Exchange Rate Expectations: Evidence from an Artificial Economy

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Abstract

Survey studies on exchange rate expectations tend to reject the rational expectations hypothesis for longer horizons. Extrapolative, adaptive and regressive expectations have been tested as alternatives, usually rejecting static expectations. The purpose of this paper is to investigate the plausibility of these alternative exchange rate expectations mechanisms in an artificial economy with traders which are heterogeneous in initial endowments, risk aversion and use of information. Artificial markets which consist of either extrapolative or adaptive expectations traders fail to reproduce statistical properties that are characteristic for empirical quarterly exchange rate series, while artificial markets with regressive expectations traders often succeed. Adaptive expectations markets are rarely weak-form efficient. Extrapolative expectations markets may be weak-form efficient, but generate too many extreme returns to be empirically plausible. Regressive expectations markets often produce exchange rate series that are similar to empirical data. The (perceived) existence of an 'anchor' seems to play an important role in the functioning of foreign exchange markets, determining the frequency of extreme exchange rate returns. (JEL F31)

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1. Exchange rate expectations

Survey data studies indicate that long term exchange rate expectations are heterogeneous (Taylor & Allen (1992), Ito (1990)) and are not adequately described by rational expectations (Dominguez (1986), Frankel & Froot (1987a), Froot & Frankel (1989), Ito (1990), Cavaglia, Verschoor & Wolff (1993)). Extrapolative, adaptive and regressive expectations have been tested as alternatives, against which static expectations are usually rejected (Frankel & Froot (1987a) and (1987b), Bank of Japan (1989), Froot & Frankel (1990), Cavaglia, Verschoor & Wolff (1993)). The objective of this paper is to investigate the plausibility of these alternative exchange rate expectations mechanisms in an artificial economy with traders which are heterogeneous in initial endowments, risk aversion and use of information. We will focus on three months ahead exchange rate expectations, as survey evidence is most elaborate for this horizon. The alternative expectations schemes (extrapolative, adaptive and regressive expectations) can be summarized (Frankel & Froot (1987a)) as:

$$E_{t,i}(s_{t+1}) = (1-\alpha_{t,i})\, s_t + \alpha_{t,i}\, z_{t,i} \qquad (1)$$

where s_t is the natural logarithm of the spot exchange rate S_t in period t and $E_{t,i}(.)$ denotes trader i's period t (not necessarily mathematical) expectation with respect to the variable between brackets. If $z_{t,i} = s_{t-1}$ we speak of extrapolative expectations. We distinguish between three cases: $\alpha_{t,i} < 0$ (bandwagon expectations), $\alpha_{t,i} = 0$ (static expectations) and $\alpha_{t,i} > 0$ (distributed lag expectations). Another scheme that is "technical" or "chartist" in nature is adaptive expectations: $z_{t,i} = E_{t-1,i}(s_t)$. It is straightforward to show that such traders use the entire history of the exchange rate and that the forecast follows from a geometric series.

If $z_{t,i} = q_{t,i}$ is the natural logarithm of some “fundamental” or “long run equilibrium” exchange rate $Q_{t,i}$, we speak of regressive expectations.

The paper is organized as follows. In section 2 we introduce the artificial economy approach that will be used to perform controlled experiments with exchange rate expectations. In section 3 we describe a theoretical model of foreign exchange between heterogeneous traders with extrapolative, adaptive and regressive expectations. In section 4 we assign empirically plausible parameter and initial values to the theoretical model, which is thus transformed into an artificial economy. In section 5 we use this artificial economy as a foreign exchange laboratory: we perform simulations for different specifications of exchange rate expectations. In section 6 we draw conclusions from the controlled experiments in section 5.

2. The artificial economy approach

An important objection against the use of survey data is the uncertainty concerning the truthfulness of the participants’ reporting. As Frankel & Froot (1987a) put it:

“Economists generally distrust survey data. It is a cornerstone of “positive economics” that we learn more by observing what people do in the marketplace than what they say.”

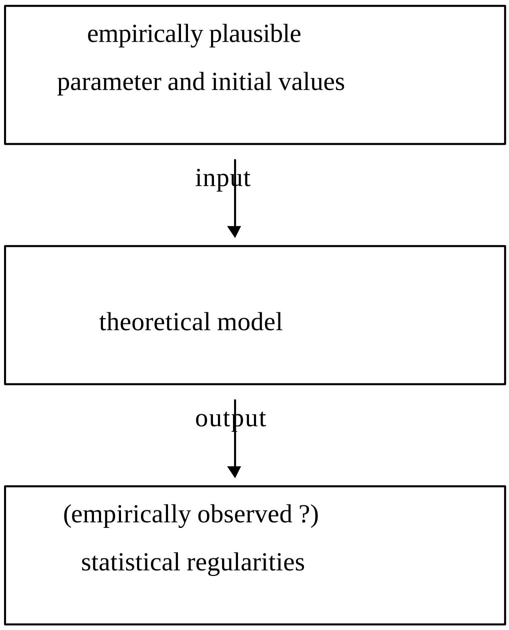
A related objection may be that the expectations of a participant at the time that he answers the survey, may differ from his expectations at the time at which he performs a market transaction. Especially when we are interested in the relationship between exchange rate expectations and the actual exchange rate, this nonsynchronicity with market data may limit the appropriate use of survey data.

A more controlled environment could be obtained with an experimental foreign exchange market. We could then solve the nonsynchronicity problem by asking participants about their expectations at the moment that they perform a transaction. However, we would still be uncertain about the truthfulness of the participants' reporting. In other words, with real people a perfectly controlled experiment is not feasible when expectations, perceptions and preferences are variables that we are interested in.

This problem can be solved by performing experiments with "artificial people", i.e. by performing computer simulations with agents whose behavior exactly follows the mathematical description that we impose via the program. In this way we know with certainty what the expectations of the participants are at the time of transaction.

The idea of this "artificial economy approach" is to simulate a theoretical model for empirically plausible parameter and initial values and study which theoretical assumptions lead or do not lead to statistical regularities in the generated time series that match empirically observed regularities. We have depicted this in figure 1:

Figure 1: Artificial economy methodology



Backus, Gregory & Telmer (1993) formally define the concept of an artificial economy:

“ . . . we build what has come to be called an artificial economy - a numerical representation of the theory whose properties can be compared to those observed in the data.”

Notice that not all numerical representations of theoretical models are artificial economies: only if the assigned numerical values are empirically plausible, do we speak of an artificial economy.

