Accounting for Swedish wealth inequality

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Abstract

Sweden's distribution of disposable income is very even, with a Gini coefficient of just 0.31. Yet its wealth distribution is extremely unequal, with a Gini coefficient of 0.79. Moreover, Swedish wealth inequality is to a very large extent driven by the large fraction of households with zero or negative wealth. In this paper, we ask to what extent the ambitious public pension scheme is responsible for these features of the data. To address this question, we study the properties of two overlapping generations economies with uninsurable idiosyncratic income risk. The first has a pension system modeled on the actual one, the second has no public pension scheme at all. Our findings support the view that the public pension scheme is to a large extent responsible for the features of the data that we focus on.

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1 Introduction

The distribution of wealth tends to be more unequal than the distribution of income. This is especially true of Sweden, indeed spectacularly so. Sweden's distribution of disposable income is second only to Finland in terms of equality among the 15 OECD countries studied by Atkinson (1995). Yet its wealth distribution is more unequal than that of the United States, whose income distribution is the most unequal among the countries in Atkinson's study. Using data from 1992 (see Domeij & Klein (1998)), we find that Sweden's distribution of disposable income by household exhibits a Gini coefficient of just 0.31 but that of wealth has a Gini coefficient of 0.79 (or even as high as 0.86 when the very rich are oversampled).

For this discrepancy to arise, it has to be that low-income earners save less as a proportion of income than high-income earners, and of course they do (see Huggett and Ventura (1997) for a survey of the evidence on US savings rates across households). The question is why. Several recent papers have tried to account for this fact as it applies to the United States. Quadrini (1997) studies the effects on savings of giving agents the opportunity to set up their own business. He finds that those who choose to become entrepreneurs tend to earn more and to save a higher fraction of their income than others.

Hubbard, Skinner & Zeldes (1995) focus on the other end of the income distribution and stress the importance of means-tested social insurance programs in accounting for the low savings rates of low-income households. Huggett & Ventura (1997) stress the demographic structure and the structure of social security payments and find that these features are quantitatively important in accounting for differences in savings rates across households. Huggett (1995) finds that it is possible to replicate many (but not all) of the features of the US wealth distribution by using a life-cycle model featuring uninsurable earnings and longevity risk as well as a simple social security scheme.

Cubbedu & Ríos-Rull consider an OLG model with shocks to household formation. They focus on the impact of changes in household structure on aggregate savings, and find that it is very small. However, they also suggest that the explicit modeling of household arrangements can go a long way towards accounting for wealth inequality. Using the same data as in Domeij & Klein (1998), we find that the wealth/income ratio of married households in 1992 was on average 37 percent greater than that of unmarried households. A possible reason for this is that married households face more downside risk than unmarried households. The dissolution of a household typically leads to a fall in the ratio of income to the number of "mouths to feed" (and hence a rise in marginal utility); this especially true for women.

This paper asks how far Sweden's ambitious public pension scheme can account for the big difference between the degree of inequality in its income and its wealth distribution. Our reason for focusing on the public pension scheme is the following. Wealth inequality in Sweden is to a large extent driven by the large fraction of households with negative or zero net wealth. This fraction is about 24 percent.[1] Given that, it makes sense to focus first on mechanisms that reduce incentives for low-income earners to save. There are several reasons for thinking that a public pension scheme is such a mechanism. In particular, a common benefit payable to each senior citizen will reduce the savings of low-income earners proportionately more than for high-income earners and thus increases the inequality of wealth (provided that claims on future pensions are not included in measured wealth). An upper limits to benefits from an earnings-based pension scheme has the same effect. In Sweden, both these elements are present, indeed very much so. For example, the minimum (cash) benefit level is about a third of GDP per adult and the upper limit is such that earnings above the median among the full-time employed do not generate further entitlements to earnings-based pension benefits.[2]

In order to assess the impact on the wealth distribution of the public pension

[1] This is a very large number by international standards; see Díaz-Gímenez, Quadrini and Ríos-Rull (1997). However, Bager-Sjögren & Klevmarken (1996) have raised doubts about the reliability of the HINK database in this respect. In particular, they claim that consumer durables are underreported in HINK. Their own findings, based on the HUS database, which includes more detailed survey data than HINK does on consumer durables, suggest that only about 15 percent of households have zero or negative net wealth. Huggett (1995), using data from the Survey of Consumer Finances, reports that 15 percent of US households have zero or negative wealth when consumer durables are excluded. Adding consumer durables reduces the number to 6 percent. We conclude from this that the share of households with zero or negative wealth in Sweden is high by US standards.

[2] According to Huggett and Ventura (1996) social security in the United States is characterized by a common benefit level including various health-care benefits of about an eighth of GDP per capita and an upper limit such that earnings above 2.47 times average earnings do not generate further entitlements to earnings-based pension benefits.

scheme, we build and calibrate a life-cycle model for a small open economy. [3] Agents face uninsurable idiosyncratic shocks to earnings and marital status and also an age-dependent probability of surviving into the next period. Earnings depend on age, sex, marital status, and a persistent but transitory shock. We estimate the parameters of the process using panel data, as in Flodén and Lindé (1999). Individuals supply labor inelastically and household saving is chosen optimally.[4] There are no aggregate dynamics. First we solve the model for the invariant wealth distribution in the presence of a Swedish-style public pension scheme. In order to assess the impact of public pensions, we re-solve the model without them.[5]

Our main results are the following. In the first place, we are able to replicate the degree of disposable income inequality with a reasonable degree of accuracy. In the baseline economy with a pension scheme present, the Gini coefficient for disposable income is 0.35. Secondly, the pension system goes a very long way in accounting for wealth inequality. Without the pension scheme, the model Gini coefficient for net wealth is just 0.54. With the pension scheme, it is 0.71. In addition to this, we are able to replicate fairly well the fraction of households with zero or negative net wealth. In the presence of the pension scheme, the model generates an economy where 29 percent of households have non-positive net wealth.

The paper is organized as follows. Section 2 describes our model economies. Section 3 presents the results. Section 4 concludes. In Appendix A, we discuss the numerical solution method that we use. Tables and figures are found in Appendix B.

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[3] Hugget (1995) and Quadrini & Ríos-Rull (1997) note that OLG economies where agents never experience zero earnings can generate a large fraction of households with zero or negative wealth.

[4] The settlement of disagreements within households is discussed in section 2.12.1.

[5] In 1998 a reform of the pension system was passed which will come into force gradually in the coming decades. The current paper investigates the consequences of the new system. In a sequel, we intend to study the extent to which the two systems differ with respect to their implications for the wealth distribution.

2 The model

We study a small open economy with 80 overlapping generations. Individuals enter the economy at the age of 20 and work until the age of 64. They face idiosyncratic shocks to household formation (marriages and separations),[6] longevity and earnings. There is no aggregate risk. All insurance markets are closed, and the only way to self-insure is by purchasing a single asset (a one-year bond), whose return is given exogenously by the world market. We consider two economies, one with a public pension scheme modeled on that of Sweden, and another with no public pension scheme at all. Especially in the latter economy, there is a strong life-cycle savings motive in addition to the precautionary savings motive.

We consider only steady states, where the distribution of individuals with respect to age, sex, marital status, earnings and assets is invariant over time.

2.1 Demographics

The demographic dynamics are similar to those of Cubbedu & Ríos-Rull (1997). The economy is inhabited by individuals that differ by sex, age, and marital status. They face mortality risk which depends on age, sex, and marital status. The survival probability of an individual indexed by age $i \in \mathcal{I} = \{20, 21, \ldots, 99\}$, sex $s \in \mathcal{S} = \{\text{male}, \text{female}\}$ and marital status $g \in \mathcal{G} = \{\text{unmarried}, \text{divorced,widowed,married}\}$ is $\gamma_{i,s,g}$. We sometimes write, abusing the notation slightly, $g =$ single when we mean $g \in \{\text{unmarried}, \text{divorced,widowed}\}$. At age 99, the probability of death is 1. We assume that individuals can only marry a partner of the same age and opposite sex. By marriage, we mean that partners share assets and current income and enjoy the same level of consumption. They also have the same current period utility function. Married individuals, by definition, have partners. Variables pertaining to a partner are denoted by a star subscript. In particular, if $s =$male, then $s^* =$female and vice versa.

We assume that the process for marital status is exogenous where the probability (conditional on survival) of an individual of age i, sex s and marital status g of transiting to marital status g' is $\pi_{i,s}(g'|g)$. Define transition probabilities

[6] By *marriage* we mean two people of the opposite sex moving in together, whether or not they literally marry.

$\pi_{i,s}(g'|g)$ so that

$$\sum_{g'} \pi_{i,s}(g'|g) = \gamma_{i,s,g}.$$

We also define the transition *measure* via

$$\Pi_{i,s}(G'|g) = \sum_{g' \in G'} \pi_{i,s}(g'|g)$$

where $G' \subset \mathcal{G}$.

A stable population is characterized by constant ratios over time across the different demographic groups. Assume the population grows at rate χ. This implies that the measure of different types $\mu_{i,s,g}$ satisfies the following difference equation.

$$\mu_{i+1,s,g'} = \sum_{g} \frac{\pi_{i,s}(g'|g)}{1+\chi} \mu_{i,s,g}. \tag{1}$$

Below we will need the transition measure A consistency requirement is then that, for each i, we have

$$\mu_{i,\text{male},\text{married}} = \mu_{i,\text{female},\text{married}}.$$

We normalize so that

$$\sum_{i,s,g} \mu_{i,s,g} = 1.$$

2.2 Preferences

Following Regalia & Ríos-Rull (1999), a household member enjoys consumption according to a period utility function of the form $U(c/\eta)$ where η is the number of consumer equivalents in the household. A dead individual has a period utility function equal to a constant, so that the marginal value of wealth is zero.

