Efficient Public Good Provision with Nonlinear Income Taxation

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Abstract

Due to the use of distortionary taxation, many believe that real-world economies should attain a lower level of public expenditures than one might suspect from the analysis of artificial models where lump-sum taxes are assumed to be available. The paper examines this popular hypothesis by means of the two-type self-selection model of income taxation. I provide sufficient conditions for both a lower and a higher level of public expenditures in second best than in first best. In particular, it is shown that the widely employed assumptions of Christiansen (1981) lead to under-provision of the public good in the income tax optimum.

Keywords: income taxation, public goods

JEL-Classification: H41, H21

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1 Introduction

In his well-known statement on the principles of public good provision, Pigou (1947, pp. 33-4) argued that the total welfare cost of a public project does not only contain the direct cost of production, but also the indirect burden imposed on the taxpayers on grounds of distortionary taxation. Real-world economies should thus attain a lower level of public expenditures than one might suspect from the analysis of artificial models where lump-sum taxes are assumed to be available.

During the last fifty years, Pigou's argument has often been employed not only in the theoretical but also in the political debate on the "appropriate" level of public sector activities. The *validity* of his reasoning, however, is usually justified by means of Ramsey's tax model, where linear taxes and a representative consumer are assumed. Although this framework is convenient for many issues of tax analysis, it does not explain the government's inability to raise non-distortionary (i.e. lump-sum) taxes. Considering this weakness, the task of providing a sound foundation for Pigou's reasoning has only partially been realized.

This paper is concerned with the validity of Pigou's claim within the framework of nonlinear income taxation where the unavailability of lump-sum taxes is explained by the government's imperfect information with regard to the households' abilities. The analysis builds on the findings of Boadway and Keen (1993) who employ the self-selection approach to income tax analysis in order to characterize the potential deviations between the public good provision rules in second best and first best. Expanding the results of Boadway and Keen, I provide sufficient conditions for both under- and over-provision of the public good in second best. These conditions are applied to specific preference structures known from the literature. In particular, it will be shown that the widely employed separability assumptions of Christiansen (1981) imply under-provision of the public good in second best. Although over-provision is shown to be possible, the results make clear that such an outcome is not very likely to happen as long as the public good is strictly normal.

It is well understood by researchers that Pigou's reasoning - though intuitively convincing - should be rigorously examined. The first result is due to Atkinson and Stern (1974). They analyze an economy with linear taxes and a representative consumer (i.e. the Ramsey tax model) and provide an example which is consistent with Pigou's claim. This example has been generalized by Wilson (1991b), Chang (1998), and Gaube (1999).[1] These results follow Pigou's intuition in assuming that only dis-

[1] These results basically assume that all commodities are normal and that the elasticity of labor supply is positive. If, however, a backward bending labor supply curve is assumed, counterexamples

tortionary taxes are available and that distributional objectives do not affect the optimal level of public consumption. These premises, however, have been challenged by Wilson (1991a) and Mirrlees (1994). They analyze an economy with heterogeneous households and point out that the government's informational constraints allow for the introduction of a poll tax in addition to linear consumption taxes. Within such a setting, Wilson and Mirrless provide counterexamples to Pigou's claim. These examples are explained by the observation that the poll tax accomodates the public sector with a non-distortionary source of marginal finance and may hence lead to a higher level of public expenditures in second best than in first best.

The analysis of Wilson and Mirrlees is induced by the point of view that the set of feasible tax instruments has to be explained by the government's imperfect information with respect to the households' abilities. This argument has originally been used to motivate the analysis of nonlinear income taxation. So far, however, the literature on public good provision with income taxation has been confined to the derivation of second-best provision rules: Analyzing Mirrlees' (1971) model with a continuum of households, Christiansen (1981) and Tuomala (1990) explore the conditions under which Samuelson's rule also hold in the second-best optimum. Boadway and Keen (1993) demonstrate that this analysis becomes far more intuitive by using the two-type self-selection approach first introduced by Stiglitz (1982), Stern (1982), and Nichols and Zeckhauser (1982).[2] Boadway and Keen provide surprisingly simple and clearcut explanations for the potential deviations between the public good decision rules in second best and first best. However, they also point out that their investigation is restricted to provision rules and does not in itself allow for a comparison between the expenditure levels in first best and second best.

The paper is organized as follows. Section 2 reviews the model. The main results and their relationship to the findings of Boadway and Keen (1993) are presented in section 3. Section 4 concludes.

2 The model

The economy consists of two types of households $i = 1, 2$, a private production sector, and the government. The N_1 households of type 1 differ from the N_2 households of type 2 only with respect to ability, but not with respect to preferences or endowments. Without loss of generality, it is assumed that each household is endowed with

can be constructed (see de Bartolomé (1998) and Gaube (1999)).

[2]See also Edwards et al. (1994), Nava et al. (1996), and Blomquist and Christiansen (1998) for investigations closely related to the approach of Boadway and Keen (1993).

one unit of time. Preferences are represented by a strictly quasiconcave and twice continuously differentiable utility function $U(X_i, L_i, G)$, where X_i, L_i, and G denote private consumption,[3] labor supply, and the public good respectively.

The commodities X and G can be produced by means of a linear technology where L_1 and L_2 serve as the only inputs of production. The quantities are normalized such that the set of feasible production plans is described by the condition $N_1 w_1 L_1 + N_2 w_2 L_2 \geq N_1 X_1 + N_2 X_2 + cG$. The marginal productivity w_2 of the second type of households is assumed to exceed that of the first type, i.e. $w_2 > w_1$. The parameter c denotes the constant marginal cost of the public good measured in terms of the effective labor input $N_1 w_1 L_1 + N_2 w_2 L_2$.

The pre-tax income levels of the households are denoted by $Y_i := w_i L_i$. Income taxation is represented by a possibly nonlinear tax function $T(Y_i)$. Normalizing the price of the private consumption good to unity, the budget constraint of a household of type i is thus given by $Y_i - T(Y_i) = X_i$. The government uses its tax revenues to finance the provision of the public good G. This implies the budget constraint

$$N_1(Y_1 - X_1) + N_2(Y_2 - X_2) - pG \geq 0, \tag{1}$$

where p denotes the producer price of the public good.[4] The government can observe the pre-tax incomes Y_i, but not their components w_i and L_i. Due to the revelation principle, these informational restrictions can be formalized by means of the self-selection constraints of the two types of households. To do so, the utilities $U(X_i, L_i, G)$ have to be expressed in terms of the observable variables X_i, Y_i and G. Using the definition $V^i(X_i, Y_i, G) := U(X_i, Y_i/w_i, G)$, it is clear that the agents will only reveal their types if the conditions

$$V^1(X_1, Y_1, G) \geq V^1(X_2, Y_2, G) \quad \text{and} \quad V^2(X_2, Y_2, G) \geq V^2(X_1, Y_1, G) \tag{2}$$

are satisfied. Hence, the government may restrict itself to consider only those values of $X_i, Y_i,\ i = 1, 2$ and G which meet the inequalities (1) and (2).

