Optimal Monetary Policy in an Economy with Incomplete Markets and Idiosyncratic Risk

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January 2000

Abstract

This study investigates an incomplete markets economy in which the saving behavior of a continuum of infinitely lived agents is influenced by precautionary saving motives and borrowing constraints. Two types of assets (interest bearing IOUs and money) enhance the liquidity of agents by providing an additional means of smoothing consumption and by effectively loosening borrowing constraints. Money is valued because of a timing friction in the bond market. In particular, the bond market closes before agents observe their idiosyncratic productivity shock. High inflation rates will transfer resources from agents with high endowments to those holding bonds which can increase welfare. However, in an inflationary environment, agents economize on money holdings, causing a reduction in welfare. Furthermore, different monetary growth rates will imply different seigniorage revenues for government. The level of seigniorage revenues will determine the interest rate on government bonds, and the effective borrowing constraint. This study quantitatively examines the welfare implications of different monetary growth rates. Initial results indicate that higher inflation rates increase welfare.

* I am grateful to my advisor, Steve Williamson, for his guidance, encouragement, and comments. I also thank Kartik Athreya, Nicole Bundy, Beth Ingram, Ellen McGrattan, and seminar participants at Iowa, South East Economic Theory Conference (Georgetown, 1999), and especially Mark Huggett for suggestions and comments. email: ahmet-akyol@uiowa.edu.

1. Introduction

One of the most celebrated propositions in modern monetary economics is the Milton Friedman's (1969) doctrine regarding the "optimum quantity of money". Friedman argued that an optimal monetary policy involves a steady contraction of the money supply at a sufficient rate (usually at the discount rate) so that the nominal interest rate is zero. The main idea behind the Friedman Rule is that a positive nominal interest rate would encourage people to economize on their cash holdings and thus decrease welfare. Several studies try to model the environments in which the Friedman Rule is optimal (e.g. Lucas and Stokey (1983), Kimbrough (1986a,1986b), see also Woodford (1990) for an excellent survey), and compute the welfare implications of growth rates of money supply other than proposed by Friedman (e.g. Dotsey and Ireland (1996), Aiyagari, Braun, Eckstein (1998)).

One class of models in which the optimality of the Friedman Rule does not necessarily hold involves the absence of complete insurance markets and borrowing constraints. In these environments, agents hold fiat money (or any other asset) to self-insure themselves against stochastic endowments and/or preferences. The consumption smoothing role of money goes back to Bewley (1980, 1983) who analyzes the optimality of the Friedman rule in this framework. He shows that there may not exist any monetary equilibria in which real balances remain bounded away from zero if the money supply is contracted at or even near the rate called for by the Friedman Rule. That is, a contraction of the money supply may prevent the existence of a monetary equilibrium in which real money balances provide "liquidity" to agents. It is important to note that the choice of a low rate of money growth implies *demonetization* of the economy - exactly the opposite of Friedman's objective. This possibility arises from the positive probability that an agent receives a long stream of bad shocks to his endowments. Agents would like to hold an infinite amount of real balances against such a possibility if the return on real balances is sufficiently high (or if the contraction of the money supply is at or near the rate called for by the Friedman Rule).

Imrohoroğlu (1992) quantitatively examines the welfare cost of inflation under imperfect insurance and finds that the welfare cost of moderate rates of inflation in the incomplete markets economy may well exceed that of the Arrow-Debreu economy. Thus, traditional estimates of cost of inflation may be misleading since the area under the empirical money demand curve is a poor measure of the welfare cost of inflation in the first place.

In a similar framework, many studies concentrate on asset price anomolies such as the low risk free rate on Treasury Bills and the equity premium. For instance, Huggett (1993) shows quantitatively that the low risk free rate can be explained when people cannot trade contingent claims on their endowments and face borrowing limits. Diaz-Gimenez and Prescott (1997) examine the behavior of real interest rates on government debt, and report that the average rate of return on government debt in the model is close to what is observed in the data. The friction in their model that gives rise to a valued currency is the large denomination of bonds (and the assumption that agents cannot pool their savings and share the proceeds of T-Bills). Monetary policy affects the real rate of return on government debt, and thereby changes the tax rate on the insurance services provided by the interest-bearing assets.

