Is There a Positive Intertemporal Tradeoff between Risk and Return After All?

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ABSTRACT: This paper develops an extended version of Turner, Startz, and Nelson's (1989) Markov-switching model of stock returns. The model is motivated as an alternative version of Campbell and Hentschel's (1992) volatility feedback model, with news about future dividends subject to a two-state Markov-switching variance. We are able to identify an endogenous volatility feedback effect by assuming that economic agents acquire information about market volatility that is not directly available to econometricians. Using this model, we find strong evidence for a positive tradeoff between volatility and the equity risk premium, especially for post-War stock returns.

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1. Introduction

A central issue in the modern empirical finance literature is the intertemporal relationship between stock market volatility and stock returns. Interest in this relationship stems from the idea that market volatility is the most easily motivated measure of aggregate risk. To the extent that it is a good measure of risk, and investors are rational and risk-averse, higher than normal volatility should increase the equity premium, defined as the expected excess return on a market portfolio over the risk-free interest rate. But higher market volatility, if unanticipated, should also initially generate a negative feedback effect on the realized excess return as investors bid stock prices down until future expected excess returns are high enough to compensate for the increase in non-diversifiable risk. Both effects reflect the same underlying tradeoff between risk and return.

In this paper, we develop a Markov-switching model of stock returns with endogenous volatility feedback and use it to estimate the intertemporal relationship between volatility and the equity premium. Contrary to findings in numerous studies, including Glosten, Jagannathan, and Runkle (1993) and Whitelaw (1994), we find evidence of a statistically significant positive tradeoff. Meanwhile, our approach builds on French, Schwert, and Stambaugh (1987), Turner, Startz, and Nelson (1989), and Campbell and Hentschel (1992) by incorporating endogenous volatility feedback in the presence of a two-state Markov-switching variance process for news about future dividends. Like Campbell and Hentschel, we derive an explicit link between the feedback effect and the relationship between volatility and the equity premium. But, whereas Campbell and Hentschel employ a quadratic autoregressive conditional heteroskedasticity (QGARCH) specification, we follow Turner, Startz, and Nelson and argue for a Markov-switching specification. The Markov-switching specification allows us to assume that economic agents observe information about volatility unavailable to econometricians, thus avoiding estimation complications, discussed in Campbell and Hentschel, that arise from the quadratic relation between the excess return and dividend news for their QGARCH specification. The Markov-switching specification is also more consistent with findings in Hamilton and Susmel (1992) that once Markov switching is accounted for, persistent ARCH effects all but disappear at the monthly return horizon considered in this paper.

We extend Turner, Startz, and Nelson's (1989) Markov-switching model by employing Campbell and Shiller's (1998a,b) log-linear present value framework to derive a theoretical link between volatility feedback and the relationship between volatility and the equity premium. The link makes it clear that any evidence of a negative feedback effect is equivalent to evidence of a positive tradeoff between volatility and the equity premium. It also reveals why it may be easier in practice to detect a volatility feedback effect than the contemporaneous effect of volatility on the equity premium. Deriving the link allows us to extend Turner, Startz, and Nelson's model by imposing a testable restriction on the volatility feedback effect. We also extend their model by considering an alternative assumption about the evolution of information available to economic agents. The assumption used in Turner, Startz, and Nelson, which we refer to as 'partial revelation,' has economic agents, like econometricians, observing only past returns at the beginning of a trading period, but observing information that is approximated by the current volatility regime by the end of the period. The assumption is time inconsistent since the current volatility regime contains information about next period's volatility regime that is not inherent in observed returns. Our assumption, which we refer to as 'full revelation,' is time consistent since agents observe the previous volatility regime at the beginning of the trading period and the current volatility regime by the end of the period.

To motivate and build support for our Markov-switching model of stock returns with endogenous volatility feedback, we estimate a series of models using CRSP data on excess returns for a value-weighted market portfolio to answer the following questions: Is there Markov-switching volatility? Is there a Markov-switching equity premium? Is there a volatility feedback effect? Is there a positive relationship between volatility and the equity premium? Are changes in volatility exogenous? Most notably, we find strong evidence of a volatility feedback effect for the post-War (1952-96) period, although the evidence for the pre-War (1926-51) period is weaker. Also, for both sample periods, we find support for a positive relationship between volatility and the equity premium. The evidence is particularly strong when we impose the restriction on the empirical model based on the theoretical link implied by the present-value framework, although this restriction can be rejected for the pre-War sample. These results hold for both information specifications.

The rest of this paper is organized as follows. Section 2 reviews the previous literature on the intertemporal relationship between volatility and the equity premium in more detail. Section 3 motivates and develops our model. Section 4 reports empirical results. Section 5 concludes.

2. Previous Literature

Past studies have produced mixed evidence on the intertemporal relationship between volatility and the equity premium. In their seminal paper, French, Schwert, and Stambaugh (1987) find a positive relationship using basic time-series models, including a generalized autoregressive conditional heteroskedasticity in mean (GARCH-M) model inspired by Engle, Lilien, and Robins (1987). Likewise, Bollerslev, Engle, and Wooldrige (1988), Harvey (1989), and Campbell and Hentschel (1992) find a positive relationship. However, other empirical studies using similar data and methodologies have produced conflicting results. For example, Glosten, Jagannathan, and Runkle (1993) find a negative relationship between the conditional expected excess market return and its conditional variance. They employ a modified version of the GARCH-M model that incorporates the nominal interest rate in the calculation of the conditional variance. Similarly, Whitelaw (1994) finds a negative relationship between the conditional expected market return and its conditional variance calculated by generalized method of moments (GMM) estimation with multiple financial variable instruments. Other studies that find a negative relationship include Fama and Schwert (1977), Campbell (1987), Breen, Glosten, and Jagannathan (1989), and Nelson (1991). Furthermore, theoretical studies by Abel (1988), Backus and Gregory (1993), and Gennotte and Marsh (1993) have convincingly demonstrated that modern general equilibrium models of stock prices can imply a negative relationship between volatility and the equity premium for at least some range of structural parameter values.

