Is Entrepreneurship Always Good For Growth?*

Baochun Peng†

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Abstract

Entrepreneurship may not be good for growth. We study the role of entrepreneurship in the context of an endogenous growth model. The exercise of entrepreneurship has two features in our model: (i) Entrepreneurs do not carry out research, instead, they select projects from the researchers, and (ii) Entrepreneurs' ability levels are heterogenous and mutually unobservable. We find that an exogenous rise in the number of high ability entrepreneurs or their ability level may lead to a lower equilibrium growth rate. This negative relationship is caused by the rent seeking element in the exercise of entrepreneurship. Thus our finding challenges the commonly held belief that innovative entrepreneurship is rent creating.

1 Introduction.

One criticism of formal economic theory is its omission of the entrepreneur[1]. The generic firm represented in economic theory must choose among alternative values for a small number of rather well defined variables like price, output, and occasionally a few others. Implicitly, the management performs a mathematical calculation which yields optimal decisions. The role of the entrepreneur is replaced by a well defined maximization problem, so there is no room for enterprise or initiative. However, in reality the range of possible

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†University of Oxford. E-mail: baochun.peng@economics.ox.ac.uk

[1] See, for example, Machovec (1995) and Kirzner (1973, 1997).

actions open to an enterprising individual are much wider than few variables allowed the in a simple maximizing model.

One immediate implication of this awareness is represented in rent-seeking theory. From the point of view of social efficiency, economic activities may be classified under two broad categories: productive and non-productive activities. Typically, activities such as R&D, labour, and fair trading under normal market conditions are usually classified as productive; while there are other activities which are classified as non-productive, such as firms lobbying for preferential treatment from the government or protection of its monopoly status. When activities bring profit, profit seeking individuals choose to engage in them even when they are socially unproductive. Baumol(1990, 1993) compared the impact of different economic institutions in ancient Rome, Medieval China and the Dark and high Middle Ages in Europe. He concludes that entrepreneurial resources, i.e., individuals who are endowed with initiative and creativity, exist in every historical period, but economic institutions provide them with different opportunities. Therefore the allocation of entrepreneurship between productive and unproductive activities can have a profound effect on the innovativeness of the economy and the degree of dissemination of its technological discoveries. Baumol points out that the number of lawyers as a proportion of total population is much smaller in Japan than in the U.S., while the number of engineers is much higher. Using data from 91 countries, Murphy, Shleifer and Vishny (1991) tested the Baumol's hypothesis, and found the rate of economic growth to be positively correlated with the number of engineering students and negatively correlated with the number of law students.

In this paper we tackle the following questions: What is the implication of introducing the role of entrepreneurs in an R&D based growth model? Does an increase in the number of entrepreneurs always increase the rate of economic growth? Is entrepreneurship in seemingly productive activities always rent-creating, or are there circumstances when even such entrepreneurship have rent-seeking elements in them?

Our findings show that entrepreneurship is not always rent creating. Following Romer (1990), we construct a model where economic growth is endogenously driven by the R&D efforts of researchers. Each new invention has a certain probability of success. Entrepreneurs purchase inventions from researchers and implement them for production. More crucially, agents have different endowments of entrepreneurial abilities, which are only observable to themselves. We show that because high-ability entrepreneurs are able to conceal their ability level, their participation in the entrepreneurial sector drive down the income of low ability entrepreneurs, and indirectly reduce the overall incentive for R&D. Thus even in the exercise of innovative entrepreneurship there is an element of rent-seeking involved, and having a greater number of high-ability entrepreneurs may in fact slow down the growth rate.

The remainder of the paper proceeds as follows. Section 2 discusses the concept of entrepreneurship used in our model; Section 3 describes the basic structure of the economy; Section 4 derives the competitive equilibrium of the economy; Section 5 studies the effect of an exogenous increase in the number of high ability entrepreneurs on the growth rate; Section 6 concludes.

2 Schumpeterian and Knightian Entrepreneurship

In endogenous growth theories where growth is driven by R&D, entrepreneurs are present. In what sense is our notion of entrepreneurs different from what is normally defined in those models? In our model, entrepreneurship has three features: Firstly, entrepreneurs in our model are not involved in R&D, but rather in invention evaluation, selection and implementation. Secondly, agents have different levels of endowed entrepreneurial ability, and thirdly, agents' level of entrepreneurial ability are mutually unobservable. In this section, we show from the writings of several recent writers support for the first feature; we then show that the second and the third features are implicit in the Frank Knight's concept of entrepreneurship.

2.1 Research vs. Entrepreneurship

The Schumpeterian entrepreneur introduces new processes of production. The innovator-entrepreneur disturbs the original circular flow of production and of the market by creating new technology. The temporary technological leadership allows the entrepreneur to make a profit, by creating a gap between the price of input and price of output. Endogenous growth models such as Romer (1990), Aghion and Howitt (1992) and Grossman and Helpman (1991) essentially adopts this definition. In their models, the entrepreneur is the researcher, who invents new intermediate products and is rewarded with the profit from producing the new product. In this way the level of technology in the economy grows.

While economic growth is indeed characterized by innovative behaviour and technological advancement, casual observation suggests that not all entrepreneurs who profit from the innovations are the original inventors. To be involved in inventive activities is not identical to the exercise of entrepreneurship. In emphasizing this difference, Kirzner (1973) wrote: "the function of the entrepreneur consists not of shifting the curves of cost or of revenues which face him, but of *noticing* that they have in fact *shifted*." The process of exploiting the commercial benefit of a new invention requires more than technical competence, other abilities such as the ability to recognize the demand, to market a new product, to organize production are no less important. As Rosenberg (1994) observes:

> "the ultimate impact of some new technological capability is not just a matter of technical feasibility or improved technical performance; rather, it is a matter of identifying human needs in ways, or in contexts that have not yet been articulated. What is called for is not just engineering expertise or high-quality professional analysis, but imagination. Sony's Walkman is an excellent example. The design of its component parts required engineering expertise, to be sure. But more impressive was the imaginative leap that identified an important market niche." (p. 5)

The ability to recognize potential demand, and to match it with available technology, is no less important than mere technical competence.

In real life inventor-entrepreneurs can only be regarded as coincidences. Generally, the role of the inventor and the entrepreneur are separate. Researchers produce new inventions, but new inventions by themselves are not sufficient to guarantee commercial success. Entrepreneurs select new inventions for implementation, and his success in doing so is a measure of his entrepreneurial ability. While the entrepreneur may not produce any invention, his work is nevertheless a part of the innovative process.

2.2 Heterogeneity and Mutual Unobservability

According to Knight (1921), profit is the reward for bearing uncertainty: "It is this true uncertainty which gives the characteristic form of 'enterprise' to economic organization as a whole and accounts for the peculiar income of the entrepreneur."(p.232.) The profit won by any particular entrepreneur depends on his own ability and good luck as well as upon the general level of initiative and ability of the market. This is sometimes understood that profit is a reward for risk-bearing. Several recent studies such as Kanbur (1979) and Evans and Jovanovic (1989) adopt this view. Kihlstrom and Laffont (1979) treats the Knightian entrepreneur as a risk bearer, and constructs a model of entrepreneurship, where agents choose between supplying labour without risk, or become an entrepreneur and operating risky technology, which is accessible to all agents.