Aiyagari (1994, 1995) shows that the precautionary savings motive in these models can lead to overaccumulation of capital, and capital taxation may necessary to maximize welfare. Similar to the inexistence result of Bewley (1983), he shows that the competitive equilibrium in an incomplete markets economy cannot sustain an interest rate of the Arrow-Debreu economy with identical preference parameters. That is, the interest rate in an incomplete markets economy is strictly lower than that of the Arrow-Debreu economy.

In a recent study, Aiyagari and McGrattan (1998) examine the optimal level of public debt in a model with incomplete markets and idiosyncratic shocks to labor productivity. On the benefit side, since agents cannot borrow against their labor income, higher levels of government debt enhance the liquidity of households by providing an additional means of smoothing consumption and by effectively loosening borrowing constraints. On the cost side, the implied taxes have adverse wealth distribution and incentive effects. They find that the current level of public debt in the U.S. economy is close to optimal; and, the welfare function is flat with respect to various levels of public debt. In other words, concerns about the level of public debt may be misplaced.

Obviously, interest bearing government debt is not the only asset in the economy providing partial insurance. One possibility is the privately issued IOUs of which the supply critically depends on the *ad hoc* borrowing constraints. Given that fiat currency has a consumption-smoothing role similar to that of government debt, public policies should take monetary policy into account as well. Furthermore, interest payments on government debt can be financed using seigniorage revenues from inflationary policies.[1] Thus, monetary policy and taxation schemes

[1] Alternatively, the equilibrium allocation may be such that the government holds privately issued bonds and use the interest income form these securities to ¯nance the contraction in the

have two similarities. First, money has insurance effects as the income taxation. That is, with higher inflation the variation in the after-tax-income is lower.[2] Second, a high growth rate of money (thus a high rate of inflation) distorts intertemporal decisions, and limits the insurance role of money. Furthermore, the seigniorage revenues of the government also determine the steady state level of interest payments on government debt, and therefore, determine the effective borrowing constraint. Alternatively, deflationary monetary policies can be conducted when the government holds privately-issued debt and uses the interest income to finance the positive return on real balances.

Thus, different growth rates of money supply imply different interest rates on public debt. Both the return on public debt and how the public debt is being financed is crucial for distributional aspects of the public policies.[3] Given that different money supply rules have welfare consequences, one can conduct welfare calculations similar to those in Aiyagari and McGrattan (1998).[4]

The important innovation in this paper is the fact that agents can use assets with different returns to self-insure against idiosyncratic uncertainty.[5] Thus, it allows us to understand the tension between interest bearing bonds and money when both assets are held for precautionary savings purposes. However, given that money is dominated by other assets in rates of return, there must be a friction in the economy which will give rise to a monetary equilibrium. Here, following Aiyagari and Williamson (1997) (see also Lucas (1990) and Fuerst (1992)), I assume limited participation in the financial market; agents do not know their current state variables (i.e. their endowment) before the bond market closes. That is, the timing within the period is such that an agent makes his decision about the bond purchase before he observes his endowment. Money market transactions

money supply. The crucial assumption here is how much agents are allowed to borrow. For instance, if agents cannot issue bonds at all, then deflationary policies cannot be sustained in equilibrium.

[2] Here after-tax income equals endowment plus interest income from bonds and real return on money holdings.

[3] Note that, unlike in representative agent models, lump-sum transfers of money is not equivalent to open-market operations.

[4] Aiyagari and McGratten compare the welfare effects of different levels of public debt whereas this study investigates the welfare implications of different rates of inflation.

[5] İmrohoroğlu and Prescott (1991) also study an economy with both flat currency and bonds. However, agents cannot hold interest bearing government debt which has to be intermediated by banks. Their main finding is that all the agents care about is the after-tax level of interest rates; thus, different growth rates of money supply with different reserve requirements have identical welfare consequences as long as the after-tax interest rates do not change.

allow him to smooth consumption after observing his endowment. Thus, agents who receive a high shock to their endowment may use money to smooth their consumption over time. Given this structure of the economy, a high rate of inflation taxes money holdings and limits the ability of agents with high endowment shocks to smooth consumption. At the same time, however, it allows the government to sustain a higher interest rate on bonds, thereby enhances the agents' ability to insure themselves against low endowment shocks.

The numerical results of this study indicate that higher rates of inflation lead to larger levels of seigniorage revenues which is used to finance higher rates of interest on government bonds. In equilibrium, these higher risk-free rates of return allow agents to self-insure themselves more effectively. The negative effects of higher inflation rates on welfare, as emphasized by Friedman and others, are dominated by the revenue effects of higher inflation tax on the return of government bonds.