The empirical results have been more uniform for studies incorporating volatility feedback. Volatility feedback—originally proposed by Pindyck (1984) as an explanation for the lackluster performance of the stock market in the 1970s—is the idea that expectations of high volatility put immediate downward pressure on stock prices. This effect should be easier to detect empirically than the direct effect of volatility on a contemporaneous expected excess return since it reflects the cumulative effect of changes in expectations about future volatility on all future discounted expected excess returns. Thus, when French, Schwert, and Stambaugh (1987) employ a two-step approach that regresses the excess market return on the predictable and unexpected components of an integrated autoregressive moving average (ARIMA) model of volatility, they are able to find strong evidence for a negative volatility feedback effect. Similarly, Turner, Startz, and Nelson (1989) find a negative feedback effect using a special case of the Markov-switching model employed in this paper. Campbell and Hentschel (1992) find a negative feedback effect using QGARCH model of volatility. But their paper is most notable for developing a theoretical model of volatility feedback. They show that, if dividend news is subject to a QGARCH process and there is a linear relationship between news volatility and the expected excess market return, Campbell and Shiller's (1988a,b) log-linear present value model of stock prices endogenously generates volatility feedback. Furthermore, the explicit solution for the volatility feedback term implies that any empirical evidence of a negative volatility feedback effect is equivalent to evidence of a positive relationship between volatility and the equity premium. More recently, Bakaert and Wu (1999) find empirical support for a negative volatility feedback. They employ a model that compares the volatility feedback effect with the leverage effect proposed by Black (1976) and Christie (1982).

3. The Model

3.1 Background

Stock returns are related to prices by the following identity:

$$R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} - 1, \tag{1}$$

where R_{t+1} denotes the return on a stock or portfolio held from time t to $t+1$, P_{t+1} is the (ex-dividend) price of the stock or portfolio at the end of period $t+1$, and D_{t+1} is dividend at time $t+1$, claimed by the owner at the beginning of time $t+1$. Solving recursively for P_t, applying the expectations operator, and imposing a transversality condition to rule out the existence of rational bubbles in asset prices, obtains the following present-value relation:

$$P_t = \mathrm{E}\left[\sum_{i=1}^{\infty}\left\{D_{t+i}\Big/\prod_{j=1}^{i}(1+R_{t+j})\right\}\Big|\, y_t\right], \qquad (1')$$

where y_t denotes information available at time *t*. Then, solving for the log-linear approximate present-value relation developed by Campbell and Shiller (1988a,b) and Campbell (1991), obtains the following:

$$p_t = \frac{k}{1-r} + \mathrm{E}\left[\sum_{j=0}^{\infty} r^j[(1-r)d_{t+1+j} - r_{t+1+j}]\Big|\, y_t\right], \qquad (1'')$$

where lower-case letters denote log values and r and k are parameters of linearization (see Campbell and Shiller, 1988a). This last representation allows us to simultaneously examine the effects of changes in expected dividends and changes in future expected returns.

3.2 Volatility Feedback

Campbell and Hentschel's (1992) volatility feedback model is a partial equilibrium model of stock returns that relies on the log-linear present-value relation given in equation (1'') and two simple assumptions. The first assumption is that news about dividends follows a QGARCH process. The second assumption is that the expected return is a linear function of the conditional variance of news about future dividends. As mentioned in Campbell and Hentschel, the first assumption can be amended to allow news to follow a Markov-switching process. This is the approach we take in this paper. In particular, we make the following two assumptions:

(i) news about dividends is subject to the following zero-mean, two-state Markov-switching variance process:

$$e_t \sim \mathrm{N}(0, s^2_{S_t}), \tag{2}$$

$$s^2_{S_t} = s^2_0(1 - S_t) + s^2_1 S_t,\ s^2_0 < s^2_1,$$

$$\Pr[S_t = 0 | S_{t-1} = 0] = q \text{ and } \Pr[S_t = 1 | S_{t-1} = 1] = p,$$

where e_t denotes new information about dividends that arrives during trading period *t*, $s^2_{S_t}$ is the variance of e_t, S_t is a Markov-switching state variable that takes on discrete values of zero or one according to the prevailing volatility regime, and *q* and *p* are the transition probabilities governing the evolution of S_t;

(ii) the expected return is a linear function of market perceptions—formed rationally in the sense of Muth (1960)—about the volatility of news about dividends:

$$\mathrm{E}[r_t | y_t] = m_0 + m_1 \Pr[S_t = 1 | y_t], \tag{3}$$

where both m_0 and m_1 are hypothesized to be positive, reflecting a positive price of risk.

Numerous studies have used a Markov-switching variance assumption to model stock returns (Schwert, 1989, Turner, Startz, and Nelson, 1989, Hamilton and Susmel, 1994, Schaller and van Norden, 1997, Kim, Nelson, and Startz, 1998, and Kim and Nelson, 1998). In terms of motivating its use here, Hamilton and Susmel's findings are of particular interest. They apply a Markov-switching autoregressive conditional heteroskedasticity (SWARCH) model to weekly stock returns and find that, once ARCH parameters are allowed to switch between regimes according to an unobserved Markov-switching state variable, estimated ARCH effects are much less persistent than for a standard ARCH model. Indeed, their results imply that monthly stock returns can be reasonably modeled as having only a Markov-switching conditional variance. Furthermore, the Markov-switching assumption avoids estimation complications associated with a nonlinear relation between excess returns and dividend news for the

QGARCH specification used in Campbell and Hentschel (1992). Instead, estimation of our model is a straightforward application of the procedure discussed in Hamilton (1989).

Following Campbell and Hentschel (1992), we can manipulate the log-linear present value model given in equation (1’’) to show that a return has three components reflecting the expected return, news, and a volatility feedback effect:

$$r_t = \mathrm{E}[r_t | y_t^b] + e_t - f_t , \tag{4}$$

where

$$e_t \equiv \mathrm{E}\left[\sum_{j=0}^{\infty} r^j \Delta d_{t+j} \Big| y_t^e\right] - \mathrm{E}\left[\sum_{j=0}^{\infty} r^j \Delta d_{t+j} \Big| y_t^b\right]$$

denotes news about dividends (y_t^b is information available at the beginning of the trading period, while y_t^e is information available by the end of the trading period) and

$$f_t \equiv \mathrm{E}\left[\sum_{j=1}^{\infty} r^j r_{t+j} \Big| y_t^e\right] - \mathrm{E}\left[\sum_{j=1}^{\infty} r^j r_{t+j} \Big| y_t^b\right]$$

denotes revisions in future expected returns and turns out to be the volatility feedback term.

