Speculation and the Decision to Abandon
a Fixed Exchange Rate Regime

by

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Abstract

This paper investigates the extent to which it is possible for speculative attacks to be predictable given information on economic fundamentals. A standard model of predictable attacks is extended to incorporate an optimizing monetary authority. It is shown that while incorporating a forward-looking monetary authority improves our understanding of many observed phenomena, it also implies that the branch of the literature that places emphasis on predictable movements in fundamentals cannot generate predictable speculative attacks. In addition, the model provides useful insights into the viability of temporary nominal anchor policies, and a theoretical foundation for an important empirical methodology.

Keywords: speculative attacks, nominal anchor, optimizing BOP crises

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1. Introduction

One of the central questions about speculative attacks on fixed exchange rate regimes is the degree to which they are predictable given information on the fundamentals of the economy. Much discussion centers around whether a particular crisis such as the breakdown in the European Monetary System (EMS) in 1992-93, the Mexican peso in 1994, or the Thai baht in 1997, was the result of predictable deterioration of fundamentals or a sudden switch from one equilibrium to another based on self-fulfilling expectations. The question is theoretically interesting, and of central practical importance. If most attacks are due to predictable movements in fundamentals then fixed exchange rates may be attractive as long as the monetary authority is willing to subordinate its other goals to the exchange rate policy. On the other hand, if self-fulfilling expectations of crises are common then even if the monetary authority does everything right it may still not be able to avoid a speculative attack. Both types of attacks are supported by

extensive theoretical literatures.[1] Since theory has provided little guidance on this issue, economists have had to address the very difficult task of trying to differentiate these views empirically.[2]

This paper reexamines the possibility of predictable speculative attacks. It argues that the theoretical literature that finds that speculative attacks may be predictable suffers from an important shortcoming. It relies heavily on the assumption of a myopic monetary authority. Once a forward-looking central bank is incorporated very different, but equally interesting conclusions arise.

This literature grew from a framework developed by Krugman (1979) based on a mechanism by Salant and Henderson (1978). This framework posits a central bank with limited resources which is pursuing a fixed exchange rate policy but also has other, higher priority, policies that are fundamentally inconsistent with the exchange rate policy in the long run. Speculators are forward looking. They foresee the eventual abandonment of the fixed rate regime and the potential profit opportunities that might occur. As soon as profit opportunities begin to manifest themselves they attempt to exploit them by buying the central bank's foreign currency reserves in a sudden attack, forcing the early abandonment of the fixed exchange rate regime.

This structure is intuitive, elegant, and empirically tractable. Moreover, many countries which have experienced foreign exchange crises do indeed seem to have been pursuing other policies which appear to have been at odds with the fixed exchange rate system. Therefore the basic structure is quite attractive, and it has fared well empirically. It suffers, however, from an important shortcoming. In these models speculators are rational, forward-looking agents who are able to foresee the inevitable collapse of the fixed exchange rate regime, whereas the central bank is not. Even if we assume that the central bank makes maintaining the fixed exchange rate regime a high priority, it would still realize that indefinite maintenance is impossible given its other

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[1]The predictable attack literature was started by Krugman (1979) and Salant and Henderson (1978). The multiple equilibria explanation of speculative attacks is due to Flood and Garber (1984a) in the natural resource context, and Obstfeld (1986) in the fixed exchange rate context. Surveys of this extensive literature are given by Agénor, Bhandari, and Flood (1992), Blackburn and Sola (1993), and Flood and Marion (1997).

[2]For example, see the innovative work of Jeanne (1997).

objectives. It would therefore take steps to minimize the costs associated with the inevitable collapse of the fixed exchange rate system.

It is not clear that the results of these models are robust to the inclusion of an optimizing central bank. If the structure is extended to allow for a forward-looking, optimizing central bank then strategic interaction between speculators and the monetary authority becomes critical to the analysis. Since the central bank would foresee the speculative attack, it would be tempted to abandon the fixed exchange rate just before the crisis, thereby avoiding large reserve losses. However, if it was optimal for the central bank to do so, rational speculators would take this into account and change their strategies, potentially altering the date or nature of the attack. I examine this issue, analyzing strategic interaction between speculators and an optimizing central bank in a linear version of the Krugman model developed by Flood and Garber (1984b). There are five reasons why it is important to do so.

1.1. Theoretical Necessity

First, it is necessary to include a forward-looking monetary authority on purely theoretical grounds. The most widely used, most often cited, and most empirically tractable class of speculative attack models do not tell a consistent story. The driving force of these models is sophisticated, forward-looking behavior on the part of speculators, and completely myopic behavior on the part of the central bank. Therefore it is important to examine whether the general conclusions of this class of models are robust to a more realistic specification of central bank behavior.

There has been some recent work examining strategic interaction between optimizing central banks and foreign currency speculators. However, these papers do not permit an examination of the predictability of speculative attacks. They either focus on self-fulfilling

expectations of attacks,[3] or they construct models which bypass the issue. In Andersen (1994) and Morris and Shin (1998) there is no element of predictability in the economy so the issue of attack predictability does not arise. Ozkan and Sutherland (1994 and 1995) examine the behavior of optimizing central banks in models where the central bank cares about speculation through its influence on the interest rate, but it does not find attacks themselves detrimental. While these paper capture an important element of reality, central banks also have a strong aversion to speculative attacks themselves. Attacks have significant, and long-lasting political and economic consequences that go far beyond instantaneous, momentary increases in interest rates.

In this paper I construct an optimizing version of a standard, and widely used model of predictable speculative attacks. The central bank has an objective function which gives it an incentive to avoid sudden speculative attacks, if possible. It is shown that when the central bank dislikes speculative attacks a very different type of strategic interaction arises. Rather than simply choosing a critical level of fundamentals to abandon the fixed exchange rate regime and accepting a speculative attack, the monetary authority has an incentive to try to preempt an expected attack by abandoning the fixed exchange rate regime before it occurs. Of course speculators will try to predict this preemptive abandonment and exploit it through their foreign currency purchases. In equilibrium, the central bank will deliberately introduce uncertainty into the decisions of speculators. By making it difficult for speculators to predict the conditions under which it will change the exchange rate policy, the central bank can hope to avoid a speculative attack.

This incentive to introduce uncertainty into the decisions of speculators has not previously been analyzed. It has significant implications for our understanding of the period leading up to changes in exchange rate policy. It implies that the uncertainty associated with these policy changes may not be an exogenous feature of fixed exchange rates, but may be introduced endogenously, and deliberately by the monetary authorities in an attempt to avoid speculative attacks. Note, however, that one of the main arguments in favor of fixed exchange

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[3]These papers include Cole and Kehoe (1996a and 1996b), Davies and Vines (1995), Obstfeld (1994 and 1996), Ozkan and Sutherland (1998), and Velasco (1997).

rates is that they may decrease the exchange rate uncertainty inherent in a floating rate system. The results of this paper suggest that, in practice, the incentives of the monetary authorities may lead them to deliberately reintroduce much of this uncertainty in an attempt to avoid speculative attacks.

1.2. Explains Observed Behavior

Secondly, the inclusion of an optimizing monetary authority leads to theoretical results which are closer to observed behavior. Typically breakdowns of fixed exchange rate regimes are preceded by increasing interest differentials, and increasing forward exchange rate premia. The breakdowns are accompanied by large depreciations of the domestic currency. None of these features are present in non-stochastic versions of these models. They can be generated in stochastic versions but then the length of the period of increasing interest rates and forward exchange rate premia is directly related to the size of the expected shocks. Moreover, interest differentials and forward rate premia must be less than they would be in a floating exchange rate regime. Additionally, in the traditional models the size of a jump in the exchange rate is directly related to the size of the last shock. In fact the magnitude of the jump must be less than the jump that would occur if the same shock hit in a flexible exchange rate environment. So, while these models are not inconsistent with observed behavior, it seems difficult to believe that the ubiquitous and dramatic depreciations which accompany fixed exchange rate abandonments are all driven by large exogenous shocks.

However, when we include an optimizing monetary authority, the incentive to introduce uncertainty into the decisions of speculators implies that these empirically observed phenomena (increasing interest rates, forward exchange rate premia, and large jumps in the exchange rate after the collapse of the fixed rate) arise in equilibrium even in a completely non-stochastic framework. Therefore, it is no longer necessary to argue that exogenous shocks are large in order to explain observed crises using this class of model.