The paper is organized as follows. In section 2 I lay out the model environment. In subsection 2.1, the equilibrium is defined. Welfare and parametrization are discussed in subsections 2.2 and 2.3. Section 3 contains a discussion of results and their comparison with those in the literature. Section 4 concludes.

2. Model

The economy has a continuum of infinitely lived agents of measure one who receive idiosyncratic shocks to their endowments. Each period, agents receive an endowment, e_t, in terms of the one perishable consumption good. Let e_t be independently and identically distributed across agents and follow a two-state Markov process over time with $e \in \{e^l, e^h\}$ with stationary transition probability $\pi(e' | e) = \Pr[e_{t+1} = e' | e_t = e] > 0$. There is no aggregate shock to the endowment process.

There are two assets in the economy: an interest bearing bond, b, issued by both private agents and the government, and fiat money, M. Agents are able to issue privately issued bonds (IOUs) up to an exogenous limit which I call the borrowing constraint. It is assumed that IOUs and government bonds have the same nonstochastic rate of return. Each period the bond market closes *before* agents observe the shock to their endowment. Therefore, agents have to make decisions about future bond holdings before they observe their endowment shock. After the realization of the shock, agents make decisions how much to consume this period and how much to save in real money balances. That is, in some periods, when the shock is high, they may prefer to substitute current consumption with future consumption by holding fiat money. In these high endowment states, agents may also find it optimal to self-insure themselves by holding fiat money. This timing friction gives rise to a monetary equilibrium even when money is dominated by bonds in rate of return. In essence, it implicitly captures the transaction costs of trades in bond markets, and implies a "liquidity premium" on the return of real money balances.

The trading in the bond market is nothing but promises of payments which are delivered after the realization of the endowment shocks in the same period. Furthermore, promises of payments are not reversible. Consequently, to guarantee the delivery of bonds, an agent cannot promise to purchase more bonds than the minimum possible resources available to him in that period. That is, there is an *endogenous* upper bound on bond purchases because of the non-negativity assumption on consumption. Therefore, all promised payments (and delivery) of bond trades take place with probability one.

The agent's problem is to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\} \quad \text{where } \beta \in (0,1) \tag{2.1}$$

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma} \text{ where } \sigma > 1. \tag{2.2}$$

subject to

$$c_t(e_t) + \frac{M_{t+1}(e_t)}{p_t} + b_{t+1} \leq e_t + (1+r_t)b_t + \frac{M_t}{p_t} \qquad \forall e_t, \tag{2.3}$$

$$b_t \geq \underline{b}, \; M_t \geq 0, \qquad \forall t. \tag{2.4}$$

where p_t is the price level, r_t is the state non-contingent return on bonds in terms of the consumption good. The exogenous borrowing constraint, $\underline{b}$, and the non-negativity constraint on fiat money are given by (2.4). Equation (2.3) has to be satisfied for all possible values of e_t so that bond purchase agreements are realized with probability one. It also involves the timing friction; the choice of bond holdings of period $t+1$ takes place prior to the realization of the endowment shock of period t whereas the choice of consumption of period t and money of period $t+1$ takes place after the realization of the endowment shock of period t. Consequently, savings in bonds are bounded by the minimum possible available resources for agents in that period. Note that the net nominal interest rate equals $[(1 + r_t)\, p_{t+1}/p_t] - 1$ and is positive unless the money supply is deflated at a sufficiently high rate.[6] Therefore, agents will not hold money balances ($M_{t+1} = 0$ in (2.3)) if they receive the minimum endowment, e^l. Intuitively, agents ex ante determine bond holdings such that if they end up receiving e^l, any positive savings in real balances would imply a loss of nominal interest income, therefore they save *only* in bonds in the worst endowment state.

