Influencing the Misinformed Misbehaver*

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Abstract

In many instances where an authority tries to influence behaviors, ensuring an adequate perception of the agents is not less important than setting ideal incentives. A difficulty arises when the authority would like to improve their imperfect coercive instruments by delivering biased information (exaggeration or attenuation) to the misbehaver. We develop the case of consumption choice in the presence of uncertain external effects. We study the constraints that the conflict of interest puts on equilibrium policies. Anti-smoking campaigns and policy against antibiotics over-consumption serve as illustrations. Technically, we solve a model of signal cum cheap talk.

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1 Introduction

In classical public economic theory, the Pigouvian tax is the perfect tool against external effects. By choosing the appropriate rate, the Government can leave economic agents free to choose their actions with the same degree

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of selfishness, and nevertheless obtain the ideal outcome, as if people were internalizing the externality. Applied to the environment, this idea has given a basis for taxes on emissions. In practice, the applicability of the principle is far from obvious when we consider that people react to the taxes they are imposed as well as to their (less than ideal) understanding of the laws of nature (or of the economy)...

This paper builds on two observations. The first is that tax policy or any other coercive arrangement generate distortions away from the first-best (this may be rationalized in more than one way, in particular with various species of asymmetric information); the second is that, under uncertainty, consumers are responsive to new information on the impact of their actions. Provided information comes from sources which are not all independent of the Government, tax policy and information campaigns are not independent, and should be understood as a whole. We illustrate how the Government, even in the hypothetical case where it defends public interest (for clarity, this expression bears no ambiguity in our approach), may try to produce with biased beliefs what it can't attain with imperfect instruments. The argument is general, but in our model, this implies economizing public money by affecting the public's mental representation of the issue. At the limit, the first-best allocation can be attained with a vast imposture...

Many examples can be found in environmental and health economics. Anti tobacco policy relies both on financial consequences (prohibition being seen as a mere extreme form of taxation) and on education: the consumer reacts to the price (comprised the tax) but also to his perception of the impact on health (his and others'). Compared to the former, the latter effect is intuitively not negligible. Adequacy of perception at the individual level is not guaranteed, and a population may be significantly heterogenous in this respect. It seems that dramatic communication campaigns have partially missed their points. Two typical attitudes of the public are encountered: quite often, people take extreme messages for granted ('tobacco kills') and become intolerant (in the sense that they devote too many resources to this cause), or, quite often too, they become exaggeratedly skeptic and keep smoking too much.

Antibiotics overconsumption is also a valuable illustration of a category of problems posed to the health authorities.[1] As it is well documented, broad spectrum antibiotics have a double negative impact, besides the obvious beneficial effects. At the individual level, they have secondary effects (notably they clear the way to more harmful germs);[2] at the social level, they enhance

[1] See Levy (1992) for the medical viewpoint, and Brown and Layton (1996) for an excellent economic analysis of the external effects involved.

[2] In general broad-spectrum antibiotics decrease individual's immunology reaction and,

resistance of the germs involved in epidemic diseases. As a consequence, at every subsequent illness episode, the individually optimal treatment may tend to increase faster than the socially optimal one.[3] This threat is already a hot issue for health authorities and organizations worldwide, and professional are fairly aware of the need for public policy. Nevertheless, the stakes remain somewhat vague and remote for the general public.[4] As our work wants to suggest, asymmetric information between the professionals and the patients complicates the elaboration of an efficient policy combining coercive and informative dimensions.

As economists accustomed to model rational solutions to social interactions, we wish to avoid explaining policy success or failure with irrational behavior of the public. Our approach is essentially based on equilibrium notions, where people are not systematically fooled (they interpret actions in a Bayesian way) and where, under this "constraint", the Government tries, to get the best of its available instruments. This work characterizes public policies (tax rates *and* information campaigns à la Crawford and Sobel 1982) that resist the public's sophisticated understanding of propaganda.

Before going on, we would like to clarify the scope we give to information campaigns in our model. First of all, we treat information campaigns as statements used by the authority to influence the public's perception of the problem (e.g. smoke damages or antibiotics induced resistant bacteria). A fundamental characteristic of information campaigns in our model is that they have no direct consequence on the authority's and on the consumers' utility: they are completely costless and, as a consequence, they enter into the category of cheap talk messages. This implies that the literal meaning of the information provided is sufficiently vague not to be falsifiable. As an example, the reader can think about what tobacco firms are obliged to write on cigarettes packets in France: 'Nuit gravement à la santé' (Harmful to health). This is not false, but the exact nuance it bears is a matter of social convention (specifically the way cheap talk is interpreted). Thus we make here a distinction between *information campaigns* and what we can call *hard*

as a consequence, new diseases can arise. Many antibiotics based on penicillin are used to treat diseases like bronchitis, otitis and tonsillitis caused by different bacteria (staphilococcus aureus, haemofilus influenzae, streptococcus pneumoniae). Possible secondary effects of penicillin consumption are Candida Albicans and Herpes. Both skin diseases need specific treatment: antifungal and antiviral agents respectively. In this case the decrease in individual's utility due to broad-spectrum antibiotic secondary effects is particularly evident.

[3] An aspect of this overconsumption is the massive use of antibiotics for breeding cattle.

[4] See Okeke *et al.* (1999) for the reasons, in developing countries, for antibiotic misuse. No doubt that the phenomena they point at are also active in developed economies.

information. The first takes the form of 'free' advertising,[5] while the second implies many costs (in particular time) for both parts. In fact, when hard evidence is provided, the authority collects detailed scientific knowledge and many data from the experts such that credibility becomes an issue that can only be circumvented at a cost (consumers have to spend time hearing and processing the new information). In other terms, we take the problem at the point where cheap, trustworthy by nature, possibilities in the short-run are exhausted. We will come back on the difference between information campaigns and education in our concluding remarks.

To be as simple as possible, and to raise the main issues in elementary terms, this model analyses an economy where consumers' current consumptions affect future utilities. The first effect is caused by the consumer's own consumption, and can be seen as a tolerance effect (in the terminology of drug addiction) or *secondary effects.* The other effect depends on aggregate consumption and is a typical *externality.* Consumers only have probabilistic beliefs on the parameter driving the intensity of secondary effects; at the same time, they do not internalize the external effect. In the absence of an appropriate public policy, consumers choose inadequate current consumption with respect to the individual and social optima.

The dynamic interpretation fits a series of problems where an exhaustible resource (e.g. the efficacy of antibiotics) may be exploited too fast if externalities are ignored. Smoking prevention policy is also constrained by similar dynamics: the effects of tobacco are not always precisely known (they may be under- or over-estimated, see Viscusi 1990), and the external consequences (second-hand smoking and additional health care costs supported by the community) are not negligible. Another interpretation of the model is to separate the goods consumed by the consumer into two groups: manufactured commodities, and commodities that are complementary to the quality of the environment. In this view, consumption and production of manufactured goods have adverse effects on the environment, i.e., indirectly, on the utility derived from the other goods. These effects are partly local and partly global, hence the need for information on local effects and coercion for curbing global effects.

We examine the power of taxes coordinated with (cheap talk) information campaigns. The Government (or the social planner, to summarize the various bodies in charge of the public's interest) is the informed principal in

[5] Note that our approach is valid if information campaigns are costly, but their costs are independent of the message the authority decides to send. This means that the diffusion cost associated to an information as "smoke is detrimental for one's health" is the same of an information as "smoke is *very* detrimental to one's health". As a consequence, if no-message is not a choice, we can, without generality loss, normalize this cost to zero.

this model. Two arguments support this assumption. Firstly, the state is in a position, by nature, to appeal to experts (civil servants, professionals, academics,...) who are able to transform dispersed data and results in operational knowledge. (We don't need to believe that this operation is perfect, since it remains reasonable to think that it is better done by the experts than by the majority of the public.) Secondly, it is likely that a researcher, if he obtains new results, will try to convince the authorities first so as to maximize the impact.

To eliminate, in the model, the consequences of a bad administration, we assume that the Government is benevolent (it maximizes the utility of the representative consumer). We propose a solution concept (a Bayesian equilibrium) to predict the equilibrium policies and allocations. We point at the fundamental equilibrium trade-off: more precision in the messages implies more distortion in the allocation. We show the following: (1) the equilibrium is never efficient since the Government is irresistibly induced to manipulate information to get the desired responses at a lower cost; (2) there exists a unique fully informative equilibrium, in which cheap talk is useless; (3) the most informative equilibrium may not be the most desirable ex ante: given credibility costs, delivering rough messages (by treating identically slightly different situations) may be optimal. Our approach differs from the standard. Finding all the equilibria for a given economy is impossible in general (save for quadratic examples in Crawford and Sobel 1982). We solve the inverse problem: the Theorem of the skeleton characterizes the set of distribution which supports a given allocations. We prove that this set is typically empty, or constrained by a certain number of conditional expectations. This facilitates discussions on efficiency.

