Decentralizing Incentive Efficient Allocations of Economies with Adverse Selection

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Abstract

We study competitive economies with adverse selection and fully exclusive contractual relationships. We consider economies where agents are privately informed over the probability distribution of their endowments, and trade to insure against this uncertainty. As in Prescott-Townsend (1984), we model exclusivity by imposing the incentive compatibility constraints directly on the agents consumption possibility set. In this set-up, we identify the externality associated with the presence of adverse selection as a special form of consumption externality. We consider a structure of markets which allows to internalize such externality, for which we show that competitive equilibria exist and are incentive efficient. On the other hand, when this expanded set of markets required to internalize such externality does not exist, competitive equilibria are shown to be, typically, not incentive efficient, but to satisfy an appropriately de ned notion of third best efficiency. Appropriate versions of the second welfare theorem for these two market structures are also established.

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1. Introduction and Motivation

We study competitive economies with adverse selection and fully exclusive contractual relationships. We consider pure exchange economies where agents have private information regarding the probability distribution of their endowments; uncertainty is purely idiosyncratic.

Many different equilibrium concepts have been introduced to study adverse selection economies. The standard strategic analysis of such economies, due to Rothschild-Stiglitz (1976), considers Nash equilibria of a game in which insurance companies choose the contracts they issue, and the competitive aspect of the market is captured by the free entry of insurance companies. Such equilibrium concept does not perform too well: equilibria in pure strategies do not exist for robust examples (Rothschild-Stiglitz (1976)), while equilibria in mixed strategies exist (Dasgupta-Maskin (1986)) but, in this set-up, are of difficult interpretation. Even when equilibria in pure strategies do exist, it is not clear that the way the game is modelled is appropriate for such markets, since it does not allow for dynamic reactions to new contract offers (Wilson (1977) and Riley (1979)). Moreover, equilibria are not robust to minor perturbations of the extensive form of the game (Hellwig (1983)).

This paper studies instead Walrasian equilibrium concepts, where agents and insurance companies act as price takers in competitive markets for contracts. Contracts provide insurance to the agents against the realization of their individual uncertainty; since uncertainty is purely idiosyncratic, we can argue that agents, though not informationally small, are small as far as the level of their trades is concerned, so that their price-taking behavior is justi ed. Moreover, insurance rms are also price-taker and endowed with a constant return to scale technology. As a consequence, the problem of dynamic reactions to new contract offers does not arise since at equilibrium there a price is quoted for all possible contracts and rms make zero pro ts for any choice of contracts offered.

We assume that agents trades can be fully monitored, so that exclusive contractual relationships are possible (as in Rothschild and Stiglitz). In such situation, the agents private information is the only friction to the operation of markets.[1] We intend therefore to address the following questions. What is the structure of markets, and what are the properties of allocations attainable as competitive equilibria? And in particular, are competitive equilibria incentive efficient, i.e., do competitive markets allow, in this set-up, to attain efficient allocations, subject to the only constraint imposed by the agents private information?

[1] Exclusivity is clearly a strong assumption. It provides however an important benchmark as we argued, and is the case typically considered in contract theory. The properties of competitive equilibria when this condition is violated, or only non-exclusive contractual relationships are available, are investigated in Bisin-Gottardi (1999).

The possibility of implementing exclusive contracts can also be viewed as the possibility of implementing fully non-linear price schedules, where the price of each contract depends on the level of trades in the same contract as well as in all other markets. Evidently, with fully non-linear price schedules, incentive efficient allocations can be easily decentralized as competitive equilibria. However, a large set of other, inefficient equilibria also exists. On the face of this fact, a possible route is to consider indeed a model with fully non-linear price schedules (or where the dimension of possible contracts is expanded along an additional dimension, the quantity traded), introducing re nements to restrict admissible equilibrium prices, and in particular the prices of contracts not traded at equilibrium. This route has been explored by Gale (1992), (1999) and Dubey, Geanakoplos, Shubik (1995) for economies similar to the one considered here.

On the other hand, we follow here Prescott-Townsend (1984) in modelling exclusive contractual relationships by restricting the consumption possibility set to include only incentive compatible allocations (prices are then linear over this restricted domain): allocations which are not incentive compatible are excluded from the consumption possibility set. Contractual relationships are then exclusive in the sense that any incentive compatible trade is available to the agents, and cannot be broken by trading multiple contracts. It is important to notice that it is always possible to replace the restriction to incentive compatible allocations by an appropriately de ned non-linear price schedule for contracts (indeed ensuring that incentive compatibility is satis ed). Thus the structure of markets and prices considered by Prescott and Townsend can also be viewed as a particular form of re nement on the form of admissible non-linear price schedules for contracts, meant to capture the fact that the only role of the non-linearity is to ensure incentive compatibility.

Prescott-Townsend (1984) study both economies with moral hazard and with adverse selection. While for moral hazard economies they prove existence and constrained versions of the rst and second theorems of welfare economics, their method does not succeed in analyzing adverse selection economies. They conclude (p. 7) that a difficulty emerges, however, in attempting to secure standard existence and optimality theorems for all economies consistent with the general structure , which encompasses both economies moral hazard and adverse selection.

In this paper we observe rst that, with adverse selection, when exclusive contracts are available, an externality arises and identify it as a special form of consumption externality. On this basis, we consider an (expanded) structure of markets which allows to internalize this externality, and propose an equilibrium notion which successfully replicates, for adverse selection economies, the results that Prescott-Townsend (1984),(1984-bis) obtain for moral hazard economies: existence of an equilibrium and (incentive) constrained versions of the rst and second theorem of welfare economics. We call such equilibria ALPT, for Arrow-Lindahl-Prescott-Townsend. ALPT equilibria require the existence of markets for property rights, which allow the internalization of the externality in the agents choice problem induced by adverse selection, in the same way as

Arrow-Lindahl equilibria do for standard consumption externalities and public goods.

In economies with asymmetric information, the whole set of trades each agent makes are related; it is so desirable - in fact needed for incentive efficiency - that they be part of a single contract (i.e., that contracts are exclusive). In addition, with adverse selection, the contracts available to each type of agent are not independent of the contracts chosen by other types. Thus, incentive efficiency requires in this case that also the contracts traded by the different types of agents be part of a single bundle of contracts. Beyond the abstraction, the structure of markets in ALPT equilibria is indeed meant to capture such further bundling of contracts, and hence to allow for the possibility that some degree of cross-subsidization among contracts takes place at equilibrium.

Having identi ed adverse selection with a form of consumption externality, we also study the equilibria of adverse selection economies when the markets required to internalize such externality are not present (we call such equilibria, EPT, for Prescott-Townsend with externalities). We show that EPT equilibria exist and characterize their properties. Evidently, EPT are not, typically, incentive efficient. However, EPT equilibria satisfy an appropriately de ned notion of third best efficiency (for which a version of the second welfare theorem also holds). In this case, the contract traded by each type is treated as a separate contract, independent of the other contracts, and no cross-subsidization takes place at equilibrium. For the economies considered in this paper, the set of EPT equilibria coincides with the set of competitive equilibria with non-linear prices, when an appropriate re nement is imposed on admissible price schedules.

The analysis will be developed for a simple insurance economy with adverse selection as the one considered by Rothschild and Stiglitz (1976). This constitutes, as we argued, a very important test case for any equilibrium notion for adverse selection economies. In addition, by exploiting the simple structure of the economy, we will be able to clearly illustrate the features and to provide a complete characterization of the various notions of competitive equilibria which will be considered. The extension of the analysis to general adverse selection economies poses various technical problems which we intend to address in another paper.

The paper is organized as follows. In the next two sections, the structure of the economy considered is presented, and incentive efficient allocations are characterized. In the following section, rst EPT equilibria and then ALPT equilibria are presented, their existence established and their efficiency properties characterized.

2. The Economy

Consider a competitive economy with adverse selection. There is a continuum of agents of two different types, b and g. Let ξ^b denote the fraction of agents of type b and ξ^g the fraction of agents of type g in the population.

There is a single consumption good. Uncertainty enters the economy via the level of the agents endowment and is purely idiosyncratic. There are two possible states, H, L, for every individual, and his endowment when H (resp. L) is realized is ω_H (resp. ω_L). Let π_s^i be the probability that individual state s, $s \in S \equiv \{H, L\}$, is realized for an agent of type i, $i \in \{g, b\}$ (it will sometimes be convenient to simply write π^i for π_H^i and $1 - \pi^i$ for π_L^i). These random variables are independently distributed across all agents (and identically distributed across agents of the same type).

With no loss of generality, let $\omega_L < \omega_H$ (state H is then the good state), and $0 < \pi^b < \pi^g < 1$ (type b is the high-risk type, and type g the low-risk).

The preferences of each agent are described by von Neumann-Morgernstern utility function, with (type independent) utility index $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, de ned over consumption in each (idiosyncratic) state $s \in S$. Let then $U^i(x^i) \equiv \sum_{s \in S} \pi_s^i u(x_s^i)$, for $i \in \{g, b\}$.

Each agent is privately informed about his type.

We assume:

Assumption 1. *Endowments are strictly positive, for all agents:* $\omega_L, \omega_H > 0$. *Preferences are strictly monotonic, strictly concave, twice continuously differentiable, and* $\lim_{x \to 0} u'(x) = \infty$.

Under the above assumptions on agents preferences, the *single crossing property* holds: the indifference curves of the two agents types intersect at most in one point. While this property is exploited in some parts of the following analysis, for many results the following, weaker, property, requiring that preferences are sufficiently diverse across agents types, is sufficient: for all $x \in \mathbb{R}_+^2$ the matrix:

$$\begin{pmatrix} DU^g(x)^T \\ DU^b(x)^T \end{pmatrix} \tag{2.1}$$

where $DU^i(x)$ denotes the gradient of $U^i(x)$, has full row rank.

Note that the economy is essentially the same as the insurance economy with adverse selection considered by Rothschild and Stiglitz (1976).

3. Incentive Efficient Allocations

Let $x^i \equiv (x_H^i, x_L^i)$, $i \in (g, b)$.

A (symmetric) *feasible allocation* is a pair $\{x^b, x^g\}$ which satis es the following resource feasibility constraint:

$$\sum_{s \in S} \left[\xi^g \pi_s^g (x_s^g - \omega_s) + \xi^b \pi_s^b (x_s^b - \omega_s) \right] \leq 0 \tag{3.1}$$

where the purely idiosyncratic nature of the uncertainty and the Law of Large Numbers have been used to take the sum of the excess demand for the commodity contingent on each individual state, weighted by its probability.

Since agents types are only privately observable, additional constraints, incentive compatibility, should also be imposed:

$$-\sum_{s\in S}\pi_s^g u\left(x_s^g\right)+\sum_{s\in S}\pi_s^g u\left(x_s^b\right)\leq 0, \tag{3.2}$$

$$-\sum_{s\in S}\pi_s^b u\left(x_s^b\right)+\sum_{s\in S}\pi_s^b u\left(x_s^g\right)\leq 0 \tag{3.3}$$

Remark 1. *As shown by Prescott and Townsend (1984) (see, also, Cole (1986)), in the presence of asymmetric information it may be desirable to expand the consumption space so as to allow for random allocations of contingent commodities, or lotteries over consumption bundles. This is not the case for our simple economy, as allocations involving nondegenerate lotteries are always suboptimal (see the proof of Proposition 1 in the Appendix and, also, the proof of Lemma 3). To keep the notation simpler, de nitions are then stated for the case of non random allocations..*

De nition 1. *An allocation $\{x^b, x^g\}$ is incentive efficient if it is feasible, incentive compatible (i.e., satis es ((3.1), (3.2), (3.3)) and there does not exist another allocation $\{\hat{x}^b, \hat{x}^g\}$, also feasible and incentive compatible, such that: $U^b\left(\hat{x}^b\right) \geqslant U^b\left(x^b\right)$, $U^g\left(\hat{x}^g\right) \geqslant U^g\left(\hat{x}^g\right)$, with at least one inequality being strict.*

For the simple adverse selection economy under consideration, we can provide a complete characterization of the set of incentive efficient allocations:

Proposition 1. *Under Assumption 1, all (symmetric) incentive efficient allocations are of one of the following forms:*
i. $x^b = x^g = (\hat{x}, \hat{x})$, *for* $\hat{x}$ *satisfying* $\sum_{s\in\{H,L\}}\left[\xi^g\pi_s^g(\hat{x}-\omega_s)+\xi^b\pi_s^b(\hat{x}-\omega_s)\right]=0$;
ii. $x^b = (\tilde{x}, \tilde{x}), x_L^g < x_H^g$, where $\tilde{x} < \hat{x}$, x^g satisfy

$$\begin{aligned}\sum_{s\in\{H,L\}}\left[\xi^g\pi_s^g(x_s^g-\omega_s)+\xi^b\pi_s^b(\tilde{x}-\omega_s)\right] &= 0\\ -u\left(\tilde{x}\right)+\sum_{s\in\{H,L\}}\pi_s^b u\left(x_s^g\right) &= 0\end{aligned}$$

iii. $x^g = (x^*, x^*), x_L^b > x_H^b$, where $x^* > \hat{x}$, x^b satisfy

$$\begin{aligned}\sum_{s\in\{H,L\}}\left[\xi^g\pi_s^g(x^*-\omega_s)+\xi^b\pi_s^b(x_s^b-\omega_s)\right] &= 0\\ -u\left(x^*\right)+\sum_{s\in\{H,L\}}\pi_s^g u\left(x_s^b\right) &= 0\end{aligned}$$

Essentially the same result appears already in Prescott and Townsend (1984)[2]. We present, for completeness, a slightly different proof in the Appendix.

