

# Wealth Inequality and Intergenerational Links

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## Abstract

Empirical studies have shown that, for many countries, the distribution of wealth is much more concentrated than the one of labor earnings and that households with higher levels of lifetime income have higher lifetime saving rates. Previous models have had difficulty in generating these features. I construct a computable general equilibrium model with overlapping generations in which parents and children are linked by bequests and earnings persistence within families. I show that voluntary bequests are important to explain the emergence of large estates that characterize the top of the wealth distribution, while accidental bequests are not. In addition, the introduction of a bequest motive generates lifetime saving profiles more consistent with the data. Allowing for earnings persistence within families generates an even more concentrated wealth distribution. A cross-country comparison between the U.S. and Sweden shows that intergenerational linkages are important to explain the upper tail of the wealth distribution also in economies where redistribution programs are more prominent and there is less inequality. Moreover Sweden, with its generous social safety net, has a larger fraction of people with zero or negative wealth. The model is capable of reproducing this feature as well.

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# 1 Introduction

Empirical studies (e.g. Hurst, Luoh and Stafford [19]; Wolff [37]; Lillard and Willis [27]; Díaz-Giménez, Quadrini and Ríos-Rull [9]) have shown that labor earnings, income and wealth are significantly concentrated, with distributions skewed to the right. However, wealth is the most concentrated of the three variables with a Gini coefficient of .72, earnings rank second with a Gini coefficient of .46, and income is the most dispersed of the three with a Gini coefficient of .44. While these empirical regularities are observed in many countries, previous models do not provide a satisfactory explanation of how the observed earnings distribution leads to the observed distribution of wealth (Quadrini and Ríos-Rull [33]).

In the data, a significant fraction of the dispersion in earnings, income and wealth across households is attributable to their different positions in the life cycle. Moreover, the intergenerational transmission of wealth is substantial. Kotlikoff and Summers [23] calculate that the majority of the current value of the US capital stock (at least 80%) can be attributed to intergenerational transfers rather than to accumulation out of earnings, which is the emphasis of the basic life-cycle model of capital accumulation. Gale and Scholz [12] use direct measures of intergenerational flows and attribute 63% of the current value of the U.S. capital stock to intergenerational transfers. Mulligan [28] shows that intergenerational links are essential to explain the emergence of very large estates.

Other empirical work (e.g. Dynan, Skinner and Zeldes [11] and Lillard and Karoly [26]) highlights the fact that households with higher levels of lifetime income have higher saving rates. Carroll [7] shows that it is difficult to explain the behavior of these consumers using either a standard life-cycle model or a dynastic model.

The goal of this research is to study how wealth is accumulated in a life-cycle economy with intergenerational links and how the characteristics of the accumulation process influence the distribution of wealth, given the distribution of labor earnings. I consider a life-cycle model with earnings uncertainty, life-span risk, and links between parents and children: the parents care about leaving bequests to their offspring, and the children partially inherit their parents' productivity. In this setup households have several motives to save: to self-insure against income and life-span risk, for retirement and possibly to leave a bequest to their children. The characteristics of the accumulation process impact the life-cycle pattern of wealth accumulation, the dispersion of wealth within cohorts and the overall wealth distribution.

I construct a computable general equilibrium model to study how these different savings motives help in understanding the dispersion of wealth across households, both in the U.S. and Swedish economies, taking as given the distributions of labor earnings. I also calibrate the model to the Swedish economy to investigate whether intergenerational linkages are important even in economies where redistribution programs are more prominent and there is less inequality.

My results show that voluntary bequests are important to explain the emergence of large estates that are usually accumulated in more than one generation and that characterize the upper tail of the wealth distribution in the data, while accidental bequests do not generate more wealth concentration. I also show that the introduction of a bequest motive generates lifetime saving profiles more consistent with the data. Saving over the life cycle is the primary factor in understanding how wealth is accumulated at the lower tail of the distribution, while intergenerational

links significantly affect the shape of the upper tail. Moreover, the introduction of a human-capital link in which the children partially inherit the productivity of their parents can generate a yet more concentrated wealth distribution. In this case more productive parents accumulate larger estates and leave larger bequests to their children who, in turn, are more successful than average in the workplace. The cross-country comparison between the U.S. and Sweden shows that intergenerational linkages are also important in economies where redistribution programs are more prominent and there is less inequality. The calibration to the Swedish data also reveals that the model reproduces well the fact that Sweden, with its more generous social insurance net, has a larger fraction of people with zero or negative wealth.

Section 2 reviews some related literature, section 3 briefly discusses the main features of the U.S. and Swedish data, and section 4 describes the model. Section 5 is a road map of the experiments that I run in order to understand the quantitative importance of each intergenerational link. Sections 6 and 7 describe the calibration and the results of the various experiments for the U.S. and the Swedish economies respectively. Section 8 discusses other factors that may be important to explain the distribution of wealth and explains how the assumptions I make are likely to affect my results. Section 9 concludes and discusses some directions for future research.

## 2 The Literature

Previous attempts at studying how the distribution of wealth is determined fall broadly into two categories. The first group of papers studies overlapping-generations economies where all savings arise over the life cycle.<sup>1</sup> The second group of paper studies economies with infinitely lived dynasties.

Huggett [18] and Gokhale et al. [13] are the only papers within the first group to focus primarily on the distribution of wealth.

Gokhale et al. [13] aim at evaluating how much wealth inequality arises from inheritance inequality. To do so, they construct an overlapping-generations model and focus on intra-generational inequality of households whose head is age 66. Their model allows for random death, random fertility, assortative mating, heterogeneous human capital, progressive income taxation and social security. All of these elements are exogenous and calibrated to the data. The families are assumed not to care about their offspring, hence all bequests are involuntary. To solve the model, they impose that individuals are infinitely risk averse and that the rate of time preference equals the interest rate. As a consequence, the families in the model have a constant per capita consumption profile, resulting in a large aggregate flow of bequests from people who die before reaching the maximum lifespan. Moreover, families do not take into account expected bequests when making consumption and saving decisions. Gokhale et al. find that inheritances in the presence of social security play an important role in generating intra-generational wealth inequality in the cohort they consider. The intuition is that social security annuitizes completely the savings of poor and middle-income people but is a very small fraction of the wealth of richer people, who thus keep assets to insure against life-span risk. In this setup, were a market for

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<sup>1</sup>Cf. İmrohoroğlu, İmrohoroğlu and Joines [20], Hubbard, Skinner and Zeldes [17].

annuities available, rich people would completely annuitize their wealth and no bequest would be left.

In Huggett [18], the workers face uninsurable income shocks and uncertain life span. The government taxes bequests at 100% and redistributes them equally to all agents alive. As in most papers that address the distribution of wealth, the skewness is generated by the introduction of a borrowing constraint. The paper succeeds in matching the U.S. Gini coefficient for wealth, but the concentration is obtained by having more people with zero or negative wealth and a much thinner upper tail than observed in the actual distribution.

The fact that people hit the borrowing constraint too often, leading to a large fraction of people at zero or negative wealth, is a common problem of models with idiosyncratic income shocks. In overlapping-generations models this problem is aggravated by the assumption that young agents are born without wealth and hence need time to accumulate precautionary savings to hedge against income shocks.

The proportion of people at zero wealth is less of a problem for the second group of papers, which tries to explain the distribution of wealth in economies populated by infinitely lived dynasties. In this case the precautionary savings have already been accumulated in steady state, hence the borrowing constraints bind less often. These models disregard the fact that the lower tail of the distribution of wealth is mainly comprised of young and old households; they succeed in lowering the proportion of households at zero or negative wealth by treating all agents as if they were middle aged.

As for the second group of papers, Krusell and Smith [24] study an economy populated by infinitely lived dynasties that face idiosyncratic income shocks. These dynasties also face a stochastic process for their discount factor and thus have heterogeneous preferences. The discount factor changes on average every generation and is meant to recover the fact that parents and children in the same dynasty may have different preferences. Krusell and Smith find that allowing for different discount factors among agents helps in matching the cross-sectional wealth distribution.

Castañeda, Díaz-Giménez and Ríos-Rull [8] consider a model of earnings and wealth inequality and use it to study the effect of tax reforms. Their model economy is populated by dynastic households that have some life-cycle flavor; workers have a constant probability of retiring at each period and once they are retired they face a constant probability of dying. They care about leaving bequests to their offspring. These newborns enter the model as workers and inherit the family's after-tax capital; in equilibrium their utility is the same as that of old workers. The paper employs a large number of free parameters to match some features of the U.S. data that are considered particularly significant, which include measures of the wealth distribution. However, the simple structure of the model does not allow proper accounting for the life-cycle pattern of savings and the role of bequests in generating wealth inequality.

Quadrini [32] constructs an infinitely-lived agent model in which agents at each period decide whether to be entrepreneurs or not. Three elements in the model are crucial. First, the existence of capital market imperfections induces workers that have entrepreneurial ideas to accumulate more wealth to reach minimal capital requirements. Second, in the presence of costly financial intermediation, the interest rate on borrowing is higher than the return from saving, therefore an

entrepreneur whose net worth is negative faces a higher marginal return from saving and reducing his debt. Third, there is additional risk associated with being an entrepreneur, hence risk averse individuals will save more. As in Castañeda, Díaz-Giménez and Ríos-Rull [8] the model uses a large number of free parameters to match features of the earnings and wealth distribution.

In a recent paper Heer [16] adopts a life-cycle setup in which parents care about leaving bequests to their children. In his framework the bequest motive does not affect much the distribution of wealth. His results differ from mine because his income process is much less representative of the actual process faced by households and because he assumes that children can perfectly observe their parent’s characteristics and wealth.

In contrast with the papers that study economies with infinitely lived dynasties, I explicitly model the life-cycle structure, which contributes to a significant fraction of the dispersion in earnings, income and wealth across households. Compared to Huggett’s [18] paper I add intergenerational transmission of wealth and ability. In contrast with Gokhale et al. [13], and consistent with the data, my model generates higher saving rates for people with higher lifetime income and age-savings profile consistent with the empirical observations. Compared to Heer [16], my paper does a better job of modeling the earnings process and the bequest motive and also explores the relevance of intergenerational transmission of ability.

Rather than focussing on the wealth distribution, Carroll [7] concentrates on the fact that in the data households with higher levels of lifetime income have higher lifetime saving rates (see Dynan, Skinner and Zeldes [11] and Lillard and Karoly [26]). He shows that neither standard life-cycle, nor dynastic models can recover the saving behavior of rich and poor families at the same time. To solve this puzzle he suggests a “capitalist spirit” model, in which finitely lived consumers have wealth in the utility function. This can be calibrated to make wealth a luxury good, thus rendering nonhomothetic preferences. In my model, nonhomotheticity arises because parents care about leaving bequests to their children (I calibrate this bequest motive taking into account the children’s utility of receiving the bequest). This setup allows me to test whether the assumptions I make are consistent not only with the saving behavior of single individuals but also with the wealth distribution as a whole.

### 3 On the Empirical Facts

#### 3.1 The U.S. Economy

Capital output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top						Percent with negative or zero wealth
			1%	5%	20%	40%	60%	80%	
3.0	.63	.72	28	49	75	89	96	99	5.8-15.0

Table 1: U.S. wealth data.

In table 1 I present various statistics on wealth and wealth distribution in the U.S.

The measure of the capital-output ratio depends on the concept of capital one has in mind. The most comprehensive notion of capital typically used includes residential structures, plant and equipment, land and consumer durables. This implies a capital output ratio of about 3 for the period 1959-1992 (Auerbach and Kotlikoff [2]). A narrower definition of capital obviously implies a lower capital-output ratio. Stokey and Rebelo [35] exclude land, consumer durables and residential structures owned by the government and obtain a ratio below 2.

The notion of transfer wealth as a fraction of total wealth is the one computed by Kotlikoff and Summers [23]. The authors distinguish two components of total wealth: transfers and life-cycle savings. They compute the transfer wealth component for a person alive at a given point in time as the current value of all non-government transfers received by that person. The current value is computed using the realized after-tax rates of return on wealth holdings. The life-cycle component is the residual one. They estimate that transfer wealth accounts for at least 80% of total wealth. A more recent study by Gale and Scholz [12] uses direct measures of intergenerational flows from the Survey of Consumer Finances and finds that intergenerational transfers account for about 63% of the current value of wealth.

The data on the wealth distribution are from Wolff [37]. His estimates are based on the 1983 Survey of Consumer Finances. Wealth is defined as owner-occupied housing, other real estate, cash, financial securities, unincorporated business equity, insurance, and pension cash surrender value, less mortgage and other debt. Wolff makes adjustments to account for consumer durables, household inventories and underreporting of financial assets and equities. Hurst, Luoh and Stafford [19] provide data on the wealth distribution using different data sources for 1989. They piece together the 1989 PSID household wealth up to the 98.6 percentile and then use the IRS data for the 98.2 to the 99.6th percentile and the Forbes data for the balance of the 32 most wealthy families. They adopt this strategy because the PSID oversamples poor people (Juster, Smith and Stafford [21] compare the PSID with the SCF data and find that the PSID is accurate only up to the richest 5% of the population). The wealth measure obtained by Hurst, Luoh and Stafford is very close to the one adopted by Wolff, but they do not perform any adjustment. They find similar numbers for the wealth distribution.

Gini coeff.	Percentage earnings in the top							Percent with negative or zero income
	1%	5%	10%	20%	40%	60%	80%	
.46	6	19	30	48	72	89	98	7.7

Table 2: U.S. data on gross earnings.

Table 2 is computed using data from the Luxembourg Income Study (LIS) data set, which collects income data sets from different countries (it is based on the CPS for the U.S.) and makes them comparable. The table is computed using data for households whose head is 25 to 60 years of age, and the definition of gross earnings includes wages, salaries and self-employment income. We can see from tables 1 and 2 that earnings display much lower concentration than wealth.

### 3.2 The Swedish Economy

Capital output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top						Percent with negative or zero wealth
			1%	5%	20%	40%	60%	80%	
2.0	> .51	.73	14	32	64	88	99	100	30

Table 3: Swedish wealth data.

Table 3 presents various statistics on wealth and wealth distribution as in the U.S. section.

The capital/gdp ratio is computed analogously that of U.S. (see Hansson [15]). The wealth distribution, the Gini coefficient and the number of people at zero or negative wealth were kindly provided by Domeij and Klein (see [10] for more information). Since these data come from the HINK 1996 database, which does not oversample the very wealthy, they somewhat understate the fraction of total wealth held by the upper tail of the distribution.<sup>2</sup>

The inheritance-wealth ratio number is from Laitner and Ohlsson [25]. They compute the current value of household inheritances in Sweden as a fraction of household wealth; the result they obtain, conditional on the age of the household head varies from a low of .34 (for households 50-59) to a high of .85 (for households 70 and older). Their number for the economy as a whole is .51. However, this number does not include inter-vivos transfers, therefore the actual present value of intergenerational transfers to wealth is higher.

Table 4 is also computed from the LIS data set, in the same way described for Table 2.

Gini coeff.	Percentage earnings in the top							Percent with negative or zero income
	1%	5%	10%	20%	40%	60%	80%	
Gross earnings								
.40	4	15	25	42	68	86	98	7.6

Table 4: Swedish data on gross earnings.

