Exhuming Q: Market Power vs. Capital Market Imperfections*

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Abstract
Evidence of the statistical significance of profits in Q regressions remains one of the principal findings in the empirical investment literature. This result is taken to support the view that capital market imperfections are an important element for understanding investment. This paper challenges that conclusion. We argue that allowing the profit function at the firm level to be strictly concave, reflecting, for example, market power, is sufficient to replicate the Q theory based regression results in which profits are a significant factor influencing investment. To be clear, our ability to replicate the existing results does not require the specification of any capital market imperfections. Thus the friction that explains the statistical significance of profits could be market power by sellers rather than capital market imperfections.

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1 Motivation

In the large empirical literature on models of capital accumulation, there is ample evidence that financial variables, such as profits, are significant regressors for current investment.\(^1\) To many economists, this finding is taken as *prima facie* evidence of capital market imperfections.

The point of this paper is to argue that this conclusion may not be warranted. In particular, the presence of market power creates measurement error in traditional investment regressions leading to the significance of profits in these empirical exercises even if there are no capital market imperfections. This point is important since these basic investment regressions provide the underpinnings for numerous theories of credit frictions.

The empirical finding of significant profits often appears in empirical investment studies based upon "Q theory". The basic idea of this approach is to solve the dynamic optimization problem of a firm with convex costs of adjusting its capital stock. The firm will optimally weigh the current marginal costs of investment against the future marginal returns. Under some assumptions (essentially homogeneity restrictions on the profit and adjustment cost functions), this marginal gain can be proxied for by the value of the firm relative to its capital stock, a value called "average Q". The power of this approach to investment is that average Q completely summarizes the discounted value of additional investment. Therefore, under this theory, current profits should not explain current investment.

Since, in practice, current profits do matter for investment, this is taken as a basis for arguing that capital markets must be imperfect. Further, this observation, along with the large costs of adjustment generally found in these empirical papers have lead to the conclusion that the Q-theory approach is an empirical failure.

We argue here that one must be cautious in reaching these conclusions since current profits may "matter" for investment for other reasons.\(^2\)

\(^1\)Surveys of this literature are numerous. See, for example, the discussion in Chirinko [1993] and Caballero [1997] and the references therein. Noteworthy recent papers discussing this evidence are Gilchrist and Himmelberg [1995, 1999] and Cummins, Hasse and Oliner [1999].

\(^2\)One must be careful to distinguish the informational role of profits in forecasting the marginal profitability of capital from their role in constraining current investment expenditures. Gilchrist-Himmelberg [1995, 1998] have investigated this point and provide evidence that profits matter beyond their ability to forecast future profitability of investment.
particular, much of the existing empirical work rests upon the substitution of 
average Q in place of marginal Q since the former is observable. However, this 
is appropriate only under very strict assumptions concerning the profit and 
cost of adjustment functions. Thus the question this paper addresses is: can 
the significance of profit flows in Q-based investment regressions be explained 
by an empirically relevant model without capital market imperfections? 3

Our analysis studies investment models which do not satisfy the Q-theory 
assumptions. In particular, firms may have market power as sellers and costs 
of adjustment may not be homogenous in the levels of investment and the 
capital stock.4 Hence, marginal and average Q are not identical so that 
empirical models using average Q are misspecified.

The difficult aspect of this approach is that the lack of analytic results 
for the types of investment models we wish to study: i.e. those in which 
the specification of technology and adjustment costs do not satisfy the re-
strictions of Q theory. Evaluation of these competing models is difficult 
empirically since these alternatives are not easily reduced to simple linear 
relationships.5 Our empirical approach is structural in nature: we propose 
a dynamic programming problem that we solve numerically and compare to 
the data. This is a methodological innovation that complements the more 
general approach we are taking to understanding investment. We structure 
this dynamic programming problem to include non-constant returns to scale.

We estimate relevant parameters by comparing the moments generated by 
our simulated model with the data. In particular, we use an indirect inference 
approach so that the parameters of our models are selected to match observed 
Q-theory regressions augmented by cash flow.6

3 We use the term "empirically relevant" here to constrain our search for parameteriza-
tions that are not at odds with other investment facts.

4 The fact that marginal and average Q will diverge when firms have market power 
is discussed by Hayashi [1982, Proposition 2]. Galeotti and Schiantarelli [1991] estimate 
an investment model allowing for market power and find support for it Their analysis, 
however, does not attempt to "explain" the findings in the more traditional Q theory 
based empirical literature.