The following section describes the model and define the equilibrium notion. Section 3 determines optimal allocations and explores the reasons that prevent them to be implemented. The properties of the equilibria are characterized in Section 4. Section 5 concludes.

2 The Model

2.1 The Consumer

The representative consumer lives two periods and the value of period-2 consumption is negatively affected by period-1 consumption. The consequences of period-1 consumption on period-2 consumption pass through two distinct channels.

- Secondary effects are due to the consumer's own consumption in period 1, x_1. These effects is measured by $-\theta x_1$ where the intensity θ which is not precisely known to the consumer. The cumulative distribution function $F(\theta)$ supported in $[\underline{\theta}, \overline{\theta}]$, and its density $f(\theta)$, correspond to the consumer's priors on θ. In general, f is continuous and non negative on the support. Special mention to the case of a finite support for θ will be made when interesting.

- The externality depends on *average* period-1 consumption in the economy, $\overline{x}_1$. The externality is measured by $-\eta\overline{x}_1$, where the intensity η is supposed to be known to all the agents.[6] The consumer does not internalize the social consequence of his period-1 consumption. This happens because there is a large number of atomistic consumers in the economy: each consumer knows that the consequence on the external effect of reducing his first-period consumption would be only marginal.

In general, the program of the consumer confronted to policy P (specified below):

$$(1) \qquad \begin{cases} \max\limits_{x_1,x_2} E\left\{U_1(x_1) + \beta U_2(x_2 - \theta x_1 - \eta\overline{x}_1)\,|P\right\} \\ \text{s.t.} : (p_1 + t)x_1 + p_2 x_2 = W + T \end{cases}$$

where prices $p_1 + t$ (t for the tax rate) and p_2 are the prices for first- and second-period consumptions; W is the consumer's endowment; T represents the transfer redistributing tax proceeds; U_1 and U_2 are concave increasing functions, and β is the standard discount factor.

2.2 Tax and Information Campaigns

The social planner is the informed principal which knows perfectly θ. We analyze a situation where the social planner applies a policy $P = (t, m) \in \mathbb{R}_+ \times M$ composed of the non-negative tax rate and a (cheap talk) message taken in a certain large set M. Through the choice of a policy P, it wishes to provide incentives *and* information to the consumer. We can think of m as composed of "sentences". We assume that M, the set of all possible phrases, is rich enough to say what needs to be said; for example it can be composed by all reasonably short utterances in English (see Farrell and Rabin 1996 on what cheap talk is and is not). It is useful, at this point, to make a distinction

[6] An alternative model could put the uncertainty on η. In general, though, this uncertainty would not exhibit the sort of conflict we are pointing at since the consumer's behavior is not affected by the intensity of the externality. In our specification, consumption does not even depend on η.

between the message the sender sends and the interpretation that the receiver gives to such a message at the equilibrium. What really matters is not the message m itself, but just the way the receiver understands it. To be clearer, whatever the language that is used to communicate, we will concentrate on the meaning the receiver assigns to every message in equilibrium.[7]

After observing the policy, the consumer updates his priors; his new beliefs are denoted by $\mu(P)$ (with $\mu(P) \in \Delta([\underline{\theta}, \overline{\theta}])$, the set of probability distributions over $[\underline{\theta}, \overline{\theta}]$).[8] Therefore, the consumer constrains the social planner to certain actions since he responds in a rational way to the incentives given by the tax, as well as to the information borne by the tax and the message.

The fiscal revenue collected by the social planner is redistributed in the form of a positive lump-sum transfer. Notice that the transfer T is not included as a message: it is constrained to redistribute all the tax proceeds, and, as a result, T is directly dependent of t and μ and does not bear any independent information. In general, there is a deadweight loss attached to taxes since a part of fiscal resources is lost in the collecting process. Let λ $(0 < \lambda < 1)$ be the cost of public funds, meaning that, λtx is lost, or used for other purposes: $T = (1 - \lambda)\, tx$ is the maximal lump-sum transfer redistributed to the consumer.

For simplicity, we assume that the discount rate β is equal to 1, and we normalize p_2 to 1 and p_1 to 0 without loss of generality since the support of θ may be translated to account for the price (notice however that this implies that $\underline{\theta} > 0$). Now we drop the index of x_1 to write the first-period consumption as x. We simplify further the model by adopting a quasi-linear utility function, where $U = \log$, and where the linear part implicitly represents the utility of the goods other than x. The consumer chooses

$$(2) \quad x(P) = \arg\max_x U(x) - \{E(\theta\,|\mu(P)) + t\}\, x \underbrace{-\eta\bar{x} + (1-\lambda)t\bar{x} + W}_{\text{Ignored by the consumer}}$$

Most propositions in Section 4 (characterization of the equilibria) do not rely on the restrictions on the specifications, as can be seen in the proofs. The explicit calculations of the first- and second-best that we can perform in Section 3 are more agreeable (at least we hope so) for the reader.

[7] As an example, let m_1 and m_2 denote two messages sent in a perfectly revealing equilibrium. Assume that m_1 corresponds to the word "dog" and m_2 corresponds to the word "cat". This is an equilibrium as long as the receiver understands this language and assigns to the message "dog" the meaning, say "$\theta = \theta_1$", and to the message "cat" the meaning, say "$\theta = \theta_2$", where θ_1 and $\theta_2 \in [\underline{\theta}, \overline{\theta}]$.

[8] In Maskin and Tirole (1995), the informed principal offers a mechanism which is a signal; in Villeneuve (2000) where a monopolistic insurer evaluates risk better than his customers, the contract offered is a signal. The contribution of cheap talk is not studied per se in these articles.

2.3 The Social Planner's Program

The social planner is assumed to be *benevolent* in the sense that it evaluates consumption like the consumer (it maximizes the consumer's utility), and *paternalistic* since it doesn't value truthful information per se, but only to the extent that it induces correct behavior. Tax policy and information campaigns are chosen so as to influence the consumer's behavior by improving his knowledge of secondary effects, and by discouraging over-consumption. Notice that there is no "insurance against θ", which is here as a global uninsurable risk, in other words, the social planner is constrained by its budget state by state. It solves, for each θ,

$$\left\{ \begin{array}{l} \max\limits_{P} U(x) - (\eta + \theta + \lambda t)x + W \\ \text{s.t. :} \\ x = x(P) \end{array} \right. \tag{3}$$

There are two differences between its program and the consumer's one: the superior information on the intensity of the secondary effects θ; and the internalization of the external effects. Notice that, consumers being all identical, in equilibrium $\bar{x} = x$.

Constant terms ignored by the decision makers are dropped in the sequel: we define $U^S(x,t,\theta) \equiv U(x) - (\eta + \theta + \lambda t)x$ for the social planner's objective, and $U^C(x,t,\theta) \equiv U(x) - (\theta + t)x$ for the consumer's (we drop terms that do not affect the preference orderings).

2.4 The Information Game

Definition 1 *A Perfect Bayesian Equilibrium (PBE) of the game is a pure strategy $\mathcal{P}$ mapping $[\underline{\theta}, \overline{\theta}]$ into $I\!R_+ \times M$ and a belief μ mapping $I\!R_+ \times M$ into $\Delta([\underline{\theta}, \overline{\theta}])$ such that:*

1. *Policies are optimal given beliefs: for each $\theta \in [\underline{\theta}, \overline{\theta}]$, $\mathcal{P}(\theta)$ solves $\max\limits_{P} U^S(x(P), t, \theta)$.*
2. *Beliefs are rational given equilibrium policy: for each P, $x(P)$ solves $\max\limits_{x} \int_{\underline{\theta}}^{\overline{\theta}} U^C(x,t,\theta)\,\mu(\theta|P)d\theta$, where $\mu(\theta|P) \equiv \frac{\mathbb{I}_{\{\mathcal{P}(\theta)=P\}} \cdot f(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \mathbb{I}_{\{\mathcal{P}(\theta)=P\}} \cdot f(s)ds}$, $\mathbb{I}$ the indicator function.*

In the following, $\widehat{\theta}$ will often be used for $E(\theta|\mu(P))$; by a slight abuse of language, we will also refer to $\widehat{\theta}$ as a belief; $x^*(\widehat{\theta}, t)$ will be used for the best response of the consumer to his beliefs and the tax.

