Nonparametric identification of latent competing risks and Roy duration models

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Abstract: This paper considers nonparametric identification of "latent" competing risks and Roy duration models in which one does not know which process has been observed. It is shown that these models are identifiable without the usual conditional independence and exclusion restrictions.

1. Introduction

Competing risks models have been widely applied in the statistical sciences. As is often the case with nonlinear and multivariate models, identification can be problematic and a number of authors have addressed this issue in various contexts. Cox (1962) and Tsiatis (1975) showed that, in the absence of independence, the underlying joint distribution is, in general, not identifiable. Heckman and Honoré (1989) and Abbring and van den Berg (1999) demonstrated that, with observable covariates, identification of these models is possible. Omori (1998) showed identifiability of independent competing risks with multiple spells. A feature of these studies, and that of most empirical work on competing risks, is that the researcher, in addition to observing the minimum survival time, also knows the cause of failure. However, this is not always the case, and there are many situations when an outcome is the minimum (or maximum) of several processes, but is inappropriately modeled as a univariate process. At best, this results in estimates of a "reduced form" whose parameters may be uninterpretable. At worst this leads to misspecification and inconsistent estimates of the underlying "structural" models. The purpose of this study is to consider situations when these "latent" competing risk models are identified.

It is useful to consider some examples. In epidemiology, (see Lee, 1992, for various studies), a subject's death may be attributed to one particular disease when in fact it may have been caused by any one of several disorders. Likewise, in reliability research, (see Meeker and Escobar, 1998, for various studies), one may observe an equipment failure, but be uncertain as to what actually caused the breakdown. A third example is when a data set has been subjected to right censoring, but the researcher is unaware which of the

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spells have been censored. In econometrics, the length of strikes is typically modeled as a univariate process (Kennan, 1985) but, under collective bargaining, strikes only end when both employers and unions agree to return to work. The timing of financial transactions has been modeled as a univariate process (Darolles, Gourieroux and Le Fol, 1998), but occurs only when buyer and seller have agreed on a price. In health economics the length of hospital stays is typically modeled as univariate (Holt, Merwin and Stern, 1996) although in some circumstances the patient may be discharged only when both he or she and the caregiver are agreed they are ready.

These examples have two common characteristics. The first is that, underlying the duration outcome, there are in fact two or more known random processes and one observes the minimum (or the maximum) of these. In this sense they are competing risks models. However, the second characteristic is that the researcher is unaware which process has actually been observed. For this reason we refer to these as latent competing risks. In many situations, the structural parameters of interest may be those of the joint distribution rather than those of the reduced form mixture distribution. The inference problem is to estimate the parameters of this joint distribution when one only observes the minimum (or maximum) of these variables. The logical starting point and the focus of this paper is whether and under what circumstances one can even identify the parameters of this joint distribution.

One difference amongst the above examples is that the first three, from epidemiology, reliability analysis and censoring, represent the minimum of two or more processes and are like standard competing risk models except one does not know which risk is observed. In the other three examples, of strikelength, timing of financial transactions and length of stay, the outcome is the maximum of two processes. From a formal perspective one can clearly take the negative of the maximum of several processes and view this as a competing risks model. However, we examine both the competing risks and "Roy duration" models. In the economics literature, the Roy (1951) model is used to analyze markets in which one observes the maximum of two or more random variables (typically wages and not duration variables) and numerous statistical results have been established with respect to the Roy model which are relevant to competing risk models. In discussing the identification of these models it also makes more intuitive sense to refer to the maximum of several duration variables rather than the minimum of the negative of these variables. The conditions we use for identification of these models, while symmetric, are different and it is of interest to consider their meaning in specific empirical situations.

Identification of the "latent" version of the Roy model[1] has been shown under certain conditions by Heckman and Honoré (1990), but their results do not cover all interesting situations. Specifically, their results apply when, conditional on covariates, (possibly unobservable) the durations are independent and exclusion restrictions are applied to achieve identification. This excludes situations where one wants to consider the direct interaction between durations. This is clearly important when examining, say, diseases or timing of marriage. In certain circumstances it may not be plausible to employ exclusion restrictions. For example, in the case of strikes, it may be argued that both parties condition on the same information and it makes little sense to include a covariate in one hazard rate

[1] i.e. for the case when a person's salary is known but not the sector they work in.

and exclude it from the other. In this paper we show how, in a standard duration context, each of these assumptions can be relaxed, using other restrictions which may be plausible in various situations.

This discussion proceeds as follows. In the next section we summarize the traditional approach to identification of duration models, using the approach of Elbers and Ridder (1982) and cast Heckman and Honoré's (1990) identification results into a duration context. In Section 3 we allow for direct dependence amongst the risk sets and show when identification is possible. In Section 4 we show how in some cases identification can be obtained even without exclusion restrictions. In each of these cases, our results provide sufficient conditions for identification. We provide specific, intuitive, examples of these.

As is standard in this literature, our results are nonparametric, in that they rely on exclusion, shape and normalization assumptions and do not rely on restricting the distributions to families indexable by a finite parameter space. It is most certainly the case that other conditions are sufficient as well. Our results may be indicative of other sufficient conditions. As is also quite standard we allow for unobservable frailty or heterogeneity. In the present context, there are unobservables entering into each hazard rate. These are allowed to be dependent although, as is typical, we assume the unobservable and observable covariates are independent. For clarity, we assume throughout that there are only two processes, although this can clearly be generalized.

2. Univariate identification and identification by exclusion restrictions

Our method of identification follows that of Elbers and Ridder (1982) for univariate duration models. Assume that the hazard rate, conditional on the observable covariates, x, and an unobservable, is separable so that

$$h(t|x,\nu) = \phi(t)\theta(x)\nu \tag{2.1}$$

where $\nu \geq 0$ has distribution function G. x and ν are assumed to be independent so that the observable survivor function is written

$$S(t|x) = \int_0^\infty e^{-\Phi(t)\theta(x)\nu} dG(\nu) \equiv L\left(\Lambda(t,x)\right) \tag{2.2a}$$

where

$$\Phi(t) = \int_0^t \phi(u)du, \qquad \Lambda(t,x) = \int_0^t \lambda(u,x)du, \qquad \lambda(t,x) = \phi(t)\theta(x) \tag{2.2b}$$

$$\lambda, \theta \geq 0$$

and L is the Laplace transform of G. Let $\theta'(x)$ denote the derivative of θ with respect to its first element, denoted x_1. To identify the latent competing risks/Roy models we will be using assumptions about the hazard rates for limiting values of the observable covariates. It is useful to prove a (weaker) variant on the basic Elbers and Ridder (1982) result using these kinds of conditions. First we formalize what is meant here by identification, adopting the definition of Roehrig (1988). In what follows, a "structure" is a set of functions thought to underlie the population distribution function. In the usual case of the univariate proportional mixed hazard rate model, a structure is a triple $\Gamma = \{\phi(t), \theta(x), G(\nu)\}$ such that $S(t|x) = \int \exp\{\phi(t)\theta(x)\nu\}dG(\nu)$. It is assumed that Γ lies in some set of triples, Ω, which satisfy certain regularity conditions.