As we can see from tables 1-4, the Swedish distribution of earnings is more equally distributed than the U.S. distribution of earnings, but the Gini coefficient for wealth is the same in the two countries (.72 in the U.S. and .73 in Sweden). However, the high Gini coefficient for wealth in Sweden results from different reasons than in the U.S. In the U.S. the top 1-5% of people hold a large fraction of total wealth, while the fraction of people with zero or negative wealth is relatively small. On the contrary in Sweden the top 1-5% of people do not hold not as much of total wealth, but a much larger fraction of people is at zero or negative wealth. This may be due to the fact that social security and unemployment benefits are more generous in Sweden than in the U.S.,

<sup>2</sup>The numbers reported above do not include households whose head is younger than age 25 because in this data set people 18 years and older are considered independent units, even if they live with their parents. Including these people would increase further the number of poor people.

and these social insurance programs are a disincentive to save, especially for poorer people. In fact, individuals for whom social security benefits are high compared to their life-time income will not save for retirement in presence of a redistributive social security system. Moreover if security nets (such as unemployment insurance) are substantial, precautionary savings will be lower.

## 4 The Model

The economy is populated by overlapping generations of people and an infinitely lived government. The agents may differ in their productivity level. The members of successive generations are linked to one another by the altruism of the parents toward their children and the offspring's inheritance of part of their parent's productivity. At age 20 each person enters the model and starts consuming, working and paying labor and capital income taxes. At age 25 the consumer procreates and he cares about leaving bequests to his children when he dies. After retirement the agent no longer works but receives social security benefits from the government and interest from accumulated assets. The government taxes labor earnings, capital income, and estates and pays pensions to the retirees.

### 4.1 Demographics

During each model period, which is five years long, a continuum of people<sup>3</sup> is born. I assume that each person does not make saving or consumption decisions until he is 20, when he begins working. Thus, I model the agent's behavior starting from age 20 and define age  $t = 1$  as 20 years old, age  $t = 2$  as 25 years old, and so on. After one model period, at  $t = 2$ , the agent's children are born, and four periods later (when the agent is 45 years old) they are 20 and start working. Since there are no inter-vivos transfers in this model economy, all individuals start off their working life with no wealth. Total population grows at a constant, exogenous, rate ( $n$ ), and each agent has the same number of children.<sup>4</sup> The agents retire at  $t = t_r = 9$  (i.e., when they are 65 years old) and die for sure by the end of age  $T = 14$  (i.e., before turning 90 years old). From  $t = t_r - 1$  (i.e., 60 years of age) to  $T$ , each person faces a positive probability of dying given by  $(1 - \alpha_t)$ ; since death is assumed to be certain after age  $T$ ,  $\alpha_T = 0$ . The assumption that people do not die before 60 years of age reduces computational time and does not influence the results because the number of people dying between the ages of 20 and 60 is small.

Since I consider only stationary environments, the variables are indexed only by age,  $t$ , and the index for time is left implicit.

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<sup>3</sup>In the theoretical sections, I will use the terms "agent", "person", "consumer" and "household" interchangeably. Each household is taken to be composed of one person and dependent children.

<sup>4</sup>The number of children is thus  $n^5$  if  $n$  is the growth rate of the population over 5 years, or  $n^{25}$  if  $n$  is expressed in yearly terms.



## 4.2 Preferences and Technology

Preferences are assumed to be time separable, with a constant discount factor. The utility from consumption in each period is given by  $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$ .

Parents care about their selfish children. The particular form of altruism I consider is called “warm glow”: the parents’ derive utility from leaving a bequest (net of estate taxes) to their offspring. The utility from leaving a net bequest  $b_t$ , is  $\phi(b_t)$ . Considering a more sophisticated form of altruism would increase the number of state variables (already 4 in this setup) and, in some cases, would generate strategic parent/child interaction.

In this economy all agents face the same exogenous age-efficiency profile,  $\epsilon_t$ , during their working years. This profile is estimated from the data and recovers the fact that productive ability changes over the life cycle. Workers also face stochastic shocks to their productivity level. These shocks are represented by a Markov process  $\{y_t\}$  defined on  $(Y, \mathcal{B}(Y))$  and characterized by a transition function  $Q_y$  where  $Y \subset \mathfrak{R}_{++}$  and  $\mathcal{B}(Y)$  is the Borel  $\sigma$ -algebra on  $Y$ . This Markov process is the same for all households. The total productivity of a worker of age  $t$  is given by the product of his stochastic productivity in that period and his deterministic efficiency index at the same age:  $y_t \epsilon_t$ . The parent’s productivity shock at age 40 is transmitted to children at age 20 according to a transition function  $Q_{yh}$ , defined on  $(Y, \mathcal{B}(Y))$ . What the children inherit is only their first draw; from age 20 on, their productivity  $y_t$  evolves stochastically according to  $Q_y$ . An alternative possibility is to assume that agents face heterogeneous income processes (both ex-ante and ex-post heterogeneity as opposed to only ex-post heterogeneity) or “education levels” and that children partially inherit from their parents these different income processes. While this is a sensible extension, it would introduce an additional state variable and is left for future research.

After retirement, the agents do not work any more but live off pensions and accumulated assets.

I assume that children cannot observe directly their parent’s assets, but only their parent’s productivity when the parent is 40 and the offspring are 15, i.e., the period before they “leave the house” and start working. Based on this information, they infer the size of the bequest they are likely to receive.<sup>5</sup> I will discuss the relevance and the qualitative effects of relaxing all of these assumptions in section 7.

The household can only invest in physical capital, at a rate of return  $r$ . The depreciation rate is  $\delta$ , so the gross-of-depreciation rate of return on capital is  $r + \delta$ .<sup>6</sup> I assume also that the agents face borrowing constraints that do not allow them to hold negative assets at any time.

## 4.3 Government

The government is infinitely lived and taxes labor earnings, capital income and estates to finance the exogenous public expenditure and to provide pensions to the retired agents.

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<sup>5</sup>Once again, this is done to keep the number of state variables at a minimum.

<sup>6</sup>Given this linear technology,  $\delta$  is irrelevant for the agents in the economy if we hold  $r$  fixed; it is only important for measuring gross revenues pertaining to capital that are included in the definition of GNP.

Labor earnings are taken as exogenous and calibrated to the data, matching the after-tax Gini coefficient. Since the U.S. tax system is progressive, this Gini coefficient is lower than the one computed from pre-tax labor earnings. In the model, I introduce a constant tax rate  $\tau_l$ , in order to balance the government budget, while all the progressive features of the tax system are already reflected in the calibrated earnings distribution.

Income from capital is taxed at a flat rate  $\tau_a$ .

Estates larger than a given value  $ex_b$  are taxed at rate  $\tau_b$  on the amount in excess of  $ex_b$ .

The structure of the social security system is the following: the retired agents receive a lump-sum transfer from the government each period until they die. The amount of this transfer is linked to the average earnings of a person in the economy.

#### 4.4 The Household's Problem

I consider an environment in which, during each period, a  $t$ -year-old individual chooses consumption  $c$  and risk-free asset holdings for the next period,  $a'$ . The state variables for an agent are denoted as  $x = (t, a, y, yp)$ , where  $t$  is his age,  $a$  are the assets he carries on from the previous period,  $y$  is the current realization of his productivity process, and  $yp$  is the value of his parent's productivity at age 40 until the worker inherits and zero thereafter. This latter variable takes on two purposes. First, when it is positive, it is used to compute the probability distribution on bequests that the household expects from his parent. Second, it distinguishes the agents that have already inherited, for whom we set  $yp = 0$ , from those who have not, for whom  $yp$  is strictly positive. The agents inherit bequests only once in a lifetime, at a random date which depends on their parent's death. Since there is no market for annuities, part of the bequests the child receives are "involuntary bequests," linked to the fact that people's life span is uncertain and therefore they accumulate precautionary savings to offset the life-span risk. The optimal decision rules are functions for consumption,  $c(x)$ , and next period's asset holding,  $a'(x)$ , that solve the dynamic programming problem described below.

Let's consider the agent's recursive problem distinguishing four subperiods in his life to clarify the problem he faces in each phase of his life.

- (i) From age  $t = 1$  to age  $t = 3$ , (from 20 to 30 years of age) the agent works and will survive with certainty until next period. Moreover, he does not expect to receive a bequest because his parent is younger than 60 and will survive at least one more period for sure. Since the law of motion of  $yp$  is dictated by the death probability of the parent, for this subperiod  $yp' = yp$ .

$$V(t, a, y, yp) = \max_{c, a'} \left\{ u(c) + \beta E_t V(t+1, a', y', yp) \right\} \quad (1)$$

subject to:

$$c \leq \left[ 1 + r(1 - \tau_a) \right] a + (1 - \tau_l) \epsilon_t y \quad (2)$$

$$a' = \left[ 1 + r(1 - \tau_a) \right] a - c + (1 - \tau_l) \epsilon_t y \quad (3)$$

$r$  is the interest rate on assets, and the evolution of  $y'$  is described by the transition function  $Q_y$ .

- (ii) From  $t = 4$  to  $t = 8$ , i.e. from 35 to 55 years old the worker will survive for sure up to next period. However, his parent is at least 60 years old and faces a positive probability of dying any period; hence a bequest might be received at the beginning of next period.  $I_{yp>0}$  is the indicator function for  $yp > 0$ ; it is 1 if  $yp > 0$  and zero otherwise.

$$V(t, a, y, yp) = \max_{c, a'} \left\{ u(c) + \beta E_t V(t+1, a', y', yp') \right\} \quad (4)$$

subject to (2) and :

$$a' = \left[ 1 + r(1 - \tau_a) \right] a - c + (1 - \tau_l) \epsilon_t y + b' I_{yp>0} I_{yp'=0} \quad (5)$$

$$yp' = \begin{cases} yp & \text{with probability } \alpha_{t+5} \\ 0 & \text{with probability } (1 - \alpha_{t+5}) \end{cases} \quad (6)$$

where  $E_t$  is the conditional expectation based on the information available at time  $t$ .

The conditional distribution of  $b'$  is given by  $\mu_b(x; \cdot)$ .<sup>7</sup>  $\mu_b$  represents the bequest distribution a person expects if his parent dies; in equilibrium it will have to be consistent with the behavior of the parent. Since the evolution of the state variable  $yp$  is dictated by the death process of the parent,  $yp'$  jumps to zero with probability  $\alpha_{t+5}$  (5 periods is the difference in age between each parent and his children).<sup>8</sup> I assume the following processes to be independent: the survival/death of the decision maker; the survival/death of his parent; the size of the bequest received from the parent, conditional on the parent dying; and the future labor income, conditional on the current one.

- (iii)  $t_r - 1$ , i.e., 60 years old: the period before retirement. The individual faces a positive probability of dying and hence has a bequest motive to save. Define after tax bequests as  $b(a') = a' - \tau_b \cdot \max(0, a' - ex_b)$ .

$$V(t, a, y, yp) = \max_{c, a'} \left\{ u(c) + \alpha_t \beta E_t V(t+1, a') + (1 - \alpha_t) \phi(b(a')) \right\} \quad (7)$$

where

$$\phi(b) = \phi_1 \left( 1 + \frac{b}{\phi_2} \right)^{1-\sigma} \quad (8)$$

subject to (2), (5) and (6).

I choose the functional form for  $\phi(b)$  to make it roughly consistent with the child's utility from receiving the bequest. To derive it, assume that the child consumes a constant amount

<sup>7</sup>The probability distribution  $\mu_b$  depends on  $x$  only through  $t$  and  $yp$ , not through  $y$ .

<sup>8</sup>If  $yp = 0$ , eq.(7) implies  $yp' = 0$  for sure, which we want.

(say the average labor income, 1) in each period, and if he inherits, he consumes the bequest in  $\phi_2$  periods, in equal amounts. Under these assumptions, the additional utility he would derive from receiving a bequest  $b$  would be:

$$\frac{(1 + \frac{b}{\phi_2})^{1-\sigma}}{(1-\sigma)} + \beta \frac{(1 + \frac{b}{\phi_2})^{1-\sigma}}{(1-\sigma)} + \dots + \beta^{\phi_2-1} \frac{(1 + \frac{b}{\phi_2})^{1-\sigma}}{(1-\sigma)} - \frac{1}{(1-\sigma)} - \beta \frac{1}{(1-\sigma)} - \dots - \beta^{\phi_2-1} \frac{1}{(1-\sigma)}.$$

Collecting terms, dropping the constant and defining  $\phi_1$  appropriately we obtain  $\phi(b)$ .  $\phi_1$  can be thought of as a measure of how much the parent values the child's utility. I will discuss how I chose  $\phi_1$  and  $\phi_2$  in sections 5 and 6.

- (iv) From  $t_r$  to  $T$ , i.e. from 65 to 85, after retirement. In the model economy, the agent will not inherit after turning 65 years old because his parent is dead at that time. Moreover, after retirement we assume that people no longer work and just live off pensions and interest. This implies that we can drop two state variables from the retired people's value function,  $y$  and  $yp$ , and that the only uncertainty the retired agents face is the time of their death.

$$V(t, a) = \max_{c, a'} \left\{ u(c) + \alpha_t \beta V(t+1, a') + (1 - \alpha_t) \phi(b(a')) \right\} \quad (9)$$

subject to (8) and:

$$c \leq \left[ 1 + r(1 - \tau_a) \right] a + p \quad (10)$$

$$a' = \left[ 1 + r(1 - \tau_a) \right] a - c + p \quad (11)$$

$p$  is the pension payment from the government. The terminal period value function  $V(T+1, a)$  is set to equal  $\phi(b(a))$ .

## 4.5 Transition Function

From the policy rules, the bequest distribution and the exogenous Markov process for productivity, we can derive a transition function  $\tilde{M}(x; \cdot)$ .  $\tilde{M}(x; \cdot)$  is thus the probability distribution of  $x'$  (the state in the next period), conditional on  $x$ , for a person that behaves according to the policy rules  $c(x)$  and  $a(x)$ .<sup>9</sup> The measurable space over which  $\tilde{M}$  is defined is  $(\tilde{X}, \tilde{\mathcal{X}})$ , with:

$$\tilde{X} \equiv \{1, \dots, T\} \times \mathfrak{R}_+ \times Y \times (Y \cup \{0\}),$$

$$\tilde{\mathcal{X}} \equiv \mathcal{P}(\{1, \dots, T\}) \times \mathcal{B}(\mathfrak{R}_+) \times \mathcal{B}(Y) \times \mathcal{B}(Y \cup \{0\}),$$

<sup>9</sup>For simplicity of notation I keep the dependence of  $\tilde{M}$  on  $c, a, \mu_b$  implicit.

$$\tilde{X} \equiv X \cup \{D\}$$

$$\tilde{\mathcal{X}} \equiv \left\{ \tilde{\chi} : \tilde{\chi} = X \cup d, X \in \mathcal{X}, d \in \{\emptyset, \{D\}\} \right\}$$

where  $\mathcal{P}$  is the cardinal set of  $\{1, \dots, T\}$ , and  $D$  indicates that a person is dead. To characterize  $\tilde{M}$ , it is enough to display it for the sets

$$L(\bar{t}, \bar{a}, \bar{y}, \bar{y}p) \equiv \{(t', a', y', yp') \in X : t' \leq \bar{t} \wedge a' \leq \bar{a} \wedge y' \leq \bar{y} \wedge yp' \leq \bar{y}p\}$$

On such sets  $\tilde{M}$  is defined by  $\tilde{M}(x, L(\bar{t}, \bar{a}, \bar{y}, \bar{y}p)) =$

$$\begin{cases} \alpha_t I_{t+1 \leq \bar{t}} \left[ I_{a(x) \leq \bar{a}} (I_{yp=0} + I_{yp \leq \bar{y}p} \alpha_{t+5}) + \right. & \text{if } x \neq D \\ \left. \mu_b(x; [0, \bar{a} - a(x)])(1 - \alpha_{t+5}) I_{yp > 0} \right] Q_y(y, [0, \bar{y}] \cap Y) & \\ 0 & \text{if } x = D \end{cases}$$

where  $I$  is an indicator function, which equals 1 if the subscript property is true and zero otherwise.