5 Tractability, of course, is one of the arguments in favor of the linear quadratic struc-
ture. Unfortunately, it appears that the results based upon this structure may be mis-
leading.

6 This approach is presented in Gourieroux, Monfort and Renault [1993]. Cooper and 
Haltiwanger [1999] use this approach to study investment with nonconvex costs of adjust-
ment. Adda and Cooper [1999] use a structural estimation approach to study the impact

3
Our findings are first that with the addition of a reasonable amount of curvature in profit functions, one can easily reproduce the regression results commonly found in the Q theory based empirical investment literature. In particular, profits enter the regression significantly and with a coefficient close to that reported by others without the introduction of borrowing restrictions into the firm’s optimization problem. Second, the parameterization of the quadratic adjustment costs function is quite reasonable: the estimated cost of adjustment function is close to the quadratic model. Third the level of adjustment costs is much lower than that inferred by other researchers. Finally, we find that our unconstrained model can also match empirical results based upon sample splits which were intended to partition the sample into constrained and unconstrained firms. In our results, no firms are constrained and differences between "large" and "small" firms reflect small differences in adjustment costs.

Overall, our findings challenge the prevailing wisdom that Q theory based investment regressions support the view that firm’s face borrowing restrictions. In fact, our results do not indicate that Q theory is alive and well: only that is has been buried for the wrong reasons.

2 Dynamic Capital Accumulation

Our approach to the neoclassical investment model is easily understood from examining a dynamic optimization problem in which a firm chooses the level of capital that maximizes the discounted expected value of its profits. The firm incurs adjustment costs when investing a nonzero amount. New capital is productive in the following period and depreciates at an exogenous rate, \( \delta \).

Letting \( K \) denote the current stock of capital, \( A \), a shock to productivity or demand, \( \pi(K, A) \) the profit level in state \( (K, A) \), the optimization problem can be expressed as a dynamic programming problem. The value function for the firm \( V(K, A) \) is the solution to:

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[^1]: Willis [1999] estimates the distribution of price adjustment costs using indirect inference as well.

[^2]: However, our specification does not allow for nonconvex costs of adjustment as in, for example, Cooper and Haltiwanger [1999].

[^3]: This representation of the firm’s problem does ignore variations in the cost of capital which are more likely to be relevant for a time series analysis, as in Abel-Blanchard [1986], than for our study which is based largely on cross sectional variations.
\[ V(K, A) = \max_{K'} \pi(K, A) - p(K' - K(1 - \delta)) - C(K', K) + \beta E_{A'|A} V(K', A') \]  

(1)

Here \( \pi(K, A) \) represents a reduced form profit function generated by the firm’s solution over other, freely adjustable factors of production.

In this problem, the firms faces no liquidity constraints: investment expenditures do not have to be financed out of current profits. The firm chooses tomorrow’s capital \( (K') \) using its conditional expectations of future profitability, \( A' \). Of course, to the extent that \( A' \) is correlated with \( A \), current profits will be correlated with future profits.

Assuming that \( V(K, A) \) exists, an optimal policy, denoted by \( K' = h(K, A) \) must satisfy:

\[ C_{K'}(K', K) + p = \beta E_{A'|A} V_{K'}(K', A') \]

(2)

where subscripts on the functions denote partial derivatives. The right side of this expression is conventionally termed "marginal Q" and denoted by \( q \). Note the timing: the appropriate measure of marginal Q is the expected discounted value for the following period due to the one-period investment delay. Using (1), this expression can be simplified to an Euler equation:

\[ C_{K'}(K', K) + p = \beta \{ E_{A'|A} \pi_{K'}(K', A') + p(1 - \delta) - C_{K'}(K'', K') \}. \]

(3)

The difficult aspect of this theory is its empirical implementation. As the value function and hence its derivative is not observable, (2) cannot be directly estimated. Thus the theory is tested either by finding a suitable proxy for the derivative of \( V(A, K) \) or by estimating the Euler equation, (3). We focus here exclusively on estimates based upon using the average value of the firm as a substitute for the marginal value of an additional unit of capital.\(^9\)

2.1 Q Models

The traditional Q theory model places additional structure on (1). In particular, following Hayashi [1982], assume that: \( \pi(K, A) \) is proportional to \( K \),

\(^9\)Given the prominence of this approach in the literature, it is natural to focus our analysis on these results.
and that the cost of adjustment function is quadratic:

\[ C(K', K) = \frac{\gamma}{2} \left( \frac{K' - (1 - \delta)K}{K} \right)^2 K. \]