Artificial economies based on the representative agent model of asset pricing - such as Backus, Gregory & Zin (1989), Macklem (1991) and Backus, Gregory & Telmer (1993) - have not (yet) been very successful in reproducing empirically plausible time series. One way to investigate the plausibility of alternative exchange rate expectations mechanisms is to relax the rational expectations assumption of the representative agent model. However, the survey evidence that we mentioned in section 1 suggests that exchange rate expectations are neither rational nor homogeneous. Hence we will also relax the representative agent assumption and build a model of foreign exchange between heterogeneous traders that do not necessarily form rational expectations. Since we are interested in the plausibility of alternative exchange rate expectations mechanisms, we would like to have a model that has the flexibility to incorporate all three alternatives (extrapolative, adaptive and regressive). We can then perform simulations with different versions of the model, corresponding with different specifications of the expectations mechanism. With the validity of the model as a maintained hypothesis, we can interpret the (in)ability of a model version to generate empirically plausible output as evidence of the (im)plausibility of the corresponding exchange rate expectations mechanism. The artificial economy can be used as a laboratory of foreign exchange that allows us to perform controlled experiments with exchange rate expectations.

3. A model of foreign exchange between heterogeneous traders

3.1 Traders, currency positions and interest rates

Consider an international banking system that organizes a one period domestic money market, a one period foreign money market and a spot foreign exchange market that allows traders to exchange domestic currency money market accounts for foreign currency money market accounts and vice versa. Suppose domestic trader i (foreign trader j) starts the trading session in period t with an account of $A_{t,i}$ ($A^*_{t,j}$) domestic currency units and $B_{t,i}$ ($B^*_{t,j}$) foreign currency units. Trading allows him to change his idle account ($A_{t,i}$, $B_{t,i}$) (($A^*_{t,j}$,$B^*_{t,j}$)) into an interest-bearing account ($X_{t,i}$, $Y_{t,i}$) (($X^*_{t,j}$,$Y^*_{t,j}$)). In period t+1 the domestic currency wealth of domestic trader i will be

$$W_{t+1,i} = (1 + I_t) X_{t,i} + (1 + I^*_t) Y_{t,i} S_{t+1} \tag{2}$$

The period t+1 foreign currency wealth of foreign trader j will be

$$W^*_{t+1,j} = (1 + I^*_t) Y_{t,i} + (1 + I_t) X^*_{t,j} / S_{t+1} \tag{3}$$

The participation of domestic trader i in the foreign exchange market will have to satisfy

$$X_{t,i} + S_t Y_{t,i} = A_{t,i} + S_t B_{t,i} \tag{4}$$

Therefore we may rewrite the domestic currency wealth of domestic trader i in period t+1 as follows

$$W_{t+1,i} = [(1 + I^*_t) S_{t+1} - (1 + I_t) S_t] Y_{t,i} + (1 + I_t) (A_{t,i} + S_t B_{t,i}) \tag{5}$$

Analogously, the exchange of foreign trader j has to obey

$$X^*_{t,j}/S_t + Y^*_{t,j} = A^*_{t,j}/S_t + B^*_{t,j} \tag{6}$$

We can therefore rewrite his foreign currency wealth in period t+1 as

$$W^*_{t+1,j} = \left(\frac{1+I_t}{S_{t+1}} - \frac{1+I^*_t}{S_t}\right)X^*_{t,j} + \left(1+I^*_t\right)\left(B^*_{t,j} + \frac{A^*_{t,j}}{S_t}\right) \tag{7}$$

Before trading the international banking system as a whole (we assume that there are m domestic traders and n foreign traders) has a domestic currency balance

$$A_t = \sum_{i=1}^{m} A_{t,i} + \sum_{j=1}^{n} A^*_{t,j} \tag{8}$$

and a foreign currency balance

$$B_t = \sum_{i=1}^{m} B_{t,i} + \sum_{j=1}^{n} B^*_{t,j} \tag{9}$$

After trading the international banking system as a whole has a domestic currency balance

$$X_t = \sum_{i=1}^{m} X_{t,i} + \sum_{j=1}^{n} X^*_{t,j} \tag{10}$$

and a foreign currency balance

$$Y_t = \sum_{i=1}^{m} Y_{t,i} + \sum_{j=1}^{n} Y^*_{t,j} \tag{11}$$

The international banking system guarantees its account holders that they will be able to attain any interest-bearing domestic and foreign currency account balance they like. This requires that

$$X_t = A_t \;\wedge\; Y_t = B_t \tag{12}$$

For simplicity we assume that domestic and foreign interest rates are exogeneously given. Hence the exchange rate S_t simultaneously clears the two money markets and the foreign exchange market if it ensures that both conditions in (12) are met. Since the domestic and foreign currency positions of domestic and foreign traders are interdependent via their respective budget equations, Walras' Law tells us that we only have to solve S_t from one of the two conditions. We assume that the international banking system knows the foreign currency supply and demand schedules of all its clients and is thus able to act as a Walrasian auctioneer by announcing a clearing exchange rate S_t.

3.2 Currency endowments

For simplicity we assume that for all $t > 1$:

$$A_{t,i} = (1+I_{t-1})X_{t-1,i} \tag{13}$$

$$B^*_{t,j} = (1+I^*_{t-1})Y^*_{t-1,j} \tag{14}$$

$$A^*_{t,j} = (1+I_{t-1})X^*_{t-1,j} \quad (15)$$

$$B_{t,i} = (1+I^*_{t-1})Y_{t-1,i} \quad (16)$$

where $A_{1,i}$, $B^*_{1,j}$, $A^*_{1,j}$, $B_{1,i}$ are exogeneously given initial values.

It follows from the market clearing conditions that between period t-1 and t the domestic and foreign currency stocks in the international banking system evolve according to

$$X_t = (1+I_{t-1})X_{t-1} \quad (17)$$

$$Y_t = (1+I^*_{t-1})Y_{t-1} \quad (18)$$

The international banking system creates domestic (foreign) money at a rate that equals the domestic (foreign) interest rate.

3.3 Trader preferences and demand functions

Let $E_{t,i}(S_{t+1})$ denote the expectation of domestic trader i in period t concerning the yet unknown future value S_{t+1} and let $Var_{t,i}(S_{t+1})$ be the variance of S_{t+1} that domestic trader i predicts in period t. Given a certain value of exchange rate S_t, it follows from (5) that domestic trader i expects his domestic currency wealth in period t+1 to be

$$E_{t,i}(W_{t+1,i}) \;=\; [\,(1 + I^*_t)\,E_{t,i}(S_{t+1}) \;-\; (1 + I_t)\,S_t\,]\,Y_{t,i} \;+\; (1+ I_t)\,(\,A_{t,i} + S_t\,B_{t,i}) \qquad (19)$$

and he expects the variance of his period t+1 domestic currency wealth to be

$$Var_{t,i}(W_{t+1,i}) \;=\; (1+I^*_t)^2\,Var_{t,i}(S_{t+1})\,Y_{t,i}{}^2 \qquad (20)$$

Assume that domestic traders are only interested in their one period ahead domestic currency wealth. The one period horizon can be induced by an overlapping generations structure in the spirit of De Long, Shleifer, Summers & Waldmann (1990):

"Such an overlapping generations structure may be a fruitful way of modeling the effects on prices of a number of institutional features, such as frequent evaluations of money managers' performance, that may lead rational, long-lived market participants to care about short-term rather than long-term performance."