The assumptions given in equations (2) and (3) directly provide us with expressions for the first two components of equation (4). However, we need to solve for the third component, f_t. Note that the expected return at some arbitrary point in the future is given by

$$\mathrm{E}[r_{t+j} | y_t] = m_0 + m_1 \Pr[S_t = 1] + m_1 l^j (\Pr[S_t = 1 | y_t] - \Pr[S_t = 1]) , \tag{5}$$

where $l \equiv p+q-1$ (see Hamilton, 1989). This expression can be used to show that the discounted sum of future expected returns is

$$\mathrm{E}\left[\sum_{j=1}^{\infty} r^{j} r_{t+j} \middle| y_{t}\right] = \frac{m_0}{1-r} + \frac{m_1}{1-r}\Pr[S_t = 1] + \frac{m_1}{1-r\,l}(\Pr[S_t = 1|y_t] - \Pr[S_t = 1]), \quad (6)$$

which, in turn, allows us to solve for f_t:

$$f_t = \frac{m_1}{1-r\,l}(\Pr[S_t = 1|y_t^{e}] - \Pr[S_t = 1|y_t^{b}]). \quad (7)$$

Given these specific expressions for the three components of equation (4), we can write the our model as follows:

$$r_t = m_0 + m_1 \Pr[S_t = 1|y_t^{b}] + d\{\Pr[S_t = 1|y_t^{e}] - \Pr[S_t = 1|y_t^{b}]\} + e_t, \quad (4')$$

where

$$d \equiv -\frac{m_1}{1-r\,l}. \quad (8)$$

Thus, a positive price of risk directly implies that, as long as volatility is persistent (i.e., $p+q>1$), the coefficient on the volatility feedback term, d, is negative. Furthermore, persistent volatility implies that a change in the discounted sum of future expected returns is much larger—and, therefore, easier to detect empirically—than a change in the contemporaneous expected return. Note that the parameter of linearization, r, which is the average ratio of the stock price to the sum of the stock price and the dividend, is slightly less than one (0.997) in practice.

3.3 Information about Volatility

Before we proceed to estimation of the model, we need to specify exactly how economic agents acquire information over time. First, the assumption of rational expectations implies that both y_t^b and y_t^e in equation (4') should contain information available to econometricians, including past returns ($r_{t-1}, r_{t-2,} \ldots$). Second, volatility feedback occurs only if the information available to agents changes within trading periods—that is, y_t^e contains information not in y_t^b. Thus, we can only generate a volatility feedback effect by assuming that agents acquire information about the volatility regime (S_t) that is not directly available to econometricians.

In the next section, we consider the following information specifications when estimating models based on equations (2) and (4'). First, we assume that there is no volatility feedback and the information available to economic agents does not change within each trading period—i.e., $y_t^b = y_t^e = y_t$. Initially, we also assume that expected excess returns are constant over time ($m_1 = 0$) to demonstrate that evidence of Markov-switching volatility is not a consequence of a switching mean. Then, we allow expected excess returns to switch according to the volatility regime, with agents observing past returns ($y_t = r_{t-1}, r_{t-2}, \ldots$) and the true state ($y_t = S_t$), respectively. These two specifications provide benchmark polar cases for information available to agents, given rational expectations. Then, we assume that there is volatility feedback and the information available to economic agents does change within each trading period—i.e., $y_t^b \neq y_t^e$. Initially, we assume that there is only 'partial revelation,' as in Turner, Startz, and Nelson (1989). That is, $y_t^b = r_{t-1}, r_{t-2}, \ldots$ and $y_t^e = S_t$, where the true regime S_t acts as a proxy for what is actually known by agents at the end of time *t*. We consider both the case where d is freely estimated and where it is restricted according to equation (8). Then, we assume that there is 'full revelation.' That is, $y_t^b = S_{t-1}$ and $y_t^e = S_t$. Again, we consider both cases for d. This latter information specification has the advantage of being time consistent.

4. Empirical Results

4.1 Data

The data are excess stock returns on a market portfolio. In particular, we employ continuously compounded total monthly returns for a value-weighted portfolio of all NYSE-listed stocks in excess of continuously compounded one-month U.S. Treasury bill yields. The data are drawn from the CRSP data files for the sample period of January 1926 to December 1996. Total returns represent capital gains plus dividend yields. Continuously compounded returns are calculated by taking natural logarithms of simple gross returns.

The use of excess returns means that 'news' refers to information about future dividends relative to future interest rates. A relative measure of this kind makes sense since the theoretical effects of volatility on real returns alone are ambiguous, even if we assume a positive relationship between volatility and the equity premium. In particular, an increase in risk could cause investors to substitute away from riskier assets, putting downward pressure on interest rates. Using excess returns allows us to avoid misinterpreting the effects of this downward pressure on the equity premium.

For the empirical exercises, we split the sample into two subperiods: 1926-51 and 1952-96. The breakpoint corresponds to the Fed-Treasury Accord and is also used in Campbell and Hentschel (1992). We look at the periods separately since using the full data sample may produce misleading implications about the effects of volatility on stock returns if the behaviour of volatility has changed dramatically since the Great Depression. There is considerable reason to believe that the behaviour of volatility has changed (see, for example, Pagan and Schwert, 1990). Pre-War excess returns underwent episodes in which volatility was considerably higher and more persistent than ever occurred in the post-War period, suggesting the need for a more complicated model of volatility than is employed in this paper. Kim, Nelson, and Startz (1998) and Kim and Nelson (1998) provide such a model, which allows for three volatility regimes. Nevertheless, their findings suggest that the two-regime model employed here and in Turner, Startz, and Nelson (1989) is sufficient to describe pre- and post-War returns separately.

4.2 Is there Markov-Switching Volatility?

Tables 1a and 1b report maximum likelihood estimates for constant and switching variance models of stock returns for the sample periods of 1926-51 and 1952-96, respectively.[1] We can use these estimates to examine the evidence for Markov-switching volatility in stock returns. For both samples, there is a huge improvement in the log likelihood values for the Markov-switching specification. For the 1926-51 sample, the likelihood ratio statistic the null hypothesis of no Markov switching ($H_0 : s_0 = s_1$) is 145.94. For the 1952-96 sample, the likelihood ratio statistic is 47.04. While the distribution of these test statistics is non-standard since the transition probabilities q and p are not identified under the null (see Hansen, 1992, and Garcia, 1995), their values far exceed asymptotic critical values reported in Garcia (1995). Therefore, the results are at least suggestive of the presence of Markov-switching volatility in stock returns. They also provide the basis for our claim, discussed below, that evidence of Markov switching is not merely a consequence of allowing for a switching mean, but is largely driven by changes in volatility. This claim is important since our theoretical model treats Markov-switching volatility as exogenous and the mean return as endogenous.

The estimates also confirm our claim that pre-War returns underwent episodes of higher and more persistent volatility than in the post-War period. In particular, note that the standard deviation of monthly returns in the high volatility regime is 0.12 in the 1926-51 sample period versus 0.06 in the 1952-96 sample period. Likewise, the probability of staying in the high volatility regime is 0.96 versus 0.91, corresponding to an expected duration of about 25 months versus 11 months. The numbers are implicit in Figures 1a and 1b, which display excess returns and smoothed probabilities of a high volatility regime for the sample periods of 1926-51 and 1952-96, respectively. These findings all suggest that the separation of the entire 1926-96 sample period into the two subsamples is appropriate, although they do not pinpoint with any great precision the exact date at

[1] All maximum likelihood estimation was conducted using the OPTMUM procedure for the GAUSS programming language. All models with Markov switching were estimated using the filter presented in Hamilton (1989). Numerical derivatives were used in estimation, as well as for calculation of asymptotic standard errors. Parameters were appropriately constrained (e.g., variances were constrained to be non-negative). Inferences appear robust to a variety of starting values.

which the behaviour changed. The Fed-Treasury Accord merely provides a convenient dividing line.