To understand  $\tilde{M}$ , notice that  $\alpha_t$  is the probability of surviving into the next period. Conditional on survival, a person currently of age  $t$  will be of age  $t+1$  next period, hence the presence of  $I_{t+1 \leq \bar{t}}$ . If his parents are already dead, i.e.,  $yp = 0$ , he cannot receive bequests anymore, and his assets next period are  $a(x)$  for sure (as discussed above, this is always the relevant case for people 65 and older). If, instead, his parents are still alive, i.e.,  $yp > 0$ , they can survive into the next period with probability  $\alpha_{t+5}$ ; in that case, tomorrow's assets for the worker will be  $a(x)$  and  $yp' = yp$ . Alternatively, the parents may die, with probability  $1 - \alpha_{t+5}$ ; under this scenario, the person inherits next period,  $yp' = 0$ , and the probability that next period's assets are no more than  $\bar{a}$  is the probability of receiving a bequest between 0 and  $\bar{a} - a(x)$ .  $Q_y$  describes the evolution of income; note that the evolution of income, one's survival and the survival of the parent are independent of each other. Finally, death is an absorbing state.

Based on  $\tilde{M}$ , I can define an operator  $R_{\tilde{M}}$  that maps probability distributions on  $(\tilde{X}, \tilde{\mathcal{X}})$ :

$$(R_{\tilde{M}} \tilde{m})(\tilde{\chi}) \equiv \int \tilde{M}(x, \tilde{\chi}) \tilde{m}(dx), \forall \tilde{\chi} \in \tilde{\mathcal{X}}.$$

This operator describes the probability distribution of finding a person in state  $x'$  tomorrow, given the probability distribution of the state today. Such an operator has a unique fixed point, which is the probability distribution that attributes probability 1 to  $\{D\}$ : everybody dies eventually. However, in the economy as a whole, we are not interested in keeping track of dead people, so I will define a modified operator on measures on  $(X, \mathcal{X})$ . Furthermore, it is necessary to take into account that new people enter the economy in each period. The transition function corresponding to the modified operator  $R_M$  is thus:

$$M(x, L(\bar{t}, \bar{a}, \bar{y}, \bar{y}p)) = \frac{\tilde{M}(x, L(\bar{t}, \bar{a}, \bar{y}, \bar{y}p)) + n^5 I_{t=5} Q_{yh}(y, [0, \bar{y}] \cap Y) I_{y \leq \bar{y}p}}{n}$$

$M$  modifies  $\tilde{M}$  in two ways. First, it accounts for population growth; when population grows at rate  $n$ , a group that is 1% (say) of the population becomes  $1/n\%$  in the subsequent period. Second, it accounts for births, which explains the second term in the numerator. If a person is 40 years old ( $t = 5$ ), his children (there are  $n^5$  of them), will enter the economy next period. All of those children have age  $t = 1$  and zero assets.<sup>10</sup> Their stochastic productivity is inherited from their parent's at 40, according to the transition function  $Q_{yh}$ ;  $y$  (which is part of  $x$ ) is their parent's productivity at 40.

The operator  $R_M$  is thus defined as

$$(R_M m)(\chi) \equiv \int M(x, \chi) m(dx) \quad \forall \chi \in \mathcal{X}$$

$R_M$  maps measures on  $(X, \mathcal{X})$  into measures on  $(X, \mathcal{X})$ , but it does not necessarily map probability measures into probability measures. Unless the population has reached a demographic steady state, the total measure of people alive may grow at rate faster or slower than  $n$ , which implies that  $(R_M m)(x) \neq 1$  even if  $m(x) = 1$ .

## 4.6 Definition of Stationary Equilibrium

A stationary equilibrium is given by:

$$\left\{ \begin{array}{l} \text{an interest rate } r, \\ \text{allocations } c(x), a(x), \\ \text{government tax rates and transfers, } (\tau_a, \tau_l, \tau_b, ex_b, p), \\ \text{a family of probability distributions for bequests } \mu_b(x; \cdot), \\ \text{and a constant distribution of people over the state variables } x: m^*(x) \end{array} \right.$$

such that, given the interest rate and the government policy:

- (i) the functions  $c$  and  $a$  solve the maximization problem described above, taking as given the interest rate, the government tax rates and transfers, and the bequest distribution he expects to receive from his parent, given as a function of his characteristics  $x$ ;
- (ii) given a per capita exogenous government expenditure  $g$  and the structure of the social security system, the government policy is such that the government budget constraint balances at every period:

$$g = \int \left[ \tau_a r a + \tau_l \epsilon_t y I_{t < t_r} - p I_{t \geq t_r} + \tau_b (1 - \alpha_{t-1}) \cdot \max(0, a' - ex_b) \right] dm^*(x);$$

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<sup>10</sup>Since  $\bar{t} \geq 1$  and  $\bar{a} \geq 0$ , I do not need to include  $I_{1 \leq \bar{t}}$  and  $I_{0 \leq \bar{t}}$ .

- (iii)  $m^*$  is an invariant distribution for the economy, i.e. it is a fixed point of the operator  $R_M$  defined in subsection 1.4.5:

$$R_M m^* = m^*.$$

I normalize  $m^*$  so that  $m^*(X) = 1$ , which implies that  $m^*(\chi)$  is the fraction of people alive that are in a state  $\chi \in \mathcal{X}$ .

- (iv) the family of expected bequest distributions  $\mu_b(x; \cdot)$  is consistent with the bequests that are actually left by the parents. Let's now characterize this statement using formulas. Define first the marginal distribution of age and income in the population, which is a probability distribution on

$$(\{1, \dots, T\} \times Y, \mathcal{P}(\{1, \dots, T\}) \times \mathcal{B}(Y)) :$$

$$m_{t,y}^*(\chi_{t,y}) \equiv m^*(\{x \in X : (t, y) \in \chi_{t,y}\}) \quad \forall \chi_{t,y} \in \mathcal{P}(\{1, \dots, T\}) \times \mathcal{B}(Y)$$

Define  $m^*(\cdot|t, y)$  as the conditional distribution of  $x$  given  $t$  and  $y$ . For any given  $(t, y)$ ,  $m^*(\cdot|t, y)$  is a probability distribution<sup>11</sup> on  $(X, \mathcal{X})$ . For any set  $\chi \in \mathcal{X}$ ,  $m^*(\chi|t, y)$  is measurable with respect to  $\mathcal{P}(\{1, \dots, T\}) \times \mathcal{B}(Y)$  and is such that

$$\int_{X_{t,y}} m^*(\chi|t, y) m_{t,y}^*(dt, dy) = m^*(\chi) \quad \forall \chi \in \mathcal{X} \quad \forall \chi_{t,y} \in \mathcal{P}(\{1, \dots, T\}) \times \mathcal{B}(Y)$$

The child observes his parent's income at 40. The conditional distribution of the characteristics of the parent at age 40, given an income level  $y_p$ , is<sup>12</sup>  $m^*(\cdot|t=5, y=y_p)$ . I want the characteristics of the parent at later ages, conditional on his income as of age 40 being  $y_p$  and conditional on not having died. Denote by  $l(\cdot|t, y_p)$  these conditional distributions. They can be obtained recursively as follows:  $l(\chi|5, y_p) = m^*(\chi|5, y_p)$  and

$$l(\chi|t+1, y_p) \equiv \frac{\int_x \tilde{M}(x, \chi) l(dx|t, y_p)}{\alpha_t}.$$

The conditional distributions  $l(\cdot|t, y_p)$  imply conditional distributions of assets  $l_a(\cdot|t, y_p)$  on  $(\mathfrak{R}_+, \mathcal{B}(\mathfrak{R}_+))$  which are given by

$$l_a(\chi_a|t, y_p) \equiv l(\{x \in X : a \in \chi_a\}|t, y_p).$$

Since the probability of death is independent of income and assets, the distribution of assets that are bequeathed by dying parents is the same as the distribution of assets of surviving

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<sup>11</sup>  $\{m^*(X|t, y)\}$  is uniquely defined up to sets of  $m_{t,y}^*$ -measure zero.

<sup>12</sup> I use the letter  $y_p$  to distinguish both from  $y$  and  $yp$ :  $y_p$  plays the role of income for the parent (state variable  $y$ ) and the parent's income for the child (state variable  $yp$ ).

parents. We thus have

$$\begin{aligned} & \mu_b((t, a, y, yp); \chi_a) = \\ & l \left( a \in \mathfrak{R}_+ : \left[ n^5 a \left( \prod_{s=1}^{t-1} \alpha_s \right)^{-1} I_{a \leq ex_b} + \frac{n^5 a \left( \prod_{s=1}^{t-1} \alpha_s \right)^{-1} I_{a > ex_b}}{(1 - \tau_b)} \right] \in \chi_a | t + 5, yp \right) \quad (12) \\ & \forall \chi_a \in \mathcal{B}(\mathfrak{R}_+) \quad \forall a \in \mathfrak{R}_+ \quad \forall y, yp \in Y, \quad t = 1, \dots, T - 5 \end{aligned}$$

In equation (12), I take into account the assumptions made before about the structure of bequest taxation and the assumption that the bequest is distributed evenly among surviving children.

I need now to define  $\mu_b$  when  $t = T - 4$ , which is the last age a person can inherit. Since there are no survivors at age  $T + 1$ , I cannot use the survivor's assets to compute the assets that are bequeathed. Instead, let's use the policy function  $a(x)$  to define:

$$l_a(\chi_a | T + 1, yp) \equiv \int_X I_{a(x) \in \chi_a} l(dx | T, yp) \quad \forall \chi_a \in \mathcal{B}(\mathfrak{R}_+).$$

With this definition, equation (12) can be extended to  $t = T - 4$  as well. Equation (12) is thus the formal requirement of consistency on  $\mu_b$ .

## 4.7 The Algorithm

The following steps are used to solve the model:

- (i) Solve the household's value functions.

Assume a functional form for  $\phi(b)$  (the utility of leaving a bequest) and start from the last period,  $T$ ; next period the agent will be dead for sure, hence he will derive utility only from the bequests he will leave. Solve backward for the value function at  $T - 1$ . Continue analogously, taking as given the value function for next period until the first period is reached.

The difficult part of solving this model is linked to the curse of dimensionality; there are four state variables. To manage this problem, keep track of the value function on a coarser grid (90-150 points) for capital (the grid is not uniform and has more points concentrated at low levels of capital). The maximization problem is solved for a household that starts with an initial level of assets on this grid. However, future investment is allowed to lie on a finer grid; this requires the household to evaluate the value function at points that do not lie on the initial grid, which is accomplished by interpolation. The resulting investment policy is thus defined on a finer capital grid.

Keep track of the transition function and invariant distribution for this economy on the coarser grid. To do so, take the asset level given by the investment policy function, find the two closest asset levels that include it on the coarser capital grid, and attribute to each



of these points a weight according to their relative distance from the original capital level on the investment policy grid.

Choose the number of grid points for capital so that the results are neither sensitive to the number of grid points nor to the linear interpolation procedure.

- (ii) Taking as given the Markov processes for the productivity and productivity inheritance and the agents' policy functions, compute the transition matrix and the associated invariant distribution. Since the agents' policy functions are defined on a finer grid, we need to map them to the coarser grid used for the value function, transition matrix and invariant distribution. To do so, take the agents' optimal decision, given his state variables, and find the adjacent values that include his optimal choice in the coarser grid. Then attribute to these two points a weight given by the relative distance between each of the two points and the agent's optimal choice.
- (iii) Iterate on the tax rate on labor income until the government budget constraint is balanced.
- (iv) Iterate on bequests until the equilibrium condition described by equation (12) is met.

## 5 The Experiments

To understand the quantitative importance of these intergenerational links, I construct several simulations that I run both for the U.S. and the Swedish economies.

I start with an experiment in which the model is stripped of all intergenerational links: an overlapping-generations model with lifespan and earnings uncertainty. The involuntary bequests left by the people who die prematurely are seized by the government and equally redistributed to all people alive.<sup>13</sup> The idea is to see how much wealth inequality can be generated by the life-cycle structure when only lifespan and earnings uncertainty are activated.

The second experiment modifies the first one: the involuntary bequests left are distributed to the children of the deceased, rather than equally to everybody alive. This experiment is meant to assess whether an unequal distribution of estates is quantitatively important when all bequests are involuntary.

The third experiment introduces the bequest motive: parents care about their children and leave them bequests. This allows us to see whether the fact that some of the bequests left are voluntary matters.

The fourth exercise activates both the bequest motive and parent's productivity inheritance in order to evaluate the importance of the family background.

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<sup>13</sup>This exercise uses Huggett's setup but adapts it to the length of the periods and the productivity process that I use throughout this paper in order to make the results comparable to the other simulations I run. I cannot use the same time period and income process as Huggett, since the simulations with altruism require a higher number of state variables and the model would require huge computing resources to solve.

Fixed		
Parameter	Value	Source(s)
$\alpha_t$	*	Bell, Wade and Goss [5]
$\epsilon_t$	*	Hansen [14]
$\sigma$	1.5	see text
$n$	1.2% yearly	Econ. Report of the President [30]
$g$	19% of GDP	Econ. Report of the President [30]
$\tau_a$	20%	Kotlikoff et al. [22]
$r$	6%	see text
$p$	40% average income	Kotlikoff et al. [22]
$Q_y$	+	Huggett [18], Lillard et al. [27]
$Q_i$	+	Zimmerman [38]
Calibrated		
Parameter	Value	Chosen to Match
$\tau_b$	10%	see text
$ex_b$	40 years of average earnings	see text
$\beta$	.95–.97	capital-output ratio
$\phi_1$	-9.5	intergenerational transfers share
$\phi_2$	8	“altruistic feedback”, see text

Table 5: Parameters for the U.S. economy and their sources.

\* refers to a vector

+ see description in the text

## 6 Numerical Simulations for the U.S. Economy

Most of the parameters of the model are taken from other sources, while few of them are chosen to match some aspects of the data. I summarize these choices in Table 5.

For people older than 60,  $\alpha_t$  is the vector of conditional survival probabilities. The series I use corresponds to the conditional survival probabilities of the cohort born in 1965. People 60 years old and younger survive for sure into the next period.

$\epsilon_t$  is the age-efficiency profile vector.

I take the risk aversion parameter,  $\sigma$  to be 1.5, this value falls in the range (1-3) widely used in the literature.

The rate of population growth,  $n$ , is set to equal the average population growth from 1950 to 1997, 1.2%

$g$  is government expenditure excluding transfers (about 19% of GDP).

$\tau_a$  is the capital income tax, 20%.

$r$  is the interest rate on capital, net of depreciation and gross of taxes. In models without aggregate uncertainty it is commonly chosen to be between the risk free rate and the rate of return on risky assets. I assume an interest rate of 6% so that the capital share of output is

about .36.

Pensions ( $p$ ) are such that the social security replacement rate is 40% and the implied government transfers to GDP ratio in the model is consistent with the one reported in the Economic Report of the President [30].

The logarithm of the productivity process is assumed to be an AR(1). I choose its persistence to be consistent with the one used by Huggett [18] adapted to a five year period model and its variance to match a Gini coefficient for earnings of workers of about .43 (Lillard and Willis [27]). The implied autocorrelation parameter is .83 and its variance .41.

The logarithm of the productivity inheritance process (for  $yp$ ) is also assumed to be an AR(1). I take its persistence from Zimmerman's [38] estimates and its variance so that the standard deviation of the logarithm of earnings is in the ballpark provided by Zimmerman [38]. The resulting autocorrelation parameter is .67 and its variance is .42.

I convert both the productivity and the productivity inheritance processes to a discrete Markov chain according to Tauchen and Hussey [36]. I use three values for the income process. The resulting income distribution is reported and compared with the data in appendix A.

The remaining parameters are chosen to match features of the U.S. economy as follows.