Definition. Let $S(t|x)$ and $S^{\dagger}(t|x)$ be conditional survivor functions for T given the covariate $X = x$ implied by the structures Γ and $\Gamma^{\dagger}$. Then Γ and $\Gamma^{\dagger}$are observationally equivalent if $S(t|x) = S^{\dagger}(t|x)$

Definition. Γ is identifiable (in Ω) if there exists no other $\Gamma^{\dagger}$ (in Ω) which is observationally equivalent to Γ.

Remark 1. The definitions actually refer to identifiability of each of the "structural" functions at a point. In fact, with the exception of the last two of our results, all of the functions we consider are identifiable at each point of their domains. When this is not the case, we indicate for which values of their arguments each function is identified. ¤

Proposition 1. *Let the survivor function be as in (2.2). Suppose that $E[\nu] = L(0) < \infty$, $\phi(t) > 0$, $\Phi(0) = 0$, $\Phi(1) = 1$ and $\Phi(\infty) = \infty$. For some $x = x^1$ and $x = x^0$, $\theta(x^1) = 1$ and $\theta(x^0) = \lim_{x \to x^0} \theta(x) = 0$. $\theta(x)$ is continuously differentiable with respect to x_1 and $\theta'(x) > 0$, $x \neq x^0$. Then, θ, Φ and L are identifiable.*

Remark 2. The assumptions imply that the unobserved heterogeneity is assumed to have finite (not necessarily known) mean. Elbers and Ridder (1982) normalize $E[\nu]$. We find it more convenient, particularly with the multivariate generalizations below, to impose the normalization $\Phi(1) = 1$. (Any other positive value will do.) This is easily imposed in practice, for example with a Weibull baseline hazard, $\Phi(t) = t^{\alpha}$. The restrictions on the covariates are stronger than Elbers and Ridder (1982), but note that they are easily satisfied. A simple example is if $\theta(x) = \exp(x'\beta)$ with one of the x's taking values on the real line. ¤

Proof of Proposition 1. With $E[\nu]$ finite and $\theta(x^1) = 1$,

$$\begin{aligned} \frac{\frac{\partial}{\partial t}S(t|x)}{\frac{\partial}{\partial t}S(t|x^1)} &= \frac{\phi(t)\theta(x)L^{(1)}\left(\Lambda(t,x)\right)}{\phi(t)L^{(1)}\left(\Phi(t)\right)} \\ &= \theta(x)\frac{L^{(1)}\left(\Lambda(t,x)\right)}{L^{(1)}\left(\Phi(t)\right)} \end{aligned} \tag{2.3}$$

and we can identify θ by observing that

$$\theta(x) = \lim_{t \to 0} \frac{\frac{\partial}{\partial t}S(t|x)}{\frac{\partial}{\partial t}S(t|x^1)}. \tag{2.4}$$

(Note that $\Phi(0) = \Lambda(0, x) = 0$.) Next, we have

$$\frac{\frac{\partial}{\partial x_1}S(t|x)}{\frac{\partial}{\partial x_1}S(1|x)} = \frac{\Phi(t)\theta'(x)L^{(1)}\left(\Lambda(t,x)\right)}{\theta'(x)L^{(1)}\left(\theta(x)\right)}$$

$$= \Phi(t)\frac{L^{(1)}\left(\Lambda(t,x)\right)}{L^{(1)}\left(\theta(x)\right)} \tag{2.5}$$

and

$$\Phi(t) = \lim_{x\to x^0} \frac{\frac{\partial}{\partial x_1}S(t|x)}{\frac{\partial}{\partial x_1}S(0|x)}. \tag{2.6}$$

Since we can observe Φ for all values from zero to infinity we can evaluate L over its domain (and G, since there is a unique relationship between the two) by varying t and observing $L(\Phi(t)) = S(t|x^1)$. Since θ, Φ and L can be expressed directly as functions of S, they are identifiable. (Any other values of θ, Φ and L would necessarily imply a different S and hence could not be observationally equivalent.) ¤

To adapt the Heckman and Honoré (1990) results for duration models we consider bivariate models with hazard rates analogous to the univariate case so that

$$h_j(t|x,\nu) = \phi_j(t)\theta_j(x_j,z)\nu_j \equiv \lambda_j(t,x_j,z)\nu_j, \qquad j = 1,2. \tag{2.7}$$

We index the x's to allow for exclusion restrictions. The vector of observed covariates is decomposed as $x = (x_1, x_2, z)$ where x_j appears only in θ_j, $j = 1,2$. z can be common to both hazard rates. This is similar to exclusion restrictions in simultaneous equations estimation. We make normalizations completely analogous to the univariate case.

The joint distribution of the two duration variables can be modeled the same way for both the competing risk and Roy duration models. The difference is the way the max/min operator affects the observed duration. In the first case we look at, the two processes are independent, conditional on the observable and unobservable covariates. We formalize the structure as follows.

Assumption 1. *The marginal survivor functions for the two processes are written as*

$$S_j(t|x,\nu) = \exp(-\Phi_j(t)\theta_j(x_j,z)\nu_j), \tag{2.8}$$

$$\Phi_j(t) = \int_0^t \phi_j(u)du, \qquad \Lambda_j(t,x_j,z) = \int_0^t \lambda_j(u,x_j,z)du,$$

$$\lambda_j(t,x_j,z) = \phi_j(t)\theta_j(x_j,z) \tag{2.9}$$

$$\lambda_j, \theta_j \geq 0, \qquad j = 1, 2$$

$$(\nu_1, \nu_2) \sim G_{12} : \Re^+ \times \Re^+ \to [0, 1]$$

This is the bivariate analogue to what is usually assumed in econometrics and biometrics for continuous univariate processes. The observed dependence between T_1 and T_2 in this case comes from the unobservables. This is relaxed below to allow for direct dependence between the two processes. Let $\theta_j'(x_j, z)$ denote the derivative of $\theta_j(x_j, z)$ with respect to its first element, $j = 1, 2$. We also use some smoothness restrictions and normalizations comparable to those in the univariate case. These are given in the following.