Individuals rank stochastic consumption sequences according to the following intertemporal utility function, where $y_i = 1$ means that the agent is alive at age i and $y_i = 0$ means that she is dead.

$$\mathrm{E}\left[\sum_{i=20}^{99} \beta^{i-20} U(c_i/\eta_i, y_i)\right], \tag{2}$$

where β is the subjective discount factor and

$$U(x, 1) = \frac{x^{1-\sigma} - 1}{1-\sigma} \tag{3}$$

where σ is the reciprocal of the intertemporal elasticity of substitution and

$$U(x,0) = \kappa \tag{4}$$

where κ is an arbitrary constant.[7]

2.3 Process for productivity

Each individual under the age of 65 supplies one unit of labor inelastically. Labor earnings are given by $w \cdot e(i,s,g,z)$ where w is the economywide wage rate. Here z is a persistent stochastic process which is independent of i, s, and g. The function $e(i,s,g,z)$ is designed in such a way that $e(i,s,g,z) = 0$ for $i > 64$ and that average idiosyncratic earnings among those below 65 is 1, i.e.

$$\sum_{i,s,g} \mu_{i,s,g} \int_{\mathcal{Z}} e(i,s,g,z)\, d\mu_z = \sum_{i,s,g} I_{\{i<65\}} \mu_{i,s,g}.$$

where μ_z is the stationary measure over $\mathcal{Z} = \{z_1, \ldots, z_9\}$. An equivalent way of writing this is that

$$\int_{\mathcal{I}\times\mathcal{S}\times\mathcal{G}\times\mathcal{Z}} e(i,s,g,z)\, d\mu_{i,s,g,z} = \int_{\mathcal{I}\times\mathcal{S}\times\mathcal{G}\times\mathcal{Z}} I_{\{i<65\}} d\mu_{i,s,g,z}.$$

where $\mu_{i,s,g,z}$ is the measure over $\mathcal{I} \times \mathcal{S} \times \mathcal{G} \times \mathcal{Z}$, i.e.

$$\mu_{i,s,g,z}(A \times Z) = \mu_z(Z) \sum_{(i,s,g)\in A} \mu_{i,s,g}.$$

where $A \subset \mathcal{I} \times \mathcal{S} \times \mathcal{G}$ and $Z \subset \mathcal{Z}$.

The transition probabilities for z are denoted as follows.

$$P[z' \in Z | z = x] = Q(Z,x).$$

[7] Given this specification, marginal utility is $\frac{1}{\eta_t}\left(\frac{c_t}{\eta_t}\right)^{-\sigma}$. This means that behavior is the net result of two competing forces. On the one hand, as expressed by the first factor, utility is spread out more thinly in periods with a high value of η_t, making it more attractive to consume in periods with a low value of η_t. On the other hand, the second factor means that the agent wants to smooth consumption per consumption equivalent. An intertemporal elasticity equal to one means that the two effects cancel out, and hence that consumption decisions are independent of η_t. An intertemporal elasticity of substitution less than one means that the second effect dominates so that a household tends to consume more in periods with high values of η_t and less in periods with a low value η_t. However, even when the intertemporal elasticity of substitution is less than one, consumption per consumption equivalent tends to be lower in periods with a high value of η_t.

2.4 The public pension scheme

Under the public pension scheme, an individual of age 65 or above is entitled to a pension benefit given by $p(g_i, h_i, \tau^n)$ where g_i is the individual's marital status at age i, h_i is the individual's pension claim at age i and τ^n is the labor income tax, payable on part of the pension benefit. Pension claims evolve according to $h_i = \widetilde{H}(h_{i-1}, e_{i-1})$ where e_i is the idiosyncratic earnings component at age i. The set of all possible pension claims is denoted by $\mathcal{H} \subset \mathbb{R}_+$.

2.5 Distributions

The joint distribution of marital status, productivity and pension claims $\mu_{i,s,g,z,h}$ satisfies the equation

$$\mu_{i,s,g,z,h}(i+1, s, G' \times Z' \times H') = \int\limits_{\mathcal{G}\times\mathcal{Z}\times\mathcal{H}} \frac{\Pi_{i,s}(G'|g)}{1+\chi} Q(Z', z) I_{\left\{\widetilde{H}(h,e(i,s,g,z))\in H'\right\}} d\mu_{i,s,g,z,h}(i, s, \cdot),$$

where $H' \subset \mathcal{H}$ is a Borel measurable set and $I_{\{x\in X\}}$ is a function that takes the value 1 if $x \in X$ and 0 otherwise. We normalize so that

$$\sum_{i,s} \mu_{i,s,g,z,h}(i, s, \mathcal{G} \times \mathcal{Z} \times \mathcal{H}) = 1.$$

This defines the measure for rectangles; by Hahn's theorem, there is a unique extension to all the measurable sets.

As a further preliminary, we need to define the (marginal) distribution of (z, h) among individuals with a given (i, s, g). We have

$$\mu_{z,h}^{i,s,g}(Z \times H) = \frac{\mu_{i,s,g,z,h}(i, s, \{g\} \times Z \times H)}{\mu_{i,s,g,z,h}(i, s, \{g\} \times \mathcal{Z} \times \mathcal{H})}.$$

We use this to derive the distribution of (z, h) among newlyweds of a certain age and sex. Since the event of getting married is independent of shocks to z, this is just the distribution of (z, h) among those who were single in the previous year. We have

$$\widetilde{\mu}_{z,h}^{i+1,s}(Z' \times H') = \int\limits_{\mathcal{Z}\times\mathcal{H}} Q(Z', z) I_{\left\{\widetilde{H}(h,e(i,s,\text{single},z))\in H'\right\}} d\mu_{z,h}^{i,s,\text{single}}.$$

Using these definitions, we can define the distribution over all the exogenous variables (i,s,g,z,z^*,h,h^*). We have

$$\mu_{i,s,g,z,z^*,h,h^*}\left(i+1,s,G'\times Z'\times (Z^*)'\times H'\times (H^*)'\right)=$$
$$\int\limits_{\mathcal{G}\times\mathcal{Z}\times\mathcal{Z}\times\mathcal{H}\times\mathcal{H}} \tfrac{\Pi_{i,s}(G'|g)}{1+\chi} I_{\left\{\widetilde{H}(h,e(i,s,g,z))\in H'\right\}}\bullet$$
$$\Big[I_{\{g=\text{married}\}}Q\left(Z',z\right)Q\left(\left(Z^*\right)',z^*\right)I_{\left\{\widetilde{H}(h^*,e(i,s^*,g,z^*))\in (H^*)'\right\}}+$$
$$I_{\{g=\text{single}\}}Q\left(Z',z\right)\widetilde{\mu}_{zh}^{i+1,s^*}\left((Z^*)'\times (H^*)'\right)\Big]\,d\mu_{i,s,g,z,z^*,h,h^*}\left(i,s,\cdot\right).$$

Note that if an individual of age i and sex s is currently single, then the distribution of a prospective partner's z and h in the subsequent period is given by $\widetilde{\mu}_{zh}^{i+1,s^*}$. On the other hand, if the individual was previously married, then the distribution of the partner's z and h depends on the current values of these variables. Again, we normalize so that

$$\sum_{i,s}\mu_{i,s,g,z,z^*,h,h^*}\left(i,s,\mathcal{G}\times\mathcal{Z}\times\mathcal{Z}\times\mathcal{H}\times\mathcal{H}\right)=1.$$

A consistency requirement is that

$$\mu_{i,s,g,z,z^*,h,h^*}\left(i,\text{male},\{\text{married}\}\times Z\times Z^*\times H\times H^*\right)=$$
$$\mu_{i,s,g,z,z^*,h,h^*}\left(i,\text{female},\{\text{married}\}\times Z^*\times Z\times H^*\times H\right)$$

for all $Z,Z^*\subset\mathcal{Z}$ and all Borel sets $H,H^*\subset\mathcal{H}$.

2.6 State vector

A individual is characterized by eight characteristics; namely (i) whether the individual is alive or not, (ii) age, (iii) sex, (iv) marital status, (v) asset holdings, (vi) own stochastic productivity component, (vii) own pension claims, (viii) partner's stochastic productivity component, and (ix) partner's pension claims. We write $x=(y,i,s,g,a,z,h,z^*,h^*)$. Of course, the final two components only matter for a married individual and may be defined arbitrarily for single individuals.

2.7 Markets

The asset structure is very simple; there is only a one-period bond, traded on a world market so that the rate of return r is given exogenously. All other financial markets are closed. There is a domestic spot market for labor and a world spot market for capital. There are no borrowing constraints, except that an individual of age 99, who (together with any possible partner) dies with probability one, is not allowed to borrow. This implies that there exists a lower bound for assets, $\underline{a}(h, h^*)$, such that if, at age 99, $a \leq \underline{a}(h, h^*)$, then consumption is zero or negative and hence marginal utility is infinite. By definition, this rules out Ponzi schemes.

If a single agent dies, her assets are put into the bequest pool and handed out in equal amounts $\overline{b}$ to a share p_b of 51-year-olds. The probability of receiving a bequest is p_b; all 51-year-olds are equally likely to receive a bequest. If a married individual dies, assets are retained by the surviving spouse. For simplicity, we assume that at most one member of a married couple can die in any given year.

2.8 Production

Output, Y, is produced according to a Cobb-Douglas production function with capital share θ. That is,

$$Y = K^{\theta} N^{1-\theta}$$

where K is the domestically employed capital stock and N is the domestic labor force, which is given by

$$N = \sum_{i,s,g} I_{\{i<65\}} \mu_{i,s,g}.$$

The exogenously given rate of return r together with the labor force N implies a unique value of K. What remains to be determined is what fraction of K is held by domestic residents. We have

$$K = K^d + K^f.$$

where K^d denotes the wealth owned by domestic residents and K^f is the foreign debt. K^d is given by the following equation.

$$K^d = \int_{\mathcal{A}\times\mathcal{S}\times\mathcal{G}} \left[I_{\{s=\text{male}\}} + I_{\{s=\text{female}\}\cap\{g=\text{single}\}} \right] a d\mu_{a,s,g} \tag{5}$$

where $\mu_{a,s,g}$ is the distribution across individuals of assets, sex and marital status. The indicator functions are there to avoid double counting; the assets of married couples are only counted once.