4.3 Is there a Markov-Switching Equity Premium?

Tables 2a and 2b report maximum likelihood estimates for the model of the effects of Markov-switching volatility on stock returns with benchmark specifications for information about volatility available to economic agents. We can use these estimates to examine the evidence for a Markov-switching equity premium. When agents are assumed to observe past returns, there is only weak evidence of a switching mean. The likelihood ratio statistic for the null hypothesis of a constant mean return ($H_0 : m_1 = 0$) is 0.78, with a *p*-value of 0.38, for the 1926-51 sample and 2.16, with a *p*-value of 0.14, for the 1952-96 sample. However, when agents are assumed to observe the true volatility regime, there is stronger evidence of a switching mean. The likelihood ratio statistics are 4.76, with a *p*-value of 0.03, for the 1926-51 sample and 6.96, with a *p*-value of 0.01, for the 1952-96 sample. But, the estimated relationship between volatility and the equity premium is actually negative for this specification, with *t*-statistics for m_1 of –2.18 and –2.13 for the 1926-51 and 1952-96 samples, respectively. These results suggest the presence of a Markov-switching equity premium. They also suggest that economic agents act upon information inherent in the true volatility regime. But, the results leave open the question of whether the negative correlation between volatility and mean return is the result of a negative price of volatility or a volatility feedback effect.

4.4 Is there a Volatility Feedback Effect?

Tables 3a and 3b report maximum likelihood estimates for the model of the effects of Markov-switching volatility on stock returns with feedback due to partial revelation. We can use these estimates to examine the evidence of a volatility feedback effect. The model nests both benchmark specifications for information about volatility available to economic agents. For the null hypothesis of no feedback with agents observing only past returns ($H_0 : d = 0$), the likelihood ratio statistics are 4.85, with a *p*-value of 0.03, for the 1926-51 sample and 21.57, with a *p*-value of <0.01, for the 1952-96 sample. The *t*-statistics for the feedback term d are –2.23 and –6.59 for the 1926-51 and

1952-96 samples, respectively. These results suggest the presence of volatility feedback. For the null hypothesis of no feedback with agents observing and only acting upon the true volatility regime ($H_0 : m_1 - d = 0$), the likelihood ratio statistics are 0.87, with a *p*-value of 0.35, for the 1926-51 sample and 16.56, with a *p*-value of <0.01, for the 1952-96 sample. These results suggest the presence of a volatility feedback effect in post-War returns, although the evidence for pre-War returns is weaker.

Tables 4a and 4b report maximum likelihood estimates for the model of the effects of Markov-switching volatility on stock returns with feedback due to full revelation. Again, we can use these estimates to examine the evidence of a volatility feedback effect. The model only nests the benchmark specification that agents observe and only act upon the true volatility regime, but it does assume a time consistent evolution of information. For the null of no feedback with agents observing the true state ($H_0 : m_1 - d = 0$), the likelihood ratio statistics are 2.44, with a *p*-value of 0.12, for the 1926-51 sample and 14.43, with a *p*-value of <0.01, for the 1952-96 sample. As with the partial revelation model, these results suggest the presence of a volatility feedback effect in post-War returns, with weaker pre-War evidence.

4.5 Is there a Positive Relationship between Volatility and the Equity Premium?

Having established the presence of a volatility feedback effect, at least for post-War returns, we examine the evidence of a positive relationship between volatility and the equity premium. Looking at the results for the model with feedback due to partial revelation (Tables 3a and 3b), it is apparent from the *t*-statistics on the feedback term d reported in the previous subsection that the feedback effect is negative and significant for both sample periods when the feedback parameter is unrestricted. The estimated direct effect m_1 is even positive, although not significant (*t*-statistic is 0.32), for the 1952-96 sample. Since volatility is very persistent ($p + q > 1$), these results provide support for a positive tradeoff. Similarly, when the volatility feedback parameter is restricted by theory according to equation (8), the estimated relationship between volatility and the equity premium is always positive, with *t*-statistics for m_1 of 1.10 and 3.17 for the 1926-51 and 1952-96 samples, respectively. The restriction given in equation (8) can be rejected at the 10 percent level for the 1926-51 sample—the likelihood ratio statistic is 3.22, with *p*-

value of 0.07—but, it cannot be rejected for the 1952-96 sample—the likelihood ratio statistic is 0.27, with *p*-value 0.60. Thus, the results for the constrained model provide support for a positive tradeoff, especially for post-War returns.

Looking at the results for the model with feedback due to full revelation (Tables 4a and 4b), the feedback effect is always negative and significant when the feedback parameter is unrestricted. The *t*-statistics are –3.08 and –6.07 for the 1926-51 and 1952-96 samples, respectively. Similarly, when the volatility feedback parameter is restricted by theory according to equation (8), the estimated relationship between volatility and the equity premium is always positive, with *t*-statistics for m_1 of 1.67 and 2.14 for the 1926-51 and 1952-96 samples, respectively. However, the restriction can be rejected at conventional levels for both sample periods. The likelihood ratio statistics are 3.96, with a *p*-value of 0.05, for the 1926-51 sample and 6.86, with a *p*-value of 0.01, for the 1952-96 sample. Still, the results are supportive of a positive relationship between volatility and the equity premium.

4.6 Is Volatility Exogenous?

The robustness of the estimates of the Markov-switching variance process across all of the models is notable. Again, an underpinning of our theoretical model is the idea that volatility is exogenous, with returns endogenously reacting to its changes. The fact that the estimates of the Markov-switching process reported in the tables change little when we allow the mean, as well as the variance, to switch provides informal support for the idea the volatility is exogenous. Further support comes from the similarity between the smoothed probabilities of a high volatility regime across all of the models. For example, Figures 2a and 2b display excess returns and smoothed probabilities for the model with restricted feedback due to full revelation. The timing of changes in regime is virtually indistinguishable from the timing given in Figures 1a and 1b. The differences that do exist are subtle, but there does appear to be a stronger relationship between periods of high volatility and NBER-dated recessions for the more complicated models. In particular, the occurrence of high volatility outside of recessions is rare, and when it does occur, it is typically short-lived. Future research will investigate the relationship between volatility, the equity premium, and the business cycle.