With this specification, one can show that \( V(K, A) \) is proportional to \( K \) so that marginal \( q \) equals \( V(K, A)/K \), a term that is called "average \( Q \)" and denoted here as \( \bar{q} \). \(^{10}\)

Using this relationship between average and marginal \( Q \), (2) implies that the investment rate is a linear function of the expected value of future \( \bar{q} \), denoted \( E\bar{q} \). Note that the theory implies that average \( Q \) contains all the information necessary to determine the firm’s optimal investment. In particular, the theory does not suggest that past investment rates or any measures of current profits and/or financial variables are needed to ascertain the optimal investment plan for the firm.

### 2.2 General Profits and Cost of Adjustment Functions

This section returns to the more general dynamic capital accumulation problem given in (1) without the added restrictions of \( Q \) theory. Instead of assuming current profits are linear in capital, as required by the \( Q \) theory model, consider

\[ \pi(A, K) = AK^\alpha \quad (4) \]

where \( \alpha \) parameterizes the curvature of the profit function. This curvature most naturally reflects market power by the seller. Further, we suppose that \( C(K', K) \) is given by:

\[ C(K', K) = (\gamma/\theta) \left( \frac{K' - (1 - \delta)K}{K} \right)^\theta K. \quad (5) \]

This is a slight generalization of the quadratic cost of adjustment though it is still homogenous in \((I, K)\).

The key step away from the traditional \( Q \) model is simply allowing \( \alpha < 1 \). This curvature of the profit function creates a measurement error in the standard investment regression model since there is a gap between average

\(^{10}\)The argument follows Hayashi [1992] though his specification does not include a stochastic term. Note too that the quadratic adjustment cost is sufficient, homogeneity of the adjustment cost function is necessary.
and marginal Q. Hayashi [1992] demonstrates that in this case marginal Q is always less than average Q.

The point can be seen by ignoring adjustment costs and assuming that the firm can simply rent capital at a rate of R. Clearly, the optimizing firm will choose a level of K such that the marginal profit from an additional piece of capital is zero:

\[ \alpha AK^*(\alpha-1) = R. \]

At this optimal level of capital, average profits are clearly positive:

\[ \frac{\pi(A, K^*) - RK^*}{K^*} = (1 - \alpha)AK^*(\alpha-1) > 0. \]

So, average profits exceed marginal profits. Moreover, the difference between them depends positively on the level of profitability at the firm.

The extension to non-quadratic costs of adjustment has a similar motivation. While the quadratic case, when combined with homogeneity assumptions, clearly makes the investment problem tractable, there is clearly no a priori logic for this curvature assumption. Our methodology allows us to explore more general specifications and thus to evaluate the quadratic restriction.\(^{11}\)

3 Empirical evidence

There are numerous surveys of the investment literature with appropriate emphasis on results using average Q as a proxy for marginal Q. Here we focus on empirical evidence using the Q framework and then turn to estimation of our more general structure.

3.1 Evidence on Q Models

The theory predicts a very specific investment equation for the Q theory models: the investment rates depends only on the expected value of average

\(^{11}\)There are two important exceptions to this point. First, Abel and Eberly [1999] and Barnett and Sakellaris [1999] allow for non-quadratic costs of adjustment. Further, there is now a large literature investigating the implications of non-convex costs of adjustment, as in Caballero, Engel and Haltiwanger [1995] and Cooper, Haltiwanger and Power [1999].
Q.\textsuperscript{12} Letting \( it \) denote period \( t \) observation for firm \( i \), tests of Q theory are frequently conducted using an empirical specification of:

\[
\frac{(I/K)_{it}}{a_0 + a_1 E^\Delta q_{it+1} + a_2(\pi_{it}/K_{it})}.
\]

(6)

The theory implies that the coefficient on average \( q \), \( a_1 \), should equal \( 1/\gamma \). The constant term is allowed to pick up any firm specific heterogeneity that may arise from differences in the adjustment processes across firms, as in Gilchrist and Himmelberg [1995]. Note that this specification includes the profit rate, \( (\pi_{it}/K_{it}) \). In fact, Q theory does not suggest the inclusion of profit rates in (6). Rather, this variable is included as a way of evaluating an alternative hypothesis in which the effects of financial constraints are not included in average \( q \). Hence researchers focus on the statistical and economic significance of \( a_2 \).\textsuperscript{13}