1.3. A Foundation for an Empirical Methodology

The third useful implication of introducing a non-myopic central bank is that it helps provide a theoretical foundation for a useful empirical methodology. Cumby and Van Wijnbergen (1989) develop an elegant method of estimating the evolution of a crisis when agents do not perfectly foresee the collapse. They capture the idea that changing the exchange rate regime is a policy decision by making the ad hoc assumption that the conditional p.d.f. of the level of reserves at the time of abandonment is distributed uniformly. This specification permits a very rich stochastic structure and has been quite successful empirically. However, the ad hoc nature of the specification has made the approach unattractive to many practitioners.

In my paper, the inclusion of an optimizing central bank allows for the explicit derivation of the probability of abandoning the fixed exchange rate regime in each state. This derived expression for the probability of abandoning the fixed exchange rate is remarkably similar to that assumed by Cumby and Van Wijnbergen. Hence it is possible to think of this paper as a theoretical foundation for their earlier empirical work, or, conversely, their empirical work as an *ex ante* test of this type of model.

1.4. Are Temporary Nominal Anchor Policies Viable?

The fourth reason why it is important to include an optimizing monetary authority in a model of fixed exchange rate abandonment is that it provides insight into the viability of nominal anchor policies. It is often suggested that a temporary fixed exchange rate policy can be useful in providing a "nominal anchor" to help stop runaway, expectations-driven inflation. The idea is that by making a highly-visible commitment to maintaining one nominal price the monetary authority can encourage rapid expectations adjustment, and hence control inflation quickly.

Of course, fixed exchange rate policies in highly inflationary environments are inherently dangerous. The risk of a speculative attack is always present. If the policy works, however, it would only need to be in place for a short time. As soon as inflation expectations adjust it should be possible to abandon the fixed exchange rate policy. Unfortunately, if the central bank is planning to abandon the fixed exchange rate regime, speculators would attempt to exploit this through their foreign currency purchases. Therefore, it is not clear that a smooth transition out of a temporary nominal anchor policy is consistent with rational expectations on the part of speculators. While these policies are often recommended for high inflation countries, no theoretical work has been done to investigate these difficulties involved in ending a fixed exchange rate policy gracefully.

By introducing an optimizing central bank into the Krugman model of speculative attacks, one can analyze this issue in a consistent manner. The results suggest that it may, in fact, be possible for central banks to extricate themselves from fixed exchange rate regimes without speculative attacks, but it will be a chancy undertaking. The line between success and failure will be very fine indeed.

1.5. Are Predictable Attacks Possible?

Lastly, the inclusion of an optimizing central bank in this framework permits new insights into the nature of speculative attacks themselves. It is shown that with an optimizing monetary authority speculative attacks based on predictable movements in fundamentals are not possible. This is true even when the central bank has only a single policy instrument available to avoid speculative attacks. The branch of the speculative attack literature that places its emphasis on predictable movements in fundamentals cannot generate speculative attacks when we allow for forward-looking monetary authorities. This suggests that any observed speculative attacks must be due to multiple equilibria.

Moreover, these results also have important implications for what types of events can trigger an attack. In the multiple equilibria explanation for speculative attacks it has been difficult to pin down what events might trigger a switch between equilibria. While my paper does not solve this problem, it does offer strong intuition about what types of events *cannot* trigger an attack. In equilibrium even with a single policy instrument, forward-looking central banks can avoid predictable speculative attacks, albeit at considerable cost. This suggests that the timing of speculative attacks must be unpredictable. Hence, with multiple equilibria, the mechanism used by speculators to coordinate their expectations on the attack equilibrium must be based on non-predictable variables.

The starting point for this paper is a widely used version of the Krugman (1979) model of speculative attacks on fixed exchange rates. The goal here is not to provide a realistic description of an attack episode. Rather the aim is to develop a clear understanding of the strategic interaction between speculators and a monetary authority which wishes to avoid speculative attacks. To this end a very simple and transparent version of the model is chosen in order to make the mechanism at work as clear as possible. The reader should be aware, however, that this stylized structure has been greatly extended by many authors, so a large number of more realistic and detailed models of speculative attacks use essentially the same mechanism. The standard model is developed in some detail in section 2. Then, in section 3, it is expanded to include an explicitly optimizing central bank. The paper is concluded in section 4, where the implications for our understanding of the breakdown of fixed exchange rates are discussed.

2. The Model with a Myopic Central Bank

Before introducing the optimization problem of the central bank, it is useful to consider the problem as it is usually presented, with fully rational speculators and a myopic central bank.

This framework posits a central bank with limited reserves which is pursuing a fixed exchange rate policy but also has other, higher priority, policies which are fundamentally inconsistent with the exchange rate policy in the long run. The model used here is a non-stochastic version of a well-known model by Flood and Garber (1984b). It is a single good, small open economy model in which purchasing power parity implies that,

$$P_t = S_t P_t^* \tag{1}$$

at all times t, where S_t is the domestic currency price of foreign currency, P_t is the domestic price level, and P_t^* is the foreign currency price of output which is assumed constant and normalized to 1. Uncovered interest parity is assumed to hold in each period,

$$i_t = i^* + E_t [(S_{t+1}/S_t) - 1] \tag{2}$$

where i_t is the nominal interest rate on domestic securities and i^* is the nominal interest rate on foreign securities, which is assumed to be constant. Money demand is given by,

$$M_t^d / P_t = \alpha - \beta i_t \tag{3}$$

It is assumed that money demand is positive when the domestic interest rate is equal to the world rate, $(\alpha - \beta i^*) > 0$. The money supply consists of the book value of central bank foreign currency reserves R_t and domestic credit D_t,[4]

$$M_t^s = R_t + D_t \tag{4}$$

and the evolution of domestic credit follows the exogenous process,

$$D_t = D_{t-1} + \mu \tag{5}$$

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[4]This is the money supply specification which is standard in the speculative attack literature. In this context if the money supply is set to $\overline{R} + D_t$ during the floating exchange rate period, this specification is equivalent to a specification in differences $M_t^s - M_{t-1}^s = S_t(r_t - r_{t-1}) + (D_t - D_{t-1})$ where r_t is central bank holdings of foreign currency denominated in foreign currency. This is because in the fixed exchange rate period the exchange rate is constant, and in the floating rate period foreign currency reserves are constant. To facilitate direct comparison with the existing literature this standard and innocuous short cut will be used.

where μ is a positive constant. In this model the parameter μ represents the other higher priority policies of the central bank which are inconsistent with the fixed exchange rate policy in the long run. μ is usually interpreted as a need to monetize a fiscal deficit.

In this myopic case, as long as its reserves are above a critical level $\overline{R}$ the central bank pursues a fixed exchange rate policy, buying and selling foreign currency on demand, at the exchange rate $\overline{S}$. Once its reserves fall to $\overline{R}$ the central bank must leave the foreign exchange market forever and the exchange rate will float freely. The assumption of limited reserves has received considerable criticism since it implies a limit to the ability of countries to borrow. This criticism is particularly relevant to the breakdown in the EMS, since the European countries abandoned the exchange rate mechanism long before their borrowing capacities were exhausted. One of the interesting implications that will emerge from the analysis here is that with an optimizing monetary authority the fixed exchange rate mechanism will usually be abandoned before reserves fall to their lower limit. With an optimizing central bank reserves can be substantially above $\overline{R}$ at the time of the move to a floating exchange rate system. However, for now consider the traditional case where the central bank abandons the fixed exchange rate system if, and only if, reserves fall to $\overline{R}$.

Suppose that speculators expected that the central bank would never abandon the fixed exchange rate regime, $E_t[S_{t+1}] = \overline{S}$ for all t. In that case from equations (1)-(4),

$$R_t + D_t = (\alpha - \beta i^*)\overline{S} \qquad (6)$$

since (2) would imply that $i_t = i^*$ at all times. Domestic credit is increasing over time so reserves must be decreasing and the fixed exchange rate will be abandoned at the state $\tilde{T}$ defined by,

$$D_{\tilde{T}} = (\alpha - \beta i^*)\overline{S} - \overline{R} \qquad (7)$$

Therefore it is not rational for speculators to believe that the fixed exchange rate will be maintained indefinitely. Hence, to solve the speculators' optimization problem, it is necessary to determine the exchange rate that will prevail after the fixed rate regime is abandoned.