5. Conclusions

In this paper, we build on Campbell and Hentschel's (1992) volatility feedback model by assuming a two-state Markov-switching variance process for news about future dividends, rather than the harder to estimate QGARCH process assumed in their paper. We also extend Turner, Startz, and Nelson's (1989) Markov-switching model of stock returns by formally deriving an underlying theoretical model and explicit link between volatility feedback and the effect of volatility on the equity premium. We use CRSP data on excess returns for a value-weighted market portfolio to estimate a series of models that provide support for Markov-switching volatility, a Markov-switching equity premium, a volatility feedback effect, and a positive relationship between volatility and the equity premium. The findings are particularly strong for post-War (1952-96) returns and hold for both of the assumptions we consider about how information is revealed to economic agents, including a time consistent process not used in Turner, Startz, and Nelson. Finally, our results are consistent with the idea underlying volatility feedback that exogenous changes in volatility drive the return process.

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TABLE 1a

Stock Market Volatility:

Constant and Markov-Switching Models, 1926-51

	Model	
Parameters	Constant Variance $\{s_0 = s_1\}$	Switching Variance $\{s_0 < s_1\}$
m	0.00520	0.01201
	(0.00412)	(0.00268)
s_0	0.07273	0.03927
	(0.00291)	(0.00210)
s_1	-	0.12086
	-	(0.01058)
q	-	0.98597
	-	(0.01058)
p	-	0.96050
	-	(0.02926)
Log Likelihood	375.05524	448.02651

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of January 1926 to December 1951. Asymptotic standard errors are reported in parentheses. The models have the following general form:

$$r_t = m + e_t,$$

where

$$e_t | S_t \sim N(0, s^2_t),$$
$$s^2_t = (1 - S_t) s^2_0 + S_t s^2_1,$$
$$\Pr[S_t = 0 | S_{t-1} = 0] = q, \text{ and } \Pr[S_t = 1 | S_{t-1} = 1] = p.$$

TABLE 1b

Stock Market Volatility:
Constant and Markov-Switching Models, 1952-96

	Model	
Parameters	Constant Variance $\{s_0 = s_1\}$	Switching Variance $\{s_0 < s_1\}$
m	0.00508	0.00695
	(0.00176)	(0.00161)
s_0	0.04070	0.03155
	(0.00124)	(0.00175)
s_1	-	0.05999
	-	(0.00665)
q	-	0.96922
	-	(0.01760)
p	-	0.90807
	-	(0.06763)
Log Likelihood	962.57177	986.09394

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighte excess returns of all NYSE-listed stocks for the sample period of January 1952 to December 1996 Asymptotic standard errors are reported in parentheses. The models have the following general form:

$$r_t = m + e_t,$$

where

$$e_t | S_t \sim N(0, s^2_t),$$
$$s^2_t = (1 - S_t) s^2_0 + S_t s^2_1,$$
$$\Pr[S_t = 0 | S_{t-1} = 0] = q, \text{ and } \Pr[S_t = 1 | S_{t-1} = 1] = p.$$

TABLE 2a

Volatility and the Equity Premium:
Benchmark Specifications, 1926-51

	Model Specification	
Parameters	Agents observe past returns $\{y_t = r_{t-1}, r_{t-2}, ...\}$	Agents observe true regime $\{y_t = S_t\}$
m_0	0.01318	0.01353
	(0.00300)	(0.00278)
m_1	-0.01164	-0.02826
	(0.01289)	(0.01296)
s_0	0.03928	0.03900
	(0.00206)	(0.00212)
s_1	0.11886	0.11721
	(0.01007)	(0.01005)
q	0.98704	0.98533
	(0.00954)	(0.01152)
p	0.96714	0.95977
	(0.02587)	(0.03085)
Log Likelihood	448.41838	450.40496

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of January 1926 to December 1951. Asymptotic standard errors are reported in parentheses. The model has the following general form:

$$r_t = m_0 + m_1 \Pr[S_t = 1 | y_t] + e_t,$$

where

$$e_t | S_t \sim N(0, s^2_t),$$
$$s^2_t = (1 - S_t) s^2_0 + S_t s^2_1,\ s^2_0 < s^2_1,$$
$$\Pr[S_t = 0 | S_{t-1} = 0] = q \text{, and } \Pr[S_t = 1 | S_{t-1} = 1] = p.$$

TABLE 2b

Volatility and the Equity Premium:
Benchmark Specifications, 1952-96

	Model Specification	
Parameters	Agents observe past returns $\{y_t = r_{t-1}, r_{t-2}, \ldots\}$	Agents observe true regime $\{y_t = S_t\}$
m_0	0.00456	0.00956
	(0.00242)	(0.00194)
m_1	0.01550	-0.01955
	(0.02138)	(0.00918)
s_0	0.03253	0.03143
	(0.00352)	(0.00191)
s_1	0.06445	0.06007
	(0.01853)	(0.00636)
q	0.97172	0.95681
	(0.01583)	(0.02384)
p	0.88038	0.85317
	(0.15691)	(0.09904)
Log Likelihood	987.17438	989.57509

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of January 1952 to December 1996. Asymptotic standard errors are reported in parentheses. The model has the following general form:

$$r_t = m_0 + m_1 \Pr[S_t = 1 | y_t] + e_t,$$

where

$$e_t | S_t \sim N(0, s^2_t),$$
$$s^2_t = (1 - S_t) s^2_0 + S_t s^2_1,\ s^2_0 < s^2_1,$$
$$\Pr[S_t = 0 | S_{t-1} = 0] = q \text{, and } \Pr[S_t = 1 | S_{t-1} = 1] = p.$$

TABLE 3a

Volatility and the Equity Premium:
Feedback Due to Partial Revelation 1926-51

	Model Specification	
	d is freely estimated. Agents observe past returns, but regime is partially revealed during period t $\{y^b_t = r_{t-1}, r_{t-2}, \ldots\}$, $\{y^e_t = S_t\}$	d is restricted by theory.* Agents observe past returns, but regime is partially revealed during period t $\{y^b_t = r_{t-1}, r_{t-2}, \ldots\}$, $\{y^e_t = S_t\}$
Parameters		
m_0	0.01242 (0.00300)	0.01057 (0.00285)
m_1	-0.02314 (0.01401)	0.00162 (0.00147)
d	-0.04016 (0.01804)	-0.02652 (0.01710)
s_0	0.03897 (0.00214)	0.03920 (0.00210)
s_1	0.11591 (0.01024)	0.11706 (0.01018)
q	0.98601 (0.01281)	0.98745 (0.01151)
p	0.95699 (0.03410)	0.95444 (0.03209)
Log Likelihood	450.84133	449.23066

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of January 1926 to December 1951. Asymptotic standard errors are reported in parentheses. The model has the following general form:

$$r_t = m_0 + m_1 \Pr[S_t = 1 | y^b_t] + d\{\Pr[S_t = 1 | y^e_t] - \Pr[S_t = 1 | y^b_t]\} + e_t,$$

where

$$e_t | S_t \sim N(0, s^2_t),$$
$$s^2_t = (1 - S_t) s^2_0 + S_t s^2_1, \; s^2_0 < s^2_1,$$
$$\Pr[S_t = 0 | S_{t-1} = 0] = q, \text{ and } \Pr[S_t = 1 | S_{t-1} = 1] = p.$$

* The following parameter restriction applies: $d = -m_1/(1 - r\,l)$, where $r = 0.997$ and $l = p + q - 1$.