2.9 A single individual's decision problem

Let $v(x)$ denote the maximized expected utility of an individual characterized by x. In this case, $x = (y, i, s, \text{single}, a, z, h, z^*, h^*)$ where z^* and h^* are irrelevant since there is no partner. Note that the distinction between being currently unmarried, widowed, or divorced is irrelevant from the point of view of decision-making. We have

$$v(x) = \max_{c \geq 0} \left\{ U\left(\frac{c}{\eta(x)}, 1\right) + \beta E\left[v(x') | x\right] \right\}$$

subject to

$$c + d = f + x_5$$

and

$$x' = J\left(x, d, g', z', (z^*)', d^*, b', (b^*)', (h^*)', y'\right)$$

where f is disposable income, which is given by

$$f = \left(1 - \tau^k\right) r x_5 + (1 - \tau^n) we(x_2, x_3, x_4, x_6) \qquad \text{if } x_2 < 65$$

or

$$f = \left(1 - \tau^k\right) r x_5 + p(x_4, x_7, \tau^n) \qquad \text{if } x_2 \geq 65$$

and where d is the savings of the individual, d^* is the savings of a new partner (if there is one), b' is the bequest received, $(b^*)'$ is the new partner's bequest (again if there is one), $(h^*)'$ is the pension claims of a new partner and $y' = 1$ if the individual survives and 0 otherwise. More precisely, the function J is defined via $x' = (0\ x_2\ x_3\ x_4\ 0\ x_6\ x_7\ x_8\ x_9)'$ if $y' = 0$ and

$$x_1' = y'.$$

$$x_2' = x_2 + y',$$

$$x_3' = x_3,$$

$$x_4' = g',$$

$$x_5' = d + b' \qquad \text{if } g' = \text{single}$$

$$x_5' = d + d^* + b' + (b^*)' \qquad \text{if } g' = \text{married}$$

$$x_6' = z'$$

$$x_7' = \widetilde{H}(x_7, e(x_2, x_3, x_4, x_6))$$

$$x_8' = (z^*)'$$

$$x_9' = (h^*)'$$

otherwise.

The probability distributions over (g', y'), z', b', $(b^*)'$ and $\left((z^*)', d^*, (h^*)'\right)$ are mutually independent, so we will describe these probability distributions separately. The required joint probabilities are then found by just multiplying the marginal probabilities.

The distribution of (g', y') conditional on g is described in section 2.1. The distribution of z' conditional on z is described in section 2.3. The distribution of bequests is as follows. If $x_2' \neq 51$, $b' = (b^*)' = 0$ with probability one. If $x_2' = 51$, then $b' = \bar{b}$ with probability p_b and $b' = 0$ with probability $1 - p_b$. The distribution of $(b^*)'$ is independent and identical. The value of $\bar{b}$ is determined in equilibrium so that total bequests received equals total legacies left behind. The distribution of $\left((z^*)', d^*, (h^*)'\right)$ conditional on x is not exogenously specified, but is a part of the equilibrium. We impose exogenously, however, that the conditional distribution of $\left((z^*)', d^*, (h^*)'\right)$ depends only on age x_2 and sex x_3.

2.10 A married household's decision problem

In our economy, married couple shares assets and enjoys equal amounts of consumption. The partners will typically not agree on the savings choice. This is because of the possibility of separation and death. For example, women expect to earn less and to live longer than men, so they would want to save more. As in Regalia & Ríos-Rull (1997), the solution is a compromise, i.e. the choice maximizes a weighted sum of the two partners' expected utility.

Denote a living man's state by $x = (1, i, \text{male}, \text{married}, a, z, h, z^*, h^*)$. Then his wife's state is $x^* = (1, i, \text{female}, \text{married}, a, z^*, h^*, z, h)$. The savings choice solves the following maximization problem, where π is the weight assigned to men.

$$\max_{c \geq 0} \left\{ U\left(\frac{c}{\eta(x)}, 1\right) + \beta E\left[\pi v(x') + (1 - \pi) v\left((x')^*\right) \middle| x\right] \right\}$$

subject to

$$c + d = f + x_5$$

and

$$x' = J_m\left(x, d, g', z', (z^*)', b', (b^*)', y', (y^*)'\right)$$

$$(x^*)' = J_m^* \left(x, d, g', z', (z^*)', b', (b^*)', y', (y^*)'\right)$$

where f is given by

$$f = \left(1 - \tau^k\right) r x_5 +$$

$$\left(1 - \tau^n\right) w \left[e\left(x_2, \text{male}, x_4, x_6\right) + e\left(x_2, \text{female}, x_4, x_8\right)\right] \qquad \text{if } x_2 < 65$$

or

$$f = \left(1 - \tau^k\right) r x_5 + p\left(x_4, x_7, \tau^n\right) + p\left(x_4, x_9, \tau^n\right) \qquad \text{if } x_2 \geq 65$$

and where d is household savings, b' is the husband's bequest, $(b^*)'$ is the wife's bequest, $y' = 1$ if the husband survives (0 otherwise) and $(y^*)' = 1$ if the wife survives (0 otherwise). More precisely, the functions J_m and J_m^* are defined via $x' = (0\ x_2\ x_3\ x_4\ 0\ x_6\ x_7\ x_8\ x_9)'$ if $y' = 0$, $(x^*)' = (0\ x_2^*\ x_3^*\ x_4^*\ 0\ x_6^*\ x_7^*\ x_8^*\ x_9^*)'$ if $y' = 0$, and

$$x_2' = x_2 + y'$$
$$(x_2^*)' = x_2 + (y^*)'$$
$$x_3' = x_3,$$
$$(x_3^*)' = x_3^*,$$
$$x_4' = (x_4^*)' = g',$$
$$x_5' = d + b' \qquad \text{if } g' = \text{widowed}$$
$$x_5' = \frac{d}{2} + b' \qquad \text{if } g' = \text{divorced}$$
$$x_5' = d + b' + (b^*)' \qquad \text{if } g' = \text{married}$$
$$(x_5^*)' = d + (b^*)' \qquad \text{if } g' = \text{widowed}$$
$$(x_5^*)' = \frac{d}{2} + (b^*)' \qquad \text{if } g' = \text{divorced}$$
$$(x_5^*)' = d + b' + (b^*)' \qquad \text{if } g' = \text{married}$$
$$x_6' = (x_8^*)' = z'$$
$$x_7' = (x_9^*)' = \widetilde{H}\left(x_7, e\left(x_2, x_3, x_4, x_6\right)\right)$$
$$(x_6^*)' = x_8' = (z^*)'$$
$$(x_7^*)' = x_9' = \widetilde{H}\left(x_9, e\left(x_2, x_3^*, x_4, x_8\right)\right).$$

Note that it is not possible to separate and remarry within a single year. The probability distributions over $\left(g', y', (y^*)'\right)$, z', $(z^*)'$, b', and $(b^*)'$ are mutually independent and are described above. The required joint probabilities are then found by just multiplying the marginal probabilities.

2.11 Equilibrium

A steady state equilibrium is a wage rate w, a decision rule $\widetilde{d}(x)$ for savings, and a distribution across individuals $\mu_{i,s,g,z,z^*,h,h^*,a} = \mu_x$ such that the following conditions hold.

1. Given the wage rate, $\widetilde{d}(x)$ solves the decision problem.
2. w equals the marginal product of labor.
3. μ_x is consistent with the exogenous dynamics and the decision rule, i.e.

   (a)

$$\mu_x\left(i+1, s, G' \times Z' \times (Z^*)' \times H' \times (H^*)' \times A'\right) =$$
$$\int\limits_{\mathcal{G}\times\mathcal{Z}\times\mathcal{Z}\times\mathcal{H}\times\mathcal{H}\times\mathcal{A}} \tfrac{\Pi_{i,s}(G'|g)}{1+\chi} I_{\left\{\widetilde{H}(h,e(i,s,g,z))\in H'\right\}} *$$
$$\left[I_{\{g=\text{married}\}} Q(Z', z)\, Q\left((Z^*)', z^*\right) I_{\left\{\widetilde{H}(h^*,e(i,s^*,g,z^*))\in (H^*)'\right\}} *\right.$$
$$\int\limits_{\mathbf{\Lambda}} I_{\left\{J_{m,5}\left(x,\widetilde{d}(x),\lambda\right)\in A'\right\}} dR(\cdot, x) + I_{\{g=\text{single}\}} Q(Z', z) *$$
$$\left.\int\limits_{\mathbf{\Upsilon}} I_{\left\{(z^*)'\in (Z^*)'\right\}} I_{\left\{(h^*)'\in (H^*)'\right\}} I_{\left\{J_5\left(x,\widetilde{d}(x),\upsilon\right)\in A'\right\}} dT(\cdot, x)\right] d\mu_x(i, s, \cdot)$$

   where $\mathbf{\Lambda}$ is the set of all possible values of $\lambda = \left(b', (b^*)', g', (y^*)'\right)$. $R(\Lambda, x)$ is the fraction, among those individuals characterized by the state vector x, such that $\lambda \in \Lambda \subset \mathbf{\Lambda}$. Thus for a fixed Λ, $R(\Lambda, \cdot)$ is a real-valued function, and for a fixed x, $R(\cdot, x)$ is a probability measure. This probability measure describes the behavior of λ_m, whose properties are described in sections 2.1 and 2.7. $\mathbf{\Upsilon}$ is the set of all possible values of $\upsilon = \left(b', (b^*)', d^*, (h^*)', (z^*)'\right)$. $T(\Upsilon, x)$ is the fraction, among those individuals characterized by the state vector x, such that $\upsilon \in \Upsilon \subset \mathbf{\Upsilon}$. The probability measure T is consistent with the equilibrium distribution of $\left(d^*, (h^*)', (z^*)'\right)$ among newlyweds of age i and sex s^*. b' and $(b^*)'$ are mutually independent and are also independent of the rest of υ.

   (b) The level $\overline{b}$ of bequests is determined by the following equation.

$$p_b\overline{b} = \int\limits_{\mathcal{X}} I_{\{g=\text{single}\}} \left(1-\gamma_{i,s,g}\right) \widetilde{d}(x)\, d\mu_x$$

   where $\mathcal{X} = \mathcal{I} \times \mathcal{S} \times \mathcal{G} \times \mathcal{Z} \times \mathcal{Z} \times \mathcal{H} \times \mathcal{H} \times \mathcal{A}$.

