Public Funding of Political Parties*

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Abstract

This paper concerns public funding of parties. Two policy motivated parties receive public funds depending on their vote share. Funds finance electoral campaigns influencing voting. Two cases are investigated. In the first some voters are policy motivated and some are "impressionable" - their vote depends directly on campaign expenditures. In the second campaigning is informative and all voters are policy motivated.

Public funds increases policy convergence in both cases. The effect is larger, the more funding depends on vote shares. When campaigns are informative, there may be multiple equilibria. Intuitively, a large party can stay large since it receives large funds.

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1 Introduction

The funding of political parties is a fundamental ingredient of a democracy. While most countries have some sort of public funding, the way it is provided varies considerably. In the US, for instance, public funding is provided for the large parties on an equal footing basis. Each (large) party receives the same amount. In many other countries, however, parties receive public support depending on their vote share in the last election. Le Duc at al. 1996 investigate 27 democracies and report that in 17 of these public subsidies depend on vote (or seat) share. In Denmark, for example, one vote gives approximately 20 Danish kroner per year, (about 3 US$), see Bille (1997).[1] At a first glance, such a system certainly appears advantageous to large parties. Mair (1994) reports that in many European countries public finance is at least as important for parties as private finance and in some cases it is even more important.[2].

In this paper, we provide a simple theoretical model designed to cast light

[1]In Denmark, direct public support of political parties was enacted in 1986. In 1995 the level was four-doubled. Bille (p201) estimates that total direct public support to political parties was 76 mio d.kr. and concludes that since 1985 the fraction of the parties' income due to public funding has increased dramatically. He investigates all major Danish parties and finds that the public funds constitute 48-98% of a party's income.

[2]Mair (1994) writes (p 9-10): "In Austria and Denmark, for example, *total* state funding for the parties at the national level more or less matches the amounts which they generate from all other sources of income taken together, while in Finland, Norway and Sweden, the total state subsidies received by the parties significantly exceed their total recorded incomes from other sources....In the German case, .., state subsidies account for a sum which is more than ten times greater than accounted for by other sources. Indeed it is only in the Netherlands, the UK and the US that the "private"sources of party funds ...still constitute a larger source of revenue than that which comes from the public purse".

on the issue of public funding. While the debate about public funding has often focussed on fairness and on reducing the power of rich private lobbies, we show that an important consequence of the system prevailing in many European countries is that it promotes policy convergence. Our framework is sufficiently general to cover the case where the allocation of public funds directly depend on the vote share (the case widespread in Europe) as well as the US case, where this is not so.

There is substantive empirical evidence that electoral campaigns affect the ways voters vote (see e.g. Holbrook 1996), which of course also is their purpose[3]. The influence of the electoral campaign on voters can be modeled in several ways. Baron (1994) suggested that there are two types of voters, informed and uninformed, impressionable, voters[4]. Informed voters vote based on the policies proposed by the different political parties (or candidates). Impressionable voters are, however, poorly informed about the policies of the different parties and their voting is *directly influenced by campaign spending.* This is also the assumption made by Grossman and Helpman (1996). Alternatively, one may assume that all voters are policy motivated but not necessarily well informed about the parties' policies. Campbell et al. (1960) provide evidence that most voters cannot correctly identify the position of parties or candidates on the main political issues, while Popkin et al. (1976) try to explain why voters have such a poor level of information by arguing that information gathering is a costly investment[5]. In this

[3] For a different view of the impact of the electoral campaign on voting behavior see for example the seminal work of Lazarsfeld et al. (1944), Campbell et al. (1960) and the more recent work by Finkel (1993).

[4] McKelvey and Ordeshook (1987) also assume that population is divided on informed and uninformed voters.

[5] See Morton and Cameron (1992) and Austen-Smith (1997) for a critique to the infor-

case campaign spending may spread information about the parties' policies, campaigns are like *informative advertising* as studied by for instance Butters (1977) or Grossman and Shapiro (1984). Campaign spending will then affect the electoral outcome since voters will cast their vote depending on their information.

Since both views seem reasonable, we will investigate both within the framework of the same basic model, with two policy motivated parties and a uni-dimensional policy space. In the first case where there are two types of voters, impressionable and policy motivated,. a party will receive a larger share of impressionable votes if it spends more on campaigns. We show that if the allocation of public funds depends on the vote share, then public funding makes the parties' policies converge relative to the case without funding. The reason is intuitive. The likelihood a party's policy is implemented is assumed to depend positively on its vote share. A more moderate policy is therefore more likely to be implemented. For a politically motivated party to the left (or right) this gives a trade off. When public funds depend on the vote share, the incentive to moderate the policy is increased, since more votes induces larger public funds, which attract impressionable voters. In equilibrium, therefore, policies are more moderate than if public funds do not depend on the vote share. More responsive impressionable voters and a system where public funds are more responsive to vote shares, will make policies converge more.

In our model, we assume, for simplicity, that parties and voters are risk averse. However, this is not essential for our results. If agents are risk averse, the European style public funding system is welfare enhancing as it reduces policy polarization and therefore the risk faced by everybody. This points to

mative role of the campaign advertising.

that such a public funding system is more beneficial in countries where the other political institutions do not make for policy convergence.

We briefly consider the case where parties have some private (lump sum) funding. In this case, the effects of public funding are mitigated, as public funds become relatively less important, parties are less eager to moderate their policies in order to capture these funds.

In the second case where political campaigns are informative, all voters are policy motivated, but some are informed and some are uninformed about the policy of a party. An uninformed voter has an expectation of the policy. Evidently, the expectation is important for the results. For the usual reasons - our results should not hinge on arbitrary assumptions about expectations - we study a rational expectations solution. One can conceive of different ways in which campaign spending influences the voters' information. Voters may directly see the party's campaigns in television spots and advertisements in newspapers. However, it may also be the case that a party get more coverage in the mass media by spending money, staging events, having a staff which cater journalists, making press statements etc. Whatever the reason, we assume that the fraction of voters who become informed about a party's policy is increasing in the amount of campaign spending of the party.

The fraction of informed voters is important for the party, when it evaluates the trade off a moderation of the policy involves. Since only informed voters will learn about a policy change, the vote gain a party obtains from moderating its policy is larger, the larger is the fraction of informed voters. Therefore, when campaigns are informative it is the case that the more effective funds are in spreading information, the more will the policies of the parties converge. Hence, although for a different reason, the result is the same as in the case where some voters are impressionable. The model furthermore shows that a party which has access to a more efficient campaign technology will receive a larger share of the votes. Thus the model points to the importance of having good relations to mass media for parties.

A possibility of multiple equilibria also arises. In some cases there are at least three equilibria, a symmetric and two asymmetric. In an asymmetric equilibrium, one party is large and the other small entirely because of the public funding. The large party receives large funds, this imply that a large fraction of voters are well informed about its policy. This gives a strong incentive to moderate policy: a large fraction of voters will learn about the moderation, hence the gain in vote share will be large. The other party, on the other hand, is caught in a situation with few votes and small public funds. Hence, few voters are precisely informed about its policy. Moderating the policy will therefore not be noticed by so many voters and the gain in votes accordingly is small. In equilibrium, this party proposes a rather extreme policy and receives few votes. We identify conditions under which there are two asymmetric equilibria, in one the left party is large, in the other the right party is large.

In countries where public funds depend on the vote share, the funds are typically given when the vote share is known - after the election. This gives rise to a dynamic process, where funds earned in one election are used in campaigns for the next election. However, if the parties are able to borrow on a credit market before the election and repay the debt using the public funds received for the votes in the election, it is possible to spend the public funds before the election. In order to keep the model simple and avoid complicated dynamics, we assume that this is the case. In our model public funding depends on the expected vote share of a party. We study a rational expectations solution, where this expectation is correct.

We are not aware of many theoretical papers considering the issue of public funding. Baron (1994) considers the effect of lobbying in a model of electoral competition with informed and uninformed voters where parties seek to maximize the probability of winning. If there were only informed voters each party would choose a platform equal to the median policy motivated voter's most preferred policy. However, since uninformed voters vote according to the relative size of campaign spending, parties also have an incentive to raise money. This can be done from lobby groups, which are supposed to be extreme. This gives parties an incentive to propose more extreme policies in order to please the lobby groups. Baron shows that in equilibrium polices will be polarized because of this effect. He also shows that introducing public funding (in a lump sum way so that funding is independent of the vote share) mitigates the power of interest groups. The reason is very intuitive; the uninformed votes depend on the relative size of campaign spending. With public funding, lobby groups' contributions become relatively less important and the incentive of the parties to choose a more extreme policy in order to raise money is decreased. As a result, the policies become less polarized.

