When to leave a monetary union: now or later?

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Abstract

Using a two-country model of monetary union where policymakers minimize the continuous-time equivalent of a Barro-Gordon-type loss function, we examine the value of the option of monetary break-up when the national preference parameters associated with an inflationary surprise follow correlated geometric Brownian motions. We derive the critical level of the ratio of these parameters that triggers a move to monetary disintegration and find that a country will be willing to return to monetary independence only if the other country's relative inflation preferences are strictly, and potentially substantially, greater than a benchmark value depending on the cost of monetary break-up alone.

Keywords: monetary disintegration; inflation; option; Brownian motion; irreversible investment

JEL classification: E5, F3

1 Introduction

When is it optimal for a country to leave a monetary union? For countries whose preferences over inflation differ, conventional wisdom suggests that any one of them will generally benefit from returning to monetary independence if the supranational preferences governing policymaking in the monetary union become less inflation averse than its own.[1] This view is certainly too simplistic, as the rapidly growing literature on irreversible investment under uncertainty shows us that the decision to invest in an irreversible project with uncertain payoffs can be profoundly affected when that investment can be delayed, as the option of waiting then typically has non-zero value and needs to be accounted for.[2] Applying this particular methodology to a country's decision of whether or not to return to monetary independence, thus interpreted as largely irreversible with uncertain benefits, should therefore allow a more rigorous understanding of the importance of relative inflation preferences in this context.

To investigate these issues in more detail, we use a simple two-country model where policymakers minimize the continuous-time equivalent of a Barro-Gordon-type loss function over inflation,[3] and examine the value of the option to return to monetary independence from a situation of monetary integration when the national preference parameters associated with an inflationary surprise follow correlated geometric Brownian motions. We derive the critical level of the ratio of these parameters that triggers a move to monetary disintegration and find that a country will generally be willing to return to monetary independence only if the other country's inflation preferences are higher than its own by a factor strictly, and potentially substantially, greater than a benchmark value depending on the proportional cost of monetary break-up alone. Higher uncertainty regarding these inflation preferences increases the value of the option to wait and thereby raises the trigger value

[1] This abstracts from the other potential costs and benefits of monetary integration/disintegration; see e.g. De Grauwe (1997), Gros/Thygesen (1998).

[2] See e.g. Dixit (1992), Pindyck (1991) or, more comprehensively, Dixit/Pindyck (1994).

[3] A similar, but more general, framework is used in Strobel (1999) to study the decision of joining a monetary union.

that prompts the option of monetary disintegration to be exercised. A higher discount rate (i.e. policymakers being more short-sighted) increases the opportunity cost of leaving the option of monetary break-up unexercised for a further instant, and thus lowers the value of that option. The likelihood of the two countries' inflation preference parameters drifting apart gets smaller the more correlated these are, having the same effect. Lastly, exercising the option of monetary disintegration becomes more onerous the higher the proportional cost associated with this move, raising the trigger value that prompts a return to monetary independence.

Section 2 now sets up the model and characterizes the optimal stopping problem involved, section 3 presents the solution and discusses our results, and section 4 concludes the paper.

2 Model

2.1 General loss function

We assume the policymaker's objective involves the instantaneous loss rate

$$z(b,\pi) = \frac{1}{2}a[\pi(t)]^2 - b(t)[\pi(t) - \pi^e(t)]$$

with inflation cost parameter $a > 0$, where $\pi(t)$ and $\pi^e(t)$ represent inflation and expected inflation, respectively.[4] The time-varying inflation preference (or benefit) parameter $b(t) \geq 0$ follows a geometric Brownian motion with drift such that

$$db = \alpha b dt + \sigma b dz$$

where $dz = \varepsilon(t)\sqrt{dt}$ is the increment of a Wiener process with $\varepsilon(t) \sim \mathrm{NID}(0,1)$.