Assumption 2. *For $j = 1, 2$, suppose that $E[\nu_j] = L_j(0) < \infty$, $\phi_j(t) > 0$, $\Phi_j(0) = 0$, $\Phi_j(1) = 1$ and $\Phi_j(\infty) = \infty$.*

Assumption 3. *For $j = 1, 2$, for some $(x_j, z) = (x_j^1, z^1)$ and $x_j = x_j^0$, $\theta_j(x_j^1, z^1) = 1$ and $\theta(x_j^0, z) = \lim_{x_j \to x_j^0} \theta_j(x_j, z) = 0$. $x_1 \cap x_2 = \emptyset$. $\theta_j(x_j, z)$ is continuously differentiable with respect to x_{j1} and $\theta_j'(x_j, z) > 0$, $x_j \neq x_j^0$.*

Consider first the latent competing risks model. In this case, the observed minimum has a survivor function which is given in the following assumption.

Assumption 4. *The survivor function of the observed durations is given by*

$$\begin{aligned} S^c(t|x) &= \iint S_1(t|x, \nu_1) S_2(t|x, \nu_2)\, dG_{12}(\nu_1, \nu_2) \\ &= \iint e^{-\Lambda_1(t, x_1, z)\nu_1} e^{-\Lambda_2(t, x_2, z)\nu_2}\, dG_{12}(\nu_1, \nu_2) \\ &\equiv L_{12}(\Lambda_1(t, x_1, z), \Lambda_2(t, x_2, z)). \end{aligned} \tag{2.10}$$

We let $L_1(\Lambda_1) = L_{12}(\Lambda_1, 0)$, $L_2(\Lambda_2) = L_{12}(0, \Lambda_2)$, denote the "marginal" Laplace transforms and indicate their derivatives by $L_j^{(1)}$, $j = 1, 2$. The partials of L_{12} are indicated by $L_{12}^{(j)}$, $j = 1, 2$. θ_j' indicates the partial derivative of θ_j with respect to its first element, x_{j1}. Note that $L_j(0) = 1$, $L_j(\infty) = 0$, and $L_j^{(1)}(0) = E[\nu_j]$, $j = 1, 2$. Also define $x^{10} = (x_1^0, x_2, z)$, $x^{20} = (x_1, x_2^0, z)$.

The intuition of our results is very simple. For both the competing risk and Roy duration models, we find conditions under which the "observable" survivor function is equal to the marginal survivor function of processes. Given that, Elbers and Ridder (1982) -like results can be applied directly. For example, in the case of psychiatric patients, the physician may wish to discharge a patient, say because of overutilized resources. In this case, one can infer that if a patient is still in the hospital it is by his/her choice and one can identify the patient's survivor function. Symmetric assumptions allow us to identify the physician's survivor function. Moreover, since the parameters of interest are expressed directly in terms of the survivor function and its derivatives, these can be estimated directly by their sample analogues.

Proposition 2. *Let Assumptions 1, 2, 3 and 4 hold. Then,* θ_j *,*Φ_j*,* $j = 1,2$ *and* L_{12} *are identifiable.*

Proof of Proposition 2. With $E[\nu_1]$ finite and $\theta_1(x_1^1, z^1) = 1$,

$$\frac{\frac{\partial}{\partial t} S^c(t|x_1, x_2^0, z)}{\frac{\partial}{\partial t} S^c(t|x_1^1, x_2^0, z^1)} = \frac{\phi_1(t)\theta_1(x_1, z) L_{12}^{(1)}\left(\Lambda_1(t,x), 0\right)}{\phi(t) L_{12}^{(1)}\left(\Phi_1(t), 0\right)}$$

$$= \theta_1(x_1, z) \frac{L_1^{(1)}\left(\Lambda_1(t,x)\right)}{L_1^{(1)}\left(\Phi_1(t)\right)} \tag{2.11}$$

and we can identify θ_1 by observing that

$$\theta_1(x_1, z) = \lim_{t \to 0} \frac{\frac{\partial}{\partial t} S^c(t|x_1, x_2^0, z)}{\frac{\partial}{\partial t} S^c(t|x_1^1, x_2^0, z^1)}. \tag{2.12}$$

Next,

$$\frac{\frac{\partial}{\partial x_{11}} S^c(t|x_1, x_2^0, z)}{\frac{\partial}{\partial x_{11}} S^c(1|x_1, x_2^0, z)} = \frac{\Phi_1(t)\theta_1'(x) L_{12}^{(1)}\left(\Lambda_1(t,x), 0\right)}{\theta_1'(x) L_{12}^{(1)}\left(\theta_1(x), 0\right)}$$

$$= \Phi_1(t) \frac{L_1^{(1)}\left(\Lambda_1(t,x)\right)}{L_1^{(1)}\left(\theta_1(x)\right)} \tag{2.13}$$

and

$$\Phi_1(t) = \lim_{x_1 \to x_1^0} \frac{\frac{\partial}{\partial x_{11}} S^c(t|x_1, x_2^0, z)}{\frac{\partial}{\partial x_{11}} S^c(1|x_1, x_2^0, z)}. \tag{2.14}$$

θ_2 and Φ_2 are identified symmetrically. If we put $t = 1$, we can evaluate $L_{12}(\theta_1, \theta_2) = S^c(1|x)$ over a rectangle by simultaneously altering x_{11} and x_{21}. Since Φ_1 and Φ_2 are identifiable and take values over $\Re^+ \times \Re^+$ we can also vary t to evaluate $L_{12}(\Phi_1\theta_1, \Phi_2\theta_2) = S^c(t|x)$ over $\Re^+ \times \Re^+$. ¤

Remark 3. Note that since there is a unique relationship between L_{12} and G_{12}, the latter is also identifiable. ¤

For the case of the Roy model, the marginal survivor functions for the two duration times, conditional on x and ν_1, ν_2, are the same as for the competing risks in (2.9). However, the probability of the maximum of two variables is the sum of the marginals minus the joint probability. Therefore, conditional on x and (ν_1, ν_2), the maximum duration time has a survivor functional:

$$S^r(t|x, \nu) = e^{-\Lambda_1(t,x_1,z)\nu_1} + e^{-\Lambda_2(t,x_2,z)\nu_2} - e^{-\Lambda_1(t,x_1,z)\nu_1} e^{-\Lambda_2(t,x_2,z)\nu_2}. \tag{2.15}$$

Denoting the marginal distribution functions of ν_1 and ν_2 as G_1 and G_2 respectively, the survivor function of the maximum, conditional on the observable covariates, is as in the following assumption.