The evaluation of wealth in terms of domestic currency may be justified by assuming that domestic traders only consume domestic goods. A convenient specification of the expected utility function (Newbery (1988)) is

$$E_{t,i}(U_{t+1,i}) = E_{t,i}(W_{t+1,i}) - \tfrac{1}{2}\gamma_{t,i} Var_{t,i}(W_{t+1,i}) \qquad (21)$$

where $\gamma_{t,i}$ is the parameter of absolute risk aversion.

If we substitute (19) and (20) into expected utility function (21), we obtain

$$E_{t,i}(U_{t+1,i}) \;=\; [\,(1+I^*_t)\,E_{t,i}(S_{t+1}) - (1+I_t)\,S_t\,]\,Y_{t,i} \;+\; (1+I_t)(A_{t,i} + S_tB_{t,i}) \;+$$

$$-\;1/2\;\gamma_{t,i}\,(1+I^*_t)^2\,Var_{t,i}(S_{t+1})\,Y_{t,i}^{\;2} \qquad (22)$$

Hence the optimal foreign currency position of domestic trader i is

$$Y_{t,i} = \frac{(1+I_t^*)E_{t,i}(S_{t+1}) - (1+I_t)S_t}{(1+I_t^*)^2\gamma_{t,i}\,Var_{t,i}(S_{t+1})} \qquad (23)$$

provided that $\gamma_{t,i} > 0$ (second order condition). Let $E^*_{t,j}(S^{-1}_{t+1})$ denote the subjective expectation of foreign trader j in period t with respect to the unknown future exchange rate $1/S_{t+1}$ and let $Var^*_{t,j}(S^{-1}_{t+1})$ be the variance of $1/S_{t+1}$ that foreign trader j expects in period t. Given a certain value of exchange rate S_t, it follows from (7) that foreign trader j expects his foreign currency wealth in period t+1 to be

$$E^*_{t,j}(W^*_{t+1,j}) = \left(\left((1+I_t)E^*_{t,j}(S^{-1}_{t+1}) - \frac{1+I_t^*}{S_t}\right)X^*_{t,j} + (1+I_t^*)\left(\frac{A^*_{t,j}}{S_t} + B^*_{t,j}\right)\right) \qquad (24)$$

and he expects the variance of his period t+1 foreign currency wealth to be

$$Var^*_{t,j}(W^*_{t+1,j}) = (1+I_t)^2\,Var^*_{t,j}(S^{-1}_{t+1})X^{*\;2}_{t,j} \qquad (25)$$

Denote foreign trader j's absolute risk aversion in period t as $\gamma^*_{t,j}$. We assume that foreign traders are only interested in their one period ahead foreign currency wealth and that they maximize the following expected utility function:

$$E^*_{t,j}(U^*_{t+1,j}) = E^*_{t,j}(W^*_{t+1,j}) - \tfrac{1}{2}\gamma^*_{t,j}\mathrm{Var}^*_{t,j}(W^*_{t+1,j}) \tag{26}$$

If we substitute (24) and (25) into (26), we get

$$E^*_{t,j}(U^*_{t+1,j}) = \left((1+I_t)E^*_{t,j}(S^{-1}_{t+1}) - \frac{1+I^*_t}{S_t}\right)X^*_{t,j} + (1+I^*_t)\left(\frac{A^*_{t,j}}{S_t} + B^*_{t,j}\right) +$$

$$-\tfrac{1}{2}\gamma^*_{t,j}(1+I_t)^2\mathrm{Var}^*_{t,j}(S^{-1}_{t+1})X^{*\,2}_{t,j} \tag{27}$$

The optimal domestic currency position of foreign trader j is

$$X^*_{t,j} = \frac{(1+I_t)E^*_{t,j}(S^{-1}_{t+1}) - \dfrac{1+I^*_t}{S_t}}{(1+I_t)^2\gamma^*_{t,j}\mathrm{Var}^*_{t,j}(S^{-1}_{t+1})} \tag{28}$$

provided that $\gamma^*_{t,j} > 0$ (second order condition). The foreign trader's demand for foreign currency can now be derived by substituting (28) into budget equation (6):

$$Y^*_{t,j} = B^*_{t,j} + \frac{A^*_{t,j}}{S_t} - \frac{(1+I_t)E^*_{t,j}(S^{-1}_{t+1}) - \dfrac{1+I^*_t}{S_t}}{S_t(1+I_t)^2\gamma^*_{t,j}\mathrm{Var}^*_{t,j}(S^{-1}_{t+1})} \tag{29}$$

From (23) and (29) we obtain the aggregate excess demand for foreign currency:

$$\Phi(S_t) = \sum_{i=1}^{m}\frac{(1+I^*_t)E_{t,i}(S_{t+1}) - (1+I_t)S_t}{(1+I^*_t)^2\gamma_{t,i}\mathrm{Var}_{t,i}(S_{t+1})} - \sum_{i=1}^{m}B_{t,i} +$$

$$+\sum_{j=1}^{n}A^*_{t,j}S^{-1}_t - \sum_{j=1}^{n}\frac{(1+I_t)E^*_{t,j}(S^{-1}_{t+1})S^{-1}_t - (1+I^*_t)S^{-2}_t}{(1+I_t)^2\gamma^*_{t,j}\mathrm{Var}^{*\,2}_{t,j}(S^{-1}_{t+1})} \tag{30}$$

3.4 Exchange rate expectations

Assume the following specification of expectations formation:

$$E_{t,i}(S_{t+1}) = S_t^{\,1-\alpha_{t,i}} Z_{t,i}^{\,\alpha_{t,i}} \tag{31}$$

$$E^*_{t,j}(S^{-1}_{t+1}) = S_t^{\,\alpha^*_{t,j}-1} Z^{*\,-\alpha^*_{t,j}}_{t,j} \tag{32}$$

$$\mathrm{Var}_{t,i}(S_{t+1}) = \omega_{t,i}S^2_t \tag{33}$$

$$\mathrm{Var}^{*}_{t,j}(S^{-1}_{t+1}) = \omega^{*\;-1}_{t,j} S^{-2}_{t} \tag{34}$$

The variance specifications (33)-(34) reflect a variance of S_{t+1}/S_t which - conditional on all information in period t - does not change over time. This is appropriate if one period is interpreted as one month or longer (for our purposes: three months): Baillie & Bollerslev (1989) found that ARCH effects in daily and weekly exchange rates seem to disappear for monthly exchange rates.

Definition 1 (Heterogeneous traders market)

A heterogeneous traders market is a market consisting of

m (≥ 1) domestic traders described by

(2), (4), (5), (13), (16), (19), (20)-(23), (31), (33),

n (≥ 1) foreign traders described by

(3), (6), (7), (14), (15), (24)-(29), (32), (34),

and which has aggregate properties

(8)-(12), (17), (18), (30).