Restricting our analysis to a discretionary policy scenario, the policymaker's choice problem is then to solve for the optimal feedback rule $\pi^*(b)$ that

[4]This adapts the discrete-time setup in Barro/Gordon (1983) to a continuous-time environment; a similar, but more general, framework is used in Strobel (1999).

satisfies the loss function

$$L(b) = \min_{\pi} E_t \int_t^{\infty} \left\{ \frac{1}{2} a[\pi(\tau)]^2 - b(\tau)[\pi(\tau) - \pi^e(\tau)] \right\} e^{-\mu(\tau - t)} d\tau$$

where $\mu > 0$ is the discount rate, treating inflationary expectations $\pi^e(\tau)$ as given $\forall \tau \geq t$.

From the Bellman equation $\mu L(b) = \min_{\pi}[z(b,\pi) + \frac{1}{dt} E_t dL(b)]$, applying Ito's Lemma and noting that $E_t dz = 0$, we obtain

$$\mu L(b) = \min_{\pi}[z(b,\pi) + \alpha b \frac{\partial L}{\partial b} + \frac{1}{2}\sigma^2 b^2 \frac{\partial^2 L}{\partial b^2}]$$

as the relevant equation of optimality. Minimization of the latter expression's bracketed term implies $a\pi(t) - b(t) = 0$, and thus gives

$$\pi^*(b) = \frac{1}{a} b(t)$$

as the optimal feedback rule in question.

Imposing rational expectations such that $\pi^e(\tau) = \pi(\tau)$, $\forall \tau \geq t$ at this stage, the resulting loss function becomes

$$L(b) = E_t \int_t^{\infty} \frac{1}{2a} [b(\tau)]^2 e^{-\mu(\tau - t)} d\tau$$

in equilibrium. Noting that $E_t[b(\tau)]^2 = [b(t)]^2 e^{(2\alpha + \sigma^2)(\tau - t)}$, $\forall \tau \geq t$ from standard properties[5] of geometric Brownian motion, this reduces to

$$L(b) = \frac{1}{2a}[b(t)]^2 \int_t^{\infty} e^{(2\alpha + \sigma^2 - \mu)(\tau - t)} d\tau = \frac{1}{2a(\mu - 2\alpha - \sigma^2)} [b(t)]^2 \qquad (1)$$

as long as $2\alpha + \sigma^2 - \mu < 0$, a condition we shall assume to be satisfied.

2.2 Monetary independence & integration

We characterize the case of monetary independence such that the inflation preference parameter $b_i(t)$ in each country $i = 1, 2$ follows a geometric Brow-

[5] See e.g. Dixit (1993, eq. (2.2)).

nian motion without drift

$$db_i = \sigma b_i dz_i$$

where $\sigma \geq 0$ and $E_t(dz_1 dz_2) = \rho dt$, with ρ the coefficient of correlation between the processes z_i and thus $-1 \leq \rho \leq 1$. Using equation (1) above, in equilibrium the respective loss functions $L(b_i)$ then become

$$L(b_i) = \frac{1}{2a(\mu - \sigma^2)}[b_i(t)]^2 \tag{2}$$

as long as $\sigma^2 - \mu < 0$, where we assume a common discount rate μ and inflation cost parameter a .

For the monetary integration case we assume that the supranational policymaker's inflation preference parameter is determined symmetrically as $b_{12}(t) = \sqrt{b_1(t)b_2(t)}$, with the constituent national inflation preference parameters $b_i(t)$ evolving as above. Using Ito's Lemma and simplifying we can write

$$db_{12} = \frac{1}{4}\sigma^2 (\rho - 1) b_{12} dt + \frac{1}{2}\sigma b_{12}(dz_1 + dz_2)$$

so that b_{12} follows a geometric Brownian motion, with expected drift $E_t(\frac{db_{12}}{b_{12}})$ $= \frac{1}{4}\sigma^2 (\rho - 1) dt$ and variance $E_t(\frac{db_{12}}{b_{12}})^2 - [E_t(\frac{db_{12}}{b_{12}})]^2 = \frac{1}{2}\sigma^2(\rho + 1)dt$. Drawing again on equation (1), in equilibrium the loss function $L(b_{12})$ then becomes

$$L(b_{12}) = \frac{1}{2a(\mu - \rho\sigma^2)} b_1(t) b_2(t) \tag{3}$$

as long as $\rho\sigma^2 - \mu < 0$.