However, the concept of uncertainty in Knight can often be misunderstood, as Kirzner pointed out: "The very emphasis on uncertainty in the Knightian system has tended to mask the fact that when an entrepreneur does enter into an admittedly risky venture he does so because he believes that, on balance, it offers an attractive opportunity." According to Kirzner, profit in the Knightian system of entrepreneurship does not come from risk bearing itself, but from the entrepreneur's alertness to the opportunity of making a profit, before other agents.

LeRoy and Singell, Jr. (1987) proposes a different way of interpreting Knight's theory of entrepreneurship. They argued in Knight's theory profit is the reward for bearing uninsurable hazards. Uncertainty arises because of the lack of recognizability of a person's entrepreneurial ability, as Knight wrote: "We have assumed...that each man in society knows his own powers as entrepreneur, but that *men know nothing about each other in this capacity*. The presence of true profit, therefore, depends on the absence of the requisite organization for combining a sufficient number of instances to secure certainty through consolidation." (Knight as quoted in LeRoy and Singell, Jr.(1987)) The Knightian entrepreneur exists because there is an adverse selection problem about the quality of the entrepreneurs, and the potential insurance market for the outcome of entrepreneurship breaks down. Entrepreneurship is not supplied as other factors of production, for it is the entrepreneurs himself, and not anyone else, that bears the 'ultimate responsibility'.

From this perspective, Kihlstrom and Laffont's model of entrepreneurship differs from Knightian entrepreneurship in at least two respects. Firstly, as they acknowledge, their model only represent a special case of Knight's view because they assume that all individuals are equal in their ability, while Knight emphasizes ability differences in his definition of the entrepreneur. (p.746) Secondly, in their model, the extent of risks are commonly observable and therefore insurable, while the Knightian entrepreneurs bear uninsurable risks. As LeRoy and Singell Jr. suggests, the Knightian theory of entrepreneurship is only possible when the two features of entrepreneurial ability: heterogeneity and mutual unobservability, are present.

3 Basic Structure of the Economy

We construct a model based on a combination of the Knightian and the Schumpeterian concept of entrepreneurship. In this section we describe the environment of our model. Our model is based on Romer (1990); some of the basic assumptions of our model are the same as in the Romer's model.

3.1 Consumption

Consider an economy populated by N agents, whose aim is to maximize the expected value of a discounted sum of single period utilities. The utility function of a representative consumer takes the following functional form: where C_t is the consumption at time t and is the discount factor, is a positive constant, the elasticity of substitution for this utility function is constant at $1/\sigma$.

$$U(c) = \int_0^\infty \frac{C_t}{1-\sigma} e^{-\rho t} dt \tag{1}$$

where C_t is the consumption at time t and ρ is the discount factor, σ is a positive constant, the elasticity of substitution for this utility function is constant at $1/\sigma$.

3.2 Production

The economy uses labour and many intermediate goods to produce a final good. The number of intermediate goods is usually interpreted as the level of specialization, and can be increased through research and development. Aggregate output at time t is given by:

$$Y_t = L_{1t}^{1-\alpha} \int_0^{A_t} X_{it}^{\alpha} di$$

where L_{1t} is the amount of labour used in the production of the final good at time t, $X_{it}$is input of the intermediate good i, and A_t the existing amount of known intermediate goods. Unlike Romer's model, we make no distinction between labour and human capital. The sector producing the final product is perfectly competitive, the price for the intermediate goods and the wage for labour equal their respective marginal productivity. In the market for intermediate products, a successful innovation generates a profit stream for all subsequent time; profit- seeking agents are attracted into the innovation sector, and in equilibrium there will be sustained economic growth.

As in Romer's model, intermediate goods are produced using capital alone. Capital K_t is the measure of cumulative forgone output. Assuming, as we do, that there is no depreciation, then K_t evolves according to the rule:

$$\dot{K}_t = Y_t - C_t$$

It takes one unit of foregone consumption to create one unit of any type of durable, the measure of K_t is related to the durable good that are actually used in production by the rule:

$$K_t = \int_0^{A_t} X_{it} di$$

When all firms producing intermediate goods are producing the same amount of output X, and the interest rate is r_t, then intermediate firms' cost is $r_t X$.

3.3 Occupation Choices

Here we introduce the role of the entrepreneurs. There are three possible occupations in the economy: workers, researchers and entrepreneurs. Workers supply their endowed unit of labour, and researchers produce new inventions to sell them in the market for inventions. Not all new inventions can be successfully implemented; entrepreneurs use their endowed entrepreneurial ability to select from the invention market and implement them. We call this the innovation process. In making these assumptions we differ from the Romer model. While the workers and the inventors in our model correspond to the workers and entrepreneurs in the Romer model, our model separates the tasks of invention and entrepreneurial selection. The outcome of implementing each new invention is uncertain, and the entrepreneur bears the ultimate responsibility for the implementation.

We further assume, due to limitation of time or energy, that each individual may only enter into one of these sectors. In particular, to exercise entrepreneurial alertness is as energy consuming as other occupations. An agent who has chosen to supply labour as a worker or researcher is unable to exercise his endowed entrepreneurial ability. It follows that researchers have no special information about the quality of their own inventions, nor is it possible for them to implement their own inventions.

3.4 The Exercise of Entrepreneurship

An entrepreneur selects new inventions from the invention market, purchases the invention and implements them for production. Not all inventions are good and only good inventions can be successfully implemented. The ability level of an entrepreneur is indicated by his accuracy in identifying good inventions.

There are two types of agents, type 1 and type 2, who are identical to each other in every aspect except endowed level of entrepreneurial ability. Let M denote the total number of new inventions, of which only a subset S_0 are good. We assume that there are b_0M inventions in S_0. Type 1 agents choose from a sub-set S_1which contains b_1M inventions, and ignore all other inventions, type 2 from a subset S_2, which contains b_2M inventions. We assume:

1)$0 < b_0 < b_1 < b_2 < 1$

2)$S_0 \subset S_1 \subset S_2$

The first assumption states both types of entrepreneurs can make mistakes in selecting inventions, and a type 2 makes more mistakes than a type 1 entrepreneur; the second assumption states that all inventions considered worthless by either types of entrepreneurs are bad inventions, and rules out the case that a type 1 entrepreneur makes a mistake that a type 2 entrepreneur would not make.

3.5 Technological Improvement

Let M_t denote the number of new inventions produced by researchers at time t, the value of M_t depends positively on the number of researchers, L_{2t} , and the state of general knowledge, A_t , as in the Romer model:

$$M_t = \delta L_{2t} A_t, \quad \delta > 0$$

This research output function is widely used in R & D based endogenous growth models. δ is a constant, each researcher produce δA_t of inventions, so research productivity rise with the level of technology. Total number of inventions is just the sum of all inventions produced by researchers. In Romer's model, all inventions can be implemented, here we assume only some inventions are good, and only good inventions can be successfully implemented.

All entrepreneurs purchase the right to use the inventions from researchers. As researchers are unable to observe the ability of the entrepreneurs, the price for inventions must be uniform within each time period. Let p_t denote the price of a invention at time t, $\Pi_t$the profit from a successful innovation, and r_t the market rate of interest. Then Π_t/r_t would be the present discounted value of each successfully implemented invention. Let b_{it} denote the success rate for type i entrepreneurs, $i = 1, 2$. Then $b_{it}\Pi_t/r_t - p_t$ is the expected income an entrepreneur earns on each invention purchased from invention market.