3 The Sources of Inefficiency

These results in this section aim at showing that the social planner's actions and allegations are likely to be suspicious in the consumer's mind: the social planner systematically wants to induce more pessimistic beliefs in place of imposing distortional taxes. This explains why neither the first-best nor the second-best can be implemented, and prepares the analysis of the restrictions put by incentive compatibility in the equilibria.

3.1 The First-Best Allocation

The *first-best allocation* is the optimum of the social planner if we assume that incentives and information are free. It solves the following program (θ is supposed to be known and the externality $\bar{x}$ to be internalized):

$$x_{FB}(\theta) = \arg\max_x U(x) - (\eta + \theta)x \tag{4}$$

or alternatively

$$U'(x) = \eta + \theta \tag{5}$$

(We recognize here the Samuelson condition, with the unimportant difference that we used average rather that total consumption for the external effect.) To implement this, tax policy should entail no cost ($\lambda = 0$), in which case $t = \eta$ would be perfect. In general, the cost of public funds prevents this allocation to be implemented.

3.2 The Attraction to Propaganda

In the *hypothetical* case where the social planner perfectly controls the tax rate *and* the consumer's beliefs, the optimal policy $(\widehat{\theta}, t)$ solves

$$\begin{cases} \max\limits_{\widehat{\theta},t} \ U(x) - (\eta + \theta + \lambda t)x \\ \text{s.t. } x = \arg\max\limits_{y} U(y) - (\widehat{\theta} + t)y \end{cases} \tag{6}$$

where θ is the true type. The constraint in (6) gives: $U'(x) = \widehat{\theta} + t$. Public funds being costly, the optimal tax rate is $t = 0$. As a consequence the program (6) yields the following strategy:

$$\begin{cases} t^*(\theta) = 0 \\ \widehat{\theta}^*(\theta) = \theta + \eta \end{cases} \tag{7}$$

When the externality is negative ($\eta > 0$), telling the truth is not the optimal strategy ($\widehat{\theta}^* > \theta$) and the most economical for the sender is to *exaggerate* its type by the amount η. In this way, the externality is fully internalized and the first-best attained using the costless instrument $\widehat{\theta}$.[9] This form of policy entirely based on propaganda is obviously never consistent with a rational consumer; it nevertheless gives useful indications on the incentives perceived by the policy maker, and explains quite well certain short-term bound to fail real policies.

3.3 The Second-Best Allocation

The *second-best allocation* is defined as the optimum of the social planner if information is free but incentives are not, due to the cost of public funds. This can be written as follows:

$$\left\{\begin{array}{l}\max_t U(x) - (\eta + \theta + \lambda t)x \\ \text{s.t. :} \\ x = \arg\max_y U(y) - (\theta + t)y\end{array}\right. \tag{8}$$

We find that the consumer's choice is:

$$x^*(\theta, t(\theta)) = \frac{1}{\theta + t(\theta)} \tag{9}$$

Plugging this value into program (8) gives the following program:

$$\max_t - \log(\theta + t) - \frac{\eta + \theta + \lambda t}{\theta + t} \tag{10}$$

Solving for t, we find the second-best allocation, and the corresponding second-best tax rate:

$$\left\{\begin{array}{l} t_{\text{SB}}(\theta) = \eta - \lambda\theta \\ x_{\text{SB}}(\theta) = \frac{1}{\eta + (1-\lambda)\theta}\end{array}\right. \tag{11}$$

($\lambda = 0$ gives the first-best.) In the following, we assume that $\overline{\theta} < \frac{\eta}{\lambda}$ to avoid corner solutions. This means in particular that the externality must not be too small compared to the maximal intensity of the secondary effects. The tax rate is strictly decreasing with respect to θ, which for the moment is not contradictory with the assumption that the tax rate itself is sufficiently informative.

[9] On the contrary, when there is no externality ($\eta = 0$), the incentive to lie disappears and telling the truth becomes optimal.

Is this allocation implementable? Suppose the consumer thinks that the social planner plays the second-best strategy. Notice that t is strictly decreasing with respect to θ, therefore the individual infers θ if $t_{\mathrm{SB}}(\theta)$ is imposed. There remains some hope to implement this allocation in an appropriate Bayesian model; this is what we test now.

Remark that the fiscal revenue $t_{\mathrm{SB}} x_{\mathrm{SB}}$, hence the lost resources due to the cost of public funds, decreases as θ increases. This implies that a low θ may have interest in mimicking a high θ. To see this, consider types $\widehat{\theta}$ and $\theta \in [\underline{\theta}, \overline{\theta}]$. Given our assumptions, observing the message $\widehat{t} = t_{\mathrm{SB}}(\widehat{\theta})$, the consumer infers $\widehat{\theta}$ and chooses $\widehat{x} = x^*(\widehat{\theta}, \widehat{t})$, whereas observing the message $t = t_{\mathrm{SB}}(\theta)$, he infers θ and chooses $x = x^*(\theta, t)$. If really $U^S(\widehat{x}, \widehat{t}, \theta) > U^S(x, t, \theta)$, there exists an incentive to misrepresent the sender's type: type θ wants to mimic type $\widehat{\theta}$.

Proposition 1
1. *The second-best allocation is never an equilibrium if* $\lambda > 0$.
2. *In the case of a finite number of types, the second-best is an equilibrium if and only if the cost of public funds* $\lambda > 0$ *is sufficiently small.*
3. *When public funds are free* $(\lambda = 0)$, *the first-best allocation is a PBE. The equilibrium policy entails a Pigouvian tax* $t = \eta$, *and precise interpretations of the messages* $\widehat{\theta} = \theta$.

Proof. The first two points are proved in the Appendix. For the third, remark that, indeed, if $\lambda = 0$, the optimal tax rate is $t = \eta$ (the tax is independent of θ). No information passes through t. Given that the social planner has no incentive to lie, the tax is specifically used to internalize the externality, and cheap talk has to be used to eliminate ambiguity.■

4 Equilibria

4.1 Propaganda Fails

Interestingly, whatever the complexity of the equilibrium, there is no perverse reason why less desirable states of the world (larger secondary effects) would become preferable: confronted to rational consumers, the authorities are not able to turn lead into gold with nice communication strategies... In other words, *in any PBE*, the larger the secondary effects, the lower the social welfare.

To see this, let θ_1 and θ_2 be two possible states of the world, and let (t_1, x_1) and (t_2, x_2) be the associated tax rates and induced consumption in equilibrium. If $\theta_1 < \theta_2$, then $U(x_2) - (\eta + \theta_1 + \lambda t_2)x_2 \geq U(x_2) - (\eta + \theta_2 + \lambda t_2)x_2$. In addition, the incentive constraint of the type-θ social planner reads: $U(x_1) - (\eta + \theta_1 + \lambda t_1)x_1 \geq U(x_2) - (\eta + \theta_1 + \lambda t_2)x_2$. By transitivity, we get: $U(x_1) + W - (\eta + \theta_1 + \lambda t_1)x_1 \geq U(x_2) - (\eta + \theta_2 + \lambda t_2)x_2$.

4.2 The Skeleton of an Equilibrium

A policy P is essentially characterized by the tax rate t and the beliefs $\widehat{\theta}$ it puts in the consumer's mind. For a complete understanding of the structure of the equilibria, a description of the functions $V^S(\widehat{\theta}, t, \theta) \equiv U^S(x^*(\widehat{\theta}, t), t, \theta)$ is useful: for all θ, the shapes of the upper contours in the plane $(\widehat{\theta}, t)$ are simple, and a single crossing properties is satisfied that simplifies the analysis of incentive constraints.