Define the shadow floating exchange rate, $\tilde{S}_t$, as the exchange rate that would prevail if the exchange rate were floating at t. In order for the exchange rate to be floating, it must be that

reserves fell to $\overline{R}$ at one point, and after that point all foreign currency transaction will take place in private markets so the money supply will simply be $\overline{R}+D_t$. Equations (1)-(5) would yield,

$$[(\alpha-\beta i^*)+\beta] \tilde{S}_t = [\overline{R} + D_t] + \beta E_t[\tilde{S}_{t+1}] \tag{8}$$

Assuming no speculative bubbles in the floating rate period, this difference equation implies that,

$$\tilde{S}_t = \frac{\overline{R} + D_t}{(\alpha-\beta i^*)} + \frac{\beta\mu}{(\alpha-\beta i^*)^2} \tag{9}$$

If the shadow floating exchange rate is less than the fixed exchange rate then there is clearly no incentive for speculators to engage in a speculative attack. An attack would cause a breakdown of the exchange rate regime and speculators would make a loss. Therefore no individual speculator would hold foreign currency during an attack, and so an attack cannot occur. Speculators will therefore wait until the post-attack exchange rate equals the fixed rate. At that point if the breakdown is delayed there will be profits to be made by speculating in foreign currency. In this model, competition for speculative profits ensures that a speculative attack occurs the instant they become available. Therefore the attack must occur at the state where $\tilde{S}_t=\overline{S}$ so that there is no instantaneous jump in the exchange rate.[5]

Defining $\overline{T}$ as the date of the speculative attack in this traditional version of the model $\tilde{S}_{\overline{T}}=\overline{S}$ yields,

$$D_{\overline{T}} = (\alpha-\beta i^*)\overline{S} - \overline{R} - [\beta\mu/ (\alpha-\beta i^*)] \tag{10}$$

Together with equations (7) and (5) this means that $\tilde{T}-\overline{T}= \beta/(\alpha-\beta i^*)$, implying that the speculative attack will cause the collapse of the fixed exchange rate before it would otherwise be abandoned. Figure 1 shows the path of reserves.

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[5]Obstfeld (1986) shows that this is, in fact, the only possible "market-clearing" equilibrium in the model. If the analysis is expanded to include the full range of subgame-perfect Nash equilibria (allowing for the possibility of shortages of foreign currency during a speculative attack) additional equilibria can emerge, see Pastine (1996 and 1998). However, to facilitate comparison with the bulk of the existing literature, these equilibria are suppressed here and attention is focused solely on market-clearing equilibria.
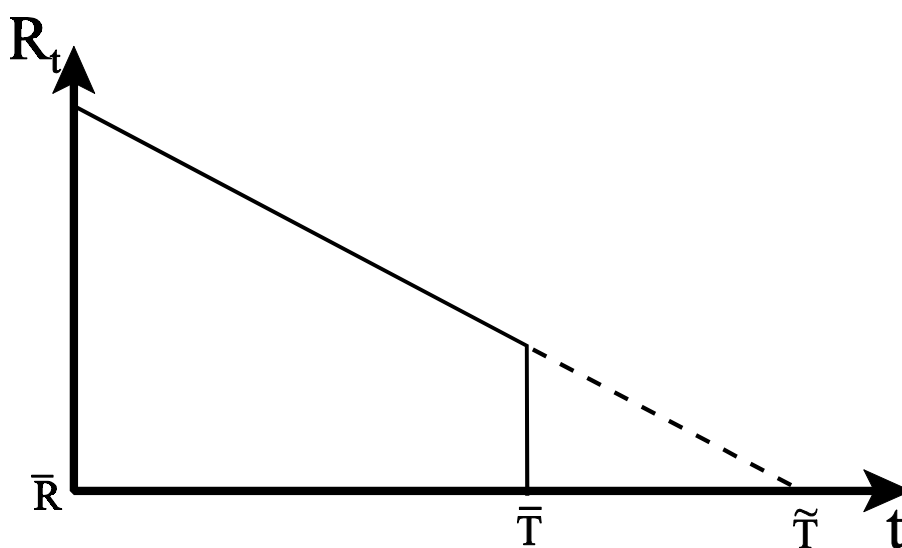
Figure 1: Reserves with a Myopic Central Bank

Initially reserves follow (6), declining one-for-one with the expansion of domestic credit. At $\overline{T}$, however, the shadow floating exchange rate rises to the fixed rate and the remaining $\mu\beta/(\alpha-\beta i^*)$ reserves are purchased in a sudden speculative attack which forces the abandonment of the fixed rate regime.

3. An Optimizing Central Bank

An important shortcoming of this traditional version of the model is that forward-looking private agents are anticipating the breakdown of the fixed exchange rate regime but the central bank is not. The speculative attack results in a sudden drop in foreign currency reserves at $\overline{T}$. If this is undesirable it can be easily avoided by simply abandoning the fixed rate at time $\overline{T}$-1. A rational central bank should realize that the move to a floating rate system is inevitable. The only open issue for the central bank is how painful that move will be and whether there are any alternative strategies which might make it somewhat less costly.

3.1. The Central Bank's Problem

To analyze this issue, an objective function for the central bank is required. There are any number of such objective functions available and any choice is necessarily imperfect. The central results of this paper require that the objective function have two features. First, the central bank must dislike speculative attacks, with the degree of distaste being continuous in the size of the attack. Second, in the absence of an attack, the central bank must prefer to maintain the fixed exchange rate.

In the proceeding analysis the focus will be on how the existence of an optimizing central bank changes the speculators' problem. Hence, it is convenient to have the central bank's objective function depend directly on speculators' choice variable. In this stylized model movements in reserves can be used as a proxy for the real costs of speculative attacks that are present in more realistic models. It is therefore assumed that the central bank prefers more reserves to less, and fixed exchange rates to floating exchange rates. The first assumption will imply that speculative attacks are undesirable for the central bank. The second assumption makes it possible to rationalize the initial existence of the fixed exchange rate regime.[6] Formally, at the end of each period t, the central bank will receive a payoff of R_t if the exchange rate is fixed, and R_t-Γ if the exchange rate is floating,[7] where $\Gamma>0$. The future is discounted at the rate $\delta\in(0,1)$.

When the fixed rate is abandoned, all foreign currency transactions take place in private markets so central bank's per period payoff in each period will be R_t-Γ if the exchange rate is abandoned in t. Hence, if the fixed rate is abandoned then the expected present value of the central bank's objective function is given by $(R_t$-$\Gamma)/(1$-$\delta)$. However, if the fixed exchange rate

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[6]Anderson (1994) presents an elegant model endogenizing central bank preferences for fixed exchange rates in a related context.

[7]Note that R_t is the book value of reserves, and (1) implies that the price level is equal to the exchange rate. Thus, in the fixed rate period the real value of central bank reserves is equal to $R_t/\overline{S}$. In the flexible exchange rate period all foreign currency transactions take place in private markets, so the real value of reserves does not change. Therefore, the central bank's one period payoff is just the real value of reserves, normalized for algebraic convenience (and minus a constant if the exchange rate is floating).

is not abandoned, future foreign currency transactions will take place with the central bank. Therefore the expected payoff from this strategy depends on what the central bank expects will happen in the future. The expected present value of the central bank's objective function is the maximum of the value if it abandons the fixed exchange rate, or the value of the game if it does not.

$$V_t = \max \left\{ \left[\frac{R_t - \Gamma}{1 - \delta} \right] , \left[R_t + \delta E_t(V_{t+1}) \right] \right\} \tag{11}$$

This maximization problem is subject to the constraint that if reserves fall to $\overline{R}$ the central bank must switch to floating exchange rates. The central bank compares the benefit from abandoning the fixed exchange rate system with the current and expected future value of maintaining it.

