence, such an approach coincides with the third welfare measure where the utility U is defined in exactly the same way as in Ulph and Ulph [1997].

4 Equivalence of the welfare measures?

We now study in which cases decisions based on the different welfare measures coincide, and therefore a distinction of these measures is redundant. For a aggregation rule $W(\cdot)$ to lead to welfare measures W_1 and W_3 that imply identical decisions, for arbitrary utility vectors $(U(e^0;s))_s$ and emission levels e^1, e^2 it must hold that

$$W((U(e^1;s))_s) > W((U(e^2;s))_s)$$
$$\Leftrightarrow \quad W((U(e^1;s) - U(e^0;s))_s) > W((U(e^2;s) - U(e^0;s))_s) \; .$$

In order to show which properties of W this implies we use the following preliminary result:

Lemma 1 Assume that a continuous monotone preference relation $\succeq$ defined on $\mathbb{R}^S$ satisfies for any $x^1; x^2 \in \mathbb{R}^S$

$$x^1 \succeq x^2 \quad \Rightarrow \quad x^1 + a \succeq x^2 + a \, , \quad \text{for all } a \in \mathbb{R}^S \; .$$

Then there exists a vector $\pi = (\pi_1; \ldots; \pi_S) \in \mathbb{R}^S_+$ where $\sum_s \pi_s = 1$, such that for all $x^1; x^2 \in \mathbb{R}^S$:

$$x^1 \succeq x^2 \; \Leftrightarrow \; \pi \cdot x^1 \geq \pi \cdot x^2 \; .$$

The vector π – given in lemma 1 – can be interpreted as a probability distribtution over the states of nature $s = 1; \ldots; S$. Then x^1 is preferred to x^2 if the expected utility of x^1 exceeds that of x^2. The proof of lemma 1 is delegated to the appendix.

Applying lemma 1 to the preference relation given by $W(\cdot)$ we directly obtain the following result:

Proposition 1 Let $W(\cdot) : \mathbb{R}^S \to \mathbb{R}$ be continuous and monotone. Maximizing the welfare measures W_1, W_2 and W_3 based on W leads to identical decisions for any given $U(\cdot)$ if and only if $W(\cdot)$ can be interpreted as risk neutral expected utility, i.e. can be represented as $W((z_s)_s) = \sum_s \pi_s z_s$ hat.

The proof is given in the appendix. Thus, equivalence of the welfare measures can only be expected if based on a risk neutral expected utility maximization. In particular, this equivalence holds if there is no uncertainty, i.e. the state of nature s is perfectly known. This case is illustrated in figure 1. For alternative decision criteria, hoever, the question arises, how the resulting decisions on the emission level differ. This will be analyzed in the following sections for risk averse expected utility maximization and the MaxiMin criterion which can also be applied to situations of where no reliable probability distribution exists.