Lemma 1 (Convexity and Single Crossing) *1. In any PBE, for all P being an equilibrium action, $\mathcal{P}^{-1}(P)$ is a convex subset of $[\underline{\theta}, \overline{\theta}]$.*

2. *For all θ, the upper contours of V^S with respect to $\widehat{\theta}$ and t are convex. The highest contour is the point $(\widehat{\theta} = \theta + \eta, t = 0)$ (see Subsection 3.2).*

3. *Let $\mathcal{V}(\theta)$ be an indifference curves for type θ passing through $(\widehat{\theta}, t)$. $\mathcal{V}(\theta)$ turns continuously anti-clockwise locally at $(\widehat{\theta}, t)$ as θ increases. The consequence is that indifference curves related to two different types cross once at most.*

Proof. See the Appendix.■

Point 1 proves in particular that a given tax rate will signal an interval (rather than any subset of the support), the interval being possibly reduced to one-point in locally revealing equilibria. This result will be used at several places, for the moment, it implies that the definition of the equilibrium is complete (Condition 1 implies that beliefs in Condition 2 are well-defined): strategies inherit the measurability of the space of types.

A description of the equilibria given the prior type distribution is difficult to perform. We solve here the inverse problem: finding the distributions that are consistent with a certain equilibrium allocation. In particular we show that understanding the fully revealing equilibria with discrete supports is sufficient to describe the possible equilibria. This is interesting in that we can simply check whether certain properties like monotonicity of the tax rate are satisfied are not.

Definition 2 (Skeleton) *Let K be an ordered set. Let $\Theta = \{\theta_k\}_{k\in K}$ be a strictly ordered complete set of types. Let $T = \{t_k\}_{k\in K}$ be a set of tax rates; $\langle\Theta, T\rangle$ designates the set of pairs $\{(\theta_k, t_k)\}_{k\in K}$.*

$\langle\Theta, T\rangle$ is said to be a skeleton *if and only if $\forall k, k' \in K, V^S(\theta_k, t_k, \theta_k) \geq V^S(\theta_{k'}, t_{k'}, \theta_k)$ (incentive compatibility).*

By convention, we denote the lowest element of Θ as θ_1, and the largest as θ_∞. Given θ_k, we define its successor in Θ as $\theta_{k+1} \equiv \min_{k'\in K}\{\theta_{k'} > \theta_k\}$ (this "+1" is just a convention, inspired by the fact that when Θ is numerable, K can be a set of successive integers). This is well defined since a skeleton is always complete.

Notice that if $\langle\Theta, T\rangle$ were not incentive compatible, then certain elements could not be implemented, and we should drop them to obtain a proper skeleton. In contrast, assuming that Θ is complete is without loss of generality: if an accumulation point of Θ is missing, we can add it, with a corresponding accumulation point in T. Due to the continuity of the incentive constraints, incentives are not reversed.

Given a skeleton, certain necessary conditions that strategies and beliefs should satisfy for the skeleton to be an equilibrium (no extra types) are straightforward. It suffices to assume that the support of the types is exactly Θ, to take $P(\theta_k) = (t_k, m_k)$ for the policy, and, consistently, to take $\mu(t_k, m_k)$ degenerate at θ_k. The role of the cheap talk message m_k is to improve information transmission by overcoming potential ambiguity in the informative contents of the tax: if a certain tax is associated to one state only, the message is not useful; if two states leading to the same tax rate, then messages should be different.

The initial set of type Θ may not be sufficient for discouraging policies with $t \notin T$. Indeed, if types are only in Θ, then beliefs are constrained to be in the convex hull of Θ. The following definition draws attention on skeletons for which one can find a sufficiently large support of types for discouraging actions outside the skeleton.

Definition 3 (Sufficient Skeleton) *A* sufficient *skeleton $\langle\Theta, T\rangle$ is one for which there exists an interval $\Xi(\langle\Theta, T\rangle) \supset \Theta$ such that, for well chosen admissible beliefs, incentive compatibility guarantees that there is no move outside T for types in $\Xi(\langle\Theta, T\rangle)$. In particular: for all $k \in K$, for all $t \notin T, \exists\widehat{\theta} \in \Xi(\langle\Theta, T\rangle)$ such that $V^S(\theta_k, t_k, \theta_k) \geq V^S(\widehat{\theta}, t, \theta_k)$; moreover, lower types prefer t_1 and larger types t_∞ to any other tax.*

is exactly the convex hull of Θ, i.e. the set of beliefs that are consistent with support Θ.

Remark 1 *Unfortunately, we are not able to provide a simple characterization of sufficient skeletons. Note, nevertheless, that any skeleton can be made sufficient by adding* one *type in* Θ *and its associated tax. There is not a unique way to do this. The simplest is to add a sufficiently large type (large secondary effects), say* $\theta_{\max}$*, associated with* $t = 0$*; and we associate belief* $\theta_{\max}$ *to any tax outside* T*. Clearly, the belief being exaggerated for types in* Θ *other than* $\theta_{\max}$*, and* $t = 0$ *being better than any other value for* $\theta_{\max}$*, then we have a skeleton. The fact that it is sufficient is by construction.*

The following Theorem explains the restrictions on the "flesh" (the distributions, possibly continuous) that can be put on the bones $\langle \Theta, T \rangle$.

Theorem 1 *Let* $\langle \Theta, T \rangle$ *be a sufficient skeleton. Let be the set of types. Let* $\mathcal{F}$ *be the set of prior probability distributions on* $\Xi(\langle \Theta, T \rangle)$ *such that there exist an equilibrium with* $\langle \Theta, T \rangle$ *as the exact set of beliefs-tax pairs.*

There exists a partition of $[\underline{\theta}, \overline{\theta}]$ *into a set of ordered intervals* $\{I_k\}_{k \in K}$ *such that for all* $F \in \mathcal{F}$*, the equilibrium strategy implementing* $\langle \Theta, T \rangle$ *satisfies almost surely* $t(\cdot) = t_k$ *over* I_k *for all* k*.*

Moreover, $\theta_k \in I_k$ *and* $\mathcal{F}$ *is non-empty and restricted only by the linear constraints:* $\forall F \in \mathcal{F}, \forall k \in K, E(\theta | I_k) = \theta_k$*.*

Proof. We reason on incentive compatibility.

If $\theta_{k+1} \neq \theta_k$, we denote by τ_k a type which is indifferent between $P(\theta_k)$ and $P(\theta_{k+1})$, i.e. $V^S(\theta_k, t_k, \tau_k) = V^S(\theta_{k+1}, t_{k+1}, \tau_k)$: given the single crossing property, and given the continuity of the social planner's welfare function with respect to the true type, τ_k is unique and belongs to $[\theta_k, \theta_{k+1}]$. We define $I_k = [\tau_k, \tau_{k+1})$. If the successor of θ_k is θ_k (this happens if θ_k is, on the right, an accumulation point in Θ), then $I_k = \{\theta_k\}$. The lower bound of the lowest interval (i.e. containing θ_1) is $\underline{\theta}$, and the upper bound of the upper interval (containing θ_∞) is $\overline{\theta}$.

If we take the off-equilibrium beliefs in $\Xi(\langle \Theta, T \rangle)$ such that $t \notin T$ is never desirable (this is always possible by definition of $\Xi(\langle \Theta, T \rangle)$), and concerning equilibrium action, we take Lemma 1 (first point) into account, then clearly, the strategy given in the proposition is incentive compatible. To ensure that the equilibrium beliefs of the consumer are such that $E(\theta | t_k) = \theta_k$, it is necessary and sufficient that $f(\cdot)$ be such that $E(\theta | I_k) = \theta_k$. ■

The weight of interval I_k is not constrained, and the conditional expectations are independent from each other: in consequence, $F \in \mathcal{F}$ can be chosen as smooth as wanted. The main limitation in the Theorem comes from off-equilibrium strategies. One may find of particular interest skeletons for which no belief outside the convex hull of Θ is needed to discourage deviation.

4.3 Fully Informative Equilibria

We prove that for a given $[\underline{\theta}, \overline{\theta}]$, there is a unique fully revealing equilibrium, which is characterized in detail.

Proposition 2 *There exists essentially a unique fully revealing equilibrium. The tax rate is the unique solution to the ordinary differential equation* $t' = \frac{\eta-(1-\lambda)t}{t-t_{SB}(\theta)}$ *with* $t(\underline{\theta}) = t_{SB}(\underline{\theta})$*. In particular:*

1. *Cheap talk is ineffective, and the strategy* $t(\cdot)$ *is strictly increasing and differentiable.*
2. *Consumption decreases with respect to* θ*.*
3. *The tax rate exhibits no distortion at* $\underline{\theta}$*, otherwise, the tax rate is larger than the second-best tax rate and lower than* $\frac{\eta}{1-\lambda}$*.*

Proof. We establish the result in two steps. The first analyzes *differentiable* fully revealing equilibria; uniqueness in this category is proved. This will provide proofs for the three points of the proposition. The second step shows that any fully revealing equilibrium is essentially identical to the differentiable one. See the Appendix. ■

As Theorem 1 implies, the corresponding allocation is the universal skeleton, i.e. the equilibrium which is consistent with *any* distribution F in $[\underline{\theta}, \overline{\theta}]$. The main constraint, compared to the second-best, is the bias towards to much taxation: taxes have to be large increasing rather than decreasing with respect to θ.