Assumption 5. *The survivor function of the observed durations is given by*

$$
\begin{aligned}
S^r(t|x) = & \int e^{-\Lambda_1(t,x_1,z)\nu_1}\, dG_1(\nu_1) + \int e^{-\Lambda_2(t,x_2,z)\nu_2}\, dG_2(\nu_2) \\
& - \iint e^{-\Lambda_1(t,x_1,z)\nu_1} e^{-\Lambda_2(t,x_2,z)\nu_2}\, dG_{12}(\nu_1,\nu_2) \\
\equiv & \; L_1(\Lambda_1(t,x_1,z)) + L_2(\Lambda_2(t,x_2,z)) - L_{12}(\Lambda_1(t,x_1,z),\Lambda_2(t,x_2,z)).
\end{aligned}
\tag{2.16}
$$

We use one additional limiting assumption here as follows.

Assumption 6. *For some $x_j = x_j^{\infty}$, $j = 1,2$, $\theta(x_j^{\infty}, z) = \lim_{x_j \to x_j^{\infty}} \theta_j(x_j, z) = \infty$.*

The identification result for the latent Roy model is as follows.

Proposition 3. *Let Assumptions 1, 2, 3, 5 and 6 hold. Then, θ_j ,Φ_j, $j = 1,2$ and L_{12} are identifiable.*

Remark 4. The difference between the assumptions in Propositions 2 and 3 is the assumption that $\theta(x_j^{\infty}, z) = \infty$, in addition to $\theta(x_j^0, z) = 0$, $j = 1,2$. This may appear as inconsistent with the competing risks case but it is actually not. The limiting assumptions on θ_j have two purposes. As in the univariate case, setting $\theta_j = 0$ allows us to identify Φ_j. In the bivariate case we also need some x to appear only in θ_i such that $S^r = L_j$. In the Roy model it is sufficient to have $\theta(x_i^{\infty}, z) = \infty$. The corresponding assumption in the competing risk case is that $\theta(x_i^0, z) = 0$. Since this assumption, for $j = 1,2$ is also used for identifying Λ_i, we appear to get away with one less assumption in the competing risks model.

Intuitively, to identify the structure we require conditions under which one knows which variable is the maximum. The assumption then, say, that $\theta_2 = \infty$ guarantees that subject two "exits" at time 0 and is never at risk. This is the opposite from the competing risk case. ¤

Proof of Proposition 3. With $E[\nu_1]$ finite and $\theta_1(x_1^1, z^1) = 1$,

$$
\begin{aligned}
\frac{\frac{\partial}{\partial t} \lim_{x_2 \to x_2^{\infty}} S^r(t|x_1,x_2,z)}{\frac{\partial}{\partial t} \lim_{x_2 \to x_2^{\infty}} S^r(t|x_1,x_2,z)} &= \frac{\phi_1(t)\theta_1(x_1,z)L_1^{(1)}(\Lambda_1(t,x))}{\phi(t)L_1^{(1)}(\Phi_1(t))} \\
&= \theta_1(x_1,z)\frac{L_1^{(1)}(\Lambda_1(t,x))}{L_1^{(1)}(\Phi_1(t))}
\end{aligned}
\tag{2.17}
$$

and we can identify θ_1 by observing that

$$
\theta_1(x_1,z) = \lim_{t \to 0} \frac{\frac{\partial}{\partial t} \lim_{x_2 \to x_2^{\infty}} S^r(t|x_1,x_2,z)}{\frac{\partial}{\partial t} \lim_{x_2 \to x_2^{\infty}} S^r(t|x_1,x_2,z)}. \tag{2.18}
$$

Next,

$$\frac{\frac{\partial}{\partial x_{11}}\lim_{x_2\to x_2^\infty} S^r(t|x_1,x_2,z)}{\frac{\partial}{\partial x_{11}}\lim_{x_2\to x_2^\infty} S^r(1|x_1,x_2,z)} = \frac{\Phi_1(t)\theta_1'(x)L_1^{(1)}\left(\Lambda_1(t,x)\right)}{\theta_1'(x)L_1^{(1)}\left(\theta_1(x)\right)}$$

$$= \Phi_1(t)\frac{L_1^{(1)}\left(\Lambda_1(t,x)\right)}{L_1^{(1)}\left(\theta_1(x)\right)} \tag{2.19}$$

and

$$\Phi_1(t) = \lim_{x_1\to x_1^0}\frac{\frac{\partial}{\partial x_{11}}\lim_{x_2\to x_2^\infty} S^r(t|x_1,x_2,z)}{\frac{\partial}{\partial x_{11}}\lim_{x_2\to x_2^\infty} S^r(1|x_1,x_2,z)}. \tag{2.20}$$

θ_2 and Φ_2 are identified symmetrically. As in Proposition 2 we can then find L_{12} for any value in $\Re^+\times\Re^+$ by simultaneously altering x_{11} and x_{21} and evaluating

$$L_{12}(\Lambda_1(t)\theta_1(x_1,z),\Lambda_1(t)\theta_2(x_2,z)) = S_1(t|x_1,z)+S_2(t|x_2,z)-S(t|x_1,x_2,z). \tag{2.21}$$ ¤

As noted after Proposition 2, this also identifies G_{12}.

3. Identification with conditionally dependent duration variables

We now generalize the above results to allow the two duration times to be dependent, conditional on the observables and unobservable heterogeneity. Doing so raises identification problems not dealt with in the Heckman and Honoré (1990) paper. A fairly general class of models can be obtained as follows. We write the joint survivor of the two duration times as the product of the marginal and conditional, i.e.

$$S_{12}(t_1,t_2|x,\nu) = S_{2|1}(t_2|t_1,x,\nu)S_1(t_1|x,\nu) \tag{3.1}$$

where S_1 is as in Assumption 1 and $S_{2|1}$ is the survivor based on a conditional hazard function of the form

$$h_{2|1}(t_2|t_1,x,\nu) = \phi_2(t_2)\rho(t_1,x_3)\theta_2(x_2,z)\nu_2. \tag{3.2}$$

A structure such as this is commonly used for modeling multiple spells in which case t_1 and t_2 represent consecutive spells. For example, suppose $b(t)$ is some positive function of t and put $\rho(t,x_3) = b(t)^{1[x_3=x_3^1]}$. In the length of stay example, $b(t)$ could correspond to the doctor's response to the patient's opinion and for some particular diagnosis, say extreme schizophrenia, this is completely ignored ($1[\cdot]=0$). We formalize this in the following assumption.