Remark 2. *The above result shows that at an incentive efficient allocation at least one of the two agents types is fully insured (has a deterministic consumption bundle). In particular, there is an incentive efficient allocation where both types are fully insured (case (i) above); this allocation is also Pareto efficient, or the incentive constraints do not bind. In case (ii), the agents of type b are fully insured, while the type g agents are partially insured (only the second of the two incentive constraints, (3.3), is binding). On the other hand, in case (iii), agents of type g are fully insured, while the b agents are overinsured (the rst incentive constraint only, (3.2), binds).*
Even though, as we already argued, lotteries over consumption bundles are allowed, the characterization obtained in Proposition 1 reveals that, in the economy under consideration, they are never optimal.

4. Competitive Equilibria

We consider in this paper the case where agents trades are fully observable, so that exclusive contracts, or equivalently fully non-linear price schedules, can be implemented. In the simple economy considered, contracts provide insurance against the realization of the agents individual uncertainty; the set of possible contracts is then the set ($\mathbb{R}^2_+$) of consumption levels in each of the two individual states. Following Prescott-Townsend (1984), we will model exclusivity of contracts by imposing incentive compatibility directly as a constraint on the set of admissible contracts each agent can trade. Evidently, with private information all contracts traded in the economy have to be incentive compatible; in this sense, to exclude contracts which fail to be incentive compatible is not a real restriction. On the other hand, the fact that any point in the set of incentive compatible allocations constitutes a contract the agent can trade implicitly requires an exclusivity condition, to ensure that the agent is unable to engage in additional trades which would lead him to a possible violation of incentive compatibility.

A budget set with linear prices de ned over such restricted consumption set is equivalent, as already argued in the Introduction, to a budget set with (appropriately chosen) nonlinear prices de ned over the (unrestricted) consumption set. Thus the restriction to incentive compatible allocations can be viewed as a way of modelling agents facing non-linear pricing schedules, when the only role of the non-linearity is to ensure incentive compatibility.

When private information is of the moral hazard type (e.g. hidden action), incentive compatibility concerns different choices (unobservable effort levels) the agent can

[2] See also Jerez (1999) for the analysis of incentive efficient allocations of a more general class of economies.

make, and the choice problem of each agent can be analyzed separately from the one of other agents. Thus, there is no ambiguity in this case: to impose an incentive compatibility restriction simply amounts to excluding the pairs of consumption bundles and effort where the effort level is not the one agents would choose at that level of consumption. Equivalently, it corresponds to having prices which vary non-linearly with the level of consumption to reflect the different choices of effort made by the agent at each level of consumption (see Kocherlakota (1998), and Magill-Quinzii (1998)). As shown by Prescott and Townsend (1984), competitive equilibria are incentive efficient in this case.[3] It is also possible to show that the same set of competitive equilibrium allocations obtained by Prescott and Townsend obtains with non-linear prices, when essentially any type of refinement on admissible prices is imposed.

On the other hand, with adverse selection, since contracts' payoff depends on the agents' type, which is unobservable, the contract traded by each type cannot be considered separately from the ones traded by the other types. This has some important consequences. The same contract, when entered by different types is, in effect, a different contract; however, all such contracts get aggregated together in the market clearing conditions and the total payoff depends on the composition of trades by the different types. An externality arises so in the feasibility conditions. In addition, as we see from (3.2),(3.3), the incentive compatibility constraints relate now the consumption levels of different types, thus an externality arises here too.

The main focus of Prescott and Townsend (1984) is on the first form of externality. They argue in fact that the source of the difficulties with adverse selection is that "agents with characteristics which are distinct and privately observed at the time of initial trading enter the economy-wide resource constraints in a heterogeneous way" (p. 7). We will show here that, if the incentive compatibility constraints given in (3.2),(3.3) is imposed on the consumption set,[4] contracts traded by different types can be treated as separate contracts, traded at (possibly) different prices. Thus the feasibility problem described above no longer arises,[5] and we are only left with the externality induced by the presence of the incentive compatibility constraints in the agents' consumption set.

These constraints imply that, when an agent, say type b, increases his consumption, this has an effect on the possible values of consumption of agents of type g, for whom it may now be easier (or more difficult, according to which incentive constraint is binding), to satisfy these constraints. Thus the admissible levels of consumption for one type

[3] See, however, Bennardo-Chiappori (1998).

[4] This possibility is briefly discussed at the end of the (unpublished, extended version of) Prescott and Townsend (1984); each type of agent is then asked to choose the consumption allocation for all types, so that a conflict typically arises, and such equilibria fail to exist.

[5] On the other hand, when exclusive contracts are not available, so that we are unable to separate agents' of different types on the basis of the different level of their trades, the feasibility problem described above cannot be avoided. Bisin and Gottardi (1999) have shown that, indeed because of this problem, with linear prices competitive equilibria may fail to exist.

depend on the level of consumption of the other type; there is so an externality in the speci cation of each agent s consumption possibility set.

Once the presence and the nature of the externality in adverse selection economies with exclusive contracts is clearly identi ed, the model can be analyzed as other competitive models with externalities in consumption.

We rst consider the notion of *EPT equilibrium,* where each agent takes as given the level of consumption of the other types, i.e., the externality is not internalized.

Next, the notion of *ALPT equilibrium* is introduced. In an ALPT equilibrium markets for consumption rights, for both the agent s own consumption and the consumption of agents of the other type, are present. These markets allow to internalize the externality the consumption of each agent imposes, via the incentive compatibility constraints, on the consumption level of the other type.

4.1. EPT Equilibria

There are separate contingent markets where each type of agent can trade at linear prices, possibly different across types, the commodity contingent on the realization of the agent s individual uncertainty; q_s^i is then the unit price at which each agent of type $i \in \{g, b\}$ can trade the commodity for delivery in his individual state $s \in \{H, L\}$. The set of admissible trades of each type is restricted by the incentive compatibility constraints, where his consumption level appears together with the consumption level chosen by the other type, which is taken as given.

We are allowing prices to depend on the agents type, even though this is only privately observed. As we will see, the presence of incentive compatibility in the consumption set ensures that the differentiation of prices according to the agents type is incentive compatible.

The choice problem of each agent of type g has then the following form. The agent chooses a consumption bundle $x^g \equiv (x_H^g, x_L^g) \in \mathbb{R}_+^2$, specifying his level of consumption in the agent s two possible individual states, subject to the budget constraint and the two incentive compatibility constraints:

$$\max_{x^g} \sum_{s \in S} \pi_s^i u\left(x_s^g\right) \qquad (P_{EPT}^g)$$

s.t.

$$\sum_{s \in S} q_s^g (x_s^g - \omega_s) \leq 0$$

$$\begin{aligned} -\sum_{s \in S} \pi_s^g u\left(x_s^g\right) + \sum_{s \in S} \pi_s^g u\left(x_s^b\right) &\leq 0, \\ -\sum_{s \in S} \pi_s^b u\left(x_s^b\right) + \sum_{s \in S} \pi_s^b u\left(x_s^g\right) &\leq 0 \end{aligned}$$

The incentive constraints require that agent g prefers (at least weakly) his own bundle to the one chosen by b and similarly that b prefers his bundle to the one g is choosing. for himself. These constraints (in particular the second one) imply that g cannot increase his level of consumption beyond a certain level, without a breakdown of the separation between the market where he and the other type can trade. The admissible choices of consumption of type g are then affected by the consumption level of agents of the other type, $x^b \equiv (x_H^b, x_L^b)$, and this is taken as given here.

The level of consumption of agents of type b has no direct effect on the agent g s utility. It only enters the agent s problem via its effect on the incentive compatibility constraints. Thus the model is, formally, a model with an externality in the consumption set. Thus we have an externality in the consumption space which is not internalized.

The choice problem of agents of type b, (P_{EPT}^b) is symmetrically de ned, now taking x^g as given.

In addition to consumers we introduce rms. Firms pool contracts of the same type, i.e., construct aggregates of the commodity contingent on the same individual state and type. The Law of Large Numbers provides then, in the economy under consideration (as already noticed), a mechanism - or a technology - for transforming aggregates of the commodity contingent on different individual states and types. Thus rms are characterized by the following constant returns to scale technology:

$$Y = \{y \in \mathbb{R}^4 : \sum_{i \in \{g,b\}} \sum_{s \in S} \pi_s^i y_s^i \leq 0\}$$

where $y \equiv [y_s^i]_{s \in S}^{i \in \{g,b\}}$.

The rms problem is then the choice of a vector y of the commodity contingent on the agents types and individual states y, lying in the set Y (i.e., which can be generated by pooling contracts of the same type and transforming them according to the Law of Large Numbers) so as to maximize pro ts:

$$\max_{y \in Y} \sum_{i \in \{g,b\}} \sum_{s \in S} q_s^i y_s^i$$

taking prices $q \equiv [q_s^i]_{s \in S}^{i \in \{g,b\}}$ as given.

Remark 3. *The rms choice of y can also be viewed as the choice of the contract to offer the agents (like insurance rms in Rothschild and Stiglitz (1976), but here rms are price-takers). Even though the set of possible contracts considered by rms (Y) is very large and contains all contracts that budget balance, since at equilibrium contracts offered have to equal demand, and agents demand is subject to incentive compatibility, this ensures that only incentive compatible contracts are offered at equilibrium. Thus we can also interpret the presence of the incentive compatibility constraints in the agents* *consumption set, rather than a constraint self-imposed by agents, as a constraint on the set of contracts rms would ever consider offering (see Jerez (1999) for a discussion of the consequences of imposing the incentive constraints directly on the rms - rather than on consumers).*

Let $V^g(q^g, x^b)$ denote the utility attained by type g at a solution of problem (P^g_{EPT}), for given $x^b, q^g \equiv [q^g_s]_{s\in S}$. Let then $V^{g,b}(q^b, x^g)$ be the maximal utility an agent of type g could reach by trading at the prices $q^b \equiv [q^b_s]_{s\in S}$, offered to agents of type b, i.e., obtained at a solution of the problem

$$\max_{\{x^{g,b}_s\}_{s\in\{H,L\}}} \sum_{s\in S} \pi^g_s u\left(x^{g,b}_s\right)$$

s. t.

$$\sum_{s\in S} q^b_s (x^{g,b}_s - \omega_s) \leq 0$$

$$\begin{aligned} -\sum_{s\in S} \pi^g_s u\left(x^g_s\right) + \sum_{s\in S} \pi^g_s u\left(x^{g,b}_s\right) &\leq 0, \\ -\sum_{s\in S} \pi^b_s u\left(x^{g,b}_s\right) + \sum_{s\in S} \pi^b_s u\left(x^g_s\right) &\leq 0 \end{aligned}$$

taking x^g as given.

By a perfectly symmetric argument we can de ne $V^b(q^b, x^b)$, $V^{b,g}(q^g, x^b)$.

De nition 2. *An EPT is given by an allocation $\{x^b, x^g\}$, a production vector y, and a price vector q such that:*
(i) x^i solves the agent s optimization problem (P^i_{EPT}), at (q^i, x^j), for $i \neq j; i, j \in \{g, b\}$;
(ii) y solves the rms pro t maximization problem (P^f_{EPT}), at q;
(iii) markets clear:

$$\xi^i (x^i_s - \omega_s) \leq y^i_s,\ i \in \{g, b\}, s \in S \tag{4.1}$$

(iv) the price differentiation across types is incentive compatible:

$$V^i(q^i, x^j) \geqslant V^{i,j}(q^j, x^i),\ \text{for } i \neq j; i, j \in \{g, b\}$$

Condition (iv) requires that, faced with prices q^g and q^b, agents of type g prefer to trade at prices q^g and type b prefers to trade at q^b. The next Lemma shows that such condition is in fact redundant (given, in particular, the presence of the incentive compatibility constraints in the speci cation of the agents admissible trades in (P^i_{EPT})).