In accordance with most analyses of the literature, the following investigation presumes that Seade's (1982) agent monotonicity (i.e. single crossing) condition is satisfied. This condition holds if the term $-L_i(U_L/U_X)$ increases with L_i. Since this

[3]Following most work on income tax analysis, the present paper assumes a single private consumption commodity. This assumption is made in order to rule out differential commodity taxation. Note that it is not clear how the results presented below would extend if differential commodity taxation were allowed (see section 4).

[4]Since technology is linear and the price of X is normalized to unity, p is constant and equal to the marginal cost c.

is true if private consumption is normal, agent monotonicity is a rather mild assumption. The single crossing property implies that at most one of the two self-selection constraints contained in (2) is binding in second best. One can thus distinguish between three possible regimes depending on whether the first, the second, or none of the two constraints is binding in the optimum. As in most work on the subject, I will confine the analysis to the second regime where only the self-selection constraint of the low-ability type is slack in second best. This is the 'normal' case discussed by Stiglitz (1982) and reflects the situation that the minimum utility level $\bar{U}^1$ which the government seeks to obtain for the less productive individuals implies redistribution from high- to low-ability households. Following Stiglitz (1982), the second-best values of X_i, Y_i, and G can thus be defined as follows:

$$\begin{aligned}(\{X_i^S, Y_i^S\}_{i=1}^2, G^S) := \underset{X_i, Y_i, G}{\operatorname{argmax}} \{V^2(X_2, Y_2, G) \mid\ & V^1(X_1, Y_1, G) - \bar{U}^1 \geq 0 \\ & V^2(X_2, Y_2, G) - V^2(X_1, Y_1, G) \geq 0 \\ & N_1(Y_1 - X_1) + N_2(Y_2 - X_2) - pG \geq 0\}.\end{aligned} \tag{3}$$

This allocation will be compared with the first-best optimum $(\{X_i^F, Y_i^F\}_{i=1}^2, G^F)$ which is based on the assumption that the planner has full information with respect to the households' abilities:

$$\begin{aligned}(\{X_i^F, Y_i^F\}_{i=1}^2, G^F) := \underset{X_i, Y_i, G}{\operatorname{argmax}} \{V^2(X_2, Y_2, G) \mid\ & V^1(X_1, Y_1, G) - \bar{U}^1 \geq 0 \\ & N_1(Y_1 - X_1) + N_2(Y_2 - X_2) - pG \geq 0\}.\end{aligned} \tag{4}$$

Note that the constraint $V^1(X_1, Y_1, G) - \bar{U}^1 \geq 0$ is binding not only in the optimum of (4), but also in any interior solution of the second-best problem (3) (see Fact 1 below). This means that (i) the low-ability households attain the same utility level in both allocations and that (ii) the first-best allocation Pareto dominates the second-best allocation. The results presented below, however, make use only of property (ii) and do not require that (i) is satisfied. [5]

The properties of the allocation $(\{X_i^S, Y_i^S\}_{i=1}^2, G^S)$ can be investigated by means of the first-order conditions corresponding to (3). Let δ, λ, and γ denote the Lagrange

[5]Note also that (3) follows Stiglitz (1982) and slightly differs from the definition used by Boadway and Keen (1993). They assume the government maximizes $V^1(X_1, Y_1, G)$ with respect to the budget constraint, the self-selection constraint $V^2(X_2, Y_2, G) - V^2(X_1, Y_1, G) \geq 0$, and the requirement $V^2(X_2, Y_2, G) - \bar{U}^2 \geq 0$. Here, the restriction $V^2(X_2, Y_2, G) - \bar{U}^2 \geq 0$ may also be binding in second best, but not necessarily so (see Brito el al. (1990)). Hence, if this definition is used the first-best allocation may not Pareto dominate the second-best allocation. Since the findings of Boadway and Keen (1993) hold for (3) as well, the results of the present paper are not at variance with any of their conclusions.

multipliers associated with the three constraints contained in (3) and define the utility level of the high-ability type 2 when mimicking the low-ability type 1 by $\hat{V}^2 := V^2(X_1, Y_1, G)$. Using this notation, the first-order conditions associated with $(\{X_i^S, Y_i^S\}_{i=1}^2, G^S)$ can be written as

$$X_1: \quad \delta V_X^1 - \lambda \hat{V}_X^2 - \gamma N_1 = 0 \tag{5}$$

$$Y_1: \quad \delta V_Y^1 - \lambda \hat{V}_Y^2 + \gamma N_1 = 0 \tag{6}$$

$$X_2: \quad (1+\lambda) V_X^2 - \gamma N_2 = 0 \tag{7}$$

$$Y_2: \quad (1+\lambda) V_Y^2 + \gamma N_2 = 0 \tag{8}$$

$$G: \quad \delta V_G^1 + (1+\lambda) V_G^2 - \lambda \hat{V}_G^2 - \gamma p = 0. \tag{9}$$

These conditions are standard and lead to the following well-known results which will be used for reference in the subsequent analysis.[6]

Fact 1: [Stiglitz (1982)]
(a) The marginal tax rate of the high-ability type is zero, i.e. $-V_Y^2/V_X^2 = 1$.
(b) The marginal tax rate of the low-ability type is positive, i.e. $-V_Y^1/V_X^1 < 1$.
(c) The constraint $V^1(X_1, Y_1, G) - \bar{U}^1 \geq 0$ *is binding.*

The difference $G^S - G^F$ reflects the influence of the government's informational constraints on the level of public activity. Fact 1 shows that these constraints also lead to distortionary taxation. Hence, according to Pigou's argument, the relationship $G^F > G^S$ should hold. In the following, I will explore whether this claim is correct.

3 The levels issue

The effect of distortionary taxation on the optimal level of public expenditures can be analyzed by means of the first-order conditions corresponding to the allocations (3) and (4). Assuming an interior solution, the first-best allocation has to satisfy Samuelson's rule. Hence, the sum of the marginal rates of substitution between the public good and *each* of the private goods is equal to the marginal rate of transformation between these commodities. Following Boadway and Keen

[6] Note that equations (5) - (9) do not directly refer to marginal tax rates. However, since the households maximize $V^i(X_i, Y_i, G)$ with respect to the budget constraint $Y_i - T(Y_i) = X_i$, we have $-V_Y^i/V_X^i = 1 - T'$.