We assume that only inventions which have been successfully implemented contribute to the growth of technology. In particular, we assume that $\dot{A}_t \equiv I_t$, where I_t is the number of good inventions implemented. If all good inventions are implemented, which is the case in our steady state equilibrium, the rate of technological growth at time t will then be described by the equation[2]:

$$\frac{\dot{A}_t}{A_t} = \frac{b_0 M_t}{A_t} = b_0 \delta L_{2t}$$

In order to obtain stationary growth equilibrium in our model, we assume the entrepreneur's capability of setting up firms also increases linearly with the aggregate knowledge A_t. More specifically, at t, each entrepreneur is able to implement A_t new inventions. In this way, as we will see, the ratio of entrepreneurs to researchers in equilibrium is constant.

[2]Alternatively, we can assume the fruitfulness of research depend on the number of inventions that is in existence, rather than the current level of technology:

$$\dot{M}_t = \delta L_{2t} M_t$$

In stationary equilibrium, the level of technology A will be a constant multiple of the number of inventions, M. A then evolve according to:

$$\frac{\dot{A}}{A} = b_0^2 \delta L_2$$

which is the rate of growth we used in our model, multiplied by b_0. This change of specification do not change our result qualitatively.

3.6 Invention Market Equilibrium

The economy has a population N of agents, e_1^* of these agents are endowed with high ability, if they enter the entrepreneurial sector, they become type 1 entrepreneurs; the rest of the population can only become type 2 entrepreneurs if they choose the entrepreneurial sector.

We assume there is full employment in the economy, this means:

$$N = L_{1t} + L_{2t} + e_{1t} + e_{2t}$$

where L_{1t} and L_{2t} are numbers of worker and researcher at t respectively, e_{1t} and e_{2t} are the number entrepreneurs belonging to the types 1 and 2.

We assume that type 1 entrepreneurs select inventions before type 2 entrepreneurs[3]. When a type 1 entrepreneur enters into the invention market, the proportion of good inventions among all inventions is b_0, thus his probability of success is:

$$b_{1t} \equiv b_1 = \frac{b_0}{b_1^*}$$

The value of b_{1t} rises as b_1^* becomes smaller, this means the more bad inventions entrepreneurs are able to reject, the more accurate is the their selection accuracy.

When type 2 entrepreneurs come to the invention market, the proportion of good inventions in the invention market would then be lower than b_0, and this affects the success probability of the type 2 entrepreneurs. Defining $B_t \equiv M_t/A_t$, we can calculate the success probability of the type 2 entrepreneurs as:

$$b_{2t} = \frac{b_0 B_t A_t - b_{1t} e_{1t} A_t}{b_2^* B_t A_t - e_{1t} A_t} = \frac{b_0 B_t - b_{1t} e_{1t}}{b_2^* B_t - e_{1t}} \tag{2}$$

In the numerator of the expression in the middle, the term $b_0 B_t A_t$ represents the number of good inventions in all inventions supplied by the researchers. Of the $e_{1t} A_t$ inventions bought by type 1 entrepreneurs, a proportion b_{1t} are good inventions, so the numerator as a whole represents the

[3]This assumption in fact makes it possible for researchers to distinguish type 1 and 2 entrepreneurs by observing when they come to the invention market, therefore undermining the assumption of single invention price, which is crucial to our result. In Appendix 1 we present a case where researchers are not able to distinguish the two types of entrepreneurs, while having identical analytical result as the case we present here.

number of good inventions left after type 1 entrepreneurs had made their choice. In the denominator, $b_2^* B_t A_t$ represents the number of inventions originally in the invention market which type 2 agents do not regard as being worthless, now the $e_{1t} A_t$ inventions selected by type 1 entrepreneurs are all contained in this set, thus the denominator represents the number of inventions type 2 entrepreneurs do not regard worthless, after type 1 agents have already made their choices. As b_{1t} is greater than b_0, it can be shown that the expression for b_{2t} is less b_0/b_2^*, which is the success rate of the type 2 entrepreneurs if they were to choose before type 1 entrepreneurs. This means that type 1 entrepreneurs reduce the success rate of type 2 entrepreneurs by choosing before type them.

In the expression above, the success rate of the type 2 entrepreneurs decreases with b_1. This is easy to understand: type 1 entrepreneurs reduce the proportion of high-quality invention from the set that is not regarded as worthless by type 2 entrepreneurs, therefore, the higher their ability is, the more distortion they cause for the type 2 entrepreneurs. On the other hand, the success rate of type 2 entrepreneurs increases with B_t, the greater is the total supply of invention, the less distortion type 1 entrepreneurs can cause.

3.7 Equilibrium Supply of Inventions

How many inventions are supplied in each period? Suppose there is excess supply of (demand for) inventions, price for invention falls (rises), income of researchers falls (rises) relative to other sectors, now as agents are free to choose between all three sectors, less (more) agents will choose the research sector, leading to less (greater) supply of inventions; therefore the market for inventions must clear as long as agents are free to choose any sector of the economy. This does not mean all inventions are bought, since a proportion $1 - b_2^*$ of the total inventions are regarded as worthless by all entrepreneurs and will not be bought. Market clearing means after invention selection, all entrepreneurs are able to find enough inventions which satisfy their requirement; and all inventions that are not bought are known to be worthless to entrepreneurs, so there is no incentive for any researchers to lower the price in order to sell the inventions.

As type 2 entrepreneurs select inventions after type 1 entrepreneurs, who only select inventions which are not considered worthless by type 2 entrepreneurs, the equilibrium condition in the market for inventions can be characterized by:

$$b_2^* B_t A_t - e_{1t} A_t = e_{2t} A_t$$

The left hand side represent the number of inventions left in the market which type 2 entrepreneurs do not regard as worthless, after type 1 entrepreneurs have selected their inventions; in other words, it is the effective supply of inventions to type 2 entrepreneurs. The right hand side represents total number of inventions type 2 entrepreneurs are capable of handling, or, demand for inventions by type 2 entrepreneurs. Rearranging the above expression and solve for B_t, we obtain:

$$B_t = \frac{e_{1t} + e_{2t}}{b_2^*} \tag{3}$$

Substituting (3) into (2), type 2 entrepreneurs' probability of success is now:

$$b_{2t} = \frac{b_0}{b_2^*}\left\{1 - \frac{e_{1t}}{e_{2t}}\left(\frac{b_2^*}{b_1^*} - 1\right)\right\} \tag{4}$$

By definition, b_2^* is greater than b_1^*, so the term in the rounded brackets on the right hand side is positive. This implies the term in the braces is less than one[4], so the value of the expression on the right hand side is less than b_0/b_2^*. Now b_0/b_2^* is the success rate of type 2 entrepreneurs if no type 1 entrepreneurs are present, thus the expression shows the presence of type 1 entrepreneurs reduces the success rate of type 2 entrepreneurs, the degrees of which is dependent on e_1/e_2.

The value of b_{2t} increases in e_{2t} and b_1^*, decreases in e_{1t} and b_2^*. This means there is complementary effect between type 2 entrepreneurs: the more ordinary entrepreneurs there is, and/or the higher is their ability, the less is the magnitude of negative effect from type 1 entrepreneurs. On the other hand, the more type 1 entrepreneurs there is, and/or the higher is their ability, the more negative effect they have on type 2 entrepreneur's success rate. We call this the competition effect between entrepreneurs of different abilities.