(c) The distribution of (d, h', z') among newlyweds of a given age and sex is determined as follows. First we write down the distribution of (d, h', z') among individuals of a given age, sex and marital status.

$$\mu_{d,z,h}^{i,s,\widehat{g}}\left(D \times Z \times H\right) = \frac{\int\limits_{\mathcal{G}\times\mathcal{Z}\times\mathcal{Z}\times\mathcal{H}\times\mathcal{H}\times\mathcal{A}} I_{\{g=\widehat{g}\}\cap\left\{\widetilde{d}(x)\in D\right\}\cap\{z\in Z\}\cap\{h\in H\}} d\mu_x\left(i,s,\cdot\right)}{\mu_{i,s,\widehat{g}}}.$$

where $d \in D \subset \mathcal{D} = \mathbb{R}$. The distribution of (d, h', z') among newlyweds of a given age and sex then satisfies

$$\widetilde{\mu}_{d,z,h}^{i+1,s}\left(D \times Z' \times H'\right) = \int\limits_{\mathcal{D}\times\mathcal{Z}\times\mathcal{H}} I_{\{d\in D\}} Q\left(Z', z\right) I_{\left\{\widetilde{H}(h,e(i,s,\text{single},z))\in H'\right\}} d\mu_{d,z,h}^{i,s,\text{single}}.$$

4. The government budget balances

$$C_g + \int\limits_{\mathcal{X}} I_{\{i>64\}} p\left(g, h, \tau^n\right) d\mu_x = \tau^n w N + \tau^k r K^d \qquad (6)$$

where $c(x)$ is the consumption function implied by the household's budget constraint and the decision rule $\tilde{d}(x)$.

5. Feasibility

$$C_g + \int\limits_{\mathcal{X}} \left[I_{\{s=\text{male}\}} + I_{\{s=\text{female}\}\cap\{g=\text{single}\}}\right] c(x) d\mu_x + \delta K + r K^f = Y \qquad (7)$$

What we have defined above is the distribution across individuals. To derive the distribution across *households,* we just exclude the married men (or women, but not both). See the derivation of domestically held assets in equation (5).

2.12 Calibration

2.12.1 Marriage, survival and household preferences

Half of the individuals enter the economy as men and half as women. A certain fraction enter as married. In each period, an individual can transit from being single to married, married to divorced or widowed, and from alive to dead. The fraction of 20-year-olds that are married as well as the transition probabilities are set according to numbers reported by Statistics Sweden (see SCB (1993)). Marriage and separation probabilities depend on age and sex. The probability of dying depends on age, sex and marital status. To make sure that these probabilities are consistent with one another, we do the following. The transition probabilities pertaining to men as well as the survival probability for single women are all taken from the data. The remaining transition probabilities are calculated as follows, using the notation of section 2.1.[8]

$$\pi_{i,\text{female}}\left(\text{widowed}|\text{married}\right) = \pi_{i,\text{male}}\left(\text{dead}|\text{married}\right)$$

$$\pi_{i,\text{female}}\left(\text{dead}|\text{married}\right) = \pi_{i,\text{male}}\left(\text{widowed}|\text{married}\right)$$

$$\pi_{i,\text{female}}\left(\text{divorced}|\text{married}\right) = \pi_{i,\text{male}}\left(\text{divorced}|\text{married}\right)$$

$$\pi_{i,\text{female}}\left(\text{married}|\text{single}\right) = \pi_{i,\text{male}}\left(\text{married}|\text{single}\right)\frac{\mu_{i,\text{male,single}}}{\mu_{i,\text{female,single}}}.$$

In order to keep the number of possible ages finite, we set the probability of dying at age 99 to one. Using data from 1912-1992, we find that the rate of population growth χ is approximately 0.005.

Recall that the period utility function takes the form $U\left(c/\eta\right)$ We calibrate the value of η as the number of consumer equivalents in the household.[9] For example, a single individual with no children is counted as 1.15, and a couple without children is counted as 1.9. A child adds between 0.45 and 0.75 consumer equivalents, depending on its age. Each household has a number of consumer equivalents that is given by a deterministic function of the household's age, marital status and, if applicable, sex. These numbers are averages across households taken from the Household Income Survey (HINK; see Domeij and Klein (1998)). For details, see Table 7.

The coefficient of relative risk aversion, σ, is set to 1.5. The calibration of the subjective discount factor β is discussed in section 2.12.6; see Table 3 for the numbers.

For simplicity, we (arbitrarily) set $\pi = 1$ so that husbands are dominant in decision-making within households, but to check the robustness of the results we intend to examine the consequences of reversing this assumption.

2.12.2 Bequests

Each 51-year old receives a fixed inheritance $\bar{b}$ with probability p_b. The number p_b is set so as to match the coefficient of variation for received inheritances.

[8] Given that $\mu_{20,s,g}$ are taken from the data, we can derive all the other $\mu_{i,s,g}$ etc recursively using equation 1.

[9] More precisely, we use the notion of *normer för baskonsumtion* as it is defined by *Socialstyrelsen.*

Using data reported in Laitner & Ohlsson (1998), we find that the coefficient of variation is $c_b = 4.874$. This implies a probability of receiving a bequest of $p_b = \dfrac{1}{1+c_b} = 0.1702$. The inherited amount is set so as to exactly exhaust the amount of funds left behind by the dead single agents.[10] See Table 3 for details.

2.12.3 Earnings process

Each individual under the age of 65 receives supplies one unit of labor inelastically. Labor earnings are given by an exogenous idiosyncratic stochastic process. In the data, we interpret earnings to mean income from work plus transfer payments except the pensions of those of age 65 and over. The earnings process is modeled as follows. The natural log of earnings at time t of individual j, e_t^j, consists of two components. The first, α_t^j, depends deterministically on age, sex and marital status. The second, z_t^j, follows an AR(1) process (which we for computational purposes approximate by a nine-state Markov process). Following Flodén & Lindé (1999), we estimate the following equations using the Generalized Method of Moments;

$$e_t^{j,obs} = \alpha_t^j + z_t^j + \xi_t^j \tag{8}$$

and

$$z_t^j = \rho z_{t-1}^j + \varepsilon_t^j \tag{9}$$

where $e_t^{j,obs}$ is observed log earnings, ξ_t^j is a white noise measurement error, ε_t^j is a white noise process and α_t^j are the fitted values from regressing $e_t^{j,obs}$ on age, age squared, and dummies for sex and marital status. We write

$$\alpha_t^j = \beta_0 + \beta_1 \text{AGE}_t^j + \beta_2 \left(\text{AGE}_t^j\right)^2 + \beta_3 I_{\{\text{SEX=male}\}} + \beta_4 I_{\{\text{MARITAL STATUS=married}\}}$$

where β_0 is adjusted so that the sample average of $\exp\left(\alpha_t^j\right)$ is 1. The data are taken from HINK ($t = 1988, 1989, 1992$). Our estimate of ρ is about 0.92. When approximated by a seven-state Markov process and normalizing average earnings to one, the process for z is characterized by

$$z \in \{-1.596, -1.242, -0.8883, -0.5346, -0.1809, 0.1727, 0.5246, 0.8801, 1.234\}$$

and a probability transition matrix with second greatest eigenvalue equal to 0.9249 (see Table 8) and whose stationary probabilites are given by

$$\mu_z = (0.0184, 0.0547, 0.1204, 0.1933, 0.2263, 0.1933, 0.1204, 0.0547, 0.0184).$$

[10] Recall that when one of the partners of a marriage dies, the surviving partner retains all the assets.

This approximation is constructed so that the mean of $\exp(z_t)$ is equal to one.

This means that the earnings function is defined via $e(i,s,g,z) = \exp(\alpha(i,s,g) + z)$ where

$$\alpha(i,s,g) = \beta_0 + \beta_1 i + \beta_2 i^2 + \beta_3 I_{\{s=\text{male}\}} + \beta_4 I_{\{g=\text{married}\}}.$$

2.12.4 The public pension scheme

The model's pension scheme, which is a somewhat stylized version of the Swedish one, has three components; one earnings-based benefit, one common benefit, and a housing subsidy.[11]

Each individual has a pension claim based on lifetime earnings in the following way. Each year, 18.5 percent of earnings up to a cut-off level is added to an account. The cut-off level is 1.05 times GDP per adult. After age 65, the account pays interest where the rate of return is stipulated by law to 1.6 percent. Retirees then receive an amount which is such as to exhaust the account exactly if the remaining lifetime is equal to its expected value at retirement. At age 65, the average expected remaining life-time is 17 years.

On top of that, each individual receives a common benefit of 0.27 GDP per adult if married and of 0.30 if single. (All numbers in this section are henceforth multiples of GDP per adult.) However, this common benefit is reduced one-for-one by the earnings-based benefit when the earnings-based benefit is in the interval $[0, 0.16]$ if married and $[0, 0.18]$ if single. When the earnings-based benefit is in the interval $[0.16, 0.39]$, and $[0.18, 0.43]$ for singles, then the common benefit is reduced by 48 percent of the difference between the earnings-based benefit and 0.16(0.18) for married (single) households.

The common benefit and the earnings-based benefit are considered as labor income and are taxed at rate τ^n. Retirees also receive a housing-subsidy of 0.14 GDP per adult,[12] which is not taxed. This somewhat intricate system is

[11] There was a reform of the Swedish pension system in 1998. Its main features were (i) pension benefits were made contingent on lifetime earnings rather than just the 15 years with the highest earnings and (ii) benefits were made contingent on the rate of growth of the economy. Keeping track of the best 15 years is computationally very costly because the state space becomes enormous. We therefore assume, in our models, that pension benefits are based on lifetime earnings as in the new system. We leave for future research the issue of whether the reform is important for the issues that we study.

[12] In fact, the housing subsidy is equal to 85 percent of housing costs up to a cutoff point which is roughly equal to 18 percent of GDP per adult. Assuming that most but not all retirees

illustrated in Figure 1. It is clear from the figure that the pension system is very redistributive indeed. The mapping from past earnings to after-tax pension benefits is nearly flat at a rather high level.

2.12.5 Government purchases and taxation

Government purchases are set so as to match the 1960-1996 average ratio of general government purchases to GDP, which is 27 percent. The capital income tax τ^k is set to 30 percent, which is the rate legally payable on personal income from capital and capital gains in Sweden. The labor income tax τ^n is set so as to balance the budget. See Table 3 for details on the numbers.