(1993), I denote the sum of the marginal rates of substitution between X and G by $\sum \mathrm{MRS}_{GX}^F := N_1(V_G^1/V_X^1) + N_2(V_G^2/V_X^2)$, where the superscript F indicates that all partial derivatives are evaluated at $(\{X_i^F, Y_i^F\}_{i=1}^2, G^F)$. Accordingly, $\sum \mathrm{MRS}_{GY}^F$ is defined by $\sum \mathrm{MRS}_{GY}^F := N_1(V_G^1/-V_Y^1) + N_2(V_G^2/-V_Y^2)$. Using these definitions, Samuelson's rule can be written as follows:

$$\sum \mathrm{MRS}_{GX}^F = p \qquad \text{and} \qquad \sum \mathrm{MRS}_{GY}^F = p. \tag{10}$$

The aim of Boadway and Keen (1993) is to derive analogous conditions for the second-best problem (3) and to compare them with (10). More specifically, they explore whether the social marginal benefit $\sum \mathrm{MRS}_{GK}$, $K \in \{X, Y\}$ of the public good exceeds its marginal production cost p in second best. This investigation corresponds to the common belief that the relationship $\sum \mathrm{MRS}_{GK} > p$ indicates under-provision of the public good in second best (i.e. $G^S < G^F$) while $\sum \mathrm{MRS}_{GK} < p$ indicates over-provision (i.e. $G^S > G^F$). However, Boadway and Keen point out that the comparison between $\sum \mathrm{MRS}_{GK}$ and p depend on the choice of the private commodity $K \in \{X, Y\}$. Their main findings can be summarized by means of the following proposition where MRS_{GX}^{1S} and MRS_{GY}^{1S} denote the marginal rates of substitution of the low-ability type 1, while $\widehat{\mathrm{MRS}}_{GX}^{2S}$ and $\widehat{\mathrm{MRS}}_{GY}^{2S}$ refer to the marginal rates of substitution of the high-ability type 2 when mimicking the low-ability type 1.

Proposition 1: [Boadway and Keen (1993)][7]
(a) $\sum MRS_{GX}^S >,=,< p \Leftrightarrow \widehat{MRS}_{GX}^{2S} >,=,< MRS_{GX}^{1S}$.
(b) $\sum MRS_{GY}^S >,=,< p \Leftrightarrow \widehat{MRS}_{GY}^{2S} >,=,< MRS_{GY}^{1S}$.

This striking result reveals that the potential deviations of the second-best decision rule from Samuelson's rule do not depend on the distortive properties of the tax system, but on how the marginal rates of substitution differ between the low-ability household and the mimicker. If a high-ability type mimicks a low-ability household by choosing the same income level, both pay the same income tax, consume the same amount of X, and differ only with respect to their supply of labor. Hence, the differences between the first-best and the second-best decision rules hinge only on assumptions concerning the households' preferences. This becomes clear by

[7] Parts (a) and (b) of the proposition follow directly from the equations (9) and (12) of Boadway and Keen (1993). Note that Proposition 1 also holds for heterogeneous preferences provided that the single crossing property is satisfied. Under the same qualification, Proposition 2, Lemma 1, and Lemma 2 could also be generalized to heterogeneous preferences. In such a framework, however, the single crossing property is not anymore implied by Seade's agent monotonicity condition.

Corollary 1: [Boadway and Keen (1993)][8]
(a) If $\partial(MRS_{GX})/\partial L <,=,> 0$ *then* $\widehat{MRS}^{2S}_{GX} >,=,< MRS^{1S}_{GX}$.
(b) If $\partial(-MRS_{GL}/L)/\partial L <,=,> 0$ *then* $\widehat{MRS}^{2S}_{GY} >,=,< MRS^{1S}_{GY}$.

Corollary 1 shows that the preference restrictions necessary to ensure $\sum \mathrm{MRS}^S_{GX} \geq p$ differ from those which have to be satisfied in order to derive $\sum \mathrm{MRS}^S_{GY} \geq p$. Hence, if for instance $\sum \mathrm{MRS}^S_{GY} > p > \sum \mathrm{MRS}^S_{GX}$ occurs in second best, the conventional intuition would suggest over-provision of G in the case of effective labor serving as the numéraire, and under-provision of G in the case of consumption serving as the numéraire.[9] One may conclude from this observation that the findings contained in Proposition 1 and Corollary 1 are not suited for a comparison between G^S and G^F. Boadway and Keen point directly at this problem and note that the common notions of over- and under-provision (which they also employ) are "merely shorthand for a central characteristic of the second-best optimum" (pp. 470 - 1). In the following I will discuss this issue and show that the levels problem can be solved as long as the first-order conditions underlying the results of Proposition 1 lead to $\sum \mathrm{MRS}^S_{GK} \geq p$ with respect to *both* private commodities $K \in \{X, Y\}$. However, if preferences imply $\sum \mathrm{MRS}^S_{GK} > p$ as well as $\sum \mathrm{MRS}^S_{GK} < p$ depending on whether X or Y is chosen as the numéraire, general results concerning the comparison between G^S and G^F do not seem to be attainable.

Before discussing this issue, it should be noted that the parts (a) and (b) of Proposition 1 and Corollary 1 are not independent of each other. This becomes clear by

Fact 2: $\sum MRS^S_{GY} > \sum MRS^S_{GX}$.

This observation follows directly from earlier findings (Fact 1) according to which the optimal marginal tax rate is zero for high-ability households and strictly positive for low-ability households.[10] Fact 2 shows that the choice of using consumption or effective labor as the numéraire is not arbitrary, but has systematic conse-

[8]Part (a) restates Corollary 2 of Boadway and Keen (1993). Part (b) follows directly from their equation (16).

[9]Within the framework of linear taxation, a similar observation has been made by Atkinson and Stern (1974). Recently, this issue has been shown to play an important role for the characterization of environmental taxes in a second-best framework (see Fullerton (1997)).

[10]Note, however, that Fact 2 contradicts Corollary 4 of Boadway and Keen (1993). This Corollary states that both conditions $\sum \mathrm{MRS}^S_{GY} = p$ and $\sum \mathrm{MRS}^S_{GX} = p$ hold in second best, if preferences can be represented by the utility function $U(X_i, L_i, G) = U(L_i/\zeta(X_i, G))$. These preferences, though, do not satisfy the agent monotonicity condition, which has been assumed to hold by Boadway and Keen.

quences with respect to eventual conclusion concerning the efficient level of public expenditures.[11] The result also implies that the analysis of the second-best optimum can be restricted to three possible occurences: First, $\sum \mathrm{MRS}^S_{GY} > \sum \mathrm{MRS}^S_{GX} \geq p$, second $p \geq \sum \mathrm{MRS}^S_{GY} > \sum \mathrm{MRS}^S_{GX}$, and third $\sum \mathrm{MRS}^S_{GY} > p > \sum \mathrm{MRS}^S_{GX}$. In the following, these three cases will be examined in detail.