Lemma 1. *If $\{x^g, x^b\}, q$ satisfy condition (i) of De nition 2, then condition (iv) of Definition 2 is also satis ed.*

Proof. Take a solution x^g of (P^g_{EPT}) at x^b, q^g. Then $V^g(q^g, x^b) = \sum_{s\in S} \pi^g_s u(x^g_s)$ and $V^{g,b}(q^b, x^g)$ is obtained by maximizing the utility of type g at the price q^b subject to, among others, g s own incentive constraint:

$$\sum_{s\in S} \pi^g_s u\left(x^{g,b}_s\right) \leq \sum_{s\in S} \pi^g_s u(x^g_s)$$

Hence $V^g \geq V^{g,b}$. A symmetric argument holds for b ■

The following result can then be immediately derived from the rst order conditions of the rms choice problem:

Lemma 2. *At an EPT equilibrium, prices of contingent commodities are fair :*

$$q^i_s = \pi^i_s, \ i \in \{g, b\}, s \in S \tag{4.2}$$

On this basis, we are able to completely characterize the EPT equilibria of the economy under consideration: the only EPT equilibrium of the economy is the Rothschild and Stiglitz separating candidate equilibrium, where type b is fully insured, at fair prices.

Proposition 2. *Under Assumption 1 there exists a unique EPT competitive equilibrium, given by a price vector q satisfying (4.2), a production plan y satisfying (4.1), and a consumption allocation $\{x^g, x^b\}$ such that:*
i. $x^b_L = x^b_H = (1-\pi^b)\omega_L + \pi^b\omega_H := x$;
ii. x^g *is obtained as a solution of* (P^g_{EPT}) *at* x^b, q.

Proof. By Lemma 2, $q^i_s = \pi^i_s$ for all i, s. At these prices, the rms optimal choice is any $y \in Y$; thus y can be picked so as to satisfy (4.1). It can be readily checked that x^b as speci ed above is a solution of (P^b_{EPT}) at x^g, q. Since, by Lemma 1, condition (iv) of De nition 2 is also satis ed, we obtain that $\{x^g, x^b, y, q\}$ indeed constitutes an EPT equilibrium.

At such equilibrium the incentive constraint of type b, (3.3), holds as equality, while the other incentive constraint, (3.2), is not binding. Also, it is immediate to see there are no other EPT equilibrium allocations in the subset of the commodity space where the incentive constraint of type g, (3.2), is not binding.

We will show that no EPT equilibrium allocation exists, with fair prices, in which the incentive constraint of the agents of type b is not binding. In this case in fact, the solution of (P^g_{EPT}) is characterized by

$$x^g_L = x^g_H = (1-\pi^g)\omega_L + \pi^g\omega_H,$$

but then there can be no vector x^b satisfying both the budget constraint and the incentive compatibility for the agents of type b, (3.3).

Finally, to prove the uniqueness of the equilibrium, it remains to show that no EPT equilibrium exists in the subset of the commodity space where both incentive constraints, (3.3) and (3.2), hold as equality. Since, for the economy under consideration, as we noticed, the single crossing property holds, the two incentive constraints can both be binding only if $x^b = x^g = x$. But then x must satisfy the budget equation both at prices π^b and at prices π^g, and at least one of them - as long as $x \neq \omega$ - as inequality. Suppose it is the one of type g, $\sum_{s \in S} \pi^g_s (x_s - \omega_s) < 0$; then - it can be readily checked - there exists another bundle $\tilde{x}$, which is also budget feasible at π^g, satis es incentive compatibility, and is strictly preferred by g. So x cannot be an EPT equilibrium allocation. A similar argument also holds when $x = \omega$. ■

Remark 4. *To gain some further intuition on the reasons why the Rothschild-Stiglitz separating candidate equilibrium is the only EPT equilibrium, let us consider the other candidate equilibrium in Rothschild and Stiglitz, the pooling allocation (described at point i. of Proposition 1). Why can t this ever be an EPT equilibrium? The formal argument, given in the proof shows that at such allocation, if prices are fair, agents of type g would bene t by modifying their choice; moreover, in this case the allocation violates b s budget equation. On the other hand, if prices were not fair, and in particular if $q^g = q^b$ so that the pooling allocation can be feasible for both types, rms would have a pro table deviation. Prices are different from probabilities so that if, for instance $q^g_L < \pi^g$, $q^b_L > \pi^b$, rms could achieve a (unboundedly large) positive pro t by buying commodity (g, L) and selling (b, L). Equivalently we can say that rms can make positive pro ts by increasing the number of contracts they would like to sell to the g types, and reducing the contracts sold to the b types (the presence of the incentive compatibility constraints in the agents budget set ensures that this is always possible), or to break the pooling by introducing some separation, as indeed was the case in the argument proposed by Rothschild and Stiglitz.*

We know from Rothschild-Stiglitz (1976) (see also Proposition 1) that, for an open set of economies, the separating candidate equilibrium is not incentive efficient. The characterization of EPT equilibria obtained in Proposition 2 then also reveals that the rst welfare theorem does not hold, that EPT equilibria may not be incentive efficient. This should not come as a surprise, given the fact that there is an externality which is not internalized by the structure of markets considered here.

It is useful however to examine more closely what is the precise source of the inefficiency. As shown by Lemma 2, at an EPT equilibrium the prices of contracts traded by each type are always fair. Thus at equilibrium there is never cross-subsidization across types: each contract traded by one type satis es a separate zero pro t condition. We show next that not only such lack of cross-subsidization is clearly responsible for the possible inefficiency of EPT equilibria, but, in the economy considered here, this is in fact the only source of inefficiency: EPT equilibrium allocations are efficient within the restricted subset of allocations which are incentive efficient and satisfy an additional no cross-subsidization condition across types. Thus the following third best version of the rst welfare theorem holds:

Proposition 3. *Under Assumption 1, all EPT equilibrium allocations are efficient within the restricted set of feasible allocations which are incentive compatible and, in addition, satisfy the condition*

$$\sum_{s\in S} \pi_s^i(x_s^i - \omega_s) = 0, \qquad i \in \{g, b\} \tag{4.3}$$

Proof. If, at a solution of the problem of maximizing a weighted average of the agents utility subject to (3.3), (3.2), and (4.3) both incentive compatibility constraints hold as equalities, under the assumptions made on agents preferences (in particular, by the single crossing property), we must have $x^g = x^b$. But then (4.3) implies $x^g = x^b = \omega$.

On the other hand, if only one of the two incentive constraints is binding, say the one for type b ((3.3)), then the optimal x^b is simply obtained by maximizing $U^b(x^b)$ over (4.3); thus it will always be at the full insurance point $x_H^b = x_L^b$ on (4.3). The level of x^g is then determined by maximizing $U^g(x^g)$ subject to (4.3) and (3.3), taking x^b as given at the full insurance level determined before. It is immediate to see that the pair (x^b, x^g) we obtain is the separating candidate equilibrium of Rothschild and Stiglitz. If we apply a symmetric argument when (3.2) is the only constraint binding, we nd that no solution exists in this case (when x^g is at the full insurance level on (4.3), no value exists for x^b which also satis es (4.3) and (3.2)). The result then follows by observing that the allocation at the separating candidate equilibrium is always preferred by both agents to autarchy.. ■

On the other hand, a second welfare theorem result holds for the present structure of markets: any incentive efficient consumption allocation can be decentralized as an EPT equilibrium with transfers (possibly dependent on the state but not the agents type).

Proposition 4. *Under Assumption 1, for any incentive efficient consumption allocation* (x^b, x^g) *there exists a set of transfers* (t_H, t_L) *which are feasible, i.e.,* $\sum_{s\in S}\left[\xi^g\pi_s^g t_s + \xi^b\pi_s^b t_s\right] \leq 0$, *and such that* (x^b, x^g) *is an EPT equilibrium allocation for the economy under consideration when each agent receives a transfer* (t_H, t_L).

Proof. Let (x^b, x^g) be an arbitrary incentive efficient allocation. By Lemma 2, at an EPT equilibrium, prices of contingent commodities are necessarily fair . If we consider then the budget equations going through x^b and x^g at prices, respectively, π^b and π^g, they will intersect at a single point; call it $\omega' := (\omega'_L, \omega'_H)$. Since (x^b, x^g) satis es the resource feasibility condition (3.1) and $\sum_s \pi^i_s(x^i_s - \omega'_s) = 0$, $i \in \{g, b\}$, ω' is also feasible:

$$\sum_{s\in S} \left[\xi^g \pi^g_s \omega'_s + \xi^b \pi^b_s \omega'_s\right] \leq \sum_{s\in S} \left[\xi^g \pi^g_s \omega_s + \xi^b \pi^b_s \omega_s\right] \tag{4.4}$$

Letting $t_s = \omega'_s - \omega_s$, (4.4) also implies that the transfers (t_H, t_L) are feasible.

We show next that (x^b, x^g) is the (unique in fact) EPT equilibrium consumption allocation of the economy with endowments $\omega' := (\omega'_L, \omega'_H)$. Suppose not, i.e., there exists another consumption bundle, say $\hat{x}^g$ for agent g, which also satis es the incentive compatibility constraints (at x^b), is budget feasible $(\sum_s(\hat{x}^g_s - \omega') \leq 0)$, and is strictly preferred to x^g by type g. But then $\sum_{s\in S} \left[\xi^g \pi^g_s \hat{x}^g_s + \xi^b \pi^b_s x^b_s\right] \leq \sum_{s\in S} \left[\xi^g \pi^g_s \omega'_s + \xi^b \pi^b_s \omega'_s\right]$; thus, by (4.4), $(\hat{x}^g, x^b)$ is also feasible, incentive compatible and it Pareto dominates (x^g, x^b), a contradiction. ■

Remark 5. *It is interesting to compare EPT equilibria with the competitive equilibria we obtain if incentive compatibility is not imposed on the consumption set but prices of contracts are allowed to be arbitrary non-linear maps. In this case, the separating candidate equilibrium of Rothschild and Stiglitz is the only equilibrium which survives the imposition of a re nement on admissible equilibrium price schedules, in the spirit of Kohlberg and Mertens (1986) stability criterion, as in Gale (1992) and Dubey-Geanakoplos-Shubik (1995). Thus, in the set-up under consideration, the separating candidate equilibrium is a robust prediction of different Walrasian equilibrium concepts. It is interesting to notice that this is an equilibrium also when the strategic analysis of Rothschild-Stiglitz (1976) found non-existence.*

4.2. ALPT Equilibria

In the de nition of EPT equilibria, the consumption set of each agent is restricted by the set of incentive compatibility constraints which relate the level of his consumption to the consumption level of the agents of the other type. This fact generates, as we noticed, an externality in consumption. To internalize such externality, we introduce here markets for rights to the other agents consumption, as in the model proposed by Arrow (1969) and Lindahl (1919) for general economies with externalities and public goods. We will then refer to competitive equilibria as Arrow-Lindahl - Prescott-Townsend (ALPT) equilibria.

The commodity space of every agent is then expanded so as to have complete markets in property rights. Each agent, say agent b has access to markets for property rights in his own consumption and in consumption of the other type, g. He consumes the rights in his consumption only; these rights can be directly transformed into consumption goods and enter then directly the agent s utility. However, to be able to consume such rights agent b must also hold the appropriate amount of rights for g s consumption, so as to ensure that incentive compatibility is satisfied; the rights for g enter then only indirectly b's utility. Similarly, agents of type g have access to markets for rights in their own and the other agents' consumption, and their allocation is also constrained by incentive compatibility.

Note that in such formulation of the commodity space, each agent's consumption is independent of the other agents' choices, since incentive compatibility relates the choice by the same agent of rights over his own and the other type's consumption. As a consequence an externality in consumption no longer arises here. Let $x^j_{i,s}$ denote agents' i allocation of rights to consumption of agents j, in state s, for $i,\ j \in \{g, b\}$, $s \in S$. The index appearing as a subscript then indicates the type of the agent who is buying the right, while the index appearing as a superscript indicates the type of agent who is the 'object' of the choice (i.e. the agent for whom the rights to consumption are purchased). The vector $x_i = \{x^j_{i,s}\}^{j\in\{g,b\}}_{s\in S}$ describes then the amount of rights purchased by agent of type i, where $x^i_{i,s}$ denote rights to the agent's own consumption in state s, while $x^j_{i,s}$ are rights to the consumption of agents of other types $j \neq i$.