Our paper differs in important ways. First, our argument does not depend on mitigating lobby groups' power. We consider partisan parties which by themselves would choose polarized policies even in the absence of lobbies. As in Baron's paper the parties have an incentive to raise campaign money. When public funding depends on the vote share, this gives the parties an extra incentive to moderate their policies. When public funding does not depend on the vote share, as in the U.S - which is the system Baron analyses - then there is no policy convergence in our model when campaigns affect impressionable voters. Hence, our analysis shows that the exact functioning of the public funding system is important for the effects on policy convergence.

An equally important difference to Baron's paper is that we provide a model of informative advertising and also analyze the effects of public funding in this framework.

The organization of the paper is as follows: section 2 presents the basic model with impressionable voters and uninformative campaigning. Section 3 derives the equilibrium with public funding. Section 4 analyzes the case of public and private funding. Informative campaigning is the subject of the rest of the paper: section 5 presents the basic model and discuss equilibria. Section 6 introduces the public funds and contains examples of different information technologies which lead to different kind of equilibria. Section 8 concludes. A few proofs are relegated to an Appendix.

2 The model with impressionable voters

We consider a society where politics is uni-dimensional. There are two parties, L and R, and a continuum of voters of measure one. The parties each propose a policy, l and r respectively. Then an election is held. The implemented policy will be the policy proposed by the winning party. There are two kinds of voters, informed who cast their ballot on the party offering the policy they like the most, and impressionable whose vote depends on the relative campaign spending of the parties. This is as in Baron (1994) and Helpmann and Grossman (1996). The fraction of informed voters is $(1-\alpha)$, where $0<\alpha<1$.

If the implemented policy is π, an informed voter with bliss point x gets utility

$$u(\pi;x)=-\left|\pi-x\right|. \tag{1}$$

The bliss points are distributed on the interval [0,1] according to the cdf $F(x)$, the corresponding density is $f(x)$. We assume that f is strictly positive and differentiable for all x in [0,1]. Parties are policy motivated and have the same type of utility function as voters (see Wittman 1990). The bliss point of party L is x_L,and the bliss point of party R is x_R. We assume that $x_L < x_R$.

Uninformed, impressionable, voters do not receive utility from policy. Their vote depends on the amount of campaign money spent by each party. Party L spends c_L, party R, c_R. The difference in spending is $\Delta \equiv c_L - c_R$. The fraction of impressionable voters in favor of party L is $H(\Delta)$. We will assume that $H\prime > 0$, $H(0) = \frac{1}{2}$ and $H\prime\prime(\Delta) \leq 0$ for all $\Delta \geq 0$. The more a party spends on campaigns, the larger its share of impressionable voters[6]. However, if a party obtains more money than the other that share does not grow at an increasing rate, and if the parties spend equally much they get a fair share of the impressionable voters. Hence, impressionable voters are not biased towards any of the parties. We further assume that the H function is symmetric: $H(\Delta) = 1 - H(-\Delta)$, so $H''(\Delta) \geq 0$ for all $\Delta \leq 0$ and, assuming that H'' is continuous, we get $H'' = 0$.

The H - function is taken as a fundamental part of the model. The formulation corresponds to the one chosen by Helpmann and Grossman (1996) and makes for simplicity. Alternatively, we could have assumed that the fraction of impressionable voters in favor of party L was a function of the relative spending as is the case in Baron (1994). Although the quantitative results would differ, this would have no impact on our qualitative results.

The parties are supposed to be committed to their policy proposals. If party L proposes l and party R proposes r, where $l \leq r$[7], an informed voter

[6] The way electoral campaigning works here is similar to the "predatory adverstising" analyzed in oligopoly theory, see Friedman (1983).

[7] In principle, the parties can of course propose policies l, r where $r < l$. This will never occur in equilibruim, however, so we will just disregard this case.

with bliss point x prefers party $L's$ policy if and only if $u(l;x) \geq u(r;x)$, i.e. if and only if

$$x \leq \frac{l+r}{2}$$

Voting is sincere, thus the fraction of informed voters in favor of party L is $F\left(\frac{l+r}{2}\right)$. If, furthermore, the difference in campaign spending of the parties is $\Delta = c_L - c_R$, the total number of voters in favor of party L is

$$v = \alpha H(\Delta) + (1-\alpha) F\left(\frac{l+r}{2}\right). \quad (2)$$

We follow Grossman and Helpman (1996) and assume that the chance that a party's policy is implemented is increasing in its share of votes. As Grossman and Helpman also notes this is slightly ad hoc although perhaps not unreasonable. This may be rationalized in several ways: it may be because the larger a party is, the larger is its influence in parliament. [8] Whatever the reason, we assume that party $L's$ policy, l , is implemented with probability $p(v)$. For simplicity, we will assume that $p(v) = v$. This simplifies formulas without having qualitative significance. Intuitively, the steeper the p function at $1/2$ is, the more incentive the parties have to moderate their policy[9].

[8] One can also assume, as in many current models of political competition, that parties are uncertain about the outcome of the election, perhaps because of polling errors or uncertainty about who will vote and who will abstain. The larger the (ex-ante) vote share a party has, the larger is the probability that it wins the election and implements its policy (see Roemer 1994 for microfoundations of this uncertainty approach) An additional alternative model that doesn't require uncertainty assumes, as in Alesina and Rosenthal (1995, 1996), Ortuno-Ortin (1997) and Gerber and Ortuno-Ortin (1998), that the implemented policy is a convex combination of the two propossed policies where the weights are given by the share of votes of each party. Since we assume that agents are risk-neutral this approach is equivalent to the one developed in the paper.

[9] We see that our linear formulation of the p function makes for less policy convergence

Parties receive public funds, which can be used for campaigning. All funds are spent on campaigns as parties are not interested in money per se. These funds (may) depend on the share of votes the party receives. In the real world it is typically case that a party receives funds depending on the share of votes it received in the last election, where the vote share is known!. If the party wants to spend money before the election, it either has to take loans or spend the money it received for the votes in the last election. If the party has to take loans, there is an issue of whether the party is creditworthy or whether it will be credit-constrained. If funds earned in the last election are used, the problem is dynamic. Although they may be important in the real world, we will disregard issues of creditworthiness. We will also avoid dynamic issues. We will study a rational expectations solution. The timing is as follows. First parties propose policies, then they receive funds depending on the expected vote share. In equilibrium, the expected vote share is correct. The rational expectations solution can also be interpreted as covering the case where parties receive public funds after the election and get credit on a perfect credit market before the election. The credit is then repaid after the election using the public funds received.

We normalize the size of the public fund available for funding of the parties to one. We will assume that the public funding treats the parties equally so the function which determines the size of a party's public funds is the same for the two parties. If party L is expected to receive the share v^e of the votes, it receives $c_L^e = \psi(v^e)$ in public funds (and party R receives

than if we assumed that p was very steep at $1/2$. On the other hand the linear formulation of the parties' utility functions makes for more convergence than if parties utility functions were strictly concave. In the latter case the marginal disutility of moderating the policy would be increasing.

$c_R^e = \psi(1 - v^e))$. The equal treatment assumption in particular imply that $\psi(\frac{1}{2}) = \frac{1}{2}$. We assume that the funding system fulfills $\psi' \geq 0$ for all v^e and $\psi''(v^e) \leq 0$ for all $v^e \geq \frac{1}{2}$ and $\psi''(v^e) \geq 0$ for all $v^e \leq \frac{1}{2}$. A party already obtaining more than 50% of the votes (weakly) increases its funds by receiving more votes but at a non-increasing rate. Notice that, assuming continuity of ψ'' it must be true that $\psi''(\frac{1}{2}) = 0$. The difference in funds is $\Delta(v^e) \equiv \psi(v^e) - (1 - \psi(v^e)) = 2\psi(v^e) - 1$, and we have $\Delta'(v^e) \geq 0$ for all v^e and $\Delta''(v^e) \leq 0$ for all $v^e \geq \frac{1}{2}$, $\Delta''(v^e) \geq 0$ for all $v^e \leq \frac{1}{2}$ and $\Delta''(v^e) = 0$ for $v^e = \frac{1}{2}$.

In the Danish system described in the introduction where a party receives approximately 20 Danish .kr. (US$ 3) per year per vote, $\psi' > 0$ and $\psi'' = 0$. The US system corresponds to the case where $\psi(v^e) = 1/2,.$ for all (relevant) v^e.