The results obtained using this approach have been mixed. Two ”problems” have emerged: (i) the relatively high value of the adjustment cost parameter and (ii) the significance of profits or other financial variables as a regressor.\textsuperscript{14}

On the first, point, while specifications and thus estimates of the coefficients certainly vary across studies, it is not uncommon to find extremely low estimates of \( a_1 \) and thus large adjustment costs. In his original study of this model, Hayashi [1982], found \( a_1 = 0.0423 \). Abel and Blanchard [1986] obtain nonsignificant coefficients for contemporaneous average Q. Fazzari Hubbard and Petersen [1988] obtain extremely low coefficients (for example, \( a_1 = 0.0065 \) in one of their specifications) while Gilchrist and Himmelberg [1995] obtain an estimate for \( a_1 \) of 0.033.

\textsuperscript{12}Again, the timing assumption is that there is a one-period delay associated with the delivery and installation of new capital. In some applications, new investment is assumed to be immediately productive so that the appropriate measure of average Q is the current one.

\textsuperscript{13}Gomes [1998] makes an important point here: even if there are borrowing restrictions, they will appear in the value of the firm and hence in marginal Q. Whether they are properly accounted for in average Q is less clear and again depends on the homogeneity of the underlying profit and cost functions and in the nature of the borrowing restrictions.

\textsuperscript{14}In fact, the view that these models ”fail empirically” is commonly held. See the concise discussion in Erickson and Whited [1999] for example. Other common results in Q regressions are that residuals are serially correlated and lagged variables are significant (Chirinko [1993], Abel and Blanchard [1986]). This is further sign that the model is misspecified, see (West [1998]).
To appreciate the magnitude of the estimates, a coefficient of \( a_1 = 0.05 \) implies a value of \( \gamma = 20 \). With an adjustment cost function of \( \frac{\gamma}{2}(I/K)^2K \), this implies an average adjustment cost of \( 10 \times (\delta)^2K \), using the steady state restriction of \( I = \delta K \). With \( \delta = 0.15 \), we get an adjustment cost relative to the steady state capital stock of 22.5%, which is very large. Put differently, at \( a_1 = 0.05 \), implies a 6% adjustment in the first period, 50% within 8 periods and of 23 periods until full adjustment, a fairly slow process.\(^{15}\)

On the second point, many studies find that \( a_2 \) is positive and significantly different from zero which is a rejection of the Q theory. For example, Fazzari, Hubbard and Petersen [1988] divide their panel into three classes of firms determined by the ratio of dividends to income. They report significant effects of cash flow on investment for all types of firms though firms with higher dividend/income ratios have smaller cash flow coefficients.\(^{16}\) However, their \( R^2 \) measures fall dramatically from the low to the high dividend firms (from 0.53 to 0.19). Both the Q variable and the cash flow variable explain more for the low dividend firms: apparently whatever makes cash flow more significant also makes Q more significant.

Gilchrist and Himmelberg [1995] obtain stronger results in favor of financial frictions. For their panel of all firms, using the conventional measure of average Q (which they term Tobin’s Q), they find that cash flow is significant in the standard investment regression model.\(^{17}\) They produce a number of sample splits using firm indicators to determine whether they are "constrained" or "unconstrained". For the most part, the firms in the constrained group appear to have investment more sensitive to financial variables.\(^{18}\)

\(^{15}\)This is derived from an experiment where \( a = 0.7, \gamma = 20, \delta = 0.15, \theta = 2, \beta = 0.94 \). There are two possible states where the transition matrix for Markov process has 0.9 on the diagonal. The firm is assumed to start at the steady state associated with the low state of profitability. The profitability shock then jumps to the high state. It takes 23 years to get to the high steady state.

These numbers change significantly (but not overwhelmingly) if we have \( a_1 = 0.5 \) or \( \gamma = 2 \). Then 14% of the adjustment occurs in the initial period and 54% within 5 periods, up to 18 periods to full adjustment.

\(^{16}\)See their Table 5, instrumental variable estimation results. Cash flow coefficients are 0.455 (0.029) for low ratios, 0.418 (0.038) for middle ratios and 0.238 (0.010) for high ratios. Low ratios are defined as less than 10% for at least 80% of the sample observations, between 10% and 20%, and more than 20%.

\(^{17}\)See Table 2 in their paper.