4 Steady State Equilibrium

We derive the steady state equilibrium of the economy in this section. When the economy is in steady state equilibrium, the number of agents in each of

[4]We need this value to be positive as well, since b_{2t} is the success rate of type 2 entrepreneurs, whose income must be positive, this requires the value of e_{2t} to be not too small relative to the value of e_{1t}. It turns out that provided the model has a solution, this condition is satisfied.

the sectors is constant. The level of technology grows at a constant rate, and all other stock variables including consumption, saving, output and capital grows at the same rate as technology. The rate of interest, invention price and the present discounted value of each successful innovation are all constant.

In principle, all agents can choose to become worker, researcher or entrepreneur. However, in this paper we focus on the case where all type 1 agents become entrepreneurs, while type 2 agents choose between the three sectors. In particular, we focus on the case where the number of type 2 entrepreneurs is positive. This implies for a type 2 agent, potential incomes of becoming an entrepreneur, or researcher or final sector worker are the same. Income of a type 1 entrepreneur is higher than that of a type 2, as they are more accurate in selecting good inventions.

The steady state equilibrium is derived by equating the supply and demand of capital in the economy. We first derive the consumption behaviour of the utility maximizing individuals, since under steady state equilibrium saving grows at the same rate as consumption, this condition can be interpreted as the rate of change for saving, or the supply of capital. On the other hand, for type 2 agents, income in these sectors are all equal. This condition allows us to derive the equilibrium number of agents in each of the sectors. Agents take the rate of interest as given, so by varying the rate of interest and examine its effect on the size of the research sector, and noting that the demand for capital is determined by the size of the research sector when the economy is in steady state, we are able to derive the demand curve for capital.

In the following section we drop all time subscripts.

4.1 Consumption

Agents allocate consumption in each period to maximize total utility in equation (1). When there is perfect capital market, and the consumers take the rate of interest r as given. Utility-maximization behaviour of the consumers yields the following condition[5]:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho)$$

When the economy is in steady state equilibrium, consumption grows at the same rate as saving. If we replace consumption by saving, the equation above can be interpreted as the rate of change in saving as a function of r, replacing C by S in the equation above, we get:

[5] See, for example, Aghion and Howitt (1998), Chapter 1.

$$\frac{\dot{S}}{S} = \frac{1}{\sigma}(r - \rho) \tag{5}$$

This function then could be thought of as a supply function for capital.

4.2 Producers of the Intermediate Goods

When an invention has been successfully implemented, intermediate producers choose the amount of output to maximize profit. As markets related to the final product are all competitive, the price for the intermediate goods equals to its marginal productivity in final good production. The cost of production is just the amount paid as interest to capital. When the economy is in equilibrium, all intermediate firms produce the same amount of output, which we denote as X, using X as the unit to measure the amount of capital required in the production of intermediate goods, the cost function a intermediate firm faces is rX. Intermediate goods producers thus choose the level of output X to solve the following maximization problem:

$$Max \quad \alpha L_1^{1-\alpha} X^{\alpha} - rX \tag{6}$$

From the first order condition, we find the output that maximizes profit:

$$X = \left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} L_1 \tag{7}$$

Substituting this into (6), a successful implementation of invention yields a profit stream of:

$$\Pi = \left(\frac{1}{r}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)\, \alpha^{\frac{1+\alpha}{1-\alpha}} L_1 \tag{8}$$

Profit increase with the number of final sector workers and decrease with the rate of interest. This is as expected: greater number of final sector workers increase the marginal productivity of the intermediate goods in the final sector, and increases the revenue of the intermediate firm; while higher interest rate increases the cost of production.

4.3 Market Equilibrium

Now the agents choose freely between the three sectors, so in equilibrium the incomes in all these sectors must be equal to each another. Let w_1 be the wage rate for the workers, the wage rate is the marginal productivity in the final sector, taking the rate of interest as given:

$$w_1 = (1-\alpha)\left(\frac{X}{L_1}\right)^{\alpha} A_t$$

or, define the variable v_1 as w_1 divided by the current level of technology A, and substitute for the value of X in (7), we obtain:

$$v_1 \equiv \frac{w_1}{A} = (1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}\left(\frac{1}{r}\right)^{\frac{\alpha}{1-\alpha}} \tag{9}$$

Wage rate in the final sector rises proportionally with the level of technology, and decreases with the rate of interest. However, it is not directly affected by the number of agents in each sector.

A researcher produces δA inventions. When both type of entrepreneurs are present, only a proportion b_2^* of all inventions will be sold[6], and each invention that is sold yields an income of p to the researcher. Let w_2 denote the income of the researchers, v_2 is defined similarly as v_1:

$$v_2 \equiv \frac{w_2}{A} = \delta p\left(\frac{e_1+e_2}{B}\right) = \delta p b_2^* \tag{10}$$

When an entrepreneur successfully implements an invention, there will be a continuous stream of profit ; with a constant rate of interest, this is evaluated by the stock market at its present discounted value Π/r. If the implementation is not successful, the entrepreneur loses the price paid for the invention. Expected income of entrepreneur type i is:

$$v_{3i} \equiv \frac{w_{3i}}{A} = b_i\frac{\Pi}{r} - p, \qquad i = 1, 2. \tag{11}$$

[6] There are uncertainties at the level of each individual invention, each invention produced by researcher may be sold or not sold, some inventions bought by entrepreneurs can be successfully implemented while others can not. Yet on the aggregate level, because of the law of large numbers, the proportion of inventions each researcher sells and the proportion of inventions each entrepreneur successfully implements can be treated as deterministic processes.

Since $b_1 > b_2$, income of the type 1 entrepreneurs is always higher than income of the type 2 entrepreneurs. If both types of entrepreneurs co-exist, all agents who are endowed with higher entrepreneurial ability chooses to become entrepreneurs. Equilibrium in this economy is characterized by the condition that low ability agents are indifferent between all three sectors, that is, $w_1 = w_2 = w_{32}$. Equating w_2 with w_{32}, we obtain the price of a invention as:

$$p = \frac{b_2}{1 + \delta b_2^*} \frac{\Pi}{r}$$

The invention price reflects the part of the income generated by a successful innovation that is allocated to the entrepreneur, it increases with the success rate of the entrepreneur, b_2, and with the present discounted value of the new innovation, Π/r. Substituting this expression for p into the expression for v_2 in (10), as well as the expression for Π in (8), we obtain the following form:

$$v_2 = J b_2 L_1 \tag{12}$$

where

$$J \equiv \frac{\delta b_2^*}{1 + b_2^*} (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{1}{r} \right)^{\frac{1}{1-\alpha}} \tag{13}$$

As the intermediate firms take the market rate of interest as given, they regard J as a constant.

When the economy is in equilibrium, because agents are indifferent between research and entrepreneurial sectors, a rise in the success rate of type 2 entrepreneurs must be followed by a rise in the income of the researchers. Also, as a rise in the present discounted value of new innovations increases the research income, and by (8) the value of L_1 is proportional to Π/r, v_2 increases with L_1 in the expression above.