TABLE 3b

Volatility and the Equity Premium:

Feedback Due to Partial Revelation, 1952-96

	Model Specification	
Parameters	d is freely estimated. Agents observe past returns, but regime is partially revealed during period *t* $\{y^b_t = r_{t-1}, r_{t-2}, \ldots\}$, $\{y^e_t = S_t\}$	d is restricted by theory.* Agents observe past returns, but regime is partially revealed during period *t* $\{y^b_t = r_{t-1}, r_{t-2}, \ldots\}$, $\{y^e_t = S_t\}$
m_0	0.00495 (0.00192)	0.00445 (0.00168)
m_1	0.00223 (0.00707)	0.00577 (0.00182)
d	-0.05327 (0.00808)	-0.05099 (0.00649)
s_0	0.03130 (0.00102)	0.03141 (0.00100)
s_1	0.05360 (0.00290)	0.05359 (0.00288)
q	0.97444 (0.00653)	0.97413 (0.00656)
p	0.91977 (0.02394)	0.91544 (0.02330)
Log Likelihood	997.85740	997.72206

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of January 1952 to December 1996. Asymptotic standard errors are reported in parentheses. The model has the following general form:

$$r_t = m_0 + m_1 \Pr[S_t=1|y^b_t] + d\{\Pr[S_t=1|y^e_t] - \Pr[S_t=1|y^b_t]\} + e_t,$$

where

$$e_t|S_t \sim N(0, s^2_t),$$
$$s^2_t = (1-S_t)s^2_0 + S_t s^2_1,\ s^2_0 < s^2_1,$$
$$\Pr[S_t=0|S_{t-1}=0]=q \text{, and } \Pr[S_t=1|S_{t-1}=1]=p.$$

* The following parameter restriction applies: $d = -m_1/(1-rl)$, where $r = 0.997$ and $l = p+q-1$.

TABLE 4a

Volatility and the Equity Premium:

Feedback Due to Full Revelation, 1926-51

	Model Specification	
Parameters	d is freely estimated. Agents observe past returns, but regime is fully revealed during period *t* $\{y^b_t = S_{t-1}\}$, $\{y^e_t = S_t\}$	d is restricted by theory.* Agents observe past returns, but regime is fully revealed during period *t* $\{y^b_t = S_{t-1}\}$, $\{y^e_t = S_t\}$
m_0	0.01118	0.00980
	(0.00301)	(0.00289)
m_1	-0.02271	0.00613
	(0.01446)	(0.00367)
d	-0.10618	-0.08538
	(0.03453)	(0.03022)
s_0	0.03955	0.03973
	(0.00209)	(0.00205)
s_1	0.12032	0.12400
	(0.01067)	(0.01103)
q	0.98437	0.98416
	(0.00984)	(0.00977)
p	0.94947	0.94678
	(0.03219)	(0.03330)
Log Likelihood	451.62328	449.64235

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of January 1926 to December 1951. Asymptotic standard errors are reported in parentheses. The model has the following general form:

$$r_t = m_0 + m_1 \Pr[S_t = 1 | y^b_t] + d\{\Pr[S_t = 1 | y^e_t] - \Pr[S_t = 1 | y^b_t]\} + e_t,$$

where

$$e_t | S_t \sim N(0, s^2_t),$$
$$s^2_t = (1 - S_t) s^2_0 + S_t s^2_1,\ s^2_0 < s^2_1,$$
$$\Pr[S_t = 0 | S_{t-1} = 0] = q, \text{ and } \Pr[S_t = 1 | S_{t-1} = 1] = p.$$

* The following parameter restriction applies: $d = -m_1/(1 - r l)$, where $r = 0.997$ and $l = p + q - 1$.

TABLE 4b

Volatility and the Equity Premium:
Feedback Due to Full Revelation, 1952-96

	Model Specification	
	d is freely estimated. Agents observe past returns, but regime is fully revealed during period *t* $\{y^b_t = S_{t-1}\}$, $\{y^e_t = S_t\}$	d is restricted by theory.* Agents observe past returns, but regime is fully revealed during period *t* $\{y^b_t = S_{t-1}\}$, $\{y^e_t = S_t\}$
Parameters		
m_0	0.00667 (0.00190)	0.00456 (0.00182)
m_1	-0.00751 (0.00803)	0.01007 (0.00471)
d	-0.08590 (0.01414)	-0.07401 (0.01611)
s_0	0.03082 (0.00155)	0.03082 (0.00154)
s_1	0.05322 (0.00461)	0.05499 (0.00506)
q	0.97159 (0.01325)	0.97246 (0.01381)
p	0.88739 (0.05972)	0.89416 (0.06600)
Log Likelihood	996.78935	993.35811

Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of January 1952 to December 1996. Asymptotic standard errors are reported in parentheses. The model has the following general form:

$$r_t = m_0 + m_1 \Pr[S_t=1|y^b_t] + d\{\Pr[S_t=1|y^e_t] - \Pr[S_t=1|y^b_t]\} + e_t,$$

where

$$e_t|S_t \sim N(0, s^2_t),$$
$$s^2_t = (1-S_t)s^2_0 + S_t s^2_1,\ s^2_0 < s^2_1,$$
$$\Pr[S_t=0|S_{t-1}=0]=q, \text{ and } \Pr[S_t=1|S_{t-1}=1]=p.$$

* The following parameter restriction applies: $d = -m_1/(1-r\,l)$, where $r = 0.997$ and $l = p + q - 1$.