3.1 Under-provision of the public good in second best

This subsection is devoted to the case $\sum \mathrm{MRS}^S_{GY} > \sum \mathrm{MRS}^S_{GX} \geq p$. According to the conventional reasoning this case suggests under-provision of the public good in second best, i.e. $G^S < G^F$. This reasoning can be motivated by a simple local argument: Assume that an allocation satisfies $\sum \mathrm{MRS}_{GY} > \sum \mathrm{MRS}_{GX} > p$ and that the government is not restricted by the households' self-selection constraints. A Pareto improvement is then possible by increasing G and decreasing (X_1+X_2) or (Y_1+Y_2) by the same amount. This local argument, however, is not sufficient for the proof of the global conjecture $G^F > G^S$. Still, the following result shows that the local intuition is correct as long as the public good is neither inferior nor a Hicksian complement for leisure.

Proposition 2: *Assume that leisure and the public good are Hicksian substitutes and that the public good is normal. Then $\sum MRS^S_{GX} \geq p$ implies $G^S < G^F$.*

Before proving this proposition, I will first provide some idea why the condition $\sum \mathrm{MRS}^S_{GX} \geq p$ alone does not suffice to derive the expected result. In order to present the basic argument, it is helpful to decompose the path between the solutions $(\{X_i^S, Y_i^S\}_{i=1}^2, G^S)$ and $(\{X_i^F, Y_i^F\}_{i=1}^2, G^F)$ by means of the allocation $(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z)$ which is characterized by the minimal amount of resources necessary to achieve the utility levels U^{iS}:

$$(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z) := \arg\min_{X_i,Y_i,G} \{N_1(X_1-Y_1) + N_2(X_2-Y_2) + pG \mid \quad V^i(X_i,Y_i,G) \geq V^i(X_i^S,Y_i^S,G^S),\ i=1,2\}. \tag{11}$$

As long as $(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z)$ is an interior allocation, it fulfills the same first-order conditions as the first-best allocation $(\{X_i^F, Y_i^F\}_{i=1}^2, G^F)$. This means that Samuelson's rule (10) is also satisfied at $(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z)$. Therefore, the decomposition of $G^F - G^S$ into $G^F - G^Z$ and $G^Z - G^S$ is analogous to the Hicksian compensation method where $G^Z - G^S$ can be interpreted as a pure substitution effect while

[11] See also Atkinson and Stern (1974) who make a similar point within the linear taxation framework.

$G^F - G^Z$ represents a pure income effect. This decomposition reveals that the transition from second best to first best affects the optimal level of public activity in two separate ways. First, it changes the marginal evaluation of public projects *relative* to private commodities. This is the substitution effect described by $G^Z - G^S$. Second, it leads to a higher overall welfare level which corresponds to the income effect $G^F - G^Z$. Since the conventional intuition refers to a change in the marginal rates of substitution, it would only suggest a positive substitution effect provided that $\sum \text{MRS}^S_{GX} \geq p$ holds in second best. However, such a positive substitution effect could be cancelled out by a negative income effect. The income effect is negative if the public good is inferior.[12] Hence, as long as the public good is not assumed to be normal, the condition $\sum \text{MRS}^S_{GX} \geq p$ cannot be sufficient for the claim $G^F - G^S > 0$.

The results of this paper are based on the assumption that the public good is not inferior. In this case, the condition $\sum \text{MRS}^S_{GX} \geq p$ does indeed imply $G^F > G^S$ if the substitution effect $G^Z - G^S$ can be shown to be positive. At first sight, the proof of this claim seems to be straightforward: Because of (10), we have $\sum \text{MRS}^S_{GX} \geq \sum \text{MRS}^Z_{GX}$ and $\sum \text{MRS}^S_{GY} > \sum \text{MRS}^Z_{GY}$. These conditions mean that the marginal rates of substitution between the public good and the private commodities are higher at $(\{X^S_i, Y^S_i\}^2_{i=1}, G^S)$ than at $(\{X^Z_i, Y^Z_i\}^2_{i=1}, G^Z)$. Since these marginal rates of substitution correspond to implicit "relative prices" between the public good and the private commodities, the transition from $(\{X^S_i, Y^S_i\}^2_{i=1}, G^S)$ to $(\{X^Z_i, Y^Z_i\}^2_{i=1}, G^Z)$ can be interpreted as the consequence of a decreasing price of the public good, which (under the assumption of constant utility levels) increases the households' demand for this commodity. However, the transition from $(\{X^S_i, Y^S_i\}^2_{i=1}, G^S)$ to $(\{X^Z_i, Y^Z_i\}^2_{i=1}, G^Z)$ also affects the relative price between leisure and private consumption. This is due to the fact that the marginal income tax of the low-ability type is positive in second best while it is zero in $(\{X^Z_i, Y^Z_i\}^2_{i=1}, G^Z)$. The increasing net wage of low-ability households raises their (compensated) demand for the public good only if leisure and the public good are Hicksian substitutes. Therefore, if these two commodities are Hicksian complements, the total effect $G^Z - G^S$ could become negative.

Proof of Proposition 2: The proof is provided in the framework of a fictitious economy where the households pay personalized prices q^i_G, $i = 1, 2$ for the public good

[12]Note that the model assumes the public good to be provided for free by the government. However, one could construct a fictitious economy where each household has to pay a personalized price for the provision of the public good G. Then, given the households' preferences, one could ask whether their demand for the public good raises with income. I consider such a fictitious economy when refering to the normality or inferiority of G.

and receive (possibly negative) lump-sum transfers I^i, $i = 1, 2$ from the government. Based on this fictitious economy it is shown that the modification of prices and transfers necessary to achieve $(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z)$ instead of $(\{X_i^S, Y_i^S\}_{i=1}^2, G^S)$ has a positive (substitution) effect on G, i.e. $G^Z - G^S > 0$. In the second step the fictitious economy is used in order to show that the income effect $G^F - G^Z$ is positive too. This completes the proof.