This specification implies that if the central bank is expecting a speculative attack, it will find it optimal to abandon the fixed exchange rate regime in the period before the attack. That is, if in the coming period the expected drop in reserves is large, the central bank will move to a floating exchange rate regime to avoid the attack. For given expectations of future reserves, a higher value of R_t implies a larger expected drop in reserves. Notice that $E_t(V_{t+1})$ includes information on the expected level of future reserves. Therefore, for a given value of V_{t+1}, high current period reserves make abandoning the fixed exchange rate system more attractive to the central bank. The central bank simply has more to lose by waiting.

Since the central bank is pursuing a fixed exchange rate policy, it must find it optimal to do so. This places a lower bound on the value of the parameter Γ, the central bank's preference for fixed exchange rates. To derive this lower bound notice that the opportunity cost of waiting one period to abandon the fixed exchange rate is given by the value of abandoning in period t minus the discounted value of abandoning in period t+1,

$$\left(\frac{R_t - \Gamma}{1 - \delta} \right) - \delta \left(\frac{R_{t+1} - \Gamma}{1 - \delta} \right) \tag{12}$$

For the choice of a fixed exchange rate to have ever been optimal, this opportunity cost must be less than the benefit of waiting when there is no speculative attack, R_t. This reduces to,

$$\delta \left(R_t - R_{t+1} \right) < (1-\delta) \Gamma \qquad (13)$$

When there is no speculative attack, reserves are given by (6). From equation (5), this implies that $(R_t - R_{t+1}) = \mu$ and (13) becomes,

$$\delta \mu < (1-\delta) \Gamma \qquad (14)$$

The benefit from having a fixed exchange rate (Γ) must be large relative to the inevitable reserve losses due to expanding domestic credit (μ). This condition must hold for the choice of a fixed exchange rate to have been optimal, and will therefore be assumed.

3.2. The Speculators' Problem

The solution to the speculators' problem is still described by uncovered interest parity, (2). However, speculators will be aware that the central bank may decide to allow the exchange rate to float before reserves fall to $\overline{R}$. If it decides to do this the floating exchange rate will no longer be described by (9). To describe the shadow floating exchange rate let $\tilde{S}_{\tau,t}$ denote the exchange rate that would prevail at t if the exchange rate was first floated in τ. So the first subscript gives the date the fixed exchange rate was abandoned and the second subscript refers to the current date. Notice that the interpretation of that shadow floating exchange rate in the optimizing model is slightly different than it is in the myopic model. In the myopic model $\tilde{S}_t$ is the exchange rate that will prevail if there is a speculative attack. In the optimizing model $\tilde{S}_{t,t}$ is the exchange rate that will prevail if the central bank abandons the fixed exchange rate. The difference arises because here the central bank may choose to abandon the fixed exchange rate without a speculative attack.

Since all foreign currency transactions in the floating rate period take place in private markets, the money supply will be $R_\tau + D_t$ and equations (1)-(5) yield,

$$[(\alpha - \beta i^*) + \beta]\, \tilde{S}_{\tau,t} = [\, R_\tau + D_t \,] + \beta\, E_t[\, \tilde{S}_{\tau,t+1}] \qquad (15)$$

Assuming no speculative bubbles in the floating rate period, this difference equation implies that the shadow floating exchange rate can be expressed as,

$$\tilde{S}_{\tau,t} = \frac{R_\tau + D_t}{(\alpha - \beta i^*)} + \frac{\beta\mu}{(\alpha - \beta i^*)^2} \qquad (16)$$

If speculators were not holding very much foreign currency at the time of the abandonment, then the money supply will be relatively high, resulting in a high path for the floating exchange rate. Thus if reserves are high at the time of the move to a floating exchange rate then the exchange rate itself will be relatively high as well. In fact, if speculators did not expect a change in the fixed exchange rate, reserves would be given by equation (6) and the shadow floating exchange rate would be strictly greater than the fixed rate,

$$\left\{ \tilde{S}_{t,t} \,\middle|\, E(q_t)=0 \right\} = \overline{S} + \frac{\beta\mu}{(\alpha - \beta i^*)^2} \qquad (17)$$

where q_t is the probability that the central bank abandons the fixed exchange rate. This implies that the shadow floating exchange rate will always be at least as high as the fixed rate since otherwise speculators would find it profitable to sell foreign currency, thereby increasing the money supply and raising the shadow floating exchange rate.

3.3. Equilibrium

In the standard model where the central bank remains passive, speculators attack the fixed exchange rate as soon as the shadow floating exchange rate rises to the fixed rate. However, this attack imposes losses on the central bank, making it attractive to abandon the fixed rate just

before the attack. By abandoning the fixed exchange rate regime one period early the central bank would avoid the speculative attack at the cost of one less period of fixed exchange rates. One might presume that this would be an equilibrium, since in the traditional model with a myopic central bank, the shadow floating exchange rate does not rise to the fixed rate until one period later. However, this is not the case. If the central bank chooses to abandon the fixed exchange rate when reserves are still above $\overline{R}$ then the shadow floating exchange rate will be correspondingly higher and speculators will find it profitable to stage a slightly smaller attack in the beginning of that period.

The subgame-perfect Nash equilibrium can be constructed by backward induction. To do so, note that reserves will be driven down to $\overline{R}$ in period $\overline{T}$ if the fixed exchange rate has not been abandoned before that time. This follows directly from the argument made in the myopic case. This means that $q_{\overline{T}}=1$. Given this, it is possible to examine the optimal central bank strategy in $\overline{T}$-1. And from this it is possible to examine the optimal central bank strategy in $\overline{T}$-2, and so on.

Since the argument will be used repetitively to work backward from $\overline{T}$, it is helpful to state it generally. Consider a time $t<\overline{T}$ where $q_{t+1}>0$. That is, in the coming period there will be a positive probability that the central bank will abandon the fixed exchange rate. The central bank must find this optimal in period t+1, which implies that it either strictly prefers to abandon the fixed rate regime in t+1 or it is indifferent between abandoning and maintaining the fixed exchange rate. In either case, from equation (11) the maximized expected present value of its objective function is given by $V_{t+1}=(R_{t+1}-\Gamma)/(1-\delta)$. The equilibrium at t can then be established by a process of elimination, starting from the potential pure-strategy equilibria.

Proposition: *With short period lengths no pure-strategy equilibrium exists.*

Proof: Consider first whether it is possible for $q_t=1$ to be an equilibrium. If so, then it must be an optimal choice for the central bank to abandon the fixed rate system, which from (11) implies

$$\frac{(R_t - \Gamma)}{(1-\delta)} \geq R_t + \frac{\delta(R_{t+1} - \Gamma)}{(1-\delta)} \tag{18}$$

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This inequality reduces to,

$$\delta(R_t - R_{t+1}) \geq (1-\delta)\,\Gamma \tag{19}$$

Speculators must be behaving optimally as well. Since $q_t=1$, there will be profit opportunities for speculators as long as the shadow floating exchange rate is above the fixed rate. Speculators exploit these profits by purchasing foreign currency which decreases the money supply and thus the shadow floating exchange rate. Competition for these speculative profits ensures that the shadow floating exchange rate is driven down to the fixed rate, $\tilde{S}_{t,t}=\overline{S}$. This yields,

$$R_t + D_t = (\alpha-\beta i^*)\overline{S} - [\beta\mu/(\alpha-\beta i^*)] \tag{20}$$

and speculation in t+1 will only continue as long as the shadow floating exchange rate is at least as high as the fixed rate, $\tilde{S}_{t+1,t+1} \geq \overline{S}$, an inequality since q_{t+1} may be less than one. This implies

$$R_{t+1} + D_{t+1} \geq (\alpha-\beta i^*)\overline{S} - [\beta\mu/(\alpha-\beta i^*)] \tag{21}$$

Therefore optimal behavior for speculators implies,

$$R_t - R_{t+1} \leq D_{t+1} - D_t = \mu \tag{22}$$

So in equilibrium both (19) and (22) must hold which implies $\delta\mu \geq (1-\delta)\Gamma$. This contradicts equation (14), so $q_t=1$ cannot be an equilibrium.

Intuitively, if the central bank plans to abandon the fixed exchange rate, speculators will try to take advantage of this by purchasing foreign currency, resulting in low central bank reserves. So when the time comes for the central bank to actually implement the switch to the floating rate, it will find that its reserves are already quite low. The damage from speculators has already been done. The additional damage that they might do if the central bank waits one more period is relatively small. So it is in the central bank's best interest to continue maintaining the fixed exchange rate. Switching exchange rate regimes at time t *with certainty* in order to avoid a speculative attack in t+1 cannot be an equilibrium.