Assumption 7. *The survivor functions for the two latent processes are written as*

$$S_{2|1}(t_2|t_1,x,\nu) = \exp(-\Lambda_{2|1}(t_2|t_1,x)\nu_1), \qquad S_1(t_1|x,\nu) = \exp(-\Lambda_1(t,x)\nu_2),$$

$$\Lambda_{2|1}(t_2|t_1,x) = \int_0^{t_2} \lambda_{2|1}(u|t_1,x)du, \qquad \Lambda_1(t_1,x) = \int_0^{t_1} \lambda_1(u,x)du,$$

$$\lambda_{2|1}(t_2|t_1,x) = \phi_2(t_2)\rho(t_1,x_3)\theta_2(x_2,z), \qquad \lambda_1(t_1,x) = \phi_1(t_1)\theta_1(x_1,z)$$

$$\rho, \phi_j, \theta_j \geq 0, \qquad j = 1,2$$

$$(\nu_1,\nu_2) \sim G_{12} : \Re^+ \times \Re^+ \to [0,1] \tag{3.3}$$

We continue to define $\Phi_j(t) = \int_0^t \phi_j(u)du$, $j = 1,2$. Note that we partition $x = (x_1,x_2,x_3,z)$ to allow for a subset of the observed covariates to enter into ρ. We show identification of this model under two different sets of assumptions (in addition to the normalizations used above.) We either assume that x_3 is a subset of x with no elements in common with either x_1, x_2 or z and x_3 has a certain limiting impact on ρ or we will show that certain shape restrictions on ϕ_2 and ρ can identify the components of this model. The rest of the structure is as in the models in the previous section. We impose the natural restriction that $\rho(0,x_3) = 1$ so that the marginal distributions with this model are as in the conditionally independent model. ρ' denotes the partial derivative of ρ with respect to t. As before, for $j = 1,2$, θ_j' indicates the partial derivative with respect to its first element, x_{j1}.

Consider first the latent competing risks model. In this case, the observed minimum has a survivor function which is written

Assumption 8. *The survivor function of the observed durations is given by*

$$\begin{aligned} S^c(t|x) &= \iint S_1(t|x,\nu_1)S_{2|1}(t|t,x,\nu_2)\,dG_{12}(\nu_1,\nu_2) \\ &= \iint e^{-\Lambda_1(t,x_1,z)\nu_1} e^{-\Lambda_{2|1}(t,t,x_2,z,x_3)\nu_2}\,dG_{12}(\nu_1,\nu_2) \\ &\equiv L_{12}(\Lambda_1(t,x_1,z),\Lambda_{2|1}(t,t,x_2,z,x_3)). \end{aligned} \tag{3.4}$$

As above, we let L_j, $j = 1,2$, denote the "marginal" Laplace transforms and indicate their derivatives by $L_j^{(1)}$, $j = 1,2$. The partials of L_{12} are indicated by $L_{12}^{(j)}$, $j = 1,2$. We define $x^{10} = (x_1^0,x_2,z,x_3)$, $x^{20} = (x_1,x_2^0,z,x_3)$.

Assumption 9. *ρ is differentiable with respect to t, $\rho(0,x_3) = 1$ and for some $x_3 = x_3^1$, $\rho(t,x_3^1) = 1$. $x_2 \cap (x_1 \cup x_2 \cup z) = \emptyset$.*

Proposition 4. *Let Assumptions 2, 3, 7, 8 and 9 hold. Then, θ_j, Φ_j, $j = 1, 2$, ρ and L_{12} are identifiable.*

Proof of Proposition 4. θ_1 and ϕ_1 can be identified in the same way as in Proposition 2 by evaluating $S^c(t|x_1, x_2^0, z, x_3)$. Also as in Proposition 2, we can identify θ_2 by first evaluating the ratio

$$
\begin{aligned}
\frac{\frac{\partial}{\partial t} S^c(t|x_1^0, x_2, z, x_3)}{\frac{\partial}{\partial t} S^c(t|x_1^0, x_2^1, z, x_3)} &= \frac{\theta_2(x) L_2^{(1)}\left(\Lambda_{2|1}(t|t,x)\right) \frac{\partial}{\partial t}\left(\Phi_2(t)\rho(t,x_3)\right)}{L_2^{(1)}\left(\Phi_2(t)\rho(t,x_3)\right) \frac{\partial}{\partial t}\left(\Phi_2(t)\rho(t,x_3)\right)} \\
&= \theta_2(x) \frac{L_2^{(1)}\left(\Lambda_{2|1}(t|t,x)\right)}{L_2^{(1)}\left(\Phi_2(t)\rho(t,x_3)\right)}
\end{aligned}
\tag{3.5}
$$

and

$$
\theta_2(x) = \lim_{t \to 0} \frac{\frac{\partial}{\partial t} S^c(t|x_1^0, x_2, z, x_3)}{\frac{\partial}{\partial t} S^c(t|x_1^0, x_2^1, z, x_3)}. \tag{3.6}
$$

To identify ρ,

$$
\begin{aligned}
\frac{\frac{\partial}{\partial x_{21}} S^c(t|x_1^0, x_2, z, x_3)}{\frac{\partial}{\partial x_{21}} S^c(t|x_1^0, x_2, z, x_3^1)} &= \frac{\rho(t,x_3)\Phi_2(t)\theta_2'(x) L_2^{(1)}\left(\Phi_2(t)\rho(t,x_3)\theta_2(x)\right)}{\Phi_2(t)\theta_2'(x) L_2^{(1)}\left(\Phi_2(t)\theta_2(x)\right)} \\
&= \rho(t,x_3) \frac{L_2^{(1)}\left(\Phi_2(t)\rho(t,x_3)\theta_2(x)\right)}{L_2^{(1)}\left(\Phi_2(t)\theta_2(x)\right)}
\end{aligned}
\tag{3.7}
$$

and consequently

$$
\rho(t,x_3) = \frac{\lim_{x_2 \to x_2^0} \frac{\partial}{\partial x_{21}} S^c(t|x_1^0, x_2, z, x_3)}{\lim_{x_2 \to x_2^0} \frac{\partial}{\partial x_{21}} S^c(t|x_1^0, x_2, z, x_3^1)}. \tag{3.8}
$$

Similarly,

$$
\Phi_2(t) = \frac{\lim_{x_2 \to x_2^0} \frac{\partial}{\partial x_{21}} S^c(t|x_1^0, x_2, z, x_3^1)}{\lim_{x_2 \to x_2^0} \frac{\partial}{\partial x_{21}} S^c(1|x_1^0, x_2, z, x_3^1)}. \tag{3.9}
$$