2.3 Optimal stopping problem

Starting from a situation of monetary integration between countries 1 and 2, the decision of, say, country 1 on whether or not to return to monetary independence involves solving the Bellman equation for the optimal stopping problem

$$F(L_{12},L_1)=\max\left\{L_{12}-(1+\tau)L_1\ ,\ \frac{1}{\mu dt}E_t[dF(L_{12},L_1)]\right\}$$

where $F(L_{12},L_1)$ is the value to country 1 of the option to give up monetary integration with country 2 and return to monetary independence, and $L_{12}-(1+\tau)L_1$ is the expected discounted benefit of such a move, where $\tau\geq 0$ is the proportional cost of monetary break-up to country 1.

In the continuation region, where the second term on the right-hand side is the larger one and postponing monetary break-up for a further instant dt is thus optimal, the relevant Bellman equation is then just

$$\mu F(L_{12},L_1)=\frac{1}{dt}E_t[dF(L_{12},L_1)]$$

Applying Ito's Lemma and ignoring terms of order $(dt)^{\frac{3}{2}}$ and $(dt)^2$, we obtain

$$\begin{gathered}2\sigma^2L_1^2\tfrac{\partial^2F}{\partial L_1^2}+\sigma^2(\rho+1)L_{12}^2\tfrac{\partial^2F}{\partial L_{12}^2}+2\sigma^2(\rho+1)L_1L_{12}\tfrac{\partial^2F}{\partial L_1\partial L_{12}}+\\+\sigma^2L_1\tfrac{\partial F}{\partial L_1}+\rho\sigma^2L_{12}\tfrac{\partial F}{\partial L_{12}}-\mu F=0\end{gathered}\tag{4}$$

as the partial differential equation satisfied by the value function $F(L_{12},L_1)$ that applies over the region of (L_{12},L_1) space where holding the option of monetary break-up unexercised is optimal.

The corresponding value-matching condition

$$F(L_{12}^*,L_1^*)=L_{12}^*-(1+\tau)L_1^*$$

and smooth-pasting conditions

$$\frac{\partial F(L_{12}^*,L_1^*)}{\partial L_{12}}=1\quad,\quad\frac{\partial F(L_{12}^*,L_1^*)}{\partial L_1}=-(1+\tau)$$

should then in principle allow derivation of the value function $F(L_{12},L_1)$ and the boundary (L_{12}^*,L_1^*) in (L_{12},L_1) space that separates the region where the option of monetary break-up remains unexercised from the one where exercise of that option is immediate; this is generally a non-trivial problem.

Noting[6], however, that the optimal decision should only depend on the ratio $\Gamma \equiv \frac{L_{12}}{L_1}$,[7] and that therefore the value function $F(L_{12}, L_1)$ should be homogeneous of degree 1 in (L_{12}, L_1) , we can write

$$F(L_{12}, L_1) = L_1 f(\frac{L_{12}}{L_1}) = L_1 f(\Gamma)$$

Using this property to rewrite the partial differential equation (4) as

$$\sigma^2(1-\rho)\Gamma^2 \frac{\partial^2 f}{\partial \Gamma^2} - \sigma^2(1-\rho)\Gamma \frac{\partial f}{\partial \Gamma} + (\sigma^2 - \mu) f = 0 \tag{5}$$

makes it an ordinary differential equation in the unknown function $f(\Gamma)$ of the scalar variable Γ , which is straightforward to solve. We then obtain the value-matching condition

$$f(\Gamma^*) = \Gamma^* - (1+\tau) \tag{6}$$

and the smooth-pasting condition

$$\frac{\partial f(\Gamma^*)}{\partial \Gamma} = 1 \tag{7}$$

as the corresponding boundary conditions.