Furthermore, bond holdings at time $t + 1$ cannot exceed the minimum labor income plus gross bond returns and money holdings purchased at time t. This restriction, which arises from the timing in the model, puts an upper bound on asset holdings with positive returns in a more severe way. Typically, the upper bound is determined by the realization of the endowment shock and the monetary policy; here, the lower bound of the support of the shock process puts an upper bound on the next period's bond holdings, b_{t+1}. This restriction implies that an agent in the worst individual state with the minimum amount of wealth will be able to finance his bond purchases with probability one. At the same time, this friction limits the ability of agents to smooth consumption through savings in bonds.[7]

Suppose that an agent decides to purchase a positive amount of bonds. After the trade in the bond market, he faces a high endowment shock so that his income *net* of promised bond payments is high. Thus, in the absence of a money market, current consumption has to be high given that the bond market is closed. In order to avoid fluctuations in consumption, agents would like to hold an asset that they can convert into consumption goods costlessly. Money serves this need, and increases welfare. Furthermore, agents do not want to commit themselves to a high level of savings in bonds when their endowment is low. Thus, bonds are relatively less liquid than money. Consequently, the demand for bonds (i.e. precautionary savings motive) decreases since agents do not want to commit themselves to hold high levels of savings against the possibility of receiving low endowment. Therefore, the consumption smoothing role of interest-bearing assets is limited compared to that of Huggett (1993), Aiyagari (1994), and Aiyagari and McGrattan (1998).

[6] Given that fiat money is more "liquid" relative to bonds, intitutively we can argue that the economy can sustain an equilbrium with $(1 + r_t) \cdot p_t/p_{t+1}$ (i.e a negative nominal interest rate) and with positive demands for both assets. In other words, money has to bear a "liquidity premium" over bonds in equilibrium.

[7] See the Results section for a discussion of this feature of the model.

It is convenient to transform the model into a stationary form. Toward this end, I define $m_t = M_t/p_{t-1}$ and $\pi_t = p_{t-1}/p_t$ where m_t is the real money balances, and π_t is the gross real return on real money balances.

Let per capita total assets in consumption units in period t be a_t. Then,

$$a_t = m_t + b_t.$$

With these changes, the dynamic programming problem solved by agents is

$$V\left(a_t, e_{t-1}\right) = \max_{b_{t+1}} \left\{ E\left[\max_{c_t, m_{t+1}} U\left(c_t\right) + \beta V\left(a_{t+1}, e_t\right) \Big| e_{t-1} \right] \right\}. \tag{2.5}$$

$$s.t.\ c_t\left(e_t\right) + m_{t+1}\left(e_t\right) + b_{t+1} \leq e_t + (1 + r_t) b_t + \pi_t m_t \quad \forall e_t, \tag{2.6}$$

$$b_t \geq \underline{b},\ M_t \geq 0, \quad \forall t. \tag{2.7}$$

In this economy, the government takes the passive role of supplying loans to the public. In particular, it sets the monetary growth rate exogenously, and the public determines, given the interest rate, its demand for bonds. The government supply the bonds according to its budget constraint. The government budget constraint in monetary units is

$$(1 + r_t) B_t + \frac{\overline{M}_t}{p_t} = B_{t+1} + \frac{\overline{M}_{t+1}}{p_t} \tag{2.8}$$

$$\overline{M}_{t+1} = (1 + \xi)\, \overline{M}_t, \tag{2.9}$$

where $\overline{M_t}$ is the aggregate supply of fiat money, and B_t is the net aggregate supply of bonds in the economy. In this economy, a money injection can be made by conducting open market operations described in (2.8) and (2.9).[8] The money supply growth, ξ, is exogenous and constant.[9] Note that the government uses the seigniorage revenues from the inflationary monetary policy (i.e. $\xi > 0$) to finance

[8] In representative agents models, lump sum transfers of money (i.e. "helicopter drop") and retiring existing debt through open market operations are equivalent. However, in this economy, an open market operation involves a transfer to agents holding government debt. Given that there is a nondegenerate distribution of agents with respect to bond holdings, there is a different level of transfer to each agent which is not the case when the government makes a lump sum transfer to each agent.

[9] I assume that there is no uncertainty about the money supply process, and the government can commit itself to a fixed money supply rule as in (2.6).

the interest payments on government bonds. Alternatively, if the government follows a deflationary monetary policy (i.e. $\xi < 0$), then it must hold private IOUs to finance the positive return on real money balances. Note that the monetary growth rate, ξ, determines the gross real return on real money balances, π_t. In particular, $\xi = 1/\pi_t - 1$.