As for EPT equilibria, good and bad types trade in separate markets; prices (here for consumption rights as well as contingent commodities) may depend then on the type of the agent. At equilibrium we will show that the incentive compatibility of such price differentiation is satisfied.

Agent g's problem has the following form (the definition of b's problem, (P^b_{ALPT}), is perfectly symmetric). He chooses a bundle $x_g = \{x^b_{g,L}, x^b_{g,H}, x^g_{g,L}, x^g_{g,H}\} \in \mathbb{R}^4_+$ of consumption rights, subject to the budget constraint and the two incentive compatibility constraints:

$$\max_{x_g} \sum_{s\in S} \pi^g_s u\left(x^g_{g,s}\right) \qquad (P^g_{ALPT})$$

s.t.

$$\sum_{s\in S} \left(p^b_{g,s} x^b_{g,s} + p^g_{g,s} x^g_{g,s}\right) \leq \sum_{s\in S} q^g_s \omega_s$$

$$-\sum_{s\in S} \pi^g_s u\left(x^g_{g,s}\right) + \sum_{s\in S} \pi^g_s u\left(x^b_{g,s}\right) \leq 0 \qquad (4.5)$$

$$-\sum_{s\in S} \pi^b_s u\left(x^b_{g,s}\right) + \sum_{s\in S} \pi^b_s u\left(x^g_{g,s}\right) \leq 0 \qquad (4.6)$$

In the budget constraint, the endowment of contingent commodities of agents of type g is evaluated at the prices $\{q^g_s\}_{s\in S} \in \mathbb{R}^2_+$, while the price of the agents' rights for consumption is $p_g = \{p^b_{g,s}, p^g_{g,s}\}_{s\in S} \in \mathbb{R}^4$. In such formulation, the agent is required to sell his entire endowment of commodities, and to buy with the revenue rights to his - and the other agents' - entire consumption.[6]

[6] This condition will ensure that the agents' budget set has a non-empty interior at all prices. See below for a further discussion of the role of this formulation.

Note that no restriction is imposed in general on the price vector p_g. On the other hand, the price q^g of the contingent commodities is always non-negative. In general, free disposal is not allowed for property rights, while it is allowed for the contingent consumption goods.

As before, to show that at equilibrium this differentiation of markets according to the agents type is incentive compatible, it is convenient to introduce the following notation. Let $V^g(p_g, q^g)$ denote the utility level agent g can achieve at a solution of problem (P^g_{ALPT}). Let then $V^{g,b}(p_b, q^b, x_b)$ be the maximal utility an agent of type g could achieve by trading at the prices (p_b, q^b) for agents of type b, i.e., by solving the problem

$$\max_{x_{b;g}} \sum_{s\in S} \pi^g_s u\left(x^b_{b,s;g}\right)$$

s.t.

$$\sum_{s\in S} \left(p^g_{b,s}x^g_{b,s;g} + p^b_{b,s}x^b_{b,s;g}\right) \leq \sum_{s\in S} q^b_s\omega_s$$

$$-\sum_{s\in S} \pi^b_s u\left(x^b_{b,s;g}\right) + \sum_{s\in S} \pi^b_s u\left(x^g_{b,s;g}\right) \leq 0$$

$$\begin{aligned} -\sum_{s\in S} \pi^g_s u\left(x^g_{b,s;g}\right) + \sum_{s\in S} \pi^g_s u\left(x^b_{b,s;g}\right) &\leq 0 \\ -\sum_{s\in S} \pi^b_s u\left(x^b_{b,s;g}\right) + \sum_{s\in S} \pi^b_s u\left(x^b_{b,s}\right) &\leq 0. \end{aligned}$$

where the last constraint describes the incentive compatibility condition with respect to the other agents operating in that market for the type b agents.

The terms $V^b(p_b, q^b)$ and $V^{b,g}(p_g, q^g, x_g)$ are then symmetrically de ned.

As standard, the supply of property rights is modelled by introducing rms - denoted type I rms - who can transform commodities into property rights for commodities. Firms of type I have the following (constant returns to scale) technology:

$$Y^I = \left\{ \left(y^j_{i,s}, r^i_s\right)^{s\in S}_{i,j\in\{g,b\}} \in \mathbb{R}^{12}_+ : y^i_{i,s} = \frac{\xi_i}{\xi_j}\, y^i_{j,s} \leq r^i_s,\ \forall s \in S,\, i,j \in \{g,b\} \right\}$$

The technology allows the transformation of any amount of contingent commodities (contingent on any agent s type and (idiosyncratic) state s) into the same amount of rights for consumption of the same contingent commodities for consumers of all types. For each i, the vector $y^I_b := \{y^{b,I}_{b,s}, y^{g,I}_{b,s}\}_{s\in S}$ denotes the - per capita - supply by the rms of rights for consumption of consumers of type b, and $r^{b,I} := \{r^{b,I}_s\}_{s\in S}$ is the vector of the contingent commodities (of type b agents) used as input to produce such rights (also on a per capita basis). Let y_g^I and $r^{g,I}$ be de ned symmetrically; then $r^I \equiv (r^{i,I})_{i\in\{g,b\}}$, $y^I \equiv (y_i^I)_{i\in\{g,b\}}$. The technology requires the consistency among the amounts of rights which can be supplied to agents of each type ($y_i^I = \frac{\xi^i}{\xi^j} y_j^I$ for all $i \neq j$), and that $y_i^{i,I} \leq r^{i,I}$, for all i.

The rms problem consists then in choosing $(y^I, r^I) \in Y^I$ so as to maximize their pro ts $\pi^I = \sum_{i\in\{g,b\}} \left(p_i \cdot y_i^I - q^i \cdot r^{i,I}\right)$.

The same rms as in the EPT model - denoted here type II rms - are also present: they purchase from consumers of all types an amount of contingent commodities which is used as input to produce another contingent commodity vector. Let $r^{II} \equiv \left(r_s^{g,II}, r_s^{b,II}\right)_{s\in S}$ be the (per capita) input of contingent commodities purchased by consumers of each type i, and $y^{II} \equiv \{y_s^{g,II}, y_s^{b,II}\}_{s\in S}$ the output of contingent commodities. The rms technology is as follows:

$$Y^{II} = \{\left(y^{II}, r^{II}\right) \in \mathbb{R}_+^8 : \sum_{i\in\{g,b\}} \sum_{s\in S} \pi_s^i (y_s^{i,II} - r_s^{i,II}) \leq 0\}$$

These rms choose then $\left(y^{II}, r^{II}\right) \in Y^{II}$ so as to maximize their pro ts
$\pi^{II} = \sum_{i\in\{g,b\}} \sum_{s\in s} q_s^i (y_s^{i,II} - r_s^{i,II})$.

Remark 6. *In economies with adverse selection, as we noticed, the contracts available to each type of agent are not independent of the contracts chosen by other types; as a consequence, some cross-subsidization across contracts may be desirable. Once this possibility is allowed, in the strategic approach - as argued by Wilson (1977) and Riley (1979) - the relationship between the contracts traded questions the sense in which pro table deviations consisting in contracts offered only to one type of agents could be considered feasible per-se, without taking into account the change in the other agents trades generated as a consequence. To capture this fact, the strategic model of Rothschild and Stiglitz (1976) has been extended to allow for possible sequences of reactions and counter-reactions by rms offering contracts (see also Hellwig (1983)); within this approach, a more explicit attempt at modelling the possibility of bundling of contracts (and hence of cross-subsidization across contracts), can be found in Miyazaki (1977).*

Beyond the abstraction of the formalization, the structure of markets in ALPT equilibria is set to capture the fact that contracts traded by different types can be bundled together and hence to allow for the possibility of cross-subsidization. Not only all contracts traded by a single agent are bundled in a single, exclusive, contract to enhance incentives (as already the case in EPT equilibria), but also the contracts traded by different types are now bundled together. In this way, the relationship between these contracts can be properly internalized, and some types may be willing to pay a subsidy to relax the incentive compatibility constraints. The role of the trading of property rights, and of the speci c formulation of the technology of rms, is simply to model the fact that traders *(in particular rms offering contracts) take into account the presence of a relationship between the contracts traded by agents of different types.*

Within the competitive approach followed here, at equilibrium all possible contracts are priced and all contracts which are not traded are not pro table. The pricing structure implies that, to offer a contract to one type, a subsidy may be needed to ensure that the other type is not willing to switch to the new contract; prices then ensure the feasibility of contracts proposed and, in a sense, can be viewed as taking into account the chain of possible reactions and counter-reactions identi ed by Wilson and Riley (paying a subsidy may so be necessary for the feasibility of the contract proposed). One may argue that the equilibrium we are considering is vulnerable to a free-riding problem. While a contract which breaks an equilibrium with cross-subsidization by taking away all, say, the g type agents, for the reasons above, may violate feasibility, if offered only to a small subset of the g agents, it would be a feasible deviation. These agents would in fact free ride on the subsidy paid by the rest of the agents of their type to ensure that feasibility holds. In this case though a similar problem arises, as we still need to prevent the majority of the g agents from switching to the new contract.

Since, as we argued, strategic analysis of competition in economies with adverse selection incur in several difficulties, we do not have a satisfactory game-theoretic foundation for the equilibrium concept proposed, ALPT. Still, the arguments in this remark aim at showing that ALPT equilibria should not be viewed as a purely normative concept.

De nition 3. *An* ALPT equilibrium *is a collection of prices $(p_i, q^i)_{i\in\{g,b\}}$, consumption and production vectors $(x_i)_{i\in\{g,b\}}$, y^I, r^I, y^{II}, r^{II}, such that:*

- *x_i is a solution of the agents maximization problem (P_i^{AL}) at prices (p_i, q^i), $i \in \{g, b\}$;*
- *$\left(y^I, r^I\right)$ solves the pro t maximization problem of rms of type I at prices $(p_i, q^i)_{i\in\{g,b\}}$;*
- *$\left(y^{II}, r^{II}\right)$ solves the pro t maximization problem of rms of type II at prices $(q^i)_{i\in\{g,b\}}$;*
- *markets for property rights clear:*

$$\xi^i x_{i,s}^j = y_{i,s}^{j,I},\ \forall s \in S,\ i, j \in \{g, b\}$$

- *markets for contingent commodities clear:*

$$r_s^{i,II} + r_s^{i,I} \leq y_s^{i,II} + \xi^i \omega_s;\ \forall s \in S,\ i \in \{g, b\}$$

- *price differentiation across types is incentive compatible:*

$$V^i(p_i, q^i) \geq V^{ij}(p_j, q^j, x_j),\ \ i \neq j \in \{g, b\}$$

Remark 7. *The market clearing conditions stated above require that - even though the agents choice problem is typically not convex - all agents of the same type i make the same choice, for all i. This is not just for notational convenience. In that case in fact, the consumption allocation $\left(x_g^g, x_b^b\right)$ which is induced by an allocation of property rights which satis es such market clearing conditions will be feasible and incentive compatible. On the other hand, if agents of the same type choose different bundles of property rights, applying the market clearing condition to the average demand of rights (i.e., requiring this to be the same across all types) does not ensure the incentive compatibility of the induced consumption allocation.*
Thus the traditional way of overcoming non-convexities by exploiting the presence of a continuum of agents cannot be followed here, as we have to show that all the possibly different choices made by agents of one type are the same across types. This constitutes the main difficulty faced in the existence argument, also with respect to the other problems discussed in the next Remark.

Remark 8. *The equilibrium concept considered here, ALPT, is an extension to economies with adverse section of the notion of Arrow-Lindahl equilibria for economies with externalities in consumption. In the present framework, as we noticed, the externality arises in the de nition of the agents consumption set; this is in fact restricted by the incentive compatibility constraints, where the consumption level of the agents of the other type enters. The particular form of the externality and the speci cation of the consumption set generate however various problems:*
- the agents feasible choice set may not be convex (it is known in fact that the set of allocations satisfying incentive compatibility is typically not convex). As a consequence agents demand may fail to be upper-hemicontinuous;
- the agents budget set may not be compact (as negative prices are also allowed);
- the boundary behavior property of demand may fail (since the rights for the other type s consumption do not directly enter the agent s utility);
- local nonsatiation may fail. Under the assumption that the agents utility functions are strictly monotone we can easily see that global nonsatiation holds: for any vector $\bar{x}_i$ we can always nd in fact an alternative level of trades x_i which is such that $x_{i,s}^i = x_{i,s}^j$, $\forall s \in S$, $i, j \in \{g, b\}$ (so that the incentive compatibility constraints are trivially satis ed), and is sufficiently large that the agent would strictly prefer x_i to $\bar{x}_i$. The same argument does not apply though if we restrict attention to an arbitrary small neighborhood of $\bar{x}_i$, in which case it may not be possible to increase, locally, the level of the agent s own consumption without violating the incentive compatibility constraints (see the example on p. 9 in Hammond (1989)).