3 Solution & discussion

We observe that the differential equation (5) becomes degenerate (i) for the non-stochastic case where $\sigma = 0$, and (ii) for the symmetric scenario where $\rho = 1$. In both cases we obtain $f(\Gamma) = 0$,[8] so that country 1's option to abandon monetary integration with country 2 and return to monetary independence has zero value throughout. Note also that $\Gamma^* = 1 + \tau$ then follows from the value-matching condition (6), giving $\frac{b_2^*}{b_1^*} = 1 + \tau$ from the definition of Γ . Thus, when there is no uncertainty about the evolution

[6] This borrows from the solution strategy in Dixit/Pindyck (1994, p. 210).

[7] Thus, $\Gamma(b_1, b_2) = \frac{\mu - \sigma^2}{\mu - \rho\sigma^2} \frac{b_2(t)}{b_1(t)}$ from equations (2) and (3).

[8] With $\sigma^2 - \mu \neq 0$, by assumption from above.

of country 1 and 2's inflation preferences, or their inflation preferences are perfectly correlated, the trigger value[9] of relative inflation preference parameters $\frac{b_2^*}{b_1^*}$ depends solely on the proportional cost of monetary break-up τ , with country 1 generally willing to return to monetary independence only when country 2's inflation preferences are higher than its own by at least a factor of $1 + \tau$.

Turning now to the more interesting non-degenerate case where $\sigma > 0$ and $\rho < 1$, we try the function $f(\Gamma) = A\Gamma^\beta$ as a solution to the differential equation (5), and confirm by substitution that it is one if β is a root of the characteristic equation

$$Q(\beta) = \sigma^2(1-\rho)\beta(\beta-1) - \sigma^2(1-\rho)\beta + (\sigma^2 - \mu) = 0$$

and thus

$$\beta_1 = 1 + \frac{1}{\sigma}\sqrt{\frac{\mu - \rho\sigma^2}{1-\rho}}$$
$$\beta_2 = 1 - \frac{1}{\sigma}\sqrt{\frac{\mu - \rho\sigma^2}{1-\rho}}$$

where the two (real) roots will satisfy $\beta_1 > 1$ and $\beta_2 < 0$, respectively.[10]

The general solution to our differential equation (5) in $f(\Gamma)$ is then $f(\Gamma) = A_1\Gamma^{\beta_1} + A_2\Gamma^{\beta_2}$ which, noting that $A_2 = 0$ in order to satisfy the boundary condition[11] $f(0) = 0$, reduces to

$$f(\Gamma) = A_1\Gamma^{\beta_1}$$

where the parameter A_1 can be determined using the value-matching condi-

[9] That is, the value of $\frac{b_2^*}{b_1^*}$ separating the region in (b_1, b_2) space where the option of monetary break-up remains unexercised (i.e. for $\frac{b_2}{b_1} < \frac{b_2^*}{b_1^*}$) from the one where exercise of that option is immediate (i.e. for $\frac{b_2}{b_1} \geq \frac{b_2^*}{b_1^*}$).

[10] Note that the graph of $Q(\beta)$ is an upward-pointing parabola where $Q(0) = \sigma^2 - \mu < 0$ and $Q(1) = \rho\sigma^2 - \mu < 0$ by assumption from above.

[11] Note that Γ follows a geometric Brownian motion, for which 0 is an absorbing barrier.

tion (6) and smooth-pasting condition (7) as

$$A_1 = (\beta_1 - 1)^{\beta_1 - 1} (1+\tau)^{1-\beta_1} \beta_1^{-\beta_1}$$

In a similar manner we obtain

$$\Gamma^* = \frac{(1+\tau)\,\beta_1}{\beta_1 - 1} = (1+\tau)(1+\sigma\sqrt{\frac{1-\rho}{\mu-\rho\sigma^2}})$$

as the critical value Γ^* . From the definition of Γ it then follows that

$$\frac{b_2^*}{b_1^*} = \frac{1+\tau}{1-\sigma\sqrt{\frac{1-\rho}{\mu-\rho\sigma^2}}} \tag{8}$$

is the trigger value of relative inflation preference parameters $\frac{b_2^*}{b_1^*}$ that separates the region in (b_1, b_2) space where the option of monetary break-up remains unexercised (i.e. for $\frac{b_2}{b_1} < \frac{b_2^*}{b_1^*}$) from the one where exercise of that option is immediate (i.e. for $\frac{b_2}{b_1} \geq \frac{b_2^*}{b_1^*}$).