4.4 The Possibilities of Cheap Talk

In less informative equilibria, cheap talk serves only to better coordinate the actions of the sender when the tax rate is ambiguous. This role is almost negligible in discrete equilibria (whenever the set of equilibrium tax rates is countable) since in case of ambiguity, it is always possible to modify as little as possible the tax rate in one of the ambiguous case to clear ambiguity, and the cost is arbitrarily low. The comparative statics exhibits a case of lack of upper hemi-continuity at $\lambda = 0$. Indeed, as $\lambda \to 0$, one can find a sequence of equilibria converging to the first-best where $t = \eta$ for all θ. The first-best is not an equilibrium if we keep restricting the signal to be supported only by the tax since no precise information on θ can be transmitted. In fact, cheap talk closes but do not substantially extend the set of equilibria.

A case in which cheap talk is useful appears when the tax rate is the same an interval of $[\underline{\theta}, \overline{\theta}]$. The analysis of this situation is analyzed now.

Again, the same effect could be (almost) attained with a small differentiation of the tax rates.

Cheap talk can be useful to transmit information only when the tax rate is constant over an interval of the type support. In this case we say that costless signalling can be *influential* if two different cheap talk messages associated to the same tax rate can be able to identify two different types (Austen-Smith and Banks 2000 introduce this terminology).

Here we study in detail the role of cheap talk when the PBE obtained using he tax only is characterized by a pooling interval (semi-pooling and pooling equilibria). Note that, the cheap talk being useful only when it is influential, we are able to treat "sequentially" the signalling and the cheap talk game. This means that we can assume a *constant* equilibrium tax over at least an interval of the type support and analyze the cheap talk incentive compatible strategies over such an interval.

This method implies that we focus on the role of cheap talk in the skeleton theory. In fact, solving the cheap talk game after the signalling equilibrium has been characterized, we are able to find a new structure of the skeleton which is incentive compatible. But, to identify an equilibrium, this new skeleton also needs appropriate out of equilibrium beliefs. This explains why existence of the equilibria cannot be easily established.

As the reader will see, considering cheap talk when the tax rate is constant allows us to apply and extend Crawford and Sobel results on costless information transmission.

Later on in this section, we will assume that the tax rate is constant over the interval $\Theta_m = [\theta', \theta'']$ with $\underline{\theta} \leq \theta' < \theta'' \leq \overline{\theta}$. As a consequence for $\theta \in \Theta_m$ the social planner solves the following program:

$$(12) \qquad \begin{cases} \max\limits_{m} U(x) - (\eta + \theta + \lambda t)\, x \\ \text{s.t.} \\ x = x^*(t, m) \end{cases}$$

Note that this is also consistent with a zero tax rate over the whole type support, that is with a policy characterized by the cheap talk only.

Before going into the details, we can give some general intuitions on the possibility of credible information transmission.

First of all, when $\eta > 0$, full information transmission is never possible. To understand why recall that when the sender' and the receiver's strategies lead to such a full transmission, the consumer perfectly infers the sender's type θ from the message m. The incentives to lie which this situation arises are particularly clear if we look to program (6). As the solution (7) shows, when the consumer believes to the sender's message, it is never optimal to say the truth: the social planner would deviate sending a message $m = \hat{\theta} > \theta$.

As it is well known, in cheap talk models transmitting no information is always incentive compatible (a pooling equilibrium always exists). In fact costless messages have no direct effect on the sender's and the receiver's payoffs. Therefore it is incentive compatible for the sender to play any pooling strategy and for the receiver to maintain the prior beliefs after whatever message. In particular, $\eta \geq \theta'' - \theta'$, no information can be transmitted. Here the sender's and the receiver's payoff are not sufficiently aligned and all the senders' types want consumers believe they are the θ'' type. Because sorting of types cannot be done through exogenous cost differences as in the signaling models, cheap talk cannot have meaning if all types would prefer to send the same message.

Second, if $\eta = 0$, perfectly informative cheap talk is always an equilibrium.

Third, in the case of a finite number of types, cheap talk is perfectly informative if the externality η is sufficiently small.[10] Saying the truth is the optimal strategy for the sender if $\eta = 0$ (see 7), since the sender's and the receiver's objective functions are perfectly aligned. If the support of types is finite we can order the values as $\theta_0 < \theta_1 < ... < \theta_i ... < \theta_M$. Consider $\theta_i < \theta_{i+1}$. The sender θ_i has no incentive to lie if his payoff saying the truth is at least equal to his payoff pretending to be the type θ_{i+1}, that is: $U(x(\theta_k)) - (\eta + \theta_k + \lambda\bar{t})x(\theta_i) \geq U(x(\theta_{i+1})) - (\eta + \theta_i + \lambda\bar{t})x(\theta_{i+1})$. Rearranging we find:

$$\eta \leq \frac{U(x(\theta_i)) - U(x(\theta_{i+1}))}{x(\theta_i) - x(\theta_{i+1})} - (\theta_i + \lambda\bar{t}) \tag{13}$$

By choosing the appropriate i (giving the smallest possible RHS), we can set an upper bound on η.

Now we give a general characterization of the incentive compatible strategies in the continuous case. To do so we refer to Crawford and Sobel (1982, henceforth CS). As the reader will see, this result is in line with Proposition 1.

Proposition 3 *1. When $\eta > 0$, the incentive compatible strategies of problem (12) are such that the type space $\Theta_m = [\theta', \theta'']$ is divided into*

[10] In line with this part of the proposition we can add the following consideration. If θ_{i+1} exceeds θ_i by a sufficiently large amount, the social planner can be counted on to reveal his type truthfully: a false announcement of θ_{i+1} would make receiver's consumption $x(\theta_{i+1})$ decrease too much, so that it is preferable to say the truth and to accept the higher consumption level $x(\theta_i)$.

intervals (which define a partition *of* Θ_m*) and all types in a given interval send the same message but types in different intervals send different messages. Moreover there is a finite upper bound* $N(\eta)$ *on the number of intervals of a partition and there exists at least one partition of each size from one to* $N(\eta)$.

2. *If for all* θ, $[1 - U''(x)]\, f(\theta) \geq [U'(x) - \theta - \bar{t}]\, f'(\theta)$, *then the maximum possible partition size* $N(\eta)$ *is non-increasing in* η.

Proof. See Appendix A.4. ■

The strategies described by the first part of the proposition characterize the equilibria in CS. From their seminal paper equilibria of this type are called *partition equilibria*.

As we said before, here we focus on the role of cheap talk in the skeleton theory and we identify incentive compatible strategies. Thus we do not check for appropriate beliefs to support the possible equilibria described by (1).

What part (1) in the previous proposition shows is that the social planner can use meaningful yet imprecise talk to communicate on the secondary effects to consumers. The social planner has interest in exaggerating the value of θ to make consumers internalize the externality: a precise information campaign gives the social planner incentives to manipulate consumers' beliefs. The problem can be partially solved through the use of signals that only specify broad ranges within which θ may lie. By restricting himself to such ambiguous statements, the social planner gives up the ability to manipulate expectations in all but a very crude way.[11] This in turn allows him to communicate some information in a credible fashion.

Part (2) shows a sufficient condition such that a decrease in η increases $N(\eta)$: more communication can occur when the players' preferences are more closely aligned. This result holds if the property of monotonicity of incentive compatibles partitions is verified. This property requires that when the partition size varies, all the intervals which compose the new partition must change in the same direction: either they all decrease or they all increase as compared to the intervals of the previous partition. The condition in part (2) on preferences and priors just implies such monotonicity.[12] Note that the

[11] In fact, with the partition of Θ_m, if the social planner wants to lie, it has to pretend that it is in a different subinterval, which changes consumers' first-period consumption by a discrete amount. Such "big lies" are less attractive than telling the truth.