\(^{18}\)However, some of their sample splits do indicate significant cash flow effects for their unconstrained firms and/or insignificant cash flow for constrained firms.
One of the important aspects of the Gilchrist-Himmelberg study is their construction of a proxy for marginal Q. As they note, one of the problems interpreting the significance of cash flow variables in investment regressions is that these factors may be forecasting future profits rather than constraining current investment. Using their panel, they estimate forecasting equations for marginal Q and argue that any remaining explanatory power of financial variables will reflect capital market imperfections.\(^\text{19}\) With this measure of Q, which they term ”Fundamental Q”, Gilchrist-Himmelberg report (see their Table 2) that for their full sample Fundamental Q is not significant and cash flow is barely significant.\(^\text{20}\) However, for their sample splits, financial variables are insignificant for their ”unconstrained” subsample and are sometimes significant for their ”constrained” subsample.

Finally, Cummins, Hassett and Oliner [1999] report similar findings in terms of their Q regressions.\(^\text{21}\) In particular, they too find that the response of investment rates to variations in average Q are quite small (implying a large value of $\gamma$). Further, cash flow is a significant regressor in their study as well. However, when they replace average Q with their version of Q based upon earnings expectations, financial variables are no longer significant.

### 3.2 Empirical Implications of the More General Model

The problems with the Q theory approach to investment fall into two categories: (i) measurement problems with average Q and (ii) problems with the specification of (1). We will argue here that these problems together imply that profits may appear as significant regressors in the standard investment regression even in the absence of borrowing restrictions.

Suppose that the profit and/or cost functions did not satisfy the conditions specified in Hayashi [1982]. As a consequence, average and marginal q diverge so that the use of $\bar{q}_{it}$ in the standard investment regression induces measurement error that may be positively correlated with profits. Hence one might find positive and significant $a_2$ in (6) in a model without any capital market imperfections.

Consider then a version of (1) using the profit and cost of adjustment

\(^{19}\) In doing so, they assume that the profit function is linearly homogenous of degree one.

\(^{20}\) In contrast, for their regressions without cash flow measures, the coefficient on fundamental Q exceeded that from their results using Tobin’s Q. Further, this coefficient was significantly different from zero.

\(^{21}\) In particular, see their Table 5.
functions given above. Our goal is to estimate the key parameters characterizing the profit and adjustment cost functions: \((\alpha, \theta, \gamma)\). The key question is whether empirically plausible profit and adjustment cost functions can reproduce the regression results from estimating (6).

Our methodology follows the indirect inference procedures described in Gourieroux, Monfort and Renault [1993]. This is a version of simulated method of moments in that the structural parameters are chosen to minimize the distance between moments generated by the data and those calculated from the simulated data. As the moments of the simulated data depend on the underlying structural parameters, minimizing this distance will, under certain conditions, provide consistent estimates of the structural parameters. The innovation associated with indirect inference is to use the coefficients of a reduced form regression to establish moments from the data and then to match these coefficients from estimating the same regression off the simulated data. The reduced form coefficients from the regression on the simulated data will be close to those from the actual data at the "true" values of the structural parameters.

The appealing feature of this approach is that it allows a researcher interested in a structural model to link results explicitly to existing less structural empirical evidence. For our purposes, we use the results of Gilchrist-Himmelberg [1995 ] as representative of the Q theory based investment literature. Denote their estimates of the parameters of the investment relationship, (6), by \((a_1, a_2)\). As noted earlier, their paper presents two sets of estimates for (6): one for the conventional measure of average Q and the other using their measure of Fundamental Q. Further, they present evidence for their full sample and for sample splits based, for example on firm size and/or the dividend behavior of a firm.

We initially focus on results from their pooled panel sample and then return to understanding their sample splits. At this stage, our goal is to understand the foundations of empirical results based upon Tobin’s Q. For this specification, they estimate \(a_1 = .03\) and \(a_2 = .24\). \(^{22}\) As these results are based upon a panel data set, our simulation/estimation exercise will be conducted within a panel structure. To do so, we decompose the shocks to profitability into two components: an aggregate shock common to all firms

\(^{22}\)These estimates are reported in their Table 2. Note that these regressions included time dummies and were estimated in first differences to remove firm fixed effects. Since we have no fixed effects build into our model, we do not need to remove them and hence focus on regression results in levels.
and a firm specific shock.

The aggregate shock process is taken from the Cooper-Haltiwanger analysis of profitability shocks in the LRD. We represent this process as a two-state Markov process with a symmetric transition matrix in which the probability of remaining in either of the two aggregate states is .8.  