We show next that, for the economy under consideration, the difficulties described above can be overcome and existence of an ALPT equilibrium established:

Proposition 5. *Under Assumption 1, an ALPT equilibrium always exists.*

We show in particular that a competitive equilibrium exists where the price of the rights for b s consumption paid by g is strictly positive, while the price of the rights for g paid by b is zero.

Proof. The existence results for Arrow-Lindahl equilibria are not applicable, as we argued in the above Remarks, to the economy under consideration. The proof is organized as follows. We rst show that the problem of nding an ALPT equilibrium can be reduced to the problem of nding a set of prices $(p_i, q^i)_{i\in\{g,b\}}$ such that the level of consumer i s demand for the rights to $j \neq i$ s consumption at those prices is the same as the demand of consumer j for rights to his own consumption (Step 1). We then show how the difficulties concerning the behavior of agents demand mentioned in Remark 8 can be overcome, in particular that local nonsatiation holds and that demand is upperhemicontinuous (Step 2). In the next step we show that boundary behavior holds and that, even though the agents choice problem fails to be convex, their demand is single-valued over the restricted price domain we consider; existence of an equilibrium price vector then follows by a standard argument (Step 3). Finally, we show that the incentive compatibility of price differentiation is also satis ed (Step 4).

Step 1. Consider the pro t maximization problem of rms of type I. Given the presence of constant returns to scale, for a solution to exist the following equalities, ensuring that pro ts are not positive for all, and zero for some, admissible production levels y^I, must hold:

$$q_s^i = \sum_{j\in J} p_{j,s}^i \frac{\xi^j}{\xi^i},\ \forall s \in S, i \in \{g,b\} \tag{4.7}$$

At these prices the output choice will be any vector y^I such that, for any i, $y_i^i = \frac{\xi^i}{\xi^j} y_j^i = r^i$.[7]

Similarly, for a solution of the pro t maximization problem of the type II rms to exist, the prices of contingent commodities must satisfy the following (zero-pro t) condition:

$$q_s^i = \frac{\pi_s^i}{\pi_{s'}^i} q_{s'}^i,\ \forall s, s' \in S,\ i \in \{g,b\} \tag{4.8}$$

i.e. they have to be fair . The optimal net output choice will then be any vector y^{II} such that: $\sum_{i\in I} \sum_{s\in S}(y_s^i - \xi^i\omega_s^i) = 0$.

[7] Strictly speaking, the second equality may also hold as inequality if the price vector q has some zero component.

Note that rms and consumers demand is homogenous of degree 0 in $(p_i, q^i)_{i\in I}$. We normalize prices in terms of contingent commodity $(1,1)$; it is then convenient to set $q_1^1 = \pi_1^1$ so that (4.8) above simpli es to:

$$q_s^i = \pi_s^i,\ \forall i \in \{g,b\},\ s \in S \tag{4.9}$$

If we premultiply by ξ^i the budget constraint of agents of type i, and sum then these expressions over $i \in \{g,b\}$, we obtain the formulation of Walras Law for the economy under consideration:

$$\sum_{i\in\{g,b\}} \sum_{j\in\{g,b\}} \sum_{s\in S} p_{i,s}^j \xi^i x_{i,s}^j \leq \sum_{i\in\{g,b\}} \sum_{s\in S} q_s^i \xi^i \omega_s \tag{4.10}$$

On the basis of the above, to nd a competitive equilibrium it suffices to nd a set of prices $(p_i, q^i)_{i\in\{g,b\}}$ satisfying (4.7) and (4.9) such that the level of consumers demand at those prices satisfy the conditions:

$$x_{i,s}^i = x_{j,s}^i, \forall i,j \in \{g,b\},\ s \in S \tag{4.11}$$

i.e., the demand of rights for all agents consumption has to be the same across all types. To see this, note that, at all prices $(p_i, q^i)_{i\in\{g,b\}}$ satisfying (4.7), the supply of property rights by rms of type I is such that, if (4.11) holds, the market for property rights clears. Furthermore, using (4.11), (4.7), (4.9), the equilibrium condition for property rights and the properties of the demand of contingent commodities by type I rms to rewrite (4.10), we nd that r^I satis es $\sum_{i\in\{g,b\}} \sum_{s\in S} \pi_s^i (r_s^{i,I} - \xi^i \omega_s) \leq 0$. Thus, given the properties of the supply of contingent commodities by type II rms at prices (4.9), we see that r^I is such that the market for contingent commodities clears too.

Let P be the set of property rights prices which satisfy the above conditions: $P \equiv \{(p_i)_{i\in\{g,b\}} \in \mathbb{R}^8 : (p_i)_{i\in\{g,b\}}$ satis es (4.7) when $(q^i)_{i\in\{g,b\}}$ is given by (4.9)$\}$.

Step 2. We will focus our attention on prices satisfying the following conditions:

$$p_{b,s}^b = \beta \pi_s^b,\ \ p_{b,s}^g = 0,\ s \in S \tag{4.12}$$

$$p_{g,s}^g = \pi_s^g,\ \ p_{g,s}^b = (1-\beta)\frac{\xi^g}{\xi^b}\pi_s^b,\quad s \in S \tag{4.13}$$

for some $\beta \in [0,1]$. We will then show the existence of a competitive equilibrium when commodity prices satisfy (4.9) and prices of property rights lie in the following restricted domain:[8]

$$P_+ \equiv \left\{(p_i)_{i\in\{g,b\}} \in P\ :\ p_i \text{ satisfy } (4.12\text{-}4.13)\right\} \subset P.$$

[8] In particular, these conditions restrict prices of property rights to be non-negative. For more general economies however, we may not be able to show that competitive equilibria exist under such restriction.

The set P_+ is convex and compact. The elements of this restricted price domain are simply identified by the parameter $\beta \in [0, 1]$.

Consider then the following truncated sets of individual consumption levels $X_M \equiv \{x \in \mathbb{R}_+ : x \leq M\}$, where $M > max_{i \in \{g,b\}, s \in S} \ \{\frac{1}{\xi^i \pi^i_s}\} \left[\sum_{i \in \{g,b\}} \sum_{s \in S} \pi^i_s \xi^i \omega_s\right]$. The feasible set of the agent i's choice problem (P^i_{ALPT}), when consumption is restricted to lie in $(X_M)^4$, is non-empty, compact, for $i \in \{g, b\}$. Moreover, under Assumption 1, we can show that the budget set has a non-empty interior, for all $\{p_i\}_{i \in \{g,b\}} \in P_+$ and $(q^i)_{i \in \{g,b\}}$ satisfying (4.9).[9] Consider in fact an agent of an arbitrary type i. Since $\omega \gg 0$, for all p_i we can find a vector x_i such that $x^i_{i,s} = x^j_{i,s}$ for all $s \in S$, $j \in \{g, b\}$, so that incentive compatibility is clearly satisfied (as equality), and $p_i \cdot x_i < \sum_{s \in S} \pi^i_s \omega_s$.

Under Assumption 1 local nonsatiation also holds. Let $\bar{x}_i$ be an arbitrary allocation of rights which satisfies the incentive constraints appearing in (P^i_{ALPT}). Differentiating i's utility and the incentive constraints with respect to x_i, we see that it is always possible to find an infinitesimal change of x_i, at $\bar{x}_i$, which improves i's utility while satisfying all incentive constraints, provided the first row of the following matrix is linearly independent of the other rows:

$$\begin{pmatrix} DU^i(x^i_i) & 0 \\ DU^i(x^i_i) & -DU^i(x^j_i) \\ DU^j(x^i_i) & -DU^j(x^j_i) \end{pmatrix}$$

Condition (2.1), implied by our assumption on preferences, ensures that this condition is always satisfied.

By a similar argument, we can also show that, for any point in the budget set which satisfies incentive compatibility, there exists, in an arbitrarily small neighborhood, another point which lies in the interior of the budget set. By differentiating the equations defining the budget set and incentive compatibility, we see that this is true if there exists $b \in \mathbb{R}^4$ such that the first element of the vector[10]:

$$\begin{pmatrix} (p^i_i)^T & \left(p^j_i\right)^T \\ (DU^i(x^i_i))^T & 0 \\ (DU^i(x^i_i))^T & -\left(DU^i(x^j_i)\right)^T \\ (DU^j(x^i_i))^T & -\left(DU^j(x^j_i)\right)^T \end{pmatrix} \cdot b \tag{4.14}$$

[9] The requirement that each agent of type i sells all of his endowment (at a price q^i), and buys rights for consumption (at p_i) plays a crucial role here. If agents could retain their own endowment while simply choosing their net trades (at p_i), their budget set would not have a nonempty interior at all prices. In this case we can find robust examples where equilibria fail to exist; see the discussion after the proof of Proposition 7.

[10] Strictly speaking, this property only needs to hold only at points x_i such that $\sum_{j,s} p^j_{i,s} x^j_{i,s} = \sum_s \pi^i_s \omega_s$ and for the submatrix of (4.14) defined by the rows associated with those incentive compatibility constraints which hold as equality at x_i.

is negative, while all its other elements are non-negative. For all price vectors p_i which are components of a vector $(p_i)_{i\in\{g,b\}} \in P_+$, this condition always holds.

Let $x_i(\beta)$ denote the solution of problem (P^i_{ALPT}), for $i \in \{g, b\}$, when x_i is restricted to lie in $(X_M)^4$, prices of rights are restricted to lie in the set P_+, and commodity prices satisfy (4.9); the agents demand correspondence can thus be written simply in terms of β. The properties of the agents choice problem we established above, and the continuity of the agents utility functions imply that, for every i, $x_i(\beta)$ is non-empty and upper-hemicontinuous for all $\beta \in [0, 1]$. It may not be convex-valued though, as argued in Remark 8, and boundary behavior also needs to be established.

Step 3. We now show that when $(p_i)_{i\in\{g,b\}} \in P_+$, and q satis es (4.9), boundary behavior holds and the demand correspondence is single-valued, so that an ALPT equilibrium exists.[11]

At prices p_b which satisfy (4.12), evidently, agents of type b always choose bundles of consumption rights providing them full insurance:

$$x^b_{b,L} = x^b_{b,H} = \min\left\{\frac{1}{\beta}\left(\pi^b\omega_H + (1-\pi^b)\omega_L\right), M\right\}$$

The smaller is β, the higher is the subsidy agents of type b receive, and the higher is their consumption.

Moreover, we show next that also the type g agents, facing prices p_g as in (4.12), always choose bundles where the agents of type b are fully insured. Three possible cases are possible according to whether only the rst (4.5), only the second,-(4.6), or both incentive constraints hold as equalities. Examine rst the case where both constraints hold as equalities; by the single crossing property, we have $x^g_g = x^b_g$. Suppose the claim is not valid, i.e., say $x^b_{g,H} > x^b_{g,L}$ and consider the alternative choice of rights for b: $\hat{x}^b_{g,H} = \hat{x}^b_{g,L}$, such that $\sum_{s\in S} \pi^g_s u\left(\hat{x}^b_{g,s}\right) = \sum_{s\in S} \pi^g_s u\left(x^b_{g,s}\right)$, while keeping unchanged the choice of rights for g. The cost of this bundle is strictly lower than the cost of x_g, at prices as in (4.12); also, the incentive constraint of b, (4.6), is obviously still satis ed, as equality, while the other incentive constraint, (4.5), holds as an inequality. By choosing this alternative bundle, agent g can then achieve the same utility level as at x_g with a lower expenditure; by local nonsatiation we get so a contradiction. A perfectly symmetric argument holds in the case $x^g_{g,H} < x^b_{g,L}$. Thus it must be $x^g_{g,H} = x^b_{g,L}$.