We now proceed by writing v_2 as an expression of L_2. Notice because of invention market equilibrium condition $e_1 + e_2 = b_2^* B$, and that $B = \delta L_2$, the size of the innovation sector, that is, the total number of researchers and entrepreneurs, is fully characterized by L_2, the number of researchers. Under full employment condition, we may express L_1in terms of L_2 as $L_1 = N - (L_2 + e_1 + e_2) = N - (1 + \delta b_2^*) L_2$. In the same way, the term e_2 in the expression for b_2 in (4) can be substituted with $e_2 = b_2^* L_2 - e_1^*$, we obtain the new expression of v_2 in terms of L_2 as:

$$v_2 = J \left\{ \frac{1}{b_2^*} - \frac{e_1^*}{\delta b_2^* L_2 - e_1^*} \left(\frac{b_2^*}{b_1^*} - 1 \right) \right\} \left[N - \left(1 + \delta b_2^*\right) L_2 \right]$$

The expressions inside the braces and the square brackets correspond to the values of b_2 and L_1 respectively. As this expression relates the number of researchers with the income they would receive, it can be understood as the demand curve for researchers.

How does income in the research sector v_2 vary with the number of researchers? The expression above shows two effects of opposing directions: On one hand, greater number of researchers reduces the distortion created by type 1 entrepreneurs, increases the success rate of type 2 entrepreneurs, cause the value of the expression in the braces to rise; on the other hand, greater number of researchers increases the size the innovation sector, thus reducing the number of workers in the final sector, and reduces the present discounted value of innovations through its effect on L_1, the value of v_2 therefore decreases. This effect can be seen through the expression in the square brackets. Overall, the value of v_2 as a function of L_2 has a bell jar shape, as Figure 1. Shows.

The two curves in Figure 1. are the values of v_1 and v_2 in terms of L_2. Provided certain regularity condition is satisfied[7], there would typically be two points of intersections, representing two potential labour market equilibriums. One of these points, L_2', is in fact unstable. Suppose there is a small increase of L_2 around the value of L_2', income of the research sector v_2 exceeds v_1, more agents choose the innovative sector, this process continues until the economy reaches the point L_2''. In the following analysis we shall focus on the stable equilibrium L_2''.

Now in drawing v_1 and v_2 in terms of L_2, we have held the rate of interest constant. How does the market equilibrium change with the rate of interest? Compare the expressions for v_1 and v_2 in (9), (12) and (13), both are decreasing in r, while v_2 has the extra term $1/r$ and is more interest sensitive than v_1. The reason that both v_1 and v_2 decreases with r can be seen

[7] The regularity condition is:

$$\delta^2 \left\{ \rho b_2^* - e_1^* b_0 \left(\sigma + \frac{b_2^*}{b_1^*} \alpha \right) - \frac{\alpha b_2^* b_0 N \delta}{1 + \delta b_2^*} \right\}^2 - 4\delta^2 \left(\sigma + \alpha\right) b_2^* b_0 e_1^* \left(\frac{\alpha b_2^* b_0 N \delta}{1 + \delta b_2^*} \frac{b_2^*}{b_1^*} - \rho \right) \geq 0$$

When the condition is hold with equality, the two curves are tangential and there is only one point of intersection.

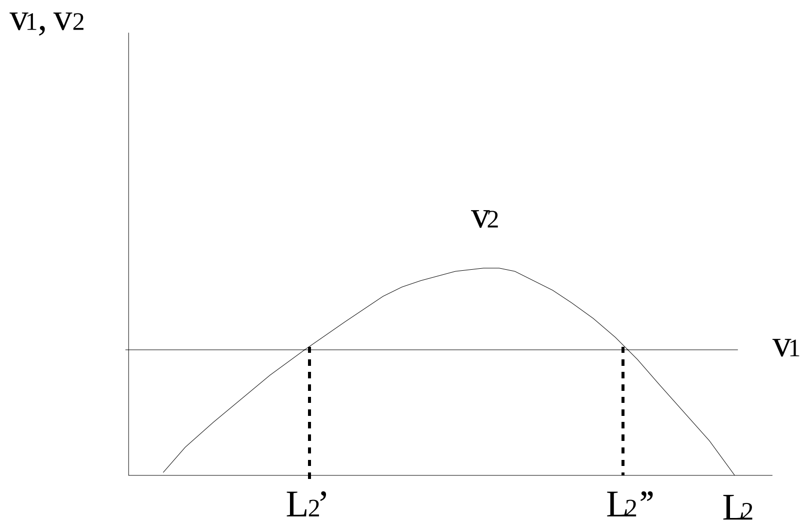

Figure 1: Research and Final Sector Income

from equation (7), that higher interest rate reduces all output of intermediate products. Since both the marginal productivity of labour and profit stream of the intermediate firms depend on X, higher r reduces both wage rate for workers and researchers. However, the value of new inventions depend on the present discounted values of successful innovations, and interest rate not only affect the size of the profit stream, but also how future values of profit stream is discounted. It is precisely this discounting effect that brings the extra $1/r$ term to v_2, and causes it to be more interest rate sensitive than v_1.

An increase in the rate of interest reduces v_2 to a greater extent than to v_1. This situation is shown in Figure 2.

As the rate of interest increase, income of the final sector workers shift down from v_1 to v_1', while income of the research sector shift from v_2 to v_2'. As the v_2 curves shift more, the number of researchers decrease. Joining all such points allows us to obtain a downward sloping curve between r and L_2.

4.4 Market for Capital

We shall confine ourselves to the steady state equilibrium of the economy. In the steady state, the numbers of agents in each of the sectors are constant, stock variables such as saving, consumption, output and the level of

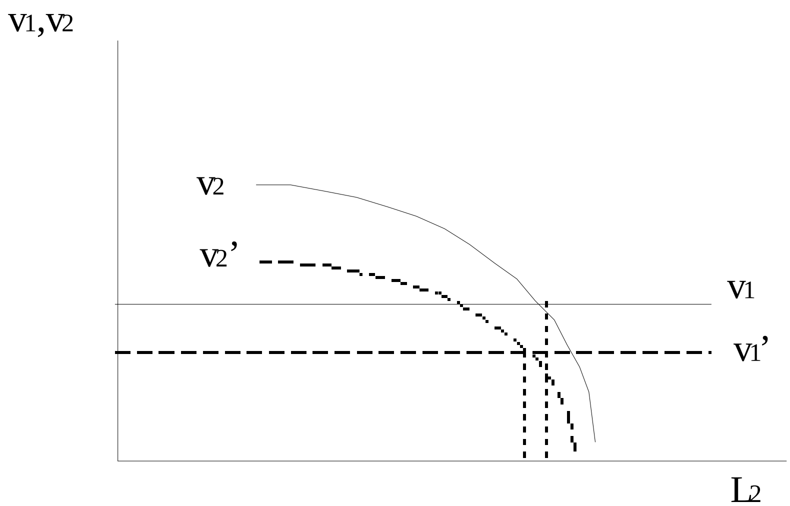

Figure 2: Effect of a Rise in Interest Rate

technology all grow at the same constant rate.

In the steady state with growth rate g, growth rate of output is the same as growth rate of knowledge level A. Now the growth rate of A can be expressed as $b_0\delta L_2$, which is the number of good inventions implemented in each period divided by A. We therefore have the relationship $g = b_0\delta L_2$. As L_2 and r are negatively related, g and r are also negatively related. In the steady state equilibrium the growth rate of the capital stock is also g, thus there is a negative relationship between the rate of change in capital stock and interest rate, we call this the demand curve for capital.