3.2.1 Estimates of \((\alpha, \gamma)\)

Our initial estimation exercise assumes the quadratic cost of adjustment specification and focuses on estimating the curvature of the profit function \((\alpha)\) and the level of the adjustment costs \((\gamma)\) with the restriction of quadratic adjustment costs \((\theta=2)\). So, the only variation from the standard Q theory model is firm market power. In order to focus the initial estimation on these key parameters, we set other parameters at levels found in previous: \(\delta=.15\) and \(\beta = .95\). This leaves \((\alpha, \gamma)\) and the stochastic process for the firm-specific shocks to profitability as the parameters remaining to be estimated. We estimate both the serial correlation \((\rho)\) and the standard deviation \((\sigma)\) of the firm specific profitability shocks.

Our approach to estimation requires two pieces: solving the dynamic programming problem and then simulating a panel data set. For each value of the vector of parameters, \(\Theta \equiv (\alpha, \gamma, \rho, \sigma)\), we solve the firm’s dynamic programming problem, using value function iteration. In order to solve the dynamic programming problem at the firm level, conditional expectations need to be formed using the parameters of the stochastic process for the firm specific shocks, \((\rho, \sigma)\). The method outlined in Tauchen [1986] is used to create a discrete state space representation of the process for any \((\rho, \sigma)\). Since the estimation makes extensive use of the cross sectional properties of the panel data set, we allowed 16 elements in the state space for the idiosyncratic profitability shock.

Once the dynamic programming problem is solved, a panel data set can be created by simulation using the estimated processes for the shocks and the policy functions derived from the solution of the dynamic programming problem. For the simulations, we assumed there were 400 firms and 50 years of data.

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In fact, our estimates are not very sensitive to the aggregate shocks. Instead, the model is essentially estimated from the rich cross sectional variation, as in the panel study of Gilchrist-Himmelberg [1995].
Given this data set, the Q theory model is estimated and other relevant moments are calculated. The regression was of the same form as (6). Thus for each value of Θ, we obtain estimates of the parameters of (6), call them $(\hat{a}_1, \hat{a}_2)$, where we have ignored the constant term. Further, we use three other moments reported by Gilchrist-Himmelberg: the serial correlation of investment rates (.4), the standard deviation of profit rates (.3) and the average value of average $Q$ (.3).^24

Let $\Psi^d$ denote the vector moments from the data and $\Psi^s(\Theta)$ denote the corresponding moments from the simulated data, given the vector of parameters $\Theta$. For our problem,

$$\Psi^d = [0.03 \ 0.24 \ 0.4 \ 0.3].$$

As in all moment matching exercises, a discussion of why these particular regression coefficients/moments were chosen to match is appropriate. Clearly, given the motivation of trying to understand the reduced form empirical evidence from investment regressions, coefficient estimates from (6) are obviously important to the exercise. The serial correlation of investment rates and the standard deviation of profit rates are necessary to pin down the parameters of the driving process. Finally, average $Q$ was included to guarantee that our estimates of the curvature of the profit function did not produce unreasonably high profit rates since average $Q$ is determined by the discounted present value of average profit rates. Beyond the economic relevance of these moments, it is also important that they are responsive to variations in the underlying parameters of our problem. This property was verified in our simulations and underlies the standard errors of our estimates.

We compute a statistic, $J(\Theta)$ defined as:

$$J(\Theta) = (\Psi^d - \Psi^s(\Theta))'W(\Psi^d - \Psi^s(\Theta)) \tag{7}$$

where $W$ is an estimate of the inverse of the variance-covariance matrix of $\Psi^d$.^25 Our problem is then to solve

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^24: The average value of average $Q$ and the standard deviation of the profit rate (measured as cash flow) comes from Table 6 in Gilchrist-Himmelberg [1995]. The serial correlation of the investment rate comes directly from Charles Himmelberg and we are grateful to him for supplying this calculation.

^25: We used a multi-stage procedure to estimate the parameters and to determine $W$. We first estimated the parameters assuming that $W$ was the identity matrix. This produces consistent estimates. We then simulated multiple panels using these estimated parameters.
\[
\min_{\theta} J(\theta).
\]

The estimate of \( \Theta \) is simply the solution to this problem, which we denote as \( \hat{\Theta} \). The difficult aspect of this problem is in characterizing the highly non-linear mapping from the structural parameters \( \Theta \) to the objective function \( J(\Theta) \). Note that this parameter vector is overidentified since we are trying to match two regression coefficients and three moments using only four parameters.