Given an expected vote share v^e, and policies l, r, the actual share of votes for party L is

$$v = \alpha H(\Delta(v^e)) + (1 - \alpha)F\left(\frac{l + r}{2}\right).$$

Under rational expectations, party $L's$ actual vote share equals its expected vote share, $v = v^e$, hence v solves

$$v = \alpha H(\Delta(v)) + (1 - \alpha)F\left(\frac{l + r}{2}\right). \tag{3}$$

If

$$\alpha\, H'(\Delta(v))\, \Delta'(v) < 1, \text{ for all } v \in [0, 1] \tag{4}$$

the solution is unique. We will assume this. It means that the responsiveness of the public funds and of the impressionable voters are sufficiently limited to make the problem well-behaved. Were this assumption not fulfilled, then an increase in votes of one percent for a party would immediately increase the number of impressionable votes for the party so much that the share of total votes increased by more than one percent again. Clearly, this would lead to an even larger increase in votes and so forth. The situation would be unstable. Notice, we are not assuming that the final equilibrium is unique, just that there is a unique vote share for each party for a given pair of policies. If the solution was not unique, the parties would be in the unpleasant situation of not being able to predict the vote share for a given pair of policies. We will denote the solution to (3) by $v(l,r)$. The implicit function theorem yields,

$$\frac{\partial v(l,r)}{\partial l} = \frac{\frac{1-\alpha}{2} f\left(\frac{l+r}{2}\right)}{1-\alpha\ H'(\Delta(v))\ \Delta'(v)} > 0 \tag{5}$$

Similarly for the vote share of party R , $1-v$

$$\frac{\partial(1-v(l,r))}{\partial r} = -\frac{\frac{1-\alpha}{2} f\left(\frac{l+r}{2}\right)}{1-\alpha\ H'(\Delta(v))\ \Delta'(v)} < 0 \tag{6}$$

When party L changes policy, the votes of some informed voters change. This changes the funds allocated to the parties, and therefore also the share of impressionable votes, which again changes the funds ect. In the rational expectations solution, the public funds are given as a fixed point to equation (3) so all these repercussions are taken into account. The assumption $\alpha\Delta\prime(v)H\prime(\Delta(v)) < 1$ ensures that the comparative statics make sense, if party L increases its policy (towards the middle) it will increase its share of votes.

3 Political equilibrium

The parties are policy motivated and each party tries to maximize its expected utility. It takes as given the policy choice of the other party and recognizes how the choice of policy may influence the distribution of funds and therefore relative campaign spending. Given the pair of policies (l, r) , party's L expected utility is given by

$$p(v(l,r))\,u(l;x_L) \;+\; (1 - p(v(l,r)))\;u(r;x_L)$$

which we can write

$$\left(\alpha H(\Delta) + (1-\alpha)F\left(\frac{l+r}{2}\right)\right)(-|l - x_L|) + \left(1 - \left(\alpha H(\Delta) + (1-\alpha)F\left(\frac{l+r}{2}\right)\right)\right)(-|r - x_L|) \tag{7}$$

Similarly, party's R expected utility is given by

$$\left(\alpha H(\Delta) + (1-\alpha)F\left(\frac{l+r}{2}\right)\right)(-|l - x_R|) + \left(1 - \left(\alpha H(\Delta) + (1-\alpha)F\left(\frac{l+r}{2}\right)\right)\right)(-|r - x_R|) \tag{8}$$

Here, $\Delta = \Delta(v(l,r))$, where $v(l,r)$ is the solution to equation (3) as described above.

A *Polical Equilibrium with uninformative campaigning* is a pair of policies (l^*, r^*), such that l^* maximizes (7) given r^* and r^* maximizes (8) given l^*.

In principle, a party's optimal policy may be equal to the party's bliss point where the utility function is non-differentiable. We will, however, only consider interior solutions where $x_L < l < r < x_R$. The first order conditions are

$$\left(\alpha H\prime(\Delta)\frac{d\Delta}{dl} + \frac{(1-\alpha)}{2}f\left(\frac{l+r}{2}\right)\right)(r-l) - \left(\alpha H(\Delta) + (1-\alpha)F\left(\frac{l+r}{2}\right)\right) = 0 \tag{9}$$

and

$$\left(\alpha H\prime(\Delta))\frac{d\Delta}{dr} + \frac{(1-\alpha)}{2}f\left(\frac{l+r}{2}\right)\right)(r-l) - \left(\alpha\,(1 - H(\Delta)) + (1-\alpha)\left(1 - F\left(\frac{l+r}{2}\right)\right)\right) = 0 \tag{10}$$

In the Appendix we show that a sufficient condition for the second order condition for maximum is that

$$\frac{f\prime}{f} < 4, \text{ and } \frac{d^2\Delta}{dl^2} \Big/ \frac{d\Delta}{dl} + \frac{H\prime\prime}{H\prime} < 2. \tag{11}$$

The latter condition is a joint condition on the responsiveness of the public funding system and the responsiveness of the impressionable voters. It is hard to give general conditions ensuring that it is fulfilled. We show however that if $H\prime\prime = 0$ and $\psi\prime\prime = 0$, so $\Delta\prime\prime = 0$, the condition is fulfilled. As is clear from the deviation, it is not necessary that $H\prime\prime = 0$ and $\Delta\prime\prime = 0$, they should just not be too large.

Now

$$\frac{d\Delta}{dl} = \Delta\prime(v(l,r))\frac{\partial v(l,r)}{\partial l} \tag{12}$$

Inserting (5) and (12), and simplifying we can rewrite the first order condition for maximum for party L, (9),

$$\frac{1}{(1-\alpha\, H'(\Delta)\,\Delta'(v))}\frac{f\left(\frac{l+r}{2}\right)}{2}(r-l) - \left(\frac{\alpha}{1-\alpha}H(\Delta) + F\left(\frac{l+r}{2}\right)\right) = 0. \tag{13}$$

In a similar way the first order condition for R (10) can be written

$$\frac{1}{1-\alpha\, H'(\Delta)\,\Delta'(v)}\frac{f\left(\frac{l+r}{2}\right)}{2}(r-l) - \left(\frac{\alpha}{1-\alpha}(1-H(\Delta)) + \left(1-F\left(\frac{l+r}{2}\right)\right)\right) = 0 \tag{14}$$

Thus, since the first parts of (13) and (14) are identical,

$$\frac{\alpha}{1-\alpha}H(\Delta) + F\left(\frac{l+r}{2}\right) = \frac{\alpha}{1-\alpha}(1-H(\Delta)) + \left(1-F\left(\frac{l+r}{2}\right)\right),$$

which implies that

$$v = \alpha H(\Delta) + (1-\alpha)\,F\left(\frac{l+r}{2}\right) = \frac{1}{2}. \tag{15}$$

Each party get half of the votes. Since, $\Delta(\frac{1}{2}) = 0$, and $H(0) = \frac{1}{2}$, this implies

$$F\left(\frac{l^* + r^*}{2}\right) = \frac{1}{2} \tag{16}$$

Let m be the median bliss point in the population, i.e. $F(m) = 1/2$, then

$$\frac{l^* + r^*}{2} = m \tag{17}$$

The expected policy equals the median (informed) voter's preferred policy. Inserting into (13) and rearranging yields

$$(r^* - l^*) = \frac{1 - \alpha\, H'(0)\ \Delta'(\frac{1}{2})}{1 - \alpha} \frac{1}{f(m)} \tag{18}$$

Using $\frac{l^* + r^*}{2} = m$, and (18) we finally get

$$l^* = m - \frac{1}{2}\frac{1 - \alpha\, H'(0)\ \Delta'(\frac{1}{2})}{1 - \alpha} \frac{1}{f(m)}. \tag{19}$$

and

$$r^* = m + \frac{1}{2}\frac{1 - \alpha\, H'(0)\ \Delta'(\frac{1}{2})}{1 - \alpha} \frac{1}{f(m)}. \tag{20}$$

The parties propose policies on each side of the median informed voter's most preferred policy. The difference in the policies depends on the density of voters at the median, $f(m)$. If the density is high, the policies are close to the median voter's preferred policy as many votes can be gained by moving the policy towards the middle. If the density is small, parties choose policies closer to their own preferred policy.

The difference also depends on the term $\frac{1-\alpha\, H'(0)\ \Delta'(\frac{1}{2})}{1-\alpha}$, which reflects the distribution of voters on informed and impressionable (as given by α) as well as on how responsive the impressionable voters are as given by $H\prime(0)$ and how responsive the public funding system is as given by $\Delta'(\frac{1}{2})$. The larger the responsiveness, the closer the policies.