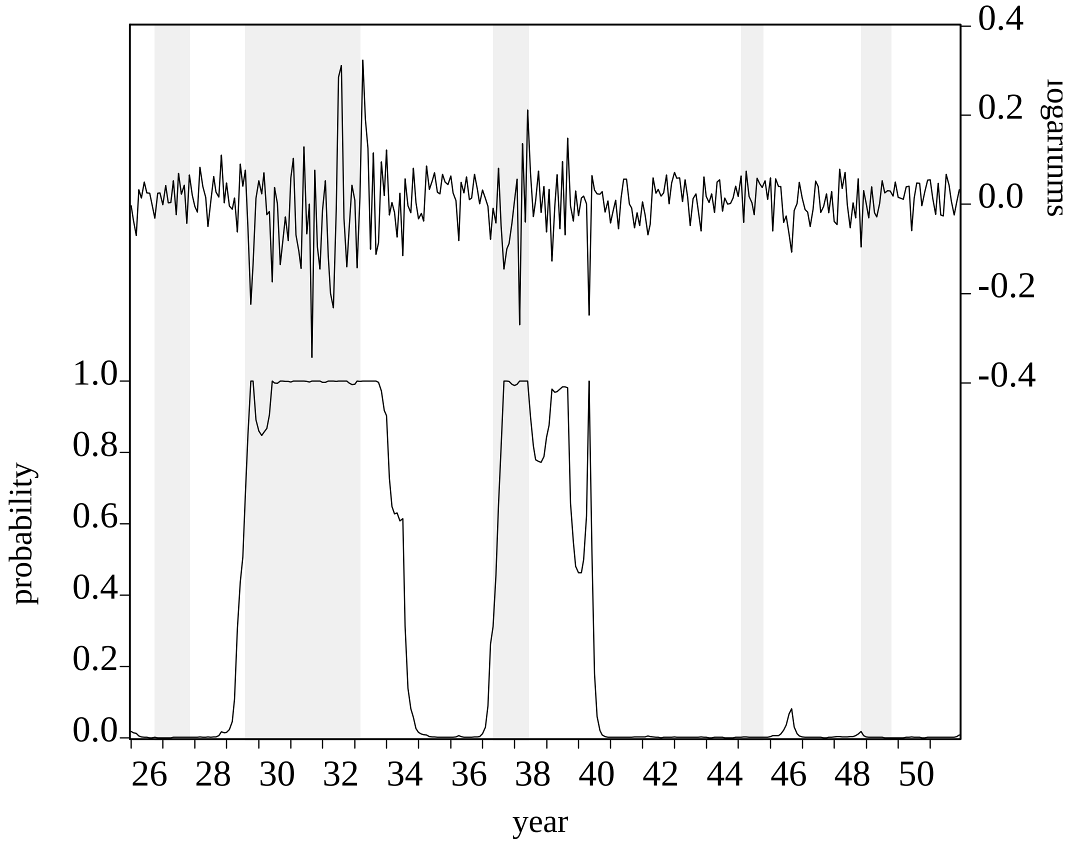

FIGURE 1a: Excess Stock Returns and Smoothed Probabilities of a High Volatility Regime for the Markov-Switching Variance Model, 1926-51.

Source: Excess stock returns are continuously compounded total monthly value-weighted returns of all NYSE-listed stocks from CRSP in excess of one-month Treasury bill yields from CRSP for the sample period of January 1926 to December 1951. Smoothed probabilities are calculated using Kim's (1994) smoothing algorithm and maximum likelihood estimates for the Markov-switching variance model presented in Table 1a. NBER-dated recessions are shaded.

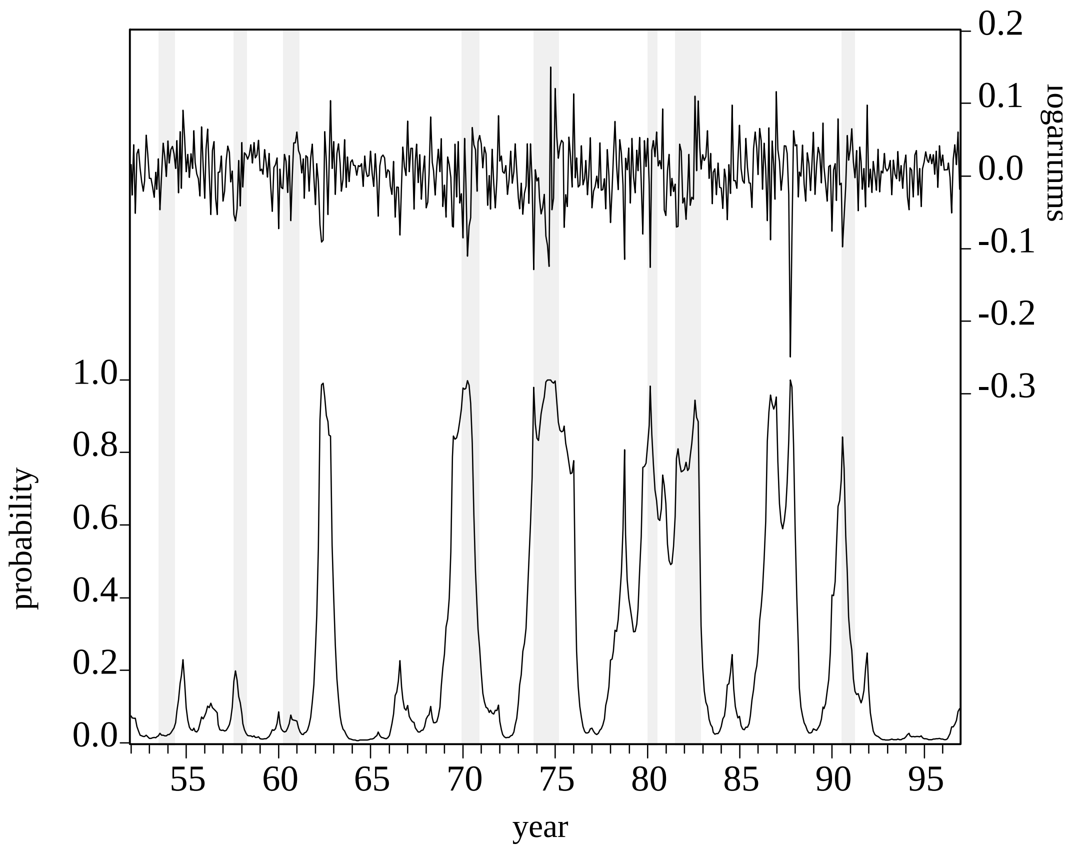

FIGURE 1b: Excess Stock Returns and Smoothed Probabilities of a High Volatility Regime for the Markov-Switching Variance Model, 1952-96.

Source: Excess stock returns are continuously compounded total monthly value-weighted returns of all NYSE-listed stocks from CRSP in excess of one-month Treasury bill yields from CRSP for the sample period of January 1952 to December 1996. Smoothed probabilities are calculated using Kim's (1994) smoothing algorithm and maximum likelihood estimates for the Markov-switching variance model presented in Table 1b. NBER-dated recessions are shaded.