The second row of Table 1 presents our results. At the value of \( \hat{\Theta} \) given in the second row of Table 1 we are able to closely match \( \Psi^d \).\(^{26}\)

<table>
<thead>
<tr>
<th>Structural Parameters</th>
<th>Reduced Form Estimates/Moments</th>
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<tbody>
<tr>
<td>( \alpha ) ( \gamma ) ( \rho ) ( \sigma ) ( \theta )</td>
<td>( a_1 ) ( a_2 ) ( sc^{I/K}_R ) ( std^{z}_R ) ( \hat{Q} )</td>
</tr>
<tr>
<td>GH95</td>
<td>.03 ( .24 ) ( .4 ) ( .25 ) ( 3 )</td>
</tr>
<tr>
<td>IC, ( \theta = 2 )</td>
<td>.689 ( .149 ) ( .103 ) ( .853 ) ( 2 )</td>
</tr>
<tr>
<td>IC, ( \theta ) free</td>
<td>.693 ( .153 ) ( .106 ) ( .846 ) ( 1.96 )</td>
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Table 1

In terms of interpreting our results, the estimated curvature of the profit function of .689 implies a markup of about 45%\(^{27}\). This is in the same range of other estimates in the literature. It is slightly more than the curvature estimate of .54 reported by Cooper-Haltiwanger [1999] for their analysis of plant-level profit functions. Gilchrist and Himmelberg [1999] estimate the marginal profit function and, by our calculations, find a curvature of between

\(^{26}\)In Table 1 and throughout, IC stands for imperfect competition (\( \alpha < 1 \)). GH95 refers to Gilchrist and Himmelberg [1995]. Quadratic adjustment costs are indicated by \( \theta = 2 \), \( sc(I/k) \) indicates the serial correlation of the investment rate, \( std(\pi/k) \) indicates the standard deviation of the profit rate, and \( \hat{Q} \) denotes average q. Standard errors and goodness of fit tests will soon follow.

\(^{27}\)Let \( p = y^{-\eta} \) be the demand curve and \( y = k \) the production function. Maximization of profit, \( \pi = py - rk \), implies that the ratio of price to marginal cost is given by \( \frac{1}{(1 - \eta)} \). From our estimation results, \( (1 - \eta) = 0.689 \) implying a markup of .451.
.5 and .8.\textsuperscript{25} Finally, Galeotti and Schiantarelli [1991] find significant market power for firms and a markup of about 33%.\textsuperscript{29}

The other interesting parameter is our estimate of the level associated with the quadratic cost of adjustment, $\gamma$. As noted above, under the null of $Q$ theory, this parameter is the inverse of the coefficient on average $Q$ in the investment regression. Hayashi initially estimated this parameter at about 25. Subsequent work has led to lower estimates, including that produced by Gilchrist and Himmelberg [1995] who find parameter estimates as high as .33 and thus $\gamma = 3$ for their "unconstrained firms".\textsuperscript{30}

The interesting point from our results is that the estimate of $\gamma$ is not identified solely from the regression coefficient on average $Q$. While this inference is correct when the profit function exhibits constant returns to scale, it is not true when the function is strictly concave. Interestingly, the estimated value of $\gamma = .149$ is far from the inverse of the coefficient on average $Q$ (about 4).

3.2.2 Sample Splits

The large empirical $Q$ literature also distinguishes between firms that are likely to be constrained in financial markets and those that are not. One distinction is often made between large and small firms with the presumption being that the former are less likely to be constrained. An interesting issue is whether our model can explain differential findings by firm size.

In Table 2 we report the regression results from Gilchrist-Himmelberg [1995] for their large and small firm splits, as well as our estimation results. Using their discussion of the data, we assume that the serial correlation of investment rates, the standard deviation of profit rates and average $Q$ do not vary by firm size.\textsuperscript{31} As in Table 2, we report the structural parameter estimates as well as the moments for each of two samples. Note that here we again impose the quadratic cost of adjustment.

\textsuperscript{25}If one uses cash flow their estimates using sales imply (see their footnote 10) a mean value of 0.76 and a range of 0.25 to 1.88, and if one uses operating income one gets a mean value of 0.49 and a range of 0.16 to 1.17.

\textsuperscript{29}This estimate is based upon their discussion of their Table 1 estimates.