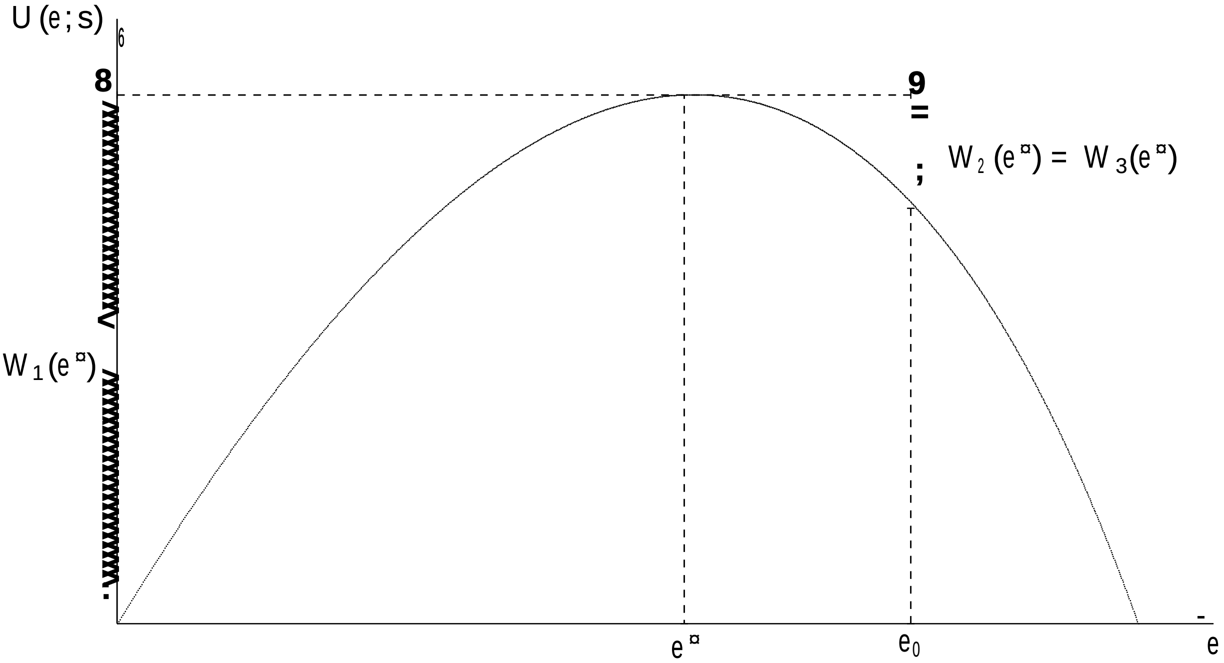

Figure 1: The equivalence of the welfare measures under certainty

5 Risk averse expected utility maximization

Let us assume that decisions are based on a risk averse expected utility maximization with respect to a probability measure π. Here W is given by

$$W((z_s)_s) = \int \phi(z_s)\, d\pi(s) ,$$

where ϕ is a concave von-Neumann-Morgenstern utility function, i.e.

$$\phi' > 0 \quad ; \quad \phi'' < 0 \ .$$

If we employ the first welfare concept,

$$W_1(e) = \int \phi(U(e;s))d\pi(s) \ ,$$

optimization leads to the following first order condition:

$$\begin{aligned} 0 &= W_1'(e) \\ &= \int \phi'(U(e;s))[U_e(e;s)]d\pi(s) \ . \end{aligned} \tag{1}$$

The second welfare functional $W_2(e) = CV(e)$ for risk-averse decision maker is implicitly defined by

$$\int \phi(U(e;s) - CV(e))d\pi(s) = \int \phi(U(e^0;s))d\pi(s) \ .$$

Maximizing W_2 we obtain

$$\begin{aligned} 0 &= W_2'(e) \\ &= \frac{\int \phi'(U(e;s) - CV(e))[U_e(e;s)]d\pi(s)}{\int \phi'(U(e;s) - CV(e))d\pi(s)} \ . \end{aligned} \tag{2}$$

Finally, the third welfare functional W_3 takes the form

$$W_3(e) = \int \phi(U(e;s) - U(e^0;s))d\pi(s) \ .$$

The first order condition for the maximization of W_3 is given by

$$\begin{aligned} 0 &= W_3'(e) \\ &= \int \phi'(U(e;s) - U(e^0;s))[U_e(e;s)]d\pi(s) \ . \end{aligned} \tag{3}$$

We denote the optimal emission levels by e^{*1}, e^{*2}, e^{*3}, respectively. To examine the impact of the choice of the welfare concept on the optimal emission level we have to compare e^{*1}, e^{*2}, and e^{*3}. Before doing this we will show the different consequences of risk-aversion depending on the criterion being employed. For this we assume that utility U and marginal utility $U_e := \frac{\partial U}{\partial e}$ decrease in s:

$$\begin{aligned} U(e;s) &> U(e;\hat{s}) \quad \text{and} \\ U_e(e;s) &> U_e(e;\hat{s}) \quad \text{for } s < \hat{s} \ . \end{aligned} \tag{4}$$

Thus, s can be interpreted as a damage parameter. The larger s, the smaller is the utility derived by the representative consumer.[5]

Comparing the risk-averse emission levels e^{*1}, e^{*2}, e^{*3} with the risk-neutral level, in the following denoted by e^{*}, we obtain the following proposition:

Proposition 2 (i) Compared with optimal decisions under risk-neutrality, risk-aversion leads to lower optimal emission levels if welfare measures W_1 or W_2 are employed, i.e. $e^{*1} < e^{*}$ and $e^{*2} < e^{*}$.
(ii) If risk-neutral welfare maximization implies a reduction of emissions with respect to the status quo level e^0, then the emission level e^{*3} based on the ex post compensation scheme exceeds the risk-neutral level, i.e. $e^{*3} > e^{*}$. If an expansion of emissions is optimal under risk-neutrality, i.e. $e^{*} > e^0$, then risk-aversion leads to less extended emissions, i.e. $e^{*3} < e^{*}$.