The effect of the public funding system can be ascertained as the case of no public funding corresponds to the special case of the model where $\Delta\prime(0) = 0$. With no public funding, the difference in policies would be

$$r - l = \frac{1}{1-\alpha}\frac{1}{f(m)}$$

Comparing with (18) we see that the existence of *public funding makes the parties' policies converge* compared with the case of no funding. The intuition is simple. The public funds give an extra incentive to moderate policies, since the extra votes gained release public funds, which can be used to gain further votes.

We may also compare with the case of no impressionable voters, $\alpha = 0$. In this case the difference in policies would be

$$r - l = \frac{1}{f(m)}$$

From (18) it directly follows that if $\frac{1-\alpha\, H'(0)\ \Delta'(\frac{1}{2})}{1-\alpha} < 1$, corresponding to $H'(0)\ \Delta'(\frac{1}{2}) > 1$, polices are more convergent than in the benchmark case with no impressionable voters, if on the other hand $H'(0)\ \Delta'(\frac{1}{2}) < 1$, policies are more divergent. In the knife edge case where $H'(0)\ \Delta'(\frac{1}{2}) = 1$, the vote share of a party responds in the same way to changes in policy whether there are impressionable voters or not. The impressionable voters then distribute their votes in exactly the same way as the informed and the difference in policies between the parties equals the difference obtained in the benchmark case when there are no impressionable voters. Hence, in this case *the publicly funded system works as if there were only informed voters*, it is as if the informed voters decide everything. The public funding system somewhat offsets the fact that a part of the electorate is poorly informed.

We have assumed that the equilibrium is interior, $x_L < l < r < x_R$. Clearly, this takes that $r - l < x_R - x_L$. As can be seen from equation

(18) this requires $f(m)$ is sufficiently large and or $\frac{1-\alpha H'(0)\ \Delta'(\frac{1}{2})}{1-\alpha}$ sufficiently small. An example fulfilling this condition is, $x_L = 0$, $x_R = 1$, $m = 1/2$ and $\frac{1-\alpha H'(0)\ \Delta'(\frac{1}{2})}{1-\alpha}\frac{1}{f(m)} < 1$. The latter condition is for example fulfilled if $H'(0)\ \Delta'(\frac{1}{2}) = 1$ and $f(m) > 1$. From our derivations it is clear that an interior equilibirum then exists, (provided the second order condition is fulfilled, which for instance is the case if $H\prime\prime = \Delta\prime\prime = 0$ and $f\prime/f < 4$). [10]

Notice that since it is the difference in campaign spending which is important, it is not the total size of the public funding per se which is important but rather how responsive the funding is to changes in vote share as given by $\Delta'(\frac{1}{2})$. This would also be the case if H depended on $\frac{c_L}{c_L+c_R}$, the relative campaign spending, and not on the difference $c_L - c_R$. The fact that the average policy $\frac{l+r}{2}$ equals m under both systems follows from our assumption that the impressionable voters are not biased towards any of the parties.

4 When parties also have private funds

Suppose that the parties also have access to private funds. For simplicity, we assume that these funds are independent of the policy of the parties. One can imagine that the parties have endowments or that some groups provide these funds since their preferences are in line with the party's and they would like to contribute to make it more likely that the party wins the election. An example would be trade unions supporting socialdemocratic parties without special reference to the exact policy chosen. This assumption is made for

[10] Interiority is not automatically guaranteed in our model since we assume that the utility functions of the parties are linear. Had we instead assumed concave utility with a derivative equal to zero at the bliss point - as would be the case with a quadratic utility function - $l - r$ would automatically be less than $x_R - x_L$ in equilibrium.

simplicity only. Furthermore, there is by now a comprehensive literature on private lobbying and the effects on policy are well-understood, see e.g. Baron (1994), Grossman and Helpman (1996), or the surveys in Persson and Tabellini (1999) and Austen-Smith (1997). Here we are just interested in the interplay of private and public funding.

Assume, therefore that party L has c_{PL} private funds and party R has c_{PR}. With public funding and a vote share of v for party L the total difference in funding between the two parties then is

$$\begin{aligned} \tilde{\Delta}(v) &= \psi(v) - (1 - \psi(v)) + c_{PL} - c_{PR} \\ &= \Delta(v) + \Delta_P \end{aligned}$$

where $\Delta_P \equiv c_{PL} - c_{PR}$ is the difference in private funds. The analysis above is then modified as follows. The vote share of party L will be the fixed point of

$$v = \alpha H(\Delta(v) + \Delta_P) + (1 - \alpha)F\left(\frac{l + r}{2}\right)$$

so

$$\frac{\partial v}{\partial l} = \frac{\frac{1-\alpha}{2} f\left(\frac{l+r}{2}\right)}{1 - \alpha \Delta' H'(\Delta(v) + \Delta_P)}$$

just as before, the only difference is that $H\prime$ is taken in a different point.

The first order conditions for maximum are unchanged, still equal to (13) and (14), (except H' is taken in $\Delta + \Delta_P$). Hence we still have in equilibrium that $v = 1/2$ and therefore that $\Delta = 0$. Suppose that party L has a smaller endowment that party R, so $\Delta_P < 0$. Then $H(0 + \Delta_P) < 1/2$, and since we still have $v = 1/2$, it must be the case that $F\left(\frac{l+r}{2}\right) > 1/2$, which means $\frac{l+r}{2} > m$. The average policy is more to the right, the reason is that the rich party R can afford to propose a more extreme policy, since it gets a larger share of the impressionable voters.

In this analysis, the relative size of the public and private funds does not matter. This may seem unreasonable. Suppose instead that the impressionable voters' votes depend on the relative campaign spending so H depends on $\frac{c_L}{c_L+c_R}$. Without private funds, the above analysis is unchanged, just let $\Delta(v) = \frac{\psi(v)}{\psi(v)+1-\psi(v)} = \psi(v)$. With private funds, we get a slight modification since now the relative funds are

$$\frac{c_{PL} + \psi(v)}{1 + c_{PL} + c_{PR}}$$

The vote share of party L is a fixed point of

$$v = \alpha\, H\left(\frac{c_{PL} + \psi(v)}{1 + c_{PL} + c_{PR}}\right) + (1-\alpha)\, F\left(\frac{l+r}{2}\right)$$

and hence

$$\frac{\partial v}{\partial l} = \frac{\frac{1-\alpha}{2} f\left(\frac{l+r}{2}\right)}{1 - \alpha\, \psi' \, \frac{1}{1+c_{PL}+c_{PR}}\, H'\left(\frac{c_{PL}+\psi(v)}{1+c_{PL}+c_{PR}}\right)}$$

We see that compared with before, the vote gaining effect from public funding is smaller the larger are the private funds. Again the richer party, will gain from being able to propose a more extreme policy and still get half of the votes. Furthermore, in this setting, the expression for the policy polarization becomes

$$(r-l) = \frac{1 - \alpha\, \psi'(\frac{1}{2})\, \frac{1}{1+c_{PL}+c_{PR}}\, H'\left(\frac{c_{PL}+1/2}{1+c_{PL}+c_{PR}}\right)}{1-\alpha} \frac{1}{f(m)}$$

We see that the larger private funds are (which really means larger relative to public funds, remember we normalized the size of public funds to one), the more divergent policy is. The policy convergence induced by the public funds is reduced as their relative size compared with private funds are reduced. To sum up, with private and public funds, the prediction of the model is that there should be more policy convergence in countries where public funds are relatively large compared with private funds.

As discussed in the introduction Baron (1994) considers a model with private and public funding where parties seek to maximize plurality. With no funding these parties would choose policies equal to the median voter's preferred policy. In his model private funding is given by extreme lobby groups, parties therefore propose more extreme policies in order to please the lobby groups. Public funds are given in a lump sum way (as in the US system). They mitigate the power of lobby groups, since the relative importance of private funds are decreased, and policies become more convergent. We see that in our model we get the exact opposite result from giving parties lump sum contributions. These contributions (whether private or public) decrease the relative importance of the funds earned through votes and this mitigates the incentive to modify the policy. Clearly, if we introduced extreme lobby groups, this would also give our parties and incentive to propose more extreme policies. The bottomline is that the exact way public and privated funding are provided is crucial for the results.