$\tau_b$  is the tax rate on estates that exceed the exemption level  $ex_b$ . According to U.S. law, each individual can make an unlimited number of tax-free gifts of \$10,000 or less per year, per recipient; therefore, a married couple can transfer \$20,000 per year to each child, or other beneficiary. For larger gifts and estates, there is a "unified credit", i.e., a credit received by the estate of each decedent, against lifetime estate and gift taxes. For the period between 1987 and 1997, each taxpayer received a tax credit that eliminated estate tax liabilities on estates valued less than \$600,000. The marginal tax rate applicable to estates and lifetime gifts above that threshold is progressive, starting from 37% (Poterba [31]). However, the revenue from estate taxes is very low (in the order of .2% of GDP in 1985-97) as there are many effective ways to avoid such taxes (see for example Aaron and Munnell [1]); moreover, only about 1.5% of decedents pay estate taxes. Therefore, in the model I set  $ex_b$  to be 40 times the median income and  $\tau_b$  to be 10% to match the observed ratio of estate tax revenues to GDP and the proportion of estates that pay estate taxes. I discuss the sensitivity of the model to the choice of these two parameters when describing the results.

I use the discount factor,  $\beta$ , to match a capital to GDP ratio of 3. In the calibrations in which the bequest motive is activated, I use  $\phi_1$  to get a reasonable share of the bequests to aggregate capital and  $\phi_2$  to make the utility from leaving bequests,  $\phi(b)$ , roughly consistent with a "truly altruistic model" in the sense that it is reasonably close to the utility of the child from receiving the bequest. Figure 22 compares the function  $\phi$  with the true value of receiving the bequest for a 65 year old at the 10% ("poor"), 50% ("median") and 90% ("rich") quantiles of the wealth distribution.<sup>14</sup> We see that  $\phi$  is a reasonable approximation of the value a truly altruistic parent would derive from the bequest he leaves. However, for the parent of a poor child, the marginal utility of leaving a bequest is higher in the fully altruistic model, and the benefit is more concave. The opposite is true for the parent of a rich child.

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<sup>14</sup>The functions are normalized by adding a constant to fit on the graph.

Table 6 summarizes the results.<sup>15</sup>

Capital output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top						Percent with negative or zero wealth
			1%	5%	20%	40%	60%	80%	
U.S. data									
3.0	.63	.72	28	49	75	89	96	99	5.8-15.0
No intergenerational links, equal bequests to all									
3.0	N/A	.64	6	23	64	90	98	100	17
No intergenerational links, unequal bequests to children									
3.0	.37	.65	6	24	65	90	98	100	17
One link: parent's bequest motive									
3.0	.57	.71	12	34	72	93	99	100	19
Both links: parent's bequest motive and productivity inheritance									
3.0	.61	.73	15	38	75	94	99	100	19

Table 6: Results for the U.S. calibration.

## 6.1 The Experiment with no Intergenerational Links and Equal Bequests to All

The results of this experiment show that an overlapping-generations model with no dynastic links and equal distribution of bequests has serious difficulties in generating enough skewness to match the distribution of the U.S. wealth. For the parameter values in Table 5, I obtain a Gini coefficient that is below what we observe in the data. Moreover, the concentration is mainly achieved by having a lower tail which is too fat and an upper tail which is far too thin.

As discussed in the introduction, overlapping-generations models tend to have a large fraction of people against the borrowing constraint. People are born without savings that could be used to absorb negative income or productivity shocks. As a consequence, all the young consumers that get a bad productivity shock hit the borrowing constraint. During their working age, the households gradually accumulate assets both as life-cycle savings for their old age and as a form of precautionary savings. As a consequence, the fraction of people with zero wealth gradually declines until retirement.

At the other end of the distribution, the absence of intergenerational links implies that it is very hard to account for large estates, as a lifetime is too short a period for most households to accumulate such large fortunes. With the current parameterization the top 1% of the population holds just about 5.5% of total wealth. With a richer income process, the top quantiles are

<sup>15</sup>For all experiments I exclude 20 year old people from the computations on the wealth distribution. I do so because in this paper I assume that people start off with zero wealth and I therefore do not propose a theory of the distribution of wealth for them. To explain the data for 20 year old people, a theory of inter-vivos transfers would, in my opinion, be required.

somewhat higher, but it is still true that few people have sufficiently high income to accumulate such large estates over a lifetime. With 7 productivity states (instead of 3) the top 1% of people hold 8% of the aggregate wealth (which is still lower than obtained in the model with links and only three income states). My main interest, however, is to show that, with the same income process, experiments with intergenerational links fare much better than an environment in which such links are not present.

In the model the richest 2% of people hold about 20 times the average annual labor income in assets or about 7 times the highest annual labor income. In the U.S. data, the richest 2% of people held about 35 years of average labor income in assets in 1994.<sup>16</sup>

The transfer-wealth ratio is not defined for this experiment, as bequests are collected by the government and redistributed lump sum to all the population, independently of family links.

The overlapping-generations model with no intergenerational links also fails to recover the age-asset profile observed in the data. All households in the model economy (figure 1) run down their assets during retirement until they are left with zero wealth at the time we assume they die for sure. This implies a much larger dissaving than we observe in the data (figure 45), especially for richer households. This also suggests that a substantial fraction of wealth that is accumulated in this economy is linked to the uncertainty over the life span. In such an economy a market for annuities would thrive and would reduce savings significantly; by purchasing annuities the households would avoid leaving large involuntary bequests when they die in their early retirement years.

## 6.2 The Experiment with No Intergenerational Links and Unequal Bequests to Heirs

In this experiment the involuntary bequests left by the deceased are inherited by their own children, rather than being redistributed by the government to all people alive. As we can see from table 6, there is little change in the distribution of wealth. The intuition is simple: some people in the economy inherit some wealth, some other people do not, but nobody cares about leaving bequests. Even high income households do not plan to share their fortune with their children but do so only if they die early in their retirement years. As a consequence, the transfer-wealth ratio is low, no persistence in wealth across families is generated and no large fortunes are accumulated in this economy.

Figure 5 depicts the probability of the parent dying for each age of the offspring, conditional on the parent dying before the offspring. In this model economy, the individuals do not die before age 60; therefore the probability of the parent dying before the offspring reaches age 35 is set to zero. Compare the saving behavior of the average agent who expects to receive some bequest, with his behavior in the hypothetical case in which he does not expect to receive a bequest (figure 6). The fact that parent's assets are not observable influences the saving behavior of people that have not yet inherited.<sup>17</sup> Since in this economy children do not become orphans

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<sup>16</sup>Sources: Economic Report of the President [30] and Hurst, Luoh and Stafford [19].

<sup>17</sup>All people have a positive probability of getting a bequest, not only those whose parent will actually leave some.

before age 40, and everybody attributes positive probability to receiving some bequest until their parent die, the comparison starts for agents of age 40. For them, we can compare the behavior of people whose parent died, and did not leave them any asset (these children do not expect an inheritance anymore), with the behavior of people that still expect to receive something. The top line refers to the age-saving profile for the average agent in the case in which he does not expect to inherit. We see that the expectation of inheriting, conditional on all other variables being the same, decreases the saving rate of the agent. However these saving rates converge over time because the parent has no bequest motive and by age 90 runs down all of his assets (figure 4). Therefore the average size of the expected bequest also goes to zero by that time.

Since the probability of dying is assumed to be independent of other individual characteristics, the distribution of bequests is simply the distribution of assets among the population of the parent's generation at age 65, rescaled because of population growth (each person has more than one child). Figure 6 shows the strictly positive range of the bequest distribution that an agent faces at 40 years of age should the parent die at that moment, conditional on the observed productivity of his parent at age 40. I do not plot the probability of receiving a bequest of zero because this would make the graph very difficult to read. This probability is about 27, 8 and 0% for a 40-year old whose parent had the lowest, middle or highest productivity level at age 40, respectively. Since age 65 is the peak of wealth accumulation, the children of those who die at age 65 (the 40 year olds) are the ones that receive the largest bequest. At this age, the average bequests are, respectively, 2.9, 6.7, and 15.3 years of average labor income. After the parents retire they run down all of their assets by age 90, so the expected bequest declines, and the people whose parents live up to the final age of the model economy do not receive any bequest.

### 6.3 The Experiment with only one Intergenerational Link: Bequest Motive

The bequest motive leads to a large increase in the concentration of wealth in the economy and explains the emergence of large estates that are accumulated by more than one generation of savers and are transmitted because of altruism. The Gini coefficient increases from .64 to .71 and the fraction of the total wealth by the people in the upper tail of the distribution increases significantly. The top 1% of the population holds 12% of total wealth, and the top 5% holds 34% of total wealth. The households at the top 2% quantile hold about 25 years of average labor income<sup>18</sup> in assets (compared with 20 years in the model with no intergenerational links), or about 8 years of the highest income level in the model.

Figure 13 compares the distribution of wealth by age conditional on having or not having received a bequest.<sup>19</sup> The model predicts that the upper tail of the wealth distribution will be mainly made of households that have already received a bequest.

The age-assets profiles for various quantiles of the wealth distribution are displayed in figure 10. From this figure we notice a substantial difference in the main motives that lead the household to save.

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<sup>18</sup>The average labor income for an agent in this economy is one over a five years period, therefore it is .2 yearly.

<sup>19</sup>The bequest a person has received may be zero if his parent dies with no assets.

The median household mainly saves for retirement; the peak in its wealth holdings occurs at age 65 and is about 6.5 times the average annual labor income in the population. If he reaches the age of 85, the median agent consumes all of his assets (before dying at 90) and does not leave any bequest. The median consumer mostly leaves unintended bequests. This becomes clearer when comparing the age-assets profiles for the bottom 10, 25 and 50% with the model with no bequest motive (figure 1): the profiles are very close.<sup>20</sup>

Those who are wealthy, either as a consequence of large inheritances or of a successful working life, plan on sharing their luck with their offspring. As we see from figure 10, at the top of the wealth distribution a lot of the accumulation is done especially in order to leave bequests, and large bequests are left even when the parents die in advanced age. Comparing these top quantiles with the ones in the model with no altruistic links, we see how the introduction of a bequest motive produces an age-wealth profile more consistent with the U.S. data (figure 45), especially in the second part of the agent's lifetime. In this setup, the absence of an annuity market is a less severe restriction on the behavior of the richest households as most of their savings are primarily accumulated to be left to the next generation.

As in the previous simulation, figure 12 shows that the expectation of receiving a bequest reduces the saving rate of the average agent. Here, however, this effect does not decrease over time. In fact the richest parents do not run down all of their assets by age 90 because of the bequest motive and, over time, the reduction in the size of the expected bequest is balanced by the increased probability of the parent dying.

The model gives a plausible value of transfer wealth defined as in Kotlikoff and Summers [23] and Gale and Scholz [12]. 57% of total wealth is transfer wealth, i.e., it is indirectly linked to bequests received in the past.

In figure 15 we can see the strictly positive range of the bequest distribution for a 40 year old person, conditional on his observed parent's productivity level, should his parent die during that period. The probabilities of receiving zero bequests are respectively 27, 11 and 0%, for individuals with parents of low, middle and high productivity level. The average bequests expected are respectively 4.7, 7.6 and 15.5 years of average labor income. Even in presence of a bequest motive, the parents run down their assets after retirement, so the expected bequest declines. The fraction of people whose parent lives up to the final age of the model economy and who do not receive a positive bequest are 93, 87 and 53%, respectively. The average bequest that they expect at that point in life is about 1.4, 1.9 and 5 years of average labor income respectively.

## 6.4 The Experiment with both Intergenerational Links

Here I explore how the results of the previous section change when parent and child are linked not only by the bequest the parent intends to leave to his child, but also through transmission

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<sup>20</sup>The model without bequest motive generates somewhat steeper profiles until retirement. This happens because to match the capital/output ratio, the model with no intergenerational links requires a higher discount factor ( $\beta = .97$  compared with  $\beta = .96$ ). The steeper consumption profile early in their lives counterbalances the steep decumulation of assets later in life, when the increased probability of dying implies very high effective discount factors. The bequest motive keeps the effective discount factor high in old age. As a consequence, we can allow for a lower  $\beta$  implying a flatter consumption profile at younger ages.

of productivity from each parent to his children.

The introduction of an additional link increases the Gini coefficient further to .73. This happens because success in the workforce is now correlated across generations: more productive parents accumulate larger estates and leave their bequests to their children who are in turn more successful than average in the workforce.

The introduction of a link in the productivity (or “human capital”) of different generations within a family tends to have two opposing effects on the accumulation of assets at the tails of the wealth distribution. First, consider the individuals that are at the lowest levels of wealth. These people tend to be the least productive in the workforce. In particular, the people who are least productive at 20 (who tend to be less productive than the others also at later ages) are more likely to have less productive and hence poorer parents. As a result, people with low productivity in their young age will, on average, receive smaller bequests, which will contribute to lowering their profile of asset accumulation. On the other hand, the anticipation of smaller bequests will lead the same people to save more to compensate for the smaller transfer wealth. The two effects will be exactly reversed at the upper end of the distribution of wealth.

The direct effect on transfer wealth dominates in our results. At the upper tail, the top 1% of the population hold 15% of total wealth, up from 12% in the previous example, and the top 5% hold 38% of total wealth, up from 34%. The agents at the top 2% of the population hold 29 times the average yearly labor income in wealth and 9 times the maximum level of labor income in this model, compared respectively with 25 and 8 times in the model with bequests only.

In figure 21 we can see the strictly positive range of the bequest distribution for a 40 year old person, conditional on his observed parent’s productivity level, should his parent die during that period. The probabilities of receiving zero bequests are respectively 29, 11 and 0%, for individuals with parents of low, middle and high productivity level. The average bequests expected are respectively 4.8, 8.1 and 16.2 years of average labor income. Comparing these numbers with the ones for the experiment with bequest motive only, we can see that introducing productivity inheritance has small effects on the average expected bequests.

The introduction of the “human capital” link in the form I consider here leads overall to modest changes in the results of the previous section. The reason for this result might stem from the weakness of the link introduced. In the current model, children inherit from their parent only their initial productivity level (they do so with probability smaller than one, according to the Markov process  $Q_{yh}$ ), and then productivity evolves independently and stochastically over time for all agents. This implies that children of poorer households tend to enter in the labor force at the lowest levels of the income process, but expect an improvement later in life and hence tend not to save. To better evaluate the productivity link, I also run an experiment in which the children’s initial productivity level is their parent’s one at 40 (inheritance with probability one). As a result, the share of wealth held by the top 1% of the population increases by a couple of percentage points, and the Gini coefficient for wealth increases to .75. Ideally, one would like the agents to be born with different income processes, to recover the fact that more-educated people have a permanent advantage over less-educated ones, and that the age-efficiency profile tends to be flatter for less-educated households, leaving them again to save more in earlier periods. Unfortunately, these considerations would require the introduction of a further state variable in



Fixed		
Parameter	Value	Source(s)
$\alpha_t$	*	Statistical Yearbook of Sweden [34]
$\epsilon_t$	*	same as U.S.
$\beta$	.95–.97	same as U.S.
$\sigma$	1.5	same as U.S.
$\phi_1$	-9.5	same as U.S.
$n$	.8% yearly	OECD Economic Surveys, Sweden [29]
$g$	25% of GDP	OECD Economic Surveys, Sweden [29]
$\tau_a$	30%	OECD Economic Surveys, Sweden [29]
$r$	6.86%	see text
$p$	50% average income	OECD Economic Surveys, Sweden [29]
$Q_y$	+	Huggett [18], Lillard et al. [27]
$Q_i$	+	Zimmerman [38]
Calibrated		
Parameter	Value	Chosen to Match
$\tau_b$	15%	see text
$ex_b$	10 years average earnings	see text
$\phi_2$	3.6	“altruistic feedback”, see text

Table 7: Parameters for the Swedish economy and their sources.