(Step 1) Assume that a household of type i faces the prices q_X for the private consumption commodity, q_L^i for labor, and q_G^i for the public good. Receiving a lump-sum transfer (or tax) $I^i \in \mathbb{R}$, the household maximizes

$$\max_{X_i, L_i, G} \quad U(X_i, L_i, G) \quad \text{s.t.} \quad q_X X_i + q_G^i G - q_L^i L_i \leq I^i.$$

This program leads to the demand functions $X_i(q_X, q_L^i, q_G^i, I^i)$, $G_i(q_X, q_L^i, q_G^i, I^i)$, and the supply function $L_i(q_X, q_L^i, q_G^i, I^i)$. Because of the relationship $L_i = Y_i/w_i$, we can define the quantities $L_i^K := Y_i^K/w_i$, $K = S, Z$ and reformulate the allocations $(\{X_i^K, Y_i^K\}_{i=1}^2, G^K)$ in terms of $(\{X_i^K, L_i^K\}_{i=1}^2, G^K)$. The latter can be implemented as an equilibrium of the fictitious economy as long as the government chooses prices and transfers $(q_X^K, q_L^{iK}, q_G^{iK}, I^{iK})$, $K = S, Z$, $i = 1, 2$, such that

$$\frac{q_L^{iK}}{q_X^K} = \frac{-U_L^{iK}}{U_X^{iK}}, \quad \frac{q_G^{iK}}{q_X^K} = \frac{U_G^{iK}}{U_X^{iK}}, \quad I^{iK} = q_X^K X_i^K + q_G^{iK} G^K - q_L^{iK} L_i^K.$$

Since the marginal tax rate of the high- (low-) ability type is zero (positive) in second best while both are zero with first-best taxation, we have

$$\frac{q_L^{1S}}{q_X^S} < w_1 = \frac{q_L^{1Z}}{q_X^Z}, \quad \frac{q_L^{2S}}{q_X^S} = w_2 = \frac{q_L^{2Z}}{q_X^Z}. \tag{12}$$

From the definition of $(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z)$ we know furthermore that Samuelson's rule has to be satisfied here. Hence,

$$N_1 \frac{q_G^{1Z}}{q_X^Z} + N_2 \frac{q_G^{2Z}}{q_X^Z} = p, \qquad N_1 \frac{q_G^{1Z}}{q_L^{1Z}} w_1 + N_2 \frac{q_G^{2Z}}{q_L^{2Z}} w_2 = p. \tag{13}$$

On the other hand, the assumption $\sum \text{MRS}_{GX} \geq p$ implies

$$N_1 \frac{q_G^{1S}}{q_X^S} + N_2 \frac{q_G^{2S}}{q_X^S} \geq p. \tag{14}$$

Without loss of generality, we may choose $q_X^Z = q_X^S$. Then equations (12) to (14) imply $q_L^{1Z} > q_L^{1S}$, $q_L^{2Z} = q_L^{2S}$, and $N_1 q_G^{1Z} + N_2 q_G^{2Z} \leq N_1 q_G^{1S} + N_2 q_G^{2S}$. Therefore, starting from $(\{X_i^S, L_i^S\}_{i=1}^2, G^S)$, the allocation $(\{X_i^Z, L_i^Z\}_{i=1}^2, G^Z)$ can be achieved by (i) raising the price q_L^{1S} to q_L^{1Z}, (ii) reducing the sum $N_1 q_G^{1S} + N_2 q_G^{2S}$ to $N_1 q_G^{1Z} + N_2 q_G^{2Z}$, and (iii) replacing the transfers I^{iS} with I^{iZ}. Condition (ii) implies that at least one of the two inequalities $q_G^{1Z} \leq q_G^{1S}$ and $q_G^{2Z} \leq q_G^{2S}$ must be satisfied. Assume first $q_G^{1Z} \leq q_G^{1S}$: Because of (i) and our assumption that leisure is a Hicksian substitute for G this leads to a positive (substitution) effect on the demand of the low-ability household for G, i.e. $G_1(q_L^{1Z}, q_X^Z, q_G^{1Z}, I^{1Z}) > G_1(q_L^{1S}, q_X^S, q_G^{1S}, I^{1S})$. Since $G_1(q_L^{1Z}, q_X^Z, q_G^{1Z}, I^{1Z}) = G_2(q_L^{2Z}, q_X^Z, q_G^{2Z}, I^{2Z})$, we thus have $G^Z > G^S$. If, on the other hand, $q_G^{2Z} \leq q_G^{2S}$ holds we get $G_2(q_L^{2Z}, q_X^Z, q_G^{2Z}, I^{2Z}) > G_2(q_L^{2S}, q_X^S, q_G^{2S}, I^{2S})$ which also implies $G^Z > G^S$.

(Step 2) Using the definition $L_i^F := Y_i^F/w_i$, the first-best allocation $(\{X_i^F, Y_i^F\}_{i=1}^2, G^F)$ can equivalently be described by $(\{X_i^F, L_i^F\}_{i=1}^2, G^F)$. Note that the latter can also be implemented within the framework of the fictitious economy and define the respective prices and transfers by $q_L^{iF}, q_X^F, q_G^{iF}$, and I^{iF}, $i = 1, 2$. Since the allocation $(\{X_i^F, Y_i^F\}_{i=1}^2, G^F)$ is first-best, we know that $(q_L^{iF}/q_X^F) = (q_L^{iZ}/q_X^Z) = w_i$, $i = 1, 2$ and $N_1(q_G^{1F}/q_X^F) + N_2(q_G^{2F}/q_X^F) = N_1(q_G^{1Z}/q_X^Z) + N_2(q_G^{2Z}/q_X^Z) = p$. Without loss of generality, we may choose $q_X^F = q_X^Z$. This implies $q_i^F = q_i^Z$, $i = 1, 2$ and $N_1 q_G^{1F} + N_2 q_G^{2F} = N_1 q_G^{1Z} + N_2 q_G^{2Z} = p$. Now define $\Delta I^i := (I^{iF} - I^{iZ})$, $\Delta q_G^i := (q_G^{iF} - q_G^{iZ})$, and let $W^i(q_X, q_L^i, q_G^i, I^i)$, $i = 1, 2$ denote the indirect utility functions of household i. It is known from Fact 1 that the constraint $V^1(X_1, Y_1, G) - \bar{U}^1 \geq 0$ is binding in second best. This implies that the first-best allocation Pareto dominates the second-best allocation. Since $W^{iZ} = W^{iS}$, $i = 1, 2$, we thus have $W^{1F} \geq W^{1Z}$ and $W^{2F} \geq W^{2Z}$. Consequently,

$$\frac{\partial W^i}{\partial I^i}\Delta I^i + \frac{\partial W^i}{\partial q_G^i}\Delta q_G^i \geq 0 \qquad i = 1, 2.$$

Using the households' budget constraints, these conditions can be transformed to $\Delta I^i \geq G_i \Delta q_G^i$. The effect of Δq_G^i and ΔI^i on household i's demand of the public good is denoted by $\Delta G_i := (\partial G_i/\partial I^i)\Delta I^i + (\partial G_i/\partial q_G^i)\Delta q_G^i$. Since the derivatives $(\partial G_i/\partial I^i)$, $i = 1, 2$, have been assumed to be positive, we get

$$\Delta G_i = \frac{\partial G_i}{\partial I^i}\Delta I^i + \frac{\partial G_i}{\partial q_G^i}\Delta q_G^i \geq \frac{\partial G^i}{\partial I^i}(G_i \Delta q_G^i) + \frac{\partial G^i}{\partial q_G^i}\Delta q_G^i = \Delta q_G^i \frac{\partial s_G^i}{\partial q_G^i}.$$