5 Informative campaigning

In this section, we will look at the case where campaigning is informative. We assume that all voters are policy interested and have preferences as described above. Voters do not learn the parties' policies automatically, but are informed through campaigns.

If a party spends c on campaigns, a fraction of voters equal to $\phi(c)$ will learn the policy of the party. We assume that parties cannot target their campaigns, so that the probability that a particular voter becomes informed through the campaign is independent of her or his bliss point. One can imagine that the party advertises in television[11] or in magazines and that only voters who see the advertisement learns about the actual policy of the party. However, it may also be the case that some voters are informed regardless of whether they see advertisements or not. These voters may read newspapers, listen to radio, watch television etc. The important thing is that some voters only learn through campaigns and that a more intensive campaign makes a larger share of the voters informed about the party's policy. See Holbrook (1996) for evidence of the information generated in the electoral campaigns and Alvarez (1996), Brians and Wattenberg (1996) and Just et al. (1990) for evidence that television advertising increases voter information.

We will assume that voters who are uninformed about a party's policy nevertheless are informed about the existence of the party, and have some expectation about the party's policy. For the usual reasons, we will assume that the uninformed voters' expectations are rational. We will briefly comment on the case where expectations are not rational but just given and perhaps determined by past experience.

With rational expectations, the timing is as follows. First uninformed voters form expectations (which in the end turns out to be correct), then parties choose policies which are seen only by informed voters, and then the election is held. The crucial aspect for the analysis is that when a party decides on its policy, it knows that only informed voters will learn the policy choice. Hence, the effect on vote shares from changes in a party's policy depends on the fraction of voters who become informed. The larger is this fraction, the more responsive is the vote share to changes in the policy. Let the uninformed voters' belief about party L's policy be denoted l^e and simi-

[11] Television advertisements represent the most important expenditure in the electoral campaigns in many countries, see West (1993).

larly the belief about party R's policy be denoted r^e.

We assume that each party makes a campaign. For the moment we will not be specific about how the funds are raised. Let ϕ_L be the fraction of voters learning about party $L's$ policy and let ϕ_R be the fraction learning party $R's$ policy. Given policies l, r and fractions ϕ_L, ϕ_R, the vote share for party L is

$$\begin{aligned} & V(l, r, l^e, r^e, \phi_L, \phi_R) \\ = \; & \phi_L \phi_R F\left(\frac{l+r}{2}\right) + \phi_L (1-\phi_R) F\left(\frac{l+r^e}{2}\right) \\ & (1-\phi_L) \phi_R F\left(\frac{l^e+r}{2}\right) + (1-\phi_L)(1-\phi_R) F\left(\frac{l^e+r^e}{2}\right) \end{aligned} \tag{21}$$

A fraction $\phi_L \phi_R$ learn both parties policies, of these the fraction $F\left(\frac{l+r}{2}\right)$ prefers the policy of party L. A fraction $\phi_L(1-\phi_R)$ only learn the policy of party L, these voters rely on their prior in assessing the policy of party R and a fraction $F\left(\frac{l+r^e}{2}\right)$ of these voters prefer the policy of party L and so forth.

When party L changes policy (taking l^e, r^e and r as given) its vote share changes with

$$\frac{\partial V}{\partial l} = \phi_L \phi_R \frac{1}{2} f\left(\frac{l+r}{2}\right) + \phi_L (1-\phi_R) \frac{1}{2} f\left(\frac{l+r^e}{2}\right)$$

Using rational expectations $r^e = r$ this reduces to

$$\frac{\partial V}{\partial l} = \frac{\phi_L}{2} f\left(\frac{l+r}{2}\right) \tag{22}$$

We see that $\frac{\partial V}{\partial l}$ depends on ϕ_L. The reason is that only informed voters will respond to the policy change. Notice that the larger is the fraction of informed voters, the more responsive is the vote share to changes in policy.

We also have

$$\frac{\partial V}{\partial \phi_L} = \phi_R \left(F\left(\frac{l+r}{2}\right) - F\left(\frac{l^e+r}{2}\right)\right) + (1-\phi_R)\left(F\left(\frac{l+r^e}{2}\right) - F\left(\frac{l^e+r^e}{2}\right)\right). \tag{23}$$

so $\frac{\partial V}{\partial \phi_L} > 0$ if $l > l^e$ and $\frac{\partial V}{\partial \phi_L} < 0$ if $l < l^e$. By a similar argument one can show that $\frac{\partial V}{\partial \phi_R} < 0$ if $r < r^e$, and $\frac{\partial V}{\partial \phi_R} > 0$ if $r^e < r$. If party $L's$ policy is more moderate than expected by the uninformed, party L gains from informing more voters about its policy, while the opposite is true if the policy is more extreme than expected.

Under rational expectations, $l^e = l^*$ and $r^e = r^*$, so equation (23) becomes

$$\frac{\partial V}{\partial \phi_L} = 0 \tag{24}$$

By a similar argument, under rational expectations

$$\frac{\partial V}{\partial \phi_R} = 0 \tag{25}$$

We also have that under rational expectations

$$V(l, r, l, r, \phi_L, \phi_R) = F\left(\frac{l+r}{2}\right) \tag{26}$$

Party $L's$ expected payoff is given by

$$V(l, r, l^e, r^e, \phi_L, \phi_R)\left(-|l - x_L|\right) + \left(1 - V(l, r, l^e, r^e, \phi_L, \phi_R)\right)\left(-|r - x_L|\right). \tag{27}$$

Given l^e, r^e and r party L seeks to maximize its expected payoff by choosing l. We will again focus on interior equilibria. Taking the derivative of (27) wrt to l we get the first order condition for party L

$$\left(\frac{\partial V}{\partial l} + \frac{\partial V}{\partial \phi_L}\frac{\partial \phi_L}{\partial l} + \frac{\partial V}{\partial \phi_R}\frac{\partial \phi_R}{\partial l}\right)(r^* - l^*) - V = 0 \tag{28}$$

Notice, that this expression includes effects on ϕ_L from changes in l. For instance under public funding, changes in l will affect the vote share and therefore the funds party L receives and ultimately ϕ_L.

Under rational expectations (24) and (25) yields that (28) reduces to

$$\frac{\partial V}{\partial l}\left(r^* - l^*\right) - V = 0$$

Inserting (22) in the first order condition gives

$$\left(r^* - l^*\right) = \frac{F\left(\frac{l^*+r^*}{2}\right)}{\frac{\phi_L}{2} f\left(\frac{l^*+r^*}{2}\right)} \tag{29}$$

In the Appendix we show that the second order condition for maximum is fulfilled under our assumption $f\prime/f < 4$.

Similarly, party R seeks to maximize

$$V(l,r,l^e,r^e,\phi_L,\phi_R)\left(-\left|l - x_R\right|\right) + \left(1 - V(l,r,l^e,r^e,\phi_L,\phi_R)\right)\left(-\left|r - x_R\right|\right) \tag{30}$$

and the first order condition for maximum R is

$$\left(\frac{\partial V}{\partial r} + \frac{\partial V}{\partial \phi_L}\frac{\partial \phi_L}{\partial r} + \frac{\partial V}{\partial \phi_R}\frac{\partial \phi_R}{\partial r}\right)\left(r^* - l^*\right) - (1 - V) = 0 \tag{31}$$

which yields

$$\left(r^* - l^*\right) = \frac{1 - F\left(\frac{l^*+r^*}{2}\right)}{\frac{\phi_R}{2} f\left(\frac{l^*+r^*}{2}\right)} \tag{32}$$

Using (29) and (32) gives us

$$F\left(\frac{l^* + r^*}{2}\right) = \frac{\phi_L}{\phi_L + \phi_R} = \frac{1}{1 + \frac{\phi_R}{\phi_L}}. \tag{33}$$

using (26), we finally get

$$V(l^*,r^*,l^*,r^*,\phi_L,\phi_R) = \frac{\phi_L}{\phi_L + \phi_R} = \frac{1}{1 + \frac{\phi_R}{\phi_L}} \tag{34}$$

Party $L's$ share of votes is determined by the relative fractions of voters informed about each party's policy. If the fractions are equal, then party L gets exactly one half of the votes. If if $\phi_L > \phi_R$, party L receives more than half of the votes. The reason is as follows. When ϕ_L is relatively large, then more voters will learn about a policy change by party L than by party R. A large ϕ_L makes party $L's$ vote share more responsive to changes in policy, $\frac{\partial V}{\partial l}$ is large, as can be seen from equation (22). When $\phi_L > \phi_R$ party $L's$ vote share is therefore more responsive to policy changes than party $R's$ vote share. This imply that party L gains more votes from moderating its policy towards the middle than party R does. Hence, in equilibrium party L's policy will be more moderate than party $R's$ and party L will consequently get a larger vote share than party R. The backside of the coin for party L is that moderating the policy makes it less attractive to party L. When $F\left(\frac{l+r}{2}\right) > 1/2$, then $\frac{l+r}{2} > m$, the average policy is to the right of the median voter's preferred policy. We have not been able to show that this is also true for the expected policy. However, in the specific example where the density function is triangular, it is true.