The critical level of country 2's inflation preferences that will induce country 1 to return to monetary independence depends both on the proportional cost of monetary break-up and the value of leaving the option of monetary disintegration unexercised for a further instant. In particular, it can be seen that $\frac{b_2^*}{b_1^*} > 1+\tau$, so that country 1 will generally be willing to return to monetary independence only if country 2's inflation preferences are higher than its own by a factor *strictly greater* than $1+\tau$, the benchmark value for the degenerate cases of $\sigma = 0$ and $\rho = 1$ discussed above. Intuitively, country 1's option to abandon monetary integration with country 2 has non-zero value in the non-degenerate case and will therefore be exercised only at a point where, in the jargon of financial options, it is sufficiently "in-the-money".

Examining more closely the directional impact of changes in σ, μ, ρ and

τ on $\frac{b_2^*}{b_1^*}$, we obtain

$$\frac{\partial \frac{b_2^*}{b_1^*}}{\partial \sigma} = \frac{\Theta\left(1+\tau\right)(1-\rho)\mu}{\left(\mu-\sigma^2\right)^2} > 0$$

$$\frac{\partial \frac{b_2^*}{b_1^*}}{\partial \tau} = \frac{1}{1-\sigma\sqrt{\frac{1-\rho}{\mu-\rho\sigma^2}}} > 0$$

$$\frac{\partial \frac{b_2^*}{b_1^*}}{\partial \mu} = -\frac{\Theta\left(1+\tau\right)(1-\rho)\sigma}{2\left(\mu-\sigma^2\right)^2} < 0$$

$$\frac{\partial \frac{b_2^*}{b_1^*}}{\partial \rho} = -\frac{\Theta\left(1+\tau\right)\sigma}{2\left(\mu-\sigma^2\right)} < 0$$

where $\Theta \equiv 2\sigma + \frac{\sigma^2(1-\rho)+\mu-\rho\sigma^2}{\sqrt{(1-\rho)(\mu-\rho\sigma^2)}} > 0$.

We note that $\frac{b_2^*}{b_1^*}$ is increasing in σ , a result familiar from the standard option pricing literature, as higher uncertainty regarding country 1 and 2's inflation preferences increases the value of the option to wait and thereby raises the trigger value that prompts the option of monetary disintegration to be exercised. The trigger value $\frac{b_2^*}{b_1^*}$ is also increasing in τ , as exercising the option of monetary break-up becomes more onerous the higher the proportional cost associated with this move. Increasing μ leads to lower levels of $\frac{b_2^*}{b_1^*}$, as a higher discount rate (i.e. policymakers being more short-sighted) raises the opportunity cost of leaving the option of monetary disintegration unexercised for a further instant, and thus decreases the value of that option. The trigger value $\frac{b_2^*}{b_1^*}$ is similarly decreasing in ρ , as the likelihood of the two countries' inflation preference parameters drifting apart gets smaller the more correlated these are, thereby decreasing the value of the option to postpone monetary break-up.

These qualitative results are illustrated in Figures 1–4, where we graph the trigger value $\frac{b_2^*}{b_1^*}$ for different parameter combinations of σ, μ, ρ and τ . While serious parameterization of the model may be somewhat ambitious due to its relative simplicity, it is nevertheless worthwhile noting (i) the substantial magnitudes of $\frac{b_2^*}{b_1^*}$ that can arise, and (ii) the dominance of the value of the option of monetary disintegration versus the proportional cost of monetary break-up in the determination of the trigger value $\frac{b_2^*}{b_1^*}$.