The proof can be found in the appendix. Proposition 2 states that for both, the first and the second welfare concept, risk-aversion leads to lower optimal emission levels compared to the risk-neutral case. This is a quite intuitive result since our assumption that marginal utility decreases in s implies that the absolute value of the difference between ex post welfare levels, $U(e; s) - U(e; \hat{s})$, increases with e. Hence, mitigation of risk requires a reduction of emissions. For the last welfare criterion this is different. Here the risk stems from the uncertain changes of ex post utility levels if the emission level is increased or decreased from the status quo emission level e^0 to e. The risk could be avoided completely by not deviating from the status quo. If a reduction of emissions under risk-neutrality is optimal, a further reduction leads to a larger deviation of the emission level from the status quo. Hence, additional risk would be generated. Thus, risk aversion implies a smaller optimal reduction of emissions compared to the optimal emission level under risk-neutrality. Differently, if e^{*} exceeds e^0, a reduction of emissions from the risk-neutral level reduces the borne risk. In this case a smaller expansion of emissions is optimal under risk-aversion. Hence, in both cases risk-aversion leads to a smaller deviation from the status quo emission level.

[5] Assumption (4), although technical, is quite common in the literature. This specification of ex post utility covers the example by Ulph and Ulph [1997] where $U(e; s) = V(e) - sd(e)$.

After discussing the impact of risk-aversion on optimal decisions based on the different welfare concepts, we will now investigate the ordering of the optimal emission levels e^{*1}, e^{*2}, and e^{*3} for the three welfare measures under risk-aversion. We obtain the following result:

Proposition 3 (i) If preferences satisfy decreasing absolute risk-aversion (DARA), $e^{*1} > e^{*2}$ holds. For increasing absolute risk-aversion (IARA) we get $e^{*1} < e^{*2}$. If preferences satisfy constant absolute risk-aversion, the decisions resulting from the first and second welfare measure coincide.[6]
(ii) The optimal emission level based on the third welfare measure, e^{*3}, exceeds e^{*1} and e^{*2} if the status quo emission level is larger than e^{*1} or e^{*2}, respectively. If an expansion of emissions is optimal for the first two welfare criteria, e^{*3} can exceed or fall short of e^{*1} and e^{*2}.

The proof is delegated to the appendix. Proposition 3 shows that for a risk-averse decision maker optimal decisions depend crucially on the choice of the welfare concept. Let us assume { and this is certainly the relevant case { that the initial status quo emission level exceeds both the optimal level based on the first and on the second welfare criterion. Then less emissions have to be abated if the third welfare measure is applied. The reason is again the different source of risk. The involved risks are smaller the closer the decision is to the status quo. Hence, for the third welfare criterion less abatement activities lead to lower risk borne by the decision maker. The differences between W_1 and W_2, however, are of different nature. What is crucial here is how the absolute risk aversion depends on income. If the absolute risk aversion is independent of the income level, the optimal decisions coincide.

Decisions based on the third welfare measure generally differ from those based on the other two welfare concepts. In particular, applying the welfare concepts employed by Ulph and Ulph [1997] and Welsch [1995], respectively, to exactly the same problem would imply different optimal decisions: An increased risk aversion would lead

[6] The measure of absolute risk aversion is defined as ¡Á''=Á'. If this measure is a constant (increasind, decreasing) function of U, the preferences show constant (increasing, decreasing) absolute risk aversion.

to stricter abatement of emissions for the U ph and U ph measure, whereas based on Welsch's criterion less abatement would result. These differences can also be illustrated for the MaxiMin criterion.

6 MaxiMin criterion

In this section we analyze the implications of the choice of welfare measures, if decisions are based on the MaxiMin criterion. This means that the probability distribution over the states of nature is not relevant here. Hence, the MaxiMin criterion can also be used in the case of complete uncertainty. It is based on the assessment of the worst case scenario. In this case the aggregation rule W is given by

$$W((z_s)_s) = \min_s z_s \ .$$

Hence, the first welfare measure takes the form:

$$W_1(e) = \min_s [U(e;s)] \ . \tag{5}$$

The ex ante compensating variation is implicitely given by

$$\min_s [U(e;s) - CV(e)] = \min_s [U(e^0;s)] \ .$$

Thus, we obtain for the second welfare concept:

$$W_2(e) = \min_s [U(e;s)] - \min_s [U(e^0;s)] \ . \tag{6}$$

Finally we get for the ex post criterion:

$$W_3(e) = \min_s [U(e;s) - U(e^0;s)] \ . \tag{7}$$

Since the last term in (6) is unaffected by the choice of e, W_1 and W_2 differ by a constant only, and thus imply the same optimal decision. It remains to investigate the differences between the optimal decisions based on W_1 (W_2) on the one hand and W_3 on the other.