2.12.6 Production

Output is produced according to a Cobb-Douglas production function with capital share $\theta = 0.36$. Capital depreciates by 10 percent per year. The return on capital (net of depreciation) is given exogenously by the world market and is calibrated to 3 percent on an annual basis. Given the stationary population, aggregate labor input is a given constant. This determines the capital stock. The subjective discount factor β is calibrated so that the ratio of foreign debt (the amount of domestically employed capital owned by foreign residents) K^f to GDP Y is 0.37 (which is the number for 1992 in the data). See Table 3 for details.

3 Results

3.1 Main results

In what follows, we will be talking about four distinct model economies. The baseline economy is the one described in section 2. The other three abstract from, respectively, (i) marriage, (ii) pensions, and (iii) marriage and pensions. Our main results are the following. In the first place, we are able to replicate the degree of disposable income inequality with a reasonable degree of accuracy. In the baseline economy, the Gini coefficient for disposable income is 0.35 (it is 0.31

receive the maximum housing benefit (which is a reasonable assumption given Swedish housing costs as reported by Statistics Sweden), a housing benefit of 14 percent of GDP per adult should not be too far from the truth.

in the data). Secondly, the pension system goes a very long way in accounting for wealth inequality. Without pensions (but with marriage), the model Gini coefficient for net wealth is just 0.54. In the baseline economy, it is 0.71 (it is 0.79 in the data). In addition to this, we are able to replicate fairly well the fraction of households with zero or negative net wealth. The baseline economy features 29 percent of households with non-positive net wealth (this number is 0.24 in the data).

These results are in line with the fact noted by Quadrini & Ríos-Rull (1997), namely that OLG models where agents receive income throughout their lives generate a lot of wealth inequality by predicting a large fraction of households with very low wealth. With US data, this is a problem, because wealth inequality there is to a large extent driven by the very rich. But with Swedish data, a large fraction of households with very low wealth is precisely what we observe. Note that, in the economies without pensions, households have no income apart from capital income after the age of 64. This explains why so few households have zero or negative wealth in this economy.

3.2 Detailed properties of the income and wealth distributions

Looking at Table 1, we see three things. In the first place, the baseline economy captures the salient properties of the income distribution. In particular, it captures the Gini coefficient and the shares earned by the bottom 40 percent, the top 20 percent, the top 10 percent and the top 1 percent. Secondly, the models without pensions do worse. In particular, they predict a higher Gini coefficient and a lower share of the bottom 40 percentiles. The reason for this, of course, is that the pension system redistributes income. It is no surprise that it is the bottom of the income that is affected the most, since the pension system, by definition, transfers income from those with labor earnings to those without. Finally, the phenomenon of marriage does not seem to matter much for the income distribution.

A shortcoming of all our models, however, is that they do not capture the double-peakedness of the disposable income distribution; see Figure 4.

Looking at Table 2, we see that the results for net wealth are qualitatively similar to those pertaining to disposable income, but they are quantitatively more striking. These results are particularly interesting given that the distribution of assets is endogenously determined in the model, whereas the lion's share of disposable income (labor earnings) is determined exogenously.

In the first place, the baseline economy captures the salient properties of the wealth distribution. Again, the models without pensions do worse; indeed, much worse. In particular, they predict a much too low Gini coefficient and too high a share of the bottom 40 percentiles and too low shares for all the top groups. Thirdly, the phenomenon of marriage does not seem to matter all that much for the wealth distribution either, but in this case modeling marriage does improve the empirical fit somewhat. (For a related result, see Cubbedu & Ríos-Rull 1997.) Notice also that the baseline economy comes very close to capturing the fraction of households that have zero or negative net wealth, whereas the models without pensions are very far from the facts in this respect.

However, as Figure 4 shows, we are not able to capture the detailed features of the lower tail of the wealth distribution. Although the models with pensions exhibit a similar fraction of households with zero or negative net wealth, the models tend to exaggerate the bunching around zero.

All the model economies tend to overpredict the correlation between income and wealth (see Table 2). This is a phenomenon common to models of this sort with exogenous labor supply; see Domeij & Heathcote (1999). If labor supply is endogenous, the predictions are typically closer to the facts in this respect; see Flodén & Lindé (1999).

3.3 Lifetime wealth and income profiles

Figure 2 displays the average disposable income of households by age (in the data, the age is that of the head of household). The baseline economy is able to capture the U-shaped aspect of the age profile of income. On the other hand, the level is underpredicted. This is because our HINK sample has a higher share of married households than the general population, from which we take our marriage probabilities. More precisely, in our simulated populations (where the transition probabilities are taken from populations in 1990 and 1991), about 42 percent of households are married (which is close to the 48 percent that we observe in 1990 population); in the HINK sample, that number is 65 percent. Clearly, if we abstract from marriage altogether, we do even worse in terms of matching the age profile of disposable income.

Apart from demographics, there are two things that account for the differences between the model-based curves in Figure 2, namely tax rates and capital income. Labor income per individual should be the same in all cases. The models without pensions feature lower tax rates; this accounts for a large fraction of the difference in disposable income; for the 20-year-olds, the difference in tax rates accounts for the whole difference. Capital income reinforces this effect, since in the absence of pensions, households accumulate more assets than in their presence, and hence their capital income is higher.

Figures 3a and 3b display age-wealth profiles for the four model economies and the data. We notice immediately that bequests lead to a noticeable jump in wealth for the 51-year-olds. It is particularly large in the absence of pensions, since accidental bequests are much larger in this case. The models with pensions, though, match the age-wealth profile fairly well. However, it does tend to overpredict the accumulation of young people and underpredict the asset holdings of old people. The latter property is not hard to understand, since the model does not feature a bequest motive.

Finally, we notice the spectacular overprediction of asset holdings in the models without pensions. One way of expressing this is that our models predict that if the public pension scheme were abolished, then, in the long run, Swedes would end up owning (at least) all of Scandinavia. See Table 3 for the exact figures.

3.4 Within-cohort inequality

In Table 4 we display the Gini coefficients for disposable income by age group. For those below the age of 65, the models predict no strong relationship between age and inequality, and this is in line with the data. The level of inequality, on the other hand, is slightly overpredicted in all the model economies.

For those above 65, the models with pensions underpredict the degree of income inequality, and the opposite is true for the models without pensions. One source of this discrepancy is that, in the data, some households with heads over 65 have members who are still working. If we remove these households, the Gini coefficient for the over-65s shrinks to 0.25. Nevertheless, it seems clear that our models exaggerate the redistributive effects of public pensions.

Concerning the Gini coefficients for net wealth by age group, our baseline economy is the only one of our models that even comes close to capturing the age profile of wealth inequality. As we can see in Table 5, the data exhibit a decline in the degree of wealth inequality in age. The baseline economy also exhibits a decline in wealth inequality for those under 65, although the level of inequality is lower than in the data. However, for the above-65s, the baseline economy predicts an *increase* in wealth inequality, which is out of line with the data.

Notice also that, in the model without pensions or marriage, where bequests are very large, wealth inequality takes a leap as we pass from the 45-49-year-olds to the 50-54-year-olds. The reason for this, of course, is that 17 percent of the 51-year-olds receive a large bequest and the others receive nothing. In the other models, bequests are too small and initial inequality too large for this effect to appear.

As Table 6 shows, in the data it is mainly the young that have zero or negative net wealth. The models with pensions predict this too. However, the models have two counterfactual predictions. First, the middle-aged are more indebted in the data than in the models. Second, the models with pensions predict too high a fraction of indebted households among those over 65. This is clearly an important source of the high wealth inequality among the old predicted by the models with pensions that we reported above. One way to understand the high predicted degree of indebtedness among the retirees is that they no longer face idiosyncratic income risk; therefore they have no precautionary motive for saving.

Given the counterfactual predictions with respect to wealth holdings of retirees, it is worth stressing that the baseline model performs reasonably well with respect to households (with heads) under 65; call these the working households. In the data, the Gini coefficient for net wealth among working households is 0.83, and the baseline model predicts 0.67. Meanwhile, the fraction of households with zero or negative net wealth in this group is 0.27, and the baseline model predicts 0.23. In summary, it may be said that we underpredict inequality and indebtedness among working households and overpredict it for the retirees.

4 Discussion

Using a calibrated overlapping generations model with a realistic demographical structure and idiosyncratic earnings risk, we found strong support for the view that the pension system goes a very long way in accounting for the difference between income and wealth with respect to inequality. It is particularly encouraging that the models seem to generate wealth inequality via the right channel: both in the models and the data, wealth inequality is mainly driven by the large fraction of households with zero or negative wealth.

The main shortcoming of the models with pensions is that they predict that old people go into debt; this is not a feature of the data. Presumably the introduction of an altruistic bequest motive could remedy this problem. Moreover, a detailed modeling of intergenerational links could help account for the upper tail of the wealth distribution, which we don't quite capture either. These points, as they apply to the United States and Sweden, are analyzed in some detail in de Nardi (1999).

A related point is that we suppress other aspects of the welfare state which may discourage savings, so that it is somewhat disturbing that we tend to overpredict the fraction of households with zero or negative net wealth. If we extended the model to incorporate these aspects, it is likely that we the models would do worse. But the inclusion of an altruistic bequest motive could mitigate this, perhaps sufficiently to bring the predictions in line with the data.

We leave for future research a detailed investigation of the above issues as they interact with public pensions as well as some sensitivity analysis. In particular, it would be interesting to see what happens if we relax the assumption that husbands are dominant in decision-making. We should expect that this should lead to higher savings, since married women expect to live longer and earn less in case of divorce than men do.

References

[1] Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving". *Quarterly Journal of Economics* **109(3)**, pp. 659-684.

[2] Castañeda, Ana, Javier Díaz-Gímenez, and José-Víctor Ríos-Rull. 1998. "Earnings and Wealth Inequality and Income Taxation", manuscript, University of Pennsylvania.

[3] de Nardi, Mariacristina. 1999. "Wealth inequality, intergenerational links and estate taxation", manuscript, University of Chicago.