Here, s_G^i denotes the Hicksian demand of household i with respect to the public good. The partial derivatives $(\partial s_G^i/\partial q_G^i)$, $i = 1, 2$ are negative. Furthermore, at least one of the Δq_G^i has to be negative too (i.e. $\Delta q_g^i \leq 0$). Since $\Delta G_1 = \Delta G_2$, this implies $G^F \geq G^Z$. ■

Remark 1 In Proposition 2 it is assumed that the public good is neither inferior nor a Hicksian complement for leisure. While the non-inferiority assumption is plausible and widely accepted, Hicksian substitutability seems to be somewhat more restrictive. Note, however, that at least one of the two private commodities has to be a Hicksian substitute for G if preferences are strictly monotone. Hence, if the public good is interpreted as a *general* public project and if public expenditures are not systematically biased in favour of one of the private commodities, both leisure and private consumption are Hicksian substitutes for G.[13]

Remark 2: Proposition 2 refers to the quantities G^S and G^F as defined in section 2. In this formulation, the utility level $\bar{U}^1$ is identical in first best and second best. Note, however, that the result is not restricted to this assumption: The proof of Proposition 2 (second step) makes it clear that the argument is valid for all first-best allocations which Pareto dominate the second-best allocation.

Proposition 2 complements part (a) of Proposition 1. Taken together, the two findings provide sufficient conditions for under-provision of the public good in second best. These sufficiency conditions are satisfied for various types of preferences. As an example, consider the following corollary which applies Propositions 1 and 2 to utility functions exhibiting the property of weak separability between labor and bundles consisting of private and public consumption.

Corollary 2: *Assume* $U(X_i, L_i, G) = U(H(X_i, G), L_i)$. *If the public good is normal, then* $G^S < G^F$.

Proof: The ratio U_G/U_X is independent of L. Using Proposition 1, this implies that the condition $\sum \text{MRS}_{GX} = p$ holds in second best. Since preferences have been assumed to be strictly monotone, the bundle (X, G) is a Hicksian substitute for leisure $(1 - L)$. It then follows from the separability between (X, G) and L that leisure $(1 - L)$ and G are Hicksian substitutes as long as G is normal (see Deaton and Muellbauer (1980), p. 129). Hence, the assumptions made in Proposition 2 are satisfied. This implies $G^F > G^S$. ■

Preferences of the type described in Corollary 2 have been analyzed extensively in the literature: If the households' utility functions are of the type $U(H(X_i, G), L_i)$, the rule $\sum \text{MRS}^S_{GX} = p$ holds in second best. This result has first been derived by Christiansen (1981) for an economy with a continuum of households. Boadway and Keen (1993) restate this property for the two-type economy analyzed in this paper. According to Christiansen (1981) and Kaplow (1996), this relationship indicates that redistributive objectives should affect taxes but not the design of public projects. This interpretation is motivated by the observation that the private consumption commodity serves as the numéraire in models of optimal income taxation. This implies that the marginal utility V^i_X is equal to the marginal utility of income V^i_I. Hence, the result $\sum \text{MRS}^S_{GX} = p$ can be interpreted in the sense that the sum of the

[13]See Starrett (1988), p.173 for this line of argument.

marginal rates of substitution between the public good and "money" is equal to the marginal production cost of the public good in second best. Corollary 2, however, makes clear that this partial analogy of "rules" does not imply identical levels of public expenditures in first best and second best.[14]

The results presented so far provide sufficient conditions for underprovision of the public good in second best. The intuition behind these findings, however, differs from the conventional argument which refers to the indirect welfare cost of distortionary taxation in order to motivate a lower level of public good provision in second best than in first best. This reasoning emerged from the analysis of linear tax systems which are distortionary by assumption. In the framework analyzed in this paper, however, distortionary taxation is due to the government's imperfect information with respect to the households' abilities. Hence, in contrast to linear tax models, the question is not how distortionary taxes affect the provision of public goods, but how public goods affect those informational constraints which induce the government to employ distortionary taxation. This becomes clear by recalling Corollary 1, Proposition 1, and Fact 2: If preferences satisfy the assumption $\partial(\mathrm{MRS}_{GX})/\partial L < 0$, we have $\widehat{\mathrm{MRS}}^{2S}_{GX} > \mathrm{MRS}^{1S}_{GX}$ and $\widehat{\mathrm{MRS}}^{2S}_{GY} > \mathrm{MRS}^{1S}_{GY}$ which in turn implies $\sum \mathrm{MRS}^{S}_{GY} > \sum \mathrm{MRS}^{S}_{GX} > p$. The first two of these inequalities mean that high-ability households - in case they try to hide their type - have higher marginal rates of substitution between the public good and the private commodities than less productive individuals. Consequently, any reform of public policy where the public good is reduced and private consumption of low-ability households is increased such that their utility remains constant, leaves the mimickers worse off. This means that a reduction of public consumption weakens the self-selection constraint of the high-ability households and reduces their incentive to hide their types. Since this effect is absent with full information, the condition $\partial(\mathrm{MRS}_{GX})/\partial L < 0$ generally implies a lower level of public consumption in second best than in first best.

3.2 The possibility of over-provision

The assumption $\partial(\mathrm{MRS}_{GX})/\partial L < 0$ means that the households are less interested in the provision of the public good if their labor supply increases. Whether this is true, however, is an empirical question and does not depend on distortions imposed

[14]Note that the separability assumption made in Corollary 2 implies another remarkable feature of second-best taxation: Optimal commodity taxes are zero if the single commodity X_i is extended to a bundle of private commodities (see Atkinson and Stiglitz (1976)). Hence, Corollary 2 is not restricted to the assumption of a composite private consumption commodity.

by income taxation. Hence, if the opposite is true, i.e. if the households' interest in the public good increases with labor supply, the second-best level of G may be higher than the first-best level even though the second-best tax system imposes a higher welfare burden on the taxpayers than lump-sum taxes. This can be seen by analyzing the case $\partial(-\mathrm{MRS}_{GL}/L)/\partial L > 0$ which - according to Corollary 1, Proposition 1, and Fact 2 - implies $\widehat{\mathrm{MRS}}^{2S}_{GX} < \mathrm{MRS}^{1S}_{GX}$, $\widehat{\mathrm{MRS}}^{2S}_{GY} < \mathrm{MRS}^{1S}_{GY}$, and $p > \sum \mathrm{MRS}^{S}_{GY} > \sum \mathrm{MRS}^{S}_{GX}$. In this situation, the social marginal benefit of the public good is below its marginal production cost irrespective of whether these values are measured in terms of effective labor or private consumption. Therefore, based on the intuition given above, one might expect a negative substitution effect, i.e. $G^Z < G^S$ in this case. The following lemma shows that this is indeed true as long as private consumption and the public good are Hicksian substitutes. The latter assumption is motivated by an argument similar to the one underlying Proposition 2: The transition from $(\{X_i^S, Y_i^S\}_{i=1}^2, G^S)$ to $(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z)$ does not only require that the (implicit) price of the public good increases with respect to both private commodities, but also that the relative price between private consumption and leisure decreases. Hence, the substitution effect could become positive if private consumption and the public good are Hicksian complements.