\* refers to a vector

+ see description in the text

our problem, making computations even more involved.

## 7 Numerical Simulations for the Swedish Economy

As in the calibration of the U.S. economy, most of the parameters for the Swedish economy are taken from other sources, and a few are chosen to match some aspects of the data. I summarize the parameter choices in Table 7.

The Statistical Yearbook of Sweden [34] provides the mortality probabilities for people at different ages in 1991-1995. In the calibration of the U.S. economy I use the mortality probabilities of people born in 1965 (which are for the most part projected, since these people are still young). Since life expectancy is increasing, the Swedish data underestimate the life expectancy at the various ages with respect to the one faced by people born in 1965 in Sweden. To correct for this problem I use the U.S. data to compute the relative increase in life expectancy for the relevant period. I then correct the Swedish data assuming that the increase in life expectancy is the same in the two countries. However, as a check, I also use the U.S. conditional probabilities in the simulation of the Swedish economy. It turns out that this has a negligible impact on the results even if the life expectancy of Swedish people is about three years longer than that of U.S.

people.

I take the age-efficiency profile  $\epsilon_t$  and the preference parameters  $\beta$ ,  $\sigma$  and  $\phi_1$  to be the same as those for the U.S.

The rate of population growth,  $n$ , is set to equal the average population growth from 1950 to 1997, .8%

The interest rate on capital, net of depreciation and gross of taxes,  $r$ , is taken to be 6.86% so that the interest rate net of taxes in the U.S. and Sweden coincide.

Pensions ( $p$ ) are such that the social security replacement rate is 50% and the implied government transfers to GDP ratio in the model is consistent with the one (net of taxes) reported in the OECD Economic Surveys, Sweden [29].

The persistence of the income and productivity inheritance processes are taken to be the same in Sweden and the U.S. Björklund and Jäntti [6] estimate the degree of intergenerational income mobility in Sweden and do not reject the hypothesis that it is the same as in the U.S. I take the ratio of the variances of the two processes to be the same one adopted for the calibration of the U.S. economy and vary their levels in order to match the Gini coefficient for the income process, which in Sweden is somewhat lower than in the U.S.

$\tau_b$  is the tax rate on estates that exceed the exemption level  $ex_b$ . I take the effective tax rate to be higher than the one for the U.S., 15%, and the exemption level to be lower, 10 years of average labor earnings. In Sweden taxes are paid on inheritances, rather than on estates, and the revenue from inheritance and gift taxes is approximately .1% of GDP. The statutory tax rate for children’s inheritance is higher than in the U.S. (for the first bracket it is about 50%) and the exemption level is much lower (in the order of \$5,000), but there are legal ways, for example bequeathing an apartment or a large firm, of obtaining a much larger exemption level. It is therefore more difficult than in the U.S. to define the statutory exemption level. The combined choice of  $\tau_b$  and  $ex_b$  matches the revenues from bequests and gift taxes.

As in the calibration of the U.S. economy, I choose  $\phi_2$  to make the utility from leaving bequests  $\phi(b)$  roughly consistent with a “truly altruistic” model, in the sense that it is reasonably close to the utility of the child receiving the bequest. The value functions of the Swedish model economy turn out to be more concave than the ones in the U.S. one; therefore the value of  $\phi_2$  that I adopt in this simulation is different. As a sensitivity check, I compare the results of the model with both intergenerational linkages when adopting the same  $\phi_2$  used in the U.S. simulation, instead. As for the level of altruism as measured by  $\phi_1$ , it turns out that over the relevant range for assets, the levels of the U.S. and Swedish value function of the bequest receiver are very close. In this sense the assumed intensity of the bequest motive is the same in both economies.

The results of the various experiments calibrated to the Swedish economy are summarized in Table 8 and discussed below.

## 7.1 The Experiments with no Intergenerational Links

Compared to the U.S. calibration, people in the Swedish model economy face less earnings uncertainty (to match a lower Gini coefficient for earnings, we need to reduce the variance of the income process) and a higher social security replacement rate. The first element tends to reduce precautionary saving and the second one to reduce life-cycle saving. The model predicts

Capital output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top						Percent with negative or zero wealth
			1%	5%	20%	40%	60%	80%	
Swedish data									
2.0	> .51	.73	14	32	64	88	99	100	30
No intergenerational links, equal bequests to all									
2.1	N/A	.64	5	23	64	89	98	100	24
No intergenerational links, unequal bequests to children									
1.9	.38	.67	6	25	67	91	99	100	26
One link: bequest motive									
2.0	.76	.71	8	29	73	95	100	100	30
Both links: bequest motive and productivity inheritance									
2.0	.77	.73	9	31	75	95	100	100	30

Table 8: Results for the Swedish calibration.

a lower wealth-to-GDP ratio than in the U.S. The ratio predicted by the model is very close to the capital-output ratio in Swedish data.

In the first experiment, the only intergenerational transfers stem from involuntary bequests; since there is life-span uncertainty and there are no annuity markets, people accumulate assets to self-insure against the risk of living for a long time. When they die earlier, they leave their assets behind. These bequests are distributed equally to all people in the economy.

Looking at the distribution of wealth, we can see that even in an economy in which there is less wealth and earnings inequality and the government redistributes more, the basic version of the model generates an upper tail of the wealth distribution which is too thin; the top 1% of people hold only 5% of total wealth, compared with 14% in the data. Unlike the results for the U.S. model economy, the model for Sweden does not generate too many people at zero wealth. In fact in the data about 30% of the population is in this situation while the model generates 24%.

When I assume that involuntary bequests are left to the children of the deceased (third row in table 8), the distribution of wealth is almost the same as in the case in which bequests are evenly distributed to all people alive. This is analogous to what I found for the U.S.

## 7.2 The Experiment with only one Intergenerational Link: Bequest motive

As in the U.S simulations, the introduction of the bequest motive helps in generating a more skewed wealth distribution by increasing the share of total wealth held by the rich. The forces discussed for the U.S. model economy that generate this result are also at work for the Swedish model economy.

The share of wealth held by the top 1% of people rises from 5% to 8%, and the share held by the top 5% increases from 22% to 29%. Moreover, we can see from figures 35 and 36 that the

wealth quantiles of the people who do receive a bequest are significantly higher than those who do not get a positive transfer of wealth from their parent.

Figure 37 shows the strictly positive range of the bequest distribution for a 40 year old person, should his parent die today, conditional on his parent's productivity at 40 years of age. Conditional on the parent's productivity level from the lowest to the highest, the probability of receiving a zero bequest is 29, 14 and 0% and the average bequests are 2.9, 4.4, and 8.5 years of average labor earnings. Compared to the U.S. simulation, therefore, the number of people expecting to receive no intergenerational transfer is slightly higher, and the average bequest size, for all parent's levels of ability, is lower.

### 7.3 The Experiment with both Intergenerational Links

The introduction of the second linkage, the intergenerational transmission of productivity, helps further in matching the top 20% of the wealth distribution: the share held by the top 1, 5, 10 and 20% increase to 9, 31, 51 and 75%. However, as discussed previously, this linkage is not very strong and hence does not change the results dramatically.

Figure 43 shows the strictly positive range of the bequest distribution for 40 year old agents, should their parent die this period.

For people whose parent was at the lowest productivity level at age 40, the average bequest is 2.6 years of average labor earnings; for these agents the probability of receiving no intergenerational transfer is 34%. For the individuals whose parent at 40 was at the middle productivity level, the average bequest is 4.4 and their probability of receiving zero bequests is 15%. For those whose parent was at the highest productivity level at age 40, these numbers are respectively 8.7 and 0%.

As mentioned in the calibration, I use different values for  $\phi_2$  in the U.S. and Swedish model economies. As a sensitivity check I report the results for the calibration of the Swedish economy using the  $\phi_2$  adopted in the U.S. simulations, all other parameters staying the same. In this case, the capital-output ratio is 1.65, the transfer-wealth ratio .46, and the fraction of people at zero wealth 31%. The top 1, 5, 10, 20, 40, 80% hold respectively 8, 28, 46, 71, 94 and 99%. However, with this parameterization, the warm-glow utility of leaving a bequest is much flatter than the value function of the children receiving it, even for the richest children. Moreover, the age-saving profiles show fast decumulation after retirement for people at all wealth levels, which is in contrast with the empirical evidence.

## 8 Discussion of the Assumptions

In order to make the model manageable and solvable I have made several simplifying assumptions. In this section I discuss the assumptions and their likely qualitative implications on the wealth distribution.

It is widely recognized (e.g., Becker and Tomes [3, 4]) that the time and resources that parents devote to children's education are very important in understanding the distribution of earnings and wealth. In this setup the simplification that children partially inherit their

parents' productivity is meant to recover the fact that education and human capital are closely related to the family background of each person. However, when explicitly modeling human capital investment, the return is commonly assumed to be decreasing, and up to a given level of investment (which may depend on the child's abilities), greater than the rate of return on physical capital. This implies that parents will begin to invest in their children's human capital and then invest in physical capital when the return from human capital reduces to the return on physical capital. In this setup, in the presence of borrowing constraints, the poorest families only invest in their children's human capital, and they may not even be able to invest up to the optimal amount. The richest families not only invest in their children's human capital but also leave them physical capital. Poorer families will tend to have poorer children, thus generating persistence in the lower end of the wealth distribution. At the upper tail of the distribution, rich children might want to save less because they expect large bequests. However, they will also be richer (because of their dominant income process and the bigger transfers they receive from their parents), hence they might want to save more. Depending on which effect dominates, this will increase or decrease persistence at the upper end of the wealth distribution. Most likely, if the altruism toward one's children is very strong for the richer people, the desire of leaving large estates to children will offset the reduction in saving because of the large bequests received. Which effect dominates thus depends on how wealth affects savings at high levels of wealth.

As discussed previously, I made restrictive assumptions on the information available to the children on their parent's wealth and income. These assumptions are made for computational reasons but are also likely to affect the results. In particular, I expect the model in the current version to display fatter tails at both ends of the wealth distribution, compared with a model in which the parent's assets and income are observable by the child. With perfect observability children of poor parents will save more, since they are aware that no bequest will be left to them. On the other hand, children of richer parents will save less. If wealth of poor parents is easier to measure than wealth of rich parents, only the lower tail of the distribution of wealth would become thinner.

Another important assumption is that there are no inter-vivos transfers. In the data these transfers often have a compensatory nature: parents tend to give when the children need money the most. This may happen when they go to college, start a new job, get married, buy a house or get a sequence of bad shocks. This assumption is probably most relevant when children are 20 to 35 years of age and are starting off their own life. Allowing for inter-vivos transfers would help reduce the number of people at zero wealth, especially among the young.

I take fertility to be exogenous and independent of the agent's wealth. This is likely to be a reasonable assumption for the U.S., but not a good one for other countries, especially developing countries. If poorer families on average are more prolific than richer families then the concentration of the wealth distribution will be increased: poor families have to divide their scarce resources among more children who, in turn, will be most likely poor.

Labor earnings are also assumed to be exogenous and people can only invest in a riskless asset. The U.S. data show that there is a noticeable correlation between high wealth and income from running a business. Introducing entrepreneurial choice in the model, for example in the form of investment in a risky asset in presence of minimum investment size and borrowing constraints,

would generate more heterogeneity in the people's income processes and more precautionary savings.

## 9 Directions for Future Research

There is considerable debate about abolishing estate taxation. In the U.S., Sweden and many other countries, estate and gift taxes produce little revenue (in the order of 0.1% of GDP) and possibly distort the savings decision of the few people that do most of the capital accumulation in the economy: the rich. I plan to study the effects of abolishing the estate taxes in the context of this model. I am interested in looking both at the macroeconomic consequences (e.g., what would happen to total capital) as well as the distributional effects (e.g., would wealth dispersion increase and by how much? Would the poor people become even poorer in absolute terms, or could they also benefit if total capital in the economy increased?).

In addition, many rich people are entrepreneurs, and entrepreneurial activity is likely to be an important factor in understanding the distribution of wealth. I plan to study entrepreneurial income and the degree of wealth inequality across different countries to better assess the importance of entrepreneurial income in evaluating the wealth distribution.

I also plan to use my model to study the demand for annuities. In fact, the market for annuities is remarkably thin. The literature provides several explanations, such as moral hazard, altruism and social security provision. In my setup, social security annuitizes all or most of the savings of poorer people and the bequest motive provides a reason for richer people not to annuitize their wealth completely. It will be interesting to introduce annuities in my setup to study the quantitative importance of these two elements.

## A Calibration of the income processes

As discussed in the calibration section, I convert both the productivity and the productivity inheritance processes to discrete Markov chains according to Tauchen and Hussey [36]. I use three values for the income process. Since I want the possible realizations for the initial inherited productivity level to be the same as the possible realizations for productivity during the lifetime, I choose the quadrature points to be the same for the two Markov processes.

Tables 9 and 10 report data on earnings distributions, respectively for the U.S. and Sweden, and compare them with the earnings distributions used in the model. The tables are computed using data for households whose head is 25 to 60 years of age. In row 1 the definition of gross earnings includes wages, salaries and self-employment income. Row 2 adds social insurance transfers to gross earnings. Row 3 refers to the earnings distribution used in the simulations.

Since the setup does not explicitly model other social insurance transfers other than social security, we should compare earnings used in the simulations to data that include social insurance transfers. From this comparison we can see that the model understates both the earnings of the upper tail, and the fraction of people at zero earnings.

Gini coeff.	Percentage earnings in the top						Percent with negative or zero income
	5%	10%	20%	40%	60%	80%	
U.S. earnings data							
.46	19	30	48	72	89	98	7.7
U.S. earnings + social insurance transfers							
.44	19	30	47	71	88	97	5.3
U.S. simulated earnings							
.43	13	25	48	72	87	96	0.0

Table 9: U.S. earnings.

Gini coeff.	Percentage earnings in the top						Percent with negative or zero income
	5%	10%	20%	40%	60%	80%	
Swedish earnings							
.40	15	25	42	68	86	98	7.6
Swedish earnings + social insurance transfers							
.32	13	22	37	62	80	94	1.0
Swedish simulated earnings							
.32	10	21	40	63	81	92	0.0

Table 10: Swedish earnings.