L_{12} is identified as in Proposition 2. $\square$

Remark 4. Without the identifying variable, x_3, it is still possible to identify the parameters using functional form restrictions. Note that in this case,

$$\frac{\frac{\partial}{\partial t}S^c(t|x_1^0,x_2,z)}{\frac{\partial}{\partial x_{21}}S^c(t|x_1^0,x_2,z)} = \frac{\frac{\partial}{\partial t}\left(\rho(t)\Phi_2(t)\right)\theta_2(x_2,z)L_2^{(1)}\left(\Lambda_{2|1}(t,x_2,z)\right)}{\rho(t)\Phi_2(t)\theta_2'(x_2,z)L_2^{(1)}\left(\Lambda_{2|1}(t,x_2,z)\right)}$$

$$= \frac{\frac{\partial}{\partial t}\log\left(\rho(t)\Phi_2(t)\right)}{\frac{\partial}{\partial x_{21}}\log\theta_2(x_2,z)} \tag{3.10}$$

We can thus identify

$$\frac{\partial}{\partial t}\log\left(\rho(t)\Phi_2(t)\right) = \frac{\partial}{\partial x_{21}}\log\theta_2(x_2,z)\frac{\frac{\partial}{\partial t}S^c(t|x_1^0,x_2,z)}{\frac{\partial}{\partial x_{21}}S^c(t|x_1^0,x_2,z)}, \tag{3.11}$$

that is, the log derivative of the product of ρ and Φ. In general, it is not possible to decompose this into the two functions. However, for certain, fairly rich parametric models one can show that, if $\rho(t) = \rho(t;a)$ and $\Phi(t) = \Phi(t;b)$, where the true values are a_0 and b_0, say, then for any other values, a' and b', say, that $\rho(t;a_0) \neq \rho(t;a')$ and $\Phi(t;b_0) \neq \Phi(t;b')$ on a set on positive measure. Consequently, it is possible to identify the two functions.

For example, suppose $\Phi(t) = t^a$, $\rho(t) = (1+t)^b$ and say for two values a_0, b_0 and a', b' these are the same. Then

$$0 = \frac{d}{dt}\left(a_0\log t + b_0\log(1+t)\right) - \frac{d}{dt}\left(a'\log t + b'\log(1+t)\right)$$

$$= \frac{a_0-a'}{t} + \frac{b_0-b'}{1+t} \tag{3.12}$$

or

$$t = -\left(\frac{1}{a_0-a'} + \frac{1}{b_0-b'}\right)(b_0-b') \tag{3.13}$$

a constant, which can only hold on a set of measure zero. The rest of the identification follows as previously. ∎

For the corresponding Roy duration model, the observed maximum has a survivor function which is written as follows

Assumption 10. *The survivor function of the observed durations is given by*

$$S^r(t|x) = \int S_1(t|x,\nu_1)\,dG_1(\nu_1) + \int S_{2|1}(t|0,x,\nu_2)\,dG_2(\nu_2)$$
$$- \iint S_1(t|x,\nu_1)S_{2|1}(t|t,x,\nu_2)\,dG_{12}(\nu_1,\nu_2)$$
$$= \int e^{-\Lambda_1(t,x_1,z)\nu_1}\,dG_1(\nu_1) + \int e^{-\Phi_2(t)\theta_2(x)\nu_2}\,dG_2(\nu_2)$$
$$- \iint e^{-\Lambda_1(t,x_1,z)\nu_1}e^{-\Lambda_{2|1}(t|t,x)\nu_2}\,dG_{12}(\nu_1,\nu_2)$$

$$\equiv L_1(\Phi_1(t)\theta_1(x)) + L_2(\Phi_2(t)\theta_2(x)) - L_{12}(\Phi_1(t)\theta_1(x), \Phi_2(t)\theta_2(x)\rho(t,x_3)). \tag{3.14}$$

Define $x^{11} = (x_1^1, x_2, z, x_3)$ and $x^{21} = (x_1, x_2^1, z, x_3)$.

Proposition 5. *Let Assumptions 2, 3, 6, 7, 9 and 10 hold. Then,* θ_j, Φ_j, $j = 1, 2$, ρ *and* L_{12} *are identifiable.*

Proof to Proposition 5. θ_1 and ϕ_1 are identified as in Proposition 3. Also as in the proof to Proposition 3 we have

$$\frac{\frac{\partial}{\partial t}\lim\limits_{x_1 \to x_1^\infty} S^r(t|x_1, x_2, z, x_3)}{\frac{\partial}{\partial t}\lim\limits_{x_1 \to x_1^\infty} S^r(t|x_1, x_2^1, z, x_3)} = \frac{\phi_2(t)\theta_2(x) L_2^{(1)}\left(\Phi_2(t)\theta_2(x)\right)}{\phi_2(t) L_2^{(1)}\left(\Phi_2(t)\right)}$$

$$= \theta_2(x)\frac{L_2^{(1)}\left(\Phi_2(t)\theta_2(x)\right)}{L_2^{(1)}\left(\Phi_2(t)\right)} \tag{3.15}$$

and

$$\theta_2(x_2, z) = \lim_{t \to 0} \frac{\frac{\partial}{\partial t}\lim\limits_{x_1 \to x_1^\infty} S^r(t|x)}{\frac{\partial}{\partial t}\lim\limits_{x_1 \to x_1^\infty} S^r(t|x^{21})}. \tag{3.16}$$

Similarly

$$\frac{\frac{\partial}{\partial x_{21}}\lim\limits_{x_1 \to x_1^\infty} S^r(t|x)}{\frac{\partial}{\partial x_{21}}\lim\limits_{x_1 \to x_1^\infty} S^r(1|x)} = \frac{\Phi_2(t)\theta_2'(x_2, z) L_2^{(1)}\left(\Phi_2(t)\theta_2(x)\right)}{\Phi_2(1)\theta_2'(x_2, z) L_2^{(1)}\left(\Phi_2(1)\theta_2(x)\right)}$$

$$= \Phi_2(t)\frac{L_2^{(1)}\left(\Phi_2(t)\theta_2(x)\right))}{L_2^{(1)}\left(\theta_2(x_2, z)\right)} \tag{3.17}$$

and

$$\Phi_2(t) = \lim_{x_2 \to x_2^0} \frac{\frac{\partial}{\partial x_{21}}\lim\limits_{x_1 \to x_1^\infty} S^r(t|x)}{\frac{\partial}{\partial x_{21}}\lim\limits_{x_1 \to x_1^\infty} S^r(1|x)}. \tag{3.18}$$