3.5 The equilibrium exchange rate

If we substitute (31)-(34) into aggregate demand function (30), we obtain

$$\Phi(S_t) = \sum_{i=1}^{m} \frac{(1+I^*_t)S_t^{1-\alpha_{t,i}} Z_{t,i}^{\alpha_{t,i}} - (1+I_t)S_t}{(1+I^*_t)^2 \gamma_{t,i}\omega_{t,i}S_t^2} - \sum_{i=1}^{m} B_{t,i} +$$

$$+\sum_{j=1}^{n} A^{*}_{t,j} S_t^{-1} - \sum_{j=1}^{n} \frac{(1+I_t) S_t^{\alpha^{*}_{t,j}-1} Z^{*\,-\alpha^{*}_{t,j}}_{t,j} S_t^{-1} - (1+I_t^{*}) S_t^{-2}}{(1+I_t)^2 \gamma^{*}_{t,j} \omega^{*\,-1}_{t,j} S_t^{-2}} \tag{35}$$

After rearranging terms we have

Proposition 1 (Equilibrium exchange rate)

In a heterogeneous traders market an equilibrium exchange rate is any real positive solution S_t to

$$\sum_{i=1}^{m} \frac{Z_{t,i}^{\alpha_{t,i}}}{(1+I_t^{*})\gamma_{t,i}\omega_{t,i}} S_t^{-1-\alpha_{t,i}} - \sum_{j=1}^{n} \frac{Z^{*\,-\alpha^{*}_{t,j}}_{t,j}}{(1+I_t)\gamma^{*}_{t,j}\omega^{*\,-1}_{t,j}} S_t^{\alpha^{*}_{t,j}} +$$

$$+\left(\sum_{j=1}^{n} A^{*}_{t,j} - \sum_{i=1}^{m} \frac{1+I_t}{(1+I_t^{*})^2 \gamma_{t,i}\omega_{t,i}}\right) S_t^{-1} + \left(\sum_{j=1}^{n} \frac{1+I_t^{*}}{(1+I_t)^2 \gamma^{*}_{t,j}\omega^{*\,-1}_{t,j}} - \sum_{i=1}^{m} B_{t,i}\right) = 0$$

An equilibrium solution is a root of a polynomial in m+n+1 factors of S_t with a constant. The polynomial of proposition 1 thanks its convenient form to the variance specifications (33) and (34). Obviously, the polynomial may have several real-valued roots. In section 3.1 we already mentioned that the international banking system - knowing the foreign exchange supply and demand schedules of all traders - acts as a Walrasian auctioneer to clear the foreign exchange market. We can now be more specific: in period t the Walrasian auctioneer uses the Newton-Raphson algorithm with initial value S_{t-1} to compute the nearest real-valued root of the polynomial of proposition 1. Although more than one equilibrium exchange rate may exist, we choose the one that is selected by the Walrasian auctioneer.

For each period t, we can supply empirically plausible numerical values for I_t, I^*_t, $Z_{t,i}$, $Z^*_{t,j}$, $\alpha_{t,i}$, $\alpha^*_{t,j}$, $\gamma_{t,i}$, $\gamma^*_{t,j}$, $\omega_{t,i}$, $\omega^*_{t,j}$, $A^*_{t,j}$, $B_{t,i}$ and compute the equilibrium exchange rate S_t from proposition 1. This will transform the theoretical model into an artificial economy.

4. The artificial economy

4.1 Number of traders and periods

We will assume that there are 200 traders: m = 100 domestic traders and n = 100 foreign traders. The length of the experiment is related to the time interpretation. Exchange rates have been floating since March 1973, hence by the end of 1997 there were 100 quarterly exchange rate observations under free float. With the use of statistical tables (Dickey-Fuller tests) in mind, we will set the length of the experiment at 100 periods.

4.2 Initial wealth

The initial currency endowments are obtained in two stages. First we draw domestic and foreign currency endowments from a uniform distribution. Then we multiply with scale parameters λ and λ^*. Their relative size reflects the relative sizes of the domestic and foreign economies. These scale parameters are also drawn from a uniform distribution:

$$\lambda \sim U(0,1) \tag{36}$$

$$\lambda^* \sim U(0,1) \tag{37}$$

The initial currency endowments of domestic traders are described by:

$$A_{1,i} \sim \lambda . 10^5 . U[0,1] \tag{38}$$

$$B_{1,i} \sim \lambda . 10^5 . U[0,1] \tag{39}$$

The initial currency endowments of foreign traders are:

$$A^*_{1,j} \sim \lambda^* . 10^5 . U[0,1] \tag{40}$$

$$B^*_{1,j} \sim \lambda^* . 10^5 . U[0,1] \tag{41}$$

4.3 Interest rates

We let the domestic interest rate follow a random walk:

$$I_t = I_{t-1} + \eta_t \tag{42}$$

with the (for quarterly interest rates) empirically plausible

$$\eta_t \sim N(0,10^{-5}) \qquad (43)$$

and an initial value drawn from a uniform distribution, such that the annualized quarterly interest rate starts at a value between 2% and 10%:

$$100\{(1+I_1)^4-1\} \sim U[2,10] \qquad (44)$$

Similarly for foreign interest rates:

$$I^*_t = I^*_{t-1} + \eta^*_t \qquad (45)$$

$$\eta^*_t \sim N(0,10^{-5}) \qquad (46)$$

$$100\{(1+I^*_1)^4-1\} \sim U[2,10] \qquad (47)$$

4.4 Risk aversion update

The measure of risk aversion in our model is the absolute risk aversion parameter $\gamma_{t,i}$. However, Friend & Blume (1975) and Landskroner (1977) found empirical evidence of constant relative risk aversion (hence decreasing absolute risk aversion).

In order to approximate constant relative risk aversion we will use the update formula

$$\gamma_{t,i} = \frac{\overline{\kappa}_i}{\left|W_{t-1,i}\right|} \tag{48}$$

with $W_{0,i} = A_{1,i} + S_0 B_{1,i}$ ($S_0 = 1$). If $W_{t-1,i} > 0$, $\overline{\kappa}_i$ is the target value of the relative risk aversion. Since $W_{t-1,i}$ is only an approximation of $E_{t,i}(W_{t+1,i})$, the actual relative risk aversion

$$\kappa_{t,i} = E_{t,i}(W_{t+1,i})\,\gamma_{t,i} \tag{49}$$

will hover around $\overline{\kappa}_i$. If $W_{t-1,i} < 0$, update formula (48) ensures a positive absolute risk aversion $\gamma_{t,i}$, given a positive $\overline{\kappa}_i$. In section 3.3 we mentioned that this is required for the second order condition for the optimal foreign currency position of domestic trader i. Notice that this update implies that the more negative a trader's wealth becomes, the more risk he is going to take: in this case we are dealing with "rogue traders". In this subsection, we have discussed only the domestic traders, but similar statements hold for foreign traders.