Note that the fraction $\frac{\phi_L}{\phi_L+\phi_R}$ is *not* the fraction of informed voters learning about party $L's$ policy. The latter fraction equals $\frac{\phi_L}{1-(1-\phi_L)(1-\phi_R)} = \frac{\phi_L}{\phi_L+\phi_R-\phi_L\phi_R}$.

Equation (34) points to the value of having access to a good information technology. Suppose the parties receive the same amount of funding, but party L has better access to mass media. This could be due to historical reasons, personal relations, perhaps the party leader does well on TV or for many other reasons. This party will be able to convert money more effectively into information, so $\phi_L > \phi_R$, and hence receive a larger vote share. Good relations to mass media makes for high vote shares in our model!

Inserting equation (33) into (32), we get that the policy polarization is given by

$$r - l = \frac{2}{(\phi_L + \phi_R)\, f\left(\frac{l+r}{2}\right)} \tag{35}$$

Now increase, ϕ_L and ϕ_R in such a way that the relative size is unaffected, i.e. ϕ_L/ϕ_R is constant. From (33), $\frac{l+r}{2}$ is unaffected, (35) directly gives that the policy polarization $l-r$ decreases. In the symmetric case where $\phi_L = \phi_R$, this is particularly clear. Here, (33) tells us that $\frac{l+r}{2} = m$, so (35) reduces to

$$r - l = \frac{1}{\phi f(m)}. \tag{36}$$

We see that, the polarization of the policy proposals decrease in the fraction of informed voters. Hence, when campaigns are informative, policies converge if the funds available for campaigning increase.

Let us briefly consider the case where expectations are not formed rationally but depend on past policy proposals. Assume that time is discrete and runs from zero to infinity. Each period is as described above. First parties propose policies, an election is held and the winner implements the promised policy. Suppose that the expectations of period t depends on the policy proposals of period $t-1$, so $l_t^e = l_{t-1}$ and $r_t^e = r_{t-1}$. Suppose further that parties are myopic, and seeks to maximize expected utility of one period only. In a stationary state, one has $l_t^e = l_{t-1} = l_t$ and $r_t^e = r_{t-1} = r_t$ for all $t \geq 0$. The rational expectations solution studied above corresponds therefore to a stationary state of the dynamic game outlined here.

6 Public funding

We will now investigate the case where parties' campaign funds come from public funds. As in the case where voters are impressionable, we will disregard potential credit problems etc. and assume that each party receives funds depending on its expected vote share. The expectation is formed rationally and is correct. Party $L's$ vote share is $V(l, r, l^e, r^e, \phi_L, \phi_R)$. As argued above, under rational expectations about the parties' policy choices, the vote share is given by equation (26), which we restate for convenience

$$V(l, r, l, r, \phi_L, \phi_R) = F\left(\frac{l+r}{2}\right).$$

The derivations above took into account that parties realize that their choice of policy changes funding and therefore ϕ_L and ϕ_R, so these derivations are still valid, see equation (28) and the related discussion. Using equation (33) we get

$$V(l, r, l, r, \phi_L, \phi_R) = \frac{\phi_L(c_L)}{\phi_L(c_L) + \phi_R(1 - c_L)}, \tag{37}$$

where we explicitly have written the $\phi's$ as functions of the funds allocated to party L. Remember we assume that the total amount of public funds available to the parties is one. As previously, we assume that the funds allocated to a party depends on its vote share, so $c_L = \psi(v)$. Hence,

$$c_L = \psi\left(\frac{\phi_L(c_L)}{\phi_L(c_L) + \phi_R(1 - c_L)}\right) \tag{38}$$

The funds allocated to party L is determined as a fixed point of this equation. Depending on the functional form of the ϕ and ψ functions there may be one or more fixed points.

In order to proceed, we therefore need to be more specific about the ϕ and ψ functions. We will assume that the public funding system is fair in the sense that $\psi(1/2) = 1/2$. A party which receives half of the votes receives half of the public funds. We also assume that a party which receives some votes receive some funds, $\psi(v) > 0$ for $v > 0$. Furthermore, the public funding system does not punish a party for receiving more votes so $\psi\prime \geq 0$.

We also assume that the parties are equally efficient in informing voters so $\phi_L(c) = \phi_R(c) = \phi(c)$. In this case equation (38) becomes

$$c_L = \psi\left(\frac{\phi(c_L)}{\phi(c_L) + \phi(1 - c_L)}\right) \tag{39}$$

Since the public funding system is fair, we see that $c_L = 1/2$ is a solution to this equation. Hence, a symmetric equilibrium exists, where each party receives half of the votes.

Advertising by parties is not the only way voters receive information about the parties policies. Some (many) voters read newspapers, listen to radio, watch TV etc., therefore some voters will be informed about a party's policy even though the party does not advertise it. This means it is reasonable to assume that $\phi(0) > 0$. As a consequence, the share of votes for party L as given by (37) is positive in equilibrium, similarly the vote share for party R is positive. By assumption a party receives funds if its vote share is positive, so $c_L = 0$ or $c_L = 1$ cannot be compatible with equilibrium.

Whether there are multiple equilibria depends on the right hand side of equation (39). Since the right hand side is positive at $c_L = 0$ and less that one at $c_L = 1$, a sufficient (but not necessary) condition for multiple equilibria is that the slope of the right hand side evaluated at $c_L = 1/2$ is larger than one. The slope of the right hand side is

$$\psi\prime \bullet \frac{\phi\prime(c)\left(\phi(c) + \phi(1-c)\right) - \left(\phi\prime(c) - \phi\prime(1-c)\right)\phi(c))}{\left(\phi(c) + \phi(1-c)\right)^2}$$

Evaluated at $c = 1/2$ we get

$$\psi\prime\left(\frac{1}{2}\right) \bullet \frac{\frac{1}{2}\phi\prime(\frac{1}{2})}{\phi(\frac{1}{2})}$$

Hence, remembering that $\psi(\frac{1}{2}) = \frac{1}{2}$, a sufficient condition for multiple equilibria is that

$$\frac{\frac{1}{2}\psi\prime\left(\frac{1}{2}\right)}{\psi(\frac{1}{2})} \bullet \frac{\frac{1}{2}\phi\prime(\frac{1}{2})}{\phi(\frac{1}{2})} > 1. \quad (40)$$

This is very intuitive, first term is the elasticity of public funding with respect to a change in the vote share. The second term is the elasticity of the fraction of informed voters with respect to campaign funds. If this elasticity is large, an increase in campaign funds leads to a large increase in the fraction of informed voters, if this furthermore leads to a large increase in public funds, multiple equilibria exist. In this case there will be a symmetric equilibrium, where $c = 1/2$ and (at least) two asymmetric equilibria. In an asymmetric equilibrium, one of the parties will be large, it will therefore receive a large fraction of the public funds and remain large. If such an equilibrium exist, there exist an other equilibrium which look the same but where the roles are reversed. Hence, in this case one can conclude that the public funding system - coupled with the advertising technology - may be the cause of asymmetric support for the two parties.

Notice that in the asymmetric equilibria the average policy is not equal to the median voter's most preferred policy. In the equilibrium where party L receives a larger share of the public funds, it receives a larger share of the votes. Using equation (26) we see that the average of the proposed polices is larger than m. The intuition is as follows, when party L receives a large share of the public funds, a larger share of the electorate is informed about $L's$ policy than about $R's$ policy. Party L therefore wins more votes by moderating its policy than party R does. Accordingly, in equilibrium party L ends up with a relatively more moderate policy than party R, which is also why party L receives a larger share of the votes.

Although, our model only contains two parties and therefore hardly is adequate for discussing entry, we may note that the feature of the asymmetric equilibrium, that the small party remains small because it is small as a consequence of the public funding system corresponds to what has been termed "petrification" in political science. According to Nassmacher (1989, p 248)" the term "petrification" refers to the absence of change in a party system. As far as political competition between parties is concerned public funding tends to favor bigger parties rather than smaller ones, and established parties over newcomers". This is exactly what happens in the asymmetric equilibrium above.