More specifically, the functional form of this demand curve can by obtained by equating v_1 with v_2, express L_2 in terms of g, and re-arrange into an expression of r in terms of g, we then obtain:

$$r = \frac{\alpha\delta b_0}{1+\delta b_2^*}\left(N - \frac{1+\delta b_2^*}{\delta b_0}\,g\right)\left[1 - \frac{e_1^*}{\frac{b_2^*}{b_0}\,g - e_1^*}\left(\frac{b_2^*}{b_1^*} - 1\right)\right]$$

This curve has a bell jar shape. The demand curve for capital is just the

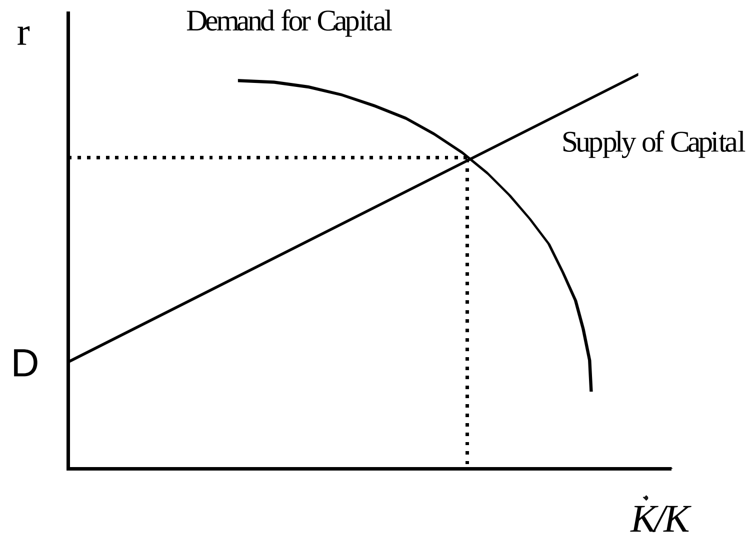

Figure 3: Steady State Equilibrium

downward sloping section of this curve[8] (see Figure 3.).

On the supply side, utility maximization by the consumer yields the equation describing saving behaviour in (3), which we re-arrange into an expression for r in terms of the saving rate:

$$r = \sigma \frac{\dot{S}}{S} + \rho$$

This is understood to be the supply curve of capital. Rate of interest in an linear expression of the saving rate, and the curve cuts r-axis at ρ.

Putting the demand and supply curves of capital together in Figure 3, the intersection point of the two curves indicates the rate of interest that clears the capital market, and the growth rate in the steady state equilibrium.

5 Comparative Statics

In this section we examine the effect of certain parameter changes on the steady state equilibrium. We first perform the more standard experiment

[8]The upward sloping section of the curve corresponds to the unstable labour market equilibriums which we have ruled out.

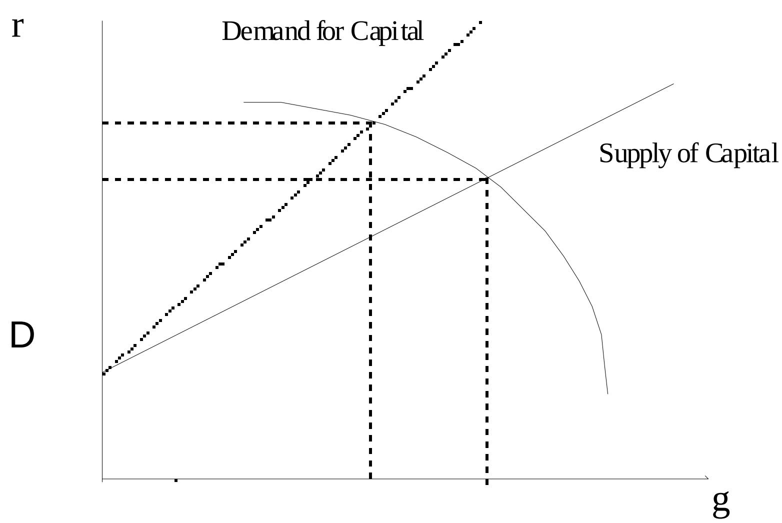

Figure 4: Exogenous Increase in σ

by increasing σ and ρ, these changes have the effect of reducing the supply of capital and reducing equilibrium growth rate. We then change the two parameters for the number and ability level of type 1 entrepreneurs, e_1^* and b_1^*. These two parameters affect the aggregate demand for capital and our analysis yields the counter-intuitive result that a greater number of type 1 entrepreneurs or higher ability level reduces equilibrium growth rate.

5.1 Effects of Changing σ and ρ

The variables $\sigma$and ρ only affect the supply side. An exogenous rise σ in increases the marginal utility of consumption, causing utility maximizing consumers to allocate a larger portion of income to consumption, therefore saving decreases. In the same way, a rise in ρ increases the consumer's preference for present consumption over future consumption, cause saving to fall. Both of these changes shift the supply curve of capital towards the left, the economy moves to a different equilibrium point. These effects are shown in Figures 4. and 5. In both cases, new equilibrium growth rate becomes lower while the steady state interest rate becomes higher.

In steady state, growth rate and the number of researchers are related through the relationship $g = b_0\delta L_2$, as the new equilibrium growth is lower,

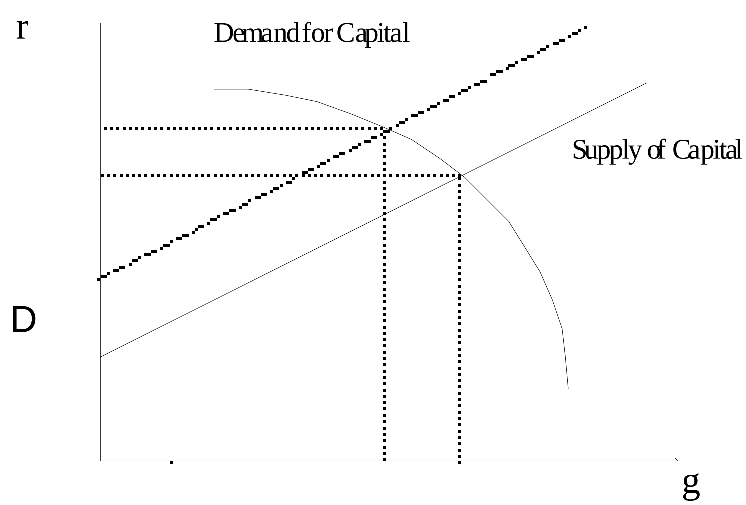

Figure 5: Exogenous Increase in ρ

the number of researchers falls. Now invention market clearing implies the condition $e_1 + e_2 = b_2^* \delta L_2$, i.e., the ratio of entrepreneurs to researchers is a constant δb_2^*. Thus, in response to the fall in researchers, total number of entrepreneurs must fall. As we have confined ourselves in the case where all type 1 agents become entrepreneurs, and as the number of type 1 agents is fixed in this case, the number of type 2 entrepreneur falls. The innovation sector, which comprises of researchers and entrepreneurs, becomes smaller. Since there is full employment, more agents become final sector workers.