4.5 Reinterpretation of risk aversion and predicted variance

In each equation the absolute risk aversion parameter and the predicted variance parameter appear as a product. For domestic traders this "risk product" is $\gamma_{t,i}\ \omega_{t,i}$ and for foreign traders it is $\gamma^*_{t,j}$ $/\omega^*_{t,j}$. Hence for the simulation output it is irrelevant whether the risk-adjustment of currency positions is caused by risk aversion or predicted exchange rate variance. We can exploit this property to neutralize the effects of second moment expectations $Var_{t,i}(S_{t+1})$ and $Var^*_{t,j}(1/S_{t+1})$. After each experiment we can reinterpret the risk product components such that all traders have rational second moment expectations. Suppose that a specific experiment generates a sample variance

$$\mathrm{Var}\left(\Delta s_t\right) = \tilde{\omega} \tag{50}$$

Then the predicted variance of domestic trader i coincides with the sample variance if

$$\omega_{t,i} = \tilde{\omega} \tag{51}$$

However, in general this will not hold for the value of $\omega_{t,i}$ that we put into the experiment. We can remedy this by redefining this parameter after the simulation is completed. We have to keep the risk product of domestic trader i the same:

$$\gamma_{t,i}\omega_{t,i} = \tilde{\gamma}_{t,i}\tilde{\omega} \tag{52}$$

This implies that we also have to redefine the absolute risk aversion parameter:

$$\tilde{\gamma}_{t,i} = \frac{\omega_{t,i}}{\tilde{\omega}} \gamma_{t,i} \qquad (53)$$

In turn this implies a reinterpretation of the relative risk aversion parameter. Since

$$\kappa_{t,i} = E_{t,i}(W_{t+1,i})\, \gamma_{t,i} \qquad (54)$$

and

$$\tilde{\kappa}_{t,i} = E_{t,i}(W_{t+1,i})\tilde{\gamma}_{t,i} \qquad (55)$$

it holds that

$$\tilde{\kappa}_{t,i} = \frac{\tilde{\gamma}_{t,i}}{\gamma_{t,i}} \kappa_{t,i} \qquad (56)$$

and equivalently

$$\tilde{\kappa}_{t,i} = \frac{\omega_{t,i}}{\tilde{\omega}} \kappa_{t,i} \qquad (57)$$

The importance of this post-experiment reinterpretation of risk is that it makes it impossible to attribute output results to irrationality in the predicted variance (assuming that the generated exchange rate series does not exhibit volatility clustering). Having neutralized the second moment expectations $Var_{t,i}(S_{t+1})$ and $Var^*_{t,j}(1/S_{t+1})$, we can attribute all simulation results to the first moment expectations $E_{t,i}(S_{t+1})$ and $E^*_{t,j}(1/S_{t+1})$, which are after all the focus of our experiments.

Landskroner (1977) estimated $\kappa_{t,i}$ between 2.4 and 8.2. Hansen & Singleton (1983) found values between 0 and 2. Dunn & Singleton (1986) found values between 1.2 and 3.5. To be safe, we assume that relative risk aversion is between 0 and 5. We will set the predicted variance at the empirically observed value (see table 2)

$$\omega_{t,i} = 4.10^{-3} \tag{58}$$

and draw $\overline{\kappa}_i$ from a uniform distribution:

$$\overline{\kappa}_i \sim U[1,5] \tag{59}$$

After a simulation is completed, we reinterpret the predicted variance and risk aversion parameters according to (51), (53) and (57). If the average value of $\tilde{\kappa}_{t,i}$ over all traders and over all periods exceeds 5, we discard the simulation on the basis of empirically implausible high relative risk aversion. Although we set the lower boundary of $\overline{\kappa}_i$ at 1 (for numerical reasons), the lower boundary for $\tilde{\kappa}_{t,i}$ is 0 (as suggested by the empirical evidence).

In this subsection, we concentrated on the domestic traders, but analogous statements hold for foreign traders.

4.6 Exchange rate expectations, trader types and market types

We distinguish between 4 types of individual traders, as summarized in table 1. For simplicity, the descriptions are stated for domestic traders. The corresponding descriptions for foreign traders are analogous.

Table 1: Types of traders

trader type	type of information $Z_{t,i}$	use of information $\alpha_{t,i}$	expectations mechanism $E_{t,i}(\Delta s_{t+1})$
distributed lag expectations	S_{t-1}	> 0	$- \alpha_{t,i} \Delta s_{t+1}$
bandwagon expectations	S_{t-1}	< 0	$- \alpha_{t,i} \Delta s_{t+1}$
adaptive expectations	$E_{t-1,i}(S_t)$	> 0	$\alpha_{t,i} (E_{t-1,i}(s_t) - s_t)$
regressive expectations	$Q_{t,i}$	> 0	$\alpha_{t,i} (q_{t,i} - s_t)$

We define $S_0 = E_{-1,i}[S_0] = 1$ and we use a random walk to represent the natural logarithm q_t of the "fundamental" exchange rate Q_t:

$$q_t = q_{t-1} + \upsilon_t \tag{60}$$

$$\upsilon_t \sim N(0,\omega) \tag{61}$$

$$q_1 = 0 \tag{62}$$

Parameter ω is set at the value indicated by equation (58). We will assume that each trader has an expectations scheme that is fixed over time: $Z_{t,i} = Z_i$ and $\alpha_{t,i} = \alpha_i$. Based on the 4 types of individual traders we will consider 5 types of markets (in definitions 2-6 the symbol α_i refers to both domestic and foreign traders):

Definition 2 (Bandwagon expectations market)

A bandwagon expectations market is a heterogeneous traders market which consists of bandwagon expectations traders with $\alpha_i \sim \overline{\alpha}.LN(-0.125,0.25)$ and $\overline{\alpha} < 0$.

With $y \sim LN(\mu,\sigma^2)$ we mean that $y = \exp(x)$ with $x \sim N(\mu,\sigma^2)$. It holds that $E[y] = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$, hence $E\left[\overline{\alpha}y\right] = \overline{\alpha}\exp\left(\mu + \frac{1}{2}\sigma^2\right) = \overline{\alpha}$ if $\sigma^2 = -2\mu$.

In other words: if we set $\sigma^2 = -2\mu$, the average alpha is given by $\overline{\alpha}$.

We have chosen $\mu = -0.125$ and $\sigma^2 = 0.25$ after inspection of the results with a random number generator. Large spreads imply implausible individual values for alpha, while small spreads exhibit implausibly little heterogeneity. The spread of individual values of alpha with $\mu = -0.125$ and $\sigma^2 = 0.25$ seemed appropriate.