Lemma 1: *Assume that private consumption and the public good are Hicksian substitutes. Then* $\sum MRS^S_{GY} \leq p$ *implies* $G^S > G^Z$.

Proof: Consider the fictitious economy introduced in the proof of Proposition 2. Because of the assumption $\sum \mathrm{MRS}_{GY} \leq p$ we now have

$$N_1 \frac{q_G^{1S}}{q_L^{1S}} w_1 + N_2 \frac{q_G^{2S}}{q_L^{2S}} w_2 \leq p. \tag{15}$$

Without loss of generality, we may choose $q_L^{1Z} = q_L^{1S}$. Then equations (12), (13) and (15) imply $q_X^Z < q_X^S$, $q_L^{2Z} < q_L^{2S}$, and

$$N_1 w_1 (q_G^{1Z} - q_G^{1S}) + N_2 w_2 \left(q_G^{2Z} \frac{q_L^{1Z}}{q_L^{2Z}} - q_G^{2S} \frac{q_L^{1S}}{q_L^{2S}} \right) \geq 0. \tag{16}$$

Hence, at least one of the two terms parenthesized in (16) has to be positive. Assume first $q_G^{1Z} - q_G^{1S} \geq 0$: Since private consumption has been assumed to be a Hicksian substitute for G, we have $G^1(q_L^{1Z}, q_X^Z, q_G^{1Z}) < G^1(q_L^{1S}, q_X^S, q_G^{1S})$. Hence, $G^Z < G^S$. If the second term of (16) is positive instead, we get $(q_G^{2Z}/q_L^{2Z}) > (q_G^{2S}/q_L^{2S})$, and $(q_G^{2Z}/q_X^Z) > (q_G^{2S}/q_X^S)$, while the relative price q_L^2/q_X remains constant. Therefore, $G^2(q_L^{2Z}, q_X^Z, q_G^{2Z}) < G^2(q_L^{2S}, q_X^S, q_G^{2S})$ which again implies $G^Z < G^S$. ■

Taken together, Proposition 1 and Lemma 1 provide sufficient conditions for a negative substitution effect $G^Z - G^S$. However, this negative substitution effect could be cancelled out by a positive income effect provided that the public good is strictly normal. Therefore, a general result for the case $\sum \mathrm{MRS}^S_{GY} \leq p$ can only be achieved if the public good is assumed to be neutral or inferior. Although this does not seem to be a very plausible assumption, it should be noted that Wilson's (1991a) counterexample to Pigou's claim also relies on it. In order to make sure that such a counterexample can be constructed within the present framework as well, consider the case where the households' preferences imply the "Y-Samuelson Rule", i.e. the condition $\sum \mathrm{MRS}^S_{GY} = p$ to be satisfied in second best. These preferences have been analyzed by Tuomala (1990) for the model with a continuum of households and by Boadway and Keen (1993) for the two type economy. Boadway and Keen (1993) show that the Y-Samuelson Rule holds in second best if (and only if) the households' preferences can be represented by a utility function of the type $U(X, L, G) = U(X, L/\zeta(X, G))$. The following corollary demonstrates that these preferences may lead to over-provision of the public good in the second-best optimum.

Corollary 3: *Assume* $U(X_i, L_i, G) = X_i + [a/(a+1)](G/L_i)^{1-1/a}$*, where* $0 < a < 1/2$*. Then* $G^S > G^F$*.*

Proof: The following properties of the example can be easily derived: (i) The agent monotonicity condition is satisfied. (ii) $-\mathrm{MRS}_{GL} = L/G$. (iii) $\mathrm{MRS}_{GX} = (1/G)(G/L)^{1-1/a}$. (iv) The commodity X is normal, while G and leisure are neutral for any $X > 0$. (v) The commodities X and G are Hicksian substitutes.

The properties (ii) and (iii) imply $\partial(-\mathrm{MRS}_{GL}/L)/\partial L = 0$ and $\partial(\mathrm{MRS}_{GX}/L)/\partial L < 0$. Using Proposition 1, this implies $p = \sum MRS_{GY} > \sum MRS_{GX}$. Hence, all assumptions made in Lemma 1 are satisfied. We thus have $G^S > G^Z$.

Using the observation that the public good G is neutral (property (iv)), the second step of the proof of Proposition 2 can easily be modified in order to derive $G^Z = G^F$. Hence, the relationship $G^S > G^Z$ implies $G^S > G^F$. ■

So far, preferences which imply the X-Samuelson Rule or the Y-Samuelson Rule served as the most important benchmarks of the literature. Corollaries 1 and 3 reveal that these two benchmarks may lead to opposite results with respect to the comparison between the level of public expenditures in first best and second best. In both cases, however, the levels G^S and G^F do not coincide. Consequently, interpretations which refer to the irrelevance of distributional objectives for the design of public projects should be treated carefully with respect to the choice of the private reference commodity.

3.3 Indeterminacy of the substitution effect

Consider now the last of the three possible occurences, i.e. $\sum \mathrm{MRS}_{GY}^{S} > p > \sum \mathrm{MRS}_{GX}^{S}$. According to the intuition presented above the substitution effect $G^Z - G^S$ is ambiguous in this situation. This ambiguity holds at least for the most plausible case where both private consumption and leisure are Hicksian substitutes for G: In transition from $(\{X_i^S, Y_i^S\}_{i=1}^2, G^S)$ to $(\{X_i^Z, Y_i^Z\}_{i=1}^2, G^Z)$, the two (implicit) relative prices between G and the private commodities move in opposite directions. Hence, we have two countervailing effects on the quantity of the public good whose magnitude cannot generally be compared with each other. The net result with regard to the difference $G^S - G^Z$ depends on the relative degree of substitutability between G and the two private commodities. This can be shown by considering two possible benchmarks where one of the private commodities is a Hicksian independent for the public good.