Next note that,

$$\frac{\partial}{\partial x_{21}} S^r(t|x^{10})$$

$$= \Phi_2(t)\theta_2'(x) \quad L_2^{(1)}\left(\Phi_2(t)\theta_2(x)\right) - \rho(t, x_3) L_2^{(1)}\left(\Phi_2(t)\rho(t, x_3)\theta_2(x)\right) \tag{3.19}$$

so that

$$\rho(t,x_3)=\lim_{x_2\to x_2^0}\frac{1}{\Phi_2(t)\theta_2'(x)}\left(1-\frac{\partial}{\partial x_{21}}S^r(t|x^{10})\right) \tag{3.20}$$

Identification of L_{12} follows as in the proof to Proposition 3. ¤

Remark 5. Note that the proof of this result does not use the additional covariate, x_3, for identification. While this may appear inconsistent with the case of competing risks, actually the result is due to the asymmetric manner in which ρ is introduced. By assumption, ρ does not appear in the marginal distribution of T_2. As we see, for limiting values of x_1 we can effectively observe both $L_2(\Phi_2\theta_2)$ and $L_2(\Phi_2\rho\theta_2)$ This allows us to identify the parameters in this model without the extra covariate x_3. With the competing risks model we can only observe $L_2(\Phi_2\rho\theta_2)$, making separate identification of Φ_2 and ρ more problematic. ¤

4. Identification without exclusion restrictions

In many situations it may not be plausible to impose exclusion restrictions. For example, in bargaining situations in economics, if both agents have access to the same information, there is no reason to expect that one agent will condition on less information than the other. However, it may be plausible to make an assumption as to how a covariate will impact on each agent's duration dependence, at least in a limiting sense. For simplicity we suppose x is a scalar. Crossley, Paric and Rilstone (1999) consider a strike situation in which work does not recommence until both parties sign a contract (a Roy duration model). They assume that firms ($j=2$) and workers ($j=1$) react asymmetrically to wage changes, x. One would expect firms to react negatively to a large wage increase, and hence θ_2 gets very small for large x, large for small (negative) x. Conversely, θ_1 gets very large for large x, small for small x. The point is that, for large enough wage changes, if a firm is still on strike, one can be fairly certain that it is of the firm's choosing; if wage changes are very small or negative, it is the workers' doing. Intuitively, we would like to simply set $\theta_1=\infty$ for values of x greater than some value, $\theta_2=\infty$ for all values of x below some level. This of course raises some technical difficulty. Rather than doing this, we simply assume that, conditional on very high wage changes, we have $S^r(t|x)=S_2(t|x)$ and for very small or negative wage changes, we have $S^r(t|x)=S_1(t|x)$. These assumptions are formalized in the following propositions for the competing risks and Roy duration models. We consider the Roy model first.

Define a Roy type survivor function as in (2.16), with one common covariate for each underlying process such that:

$$\begin{aligned} S^r(t|x) &= \int e^{-\Lambda_1(t,x)\nu_1}\,dG_1(\nu_1)+\int e^{-\Lambda_2(t,x)\nu_2}\,dG_2(\nu_2)\\ &\quad-\iint e^{-\Lambda_1(t,x)\nu_1}e^{-\Lambda_2(t,x)\nu_2}\,dG_{12}(\nu_1,\nu_2)\\ &\equiv L_1(\Lambda_1(t,x))+L_2(\Lambda_2(t,x))-L_{12}(\Lambda_1(t,x),\Lambda_2(t,x)) \end{aligned} \tag{4.1}$$

Put

$$S_j(t|x) = \int e^{-\Lambda_j(t,x)\nu_j}\, dG_j(\nu_j), \qquad j = 1,2. \tag{4.2}$$

With the asymmetric limiting assumptions on θ_1 and θ_2, the observable survivor function is given in the following.

Assumption 11. *The survivor function for the observed durations is*

$$S^{r*}(t|x) = S^r(t|x)1[x^{2\infty} < x < x^{1\infty}] + S_2(t|x)1[x \geq x^{1\infty}] + S_1(t|x)1[x \leq x^{2\infty}]. \tag{4.3}$$

By inspection of this last line it is intuitive how the model is identified. The components of S_1 are identified using values of x such that $x \leq x^{2\infty}$. Conversely the components of S_2 are identified using values of x such that $x \geq x^{1\infty}$. We require that the relevant normalizations can be imposed within these ranges. These are as follows.

Assumption 12. *For some x^{j0}, x^{j1}, $\lim_{x\to x^{j0}} \theta_j(x) = 0$, $\theta_j(x^{j1}) = 1$, $j = 1,2$, where $x^{10}, x^{11} < x^{2\infty}$ and $x^{20}, x^{21} > x^{1\infty}$. $\theta_j(x)$ is continuously differentiable with respect to x and $\theta_j'(x) > 0$, $x \neq x_j^0$, $j = 1,2$.*

Remark 6. An example of the type of functions which are implied by this assumption and the structure of S^{r*} is in Figure 1. What we have in mind are asymmetric responses to a covariate. The opposing hyperbolas in that figure, with the placement of the x^{ji}'s will give rise to the sort of limiting behaviour we have in mind, say in the strike example. Another example would be in the timing of marriage where each individual could be expected to react asymmetrically to the relative dowry of the other. ¤

Proposition 6. *Let Assumptions 2, 11 and 12 hold. Then Φ_j and L_j, $j = 1,2$ are identifiable, θ_1 is identifiable over $x < x^{20}$ and θ_2 is identifiable over $x > x^{10}$.*

Remark 7. Note that Proposition 6 only partially identifies the model (at least nonparametrically.) Without imposing other restrictions we can identify $\theta_1(x)$ ($\theta_2(x)$) only for those values of $x < x^{2\infty}$ ($x > x^{1\infty}$). L_1 and L_2 are identifed over their domains since we can identify Λ_1 and Λ_2 and vary these over $[0,\infty)$. However to identify L_{12} we need to be able to evaluate it for arbitrary values of both its arguments. This is not possible. ¤