6.1 On the $\phi-$function, two examples

First we will shortly review a foundation for the $\phi-$function proposed by Grossman and Shapiro (1984) in their work on informative advertising in monopolistic markets. Imagine that each party puts ads in magazines. A magazine has a readership consisting of a fraction r of the population. With probability r a given voter sees an ad in a specific magazine. If a party advertises in n magazines, the probability a voter does not see an ad from the party is $(1-r)^n$, this equals the fraction of the population, which the party does not reach. Hence, the party reaches a fraction equal to

$$\phi = 1 - (1-r)^n$$

Accordingly, in order to reach a fraction ϕ the party has to put an ad in

$$n = \frac{\log(1-\phi)}{\log(1-r)}$$

magazines. If the cost of an ad in a magazine is a per reader of the magazine, an ad costs ar. Hence the cost of reaching a fraction ϕ is

$$arn = ar\frac{\log(1-\phi)}{\log(1-r)}$$

If the party has funds c, it can therefore reach the fraction ϕ solving

$$c = ar\frac{\log(1-\phi)}{\log(1-r)}$$

or

$$\phi(c) = 1 - \exp\left(\frac{\log(1-r)}{ar}c\right)$$

which we write

$$\phi(c) = 1 - \exp(bc) \tag{41}$$

where $b = \frac{\log(1-r)}{ar} < 0$. The more expensive is advertising, the lower is b numerically and the lower is $\phi(c)$. Notice that this advertising technology implies $\phi(0) = 0$. The elasticity of ϕ evaluated at $c = 1/2$ is

$$\frac{\frac{1}{2}\phi\prime(\frac{1}{2})}{\phi(\frac{1}{2})} = \frac{-\frac{b}{2}\exp\left(\frac{b}{2}\right)}{1-\exp\left(\frac{b}{2}\right)} < 1$$

since b is negative. Hence with this advertising function, the sufficient condition for multiple equilibria imply that the elasticity of the public funding system, the elasticity of the ψ- function, has to be larger than 1.

Inserting this $\phi-$function in equation (38) yields

$$c_L = \psi\left(\frac{1-\exp(bc_L)}{1-\exp(bc_L)+1-\exp(b(1-c_L))}\right) \tag{42}$$

Clearly, $c_L = \frac{1}{2}$ is a solution. Similarly, by inserting it is readily seen that $c_L = 0$ and $c_L = 1$ also are solutions if we assume that a party which receives no votes, receive no funds $\psi(0) = 0$. These are the only solutions in the interval [0,1]. Thus there are multiple equilibria[12], but the corner equilibria depend crucially on the fact that with this particular $\phi-$ function, $\phi(0) = 0$ and $\phi(1) = 1$, which may seem unreasonable as also discussed above.

Intuitively, the asymmetric equilibrium works like this. If a party receives no votes, it receives no funds. No voters will learn about the party's policy, the party has therefore no incentive to moderate its policy as this will attract no voters. A party therefore chooses its bliss-point, which in the derivations above implicitly is assumed to be at the end of the line (in 0 or 1). The party therefore receives no votes. One may like these equilibria or not. They correspond to the case where extreme parties, which nobody (except a negligible fraction) has heard about, have extreme policies.

If we modify the framework of Grossman and Shapiro slightly and assume that a fraction β of the voters become informed about a party's policy regardless of whether it advertises or not, the asymmetric equilibria disappear. The remaining fraction $1-\beta$ are as described above. If a party receives funds equal to c, its policy will be learned by

$$\beta + (1-\beta)\phi(c)$$

voters. Rather than equation (42) we then get

$$c_L = \psi\left(\frac{\beta + (1-\beta)\left(1-\exp\left(bc_L\right)\right)}{\beta + (1-\beta)\left(1-\exp\left(bc_L\right)\right) + \beta + (1-\beta)(1-\exp(b(1-c_L)))}\right)$$

Here $c_L = \frac{1}{2}$ is the only solution in the interval $[0, 1]$.

[12] Remember the elasticity condition was a sufficient condition, so there is no contradiction here.

The advertising technology proposed by Grossman and Shapiro has the feature that the marginal value of advertising is overall decreasing, $\phi''(c) = -b^2 \exp(bc) < 0$. Suppose instead that advertising first features increasing and then decreasing returns to scale. This would be the case if a certain amount of advertising was needed in order to "get the message out" and it, on the other hand, was very hard to reach all voters. An advertising technology with this feature is the following

$$\phi(c) = \frac{c^2}{c^2 + (1-c)^2}$$

Here $\phi'(0)$ equals zero and so does the elasticity of ϕ evaluated in zero. ϕ' is increasing for $c < 1/2$ and decreasing for $c > 1/2$.

In this case equation (39) becomes

$$c_L = \psi\left(\frac{\frac{c_L^2}{c_L^2+(1-c_L)^2}}{\frac{c_L^2}{c_L^2+(1-c_L)^2} + \frac{(1-c_L)^2}{c_L^2+(1-c_L)^2}}\right) = \psi\left(\frac{c_L^2}{c_L^2 + (1-c_L)^2}\right)$$

which has three solutions $c_L = 0, 1/2$, and 1, if $\psi(0) = 0$.

Suppose again that some voters learn the policy of the party regardless of whether advertising occurs or not. The following formulation is consistent with this,

$$\phi(c) = \frac{a + c^2}{2a + c^2 + (1-c)^2}.$$

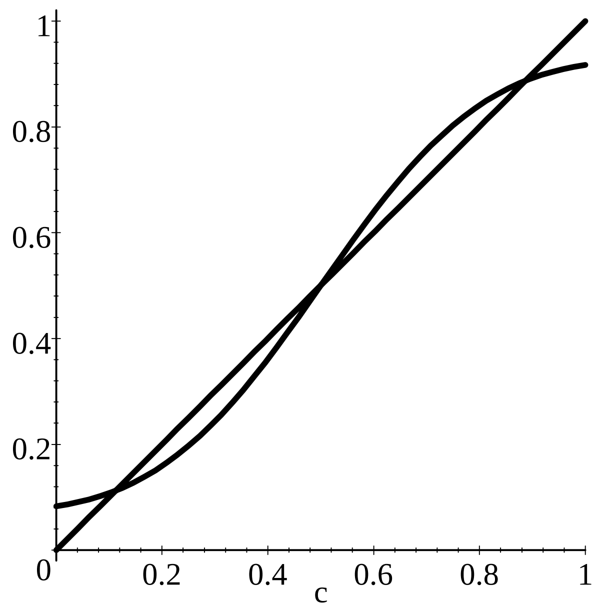
then $\phi(0) = \frac{a}{2a+1} > 0$, and $\phi'(0) = 0$. With this formulation, $\phi(c)+\phi(1-c) = 1$ for all c. The elasticity evaluated at $c = 1/2$ is

$$\left.\frac{\frac{\partial}{\partial c}\left(\frac{a+c^2}{2a+c^2+(1-c)^2}\right)c}{\frac{a+c^2}{2a+c^2+(1-c)^2}}\right|_{c=\frac{1}{2}} = \frac{1}{2a + \frac{1}{2}}$$

which is larger than one if we assume that $a < 1/4$. If therefore the elasticity of the ψ function is at least one, there are multiple equilibria. To be specific, assume that $\psi(v) = v$, the public funds are proportional to the share of votes. Now equation (39) becomes

$$c_L = \frac{\phi(c_L)}{\phi(c_L) + \phi(c_L)} = \phi(c_L) = \frac{a + c_L^2}{2a + c_L^2 + (1 - c_L)^2} \tag{43}$$

There are three solutions $c = \frac{1}{2}$, $c = \frac{1}{2} + \frac{1}{2}\sqrt{(1 - 4a)}$, and $c = \frac{1}{2} - \frac{1}{2}\sqrt{(1 - 4a)}$. For $a < 1/4$ all solutions are real and belong to the interval $[0, 1]$. The figure below clearly shows the three equilibria.



7 Concluding remarks

Public funding of political parties tends to moderate partisan policies. This conclusion holds true whether campaign spending is uninformative and directly affect the votes of impressionable voters or whether campaign spending contributes to inform a policy interested electorate about the policies of the parties.

In the first case campaign money buy votes. If public funding depends on a party's vote share, this increases a party's incentive to moderate its policy. A moderate policy gives more votes and therefore more public funds which can be used to buy even more votes. Parties face a trade off between selecting a policy they like and moderating the policy in order to make it more likely it is implemented. With public funding the trade off is changed, making moderation more attractive.