## B Figures

U.S. calibration. Experiment with no links and equal bequests to all

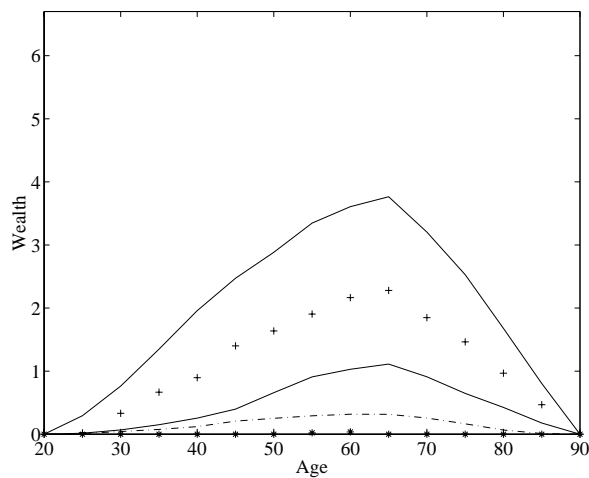


Figure 1: Wealth .1 .3 .5 .7 .9 quantiles, by age

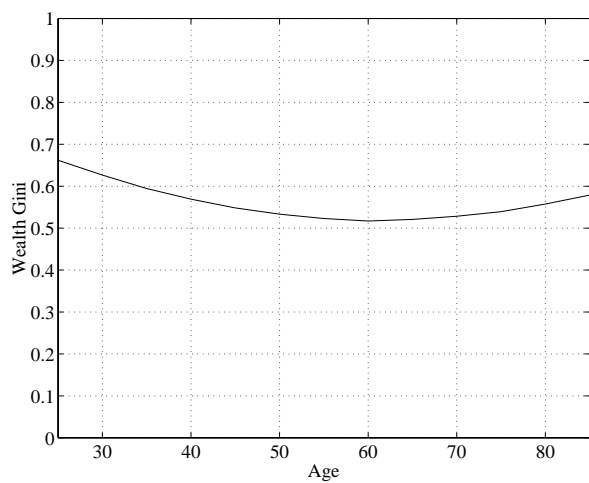


Figure 2: Wealth Gini for different cohorts



**U.S. calibration. Experiment with no links and unequal bequests to heirs**

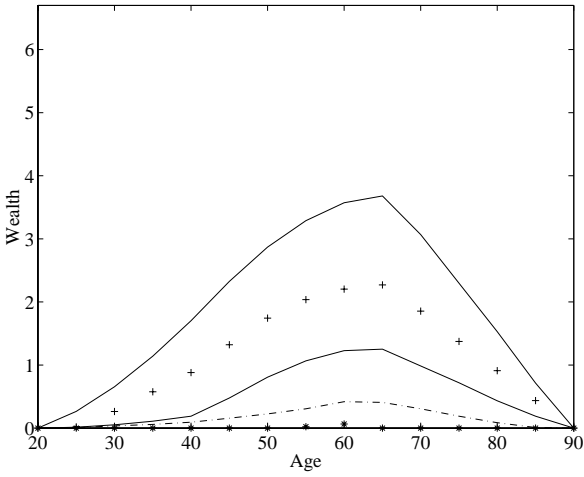


Figure 3: Wealth .1 .3 .5 .7 .9 quantiles, by age

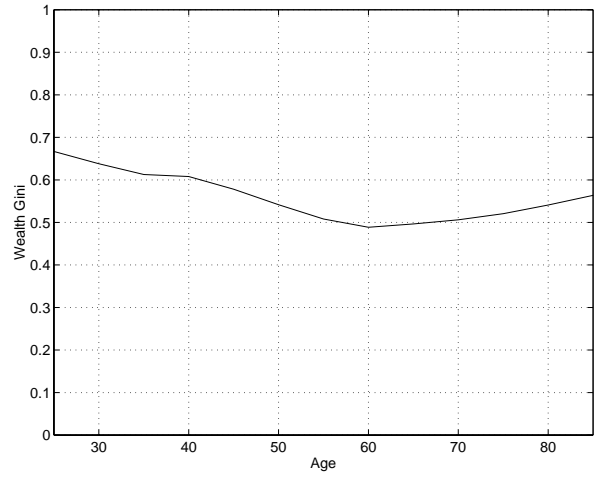


Figure 4: Wealth Gini for different cohorts

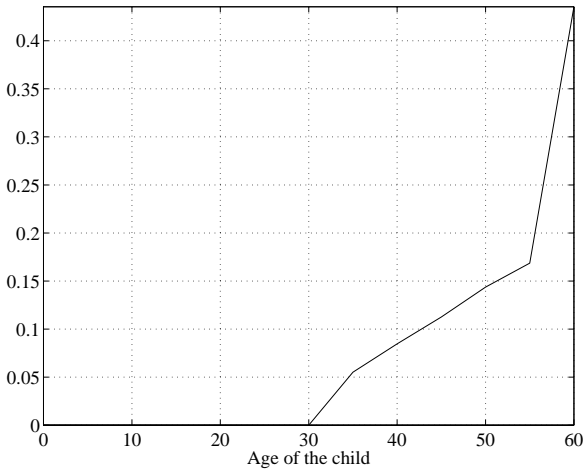


Figure 5: Probability of the parent dying at each age of the child

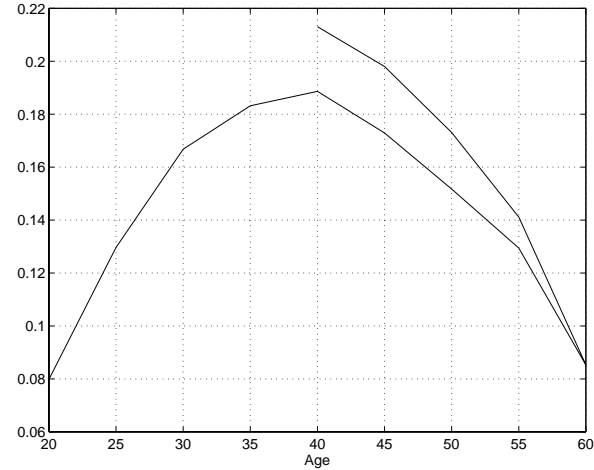


Figure 6: Saving for people who expect or not to inherit

U.S. calibration. Experiment with no links and unequal bequests to heirs

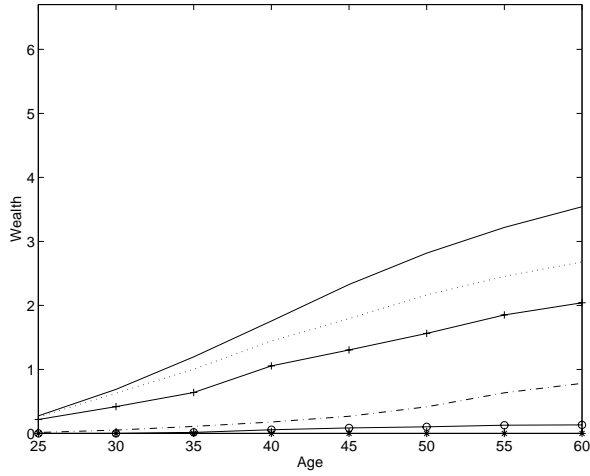


Figure 7: Wealth quantiles conditional on not having inherited

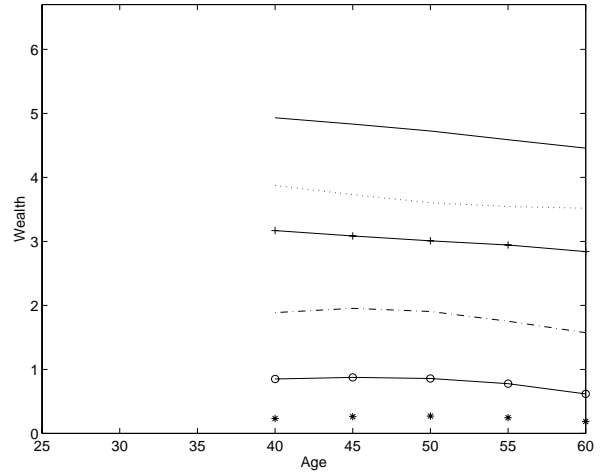


Figure 8: Wealth quantiles conditional on having inherited

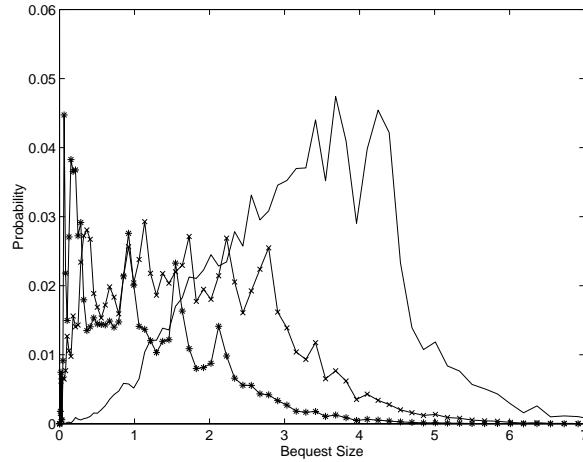


Figure 9: Strictly positive range of the expected bequest distribution at age 40, conditional on the productivity of the parent

**U.S. calibration. Experiment with bequest motive**

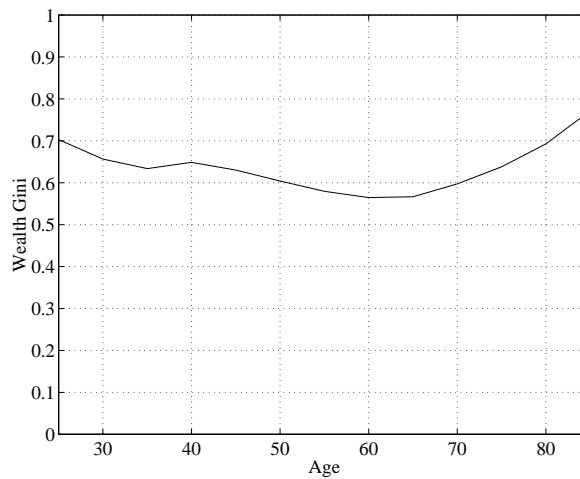
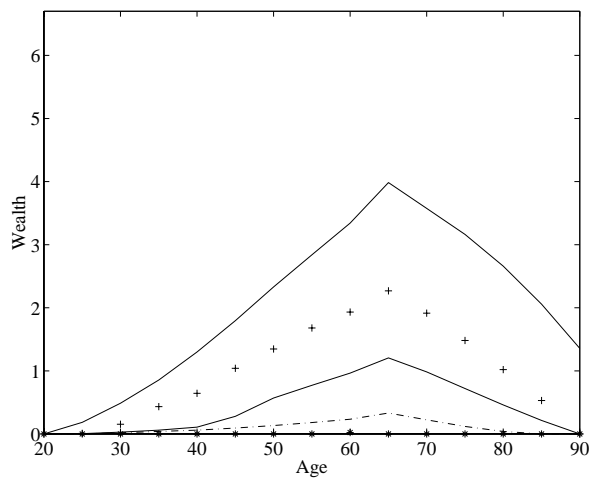


Figure 10: Wealth .1 .3 .5 .7 .9 quantiles, by age

Figure 11: Wealth Gini for different cohorts

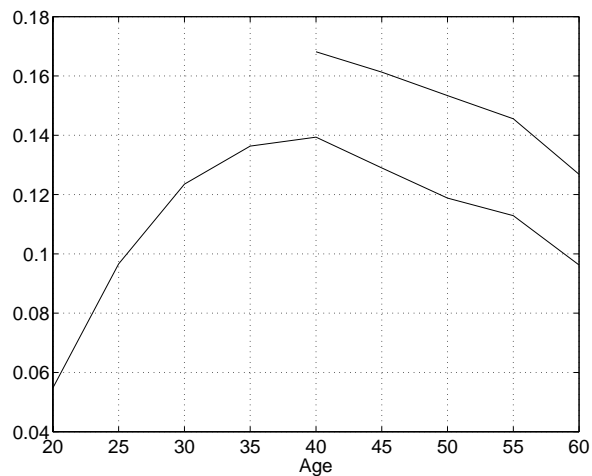


Figure 12: Saving for people who expect or not to inherit

### U.S. calibration. Experiment with bequest motive

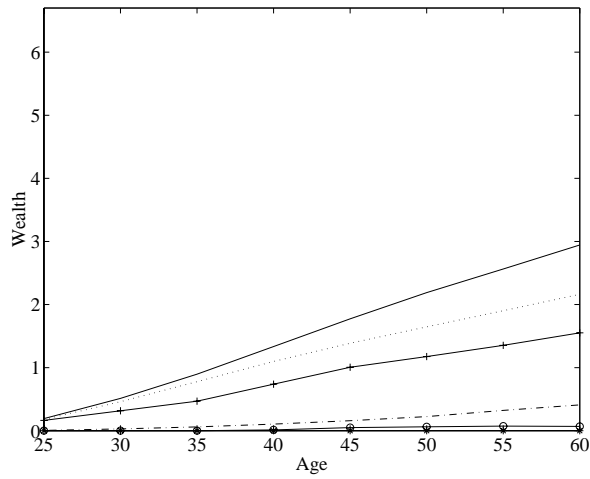


Figure 13: Wealth quantiles conditional on not having inherited

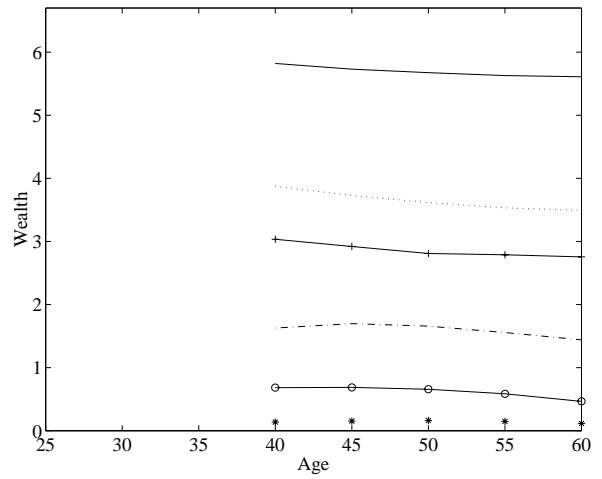


Figure 14: Wealth quantiles conditional on having inherited

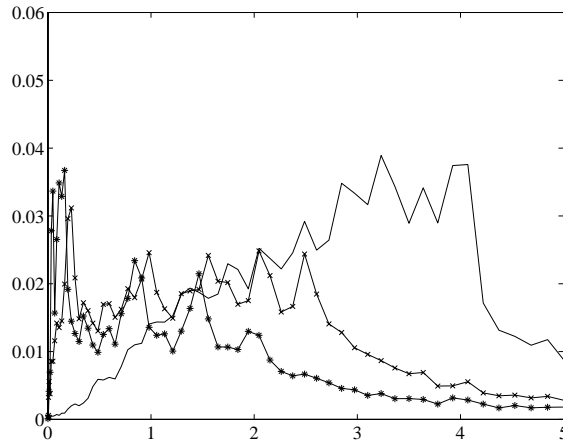


Figure 15: Strictly positive range of the expected bequest distribution at age 40, conditional on the productivity of the parent