Now consider the conditions that are necessary for $q_t=0$ to be an equilibrium. For the choice of $q_t=0$ to be optimal for the central bank it must be that,

$$\frac{(R_t - \Gamma)}{(1-\delta)} \leq R_t + \frac{\delta(R_{t+1} - \Gamma)}{(1-\delta)} \tag{23}$$

which reduces to,

$$R_t - R_{t+1} \leq (1-\delta)\Gamma/\delta \qquad (24)$$

Maintaining the fixed exchange rate will be optimal as long as in the next period reserves will not fall by more than $(1-\delta)\Gamma/\delta$.

This means that if the expected attack is very small relative to the central bank's preference for fixed exchange rates, then it can be optimal to deliberately accept the attack rather than to give up even one period of the fixed exchange rate regime. In this case $q_{\overline{T}-1}=0$ is an equilibrium. However, this equilibrium is an artifact of the period length. If we consider short periods, this equilibrium cannot exist (see Appendix). In particular, if we consider a continuous time model given by the limit of the discrete time models as the period length goes to zero, then this equilibrium with passive acceptance of the speculative attack is not possible for any set of parameter values.[8] The size and cost of the speculative attack is not dependent on the period length, but the opportunity cost of abandoning the fixed exchange rate system one period earlier is. With short enough periods the opportunity cost is negligible. Therefore if we consider short periods it will not be optimal for the central bank to remain passive in the face of a predictable speculative attack. Hence $q_{\overline{T}-1}$ will not be zero in equilibrium. Thus $q_{\overline{T}-1} \in (0,1)$ which completes the proof of the proposition.   □

Intuitively, if the central bank can predict an oncoming speculative attack then it will wish to abandon the fixed exchange rate regime just before the attack. Likewise, if speculators can predict this preemptive abandonment of the fixed exchange rate regime, then they will exploit this knowledge by buying foreign currency just before the abandonment. Thus in order to avoid a speculative attack the central bank must introduce uncertainty into the decisions of speculators.

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[8]One could of course formulate the problem in continuous time and the equilibrium would be identical to equilibrium of the discrete-time model presented here (derivation available from the author). Discrete time was chosen as it permits a somewhat clearer presentation of the intuition.

It cannot follow a predictable pure strategy, since such a strategy would result in a speculative attack.[9]

Therefore $q_{\overline{T}-1}$ cannot be one and it cannot be zero. So $q_{\overline{T}-1} \in (0,1)$, which means that the central bank must be indifferent between maintaining and abandoning the fixed exchange rate regime. From (11) and the fact that $V_{t+1}=(R_{t+1}-\Gamma)/(1-\delta)$, central bank reserves must be equal to

$$R_t = R_{t+1} + (1-\delta)\Gamma/\delta \qquad (25)$$

For the period $t=\overline{T}-1$, $R_{t+1}=\overline{R}$ which yields the level of reserves in the period $\overline{T}-1$.

Then, working backward from $\overline{T}-1$, it is possible to establish $\underline{T}$, the earliest state where abandonment of the fixed exchange rate can be an equilibrium outcome. Iterating the same argument yields reserves of

$$R_t = \overline{R} + (\overline{T}-t)(1-\delta)\Gamma/\delta \qquad (26)$$

Equation (26) describes the equilibrium level of reserves as long as it yields reserves that are less than those given by equation (6). At that point and earlier $q_t=0$ is an equilibrium and so reserves will follow (6). The path of reserves can be seen in Figure 2. After $\underline{T}$ reserves decline quickly, but continuously, until they reach their lower bound at $\overline{T}$.

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[9]Karim Abadir and Harald Uhlig independently pointed out that the central bank's problem has much in common with the classic "Surprise Quiz" game. In that game a professor promises the students that there will be a surprise quiz in the coming week. If the quiz does not occur by Thursday, then the students know that it must occur on Friday, and thus it is not a surprise. So the professor cannot play a pure strategy of having the exam on Friday. But if the exam has not occurred by Wednesday the students know it must occur on Thursday, and it will not be a surprise. So the professor cannot play a pure strategy of having the exam on Thursday. Backwards induction via the same argument eliminates any pure-strategy for the professor. Similarly, the central bank would like to have the abandonment of the fixed exchange rate regime come as a surprise to speculators, and so for the same reason it cannot play a pure-strategy.
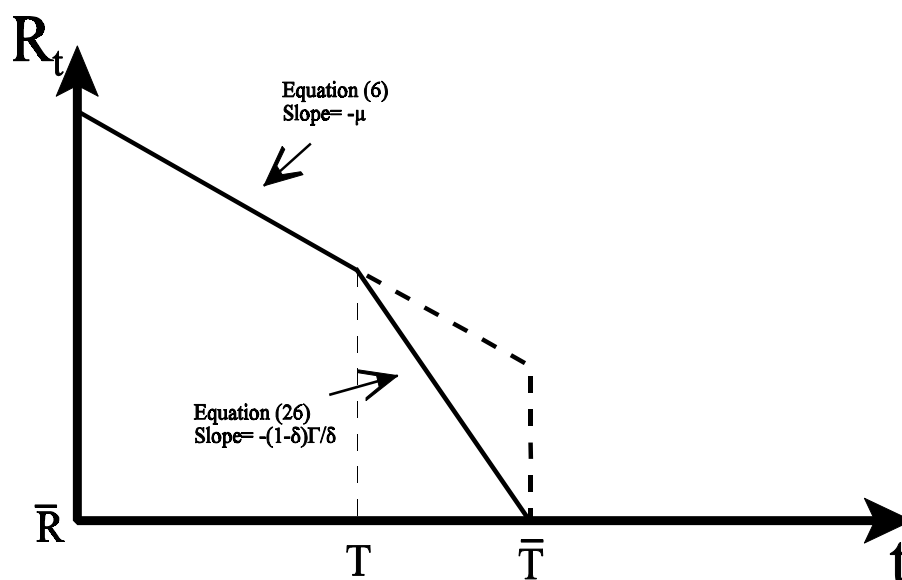
Figure 2: Reserves with an Optimizing Central Bank

Consider the problem of an individual speculator. Suppose that, at the moment, reserves are high and that in the next period they will be relatively low. The speculator therefore realizes that the central bank will prefer to abandon the fixed rate regime this period rather than permit such a large drop in reserves. So he will purchase foreign currency in this period. This means that the drop in foreign currency reserves will not be as dramatic in the coming period. Therefore, the central bank will no longer be quite as eager to abandon the fixed rate.

Each speculator will purchase foreign currency as long as the central bank prefers to abandon the fixed rate. When it is indifferent between abandoning and maintaining the fixed rate, they will realize that if they purchase additional foreign currency the drop in reserves between the current period and the next will be small enough so that the central bank will prefer not to abandon. Since speculation decreases the money supply, it raises the domestic interest rate above the foreign rate. Hence purchasing foreign currency entails an opportunity cost and they will not purchase additional foreign currency.

So optimizing behavior on the part of speculators ensures that reserves are at a level where the central bank either strictly prefers maintaining the fixed exchange rate one more period

or is indifferent between abandoning and maintaining the fixed rate. In other words, optimizing speculators will always ensure that the central bank does not *strictly* prefer to abandon the fixed exchange rate regime. If it did so, speculators could make profits by purchasing additional foreign currency, making abandonment less attractive to the central bank. Since sudden attacks on foreign currency reserves make preemptive abandonment of the fixed exchange rate attractive, in equilibrium there can be no such predictable attacks.

Since this point is important, it is worth emphasizing with a counter example. Suppose that there was a predictable speculative attack, defined as a predictable, sudden, large drop in reserves. Then just before the drop the central bank would abandon the fixed exchange rate system, thereby avoiding the attack. If it was planning on doing so, however, speculators would buy foreign currency just before it did. But this would imply that foreign currency reserves would already be low by the time of the expected attack, so the drop in reserves would be relatively small. The same logic applies to the period before, and the period before that. Therefore, in equilibrium there cannot be a predictable speculative attack.