Next, consider the possibility that only the incentive constraint of b, (4.6), holds as equality; to nd the solution of (P^g_{ALPT}) in this case, at prices p_b as in (4.12), it

[11] The presence of a continuum of agents of each type in our economy could allow us to convexify demand and show that, at some price, the average demand of each type satis es the equilibrium condition (4.11). If, however, the convexi ed demand is different from the original demand (i.e., if agents of the same type make different choices at these prices), it would not follow that demand is exactly equal across types and hence, as argued in Remark 7, we cannot guarantee that the induced consumption allocation is also incentive compatible.

suffices to look at the rst order conditions of the problem where only the constraint (4.6) is imposed. In this simpler problem, the optimal level of rights for b s consumption, $(x^b_{g,s})_{s\in\{H,L\}}$ must be such as to minimize the amount paid for such rights to attain a given utility level of utility of the agents of type b, and the solution of such expenditure minimization problem clearly has the property that $x^b_{g,L} = x^b_{g,H}$.

Finally, we observe that when, as in (4.12), $\{p^b_{g,s}\}_{s\in\{H,L\}} > 0$, it is never optimal for the type g agents to choose bundles of consumption rights where only their own incentive constraint, (4.5), is satis ed as equality (as in such case they could always increase their utility by reducing $\{x^b_{g,s}\}_{s\in\{H,L\}}$ while still satisfying the incentive constraints). Thus the last case to be considered never arises at a solution, completing the proof of the claimed property of g demand.

The existence of an ALPT equilibrium is then reduced to nding a value $\beta \in [0,1]$ such that

$$x^b_{g,L} = x^b_{g,H} = \frac{1}{\beta}\left(\pi^b\omega_H + (1-\pi^b)\omega_L\right) \tag{4.15}$$

Agents of type g face fair prices for their own consumption. Also, for $\beta \to 1$, agents of type g face prices for the rights to agent b s consumption converging to 0, and hence they choose: *i)* rights to their own consumption which in the limit support the full insurance allocation at fair prices, $x^g_{g,L} = x^g_{g,H} = (\pi^g\omega_H + (1-\pi^g)\omega_L)$; and *ii)* rights to b s consumption, $x^b_{g,s}$, $s \in S$, which in the limit satisfy incentive compatibility with respect to their own full insurance allocation. As a consequence, for $\beta \to 1$,

$$x^b_{g,L} = x^b_{g,H} > \frac{1}{\beta}\left(\pi^b\omega_H + (1-\pi^b)\omega_L\right), \tag{4.16}$$

If $\beta \to 0$, on the other hand, agents of type g face positive prices for the rights to agents of type b consumption, and hence

$$x^b_{g,L} = x^b_{g,H} < \min\left\{\frac{1}{\beta}\left(\pi^b\omega_H + (1-\pi^b)\omega_L\right), M\right\}$$

since $\lim_{\beta\to 0}\ \min\left\{\frac{1}{\beta}\left(\pi^b\omega_H + (1-\pi^b)\omega_L\right), M\right\} = M$.

We show, nally, that type g s demand correspondence is single-valued for all β:

Lemma 3. *Under Assumption 1, $(x_g(\beta))$ is single valued for all $\beta \in [0,1]$.*

Proof. We proceed as follows. We rst expand the commodity space to include all lotteries over consumption allocations. The problem which de nes the demand of agent g is then convex. Hence his demand is convex-valued for any $\beta \in [0,1]$. We then exploit the rst order conditions of agent g to show that his optimal choice is only given by degenerate lotteries. This implies, since the demand correspondence is convex-valued, that it is also single valued.

Let $[\Delta(X_M)]$ denote the space of probability measures over the (compact) set X_M, endowed with the weak* topology[12]. When lotteries are also allowed, the optimization problem of each agent of type g consists in the choice of $[\mu^i_{g,s}] \in [\Delta(X_M)]$, for $i \in \{g, b\}$, $s \in S$, to solve:[13]

$$\max_{[\mu^i_{g,s}]^{i\in\{g,b\}}_{s\in S}} \sum_{s\in S} \pi^g_s \int_{X_M} u(x) d\mu^g_{g,s}(x) \qquad (P^g_{ALPT;lot})$$

subject to[14]

$$\sum_{i\in\{g,b\}} \sum_{s\in S} \int_{X_M} p^i_{g,s}\, x\, d\mu^i_{g,s}(x) \leq \sum_{s\in S} q^g_s \omega_s$$

$$-\sum_{s\in S} \pi^g_s \int_{X_M} u(x) d\mu^g_{g,s}(x) + \sum_{s\in S} \pi^g_s \int_{X_M} u(x) d\mu^b_{g,s}(x) \leq 0$$

$$-\sum_{s\in S} \pi^b_s \int_{X_M} u(x) d\mu^b_{g,s}(x) + \sum_{s\in S} \pi^b_s \int_{X_M} u(x) d\mu^g_{g,s}(x) \leq 0.$$

The set of admissible choices in $(P^g_{ALPT;lot})$ is convex, as it is de ned by linear inequalities. In addition, it is compact, since it is a closed subset of the set $[\Delta(X_M)]^4$, which is compact as is de ned by a nite product of measures on the compact set X_M; it is non-empty, since it includes the degenerate lottery concentrated on the endowment point, for each type.

Evidently, this set contains the admissible set of (P^g_{ALPT}).

The linearity of the objective function, together with the convexity, compactness and non-emptiness of the set of admissible choices, ensure that a solution of problem $(P^g_{ALPT;lot})$ exists (see Luenberger (1969), Section 8).

To show that a solution of problem $(P^g_{ALPT;lot})$ always obtains at a degenerate lottery, when $(p_i)_{i\in\{g,b\}} \in P_+$, and q^g satis es (4.9), it suffices to look at the rst order conditions of the problem. Let ρ denote the Lagrange multiplier of the budget constraint, and λ_g and λ_b the Lagrange multipliers, respectively, of the rst and the second incentive compatibility constraint. The rst order condition with respect to, e.g., $\mu^b_{g,L}(.)$, can then be written as follows (see Luenberger (1969), Section 9.3):

$$\int_{X_M} \left[\left(\lambda_g(1-\pi^g) - \lambda_b(1-\pi^b)\right) u(x) - \rho(1-\beta)(1-\pi^b)x\right] dh(x) = 0$$

[12] The product space $[\Delta(X_M)]^4$ is then endowed with the corresponding product topology.

[13] Note that we are restricting our attention to the case of ex post randomization, as lotteries - lying in $[\Delta(X_M)]^4$, rather than, more generally, in $\Delta[(X_M)^4]$ - are conditional on the realization of the uncertainty. This is only for simplicity and, since our only aim is to prove that the agents choice is concentrated on degenerate lotteries, it entails no loss of generality.

[14] Lotteries over consumption bundles are priced by the average amount of commodities they use.

for any $h(x)$ such that $\int_{X_M} dh(x) = 0$ and $\mu^b_{g,L}(x) + h(x) \geq 0$. Since such condition must hold for all $h(x)$, it implies

$$\left[\left(\lambda_g(1-\pi^g) - \lambda_b(1-\pi^b)\right) u(x) - \rho(1-\beta)(1-\pi^b)x\right] = 0 \tag{4.17}$$

But, λ_g, $\lambda_b \geq 0$, with at most one strict inequality; and $\rho > 0$. It follows that (4.17) has at most a unique solution in x for all $\beta \in [0,1]$.[15] As a consequence, at any solution, $\mu^b_{g,L}(.)$ has all its mass concentrated on the values $x \in X_M$ such that equation (4.17) is satis ed.

A similar argument holds for $\mu^b_{g,H}(.)$, and $\mu^g_{g,s}(.)$, for $s \in S$. ■

This implies that agent g s demand is a continuous function. Since b faces a zero price for the property rights for g, his problem is unconstrained by the incentive compatibility constraints; thus his demand for rights for his own consumption is also a continuous function. Hence, by the argument at the beginning of this Step, an intermediate value of $\beta \in [0,1]$ exists which satis es (4.15) and constitutes then, as we argued, an ALPT equilibrium.

Step 4. To complete the proof it only remains to show that incentive compatibility of price differentiation also holds. This is immediate, given the way in which $V^{ij}(p_j, q^j, x_j)$ is constructed. ■

We show next that the structure of markets considered here indeed allows to solve the problem of decentralizing incentive efficient allocations:

Proposition 6. *All ALPT equilibria are incentive efficient.*

Proof. The proof is quite standard. Suppose not, i.e. there exists a feasible, incentive compatible allocation $(\hat{x}^b, \hat{x}^g)$ which Pareto dominates the equilibrium allocation (x^b_b, x^g_g). Then, given that as shown in the Proof of Proposition 5 local nonsatiation holds, it must be

$$p^b_{g,L}\hat{x}^b_L + p^b_{g,H}\hat{x}^b_H + p^g_{g,L}\hat{x}^g_L + p^g_{g,H}\hat{x}^g_H \geqslant \sum_{s\in S} q^g_s\omega_s$$

$$p^b_{b,L}\hat{x}^b_L + p^b_{b,H}\hat{x}^b_H + p^g_{b,L}\hat{x}^g_L + p^g_{b,H}\hat{x}^g_H \geqslant \sum_{s\in S} q^b_s\omega_s$$

one of the two inequalities being strict. Summing then the two inequalities and using (4.7) we get:

$$\xi^b\left(q^b_L\,\hat{x}^b_L + q^b_H\hat{x}^b_H\right) + \xi^g\left(q^g_L\hat{x}^g_L + q^g_H\hat{x}^g_H\right) > \sum_{s\in S}\;\sum_{i\in\{g,b\}} \xi^i q^g_s\omega_s$$

[15] A similar argument has been used by Prescott and Townsend (1984) to characterize incentive efficient allocations.

while

$$\xi^b \left(q_L^b x_L^b + q_H^b x_H^b\right) + \xi^g \left(q_L^g x_L^g + q_H^g x_H^g\right) = \sum_{s \in S} \sum_{i \in \{g,b\}} \xi^i q_s^g \omega_s$$

i.e. we get a contradiction to the fact that $(y^{b,II} - r^{b,II}, y^{g,II} - r^{g,II}) = (\xi^b(x_b^b - \omega), \xi^g(x_g^g - \omega))$ maximizes the pro ts of type II rms at prices q. ■

At the ALPT equilibrium we constructed in the proof of Proposition 5, agents of type b are subsidized by agents of type g: as long as $\beta < 1$, we have in fact $\sum_s \pi_s^b(x_s^b - \omega_s) > 0$. The following question then arises: is the presence of some level of cross-subsidization an intrinsic feature of ALPT equilibria? In particular, is the Rothschild and Stiglitz separating candidate equilibrium, which involves no cross-subsidization and is the unique EPT equilibrium of this economy, also an ALPT equilibrium ? Clearly for the economies for which this allocation is not incentive efficient (an open set of economies, as shown in Proposition 1) it cannot be an ALPT equilibrium, since this would contradict the efficiency property of ALPT equilibria, shown in Proposition 6. But even for those economies for which the Rothschild and Stiglitz separating candidate equilibrium is incentive efficient, it is not an ALPT equilibrium as can be easily seen from our construction.[16]

We must notice though that our de nition of ALPT equilibrium implicitly relies on a particular distribution of the endowments of property rights: each agent of type $i \in \{g, b\}$ is required to sell his entire endowment of contingent commodities at prices q^i and can then use this revenue to buy rights for consumption at prices p_i. Alternatively we could have assumed that agents have to sell only a fraction α of their endowment at prices q^i. By a straightforward extension of the argument in the proof of Propositions 5 and 6, for any $\alpha \in (0, 1]$ we can nd an ALPT equilibrium of this kind and such equilibria are always incentive efficient,. (This is not so for $\alpha = 0$, since in this case the budget set of agents of type b does not have a non-empty interior for all prices $p \in P_+$ (see footnote 9).) Moreover, since at the constructed equilibrium $p_b^b \leq \pi^b$, the lower is α the lower is the subsidy to the type b agents. We can thus take the parameter α to capture different distributions of the endowment of property rights across agents of the two types.

Let us denote by $\beta(\alpha)$ the equilibrium value of β associated to α. The next proposition reveals an important property of ALPT equilibria and in particular of the level of cross-subsidization which obtains at equilibrium. As we let α tend to 0, the sequence of ALPT equilibrium allocations $x_i(\beta(\alpha); \alpha)$ (recall that $x_g(\beta(\alpha); \alpha) = x_b(\beta(\alpha); \alpha))$ obtained by applying the argument in the proof of Proposition 5 converges to the incentive efficient allocation $\underline{x}$ characterized by the *minimum level of subsidization from agents of type g to agents of type b.*

[16] For the separating candidate equilibrium to be an ALPT equilibrium we must have in fact $\beta = 1$, since there is no cross-subsidization at such allocation. But, when $\beta = 1$, $x_{g,L}^b = x_{g,H}^b > x_{b,L}^b = x_{b,H}^b$, from equation (4.16).