\textsuperscript{30}Note though that this result does not come from a regression with Tobin’s $Q$. So, the inference from the standard $Q$ theory, which requires average and marginal $Q$ to be equal, does not apply here.

\textsuperscript{31}This point is made in the Appendix of Gilchrist-Himmelberg [1995]. For this exercise, we recomputed the $W$ according to this sample split using the simulation method described above.
<table>
<thead>
<tr>
<th>Sample</th>
<th>Structural Parameters</th>
<th>Reduced Form Estimates</th>
<th>( \frac{\Omega}{k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LARGE</td>
<td>( \alpha ) ( \gamma ) ( \rho ) ( \sigma ) ( \theta ) ( a_1 ) ( a_2 ) ( \text{sc}(\frac{\Omega}{k}) ) ( \text{std}(\frac{\Omega}{k}) )</td>
<td>( \bar{Q} )</td>
<td></td>
</tr>
<tr>
<td>GH95:</td>
<td>( .027 ) ( .124 ) ( .4 ) ( .25 )</td>
<td>( 3 )</td>
<td></td>
</tr>
<tr>
<td>I.C., ( \theta = 2 )</td>
<td>( .69 ) ( .304 ) ( .09 ) ( .81 ) ( 2 )</td>
<td>( .048 ) ( .129 ) ( .178 ) ( .24 )</td>
<td>( 3.07 )</td>
</tr>
<tr>
<td>SMALL</td>
<td>GH95:</td>
<td>( .056 ) ( .2 ) ( .4 ) ( .25 )</td>
<td>( 3 )</td>
</tr>
<tr>
<td>I.C., ( \theta = 2 )</td>
<td>( .69 ) ( .22 ) ( .11 ) ( .84 ) ( 2 )</td>
<td>( .064 ) ( .2 ) ( .078 ) ( .251 )</td>
<td>( 2.98 )</td>
</tr>
</tbody>
</table>

Table 2

It is important to note that our exercise does not make use of an artificial panel. Rather, we let the data tell us whether there are significant economic differences between large and small firms, by doing separate estimation exercises for different subsets of empirical results. As before, our inputs to the process are the moments we wish to match and our output is the same set of moments (approximately matched) and the corresponding estimated parameters.

This exercise is fairly successful. We are able to match the differential responses of investment to cash flow coefficients which is a crucial element of the financial frictions empirical literature. The estimation procedure does this by finding a larger adjustment cost parameter (\( \gamma \)) for large firms. Another interesting characteristic of these results is that the estimation procedure finds the same concavity of profits for the two sets of firms, basically unchanged from the one obtained when matching the full sample results.

3.2.3 Estimates of (\( \alpha, \gamma, \theta \))

As a final exercise, we focus jointly on the curvature of the profit and the cost of adjustment function. Instead of forcing the adjustment function to be quadratic (i.e. setting \( \theta = 2 \) in (5)), we allow the curvature of the adjustment cost function to be determined by the data. We proceed as above by finding the values of these parameters that minimize \( J(\Theta) \) where \( \Theta = (\alpha, \gamma, \rho, \sigma, \theta) \). The results are reported in the bottom row of Table 1.

From here it is quite clear that the model with quadratic costs is not a bad specification: the estimated value of \( \theta \) is quite close to 2.\footnote{Abel-Eberly [1999] report a curvature estimate such that the marginal adjustment cost function is convex as in Barnett-Sakellaris [1999] too.} The other
parameter estimates, not surprisingly, remain relatively unchanged.

4 Conclusions

Our model can produce regression results very close to those obtained in empirical studies based upon the Q theory model. In stark contrast to the conclusions reached in those studies, our model does not contain any capital market imperfections. Instead, it differs from the standard model by moving away from the linear-quadratic structure generally taken as given in those exercises. Thus, the statistical significance of profit rates in the standard investment regression may not reflect capital market imperfections.

Additional insights into these competing models can be obtained by looking explicitly at the implications of a model with borrowing constraints, a topic we turn to next in our research. Apparently, there has been no systematic study of the alternative model to determine whether the rejections of the basic Q model could reflect capital market imperfections. As in Gomes [1998], the natural way to proceed is to specify capital market imperfections that do not satisfy the homogeneity properties. Otherwise, these capital market restrictions will be summarized in marginal Q and thus, under the right assumptions, captured by average Q as well. The introduction of nonconvexities of market participation is particularly appealing if one wants to model the conjecture that firm size is important for capital market imperfections and, more generally, to allow the constraints on firms to be endogenous.

References