In the range $t < \underline{T}$ the central bank prefers to continue maintaining the fixed exchange rate so reserves decline one for one with the expansion of domestic credit. In the period $t \in [\underline{T}, \overline{T})$ speculation ensures that the central bank is indifferent between abandoning and maintaining the fixed rate regime, which implies that reserves are declining at a greater rate.

From this information on the path of reserves it is straightforward to determine the behavior of other variables. First the behavior of all the endogenous variables will be derived and then the intuition for the results will be discussed. During the period $t < \underline{T}$ there is no chance that the fixed exchange rate will be abandoned ($q_t = 0$). This implies that $i_t = i^*$ and that the shadow floating exchange rate is given by equation (17). After $\underline{T}$ reserves are given by equation (26) and domestic credit can always be written as $D_t = D_{\overline{T}} + (t - \overline{T})\mu$. The shadow floating exchange rate can be found by substituting these into (16),

$$\tilde{S}_{t,t} = \left[\frac{\bar{R} + D_{\overline{T}}}{(\alpha - \beta i^*)} + \frac{\beta \mu}{(\alpha - \beta i^*)^2} \right] + (t - \bar{T}) \frac{\mu - (1 - \delta)\Gamma/\delta}{(\alpha - \beta i^*)} \tag{27}$$

The bracketed term is equal to the fixed exchange rate by (10) so the shadow floating exchange rate is simply

$$\tilde{S}_{t,t} = \bar{S} + (t - \bar{T}) \frac{\mu - (1-\delta)\Gamma/\delta}{(\alpha - \beta i^*)} \tag{28}$$

From this, (5), (10), and the fact that $\underline{T}$ is defined as the time where (6) and (26) yield the same level of reserves, it is straightforward to show that at $\underline{T}$ the shadow floating exchange rate is $\tilde{S}_{\underline{T},\underline{T}} = \bar{S} + \beta\mu/(\alpha - \beta i^*)^2$, which is equal to (17). So the shadow floating exchange rate is constant and above the fixed rate before $\underline{T}$. Then it declines linearly, reaching the fixed exchange rate at $\bar{T}$.

To examine the behavior of the interest rate in equilibrium note that (1), (3) and (4) must hold yielding,

$$i_t = \alpha/\beta - (R_t + D_t)/\beta\bar{S} \tag{29}$$

and during $t \in [\underline{T}, \bar{T}]$ reserves are given by equation (26) so this can be expressed as,

$$i_t = \frac{\alpha}{\beta} - \frac{(\bar{R} + D_{\bar{T}})}{\beta\bar{S}} + (\bar{T} - t)\left[\frac{\mu - (1-\delta)\Gamma/\delta}{\beta\bar{S}} \right] \tag{30}$$

Using equation (10) this can be reduced to,

$$i_t = i^* + \frac{\mu}{(\alpha - \beta i^*)\bar{S}} + (\bar{T} - t)\left[\frac{\mu - (1-\delta)\Gamma/\delta}{\beta\bar{S}} \right] \tag{31}$$

From the definition of $\underline{T}$ the interest rate is equal to the world rate at $\underline{T}$. It then increases linearly until it reaches $i^* + \mu/(\alpha - \beta i^*)\bar{S}$ at $\bar{T}$.

The one remaining endogenous variable is q_t, the probability that the central bank abandons the fixed exchange regime. This can be derived by noting that (2) implies,

$$i_t = i^* + q_t [(\tilde{S}_{t,t+1}/\bar{S}) - 1] \tag{32}$$

Since $\tilde{S}_{t,t+1}=\tilde{S}_{t,t}+\mu/(\alpha-\beta i^*)$, equation (28) yields,

$$q_t \;=\; \frac{(\alpha-\beta i^*)\,\bar{S}\,(i_t-i^*)}{\mu \;+\; (t-\bar{T})\left[\mu-(1-\delta)\Gamma/\delta\right]} \tag{33}$$

At $\underline{T}$, $i_t=i^*$, so equation (33) says that the probability of abandoning the fixed exchange rate is zero at $\underline{T}$. It then increases, at an increasing rate, until it reaches 1 at $\overline{T}$.

It is also worth noting that if we made the additional assumptions of risk neutrality and perfectly competitive forward markets, the one period ahead forward exchange rate, F_t, would be equal to the expected exchange rate in the next period,

$$F_t = q_t\,\tilde{S}_{t,t+1} + (1-q_t)\overline{S} \tag{34}$$

Combining this with equation (32) yields,

$$F_t = \left[1+(i_t-i^*)\right]\overline{S} \tag{35}$$

Let f_t denote the one period ahead forward exchange rate premium, defined as

$$f_t \;\equiv\; \frac{(F_t-\bar{S})}{\bar{S}} \;=\; (i_t-i^*) \tag{36}$$

So the forward exchange rate premium is equal to the interest rate differential, starting at zero at $\underline{T}$ and rising linearly to $\mu/(\alpha-\beta i^*)\overline{S}$ at $\overline{T}$.

3.4. Interpretation
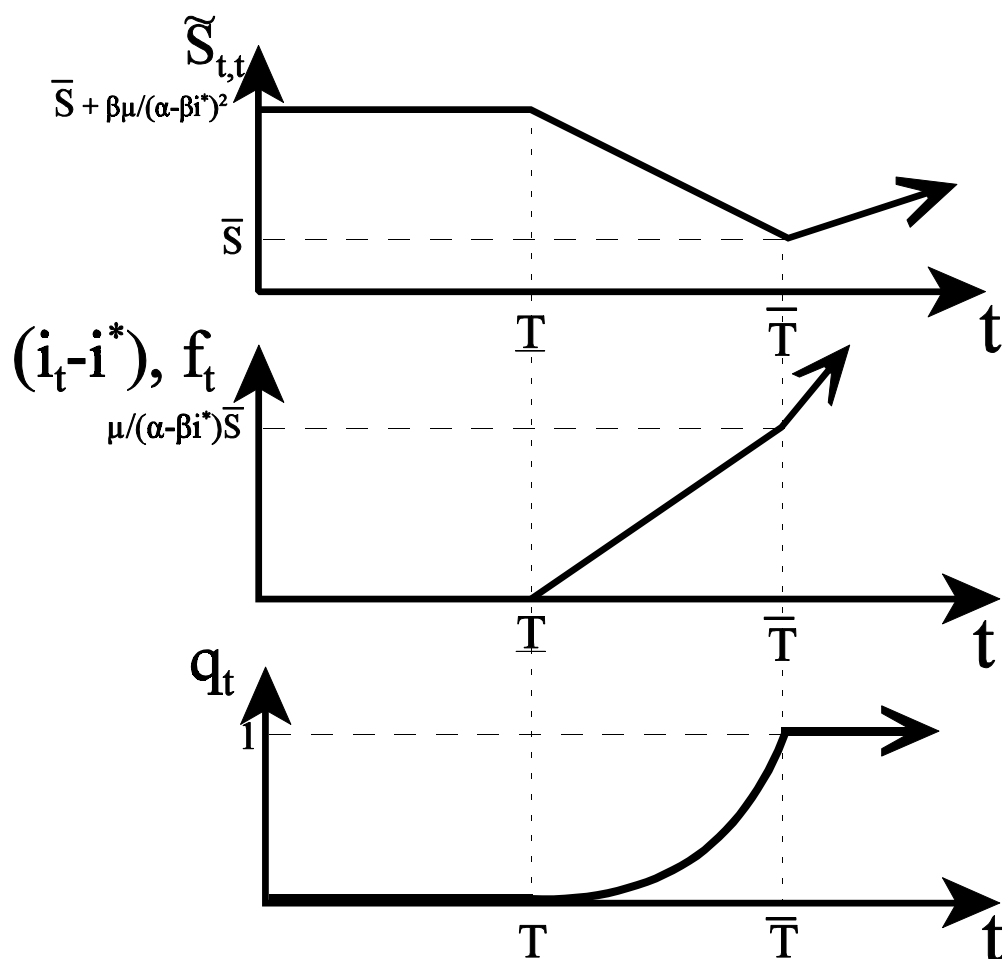
These results are summarized in Figure 3.

Figure 3: The evolution of the shadow floating exchange rate, the interest rate differential and forward exchange rate premium, and the probability of a move to a floating exchange rate regime.