Changes in the output of intermediate firms, the profit stream and the present discounted value of each invention are however ambiguous. From equations (6) to (8), the rise in r tend to increase the cost of production of the intermediate producer, leading to a smaller output and profit rate; a higher r also reduces the present discounted value of invention. On the other hand, in the new equilibrium the number of final sector workers rises, which increases the demand for intermediate goods, and tend to increase both output and profit rate.

Price for inventions falls. To see this point, notice that as r rises, equation (9) implies v_1 decreases; by the free entry condition of all sectors,v_2 also fall. Equation (10) then shows price for invention must fall in order for v_2 to fall. Total profit from each successful innovation, i.e., its present discounted value

Π/r, is allocated between the entrepreneur and researcher, invention price essentially indicates the size of researcher's portion. Now the increase in the rate of interest exerts a negative impact on Π/r, and in equilibrium, this negative impact is translated into a fall in the invention price.

We now present the main result of the paper, that a rise in the number of high ability agents or their ability level reduces the equilibrium growth rate.

5.2 Exogenous Increase in the number of High Ability Agents

What happens if there is an exogenous increase in the number of high-ability agents? The result is surprising: provided both types of entrepreneurs are still present in equilibrium, an increase in the number of high-ability agents in fact reduces the number of inventions produced, and reduces the rate of economic growth.

When the value of e_2^* rises, according to (4), success rate of the type 2 entrepreneurs falls. This leads the v_2 curve in Figure 2. to shift down. In the market for capital, the value of $\dot{K}/K$ becomes less at every given level of r, thus the demand curve shifts towards the left. This situation is shown in Figure 6. In the new steady state equilibrium, unlike the two previous cases, rate of interest becomes lower; growth rate also becomes lower.

The lower growth rate implies fewer inventions are produced each period, and fewer agents choose the entrepreneurial sector. Thus the exogenous increase in type 1 entrepreneurs crowds out more type 2 entrepreneurs than it replaces, so that in the new steady state, total number of entrepreneurs falls. As fewer agents choose the research and entrepreneurial sectors, the number of agents involved in innovative activity drops; instead, more agents become final sector workers.

The difference with the previous cases of exogenous increase in σ and ρ, is that now the rate of interest in the new equilibrium falls instead of rises. In other words, the unit cost of producing the intermediate good has fallen. At the same time, the number of final sector worker has risen, which increases the productivity of intermediate goods in the final sector, and increases the demand for intermediate goods. According to (7) and (8), both of these effects serve to increase the output and profit rate of each intermediate producer. Moreover, the fall in r also changes the way future profit are discounted, thus the present discounted value of each successful innovation rises, to a greater extent than the rise in profit rate.

Price for inventions rises. This is because as r falls, equation (9) implies v_1 increases; by the free entry condition of all sectors, v_2 also rise. Equation

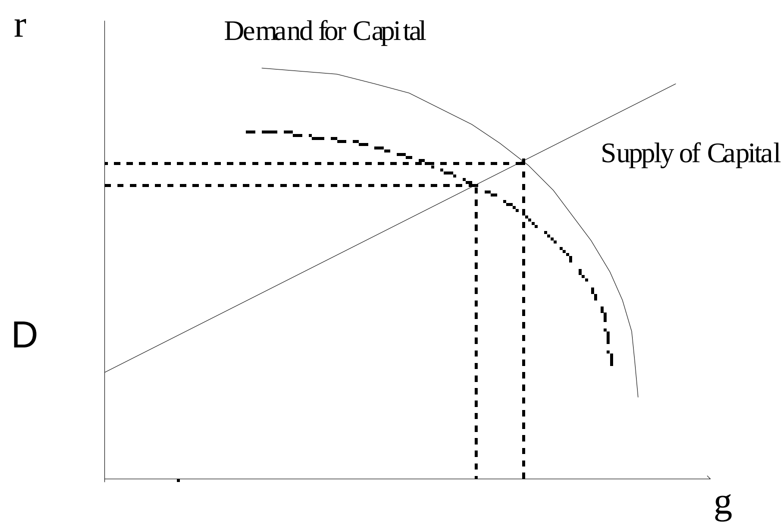

Figure 6: Rise in the Number of High Ability Agents

(10) then shows price for invention must rise in order for v_2 to rise. More heuristically, the fall in r increases the total profit to be allocated between the entrepreneur and researcher from each successful innovation, i.e., Π/r; invention price which corresponds to the size of researcher's portion rises in response to the increase in Π/r.

5.3 Exogenous Increase in Invention Selection Accuracy of Type 1 Entrepreneurs

The effect of a fall in b_1^* is almost identical to the rise in e_2^*. When b_1^* falls, success rate of the type 1 entrepreneurs rise, from (4) we can see the success rate of type 2 entrepreneurs falls. This leads the v_2 curve in Figure 2. to shift down. In the market for capital, the value of $\dot{K}/K$ becomes less at every given level of r, thus the demand curve shifts towards the left. From this point on the situation of the new steady state equilibrium is qualitatively identical to the case of exogenous increase in e_1^*, depicted in Figure 6. Growth rate becomes higher, the value of r drops. The size of the innovative sector becomes smaller while the number of final sector worker increases. Intermediate firm produces more, and the value of each successful innovation rises. Price for invention also rises.

5.4 Discussion

Our analysis of the effects of supply and demand side parameter changes show that in both cases, the rate of growth falls. However, the effects on the invention market are very different: in one case invention price falls while in the other case it rises. What amounts to this difference?

The effects of exogenous increases in σ and ρ are quite easy to understand: these parameters affect the consumption side, cause the consumers to attach greater preference to present consumption, so that r increases as the consumers become more reluctant to supply to the capital market. The rise of interest rate cause the intermediate firms to produce less, and future profit streams are also valued less at present. This can be thought of as a lower demand for the output of the innovation sectors as a whole. As a result, price for inventions falls, reflecting this fall in demand. In equilibrium, less agents become involved in research and entrepreneurship, and the growth rate falls.

The cases of fall in b_1^* and rise in e_1^* are more interesting. In these cases there is a fall in interest rate, which gives intermediate firms incentives to produce more, and raise both the profit stream and the present discounted value of each successful innovation. In the new steady state equilibrium, price for invention also rises in response to the rise in the value of innovation. It seems the exogenous change in parameters has created greater demand for the work of innovative activity, yet the size of innovative sectors and the rate of growth fall. The reason for this counter-intuitive result can be understood by examining the expressions for b_2 and v_3 in (4) and (11). When e_1^* rises or b_1^* falls, success rate of type 2 entrepreneurs falls; therefore, although the value of each successful innovation rises, the effect on income of type 2 entrepreneurs is offset by the fall in b_2. In other words, the value created by innovations are shared between researchers and type 1 and 2 entrepreneurs. When e_1^* rises or b_1^* falls, though the value of each successful innovation rises, yet the share which goes to the type 1 entrepreneurs also increase, so that the overall effect is lower level of research activity and lower level of entrepreneurial activity by the type 2 agents. The rise in the number of type 1 agents or in their ability level allows them to obtain a larger portion of the profit made from successful innovations, the fall in b_2 captures the rent seeking element in entrepreneurship, which leads to lower equilibrium growth rate in our model.