[4] Díaz-Gímenez, Javier, Vincenzo Quadrini, and José-Víctor Ríos-Rull. 1997. "Dimensions of Inequality: Facts on the US Distributions of Earnings, Income and Wealth", *Quarterly Review (Spring)*. Federal Reserve Bank of Minneapolis.

[5] Domeij, David and Jonathan Heathcote. 1999. "Capital versus labor taxation with heterogeneous agents", manuscript, Stockholm School of Economics.

[6] Domeij, David and Paul Klein. 1998. "Inequality of Income and Wealth in Sweden", manuscript.

[7] Flodén, Martin and Jesper Lindé. 1999. "Idiosyncratic risk in the US and Sweden: Is there a role for government insurance?", manuscript, Stockholm School of Economics.

[8] Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes. 1995. "Precautionary Saving and Social Insurance", *Journal of Political Economy* **103(2)**, pp. 360-399.

[9] Huggett, Mark. 1995. "Wealth distribution in life cycle economies", manuscript.

[10] Huggett, Mark and Gustavo Ventura. 1997. "Understanding why high income households save more than low income households", manuscript.

[11] Quadrini, Vincenzo. 1999. "Entrepreneurship, Saving and Social Mobility", *Review-of-Economic Dynamics,* forthcoming.

[12] Quadrini, Vincenzo and José-Víctor Ríos-Rull. 1997. "Understanding the US Distribution of Wealth", *Quarterly Review (Spring)*. Federal Reserve Bank of Minneapolis.

[13] SCB, (1993), Be15 SM 9301, Familjeförändringar kring 1990.

[14] SCB, (1994), Befolkningsstatistik del 4, 1992.

A Computation of the equilibrium

We fix grids for marital status, assets, age, own productivity, partner's productivity, own pension claims and partner's pension claims. We use piecewise multilinear interpolation to evaluate the decision rule at points not on the grid.[13]

The algorithm is initialized by guessing the values of the size of the bequest $\overline{b}$, the labor tax rate τ^n, the patience rate β, the beliefs about the distribution of prospective partners' assets, productivity and pension claims $\widetilde{\mu}^{i,s}_{d,z,h}$. In the latter case, we simplify matters by imposing that beliefs over a prospective partner's assets and pension claims are a deterministic function of sex and age and that a prospective partner's productivity is distributed according to the stationary distribution μ_z.

The decision rules are then solved for and the guesses are updated by simulating the economy. In our simulated economies, 300 individuals of each sex enter at age 20. This implies that, at any point in time, the economy is inhabited by approximately 32000 individuals.

B Tables and figures

[13] The grids for marital status, assets, age, own productivity, partner's productivity, own pension claims and partner's pension claims consists of 3, 19, 15, 3, 3, 3, 3 points, respectively. This means that, for the baseline economy, each iteration requires us to solve 28899 first order conditions.

Table 1: The disposable income distribution: Gini coefficient and shares

	Models with marriage		no marriage		Data
	pensions	no pensions	pensions	no pensions	
Gini	0.35	0.47	0.31	0.43	0.31
0-40%	0.19	0.10	0.21	0.13	0.19
80-100%	0.42	0.49	0.40	0.47	0.37
90-100%	0.26	0.29	0.25	0.29	0.22
99-100%	0.04	0.04	0.04	0.05	0.04

Table 2: The net wealth distribution: Gini coefficient and shares

	Models with marriage		no marriage		Data
	pensions	no pensions	pensions	no pensions	
Gini	0.71	0.54	0.65	0.54	0.79
0-40%	-0.02	0.06	0.01	0.08	-0.06
80-100%	0.70	0.54	0.64	0.56	0.72
90-100%	0.46	0.34	0.41	0.37	0.50
99-100%	0.08	0.05	0.07	0.06	0.13
Share($a \leq 0$)	0.29	0.04	0.18	0.02	0.24
$\rho(f,a)$	0.63	0.41	0.62	0.46	0.31

Note: Share($a \leq 0$) is the share of households with non-positive net wealth.
$\rho(f, a)$ is the correlation between disposable income and net wealth.

Table 3: Patience rate, bequests, labor tax rate and foreign debt/GDP ratio

	Models with marriage pensions	no pensions	no marriage pensions	no pensions
β	0.99	0.99	1.00	1.00
$\overline{b}$	2.11	21.23	9.15	42.94
τ^n	0.52	0.29	0.54	0.31
$\frac{K^f}{Y}$	0.37	-6.30	0.37	-5.39

Table 4: The distribution of disposable income by age: Gini coefficients

	Models with marriage		no marriage		Data
	pensions	no pensions	pensions	no pensions	
Age					
20-24	0.37	0.37	0.34	0.34	0.30
25-29	0.38	0.38	0.35	0.34	0.26
30-34	0.37	0.37	0.36	0.35	0.25
35-39	0.35	0.35	0.35	0.34	0.26
40-44	0.34	0.34	0.35	0.34	0.28
45-49	0.34	0.33	0.36	0.33	0.26
50-54	0.34	0.32	0.35	0.32	0.29
55-59	0.33	0.30	0.34	0.31	0.28
60-64	0.34	0.30	0.33	0.29	0.29
65-	0.16	0.41	0.12	0.45	0.29
All	0.35	0.47	0.31	0.43	0.31

Table 5: The distribution of net wealth by age: Gini coefficients

	Models with marriage		no marriage		Data
	pensions	no pensions	pensions	no pensions	
Age					
20-24	1.17	0.77	0.75	0.64	2.56
25-29	0.82	0.60	0.59	0.47	2.31
30-34	0.69	0.52	0.54	0.42	1.40
35-39	0.62	0.46	0.52	0.39	0.95
40-44	0.58	0.42	0.52	0.37	0.82
45-49	0.54	0.37	0.52	0.35	0.70
50-54	0.49	0.37	0.50	0.44	0.63
55-59	0.49	0.33	0.50	0.41	0.60
60-64	0.51	0.21	0.52	0.38	0.55
65-	0.84	0.41	0.75	0.45	0.56
All	0.71	0.54	0.65	0.54	0.79

Table 6: Fraction of households with non-positive net wealth

	Models with marriage pensions	no pensions	no marriage pensions	no pensions	Data
Age					
20-24	0.59	0.37	0.34	0.24	0.43
25-29	0.33	0.05	0.11	0.01	0.52
30-34	0.23	0.02	0.07	0.00	0.42
35-39	0.19	0.00	0.05	0.00	0.31
40-44	0.18	0.00	0.05	0.00	0.24
45-49	0.11	0.00	0.06	0.00	0.19
50-54	0.07	0.00	0.07	0.00	0.15
55-59	0.06	0.00	0.06	0.00	0.12
60-64	0.08	0.00	0.12	0.00	0.10
65-	0.43	0.00	0.36	0.00	0.06
All	0.29	0.04	0.18	0.02	0.24

Table 7: Average consumption equivalents by age, sex and marital status

	Single		Married
	male	female	
Age			
20-24	1.15	1.21	2.13
25-29	1.16	1.38	2.50
30-34	1.19	1.65	2.92
35-39	1.22	1.79	3.15
40-44	1.25	1.65	2.99
45-49	1.22	1.38	2.51
50-54	1.19	1.28	2.13
55-59	1.17	1.16	1.98
60-64	1.15	1.16	1.92
65-	1.15	1.15	1.90

Table 8: The probability transition matrix for the stochastic productivity component in the earnings process: z

0.644	0.337	0.019	0.000	0.000	0.000	0.000	0.000	0.000
0.114	0.573	0.298	0.014	0.000	0.000	0.000	0.000	0.000
0.003	0.136	0.590	0.261	0.011	0.000	0.000	0.000	0.000
0.000	0.004	0.163	0.600	0.226	0.008	0.000	0.000	0.000
0.000	0.000	0.006	0.193	0.603	0.193	0.005	0.000	0.000
0.000	0.000	0.000	0.008	0.226	0.600	0.163	0.004	0.000
0.000	0.000	0.000	0.000	0.011	0.261	0.590	0136	0.003
0.000	0.000	0.000	0.000	0.000	0.014	0.298	0.573	0.114
0.000	0.000	0.000	0.000	0.000	0.000	0.195	0.337	0.644