Proof to Proposition 6. First note that

$$\begin{aligned} S^{r*}(t|x, x > x^{1\infty}) &= L_1(\infty) + L_2(\Lambda_2(t,x)) - L_{12}(\infty, \Lambda_2(t,x)) \\ &= L_2(\Lambda_2(t,x)) \end{aligned} \tag{4.4}$$

so that

$$\begin{aligned} \frac{\frac{\partial}{\partial x} S^{r*}(t|x, x > x^{1\infty})}{\frac{\partial}{\partial x} S^{r*}(1|x, x > x^{1\infty})} &= \frac{\Phi_2(t)\theta_2'(x)L_2(\Lambda_2(t,x))}{\theta_2'(x)L_2(\theta_2(x))} \\ &= \Phi_2(t)\frac{L_2(\Lambda_2(t,x))}{L_2(\theta_2(x))} \end{aligned} \tag{4.5}$$

and

$$\Phi_2(t) = \lim_{x \to x^{20}} \frac{\frac{\partial}{\partial x} S^{r*}(t|x, x > x^{1\infty})}{\frac{\partial}{\partial x} S^{r*}(1|x, x > \overline{x}^{1\infty})}. \tag{4.6}$$

Similarly,

$$\begin{aligned} \frac{\frac{\partial}{\partial t} S^{r*}(t|x, x > x^{1\infty})}{\frac{\partial}{\partial t} S^{r}(t|x^{21}, x > x^{1\infty})} &= \frac{\phi_2(t)\theta_2(x)L_2(\Lambda_2(t,x))}{\phi_2(t)L_2(\Phi_2(t))} \\ &= \theta_2(x)\frac{L_2(\Lambda_2(t,x))}{L_2(\Phi_2(t))} \end{aligned} \tag{4.7}$$

and

$$\theta_2(x) = \lim_{t \to 0} \frac{\frac{\partial}{\partial t} S^{r}(t|x, x > x^{1\infty})}{\frac{\partial}{\partial t} S^{r}(t|x^{21}, x > x^{1\infty})} \tag{4.8}$$

We can similarly identify θ_1 and Φ_1. The identification of L_1 and L_2 follows by noting that

$$L_1(\Phi_1(t)) = S^{r*}(t|x^{11}), \tag{4.9}$$

$$L_2(\Phi_2(t)) = S^{r*}(t|x^{21}). \quad ¤ \tag{4.10}$$

In the competing risk case, it is also not possible in general to identify all the functions over their entire domains. (If one is willing to impose that these functions belong to a known finite dimensional family then that is, of course, a different story.) First, put

$$\begin{aligned} S^c(t|x) &= \iint e^{-\Lambda_1(t,x)\nu_1} e^{-\Lambda_2(t,x)\nu_2}\, dG_{12}(\nu_1, \nu_2) \\ &\equiv L_{12}(\Lambda_1(t,x), \Lambda_2(t,x)) \end{aligned} \tag{4.11}$$

and

$$S_j(t|x) = \int e^{-\Lambda_j(t,x)\nu_j}\, dG_j(\nu_j), \qquad j = 1, 2. \tag{4.12}$$

With the asymmetric limiting assumptions on θ_1 and θ_2, we have the following assumptions.

Assumption 13. *The survivor function for the observed durations is*

$$S^{c*}(t|x) = 1[x^{20} < x < x^{10}] + S_2(t|x)1[x \geq x^{10}] + S_1(t|x)1[x \leq x^{20}]. \tag{4.13}$$

Assumption 14. *For some $x^{21} > x^{10}$, $\theta_2(x) = 1$, for some $x^{11} < x^{20}$, $\theta_1(x) = 1$. $\theta_j(x)$ is continuously differentiable $\theta_1'(x) > 0$, $x < x_{20}$ and $\theta_2'(x) > 0$, $x > x_{10}$.*

Proposition 7. *Let Assumptions 2, 12 and 13 hold. Then Φ_j and L_j, $j = 1, 2$ are identifiable, θ_1 is identifiable over $x < x^{20}$ and θ_2 is identifiable over $x > x^{10}$.*

Remark 8. Note again that while the marginal Laplace transforms, L_1 and L_2 are identifiable, L_{12} is not. The reason is that to trace out L_{12} we need to simultaneously change both its arguments at points which are not degenerate. Note that effectively there are no x's such that both θ_1 and θ_2 are nonzero. See Figures 1 and 2 for stylized examples of the asymmetric reaction functions in the Roy and competing risks models. ¤

Proof to Proposition 7. Note first that

$$S^c(t|x, x > x^{10}) = L_{12}(0, \Lambda_2(t, x)) = L_2(\Lambda_2(t, x)) \tag{4.14}$$

so that

$$\begin{aligned} \frac{\frac{\partial}{\partial x} S^{c*}(t|x, x > x^{10})}{\frac{\partial}{\partial x} S^{c*}(0|x, x > x^{10})} &= \frac{\Phi_2(t)\theta_2'(x)L_2^{(1)}(\Lambda_2(t, x))}{\theta_2'(x)L_2^{(1)}(\theta_2(x))} \\ &= \Phi_2(t)\frac{L_2^{(1)}(\Lambda_2(t, x))}{L_2^{(1)}(\theta_2(x))} \end{aligned} \tag{4.15}$$

and

$$\Phi_2(t) = \lim_{x \to x^{20}} \frac{\frac{\partial}{\partial x} S^{c*}(t|x, x > x^{10})}{\frac{\partial}{\partial x} S^{c*}(0|x, x > x^{10})}. \tag{4.16}$$

Similarly,

$$\begin{aligned} \frac{\frac{\partial}{\partial t} S^{c*}(t|x, x > x^{10})}{\frac{\partial}{\partial t} S^{c*}(t|x^{21}, x > x^{10})} &= \frac{\phi_2(t)\theta_2(x)L_2^{(1)}(\Lambda_2(t, x))}{\phi_2(t)L_2^{(1)}(\Phi_2(t))} \\ &= \theta_2(x)\frac{L_2^{(1)}(\Lambda_2(t, x))}{L_2^{(1)}(\Phi_2(t))} \end{aligned} \tag{4.17}$$

and

$$\theta_2(x) = \lim_{t \to 0} \frac{\frac{\partial}{\partial t} S^{c*}(t|x, x > x^{10})}{\frac{\partial}{\partial t} S^{c}(t|x^{21}, x > x^{10})}. \tag{4.18}$$

Using the normalizations $\theta_j(x^{j1}) = 1$, $j = 1, 2$ we have

$$L_j(\Phi_j(t)) = S^{c*}(t|x^{j1}), \qquad j = 1, 2 \tag{4.19}$$

so that L_1 and L_2 are identifiable. ¤

5. Summary

This paper has shown that, under a variety of restrictions, a class of "latent" competing risks and Roy duration models can be identified. Since the functions we consider are written in terms of the survivor function and its derivatives, our results suggest a couple of estimation strategies. One can either consider using nonparametric analogues of the survivor function and its derivatives or parameterize the components of the model and use standard maximum likelihood procedures.

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