2.1. Equilibrium

The individual agent's policy functions that solve the dynamic program in (2.5)-(2.7) along with the stochastic process of endowments induce a joint stochastic process for bond holdings, money holdings, and endowment. Under some general conditions, this stochastic process will have a limiting distribution on bond holdings, money holdings, and endowment. Given that this is an economy with a continuum of agents, these limiting distributions describe *both* the long-run behavior of a single agent and the cross-sectional distribution of asset holdings for the whole economy.[10] Formally, the equilibrium can be defined as follows:

A stationary competitive equilibrium for this economy consists of decision rules $b(a)$, $m(a,e)$, $c(a,e)$, for bond holdings, money holdings and consumption respectively, together with the value function, V, the steady state joint distribution of assets and endowment, $H(a,e)$, the real return on bonds, r, and the real return on real balances, π, which is consistent with the exogenous rate of money supply, ξ, such that

(i) given prices $\{r, \pi\}$, $b(a)$, $m(a,e)$, $c(a,e)$ maximize utility subject to their budget constraint, the borrowing constraint, and the non-negativity of money holdings (i.e. $b(a)$, $m(a,e)$, $c(a,e)$ solve the problem in (2.7)-(2.9)).

(ii) Bond Market clears:

$$B = \iint b(a)dH(a,e). \tag{2.10}$$

(iii) Money Market clears:

$$M = \iint m(a,e)dH(a,e). \tag{2.11}$$

[10] See Magill and Quinzii (1994) for conditions on the law of motion for endowment and asset holdings.

(iv) The government budget constraint is satisfied every period:

$$rB = (1 - \pi)M. \tag{2.12}$$

By Walras' Law, the goods market is also in equilibrium; that is, the expected value of the endowment equals the consumption of the perishable good.

$$E\left[e\right] = \iint c(a,e)dH(a,e). \tag{2.13}$$

2.2. Welfare

The welfare criterion used here is similar to that in Aiyagari and McGrattan (1999). It is the expected discounted sum of utilities evaluated under the equilibrium stochastic consumption stream of an infinitely-lived agent. It is the steady state ex ante welfare of an agent. Therefore, all generations are treated equally. The welfare function is given by

$$\Omega = \iint V(a,e)\mathrm{d}H(a,e). \tag{2.14}$$

where $H(a,e)$ is the steady state joint distribution of assets and endowment.

For each money supply rule, ξ, I calculated the certainty equivalent of consumption level in complete markets economy. That is,

$$\lambda = \left[\frac{V_\xi}{V_{cm}}\right]^{1/(1-\sigma)} \tag{2.15}$$

where V_{cm}, V_ξ are the welfare of an agent in the complete markets economy, and the welfare in the incomplete markets economy with the money supply rule ξ. The closer λ gets to one, the less important becomes the frictions in the incomplete markets economy, and the higher is the welfare.

2.3. Parameterization

I follow the parametrization of Huggett (1993) for the set of parameters $\{e_h, e_l;$ $\pi(e_h|e_h), \pi(e_l|e_l); \beta; \sigma; \underline{b}\}$ and the period length. The endowment process, e, can be interpreted as employment states following Imrohoroğlu (1989). When $e_h = 1.0, e_l = 0.1, \pi(e_h|e_h) = 0.925, \pi(e_h|e_l) = 0.5$, and there are six model periods in one year, the standard deviation of annual earnings as a percentage of mean of an agent is 20% and the average duration of the low endowment (i.e. unemployment) is 17 weeks or approximately two model periods. The discount factor is set to 0.99322 which is 0.96 on an annual basis (which implies an interest rate of 4.2% *in a complete markets economy*). I studied the cases where the risk aversion coefficient, σ, is set at 3. This is consistent with Huggett (1993), Heaton and Lucas (1997). The set $\underline{b} \in \{-2, -4, -6, -8\}$ for different borrowing constraints are selected to allow for the possibility of deflationary monetary policies. A borrowing limit of -5.3 is equal to one year's average endowment.

The algorithm to compute the equilibria is described in Judd (1999) and Miranda and Fackler (1999). The dynamic programming problem for the individual agent is a function of the continuous state variable wealth, a, which is the sum of money and bond holdings. I employ smooth approximation methods to obtain the value function and the related policy functions. I am able to approximate the value function using Chebychev polynomials of degree 15 and with 50 grid points. The computation consists of the following steps. First, for a fixed rate inflation (deflation), a rate of interest rate on bonds and coefficients of the Chebychev polynomial are guessed. With the value function computed, I simulate the policy functions using a long sequence of endowment.[11] I calculate the mean of the resulting data (i.e. the mean of bond and money holdings). Using the mean of bond demand and the mean of money demand, I check whether the government budget constraint is satisfied.[12] If not, reset the interest rate on bonds using a simple bisection method. Once the government budget constraint is satisfied, I compute the welfare for the corresponding rate of inflation (deflation). In particular, I compute the certainty equivalent of the risk-free steady state level of consumption.