Figure 1: The public pension scheme for a single individual

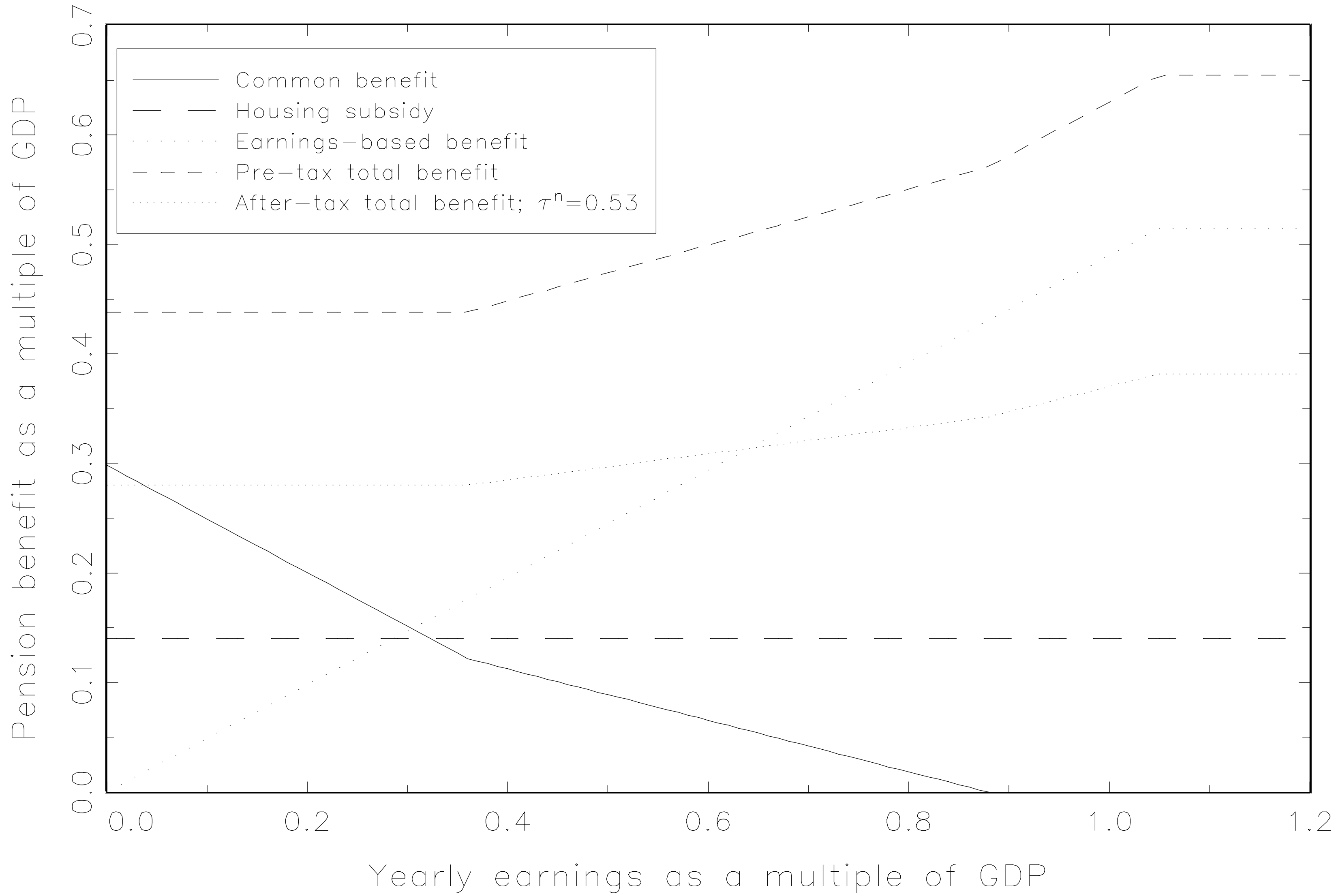

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Figure 2: Average disposable income (as a multiple of GDP per capita) by age