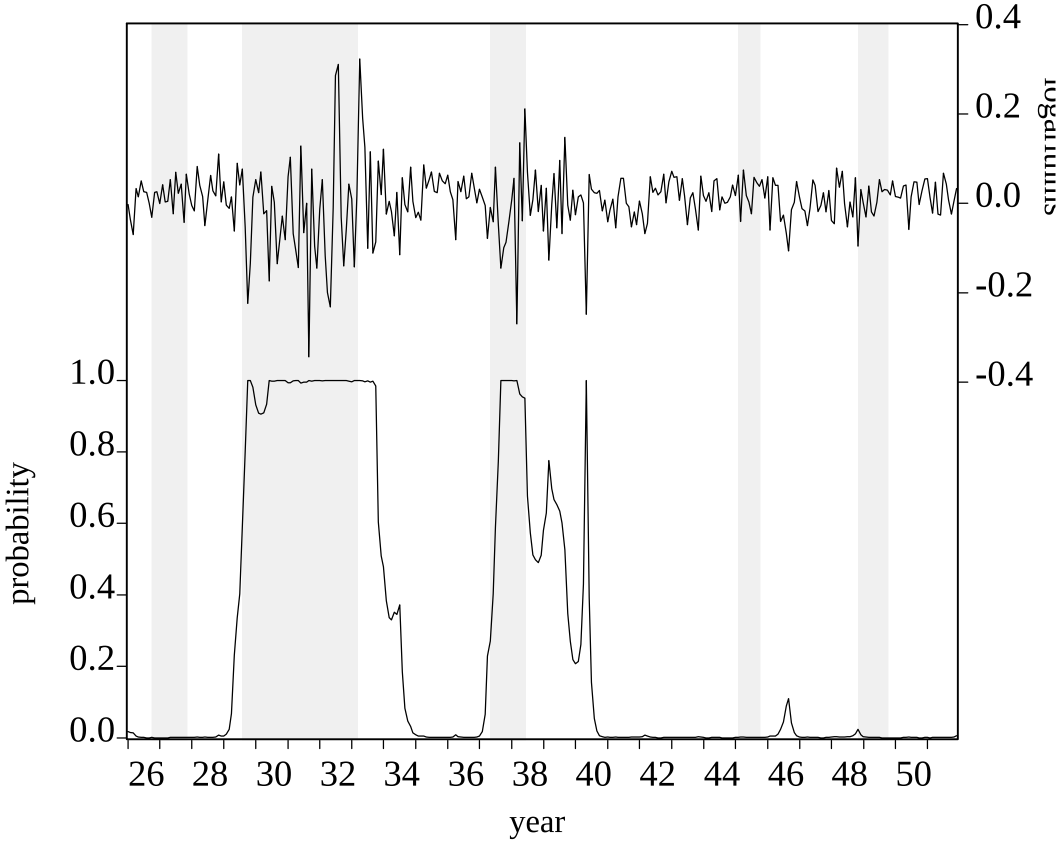

FIGURE 2a: Excess Stock Returns and Smoothed Probabilities of a High Volatility Regime for the Model with Restricted Feedback Due to Full Revelation, 1926-51.

Source: Excess stock returns are continuously compounded total monthly value-weighted returns of all NYSE-listed stocks from CRSP in excess of one-month Treasury bill yields from CRSP for the sample period of January 1926 to December 1951. Smoothed probabilities are calculated using Kim's (1994) smoothing algorithm and maximum likelihood estimates for the restricted feedback due to full revelation model presented in Table 4a. NBER-dated recessions are shaded.

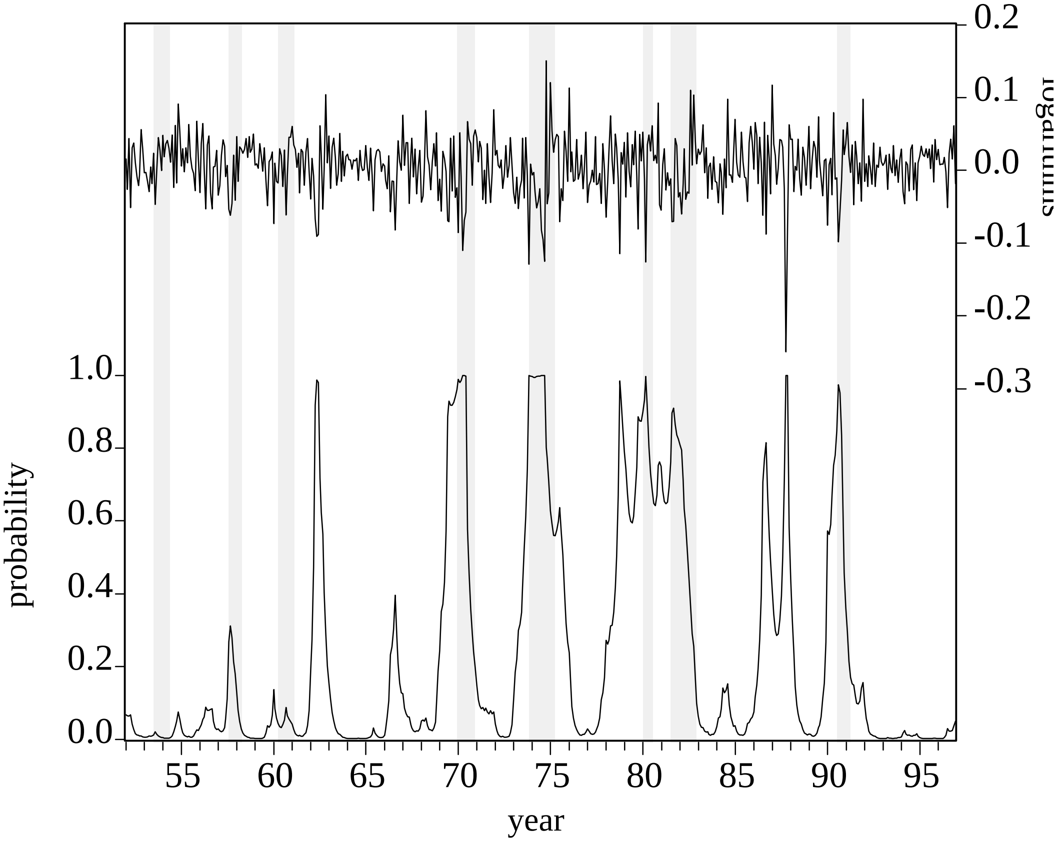

FIGURE 2b: Excess Stock Returns and Smoothed Probabilities of a High Volatility Regime for the Model with Restricted Feedback Due to Full Revelation, 1952-96.

Source: Excess stock returns are continuously compounded total monthly value-weighted returns of all NYSE-listed stocks from CRSP in excess of one-month Treasury bill yields from CRSP for the sample period of January 1952 to December 1996. Smoothed probabilities are calculated using Kim's (1994) smoothing algorithm and maximum likelihood estimates for the restricted feedback due to full revelation model presented in Table 4b. NBER-dated recessions are shaded.