Lemma 2: *Assume $\sum MRS_{GX}^{S} < p < \sum MRS_{GY}^{S}$. (a) If leisure and the public good are Hicksian independents, then $G^S > G^Z$. (b) If private consumption and the public good are Hicksian independents, then $G^S < G^Z$.*

Proof: Consider again the fictitious economy introduced in the proof of Proposition 2: Because of $\sum \mathrm{MRS}_{GX} < p < \sum \mathrm{MRS}_{GY}$ we have

$$(i) \quad N_1 \frac{q_G^{1S}}{q_X^S} + N_2 \frac{q_G^{2S}}{q_X^S} < p \quad \text{and} \quad (ii) \quad N_1 \frac{q_G^{1S}}{q_L^{1S}} w_1 + N_2 \frac{q_G^{2S}}{q_L^{2S}} w_2 > p. \tag{17}$$

(a) Without loss of generality, we may choose $q_X^Z = q_X^S$. Then equations (12), (13) and (17(i)) imply $q_L^{1Z} > q_L^{1S}$, $q_L^{2Z} = q_L^{2S}$, and $N_1 q_G^{1Z} + N_2 q_G^{2Z} > N_1 q_G^{1S} + N_2 q_G^{2S}$. Therefore, starting from the allocation (X^S, L^S, G^S) with the prices q^S and the income I^S the allocation (X^Z, L^Z, G^Z) can be achieved by (i) raising the price q_L^{1S} to q_L^{1Z}, (ii) raising the sum $N_1 q_G^{1S} + N_2 q_G^{2S}$ to $N_1 q_G^{1Z} + N_2 q_G^{2Z}$, and (iii) replacing I^{iS} with I^{iZ}. As long as leisure and G are Hicksian independents this leads to a negative (substitution) effect on G. Therefore, $G^Z < G^S$.

(b) In contrast to part (a), we may now choose $q_L^{1Z} = q_L^{1S}$. Then equations (12), (13) and (17(ii)) imply $q_X^Z < q_X^S$, $q_L^{2Z} < q_L^{2S}$, and

$$N_1 w_1 (q_G^{1Z} - q_G^{1S}) + N_2 w_2 \left(q_G^{2Z} \frac{q_1^Z}{q_2^Z} - q_G^{2S} \frac{q_1^S}{q_2^S} \right) < 0. \tag{18}$$

Assume first $q_G^{1Z} - q_G^{1S} < 0$: Since private consumption and the public good have been assumed to be Hicksian independents, we have $G^1(q_L^{1Z}, q_X^Z, q_G^{1Z}) > G^1(q_L^{1S}, q_X^S, q_G^{1S})$. Hence, $G^Z > G^S$. If instead the second term of (18) is negative, we get $(q_G^{2Z}/q_L^{2Z}) <$

(q_G^{2S}/q_L^{2S}) and $(q_G^{2Z}/q_X^Z) < (q_G^{2S}/q_X^S)$ while the relative price q_L^2/q_X remains constant. Therefore, $G^2(q_L^{2Z}, q_X^Z, q_G^{2Z}) > G^2(q_L^{2S}, q_X^S, q_G^{2S})$ which again implies $G^Z > G^S$. ■

Lemma 2 shows that the sign of the substitution effect $G^S - G^Z$ depends on whether G is more substitutive with respect to leisure or private consumption. Unfortunately, the techniques used in the proofs of this paper do not allow for more specific results. Note that this limitation also applies to the analysis of Wilson's (1991a) counterexample which has been discussed by Boadway and Keen (1993). In this example, preferences are represented by a utility function $U(X_i, L_i, G) = A(X_i, L_i) + B(G)$, where $A(X_i, L_i)$ is homogeneous of degree one in private consumption and leisure, and $B(G)$ is strictly concave. Wilson (1991a) has shown for this structure that the second-best level of public good provision exceeds the first-best level in a model with linear consumption taxes and a poll tax. Since $A(X_i, L_i)$ is homogeneous of degree one, it is also concave and implies a negative cross derivative $A_{XL} < 0$. It is easy to see that these features lead to the second-best situation $\sum \text{MRS}_{GY}^S > p > \sum \text{MRS}_{GX}^S$. Furthermore, the normality of X and leisure implies Hicksian substitutability of G with each of the private commodities, and strict concavity of the function $B(G)$ means that the public good is neutral. Hence, the substitution effect is ambiguous and the income effect is zero. Therefore, the results derived above leave open the question of whether Wilson's counterexample also applies to the case of nonlinear income taxation.[15]

4 Concluding remarks

In contrast to previous analyses where linear tax systems have been assumed, this paper examines the efficient level of public expenditures with explicit reference to those informational restrictions which prevent the government from employing lump-sum taxation. The results of this paper show that the conventional argument, according to which distortionary taxation implies a lower level of public expenditures than lump-sum taxation, is flawed if the government's informational constraints are taken seriously: First, it has been shown that the comparison between the first-best and the second-best level of public expenditures does not primarily depend on the distortions imposed by the tax system, but on how public expenditures affect the self-selection constraint of the high-ability households. Second, the analysis revealed that tax distortions do not only imply a change of the marginal rates of substitution,

[15]Note, however, that the neutrality of the public good is essential for Wilson's result. Hence, the example cannot be extended to more general utility functions which imply strict normality of all commodities.

but also of the utility levels attained in the optimum. Since the second-best utility levels are below the first-best levels, the distortionary tax system also induces income effects which may be far more important for the extent of public expenditures than the distortionary effects which are usually emphasized in the literature.

The results of this paper have been derived in the simplest model of income taxation with only two types of households and a composite consumption commodity. This framework most clearly reveals how the intuition developed from linear tax analysis might change if the agents' self-selection constraints become binding in the optimum. Note, however, that the basic results of this paper will not be affected if the model is extended to more than just two types of households. In this case, at least some of the self-selection constraints will bind in the optimum. Hence, Proposition 1 and Corollary 1 still apply.[16] Going through the proofs of Proposition 2, Lemma 1, and Lemma 2, it becomes clear that these results will also survive provided that the first-best optimum Pareto dominates the second-best optimum. Consequently, the basic intuition developed so far does not change by increasing the number of ability-types. However, it is not clear whether this conclusion also holds if the model is extended to more than a single private consumption commodity. This assumption implicitly neglects the possibility of differential commodity taxation. If the latter is allowed for, none of the basic results of income tax analysis (see e.g. Fact 1) remain valid. Furthermore, Seade's agent monotonicity condition no longer implies the single crossing property which is important for the results presented above.[17] Hence, as is the case with most work on income tax analysis, the issue of differential commodity taxation is left open here for further investigation.

[16]For a discussion of this issue, see Boadway and Keen (1993), p. 476.

[17]For a discussion of these issues see Edwards et al. (1994) and Nava et al. (1996).

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