4 Conclusion

Using a simple two-country model where policymakers minimize the continuous-time equivalent of a Barro-Gordon-type loss function over inflation, we examined the value of the option to return to monetary independence from a situation of monetary integration when the national preference parameters associated with an inflationary surprise follow correlated geometric Brownian motions. We derived the critical level of the ratio of these parameters that triggers a move to monetary disintegration and found that a country will generally be willing to return to monetary independence only if the other country's inflation preferences are higher than its own by a factor strictly, and potentially substantially, greater than a benchmark value depending on the proportional cost of monetary break-up alone. Higher uncertainty regarding these inflation preferences increases the value of the option to wait and thereby raises the trigger value that prompts the option of monetary disintegration to be exercised. A higher discount rate (i.e. policymakers being more short-sighted) increases the opportunity cost of leaving the option of monetary break-up unexercised for a further instant, and thus lowers the value of that option. The likelihood of the two countries' inflation preference parameters drifting apart gets smaller the more correlated these are, having the same effect. Lastly, exercising the option of monetary disintegration becomes more onerous the higher the proportional cost associated with this move, raising the trigger value that prompts a return to monetary independence.

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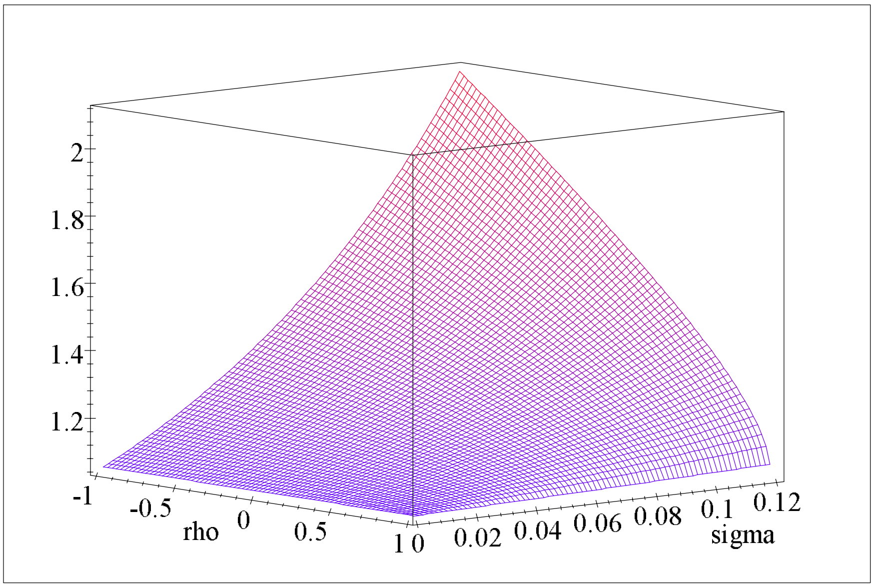

Fig. 1: Trigger value (tau=0.05,mu=0.1)

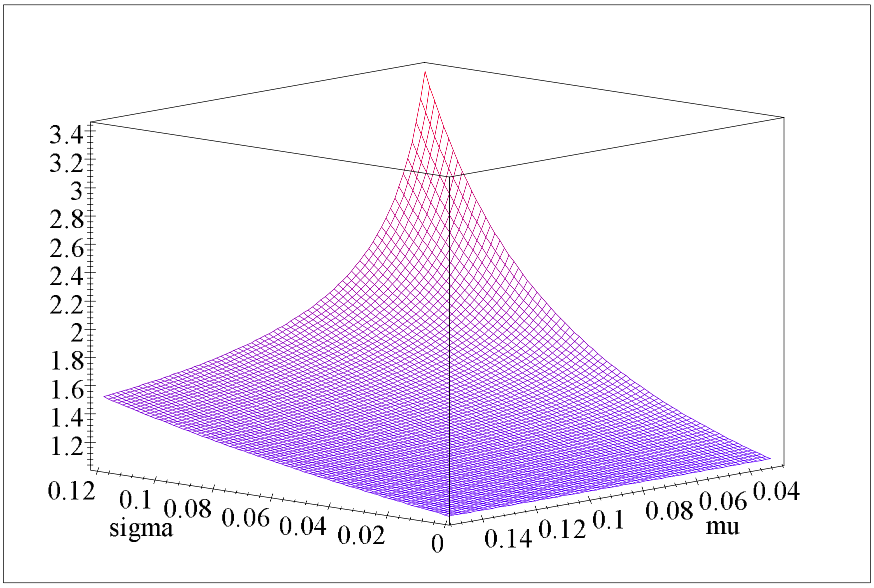

Fig. 2: Trigger value (tau=0.05,rho=0)