More formally, $\underline{x}$ is obtained as a solution of the problem of maximizing $U^g(x^g)$ subject to the resource feasibility and incentive compatibility constraints (3.1), (3.2), (3.3), and the additional constraint that $\sum_s \pi_s^b(x_s^b - \omega_s) \geq 0$, i.e., that b is not subsidizing g. It is immediate to see, given the characterization of incentive efficient allocations derived in Proposition 1, that $\underline{x}$ coincides with the Rothschild and Stiglitz separating candidate equilibrium allocation whenever this is incentive efficient, and is otherwise (when the constraint $\sum_s \pi_s^b(x_s^b - \omega_s) \geq 0$ is not binding) given by the incentive efficient allocation at which b s welfare is minimal.

Thus when α approaches 0, or the distribution of rights is the least favorable to the type b agents, the equilibrium is given by the incentive efficient allocation $\underline{x}$ where the subsidy to the b agents is minimal. It is interesting to observe that $\underline{x}$ is also the unique allocation in the *core* as de ned by Marimon (1988).

Proposition 7. *Under Assumption 1,* $\lim_{\alpha \to 0} x_i(\beta(\alpha); \alpha) = \underline{x}$.

Proof. .

At any ALPT equilibria we consider, prices p lie in P_+, and q_i satisfy (4.9), for any $i \in \{g, b\}$, by construction. Moreover the incentive constraints are not binding for the type b agents, and

$$x_{b,H}^b(\beta(\alpha); \alpha) = x_{b,L}^b(\beta(\alpha); \alpha) = \frac{(\alpha + (1-\alpha)\beta) \ \sum_{s \in S} \ \pi_s^b \omega_s}{\beta}$$

where at the numerator of the term on the right hand side we have the expression of the agents income $\alpha \sum_s q_s^b \omega + (1-\alpha) \sum_{s,i} p_{b,s}^i \omega$ when they sell a fraction α of their endowment at prices $q^b = \pi^b$ and the rest is evaluated at the prices $p_b^b = \beta \pi^b, p_b^g = 0$.

Since both sequences $x_i(\beta(\alpha); \alpha)$, and $\beta(\alpha)$ lie in compact sets, they admit convergent subsequences; let $\hat{x}, \hat{\beta}$ be their limit.

The subsidy received by agents of type b, $\sum_{s \in S} \ \pi_s^b(x_{b,s}^b(\beta(\alpha); \alpha) - \omega_s)$, is equal to $\frac{\alpha(1-\beta(\alpha))}{\beta(\alpha)} \ \sum_{s \in S} \ \pi_s^b \omega_s$. If $\hat{\beta}$, the limit of $\beta(\alpha)$, is strictly positive, the value of the subsidy converges to 0 so that x_b clearly converges to $\underline{x}$ (equal to the Rothschild and Stiglitz separating candidate equilibrium in this case).

On the other hand, if $\hat{\beta} = 0$, the problem of agents of type g, (P_{ALPT}^g), in the limit, for $\alpha = \beta = 0$, reduces to the problem of maximizing $U^g(x^g)$ subject to the resource feasibility and incentive constraints (3.1), (3.2), (3.3), whose solution is $\underline{x}$. The limit value of the subsidy to agents of type b is in this case positive or zero (but cannot be negative since is it strictly positive for all $\alpha > 0$). ■

The proof of Proposition 7 shows that, for the economies in which the Rothschild and Stiglitz separating candidate equilibrium allocation is incentive efficient, it is decentralized as an ALPT equilibrium with $\alpha = 0$, and the equilibrium prices satisfy (4.12) with $\beta = \hat{\beta} > 0$. On the other hand, for economies in which the Rothschild and Stiglitz separating candidate equilibrium is not incentive efficient, an ALPT equilibrium with $\alpha = 0$ does not exist (see footnote 9), but the sequence of ALPT equilibria allocations converges, as $\alpha \to 0$, to $\underline{x}$, the allocation with the minimum level of subsidization from agents of type g to agents of type b, and equilibrium prices in the limit satisfy (4.12) with $\hat{\beta} = 0$.

Using such a characterization of ALPT equilibria with $\alpha = 0$, it is now straightforward to extend the second welfare theorem obtained for EPT equilibria, Proposition 4, to ALPT equilibria: any incentive efficient consumption allocation can be decentralized as an ALPT equilibrium with $\alpha = 0$ and with transfers (possibly dependent on the state but not the agents type).

Proposition 8. *Under Assumption 1, for any incentive efficient consumption allocation* (x^b, x^g) *there exists a set of transfers* (t_H, t_L) *which are feasible, i.e.,* $\sum_{s\in S}\left[\xi^g \pi^g_s t_s + \xi^b \pi^b_s t_s\right] \leq 0$, *and such that* (x^b, x^g) *is an ALPT equilibrium allocation, with* $\alpha = 0$, *for the economy under consideration when each agent receives a transfer* (t_H, t_L).

It is an interesting open question whether a stronger version of the second welfare theorem, one which holds for any $\alpha \in [0, 1]$, can be proved for the class of economies we study in this paper.

Proof. Let (x^b, x^g) be any incentive efficient allocation. By Proposition 4, there exists a set of feasible transfers (t_H, t_L) such that (x^b, x^g) is an EPT equilibrium allocation for the economy with endowments $\omega' := (\omega_L + t_L, \omega_H + t_H)$. Moreover, by Proposition 2, the economy with endowments ω' has a unique EPT equilibrium, i.e. (x^b, x^g), which is the Rothschild and Stiglitz separating candidate equilibrium for that economy. Since (x^b, x^g) is incentive efficient, from Proposition 7 it follows that it can be decentralized as an ALPT equilibrium with $\alpha = 0$ of the economy with endowments ω'. ■

5. Conclusions

We have studied in this paper the existence and welfare properties of competitive equilibria of economies with adverse selection. In particular we have shown that, when exclusive contracts are available, an appropriate structure of markets (for menus of contracts or, equivalently, for consumption rights) allows to resolve the problem of decentralizing incentive efficient allocations as all competitive (ALPT) equilibrium allocations are incentive efficient in this case. We have also shown that when such markets for menus of contracts (or consumption rights) are not available, the (unique) competitive (EPT) equilibrium allocation is not incentive efficient, for an open set of economies(and coincides with the Rotschild and Stiglitz separating candidate equilibrium allocation).

Our results have been derived for a class of simple insurance economies with adverse selection where agents can be of two possible types, and the - privately observed - type of each agent only concerns the probability structure of the idiosyncratic shocks affecting the agent. The analysis of markets and equilibria for such economies constitute a basic workhorse for the economics of uncertainty, at least since the work by Rotschild and Stiglitz (1976).

The equilibrium concepts we introduced can be extended to more general classes of economies with adverse selection. A generalization of the results we obtained poses however some technical problems. The main difficulty, as we saw in this paper, arises from the interaction between the non-convexities induced by the presence of the incentive compatibility constraints in the consumption set and the feasibility requirement that, even though a large economy is considered and the Law of Large Number can be used to obtain the value of aggregate demand, markets for consumption rights have to clear exactly and not simply on average : i.e., if agents of the same time make different choices at equilibrium, all other types have to make exactly the same choice. While for the special structure of the economies studied in this paper we have been able to circumvent such problem by showing that over a relevant subset of the price domain agents optimal choice is unique and hence to restrict attention to deterministic allocations, this property hardly generalizes. For more general economies with adverse selection, we may have so to expand the commodity space so as to include lotteries over the underlying consumption space, and to deal at the same time with the stronger feasibility requirement in our framework (as at equilibrium all types have to choose exactly the same lottery).

Appendix

A Characterization of Incentive Efficient Allocations
(with Giuseppe Lopomo - Stern School, NYU)

We prove here Proposition 1, which provides a complete characterization of incentive constrained optimal allocations for the economy under consideration. The argument follows, with some minor modi cations, the one in Prescott and Townsend (1984), and is presented here for completeness.

We show rst that even if (ex post) lotteries over consumption bundles are allowed, such randomization is never optimal, as the solutions of the maximization of the agents welfare subject to the resource feasibility and the incentive compatibility constraints always obtain at degenerate lotteries. The commodity space is expanded as in Lemma 3 to allow for lotteries, and contains all probability measures on X_M, $[\mu_s^i]_{s\in\{H,L\}}^{i\in\{g,b\}} \in [\Delta(X_M)]^4$.[17] As before, we endow $[\Delta(X_M)]$ with the weak* topology and $[\Delta(X_M)]^4$ with the product topology.

Incentive efficient allocations are then the solutions of the following problem, for all $(\gamma^i)_{i\in\{g,b\}} \in \mathbb{R}_+^2 : \gamma^g + \gamma^b = 1$,

$$\max_{[\mu_s^i]_{s\in S}^{i\in\{g,b\}}} \sum_{i\in\{g,b\}} \gamma^i \sum_{s\in S} \pi_s^i \int_{X_M} u(x) d\mu_s^i(x) \tag{A.1}$$

subject to

$$\sum_{i\in\{g,b\}} \xi^i \sum_{s\in S} \pi_s^i \left(\int_{X_M} x d\mu_s^i(x) - \omega_s \right) \leq 0$$

$$-\sum_{s\in S} \pi_s^b \int_{X_M} u(x) d\mu_s^b(x) + \sum_{s\in S} \pi_s^b \int_{X_M} u(x) d\mu_s^g(x) \leq 0$$

$$-\sum_{s\in S} \pi_s^g \int_{X_M} u(x) d\mu_s^g(x) + \sum_{s\in S} \pi_s^g \int_{X_M} u(x) d\mu_s^b(x) \leq 0$$

The *utility possibility set* is then the set of pairs of utility levels for each type attainable at an incentive efficient allocation.

Lemma A. 1. *For any $\gamma^b \in [0,1]$, there is a unique solution of problem (5.1), given by a degenerate lottery (i.e., μ_s^i has all the mass concentrated on a single value $x \in X_M$, for all i, s) and the utility possibility set is convex.*

[17] Compactness of the commodity space over which lotteries are de ned is required for technical reasons. Given this, the result obtained in Lemma A.1 below implies that the particular choice we made for such commodity space (X_M) is without loss of generality.

Proof. By the same argument as in the proof of Lemma 3, we see that problem (5.1) is a convex programming problem, and has a solution for all $\gamma^b \in [0,1]$. The rst order conditions of problem (5.1), say with respect to $\mu_L^b(x)$, is (see Luenberger (1969), Section 9.3):

$$\int_{X_M} \left(1-\pi^b\right)\left[\left(\gamma^b+\lambda^b-\lambda^g\right)u(x)-\rho\xi^b x\right]dh(x)=0 \tag{A.2}$$

for any $h(x)$ such that $\int_{X_M} dh(x) = 0$ and $\mu_L^b(x) + h(x) \geq 0$, where $\lambda^g, \lambda^b, \rho$ are the Lagrange multipliers associated with the two incentive constraints and the resource feasibility constraint. Since (5.1) has to hold for all $h(x)$, it implies

$$\left[\left(\gamma^b+\lambda^b-\lambda^g\right)u(x)-\rho\xi^b x\right]=0 \tag{A.3}$$

Since ρ, $\xi^b > 0$,(5.1) has at most a unique solution in x. As a consequence at any incentive constrained optimum, $\mu_L^b(x)$ has all its mass concentrated on the only value of $x \in X_M$ such that equation (4.17) is satis ed.

A similar argument holds for $\mu_H^b(x), \mu_s^g(x)$, $s \in \{H, L\}$.

The convexity of the problem then implies that the solution of problem (5.1) is unique, for all $\gamma^b \in [0,1]$ and that the utility possibility set is convex. ■

Lemma A.1 allows us to consider a simpler problem than (5.1), where only deterministic allocations $[x_s^i]_{s\in\{H,L\}}^{i\in\{g,b\}} \in (X_M)^4$ are allowed:

$$\max_{[x_s^i]_{s\in\{H,L\}}^{i\in\{g,b\}}} \sum_{i\in\{g,b\}} \gamma^i \sum_{s\in S} \pi_s^i u\left(x_s^i\right) \tag{A.4}$$

subject to

$$\sum_{i\in\{g,b\}} \xi^i \sum_{s\in S} \pi_s^i\left(x_s^i-\omega_s\right) \leq 0 \tag{A.5}$$

$$-\sum_{s\in S} \pi_s^b u\left(x_s^b\right)+\sum_{s\in S} \pi_s^b u\left(x_s^g\right) \leq 0 \tag{A.6}$$

$$-\sum_{s\in S} \pi_s^g u\left(x_s^g\right)+\sum_{s\in S} \pi_s^g u\left(x_s^b\right) \leq 0 \tag{A.7}$$

Let us denote still by ρ, λ^b, and λ^g the Lagrange multipliers associated to the above constraints.