U.S. calibration. Experiment with bequest motive and productivity inheritance

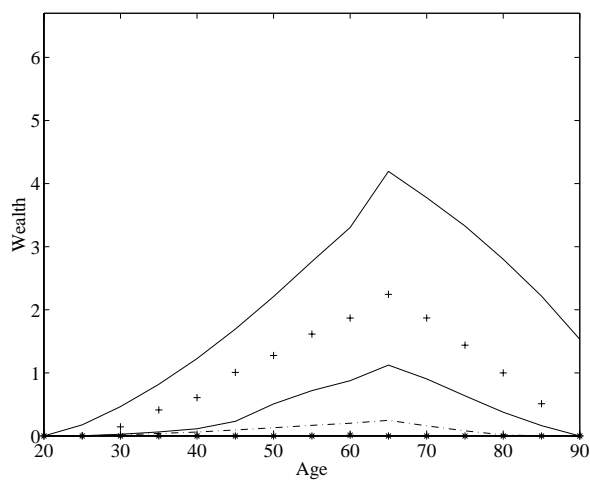


Figure 16: Wealth .1 .3 .5 .7 .9 quantiles, by age

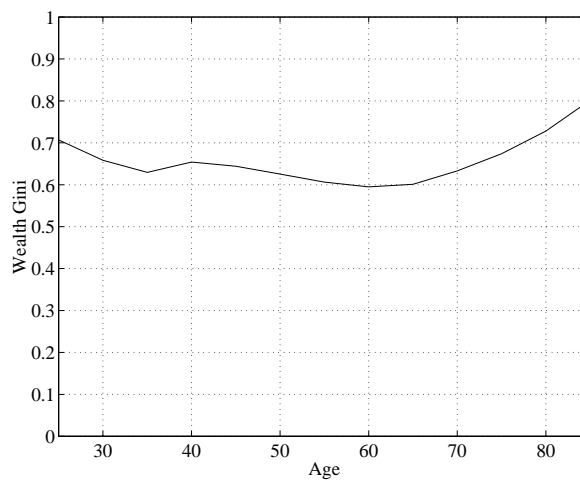


Figure 17: Wealth Gini for different cohorts

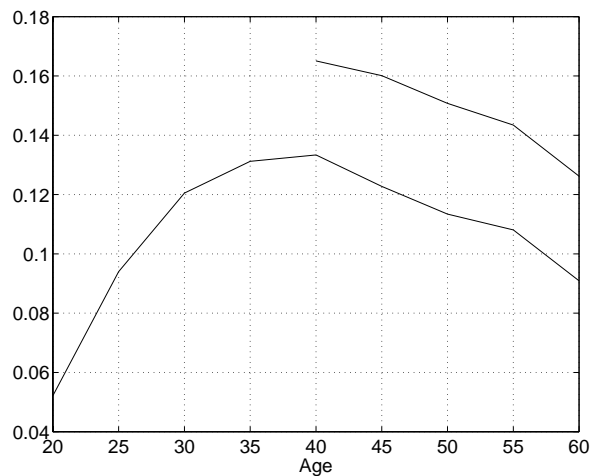


Figure 18: Saving for people who expect or not to inherit

U.S. calibration. Experiment with bequest motive and productivity inheritance

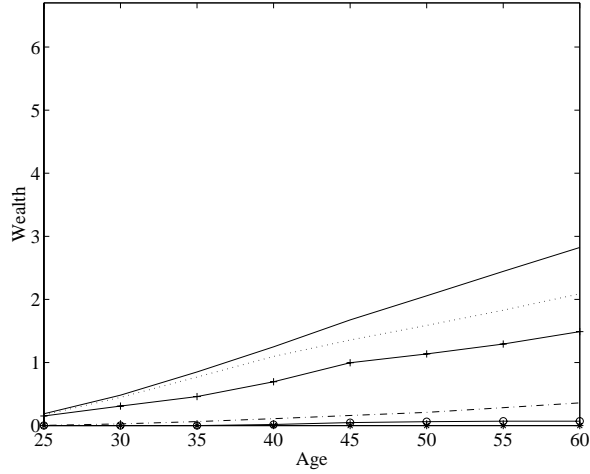


Figure 19: Wealth quantiles conditional on not having inherited

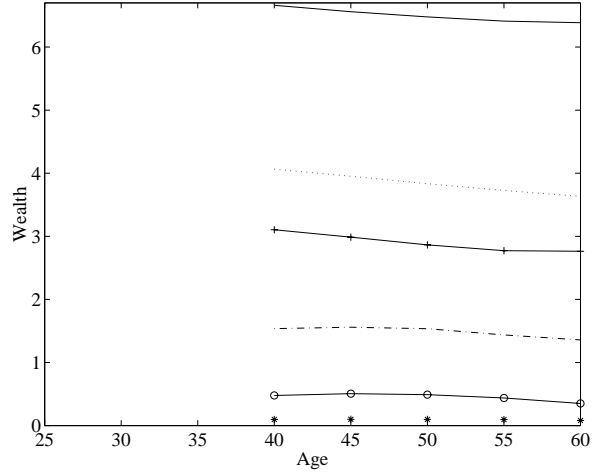


Figure 20: Wealth quantiles conditional on having inherited

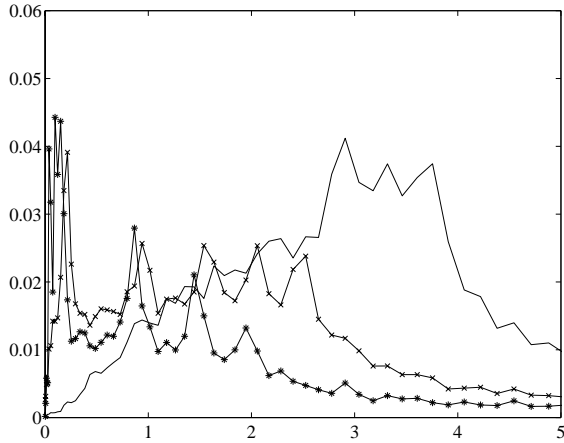


Figure 21: Strictly positive range of the expected bequest distribution at age 40, conditional on the productivity of the parent

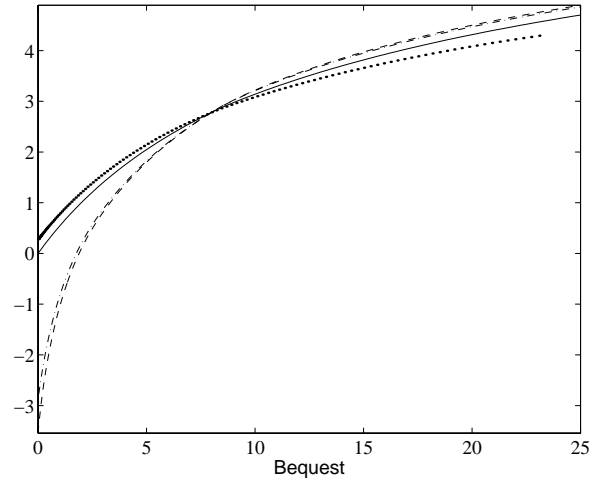


Figure 22: Warm glow and true altruism compared: warm glow (solid), poor (dashed), median (dash-dot), rich (dots)

Swedish calibration. Experiment with no links and equal bequests to all

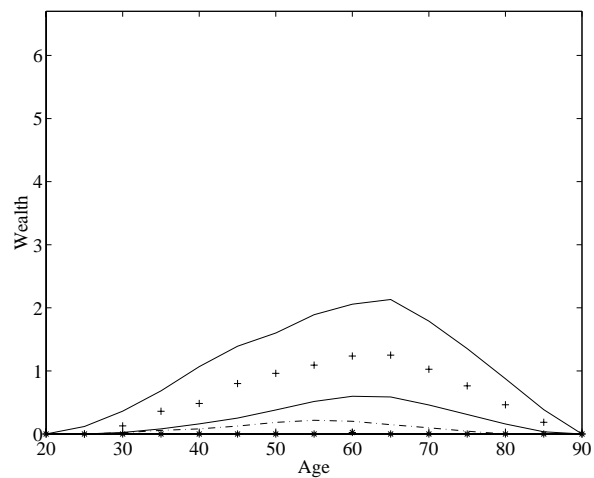


Figure 23: Wealth .1 .3 .5 .7 .9 quantiles, by age

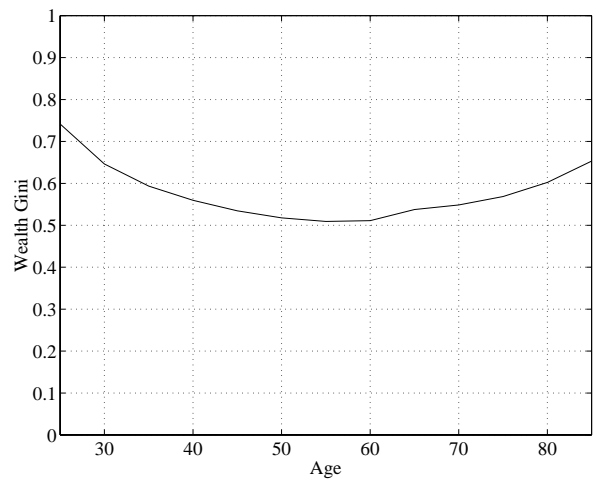


Figure 24: Wealth Gini for different cohorts

Swedish calibration. Experiment with no links and unequal bequests to heirs

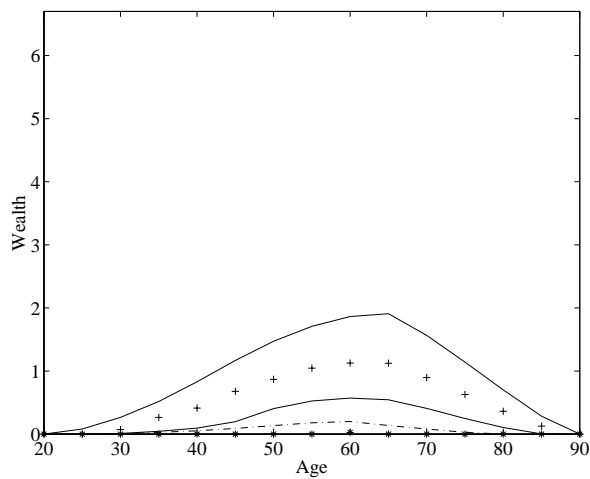


Figure 25: Wealth .1 .3 .5 .7 .9 quantiles, by age

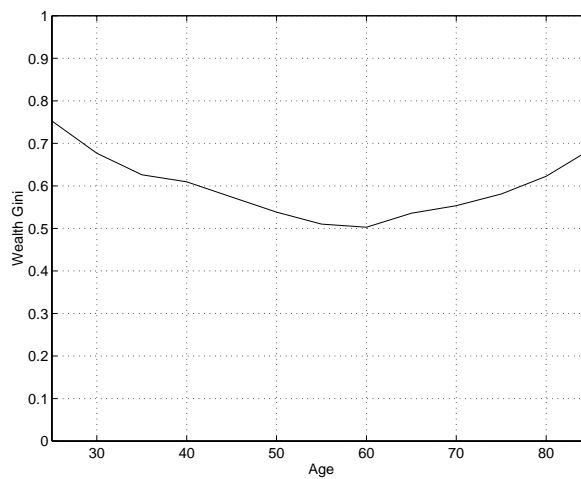


Figure 26: Wealth Gini for different cohorts

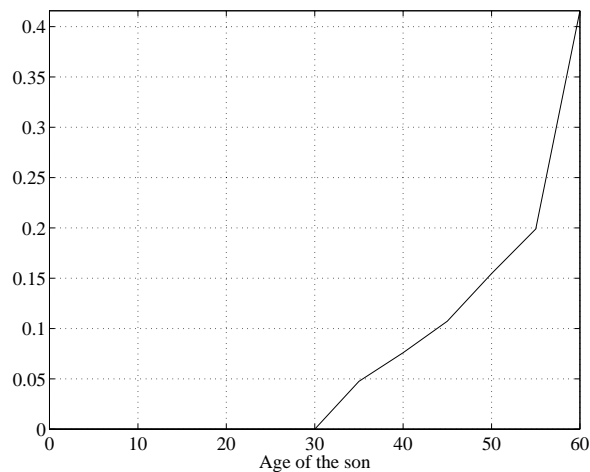


Figure 27: Probability of the parent dying at each age of the child

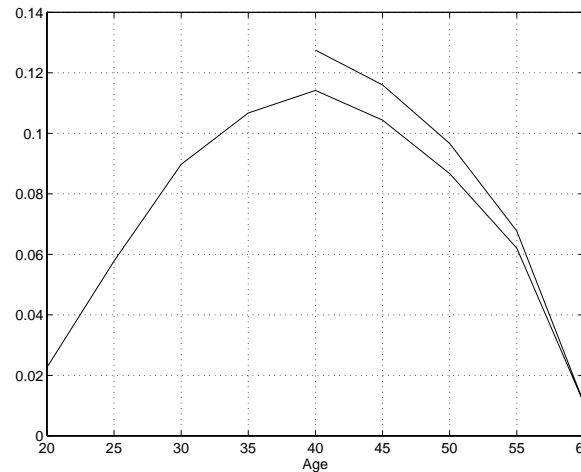


Figure 28: Saving for people who expect or not to inherit



Swedish calibration. Experiment with no links and unequal bequests to heirs

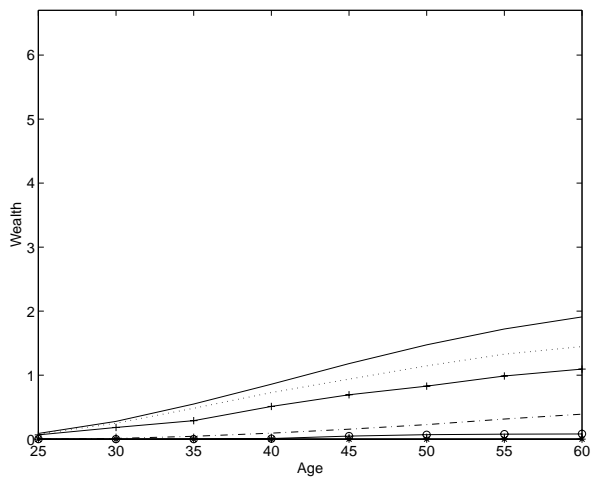


Figure 29: Wealth quantiles conditional on not having inherited

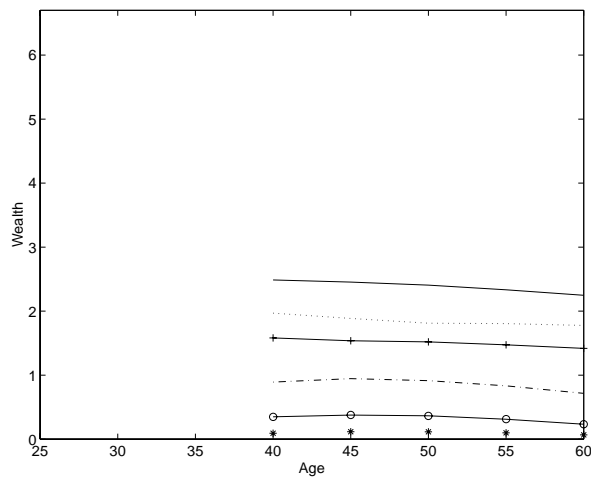


Figure 30: Wealth quantiles conditional on having inherited

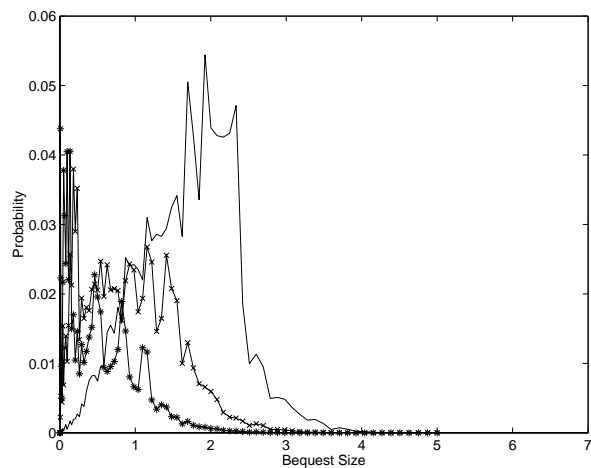


Figure 31: Strictly positive range of the expected bequest distribution at age 40, conditional on the productivity of the parent