[12] In ([5]) monotonicity assures the uniqueness of the equilibrium too. In fact, if (2) is verified it exists only one partition equilibrium for each partition size $1 \leq N \leq N(\eta)$. Roughly speaking the reason is the following: when we pass from a partition of size N to a partition of size $N-1$, if all the N intervals which compose the first partition increase as a

uniform probability distribution function verifies this condition. Moreover, when the utility function is logarithmic, the condition becomes: $\frac{f'(\theta)}{f(\theta)} \leq \frac{1+\underline{\theta}^2}{\overline{\theta}-\underline{\theta}-\bar{t}}$ for $f'(\theta), \overline{\theta} - \underline{\theta} - \bar{t} > 0$ and $\frac{f'(\theta)}{f(\theta)} \leq -\frac{1+\underline{\theta}^2}{\overline{\theta}-\underline{\theta}+\bar{t}}$ for $f'(\theta), \overline{\theta} - \underline{\theta} - \bar{t} < 0$.

To conclude this section it is interesting to remark that in CS the most informative available equilibrium (which is characterized by the $N(\eta)$ partition) is the unique ex-ante efficient equilibrium.[13] This result does not hold in our setting where the social planner uses both cheap talk and a costly signal. In fact in our model the expected cost of signal can dominate the expected gain from more precise cheap talk (see next section).

4.5 Refinements and Efficiency

The application of the above proposition clarifies the structure of the equilibria. We can start to build an equilibrium by choosing a skeleton, and we fill the rest of the distribution to preserve conditional expectations (Theorem 1). In particular, we can easily give examples in which the tax rate is not monotonic, where it is revealing on certain subsets of $[\underline{\theta}, \overline{\theta}]$ with bundles elsewhere, etc.

One must remark than refinements like the Intuitive Criterion of Cho and Kreps is particularly difficult to apply. Indeed, preferences are typically not monotonic: some exaggeration (beliefs larger than the truth) may be desirable but not too much; a lower (respectively a larger) tax may be desirable for constant beliefs, but this must no be pushed too far, and so on. The consequence is that, for a given θ, the structure of the subset of actions and messages which are desirable may be complex. Our conclusion is not that such refinements are neutral, but rather than their application is difficult.

In Crawford and Sobel (1982), a more informative equilibrium Pareto-dominates. Austen-Smith and Banks (2000) find that this is not true when burning money to signal the type is possible. Our proof is simplified by our approach by skeletons: it is easy to see that the unique fully informative allocation, which is always an equilibrium, need not be efficient. Take an equilibrium and take its skeleton. The substance of Theorem 1 is that the *relative* weights of the different parts of the skeleton do not matter: in particular, we can choose to build an equilibrium by distributing the probability

consequence of the size variation, there is at most one new partition such that the $N-1^{\text{th}}$ interval ends just at the point corresponding to $\overline{\theta}$. In others words, the last interval on the right, the N^{th} interval, slides out of the support and there is only one $N-1$ partition such that the upper bound of the $N-1^{\text{th}}$ interval coincides exactly with $\overline{\theta}$.

[13]See theorem 3 and 5 in [5] which say that both the sender and the receiver strictly prefer equilibrium partition with more steps.

masses of the types between the pieces of the skeleton arbitrarily. If the mass of a piece where the distortion is substantial is sufficiently large, then the equilibrium is necessarily inefficient.

5 Conclusion

Our model suggests that the benevolent social planner may have a hard time gaining credibility for its actions and messages, this even though its objective and the consumer's are aligned... The main constraint is that incentives to exaggerate the individual consequences of a consumer's actions to optimize the social impact are systematic. In the case of the full information equilibrium, the social planner is forced to distort incentives more, the stronger the individual consequences. In other terms, if the real impact of cigarettes is very strong, then taxes on tobacco have to be overly strong compared to the optimal taxes.

We also show that the information content of the tax is more important than the free message: equilibria can be arbitrarily approximated with strategies where cheap talk does not matter. Using ideas à la Maskin and Tirole (1992), one can construct a mechanism in which signaling costs are minimized. But they cannot be annihilated. The issue remains the same: how to inform cheaply and credibly when even the best experts may be suspected to serve particular interests, and when even the best intentions induce lies? This gives one more justification for general education: the public become able to judge by themselves the relevance of the arguments, and are less sensitive to cajolery.

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A Appendix

A.1 Proof of Proposition 1

Let $Z(\widehat{\theta}) = U^S(\widehat{x}, \widehat{t}, \theta) - U^S(x, t, \theta)$. We will consider:

$$Z'(\widehat{\theta}) = \frac{\lambda\eta + (1-\lambda)\theta - (1-\lambda)^2\widehat{\theta}}{(\eta + (1-\lambda)\widehat{\theta})^2} \tag{14}$$

Clearly, this expression changes sign only once, which implies that there exists a unique optimum:

$$\widehat{\theta} = \theta^* = \frac{\lambda\eta}{(1-\lambda)^2} + \frac{\theta}{1-\lambda} > \theta \tag{15}$$

that is, the interest to misrepresent sender's type is maximal for a finite value of $\widehat{\theta}$ that increases with λ and η. Moreover for $\lambda = 0$, there is no mimicking.

Z being single-peaked, we are sure that $I = 0$ has a maximum of two roots: the truth $\widehat{\theta} = \theta$, and the maximal lie $\widehat{\theta} = \theta_{\max}$. Unfortunately, $\theta_{\max}$ is not calculable, but we can prove that its value increases with λ since

$$\text{(16)} \quad \frac{d\theta_{\max}}{d\lambda} = -\frac{\partial Z}{\partial \lambda} \Big/ \frac{\partial Z}{\partial \widehat{\theta}}\Big|_{\widehat{\theta}=\theta_{\max}} = \frac{(\theta-\widehat{\theta})(\eta^2 + (2-\lambda)\eta\widehat{\theta} + (1-\lambda)\theta\widehat{\theta})}{(\eta + (1-\lambda)\theta)(\lambda\eta + (1-\lambda)\theta - (1-\lambda)^2\widehat{\theta})}$$

is positive for values of $\widehat{\theta}$ above the root of the denominator, i.e. above the optimal lie $\frac{\lambda\eta}{(1-\lambda)^2} + \frac{\theta}{1-\lambda}$. Situations with a higher degree of inefficiency (higher λ) are subject to stronger threats from the part of the authorities. This proves 1: the second-best is not implementable if the support is continuous.

In Figure 1, $\widehat{\theta}$ is the reported type, θ is the true type, θ^* is the value which maximizes incentive to misreport and finally $\theta_{\max}$ is the value such that the sender has no incentive to signal $\widehat{\theta} > \theta_{\max}$. If the support is finite, we can order the values as $\theta_0 < \theta_1 < \cdots < \theta_i \cdots < \theta_M$: if for all i, $\theta_{i+1} > \theta_{i,\max}$ where $\theta_{i,\max}$ is the maximal lie for $\theta = \theta_i$, then for a value of λ sufficiently low or for a distance between θ_i and θ_{i+1} sufficiently high, there is no incentive to misreport. Indeed, equation (16) proves that $\theta_{\max} - \theta$ increases with λ and, conversely, when $\lambda = 0$, $\theta_{\max}$ converges to θ. As a consequence the second-best allocation can be implemented. This proves 2.

A.2 Proof of Lemma 1

1. From (2) we see that the consumer's action is independent of the true type θ. More specifically, $x = x^*(\widehat{\theta}, t)$ where $\widehat{\theta}$ are updated beliefs. In a given equilibrium, we consider now the sender's incentive constraint; type θ will prefer policy $P_1 = \mathcal{P}(\theta_1)$ to any $P_2 = \mathcal{P}(\theta_2)$ (implying, respectively, consumptions x_1 and x_2) if and only if:

$$\text{(17)} \qquad U(x_1) - (\eta + \theta + \lambda t_1)\, x_1 \geq U(x_2) - (\eta + \theta + \lambda t_2)\, x_2$$

If we define $Z(\theta)$ as the value of playing P_1 minus the value of playing P_2, clearly, $\frac{dZ}{d\theta} = x_2 - x_1$, a constant. Let's denote by $\Gamma(P_1, P_2)$ the set of types such that policy P_1 is preferred to policy P_2. Clearly now, $\forall P_1, P_2,\ \Gamma(P_1, P_2)$ is a half straight-line, the whole real line, or an empty set. From this result it follows that if two values of θ are such that they imply the same preferred policy P, then all the types which are between these values are characterized by the same preferred policy.