Let us first consider the welfare concept W_1. Since we assumed that utility decreases in s, we obtain

$$W_1(e) = U(e; \max s) \ ,$$

and therefore the following first order condition:

$$W_1'(e) = W_2'(e) = U_e(e; \max s) = 0 \ . \tag{8}$$

The solution to this problem is denoted by e^{m1}, or e^{m2} for the welfare measure W_2, respectively.

For the third welfare measure we have to analyze the ex post change in the utility level

$$U(e; s) - U(e^0; s) \ .$$

Due to assumption (4) this difference decreases in s if $e > e^0$, it is a increasing function of the damage parameter if $e < e^0$.[7] Therefore, the minimal (maximal) damage parameter is decisive for the minimal ex post improvement of utility if a reduction (expansion) of emissions is planned. Hence, the welfare function takes the form:

$$W_3(e) = \begin{cases} U(e; \max s) - U(e^0; \max s); & \text{for } e \geq e^0 \\ U(e; \min s) - U(e^0; \min s); & \text{for } e < e^0 \ . \end{cases}$$

The maximizing emission level we denote by e^{m3}. It is characterized by the following conditions:

$$U_e(e^{m3}; \max s) = 0; \quad \text{if } e^{m3} > e^0 \tag{9}$$

$$U_e(e^{m3}; \min s) = 0; \quad \text{if } e^{m3} < e^0 \tag{10}$$

$$U_e(e^{m3}; \max s) \leq 0 \leq U_e(e^{m3}; \min s); \quad \text{if } e^{m3} = e^0 \ . \tag{11}$$

These conditions enable us to compare the emission level e^{m3} with $e^{m1} (= e^{m2})$.

Proposition 4 *The emissions based on the maximization of W_3 exceed the optimal emission level with respect to the other two criteria, i.e. $e^{m3} \geq e^{m1} = e^{m2}$. The optimal emission level e^{m3} depends on the status quo emission level e^0. It is given as the emission level within the interval of possibly ex post optimal emission levels, that requires the smallest policy change with respect to the status quo.*

Applying the MaxiMin criterion, the decision maker bases his decisions exclusively on the worst case. This is determined by the smallest ex post utility level if the first two

[7]See proof of proposition 2.

welfare measures are applied. If, however, the third criterion is taken, the minimal increase of utility is decisive. As a consequence, decisions based on W_3 show a strong aversion against a policy change. In figure 2 it is illustrated how decisions depend on the status quo level e^0. The proof of proposition 4 is given in the appendix.

The properties of the welfare measure W_3 which have been derived also imply potential problems for calculating abatement costs or benefits from the abatement of emissions in order to find a decision. To illustrate this we again study the example, where U is given as direct utility from emissions minus environmental damage, i.e. $U(e;s) = V(e;s) - D(e;s)$.

Example 1 Let $D(e,s)=d(e)$ be independent of s. Then $\arg\min_s U(e;s) = \arg\min_s V(e;s)$ on the one hand, whereas $\arg\min_s[U(e;s) - U(e^0;s)] = \arg\max_s[V(e^0;s) - V(e;s)]$ on the other. If assumption (4) holds, then a state of nature s with a high ex post utility level also implies high abatement cost. Hence, the "worst case" depends on the choice of the welfare measure: For W_1, W_2 it is given by the lowest utility level (corresponding to the lowest abatement costs), whereas for W_3 the "worst case" is given by the highest abatement costs (corresponding to the highest utility level).

Example 2 Let $V(e;s) = v(e)$ be independent of s. Then we obtain $\arg\min_s U(e;s) = \arg\max_s D(e;s)$ and $\arg\min_s[U(e;s) - U(e^0;s)] = \arg\min_s[D(e^0;s) - D(e;s)]$. Under assumption (4) a high enviromental damage corresponds to high benefits form abatement. As the "worst case" by applying W_1 or W_2 would result the maximal enviromental damage. If the welfare measure W_3 is taken, the minimal gain from reducing emissions is "worst", which corresponds to the minimal enviromental damage.