Definition 3 (Distributed Lag expectations market)

A distributed lag expectations market is a heterogeneous traders market which consists of distributed lag expectations traders with $\alpha_i \sim \overline{\alpha}.LN(-0.125,0.25)$ and $\overline{\alpha} > 0$.

Definition 4 (Extrapolative expectations market)

An extrapolative expectations market is a heterogeneous traders market which consists of bandwagon and distributed lag expectations traders with $\alpha_i \sim N(\overline{\alpha},0.25)$.

The variance parameter 0.25 was again selected on the basis of inspection of random number generator results.

Definition 5 (Adaptive expectations market)

An adaptive expectations market is a heterogeneous traders market which consists of adaptive expectations traders with $\alpha_i \sim \overline{\alpha}.LN(-0.125,0.25)$ and $\overline{\alpha} > 0$.

Definition 6 (Regressive expectations market)

A regressive expectations market is a heterogeneous traders market which consists of regressive expectations traders with $\alpha_i \sim \overline{\alpha}.LN(-0.125,0.25)$ and $\overline{\alpha} > 0$.

In each of the markets described by definitions 2-6, the traders use the same type of information Z_t to form exchange rate expectations $E_{t,i}(S_{t+1})$. However, they are heterogeneous in the way that they use the information. The individual expectations parameter α_i, which indicates the weight that is being given to the information Z_t, is drawn from a distribution around $\overline{\alpha}$, which in turn indicates how the "average" trader uses the information Z_t.

5. The experiments

5.1 Experimental design

In section 4 we transformed the theoretical model of section 3 into an artificial economy with a flexible specification of the exchange rate expectations mechanisms of individual traders. We defined 5 market types, corresponding with different exchange rate expectations mechanisms. With each market type, we will perform experiments that can be depicted in figure 2:

Figure 2: The experiment

pre-exchange currency positions: $A_{t,i}$, $B_{t,i}$, $A^*_{t,j}$, $B^*_{t,j}$

interest rate time series: I_t, I^*_t

risk aversion parameters: $\gamma_{t,i}$, $\gamma^*_{t,j}$

predicted variance parameters: $\omega_{t,i}$, $\omega^*_{t,j}$

expectations formation: $\alpha_{t,i}$, $z_{t,i}$, $\alpha^*_{t,j}$, $z^*_{t,j}$

$\downarrow$ input

model of foreign exchange between heterogeneous traders

$\downarrow$ output

exchange rate time series: s_t

exchange rate expectations: $E_{t,i}(s_{t+1})$, $E^*_{t,j}(s_{t+1})$

We described the input of the experiment in section 4 and the model of foreign exchange between heterogeneous traders in section 3. In this section we will discuss the output and investigate if there is a relationship with specific input.

5.2 Empirically plausible output

We want to evaluate the output of the experiments, in particular the artificial exchange rate series, on their empirical plausibility. This requires investigating the stylized facts of empirically observed quarterly exchange rates. Apart from the first 4 moments of Δs_t, we look at 4 test statistics that are inspired by the (weak-form) efficient market hypothesis and the statistical phenomena of volatility clusters and fat tails.

Under weak-form market efficiency, the (log) exchange rate s_t reflects all information contained in the lagged exchange rates s_{t-1}, s_{t-2}, . . . (see for example Baillie & McMahon (1989)). Consider the univariate model

$$s_t = \lambda\, s_{t-1} + \varepsilon_t \tag{63}$$

where ε_t is an error term. If we take expectations conditional on the lagged exchange rates, we obtain

$$E_{t-1}[s_t] = \lambda\, s_{t-1} + E_{t-1}[\varepsilon_t] \tag{64}$$

or equivalently

$$E_{t-1}[\Delta s_t] = (\lambda-1)\ s_{t-1} + E_{t-1}[\varepsilon_t] \tag{65}$$

Under weak-form market efficiency, it should hold that $E_{t-1}[\Delta s_t] = 0$. This implies that $\lambda = 1$ and that ε_t should not contain serial correlation. In other words, s_t should follow a martingale process. These properties can be tested with a Dickey-Fuller unit root test on s_t and a Box-Pierce Q-test for serial correlation in Δs_t.

Among the stylized facts of high frequency exchange rates are volatility clusters and fat tails (de Vries (1995)). We verify whether these properties are retained for low frequency data such as our quarterly exchange rates. We will test for volatility clusters by performing a Box-Pierce Q-test for serial correlation in $(\Delta s_t)^2$. We test for fat tails with a Jarque-Bera test for normality of Δs_t. The results are described in table 2:

Table 2: Summary statistics of empirical quarterly exchange rates 1973-1995

	mean(Δs) x10^{-3}	var(Δs) x10^{-3}	skew(Δs)	kurt(Δs)	DF(s)	BP_8(Δs)	BP_8(Δs^2)	JB(Δs)
$/AS	7.878	4.083	- 0.168	2.471	- 1.29	13.31 (0.1015)	4.67 (0.7919)	1.71 (0.4243)
$/BFr	3.400	4.240	- 0.324	2.647	- 0.57	13.64 (0.0916)	5.39 (0.7149)	0.94 (0.6259)
$/Can$	- 3.432	0.476	0.016	3.149	0.61	9.71 (0.2861)	2.68 (0.9526)	3.15 (0.2065)
$/DKr	1.237	3.779	- 0.153	2.503	- 0.33	11.88 (0.1568)	3.38 (0.9080)	0.92 (0.6327)
$/FKr	- 1.307	2.666	- 0.306	2.863	0.07	20.66 (0.0081)	7.17 (0.5185)	2.28 (0.3206)
$/FFr	- 0.836	3.779	- 0.169	2.565	- 0.05	10.55 (0.2287)	4.85 (0.7738)	1.20 (0.5478)
$/DM	7.507	4.258	- 0.147	2.629	- 1.45	13.16 (0.1066)	3.74 (0.8801)	1.10 (0.5771)
$/IP	- 4.768	3.408	- 0.063	2.501	- 1.54	22.41 (0.0042)	3.47 (0.9014)	2.10 (0.3507)
$/Lira	-11.000	3.576	- 0.556	3.474	1.67	8.21 (0.4128)	2.84 (0.9440)	20.51 (0.0000)
$/ **-**	10.437	3.741	0.213	2.791	- 1.65	12.74 (0.1212)	4.19 (0.8393)	7.57 (0.0227)
$/ ***l***	6.673	4.084	- 0.240	2.504	- 1.29	14.19 (0.0771)	3.56 (0.8944)	1.15 (0.5637)
$/NKr	- 0.745	2.843	- 0.681	3.907	- 0.01	16.65 (0.0340)	3.86 (0.8693)	12.21 (0.0022)
$/esc	-19.510	3.848	- 0.415	3.309	2.66	14.63 (0.0667)	7.30 (0.5051)	24.48 (0.0000)
$/pta	- 8.042	3.323	- 0.509	3.169	1.22	9.95 (0.2685)	6.30 (0.6136)	13.67 (0.0011)
$/SFr	11.369	5.239	0.123	2.831	- 2.02	8.80 (0.3596)	7.55 (0.4781)	5.12 (0.0772)
$/ **,**	- 5.155	3.165	- 0.228	2.884	- 1.45	18.15 (0.0201)	4.42 (0.8170)	3.84 (0.1465)

var(x) is variance of x, skew(x) is skewness of x, kurt(x) is kurtosis of x

DF(x) is Dickey-Fuller test statistic (H_0: x contains unit root) which has tabulated critical values