Before $\underline{T}$ there is no chance of the central bank abandoning the fixed exchange rate regime. Since $\overline{T}$ is far in the future, the central bank wants to maintain the fixed rate. Reserves are high enough so that if speculators attacked and drove reserves to $\overline{R}$, forcing a move to a floating exchange rate, they would contract the money supply to a point where the fixed exchange rate would

exceed the floating rate; hence they do not attack. The domestic interest rate is therefore equal to the world rate.

After $\underline{T}$ this situation changes. Both speculators and the central bank realize that $\overline{T}$ is approaching and that the fixed exchange rate must be abandoned. This gives the central bank an incentive to abandon it before speculation drives down foreign currency reserves. But speculators are aware of this incentive and purchase foreign currency in anticipation of the potential jump in exchange rates. As time goes by, speculators purchase more foreign currency, always keeping the central bank indifferent between maintaining and abandoning the fixed exchange rate regime. This drives up the domestic interest rate and forward exchange rate premium while driving down the shadow floating exchange rate. Therefore the opportunity cost of holding foreign currency is increasing, and the benefit to successful speculation is declining. But the probability that speculation will be successful is increasing rapidly, ensuring that the expected return from holding foreign currency remains constant at zero. However, in the event of a move to floating exchange rates the *ex post* profits from holding foreign currency can be substantial, since the shadow floating exchange rate is above the fixed rate until $\overline{T}$.[10]

4. Conclusion

A version of the Krugman (1979) balance-of-payments crisis model is developed that explicitly incorporates an optimizing central bank. This permits an analysis of strategic interaction between the central bank and speculators. In this model the central bank is able to

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[10]The period after $\overline{T}$ is also depicted in Figure 3. This is the behavior of the variables off the equilibrium path. These states will never be reached, the abandonment of the fixed exchange rate regime will occur with certainty at or before $\overline{T}$. However, to derive the equilibrium strategies it is necessary to specify the alternative, what would happen if the optimal strategies were not implemented. This is what is given to the right of $\overline{T}$. These strategies are precisely the off-equilibrium strategies which are implicit in the traditional analysis of the model. If the fixed exchange rate is still maintained after $\overline{T}$, then speculators will immediately attack the central bank's foreign currency reserves forcing the abandonment of the fixed exchange rate. Therefore $R_t = \overline{R}$ and $q_t = 1$. This implies that the shadow floating exchange rate and the interest rate are given by (9) and (2) respectively, which is what is depicted in this part of Figure 3, but these states will never be observed since the fixed exchange rate will always be abandoned before they are reached.

choose the timing of the move to floating exchange rates while speculators attempt to predict this date and exploit it through their foreign currency purchases. It is shown that the central bank has an incentive to introduce uncertainty into the decisions of speculators. By doing so it is possible for the central bank to avoid predictable speculative attacks. This is true even when the only policy instrument available to the central bank is the decision to abandon or continue maintaining the fixed exchange rate regime. When central banks have more sophisticated policy instruments at their disposal, avoiding predictable attacks is even easier. Thus the hypothesis of predictable speculative attacks is inconsistent with forward-looking behavior on the part of central banks.

The existence of speculators causes the central bank to abandon the fixed exchange rate much earlier than it would have in their absence. This is true in equilibrium even though there are no speculative attacks. With capital mobility and an optimizing central bank it is the threat of such attacks that causes the central bank to abandon the fixed exchange rate. Even when we do not observe significant speculation, the presence of speculators can have an important effect on the decisions of the monetary authority. In fact, the earliest abandonments of fixed exchange rates will be accompanied by low speculation. But these abandonments are the ones whose timings have been most affected by the presence of speculators.

There is an important element of time inconsistency here. Rather than enduring the protracted period of decreasing reserves that it must face in equilibrium, the central bank would prefer the equilibrium of the myopic model. In the myopic model the central bank had relatively high reserves right up to the attack. While the speculative attack is costly for the central bank, it is not as costly as the protracted period of increasing speculation.[11] The central bank would like to convince speculators that it will not abandon the fixed exchange rate unless reserves are driven down so far that continued maintenance is impossible. While it would like to be able to commit to this policy, in practice it cannot. Speculators know that the central bank has an incentive to

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[11]This can be seen by realizing that in equilibrium the central bank is indifferent between abandoning the fixed exchange rate anywhere during this period. In particular, it is willing to wait until it is forced to abandon. At that point the present value of its payoff is the same as it would be if there had been an attack. But on the path to that point the forward-looking central bank endured increasing speculative pressure, while the myopic central bank did not.

abandon the fixed exchange rate policy just before they are expected to attack. The central bank's announced policy is not credible, and so in equilibrium it must endure a protracted period of increasing speculation.

In practice, central banks almost universally claim that they will never, under any circumstances, abandon fixed exchange rate regimes. They are, in effect, claiming to be myopic. In equilibrium it is, in fact, preferable to be myopic, but it is not credible. In the absence of a device which irrevocably commits the central bank to the policy, rational speculators will not believe these claims. It should be noted that this is not an argument in favor of currency boards, in spite of the fact that currency boards cannot legally abandon fixed exchange rate regimes. Since any government with the power to institute a currency board also has the power to abolish it, currency boards do not solve the time inconsistency problem. They simply transfer the problem from the central bank to the government.

With a forward-looking monetary authority, there is an incentive to introduce uncertainty into the decisions of speculators. If speculators are certain of the conditions that will cause the central bank to abandon the fixed exchange rate regime, they will engage in a sudden speculative attack just before the abandonment. In order to avoid such attacks the central bank must ensure that speculators are not certain of what it will do. This endogenous uncertainty implies that the timing of the move to a floating exchange rate cannot be known with certainty, even in a completely non-stochastic structure. Fundamentals move deterministicly but the outcome is stochastic. However, while the date of the abandonment of the fixed exchange rate regime cannot be known with certainty, fundamentals confine the abandonment to a bounded set of dates. The variables that determined the attack date in the traditional analysis also help determine the possible dates of a breakdown with an optimizing central bank. Thus the model is consistent with the wealth of empirical evidence found using the traditional approach. Moreover, with an optimizing central bank the probability of abandonment at each date can be completely specified.

This derived expression for the probability of abandoning the fixed exchange rate is remarkably similar to that assumed by Cumby and Van Wijnbergen in their 1989 empirical work on Argentina's crawling peg. In order to capture the idea that changing the exchange rate regime

is a policy decision they assume that the conditional p.d.f. of the level of reserves at the time of abandonment is distributed uniformly. This assumption implies that the probability of abandonment evolves quite similarly to equation (33).[12] Hence it is possible to think of this paper as a theoretical foundation for their earlier empirical work, or, conversely, their empirical work as an *ex ante* test of this type of model.

The implication of this endogenous probability of abandonment is that the time near an abandonment of a fixed exchange rate is likely to be characterized by a high degree of uncertainty. This uncertainty is not an exogenous feature of fixed exchange rates but rather is introduced endogenously and deliberately by the central bank in an attempt to move to a floating exchange rate regime before a speculative attack.

The seminal work of Harsanyi (1973) provides an additional interpretation of this endogenous uncertainty. If speculators were unsure of the objective function being used by the central bank, or about the current level of reserves, then the central bank would use a pure strategy. However, equilibrium uncertainty would be present due to this exogenous uncertainty about the central bank's objective function. Harsanyi's insight is that as this exogenous uncertainty goes to zero, the equilibrium uncertainty does not. In fact, the probability of abandoning the fixed exchange rate at each time approaches the probabilities found in the model. This remarkable fact implies that even arbitrarily small uncertainty about the central bank's objective function or foreign currency reserves will lead to very large uncertainty about the conditions under which the central bank will abandon the fixed exchange rate regime. Hence, it is not realistic to presume that if speculators have reasonably good information about the central bank's objective function and about central bank foreign currency holdings, that they will be able

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[12]Abstracting from their rich stochastic structure, Cumby and Van Wijnbergen's assumption implies $q_t = \dfrac{1}{1+(\bar{T}-t)}$ in the notation of this paper. Equation (33) can be rewritten as $q_t = \dfrac{k_1}{k_2 + k_3(\bar{T}-t)} - k_4$ where

the k_is are positive functions of parameters. These expressions for the probability of abandoning the fixed exchange rate system are quite similar and evolve in much the same way but there is no set of parameters in this model which would make them identical, $k_1=k_2=k_3=1$ is not possible. Nevertheless, Cumby and Van Wijnbergen's estimation method can be easily extended to use related specifications of the probability of abandonment.

to deduce good estimates of the conditions under which the central bank will abandon the fixed exchange rate regime.