7 Conclusion

In this paper we analyzed how optimal decisions under enviromental uncertainty depend on the choice of the welfare measure. Within a simple static model we studied three different welfare concepts. The first one bases on ex ante utility, the second

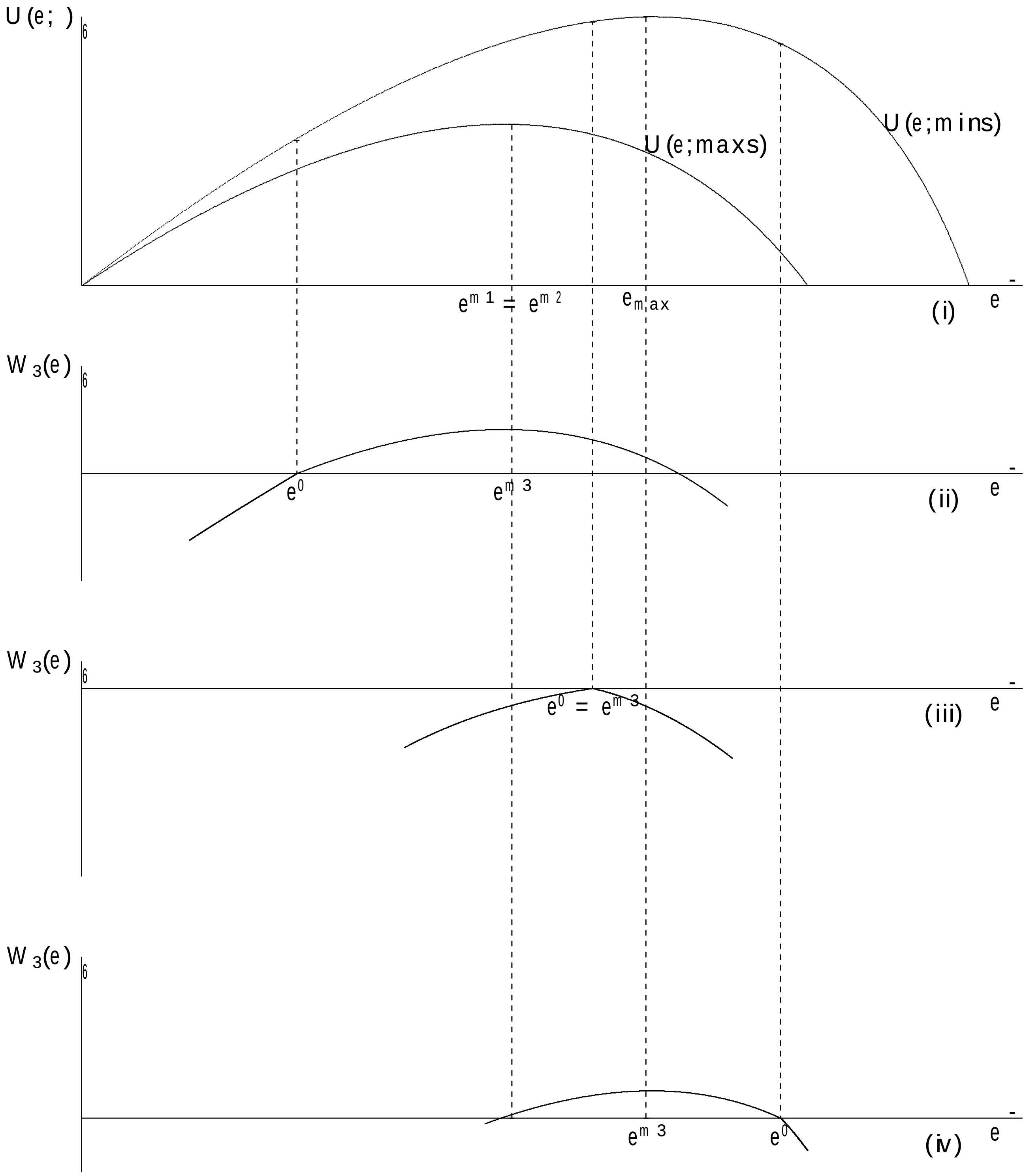

Figure 2: Optimal emissions for a MaxMin criterion. (i) shows the set ex post utility levels for the minimal and for the maximal damage parameter. e^{m1} and e^{m2} coincide with e_{min}. For the third welfare measure the optimal emission level depends on the status quo. This is illustrated in (ii)-(iv). (ii) illustrates the choice of e^{m3} given a status quo emission level below e_{min}. Here W_3 increases with e as long as $e < e_{min}$. In (iii), $e^{m3} = e^0$ holds, since changing the emission level from e^0 would decrease W_3. In (iv) the status quo level e^0 exceeds e_{max}. To get the maximal welfare W_3, e has to be decreased to e_{max}.

one on ex ante compensating variation measures, the last one on an aggregation of ex post utility changes. Thereby the concepts of abatement costs and of benefits from abatement can be embedded in the third measure since they compare the implications of the reference scenario (status quo) and the intended emission level.