BP_8(x) is Box-Pierce test statistic (H_0: first 8 autocorrelations of x are zero) which is $\chi^2(8)$ under H_0

JB(x) is Jarque-Bera test statistic (H_0: x is normally distributed) which is $\chi^2(2)$ under H_0

p-values of BP- and JB-test statistics between parentheses

Data source: Main Economic Indicators, various issues

First consider the results on weak-form market efficiency. The Dickey-Fuller test indicates a unit root for 15 out of the 16 exchange rates at the 5% significance level and for all exchange rates at the 2.5% significance level (see Fuller (1976), table 8.5.2). The Box-Pierce Q-tests indicate the absence of serial correlation in Δs_t for only 9 out of the 16 exchange rates at the usual 10% significance level, 12 out of 16 at the 5% level and 14 out of 16 at the 1% level. Since the significance level indicates the probability of mistakenly accepting the presence of serial correlation, strong believers in weak-form market efficiency may still interpret the DF(s) and $BP_8(\Delta s)$ results as evidence of weak-form market efficiency. However, it seems more sensible to interpret the results as an indication that quarterly exchange rates are close to a martingale process, while exhibiting some serial correlation. For our purposes, i.e. the comparison of statistical properties of artificial data with those of empirical data, this discussion is not very important: we are more interested in the range of values that the test statistics assume. Therefore we will call DF(s) > -1.95 (the usual 5% significance level) and $BP_8(\Delta s)$ < 22.55 (corresponding with a nice round p-value of 4.10^{-3}) empirically plausible output.

Now we look at the results on volatility clustering. The $BP_8(\Delta s^2)$ tests indicate the absence of volatility clustering for all exchange rates. This result confirms the results of Baillie & Bollerslev (1989), who found that ARCH effects in daily and weekly exchange rates seem to disappear for monthly exchange rates. Hence for empirically plausible output we require $BP_8(\Delta s^2) < 13.36$, the usual 10% significance level.

Finally we consider the results on normality: the JB(Δs) test statistics indicate normality for only 11 out of 16 exchange rates at the 5% significance level and 13 out of 16 at the 1% significance level. Hence normally distributed exchange rate returns do not seem to be a stylized fact even for quarterly data. The highest JB(Δs) value is 24.48. Therefore for empirically plausible output we require JB(Δs) < 24.86, which corresponds with a p-value of 4.10^{-6}. We summarize the results in definition 7:

Definition 7 (Empirically plausible output)

An artificial economy generates empirically plausible output, if the simulated (log) exchange rate series s_t has the following statistical properties:

$DF(s) > -1.95$

$BP_8(\Delta s) < 22.55$

$BP_8(\Delta s^2) < 13.36$

$JB(\Delta s) < 24.86$

The first two properties of definition 4 correspond with a process that approximates a martingale: an exact martingale is characterized by $BP_8(\Delta s) < 13.36$. The economic interpretation of such a process is approximate weak-form efficiency. The third property adds the restriction that volatility clustering is absent. The fourth property reflects an upper limit for the fatness of the tails that we have found for empirically observed quarterly exchange rates. The tail fatness is positively related to the occurrence of extreme exchange rate returns in the sample. In the next subsection we will use this criterion to evaluate the experimental output.

5.3 Simulation results

We simulate all market types with m = 100 domestic traders and n = 100 foreign traders. Each market type is characterized by the type of information Z and the use of information α. With respect to the type of information, we distinguish between adaptive expectations markets, bandwagon expectations markets, distributed lag expectations markets, extrapolative expectations markets and regressive expectations markets. At the same time, we consider different values for the “average” use of information $\overline{\alpha}$. For each market type we perform 100 simulations of each 100 periods and we count the number of time series which can be identified as martingales, martingales without volatility clustering, random walks and empirically plausible time series.

For the adaptive expectations market, we consider the following values of the average α: $\overline{\alpha} = 0.05$, 0.10, 0.15, 0.20, 0.25, 0.30. Empirical estimates of α range from 0.07 to 0.19 (Frankel & Froot (1987a) and (1987b), Cavaglia, Verschoor & Wolff (1993)). The simulation results are reported in table 3:

Table 3: Adaptive expectations markets (100 simulations)

average α	martingales $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$	martingales without volatility clustering $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$	random walks $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 5.99$	empirically plausible $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 22.55$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 24.86$
0.05	0	0	0	0
0.10	0	0	0	0
0.15	0	0	0	0
0.20	5	4	0	0
0.25	8	8	0	0
0.30	6	6	0	0

The results indicate that adaptive expectations markets with empirically plausible values of α are incapable of generating empirically plausible exchange rates. In a small number of cases the adaptive expectations market is able to produce a martingale process. However, in these cases the Jarque-Bera test statistics are far too high (not lower than 2026.36*)* in comparison with empirical results. Although adaptive expectations markets are sometimes weak-form efficient and exhibit no volatility-clustering, they generate too many extreme returns to be empirically plausible.

For the bandwagon expectations market, we consider the following values of the average α: $\overline{\alpha}$ = -0.25, -0.20, -0.15, -0.10, -0.05. Empirical estimates of α for extrapolative expectations range from -0.07 to 0.58 (Frankel & Froot (1987a) and (1987b), Bank of Japan (1989), Froot & Frankel (1990), Cavaglia, Verschoor & Wolff (1993)). The results are reported in table 4:

Table 4: Bandwagon expectations markets (100 simulations)

average α	martingales $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$	martingales without volatility clustering $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$	random walks $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 5.99$	empirically plausible $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 22.55$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 24.86$
- 0.25	0	0	0	0
- 0.20	0	0	0	0
- 0.15	0	0	0	0
- 0.10	0	0	0	0
- 0.05	0	0	0	0

The bandwagon expectations market performs even worse than the adaptive expectations market. For each of the 5 parameter values, there is not a single simulation that generates an exchange rate series that passes the four tests of empirical plausibility. The main deficiency of bandwagon expectations markets is that they are not weak-form efficient, which means that the most recent exchange rate return contains information on the direction and size of the next exchange rate change.