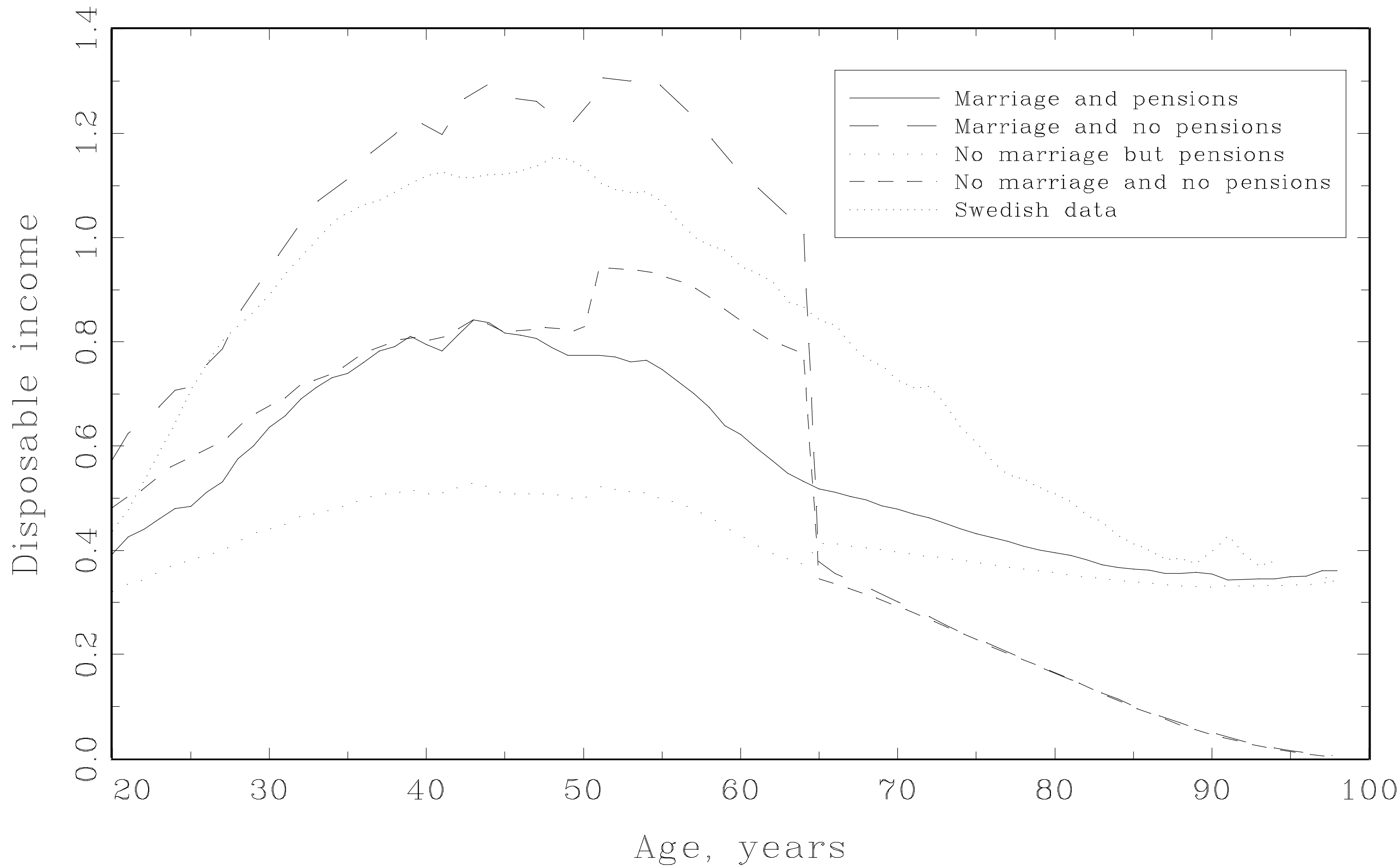

Figure 3a: Average net wealth (as a multiple of GDP per capita) by age

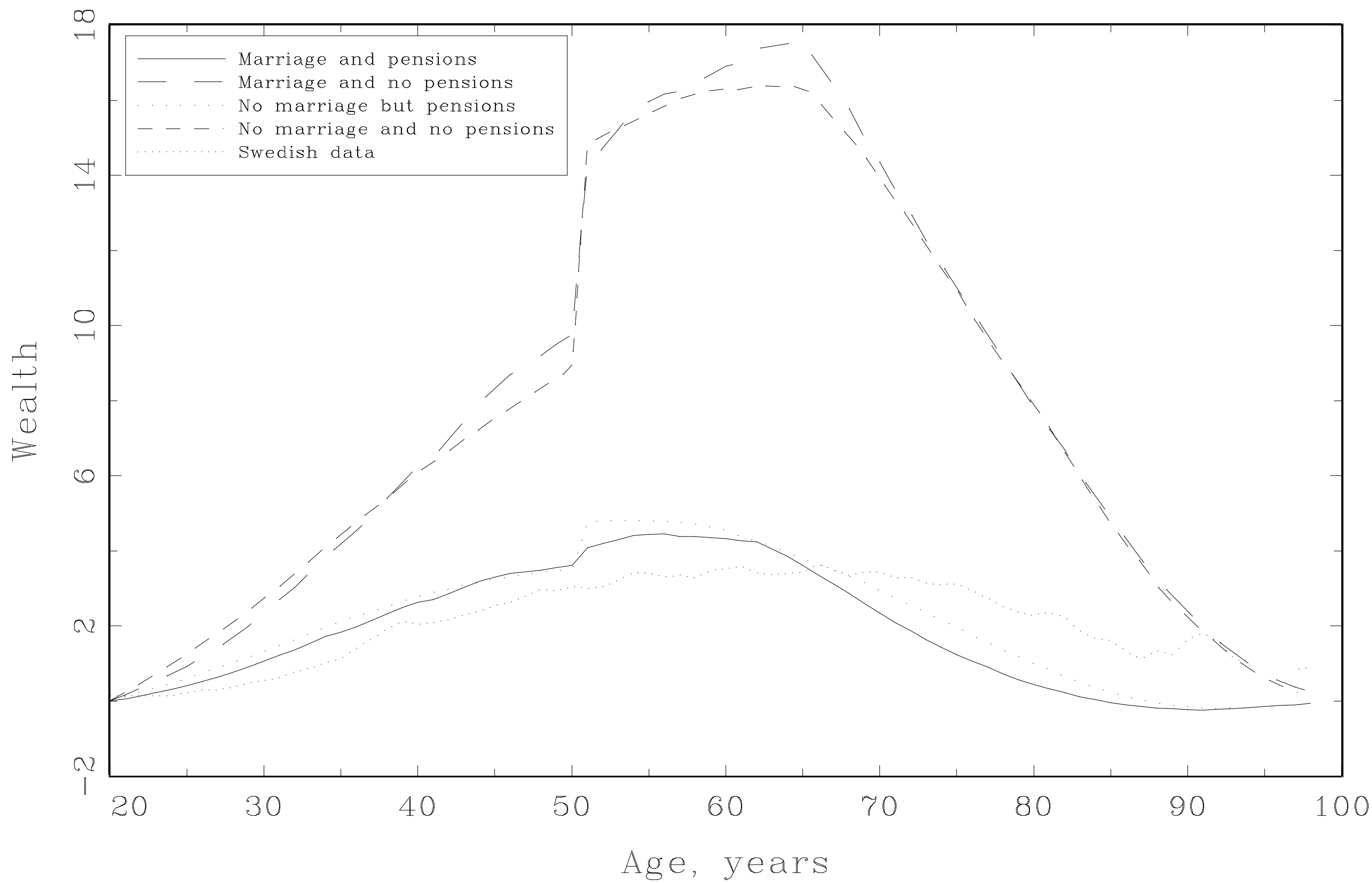

Figure 3b: Average net wealth (as a multiple of GDP per capita) by age

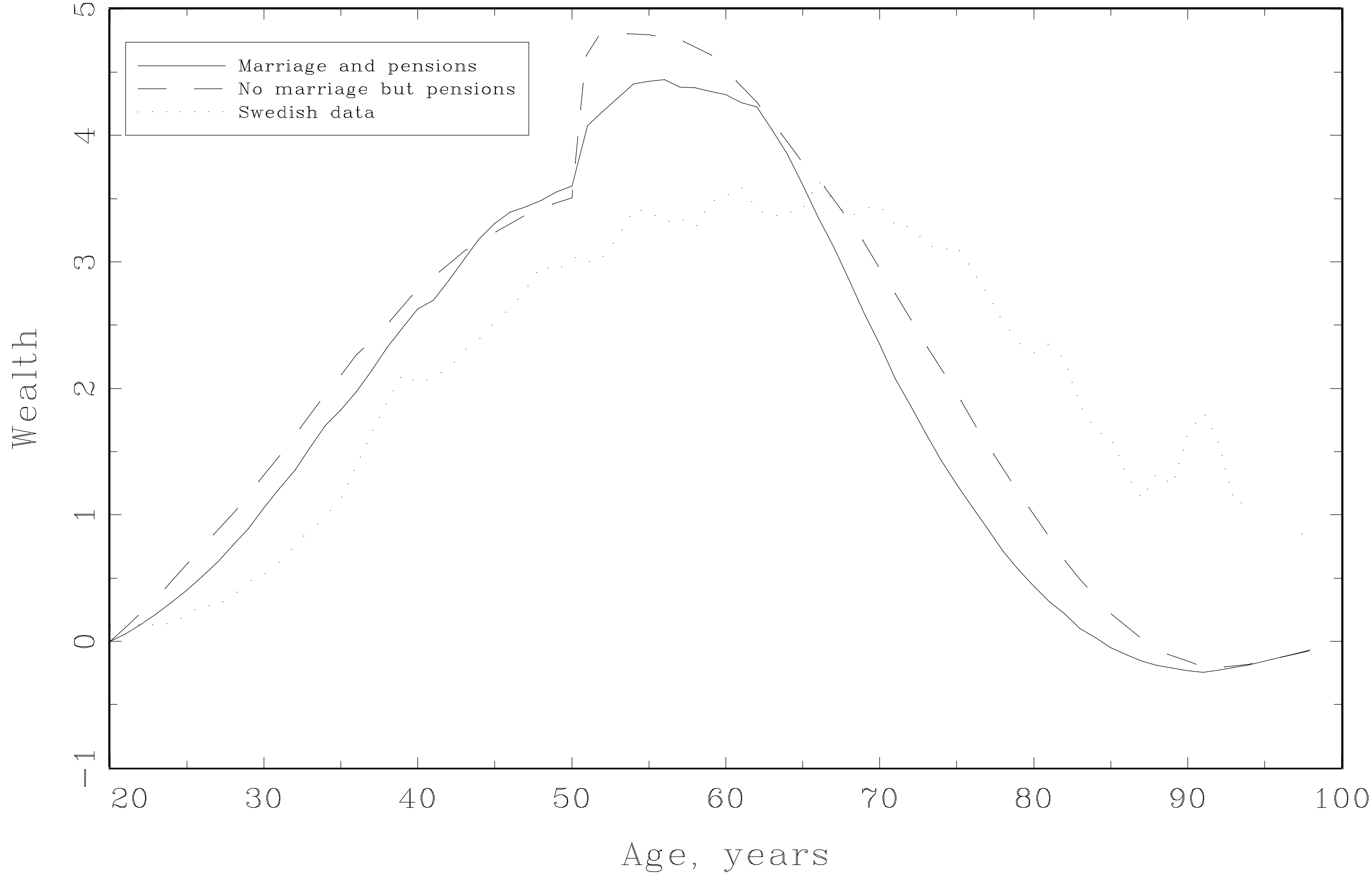

Figure 4: Disposable income distribution

Figure 4a: Disposable income distribution, marriage and pensions

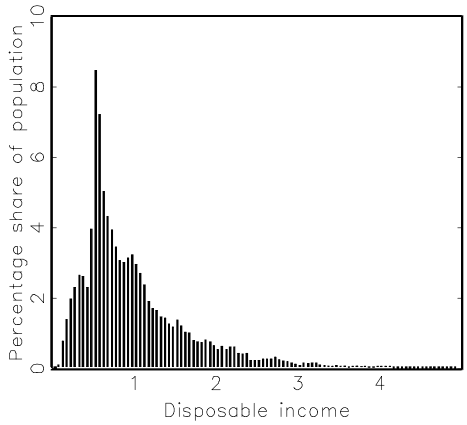

Figure 4b: Disposable income distribution, marriage and no pensions

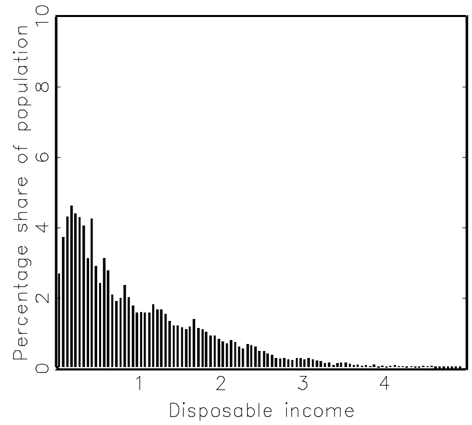

Figure 4c: Disposable income distribution, no marriage but pensions

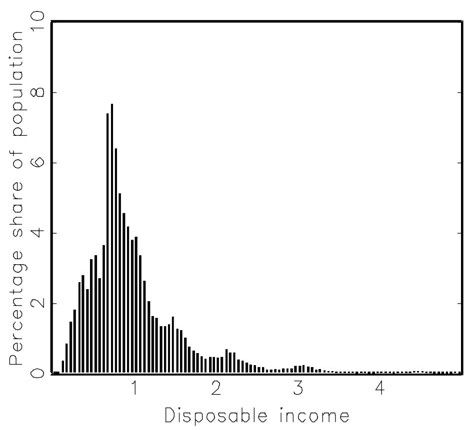

Figure 4d: Disposable income distribution, no marriage and no pensions

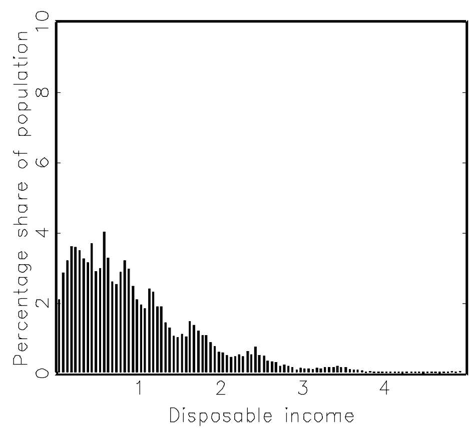

Figure 4e: Disposable income distribution, Swedish data

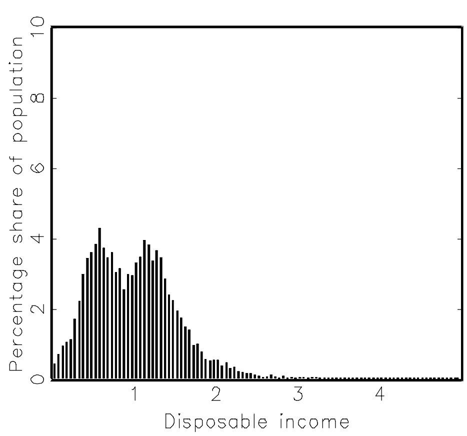

Figure 5: Wealth distribution

Figure 5a: Wealth distribution,
marriage and pensions

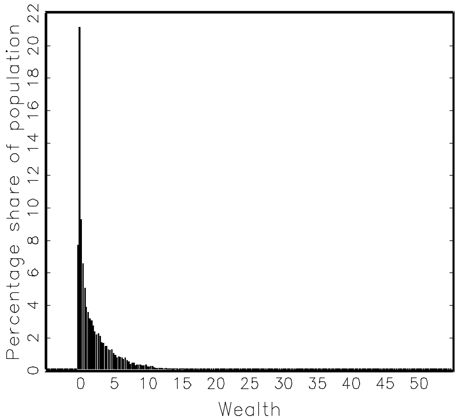

Figure 5b: Wealth distribution,
marriage and no pensions

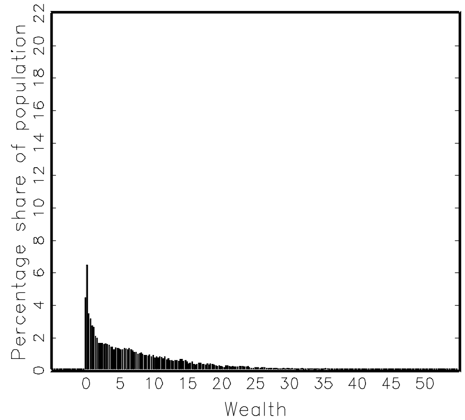

Figure 5c: Wealth distribution,
no marriage but pensions

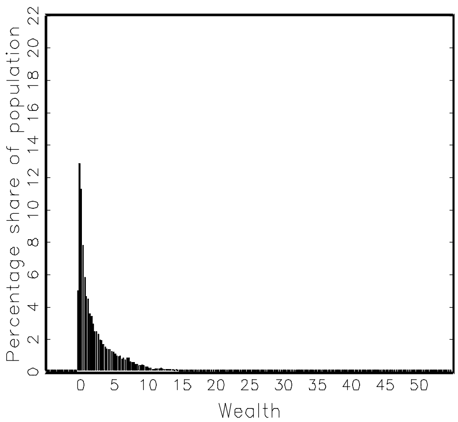

Figure 5d: Wealth distribution,
no marriage and no pensions

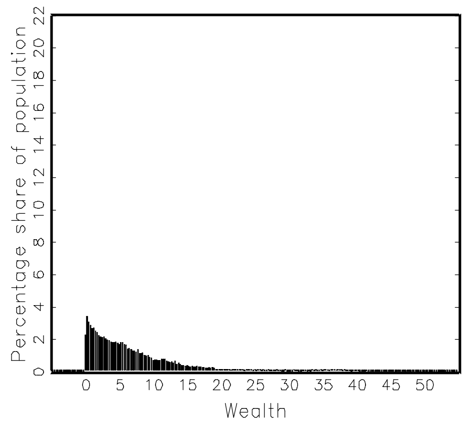

Figure 5e: Wealth distribution,
Swedish data