This endogenous uncertainty means that prior to the switch in exchange rate regimes there are increasing domestic interest rates, and increasing forward exchange rate premia over the fixed exchange rate. In addition, the exchange rate may jump during the move to the floating rate, yielding *ex post* profits to foreign currency speculators. These are attractive features of the optimizing model that in the traditional model could only be induced through exogenous shocks. The exogenous shock approach implies that the length of the period of increasing interest rates and forward exchange rate premia is directly related to the size of the expected shocks. Additionally, in the traditional model the size of a jump in the exchange rate is directly related to the size of the last shock. In fact the magnitude of the jump must be less than the jump that would occur if the same shock hit in a flexible exchange rate environment. With an optimizing central bank, however, these empirically observed phenomena are no longer the result of exogenous shocks. They arise in equilibrium even in this non-stochastic framework due to the endogenous uncertainty introduced by the central bank.

The results are encouraging for the viability of temporary nominal anchor policies to combat runaway expectations-driven inflation. By introducing uncertainty into speculators' decisions, it is in fact possible for a central bank to leave a fixed exchange rate regime gracefully, even when the abandonment is anticipated by rational speculators. Nevertheless, the results of the model also suggest that these policies are likely to be very dangerous. During the time leading up to the abandonment of the fixed rate regime, the existence of rational speculators ensures that the central bank is indifferent between maintaining and abandoning the fixed exchange rate. This means that the central bank will be very vulnerable to shifts in peoples' perceptions of its commitment to the announced policy. Any perceived weakness, or any negative shock could

cause a sudden crash.[13] Seen in this light, it is not surprising that while we do see several successful implementations of nominal anchor policies (most notably Poland and Israel), the list of dramatic failures is much longer. However, this is the first paper that addresses this issue and the reader should be aware that it uses a very stylized model that does not incorporate many of the features that are important to the success of nominal anchor policies. Work still needs to be done in more fully specified models to see how the central bank's incentive to introduce uncertainty into speculators' decisions affects the adjustment of inflationary expectations, and how this incentive interacts with the possibility of rational and self-fulfilling expectations of attacks.

An important implication of introducing an optimizing monetary authority is that it implies that speculative attacks cannot be precipitated by predictable movements in fundamentals. If speculative attacks were predictable, the central bank would avoid them by abandoning the fixed exchange rate system just before they occurred. However, this would mean that the breakdown of the fixed rate regime was predictable and rational speculators would exploit this, resulting in an earlier attack. But the same logic would apply to that attack as well. Therefore, in equilibrium, speculative attacks cannot be predictable in a world with optimizing monetary authorities.[14]

This does not imply that speculative attacks do not depend on fundamentals. Many models of speculative attacks with optimizing monetary authorities imply that the existence of multiple equilibria depends on fundamentals. See Cole and Kehoe (1996a and 1996b), Davies and Vines (1995), Obstfeld (1994 and 1996), Ozkan and Sutherland (1998), and Velasco (1997).

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[13]See, for example, Obstfeld (1995) for a model where exogenous shocks can alter the central bank's tradeoff between fixed and floating exchange rates. Typically in models of this type the central bank is initially assumed to strictly prefer fixed exchange rates. Nevertheless, large enough negative shocks can decrease the attractiveness of fixed rates, and create the potential for a speculative attack. The danger would be even greater starting from a position of indifference between the two exchange rate regimes.

[14]Actually, this result does not depend on the monetary authority per se, but rather on speculators' *perceptions* of the monetary authority, since it is speculators who engage in speculative attacks and the strategy of the central bank is not observable to them. In the model, rationality implies that perceptions are the same as reality. In the real world this may not necessarily be the case. However, as long as speculators perceive that the central bank is behaving rationally, there cannot be predictable speculative attacks.

The point is that while the *conditions* for a speculative attack may depend on predictable fundamentals, the precise *time* of a speculative attack cannot. An implication of this is that in models with multiple equilibria the coordination of speculator expectations on a particular equilibrium cannot depend on predictable variables, since otherwise the central bank could predict the attack, and therefore avoid it. The coordination of speculator expectations can, however, depend on the unpredictable components of predictable variables, including the unpredictable components of fundamentals.

Appendix

To examine the possibility that the model might be able to generate Krugman style speculative attacks with a frequently optimizing central bank, the analysis is extended to allow for an arbitrary period length, n. Low values of n imply frequent decision making on the part of the central bank and the preceding analysis is a special case where n=1. A continuous time version of the model can be examined by taking the limit of the discrete time models as n→0.

Equations (1), (3) and (4) are not dependent on the period length and can be retained without modification, noting only that i_t in the money demand equation (3) refers to the interest return over calender time 1. Therefore the per-period interest rate is given by ni_t, so equation (2) becomes,

$$ni_t = ni^* + E_t [(S_{t+1}/S_t) -1] \qquad (37)$$

and (5) is,

$$D_t = D_{t-1} + n\mu \qquad (38)$$

From these equations the shadow floating exchange rate in the traditional model can be derived using the same method to show that (9) still holds. The time of the speculative attack in the traditional model $\overline{T}$ is given by $\tilde{S}_t = \overline{S}$ which implies that reserves fall by $\beta\mu/(\alpha-\beta i^*)$ at $\overline{T}$. So the size of the speculative attack in the traditional model is not dependent on the period length. This is unsurprising since the model generates attacks in continuous time as well.[15]

Now moving to the problem of the central bank, the central bank's per-period discount rate is δ^n. It's per-period payoff is proportional to the period length: nR_t with fixed exchange rates and $n(R_t-\Gamma)$ with floating rates. Therefore the central bank's maximization problem is

$$V_t = \max \left\{ \left[\frac{n(R_t - \Gamma)}{1 - \delta^n} \right] , \left[nR_t + \delta^n E_t(V_{t+1}) \right] \right\} \qquad (39)$$

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[15]The original Krugman (1979) model was cast in continuous time.

We are interested here in whether it is optimal for the central bank to set $q_{\overline{T}-1}=0$ knowing that there will be a speculative attack in the next period. If this is so, then from the maximization of (39),

$$n(R_{\overline{T}-1}-\Gamma)/(1-\delta^n) \le nR_{\overline{T}-1} + \delta^n V_{\overline{T}} \qquad (40)$$

Since there will be a speculative attack in $\overline{T}$, reserves will fall to $\overline{R}$, and the fixed exchange rate will be abandoned. Therefore, $V_{\overline{T}}=n(\overline{R}-\Gamma)/(1-\delta^n)$. So in order for $q_{\overline{T}-1}=0$ to be an equilibrium it must be the case that,

$$\frac{n(R_{\overline{T}-1}-\Gamma)}{(1-\delta^n)} \le nR_{\overline{T}-1} + \frac{n\delta^n(R_{\overline{T}}-\Gamma)}{(1-\delta^n)} \qquad (41)$$

which reduces to,

$$R_{\overline{T}-1} - R_{\overline{T}} \le (1-\delta^n)\Gamma/\delta^n \qquad (42)$$

Since reserves fall by $n\mu$ each period just due to expanding domestic credit and since there will be a speculative attack depleting reserves by $\beta\mu/(\alpha-\beta i^*)$ in period $\overline{T}$, $R_{\overline{T}-1}-R_{\overline{T}}=n\mu+\beta\mu/(\alpha-\beta i^*)$. Therefore (42) requires,

$$\mu\beta/(\alpha-\beta i^*) \le (1-\delta^n)\Gamma/\delta^n - n\mu \qquad (43)$$

This condition must hold for it to be optimal for the central bank to remain passive in the face of an expected speculative attack. In the limit as $n\to0$ the right hand side goes to zero and the inequality cannot hold for any set of parameters. Therefore with short periods there cannot be an equilibrium where the central bank remains passive in the face of a predictable speculative attack.

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