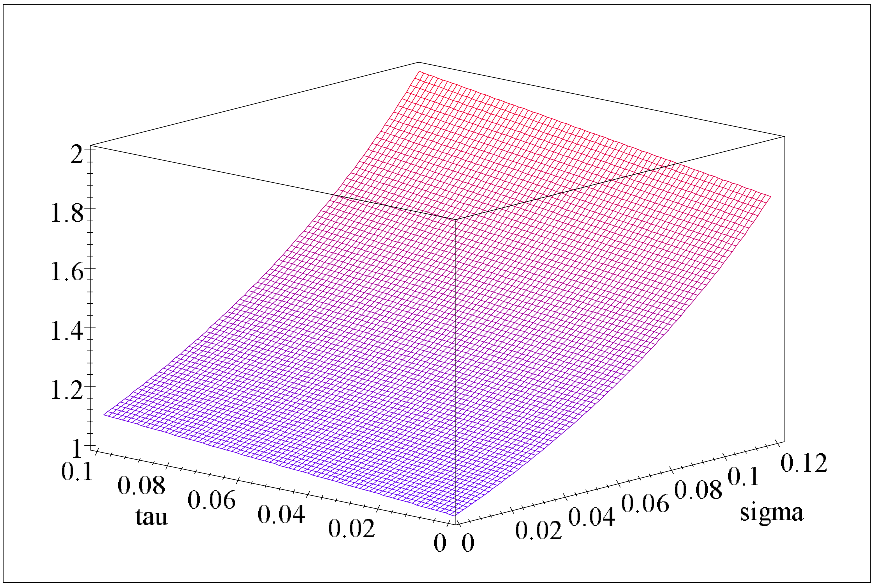

Fig. 3: Trigger value (rho=-0.5,mu=0.1)

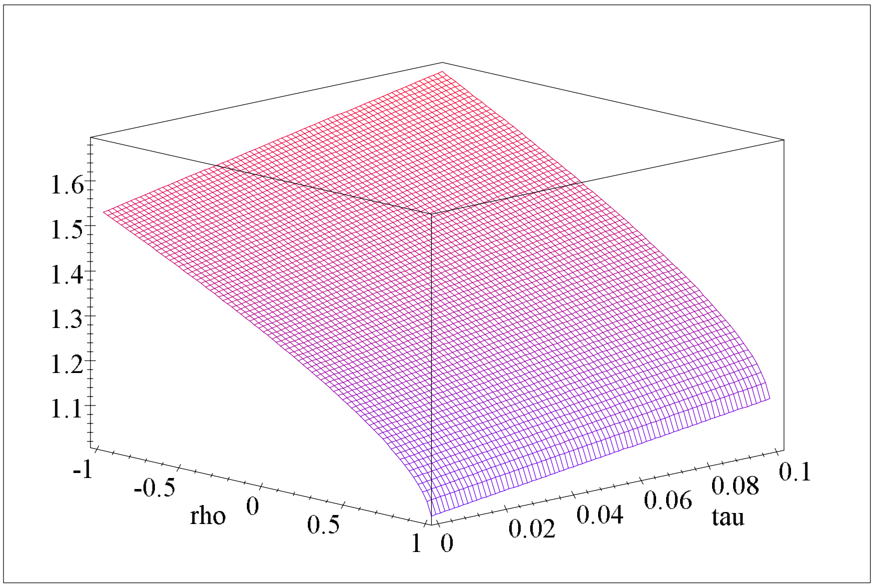

Fig. 4: Trigger value (sigma=0.08,mu=0.1)