3. Results and Discussion

Inflationary monetary policy has effects on the distribution of wealth, and therefore, on social welfare. The best government policy is to tax the agents who receive high productivity shocks and to transfer these resources to agents who receive low productivity shocks. This policy has the same outcome in a complete markets economy. However, such a policy would require that the government has a substantial amount of information about each agent's endowment. I assume that the government does not have such an informational advantage over markets,

[11] I generate a sequence of 20,000 periods long, and ignore the first 2,000 observations to avoid the effects of the initial starting value of weath on the stationary distribution.

[12] Numerically, the criterion is 10^{-4} in absolute value.

and therefore cannot implement this policy. Instead, it uses fiscal and monetary policies to maximize social welfare.

In my model, inflationary monetary policy has three effects. A positive inflation will induce agents to economize money holdings since holding cash is costly in an economy with positive nominal interest rates. However, agents with high productivity shock will hold larger amounts of money relative to agents with low productivity shock. Thus, it will also redistribute resources from agents with a high current productivity shock to agents who hold bonds (which I call the distribution-effect). The first effect calls for deflationary monetary policy (i.e. Friedman rule) whereas the second one emphasizes expansionary monetary policy. Although the former is present in models with representative agents, the latter can be analyzed only in heterogenous agents framework. Therefore, previous studies with representative agents cannot capture the trade-off of monetary policy.

Furthermore, as shown in (2.12), the exogenous rate of inflation determines, together with endogenous demand for bonds and money, the real interest rate in the stationary equilibrium. A higher rate of inflation may generate larger seigniorage revenues for the government which may imply a higher rate of real interest on bonds (which I call the revenue-effect). The opportunity cost of holding bonds is $\mu = 1/\beta$ ¡ 1. The closer the interest rate is to μ, the less costly it is to hold bonds, and the more effective bonds are in enabling *the consumer with positive bond holdings* to smooth consumption. However, higher interest rates will also penalize those with negative levels of bond holdings.

Generally, as in Aiyagari and McGrattan (1998) and others, persistence in shocks makes the assets more important to smooth consumption. In general, higher persistence implies that once an agent takes a low endowment shock it is likely that he will have low endowment for future periods; therefore, it is optimal to accumulate a larger amount of assets against such a possibility. In this model, persistence in the shock process conveys information to the constrained agents about the current shock they cannot observe. Thus, agents are less certain about their productivity which consequently decreases their demand for money balances. At the same time, however, higher persistence induces agents to hold more assets to self-insure themselves which increases the demand for interest-bearing assets. Therefore, the serial correlation parameter in shocks pins down the tension between money and interest-bearing assets when agents try to smooth consumption. If, however, shocks are i.i.d., then previous realizations of the shock process provides no information about the current productivity level to the agent, thus money serves well to smooth consumption. In some sense, when shocks are serially correlated, bonds become more effective relative to money, vice versa.

Table 1: $\sigma = 3$, $\underline{b} = -2$

inflation	1%	3%	6%	10%
welfare (λ)	0.9848	0.9859	0.9894	0.9928
public debt	2.0987	2.4139	3.1643	3.9941
money	0.6656	0.6764	0.6871	0.6927
st.dev. of con.	0.1013	0.0959	0.0853	0.0814
int. rate (ann.)	0.0034	0.0084	0.0127	0.0164

I summarize the results with the different levels of the borrowing constraint in Tables 1-4. As in previous studies, the interest rate is much lower than that of a complete markets economy with the same parameters. Yet, the results of Huggett (1993) indicate a much lower rate of interest (-23%) with a risk aversion parameter, $\sigma = 3$. In this economy, the return of an additional asset (i.e. money) puts a lower bound on the return of the risk-free bonds. In other words, given that money is more liquid relative to risk-free bonds, the return on these assets must be strictly higher than that on money.