6 Conclusion

This paper studies the implication of incorporating entrepreneurship in an endogenous growth model. There are two features in Knightian theory of entrepreneurship, that the ability levels of the entrepreneurs are heterogenous, and that they are mutually unobservable. These two characteristics lead to the break-down of insurance, and the entrepreneurs must bear ultimate responsibility in a world of uncertainty. We constructed a model containing these two aspect of Knightian entrepreneurship. In our model, the exercise of entrepreneurship is part of the innovation process that drives economic growth. Entrepreneurs select inventions produced by researchers for implementation, and only successfully implemented inventions raise the level of technology. As researchers are not able to distinguish between the high and low ability entrepreneurs, all inventions are sold at the same price. Because of this, an exogenous rise in the number of high ability entrepreneurs or their ability level reduces the success rate of low ability entrepreneurs, and unambiguously leads to a lower equilibrium growth rate.

Our finding challenges the commonly held belief that innovative entrepreneurship is always rent creating. Equilibrium growth in our model is negatively related to the number of high ability entrepreneurs precisely because high ability entrepreneurs drive down the success rate for low ability entrepreneurs. Thus we have shown that the presence of a rent-seeking element in innovative entrepreneurship reduces equilibrium growth rate of the economy.

Are there any caveats for this negative relationship between entrepreneurship and growth? Looking more closely, this negative relationship arises because of two particular features in our model: Firstly, unlike Romer's model, we have assumed the roles of entrepreneur and researcher to be separate, entrepreneurs are not directly involved in the invention process. Secondly, we assumed all inventions considered to be worthless by entrepreneurs are indeed bad inventions. The combined effect of these two assumptions ensures that all good inventions are bought by entrepreneurs and implemented in equilibrium. Having more high ability entrepreneurs do not increase the expected value of each invention, so that the growth rate can essentially be tied down to the number of researchers alone.

While these two assumptions are not unreasonable, if they are relaxed, relationship between equilibrium growth rate and entrepreneurship would cease to be unambiguously negative. For example, if we relax the first assumption and assume that high ability entrepreneurs not only select good inventions more accurately, but also make better use of each invention; (that is, there are two stages in invention, researchers complete the first, while entrepreneurs complete the second stage.) then having a greater number of high ability entrepreneurs would raise the expected value of each invention and have a positive impact on the rate of technological growth. Similarly, if we relax the second assumption and assume that it is possible for entrepreneurs to mistake good invention for worthless ones, and high ability entrepreneurs make fewer of these mistakes, then having more high ability entrepreneurs would increase the utilization rate of good inventions, and have a positive effect on growth rate. However, even in these two cases, high ability entrepreneurs would still have a negative effect on the success rate of the low ability entrepreneurs, and since for low ability agents incomes from research and entrepreneurial sectors are the same when the economy is in equilibrium, this negative effect is also passed down to the research sector. Hence there would still be a negative effect on the rate of growth.

To sum up, the unambiguously negative relationship between high ability entrepreneurs and equilibrium growth rate is due to certain simplifying assumptions we made for the purpose of presentational clarity. We do not imply that entrepreneurship and growth are always negatively related. However, when the element of entrepreneurship is incorporated into an endogenous growth model, such that research and entrepreneurship are not identical to each other, the exercise of entrepreneurship can have an negative impact on the rate of growth, and this negative impact is a logical consequence of the two features essential to entrepreneurship: ability heterogeneity and mutual unobservability.

7 Appendix: The Sequence of Invention Selection

In our model, we assumed type 1 entrepreneurs make selection before type 2 entrepreneurs. This is inconsistent with the assumption that researchers are unable to distinguish the two types of entrepreneurs. Here we present a slight modification to the invention selection process, so that the researchers are not able identify type 1 entrepreneurs by observing when they come to the invention market. Analytical results yielded in this modification are identical with those in our model.

Suppose there are e_1 and e_2 of type 1 and 2 entrepreneurs respectively. Type 2 entrepreneurs are divided into two sub-groups,γe_2 of them choose before the type 1 entrepreneurs, while the other $(1-\gamma)e_2$ after. γ is a number between 0 and 1 and is not observable to the researchers[9]. Each

[9]As it turns out, we can even assume γ is time dependent, so that at the beginning of each t, the economy is given a γ_t between zero and one, according to which type 2

type 2 entrepreneur is allocated to one of the sub-groups randomly.

Let b_1 denote the success rate of type 1 entrepreneurs, b_{21} and b_{22} denote the success rate of the two group of type 2 entrepreneurs who select before and after type 1 entrepreneurs, respectively; BA_t is the total number of inventions on the market. Type 2 entrepreneurs who choose first are not affected by type 1 entrepreneurs, thus the value of b_{21} is:

$$b_{21} = \frac{b_0}{b_2^*}$$

On the other hand, because the set S_1 strictly contains the set S_2, success rate of the type 1 entrepreneurs is not affected by type 2 entrepreneurs who chose before them, therefore the value of b_1 is:

$$b_1 = \frac{b_0}{b_1^*}$$

Following the reasoning in section 3 of the model, success rate of the second group of type 2 entrepreneurs is represented by the following expression:

$$b_{22} = \frac{b_0 B - b_{21}\gamma e_2 - b_1 e_1}{b_2^* B - \gamma e_2 - e_1} \tag{14}$$

The numerator represents the number of good inventions left, while the denominator represents the number of inventions the second group of type 2 entrepreneurs choose from.

Now we need to determine the equilibrium supply of inventions. Invention market clearing requires the total number of inventions the second group of type 2 entrepreneurs demands to be equal to the inventions reminding in market that are not considered to be worthless by type 2 entrepreneurs, that is,

$$(1-\gamma)\, e_2 = b_2^* B - \gamma e_2 - e_1$$

Re-arrange the above expression, we get,

$$B = \frac{e_1 + e_2}{b_2^*}$$

entrepreneurs are separated into two groups. Our result is not affected.

which is identical to what we obtained in the model. Substitute this equilibrium value of B into the expression of the value of b_{22} in (14), we obtain:

$$b_{22} = \frac{b_0}{b_2^*} \left\{ 1 - \frac{1}{1-\gamma} \frac{e_1}{e_2} \left(\frac{b_2^*}{b_1^*} - 1 \right) \right\}$$

Now since type 2 entrepreneurs are allocated into the two groups randomly, the aggregate rate of success b_2 is the average of b_{21} and b_{22}, weighted by the relative size of each group,

$$b_2 = \gamma b_{21} + (1-\gamma)\, b_{22}$$

Substitute values of b_{21} and b_{22} into the above expression, we obtain the average rate of success for a type 2 entrepreneur at:

$$b_2 = \frac{b_0}{b_2^*} \left\{ 1 - \frac{e_1}{e_2} \left(\frac{b_2^*}{b_1^*} - 1 \right) \right\}$$

this expression is identical to (4), which is the success rate of type 2 entrepreneurs when they all choose after the type 1 entrepreneurs. Thus, if we replace the setting for invention selection by what is described in this model, all the results of the model will remain unchanged.

In modelling the sequence of invention selection, it seems realistic to divide the two types of entrepreneurs into many sub-groups and let them take turns to choose inventions. However, expressions for the success rates of these sub-groups typically soon becomes too complicated to trace analytically. In this appendix we presented the simplest of these cases, which yielded the somewhat surprising result that allowing the two types of agents to mix to a certain degree do not affect the average success probability of each group. We conjecture that this result holds as long as some of the type 2 entrepreneurs are placed at the end of the sequence for invention selection.

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