2. It suffices to check that the utility is quasi-concave. To do this, we check that the successive principal minors of the bordered Hessian matrix

alternate signs (odd principal minors have to be positive):

$$
(18) \qquad \begin{bmatrix}
0 & \frac{\eta+\theta-t-(1+\lambda)\widehat{\theta}}{(t+\widehat{\theta})^2} & \frac{\eta+\theta-(1-\lambda)t-\widehat{\theta}}{(t+\widehat{\theta})^2} \\
\frac{\eta+\theta-t-(1+\lambda)\widehat{\theta}}{(t+\widehat{\theta})^2} & -\frac{2\eta+2\theta-t-(1+2\lambda)\widehat{\theta}}{(t+\widehat{\theta})^3} & -\frac{2\eta+2\theta-(1-\lambda)t-(1+\lambda)\widehat{\theta}}{(t+\widehat{\theta})^3} \\
\frac{\eta+\theta-(1-\lambda)t-\widehat{\theta}}{(t+\widehat{\theta})^2} & -\frac{2\eta+2\theta-(1-\lambda)t-(1+\lambda)\widehat{\theta}}{(t+\widehat{\theta})^3} & -\frac{2\eta+2\theta-(1-2\lambda)t-\widehat{\theta}}{(t+\widehat{\theta})^3}
\end{bmatrix}
$$

The first and second are equal to zero, and we find $\frac{\lambda^2}{(t+\widehat{\theta})^4}$ for the third. This gives the result.

3.Let

$$
(19) \qquad \left.\frac{dt}{d\widehat{\theta}}\right|_{V^S=\text{constant}} = -\frac{\eta+\theta-(1-\lambda)t-\widehat{\theta}}{\eta+\theta-t-(1+\lambda)\widehat{\theta}}
$$

be the MRS between t and $\widehat{\theta}$. Its derivative with respect to θ is

$$
(20) \qquad \frac{\lambda(t+\widehat{\theta})}{(\eta+\theta-t-(1+\lambda)\widehat{\theta})^2},
$$

which is positive for $t \geq 0$. (Notice that upper contours being closed, two indifference curves cross twice at least; we proved here that in the domain of interest—positive tax rate—crossing occurs once at most, which is sufficient to retrieve the standard argument based on single crossing.)

A.3 Proof of Proposition 2

A.3.1 Differentiable Equilibria

This analysis follows this plan: reasoning on local incentive compatibility, we find the ordinary differential equation satisfied by any equilibrium tax policy and we eliminate solutions with tax rates below the second-best schedule; we check global incentive compatibility along the equilibrium policy; we search for off-equilibrium beliefs (i.e. associated with off-equilibrium tax rates) that discourage deviations; this gives a unique equilibrium.

Local Incentive Compatibility We reason on incentive constraints. Starting from the fact that the social planner prefers $t(\theta)$ (and the implied $x(\theta)$) to $t(\theta+d\theta)$ and to $t(\theta-d\theta)$ and taking limits we get

$$
(21) \qquad x'U' - \lambda t'x - (\eta+\theta+\lambda t)x' = 0
$$

Given that the consumer's first-order condition is

$$
(22) \qquad U' = \theta + t,
$$

we can eliminate U' to get (after simplification)

$$tx' = \lambda t'x + (\eta + \lambda t)x' \tag{23}$$

t and x being separable, (23) is easily integrating to give

$$\forall\theta, \theta_0 : \frac{\frac{\eta}{1-\lambda} - t(\theta)}{\frac{\eta}{1-\lambda} - t(\theta_0)} = \left(\frac{x(\theta)}{x(\theta_0)}\right)^{\frac{1-\lambda}{\lambda}}, \tag{24}$$

where θ_0 and $t_0 = t(\theta_0)$ are initial conditions. Equations (24) and (9) determine implicitly but entirely the solutions $t(\theta)$ and $x(\theta)$. In particular, x solves

$$\left((\theta + \frac{\eta}{1-\lambda})x - 1\right) x^{-\frac{1}{\lambda}} = \left((\theta_0 + \frac{\eta}{1-\lambda})x_0 - 1\right) x_0^{-\frac{1}{\lambda}} = \text{Constant} \tag{25}$$

By differentiation, we get

$$x' = -\frac{\lambda x^2}{1 - (\eta + (1-\lambda)\theta)x} \tag{26}$$

Notice that the second-order condition

$$0 \geq x'^2 U'' + x''U' - \lambda t''x - 2\lambda t'x' - (\eta + \theta + \lambda t)x'', \tag{27}$$

simplified with the derivative of the first-order condition

$$0 = x'^2 U'' + x''U' - \lambda t''x - 2\lambda t'x' - x' - (\eta + \theta + \lambda t)x'', \tag{28}$$

yields

$$x' \leq 0 \tag{29}$$

Applied to (25) using (9), we can see that $x' \leq 0$ if and only if $t > \eta - \lambda\theta$. We conclude that the tax rate is larger than the second-best tax rate in any equilibrium.

Straightforward calculations prove that the differential equation satisfied by t is

$$t' = \frac{\eta - (1-\lambda)t}{t - \eta + \lambda\theta} \tag{30}$$

with $t > \eta - \lambda\theta$.

Global Incentive Compatibility It remains to be checked that the program of the principal is well-behaved, i.e. that we are *not* in a case where infinitesimal deviations are rejected (of this we are sure because of the first- and second-order conditions) whereas finite deviations are possible. Let θ be the true value of the secondary effects parameter. We calculate the derivative (with respect to $\widehat{\theta}$) of the social planner's utility contemplating offering $t(\widehat{\theta})$, thereby inducing $x(\widehat{\theta})$:

$$\begin{aligned}
(31) \qquad & x'(\widehat{\theta})U'(x(\widehat{\theta})) - (\lambda t'(\widehat{\theta})x(\widehat{\theta}) + (\eta + \theta + \lambda t(\widehat{\theta}))x'(\widehat{\theta})) \\
= \; & \frac{x'(\widehat{\theta})}{x(\widehat{\theta})} - (\lambda t'(\widehat{\theta})x(\widehat{\theta}) + (\eta + \theta + \lambda t(\widehat{\theta}))x'(\widehat{\theta}))
\end{aligned}$$

Using (23), we find that the following expression has the same sign as (31)

$$(32) \qquad x(\widehat{\theta})(t(\widehat{\theta}) + \theta) - 1$$

Given (9), it becomes clear that (31), is positive for $\widehat{\theta} < \theta$ and negative for $\widehat{\theta} > \theta$. Incentive compatibility is satisfied everywhere for equilibrium actions.

Uniqueness of the Differentiable Equilibrium The difficulty now is to find appropriate beliefs for off-equilibrium actions. Let $t(\theta)$ be a solution to (30) such that there exists $\varepsilon > 0$ with $t(\theta) > \eta - \lambda\theta + \varepsilon, \forall\theta$. Given (30), either $t(\theta)$ is systematically above $\frac{\eta}{1-\lambda}$ or $t(\theta)$ is strictly increasing, in any case $\eta - \lambda\underline{\theta} < \min_\theta t(\theta)$. We choose an arbitrary t in $(\eta - \lambda\underline{\theta}, \min_\theta t(\theta))$ and we denote by $\widehat{\theta}$ the associated belief. It is clear that when $\widehat{\theta}$ is the type of the social planner, playing t (which is closer to the second-best) is preferred to playing $t(\widehat{\theta})$. As a consequence, solutions such that $t(\theta) > \frac{\eta}{1-\lambda}$ can be eliminated as well as increasing solutions with $t(\underline{\theta}) \neq \eta - \lambda\underline{\theta}$. Therefore, the existence of ε for an admissible solution is not possible and $t(\underline{\theta}) = \eta - \lambda\underline{\theta}$ (no distortion at the bottom).

If for $t > t(\overline{\theta})$, beliefs are $\overline{\theta}$, t is not attractive: indeed, $t(\overline{\theta})$ is not attractive, and t is worse. Now we prove that the beliefs associating $\underline{\theta}$ to any tax rate below $t(\underline{\theta})$ do not induce deviations. The value for the social planner of type θ to impose $t < t(\underline{\theta})$ thereby inducing belief $\underline{\theta}$, is: $-\log(\underline{\theta} + t) - \frac{\eta+\theta+\lambda t}{\underline{\theta}+t} + W$. The root of the derivative with respect to t is $\eta + \theta - (1+\lambda)\underline{\theta} > \eta - \lambda\underline{\theta} = t(\underline{\theta})$. The value being increasing with respect to t over $[0, t(\underline{\theta})]$, $t(\underline{\theta})$ is a better move than any $t < t(\underline{\theta})$. Given that equilibrium actions are incentive compatible, neither $t(\underline{\theta})$ nor t are desirable, compared to $t(\theta)$. We have the unique equilibrium allocation.