We have demonstrated that exclusively for risk-neutral decision makers the decisions based on these welfare measures coincide. For all other decision criteria differences can arise. These were studied for risk aversion and the MaxiMin criterion. Decisions based on the first two welfare measures differ for risk-averse expected utility maximizer with nonconstant absolute risk aversion, whereas they coincide for the MaxiMinimizer. Decisions based on the third welfare concept, on the other hand, generally depart from those obtained from the first two measures. For the probably mostly relevant case of an optimal reduction of emissions we obtained the following result: A decision maker who aggregates ex post utility changes, i.e. the benefits from changing the policy from the status quo, comes out with larger optimal emission levels than would be obtained for the first two welfare concepts. The reason for this is that a state with a high utility level generally corresponds to small benefits from the policy change, and vice versa. So, the higher the environmental damage, the more benefits can be obtained from reducing emissions. This case occurs because the utility in the status quo scenario does depend on the state of nature.

Summarizing, optimal decisions of non-risk-neutral decision makers may heavily depend on the underlying welfare functional. Hence, one has to be aware of the consequences of the choice of the welfare functional on emission decisions. The question arises whether there exists one natural welfare measure which should be applied. In our model it seems to be preferable to base decisions directly on the (expected) utility of the representative consumer since this imitates the decision which is desired by the consumer himself. However, due to informational restrictions this may be complicated or even impossible. In such a case it is certainly easier to assess utility changes of consumers by analyzing the willingness-to-pay for a certain political action. Thus, there are normative reasons in favor of the first welfare measure whereas practical considerations may lead to the necessity to apply the second or the third one. In any case, a thorough assessment is necessary to inquire which welfare concept is appropriate in a particular context.

Appendix

Proof of lemma 1:

Due to the continuity of preferences they can be represented by a continuous utility function $W(x)$. This can be defined as $x \sim W(x) \cdot (1, \ldots, 1)$ because preferences are monotone.[8] Note, that $W(\alpha(1, \ldots, 1)) = \alpha$. Hence, if $x \sim W(x) \cdot (1, \ldots, 1)$, then for arbitrary vectors y we get $x + y \sim W(x) \cdot (1, \ldots, 1) + y$. It follows

$$
\begin{aligned}
W(x+y) &= W(W(x) \cdot (1, \ldots, 1) + y) \\
&= W(y + W(x) \cdot (1, \ldots, 1)) \\
&= W(W(y) \cdot (1, \ldots, 1) + W(x) \cdot (1, \ldots, 1)) \\
&= W((W(x) + W(y)) \cdot (1, \ldots, 1)) \\
&= W(x) + W(y),
\end{aligned}
$$

and therefore $W(z \cdot x) = zW(x)$ and $W(\frac{1}{z} \cdot x) = \frac{1}{z} W(x)$ for all integers z. Let $\alpha \in \mathbb{R}$ denote any real number. It can be approximated by $\frac{p^n}{q^n} \to \alpha$, where p^n, q^n are integers. Then continuity of $W(\cdot)$ implies:

$$
\begin{aligned}
W(\alpha x) &= W(\lim_n \frac{p^n}{q^n} \cdot x) \\
&= \lim_n W(\frac{p^n}{q^n} \cdot x) \\
&= \lim_n \frac{p^n}{q^n} W(x) \\
&= \alpha W(x).
\end{aligned}
$$

Therefore $W(\cdot)$ is linear and can be represented as $W(x) = \pi \cdot x$, where $\pi_s \geq 0$. Q.E.D.

Proof of proposition 1:

Assume that for all utility functions $U(\cdot)$, W_1 and W_3 lead to identical decisions. Then – applying lemma 1 – the preference $\succeq$ defined by $x^1 \succeq x^2 \Leftrightarrow W(x^1) \geq W(x^2)$ can be

[8] A (standard) proof is given by Mas-Colell et al. [1995], p. 47.

represented by $x^1 \succeq x^2 \Leftrightarrow \sum_s \pi_s x_s^1 \geq \sum_s \pi_s x_s^2$. Hence W can be interpreted as risk neutral expected utility.

Assume the other way around that the preference is represented by $W(x) = \int x_s d\pi(s)$. Then we obtain

$$
\begin{aligned}
W_1(e) &= \int U(e;s)d\pi(s), \\
W_2(e) &= \int U(e;s)d\pi(s) - \int U(e^0;s)d\pi(s), \\
W_3(e) &= \int [U(e;s) - U(e^0;s)]d\pi(s).
\end{aligned}
$$

Differentiation of these welfare measures lead to identical first order conditions

$$
0 = W_1'(e) = W_2'(e) = W_3'(e) = \int U_e(e;s)d\pi(s).
$$

Thus, the "optimal" emission levels coincide. Q.E.D.