Swedish calibration. Experiment with bequest motive

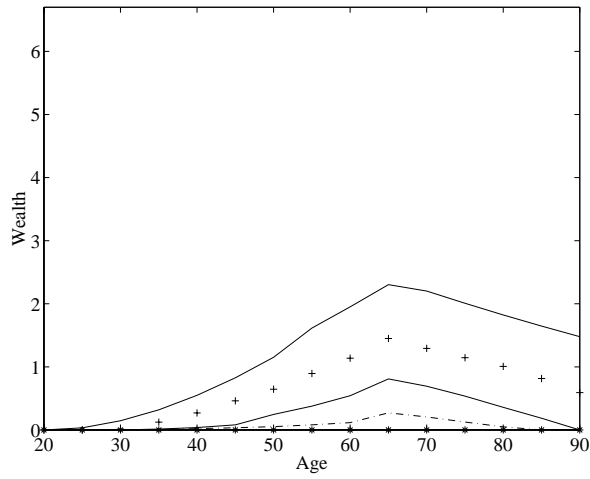


Figure 32: Wealth .1 .3 .5 .7 .9 quantiles, by age

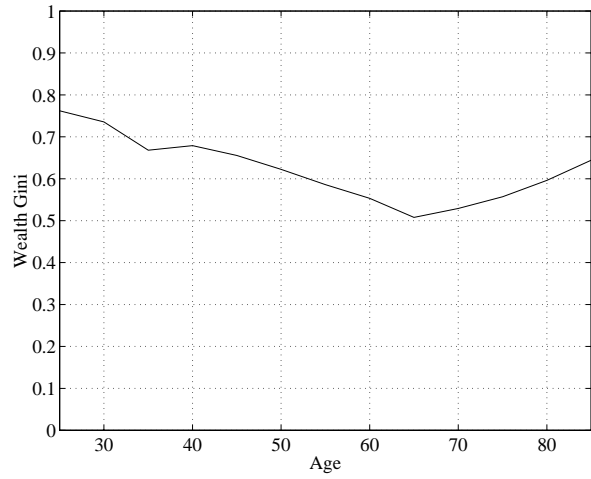


Figure 33: Wealth Gini for different cohorts

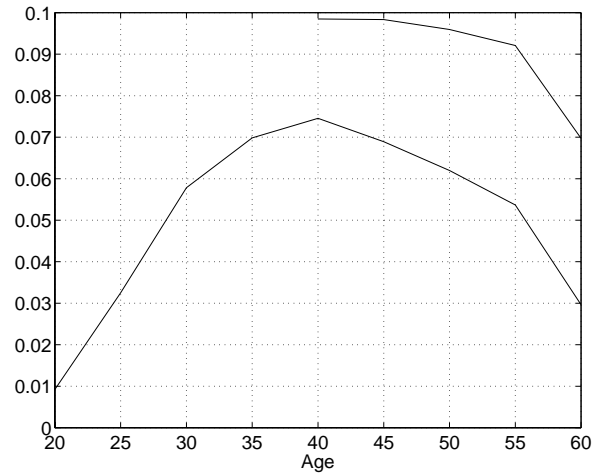


Figure 34: Saving for people who expect or not to inherit

Swedish calibration. Experiment with bequest motive

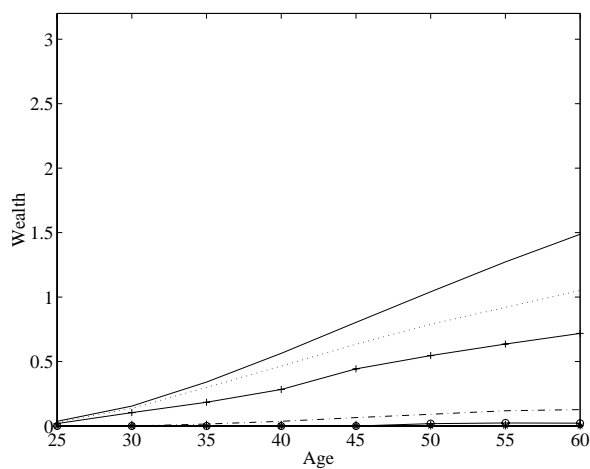


Figure 35: Wealth quantiles conditional on not having inherited

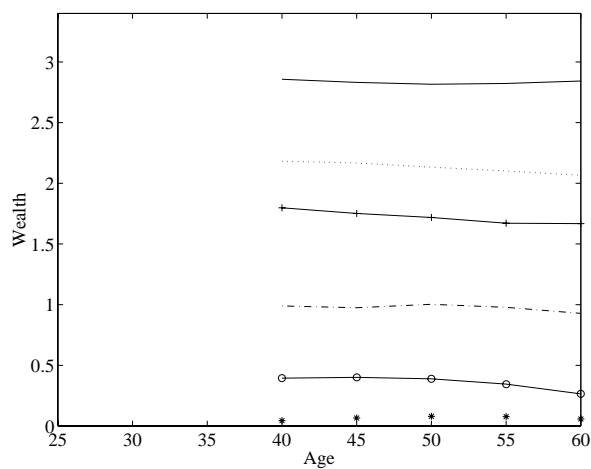


Figure 36: Wealth quantiles conditional on having inherited

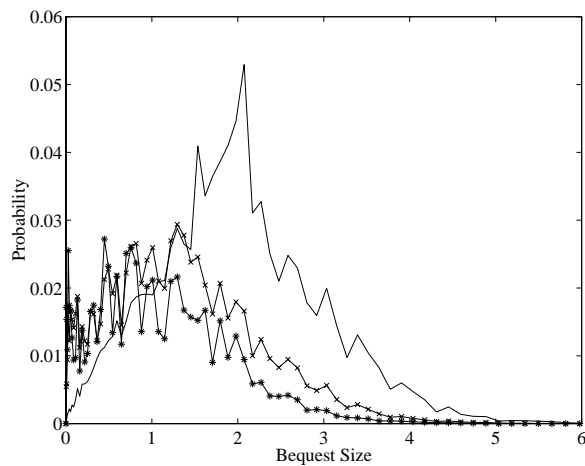


Figure 37: Strictly positive range of the expected bequest distribution at age 40, conditional on the productivity of the parent

Swedish calibration. Experiment with bequest motive and productivity inheritance

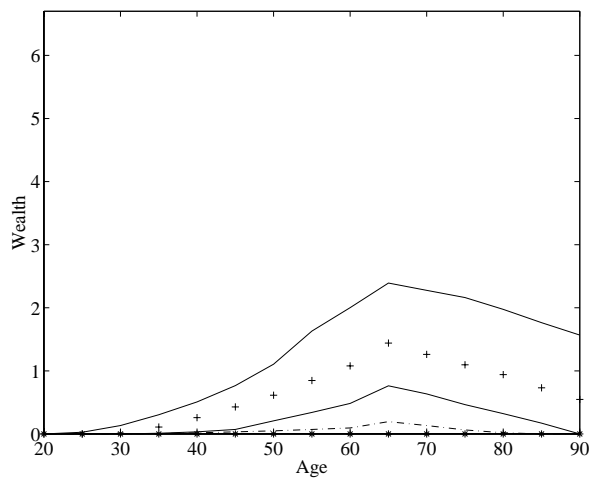


Figure 38: Wealth .1 .3 .5 .7 .9 quantiles, by age

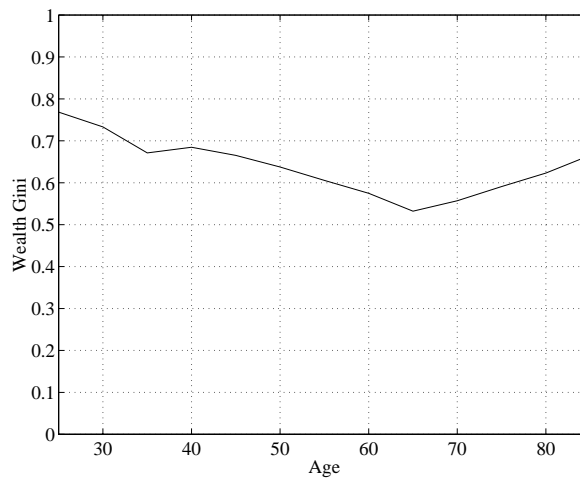


Figure 39: Wealth Gini for different cohorts

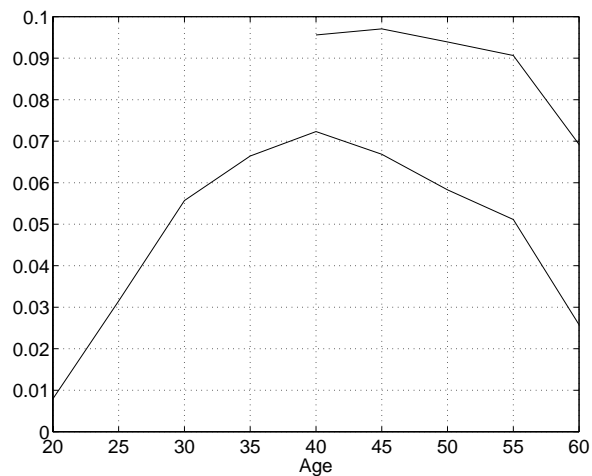


Figure 40: Saving for people who expect or not to inherit

Swedish calibration. Experiment with bequest motive and productivity inheritance

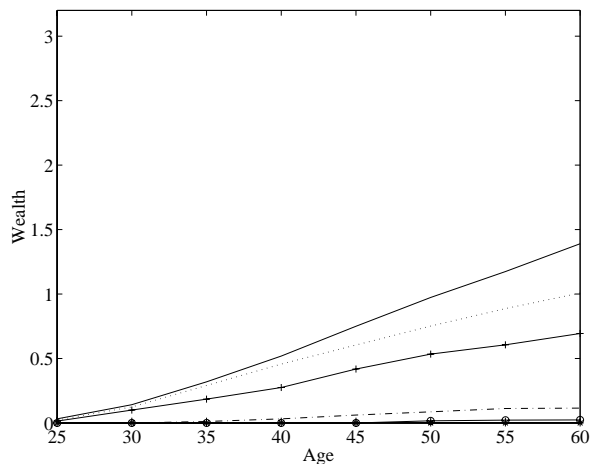


Figure 41: Wealth quantiles conditional on not having inherited

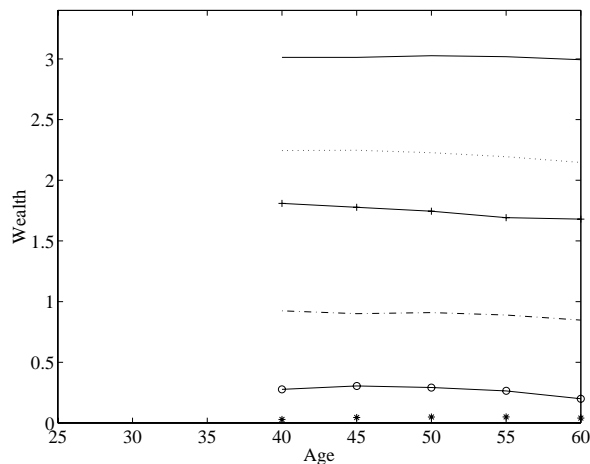


Figure 42: Wealth quantiles conditional on having inherited

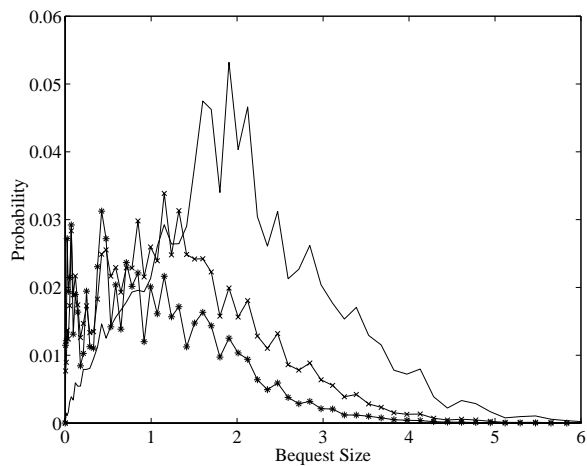


Figure 43: Strictly positive range of the expected bequest distribution at age 40, conditional on the productivity of the parent

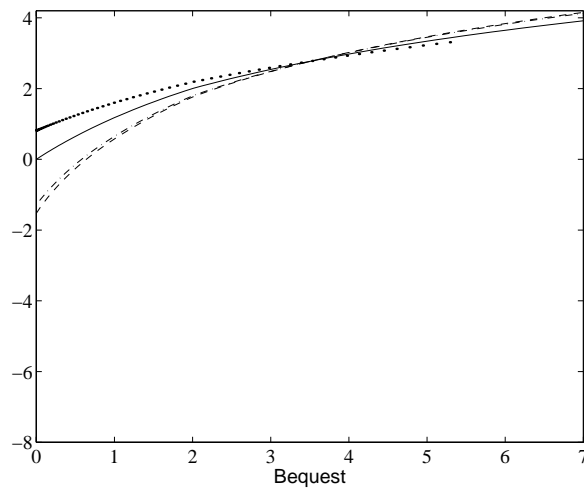


Figure 44: Warm glow and true altruism compared: warm glow (solid), poor (dashed), median (dash-dot), rich (dots)

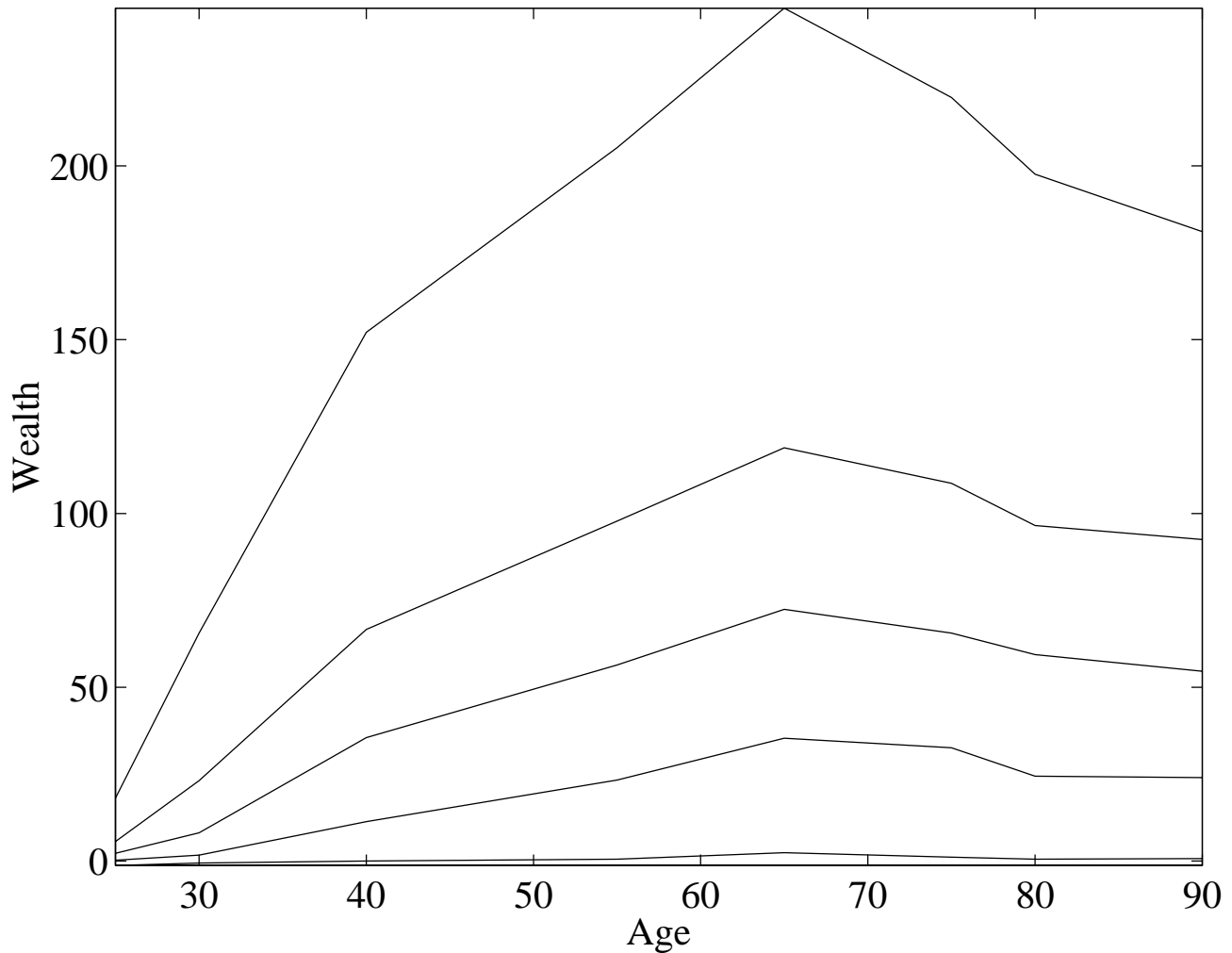


Figure 45: U.S. data, wealth quantiles: .1, .3, .5, .7, .9

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