A.3.2 Uniqueness in General

Let us take a fully revealing equilibrium. Given that the social planner's preferences, for constant beliefs, are single-peaked with respect to t (see the convexity property in Lemma 1), and given the value of its equilibrium strategy, there exist a maximum of two tax rates per θ, $t_L(\theta)$ and $t_U(\theta)$, both being suboptimal (as compared to the second-best) when different. More precisely, $t_L(\theta) \leq \eta - \lambda\theta \leq t_U(\theta)$). The Theorem of the maximum ensures that the *value* of the social planner's equilibrium strategy is continuous with respect to θ, therefore functions $t_L(\cdot)$ and $t_U(\cdot)$ are continuous with respect to θ. We denote by Θ_L and Θ_U the subsets of $[\underline{\theta}, \overline{\theta}]$ leading to a move in the lower, respectively in the upper, selection. Notice that $\Theta_L \cup \Theta_U = [\underline{\theta}, \overline{\theta}]$ but $\Theta_L \cap \Theta_U \neq \emptyset$ if mixed strategies are used.

The first step is to prove that Θ_L is not dense in any interval of $[\underline{\theta}, \overline{\theta}]$. We reason by contradiction: let us take J an interval in $[\underline{\theta}, \overline{\theta}]$ where Θ_L is dense. Let us take $\theta_0 \in J$, and a strictly monotonic sequence $(\theta_n)_{n\geq 1}$ in Θ_L converging to θ_0. We prove that for all sequence $(\theta_n)_{n\geq 1}$, $\lim_{n\to\infty} \frac{t_n - t_0}{\theta_n - \theta_0} = \frac{\eta-(1-\lambda)t_0}{t_0-\eta+\lambda\theta_0}$, where t_n denotes $t_L(\theta_n)$. Indeed, incentive constraints (θ_n wish not to mimic θ_0, and vice-versa) imply that:

$$(33) \quad -\log(\theta_n + t_n) - \frac{\eta + \theta_n + \lambda t_n}{\theta_n + t_n} \geq -\log(\theta_0 + t_0) - \frac{\eta + \theta_n + \lambda t_0}{\theta_0 + t_0}$$

$$(34) \quad -\log(\theta_n + t_n) - \frac{\eta + \theta_0 + \lambda t_n}{\theta_n + t_n} \leq -\log(\theta_0 + t_0) - \frac{\eta + \theta_0 + \lambda t_0}{\theta_0 + t_0}$$

Therefore (taking a first-order approximation, and multiplying by $(\theta_0 + t_0)^2$)

$$(35) \quad 0 \geq ((1-\lambda)t - \eta)(\theta_n - \theta_0) + (t - \eta + \lambda\theta)(t_n - t_0) + o(\theta_n - \theta_0) + o(t_n - t_0)$$

$$(36) \quad 0 \leq ((1-\lambda)t - \eta)(\theta_n - \theta_0) + (t - \eta + \lambda\theta)(t_n - t_0) + o(\theta_n - \theta_0) + o(t_n - t_0)$$

This proves that t_L is differentiable at θ_0, therefore differentiable on interval J (the limit of the rate of variations is the same for all sequences).

A solution of the differential equation (30) situated below the second-best taxes is incentive compatible at no point (the second-order solution is never satisfied, see proof of Proposition 2, Subsection A.3.1), we can conclude that strategy t_L is not incentive compatible, and that the interval J does not exist.

It is easy now to conclude that Θ_H is dense in $[\underline{\theta}, \overline{\theta}]$: the complementary set (in an interval) of a set which is nowhere dense is dense. In consequence, t_U satisfies the differential equation (30) in a dense subset of $[\underline{\theta}, \overline{\theta}]$, which suffices to guarantee that it does so everywhere. The upper selection is necessarily equal to the unique differentiable equilibrium strategy, since we can apply to $t_U(\cdot)$ the reasoning suited for differentiable equilibria.

It remains to be proved now that Θ_L contains a finite number of points. Let us take θ_1 and $\theta_2 \in \Theta_L$ (where $\theta_1 \neq \theta_2$) with corresponding tax rates t_1 and t_2. Let us denote by $t_i(\cdot)$ $(i = 1, 2)$ the solution to (30) with maximal domain passing through t_i at θ_i. Note that either $t_1(\cdot)$ and $t_2(\cdot)$ are the same, or one is systematically above the other (according to the Cauchy-Lipschitz Theorem, two different solutions to differential equation (30) never cross).

Assume for fixing ideas that $t_2(\cdot)$ is above $t_1(\cdot)$, in particular, if the two curves are sufficiently close to each other, $t_2(\theta_1)$ is defined and is larger than t_1. Notice that $t_2(\theta_1)$ is closer to the second-best than t_1. In addition, our study of the incentives when taxes are below the second-best shows that, when the type is θ_1, t_2 with belief θ_2 is preferred to $t_2(\theta_1)$ with belief θ_1 (the first-order condition selects minima, as proves the second-order condition). By transitivity, t_2 is preferable to t_1 when the true type is θ_2. This is in contradiction with incentives. If there is an infinite number of types in Θ_L, we can always exhibit θ_1 and θ_2 which are close enough to each other for the preceding reasoning to be applicable. We conclude that Θ_L contains a finite number of points.

Remark that ex-post, we can conclude that cheap talk is not influential since the tax rate is a sufficient signal of the current state.

A.4 Proof of Proposition 3

(1) We show here that problem (12) meets the assumptions of CS. The support of θ, Θ_m, is analogous to the type-support of CS, $[0, 1]$. If we make the following change of variables in CS's setting: $y = -x$, our model verifies exactly the CS's assumptions. In fact U^i, $i = S, R$, is such that $U^i_1(x, \theta, \bar{t}) = 0$ for some x and $U^i_{11}(\cdot) < 0$ (partial derivatives are denoted by subscripts in the usual way), so that U^i has a unique maximum in x for each given (θ, η). Concerning the externality, as we pointed out before, η is what essentially makes the players' utility functions to diverge and, as a consequence $\arg\max_x U^S(x, \theta, \eta, \bar{t}) \leq \arg\max_x U^R(x, \theta, \bar{t}), \forall\theta$. The "sorting condition" is $U^i_{12}(\cdot) < 0$, it ensures that the best value of x from a fully informed agent's standpoint is a strictly decreasing function of the true value of θ.

Theorem 1 in CS shows that when $\eta > 0$ all equilibria in the model are partition equilibria. As we pointed out in the text, in our model where also costly signalling is used, problems concerning existence of equilibria arise. In fact given a skeleton, roughly speaking, when we make a partition of Θ_m à la CS we change the indifference types $\tau_s = \theta'$ and $\tau_{s+1} = \theta''$ (see our

Theorem 1) such that new out of equilibrium beliefs are needed to support the signalling-only initial equilibrium. For this reason the partition of the type interval Θ_m defined by CS's Theorem 1 only characterizes incentive compatible strategies and not equilibria here.

(2) We show here a sufficient condition for monotonicity. Given that in our setting $U^S(x, \theta, 0, \bar{t}) = U^R(x, \theta, \bar{t})$, $\eta \geq 0$ and $U^S_{13} < 0$ everywhere, Theorem 2 in CS determines sufficient conditions on priors and preferences that imply the monotonicity. The first condition of the theorem, provided the change of variable $y = -x$, is always verified in our model. The second condition, when $U^R(x, \theta, \bar{t}) = U(x) + W + ((1-\lambda)\bar{t} - \eta)\bar{x} - (\theta + \bar{t})x$, means that

$$\frac{\partial}{\partial \theta}\left[\int_{\underline{\theta}}^{\theta} U''(x) f(\theta) d\theta + (U'(x) - \theta - \bar{t}) f(\theta)\right] \leq 0 \tag{37}$$

which, after derivation and integration, gives

$$(U''(x) - 1) f(\theta) + (U'(x) - \theta - \bar{t}) f'(\theta) \leq 0 \tag{38}$$

where the first term in the inequality is negative and the second can be positive or negative. When $U'(x) - \theta - \bar{t}$ and $f'(\theta)$ have opposite sign, condition (38) is always verified. While when the two previous terms have the same sign, (38) is a sufficient condition for monotonicity. ■