For the distributed lag expectations market, we consider 6 values of the average α:

$\overline{\alpha}$ = 0.10, 0.20, 0.30, 0.40, 0.50, 0.60. We just mentioned that the empirical estimates of α for extrapolative expectations range from -0.07 to 0.58. The results are reported in table 5:

Table 5: Distributed Lag expectations markets (100 simulations)

average α	martingales $DF(s_t)$ > - 1.95 $BP_8(\Delta s_t)$ < 13.36	martingales without volatility clustering $DF(s_t)$ > - 1.95 $BP_8(\Delta s_t)$ < 13.36 $BP_8(\Delta s_t^2)$ < 13.36	random walks $DF(s_t)$ > - 1.95 $BP_8(\Delta s_t)$ < 13.36 $BP_8(\Delta s_t^2)$ < 13.36 $JB(\Delta s_t)$ < 5.99	empirically plausible $DF(s_t)$ > - 1.95 $BP_8(\Delta s_t)]$ < 22.55 $BP_8(\Delta s_t^2)$ < 13.36 $JB(\Delta s_t)$ < 24.86
0.10	0	0	0	0
0.20	0	0	0	0
0.30	0	0	0	0
0.40	0	0	0	0
0.50	0	0	0	0
0.60	0	0	0	0

The distributed lag expectations market suffers from the same problem as the bandwagon expectations market: the exchange rate process is predictable (in a linear sense). Notice that these two expectations mechanisms are mirror images of each other.

The extrapolative expectations market contains both bandwagon and distributed lag expectations traders. For this market, we consider 10 values of the average α:

$\overline{\alpha}$ = -0.20, -0.10, 0.00, 0.10, 0.20, 0.30, 0.40 0.50, 0.60, 0.70. The empirically observed range [-0.07,0.58] is contained in this spectrum. The results are reported in table 6:

Table 6: Extrapolative expectations markets (100 simulations)

average α	martingales $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$	martingales without volatility clustering $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$	random walks $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 5.99$	empirically plausible $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 22.55$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 24.86$
- 0.20	40	38	0	0
- 0.10	56	53	0	0
0.00	66	61	0	0
0.10	66	62	0	0
0.20	62	56	0	0
0.30	54	54	0	0
0.40	47	41	0	0
0.50	43	40	0	0
0.60	43	41	0	0
0.70	27	25	0	0

The presence of both types of traders considerably improves weak-form market efficiency, especially in the [-0.1,0.3] region, where more than half of the simulations follow a martingale process without volatility clusters. This performance is also much better than for adaptive expectations markets. However, the simulated extrapolative expectations markets which exhibit weak-form market efficiency generate too many extreme exchange rate returns to be empirically plausible. As a matter of fact, only 1% of all simulations have a Jarque-Bera test statistic below 24.86.

For the regressive expectations market, we consider 10 values of the average α: $\overline{\alpha}$ = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10. The empirically observed nonnegative parameters α for regressive expectations are 0.02 and 0.09 (Frankel & Froot (1987a) and (1987b), Bank of Japan (1989), Froot & Frankel (1990)). The results are reported in table 7:

Table 7: Regressive expectations markets (100 simulations)

average α	martingales $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$	martingales without volatility clustering $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$	random walks $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 13.36$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 5.99$	empirically plausible $DF(s_t) > -1.95$ $BP_8(\Delta s_t) < 22.55$ $BP_8(\Delta s_t^2) < 13.36$ $JB(\Delta s_t) < 24.86$
0.01	52	36	27	43
0.02	59	47	38	54
0.03	67	50	36	54
0.04	59	51	44	58
0.05	67	56	44	63
0.06	76	64	50	64
0.07	59	45	33	49
0.08	62	48	39	57
0.09	61	47	38	54
0.10	55	39	29	42

The regressive expectations market is the only market that is able to generate empirically plausible exchange rate series. The performance is best for $\overline{\alpha}$ = 0.05-0.06, which is in the middle area of the range of empirical estimates. For these average expectations parameter values, regressive expectations markets generate empirically plausible exchange rate series in more than 60 of the 100 simulations. In other words, for about 60% of the “states of the world” (as represented by the artificial distributions of initial currency endowments, interest rates, risk aversion and expectations parameters) the regressive expectations market seems to be a plausible representation of the foreign exchange market.

Notice that the regressive expectations market is only marginally better than the extrapolative expectations market in generating exchange rate processes which are weak-form efficient and free of volatility clusters: the averages are 48.3 and 47.1 respectively. The crucial difference is that the tails of the distribution of exchange rate returns are considerably less fat in case of the regressive expectations market than in case of the extrapolative expectations market. In other words, the occurrence of extreme exchange rate returns in the regressive expectations market is comparable with empirical markets.

It should be kept in mind that regressive expectations may take several forms. The survey data studies considered purchasing power parity, some constant long run exchange rate level and a moving average of the exchange rate. For the artificial economies we simply generated a random walk and presented it as the fundamental exchange rate. Of course, this is interesting in itself. Even if this random walk has no relationship at all with the “fundamentals” in the economy, the artificial foreign exchange market exhibits the same statistical behavior as empirical currency markets. In other words: the relevant issue is *that* traders believe in a fundamental exchange rate, it does not seem to matter *what* the fundamental exchange rate is based on. The “fundamental exchange rate” may be as fictitious as the perceived patterns in the “charts”.

6. Conclusion

A clear picture emerges from the simulations: artificial economies with “technical” expectations schemes (adaptive, bandwagon, distributed lag, extrapolative) do not generate exchange rate series with statistical properties that match those of empirically observed exchange rates. Bandwagon, distributed lag and adaptive expectations markets are rarely weak-form efficient. Extrapolative expectations markets are frequently weak-form efficient and free of volatility clusters, but generate too many extreme returns compared with empirical foreign exchange markets. The artificial economies with “fundamentalist” expectations schemes perform considerably better. Regressive expectations markets generate empirically plausible exchange rate series in about 40 to 60 of the 100 simulations, depending on the average α. The simulations with the artificial foreign exchange market therefore suggest that regressive expectations are the most plausible representation of quarterly exchange rate expectations, when compared with extrapolative, bandwagon, distributed lag and adaptive expectations. The (perceived) presence of an ‘anchor’ seems to play an important role in the functioning of foreign exchange markets, determining the frequency of extreme exchange rate returns.

In this paper we performed experiments with artificial markets consisting of traders which are heterogeneous with respect to initial endowments and risk aversion. However, in each simulated market all traders base their exchange rate expectations on the same type of information z. The expectations are heterogeneous because of individual differences in the use of information, as expressed by the weight α. In other words, the artificial markets exhibit “weak-form heterogeneity” with respect to expectations. This form of heterogeneity is appropriate for a comparison with survey data studies on exchange rate expectations, such as Frankel & Froot (1987a). It allowed us to investigate the plausibility of adaptive, bandwagon, distributed lag, extrapolative and regressive expectations markets. The next step is to perform experiments with markets consisting of, for example, both “chartists” (extrapolative traders) and “fundamentalists” (regressive expectations traders).

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