Proof of proposition 2:

The risk neutral emission level e^* is given by $\int U_e(e^*;s)d\pi(s) = 0$. The right hand sides of (1)-(3) look similar, except for the different weights $\phi'(\cdot)$ attached to the marginal utility U_{e_2}. Note that under our assumptions $U_e(e;s)$ decreases in s. Hence, if the weights $\phi'(\cdot)$ are an increasing function of s, smaller values of the marginal ex post utility $U_e(e^*;s)$ get larger weights. Thus, in this case we would obtain a negative value for the right hand sides of (1)-(3) at $e = e^*$. Analogously, the right hand sides take a positive value at e^* if the weights are decreasing in the damage parameter. It is easily seen that the weights in (1) and (2) are increasing in s. Hence it follows that $W_1'(e^*) < 0$ and $W_2'(e^*) < 0$. Since W_1 is concave and since $W_2''(e) < 0$ if $W_2'(e) = 0$, this directly implies claim (i).

If $s < \hat{s}$, then our assumptions in (4) imply that $[U(e;s) - U(e;\hat{s})]$ increases in e. Therefore we obtain:

$$
\begin{aligned}
&U(e;s) - U(e;\hat{s}) \gtrless U(e^0;s) - U(e^0;\hat{s}) \quad \text{and} \\
&U(e;s) - U(e^0;s) \gtrless U(e;\hat{s}) - U(e^0;\hat{s}) \quad \text{for} \quad e \gtrless e^0;\ s \leq \hat{s}. \qquad (12)
\end{aligned}
$$

Thus, the weight $\phi'(U(e;s) - U(e^0;s))$ in (3) increases in s if $e > e^0$ and decreases if $e < e^0$. Therefore, $W_3'(e^*) < 0$ if $e^* > e^0$, and $W_3'(e^*) > 0$ if $e^* < e^0$. This implies claim (ii) since under our assumptions $W_3''(e) < 0$. Q.E.D.

Proof of proposition 3:

(i) To derive the relationship between e^{*1} and e^{*2} we consider the numerator of the right hand side of (2) at $e = e^{*1}$ as a function of CV:

$$RHS(CV) := \int \phi'(U(e^{*1};s) - CV)[U_e(e^{*1};s)]d\pi(s) \ .$$

Note that (1) implies that $RHS(0) = 0$. Differentiating $RHS(\cdot)$ we obtain

$$\begin{aligned} RHS'(CV) &= \int -\phi''(U(e^{*1};s) - CV)[U_e(e^{*1};s)]d\pi(s) \\ &= \int \left(-\frac{\phi''(\cdot)}{\phi'(\cdot)} \right) \phi'(\cdot)[U_e(e^{*1};s)]d\pi(s) \end{aligned} \qquad (13)$$

Since utility and marginal utility are decreasing in s, the third factor in (13) is decreasing, the second one is increasing in s. If preferences obey DARA, the first factor increases in s. Let $\hat{s}$ be chosen such that $U_e(e^{*1};s) \gtrless 0$ for $s \lessgtr \hat{s}$. Then we obtain

$$\begin{aligned} RHS'(CV) &= \int \underbrace{\left(-\frac{\phi''(U(e^{*1};s) - CV)}{\phi'(U(e^{*1};s) - CV)} \right)}_{\uparrow \text{ in } s} \phi'(U(e^{*1};s) - CV)[U_e(e^{*1};s)]d\pi(s) \\ &= \int_{s \le \hat{s}} \left(-\frac{\phi''(U(e^{*1};s) - CV)}{\phi'(U(e^{*1};s) - CV)} \right) \underbrace{\phi'(U(e^{*1};s) - CV)[U_e(e^{*1};s)]}_{\ge 0} d\pi(s) \\ &\quad + \int_{s > \hat{s}} \left(-\frac{\phi''(U(e^{*1};s) - CV)}{\phi'(U(e^{*1};s) - CV)} \right) \underbrace{\phi'(U(e^{*1};s) - CV)[U_e(e^{*1};s)]}_{\le 0} d\pi(s) \\ &< \left(-\frac{\phi''(U(e^{*1};\hat{s}) - CV)}{\phi'(U(e^{*1};\hat{s}) - CV)} \right) \int_{s \le \hat{s}} \phi'(U(e^{*1};s) - CV)[U_e(e^{*1};s)]d\pi(s) \\ &\quad + \left(-\frac{\phi''(U(e^{*1};\hat{s}) - CV)}{\phi'(U(e^{*1};\hat{s}) - CV)} \right) \int_{s > \hat{s}} \phi'(U(e^{*1};s) - CV)[U_e(e^{*1};s)]d\pi(s) \\ &= \underbrace{\left(-\frac{\phi''(U(e^{*1};\hat{s}) - CV)}{\phi'(U(e^{*1};\hat{s}) - CV)} \right)}_{>0} \underbrace{\int \phi'(U(e^{*1};s) - CV)[U_e(e^{*1};s)]d\pi(s)}_{=RHS(CV)} \end{aligned}$$