It is convenient to distinguish the solutions of problem (5.1) according to the values of λ^b and λ^g; there are four possible cases:

Case 1: $\lambda^b > 0$, $\lambda^g > 0$. This case is not possible. In fact, $\lambda^b > 0$, $\lambda^g > 0$ requires, by the complementary slackness conditions, that both the incentive constraints are binding:

$$-\left(1-\pi^b\right)u\left(x_L^b\right)-\pi^b u\left(x_H^b\right)+\left(1-\pi^b\right)u\left(x_L^g\right)+\pi^b u\left(x_H^g\right)=0$$

$$-\left(1-\pi^{g}\right) u\left(x_{L}^{g}\right)-\pi^{g} u\left(x_{H}^{g}\right)+\left(1-\pi^{g}\right) u\left(x_{L}^{b}\right)+\pi^{g} u\left(x_{H}^{b}\right)=0;$$

which in turn implies, given the single crossing property:

$$x_{s}^{b}=x_{s}^{g},\ s\in\{H,L\} \tag{A.8}$$

Substituting (A.8) into the rst order conditions of problem (5.1), after some algebra, we get a contradiction to the fact that both λ^b and λ^g are strictly positive.

Case 2: $\lambda^b=\lambda^g=0$. In this case, the rst order conditions with respect to x of problem (5.1) reduce to:

$$\begin{aligned} 0&=\gamma^{b} u'\left(x_{L}^{b}\right)-\rho\xi^{b}\\ 0&=\gamma^{b} u'\left(x_{H}^{b}\right)-\rho\xi^{b}\\ 0&=\left(1-\gamma^{b}\right) u'\left(x_{L}^{g}\right)-\rho\left(1-\xi^{b}\right)\\ 0&=\left(1-\gamma^{b}\right) u'\left(x_{H}^{g}\right)-\rho\left(1-\xi^{b}\right). \end{aligned}$$

Solving this system we obtain $x_L^b=x_H^b$ and $x_L^g=x_H^g$. But then the incentive compatibility and resource feasibility constraints imply $x_L^b=x_H^b=x_L^g=x_H^g=\hat{x}$; at such value the above system is only satis ed for $\gamma^b=\xi^b$.[18]

Case 3: $\lambda^b>0$ and $\lambda^g=0$. In this case, the rst order conditions of problem (5.1) become:

$$\begin{cases} 0=\gamma^{b} u'\left(x_{L}^{b}\right)-\rho\xi^{b}+\lambda^{b} u'\left(x_{L}^{b}\right)\\ 0=\gamma^{b} u'\left(x_{H}^{b}\right)-\rho\xi^{b}+\lambda^{b} u'\left(x_{H}^{b}\right)\\ \rho\left(1-\xi^{b}\right)=\left(1-\gamma^{b}-\lambda^{b}\frac{1-\pi^{b}}{1-\pi^{g}}\right) u'\left(x_{L}^{g}\right),\\ \rho\left(1-\xi^{b}\right)=\left(1-\gamma^{b}-\lambda^{b}\frac{\pi^{b}}{\pi^{g}}\right) u'\left(x_{H}^{g}\right). \end{cases} \tag{A.9}$$

together with $(A.5)-(A.7)$. From the rst two conditions of $(A.9)$ we obtain $x_L^b=x_H^b:=x^b$; moreover, since $\lambda^b>0$, $x^b<\hat{x}$. From the remaining two conditions we get then $x_L^g<x_H^g$, as $\pi^b<\pi^g$ implies $\frac{\pi^b}{\pi^g}>\frac{1-\pi^b}{1-\pi^g}$. Since $(x_s^i)_{i,s}=\hat{x}$ was shown to be the solution of problem (5.1) for $\gamma^b=\xi^b$, the allocations we obtain as solutions of system $(A.9)$ and of $(A.5)-(A.7)$ where $x^b<\hat{x}$, can only be solutions of that problem for $\gamma^b<\xi^b$.

It is convenient to parameterize the solutions of $(A.5)-(A.7),(A.9)$ with respect to x,λ^b,γ^b according to the value of $x^b=x_L^b=x_H^b$; let $(x_s^i(x^b))_{s\in\{H,L\}}^{i\in\{g,b\}}$ describe, in particular, the solution for x, and the associated utility level be $U^i(x^b):=\sum_{s\in S}\ \pi_s^i u(x_s^i(x^b))$.

[18] Note that the allocation $x_L^b=x_H^b=x_L^g=x_H^g=\hat{x}$ is also rst best efficient.

Observe that for $x^b = \hat{x}$ we get $\gamma^b(x^b) = \xi^b, x_s^i(x^b) = x^b$. Thus the set of incentive constrained optima for which $\lambda^b > 0$ and $\lambda^g = 0$ is non-empty if we can show that the utility of the g types along the solutions of $(A.5) - (A.7), (A.9)$ increases when we lower x^b below $\hat{x}$, or:

$$\frac{dU^g(x^b)}{dx^b} \;|_{x^b=\hat{x}} < 0 \tag{A.10}$$

We will now show that $(A.10)$ indeed holds. Substituting x^b for x_L^b, x_H^b, equations $(A.5), (A.6)$ can in fact be solved for x_L^g, x_H^g; applying then the Implicit Function Theorem to these two equations and substituting into $U^g(.)$, we obtain

$$\frac{dU^g(x^b)}{dx^b} = \frac{[u'(x_L^g) - u'(x_H^g)]\, u'(x^b)\left(1-\xi^b\right)(1-\pi^g)\,\pi^g}{\left(1-\xi^b\right)\left(\pi^g\left(1-\pi^b\right)u'(x_L^g) - \pi^b(1-\pi^g)\,u'(x_H^g)\right)}$$

$$+\frac{\left[(1-\pi^g)\,\pi^b - \pi^g\left(1-\pi^b\right)\right]u'(x_L^g)\,u'(x_H^g)\,\xi^b}{\left(1-\xi^b\right)\left(\pi^g\left(1-\pi^b\right)u'(x_L^g) - \pi^b(1-\pi^g)\,u'(x_H^g)\right)}$$

The denominator of this expression is always positive, since $\pi^g\left(1-\pi^b\right) > \pi^b(1-\pi^g)$ and $u'(x_L^g) \geqslant u'(x_H^g)$. The numerator, on the other hand, when evaluated at $x_L^g = x_H^g = x^b$ is negative.[CHECK IF OK!!!].

By Lemma A.1, the utility possibility set is convex and hence so is the set of incentive efficient allocations for Case 3. From the above result it then follows that there exists $\tilde{x} < \hat{x}$ such that for all x^b in the non-empty open interval $(\hat{x}, \tilde{x})$ the induced allocation $(x_s^i(x^b))_{s\in\{H,L\}}^{i\in\{g,b\}}$ constitutes a solution of problem (5.1) for some $0 < \gamma^b < \xi^b$, i.e., is incentive efficient. The lower boundary of this interval, $\tilde{x}^b$, is given by the value of x^b such that (x^b, x_L^g, x_H^g) solve the equation

$$[u'(x_L^g) - u'(x_H^g)]\, u'(x^b)\left(1-\xi^b\right)(1-\pi^g)\,\pi^g + \left[(1-\pi^g)\,\pi^b - \pi^g\left(1-\pi^b\right)\right]u'(x_L^g)\,u'(x_H^g)\,\xi^b = 0,$$

and $(A.5), (A.6)$ (i.e. such that $\frac{dU^g(x^b)}{dx^b} = 0$, resource feasibility and incentive compatibility for the type b are satis ed).

Case 4: $\lambda^b = 0$ and $\lambda^g > 0$. In this case, the rst order conditions of problem (5.1) imply

$$\rho\left(1-\xi^b\right) = \left(1-\gamma^b+\lambda^g\right)u'(x_L^g)$$

$$\rho\left(1-\xi^b\right) = \left(1-\gamma^b+\lambda^g\right)u'(x_H^g)$$

hence $x_L^g = x_H^g := x^g$. As in the previous case, it is easy to show that $x^g < \hat{x}$. Also, again from the rst order conditions of problem (5.1), we get

$$\rho\xi^b = \left(\gamma^b - \lambda^g\frac{1-\pi^g}{1-\pi^b}\right)u'\left(x_L^b\right)$$

$$\rho\xi^b = \left(\gamma^b - \lambda^g \frac{\pi^g}{\pi^b}\right) u'\left(x_H^b\right),$$

and hence $x_H^b < x_L^b$, since $\pi^b < \pi^g$.

Since, as we showed, $x_s^g = \hat{x}$, $\in \{H, L\}$ is optimal for $\gamma^b = \xi^b$, and the allocations we obtain from the solution of the rst order conditions in Case 4 have the property that $x_s^g < \hat{x}$, $s \in \{H, L\}$, it follows that such allocations can only be incentive efficient for $\gamma^b > \xi^b$.

The rest of the analysis in this case is perfectly symmetric to the one followed for Case 3, and is hence omitted. ■

References

Arrow, K. (1969): The Organization of Economic Activity: Issues Pertinent to the Choice of Market Versus Non-Market Allocation , in *Collected Papers of K. J. Arrow*, vol. 2, Cambridge, Ma: Harvard University Press, 1983.

Bennardo, A. and P. A. Chiappori (1998): Competition, Positive Pro ts, and Market Clearing under Asymmetric Information , University of Chicago, mimeo.

Bisin, A. and P. Gottardi (1999): Competitive Equilibria with Asymmetric Information , *Journal of Economic Theory*, 87, 1-48.

Cole, H. (1989): Comment: General Competitive Analysis in an Economy with Private Information , *International Economic Review*, 30, 249-52.

Dasgupta, P. and E. Maskin (1986): The Existence of Equilibrium in Discontinuous Economic Games, I: Theory , *Review of Economic Studies*, 53, 1-26.

Dubey, P., J. Geanakoplos, and M. Shubik (1995): Default and Efficiency in a General Equilibrium Model with Incomplete Markets , Yale University, mimeo.

Gale, D. (1992): A Walrasian Theory of Markets with Adverse Selection , *Review of Economic Studies*, 59, 229-55.

Hammond (1989): Perfected Option Markets in Economies with Adverse Selection , Stanford University, mimeo.

Hellwig, M. (1993): Some Recent Developments in the Thoery of Competition in Markets with Adverse Selection , *European Economic Review*.

Jerez, M. B. (1999): General Equilibrium with Asymmetric Information: A Dual Approach , UCLA, mimeo.

Kocherlakota, N. (1998): The Effect of Moral Hazard on Asset Prices when Financial Markets Are Complete , *Jornal of Monetary Equilibrium*, 41, 39-56.

Kohlberg and J. J. Mertens (1986): On the Strategic Stability of Equilibria , *Econometrica*, 54, 1003-38.

Lindahl E. (1919): *Die Gerechtigkeit der Besteuring*, Lund: Gleerup. [English translation: Just Taxation: A Positive Solution , in *Classics in the Theory of Public Finance*, edited by R. A. Musgrave and A. T. Peacock, London: Macmillan.]

Luenberger, D. (1969): *Optimization by Vector Space Methods*, New York: Wiley.

Magill, M. and M. Quinzii (1998): Incentives and Risk Sharing in a Stock Market Equilibrium , University of Southern California, mimeo.

Marimon, R. (1988): The Core of Private Information Economies , University of Minnesota, mimeo.

Miyazaki, H (1977): The Rat Race and Internal Labour Markets , *Bell Journal of Economics*, 8, 394-418.

Prescott, E. and R. Townsend (1984): Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard , *Econometrica*, 52, 21-45.

Prescott, E. and R. Townsend (1984-bis): General Competitive Analysis in an Economy with Private Information , *International Economic Review*, 25, 1-20.

Riley, J. (1979): Informational Equilibrium , *Econometrica*, 47, 331-59.

Rotschild, M. and J. Stiglitz (1976): Equilibrium in Competitive Insurance Markets: An Essay in the Economics of Imperfect Information , *Quarterly Journal of Economics*, 80, 629-49.

Wilson, C. (1977): A Model of Insurance Markets with Incomplete Information , *Journal of Economic Theory*, 16, 167-207.