Table 2: $\sigma = 3$, $\underline{b} = -4$

inflation	1%	3%	6%	10%
welfare (λ)	0.9899	0.9897	0.9927	0.9947
public debt	0.6270	1.1048	1.9945	2.8164
money	0.6705	0.6865	0.6959	0.6991
st.dev. of cons.	0.0992	0.0886	0.0857	0.0800
interest rate (ann.)	0.0105	0.0189	0.0204	0.0236

Numerical results show that under all levels of borrowing constraints welfare increases with higher rates of inflation (see Tables 1-4). Furthermore, the level of public debt is always positive and monotonically increasing with the rate of inflation. Interestingly, agents do not substantially change their demand for money for different rates of inflation. Given that, seigniorage revenues rise with higher inflation rates, and consequently, the equilibrium interest rate increases. On net, higher interest rates allow agents to self-insure more effectively, and causing welfare to increase.

Table 3: $\sigma = 3, \underline{b} = -6$

inflation	1%	3%	6%	10%
welfare (λ)	0.9929	0.9938	0.9950	0.9966
public debt	0.2647	0.7686	1.4340	2.2685
money demand	0.7026	0.7055	0.7095	0.7121
st.dev. of cons.	0.0817	0.0794	0.0787	0.0766
interest rate (ann.)	0.0262	0.0280	0.0290	0.0301

I also calculated changes in welfare when the monetary authority shifts from one money supply rule to another. In other words, I calculated the difference in welfare in consumption units when the inflation rate rises.[13] For instance, when $\underline{b} = -8$, an agent enjoys 0.24% increase in consumption in every period if the inflation has been raised from 1% to 10%. For the most severe borrowing constraint ($\underline{b} = -2$), the gain in consumption equals 0.81% for the same increase in the rate of inflation. Thus, the welfare gains from higher inflation are higher when agents are subject to more severe borrowing constraints.

These results differ from those of the existing literature in two dimensions. First, previous studies of incomplete markets report a *loss* of welfare when the economy experiences higher rates of inflation (e.g. Imrohoroğlu (1992)). Second, these welfare gains are relatively large even though the agents are infinitely-lived and can hold two types of assets. Krusell and Smith (1999) show that dynasties can protect their members very well in utility terms from idiosyncratic shocks using only one asset even in the presence of a borrowing constraint.

Table 4: $\sigma = 3, \underline{b} = -8$

inflation	1%	3%	6%	10%
welfare (λ)	0.9956	0.9960	0.9968	0.9979
public debt	0.2258	0.6414	1.2545	2.0466
money demand	0.7198	0.7213	0.7228	0.7215
st.dev of cons.	0.0737	0.0734	0.0727	0.0752
interest rate	0.0336	0.0338	0.0341	0.0337

[13] This calculation is similar to calculating λ in (2.15).

Money is valued in this economy because of the informational/timing friction in the loan market. Results may be sensitive to this specific friction. Certainly, there are transaction costs for trading in the bond markets, and money bears a liquidity premium over these assets. The question is whether results will change if the model incorporates these transaction costs explicitly.

One has to bear in mind that I abstracted from aggregate uncertainty about the monetary policy and the endowment process. In an environment with incomplete markets, aggregate uncertainty about the monetary policy and the inflation may limit the ability of government to finance interest payments on its debt since the tax base (i.e. demand for money) may be small.

4. Conclusion

In this paper I develop a dynamic, incomplete markets general equilibrium model with two types of assets (money and bond) that are held only for precautionary savings motives. The study tries to investigate the welfare consequences of different rates of inflation. In particular, welfare issues are discussed when the government can collect seigniorage revenues using the inflation tax which are used to finance the payments on public debt, and when money also serves for consumption-smoothing purposes.

Previous studies show that the inflation tax is welfare-decreasing in an environment with only fiat money, and therefore, call for deflationary policies. On the contrary, this study concentrates not only on the insurance role of money but on the revenues generated by inflationary policies. The main finding of the paper is that welfare increases when the government uses higher rates of inflation tax to finance higher returns and level of public debt. Furthermore, welfare gains in consumption units are large relative to those in models with infinitely-lived agents. When agents are subject to more severe borrowing constraints, welfare gains from higher growth rates of money are larger.

In future work, it may be useful to consider the effects of inflation on capital accumulation. Certainly, higher inflation rates, like any other distortionay taxation, will have negative incentive effects on investment and output. Incompleteness and imperfections in loan markets could be incorporated explicitly in order to understand and evaluate welfare consequences of inflation.

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