If campaigning is informative the result is the same, but the channel is different. When choosing policy a party takes into account that only a fraction of the electorate will be informed about the party's policy choice. Uninformed voters will not learn the exact policy choice of the party but will rely on their expectations. Hence, the increase in vote share from moderating the policy depends on the fraction of voters who will become informed about the policy. The larger is this fraction, the more attractive is a policy moderation. Since public funding will make a larger fraction of the electorate informed it makes for policy moderation. Notice, that here it is the level of funding which is important while with impressionable voters it is the responsiveness of public funding to changes in vote share which is important.

We have assumed that voters are risk neutral. While this makes for simplicity, it also implies that we disregard a potential important feature of informative campaigning. Namely, that information reduces uncertainty about a party's policy. If voters are risk averse this is a positive feature of campaigning (see Brock and Magee 1978, Austen-Smith 1987 and Cameron and Enelow 1992 for theoretical models with risk-averse voters, uncertainty about parties' platforms and campaign spending financed by private contributors)

In our model, a voter who is uninformed about the policy of a party has a point expectation about the policy. In equilibrium, the expectation is correct. If the voter was uncertain about the policy such that she had a non-degenerate probability distribution over possible policies and if she furthermore was risk averse, then she would tend to dislike the party just because of the uncertainty. A risk averse voter who is indifferent between l and r would prefer party L if she was not completely sure about $R's$ policy. In this case campaigning would have a positive effect from the viewpoint of the party. Even though uninformed voters may have correct expectations on average, more voters would vote for R if they were sure about $R's$ policy.

We conjecture that adding uncertainty and risk aversion to our model would just reinforce our results. Parties will be more eager to raise campaign money since reducing the uncertainty about their policies will attract more voters. This will give parties a further incentive to modify their policies towards the middle in order to raise public funds.

As it stands our model shows, that policy convergence results even though voters are risk neutral and there is not uncertainty.

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8 Appendix

8.1 With impressionable voters

We derive the sufficient conditions for the second order condition for maximum in the model with impressionable voters. For party L the second order condition for maximum is

$$-2\left(\alpha H\prime\frac{d\Delta}{dl}+\frac{1-\alpha}{2}f\right)+\left(\alpha H\prime\prime\frac{d\Delta}{dl}+\alpha H\prime\frac{d^2\Delta}{dl^2}+\frac{1-\alpha}{4}f\prime\right)(r-l)\leq 0 \tag{44}$$

Since $r-\ l\leq 1$, this is fulfilled if

$$-2\left(\alpha H\prime\frac{d\Delta}{dl}+\frac{1-\alpha}{2}f\right)+\alpha H\prime\prime\frac{d\Delta}{dl}+\alpha H\prime\frac{d^2\Delta}{dl^2}+\frac{1-\alpha}{4}f\prime\leq 0$$

or

$$\frac{(1-\alpha)}{\alpha}\left(\frac{f\prime}{4}-f\right)+H\prime\frac{d^2\Delta}{dl^2}+\frac{d\Delta}{dl}\left(H\prime\prime-2H\prime\right)\leq 0$$

assuming that $\frac{d\Delta}{dl}>0$, (which is the case if $\psi\prime>0$) we can rewrite this

$$\frac{(1-\alpha)}{\alpha}\left(\frac{f\prime}{4}-f\right)+H\prime\frac{d\Delta}{dl}\left(\frac{\frac{d^2\Delta}{dl^2}}{\frac{d\Delta}{dl}}+\frac{H\prime\prime}{H\prime}-2\right)\leq 0$$

We see that a sufficient condition for this to be fulfilled is that

$$\frac{f\prime}{f}<4,\ \text{and}\ \frac{d^2\Delta}{dl^2}\Big/\frac{d\Delta}{dl}+\frac{H\prime\prime}{H\prime}<2. \tag{45}$$

The latter condition is a joint condition on the responsiveness of the public funding system and the responsiveness of the impressionable voters. It is hard to give general conditions ensuring that it is fulfilled. If however, $H'' = 0$ and $\psi'' = 0$, so $\Delta'' = 0$, the condition is fulfilled. To see this calculate

$$\frac{d\Delta}{dl} = \Delta'(v(l,r))\frac{\partial v(l,r)}{\partial l}$$

and

$$\frac{d^2\Delta}{dl^2} = \Delta''\frac{\partial v(l,r)}{\partial l} + \Delta'\frac{\partial^2 v}{\partial l^2}$$

Hence

$$\frac{d^2\Delta}{dl^2}\Big/\frac{d\Delta}{dl} = \frac{\Delta''\frac{\partial v(l,r)}{\partial l} + \Delta'\frac{\partial^2 v}{\partial l^2}}{\Delta'\frac{\partial v(l,r)}{\partial l}} = \frac{\Delta''}{\Delta'} + \frac{\partial^2 v}{\partial l^2}\Big/\frac{\partial v(l,r)}{\partial l}$$

Using (5) we get

$$\begin{aligned} \frac{\partial^2 v}{\partial l} &= \frac{\frac{1-\alpha}{4}f'(1-\alpha H'\Delta') + \frac{1-\alpha}{2}\alpha f\Delta'' H'\frac{\partial v}{\partial l} + (\Delta')^2 H''\frac{\partial v}{\partial l}}{(1-\alpha\ H'\ \Delta')^2} \\ &= \frac{\frac{1-\alpha}{4}f'}{(1-\alpha\ H'\ \Delta')} < \frac{(1-\alpha)\,f}{1-\alpha\ H'\ \Delta'} = 2\frac{\partial v}{\partial l} \end{aligned}$$

where the first equality follows from the assumption that $H'' = 0$ and $\psi'' = 0$, so $\Delta'' = 0$. The inequality follows from the assumption that $f'/f < 4$. We now have that

$$\frac{\partial^2 v}{\partial l^2}\Big/\frac{\partial v(l,r)}{\partial l} < 2$$

together with $\Delta'' = 0$, this gives that $\frac{d^2\Delta}{dl^2}\Big/\frac{d\Delta}{dl} < 2$. Under the assumption that $H'' = 0$, this imply that the second inequality in (45) is fulfilled as claimed. As is clear from this deviation, it is not necessary that $H'' = 0$ and $\Delta'' = 0$, they should just not be too large. Party R is in a symmetric situation, its second order condition is fulfilled under the same assumptions. Finally, note that if $\frac{d\Delta}{dl} = 0$, the second order condition is fulfilled if $f'/f < 4$.

8.2 With informative campaigning

Now we show that the second order condition also is fulfilled in the model with informative campaigning.

Equation (21) implies

$$\frac{\partial^2 V}{\partial \phi_L \partial l} = \frac{\phi_R}{2} f\left(\frac{l+r}{2}\right) - \frac{\phi_R}{2} f\left(\frac{l+r^e}{2}\right) = 0 \tag{46}$$

when $r = r^e$. Similarly, again using rational expectations (21) also implies

$$\frac{\partial^2 V}{\partial \phi_R \partial l} = 0 \tag{47}$$

Party $L's$ first order condition (28) is

$$\left(\frac{\partial V}{\partial l} + \frac{\partial V}{\partial \phi_L}\frac{\partial \phi_L}{\partial l} + \frac{\partial V}{\partial \phi_R}\frac{\partial \phi_R}{\partial l}\right)(r^* - l^*) - V = 0$$

Hence, the second order condition for maximum is

$$\left(\frac{\partial^2 V}{\partial l^2} + \frac{d\left(\frac{\partial V}{\partial \phi_L}\frac{\partial \phi_L}{\partial l} + \frac{\partial V}{\partial \phi_R}\frac{\partial \phi_R}{\partial l}\right)}{dl}\right)(r^* - l^*) - \left(2\frac{\partial V}{\partial l} + \frac{\partial V}{\partial \phi_L}\frac{\partial \phi_L}{\partial l} + \frac{\partial V}{\partial \phi_R}\frac{\partial \phi_R}{\partial l}\right) \leq 0$$

Using (24), (25) (46) and (47) it reduces to

$$\frac{\partial^2 V}{\partial l^2}(r^* - l^*) - 2\frac{\partial V}{\partial l} \leq 0$$

Inserting (22) yields

$$\frac{\phi_L}{4} f\prime\left(\frac{l+r}{2}\right)(r-l) - 2\frac{\phi_L}{2} f\left(\frac{l+r}{2}\right) \leq 0$$

which is clearly fulfilled under our assumption $f\prime/f < 4$.