Hence, as long as $RHS(CV) \le 0$ the derivative $RHS'(CV) < 0$. Since $RHS(0) = 0$ this implies $RHS(CV) < 0$ at $CV(e^{*2}) (> 0)$. Since $\int \phi'(U(e;s) - CV)[U_e(e;s)]d\pi(s)$ decreases in e, it finally follows that $e^{*2} < e^{*1}$. Similarly, IARA leads to $e^{*2} > e^{*1}$ and for CARA one arrives at $e^{*2} = e^{*1}$.

(ii) To show the relation between e^{*3} and e^{*1} (e^{*2}) for $e^0 > e^{*1}$ ($e^0 > e^{*2}$) it suffices to show that $W_3'(e^{*1}) > 0$ ($W_3'(e^{*2}) > 0$). We consider the relation for e^{*1}. Let $\hat{s}$ be chosen as in (i), CV denotes $CV(e^{*1})$. Applying (12) we obtain:

$$
\begin{aligned}
W_3'(e^{*1}) &= \int \underbrace{\phi'(U(e^{*1};s) - U(e';s))}_{\downarrow \text{ in } s}[U_e(e^{*1};s)]\,d\pi(s) \\
&= \int_{s\le \hat{s}} \phi'(U(e^{*1};s) - U(e';s))\,[\underbrace{U_e(e^{*1};s)}_{\ge 0}]\,d\pi(s) \\
&\quad + \int_{s>\hat{s}} \phi'(U(e^{*1};s) - U(e';s))\,[\underbrace{U_e(e^{*1};s)}_{\le 0}]\,d\pi(s) \\
&> \phi'(U(e^{*1};\hat{s}) - U(e';\hat{s}))\left[\int_{s\le \hat{s}} U_e(e^{*1};s)\,d\pi(s) + \int_{s>\hat{s}} U_e(e^{*1};s)\,d\pi(s)\right] \\
&> \frac{\phi'(U(e^{*1};\hat{s}) - U(e';\hat{s}))}{\phi'(U(e^{*1};\hat{s}) - CV)} \underbrace{\int_S \phi'(U(e^{*1};s) - CV)\,U_e(e^{*1};s)\,d\pi(s)}_{=0} \\
&= 0\,.
\end{aligned}
$$

Analogously one can show that $W_3'(e^{*2}) > 0$ for $e^{*2} < e'$. If on the other hand an expansion of emissions is optimal ($e^{*1}; e^{*2} > e'$), then the first inequality changes into "<", implying an ambiguous sign of $W_3'(e^{*1})$ ($W_3'(e^{*1})$). Q.E.D.

Proof of proposition 4:

Let $e_{min} = e^{m1} = e^{m2}$ be defined as $U_e(e_{min}; \max s) = 0$ and e_{max} as $U_e(e_{max}; \min s) = 0$. Note that if $e' < e_{min}$, then $U_e(e'; \min s) > U_e(e'; \max s) > 0$ holds. Hence, it is optimal to raise the emission level and from (9) we obtain $U_e(e^{m3}; \max s) = 0$ which implies $e^{m3} = e_{min}$. On the other hand, $e' > e_{max}$ analogously leads to $e^{m3} = e_{max}$. Finally, if $e_{min} \le e' \le e_{max}$, it follows that $U_e(e'; \max s) \le 0 \le U_e(e'; \min s)$. Thus, any change of the emission level would reduce W_3. Hence, we obtain $e^{m3} = e'$ as the optimal emission level. Summarizing, if e' is outside the interval $[e_{min}; e_{max}]$, the optimal emission level coincides with the closest boundary of the interval. If e' lies in the interval, then policy is not changed ($e^{m3} = e'$). Note that since $e^{m1} = e^{m2} = e_{min}$, the emission level resulting from the W_3 measure is not smaller than $e^{m1} = e^{m2}$. Q.E.D.

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