Relative Consumption and Saving

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Abstract

We analyze in a simple two-period model the e¤ect of relative consumption on saving by assuming that people care about their ordinal rank in the consumption distribution at each date We outline some general properties of the model and then completely solve a simple version. We show that a rise in consumption inequalities implies a negative impact on saving.

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1. Introduction

Social status is a ranking of individuals based on their traits, assets, and actions (see Weiss and Fershtman (1998)). Among other purposes, it provides a way to allocate non-market goods such as authority or deference. Attempts of individuals to achieve a greater social status may have profound implications on consumption

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decisions as first suggested by Veblen (1922) or later by Duesenberry (1949). Consumption may signal human and physical wealth and may inform about the social position of the person which enjoys it. Consumption may also be a direct source of social rewards insofar as a relatively high standing involves admiration or envy by others. These various motivations imply that individuals are concerned by their ranking in the consumers' hierarchy.

Several empirical papers in the consumption literature have argued that some form of comparison utility may play an important role in determining consumption. Di Tella, MacCulloch and Oswald (1997) examine the evolution of happiness in the form of responses to survey questions in 13 industrialized countries since the early 70s. They find no trend in the US, a decline in Italy and Germany for example. Conventional models with absolute utility fail to explain these trends since meanwhile, real incomes have more than tripled over the period. Solnick and Hemenway (1998) use survey data to provide some empirical information about concerns regarding relative standing. Half of the respondents preferred to have 50% less real income but high relative income. Kapteyn, Van de Geers and Van de Stadt (1985) estimate a model in which both one's own past consumption and the consumption of others influence utility. They cannot reject the proposition that utility is entirely relative (see also a more recent paper by Kapteyn et al. (1997)).

A concern for relative consumption may have significant effects on saving. In this line of thought some economists have argued that it was partly responsible for the decline in household savings and in growth observed in some developed countries (for instance Knell (1999)). From a theoretical point of view, we need an intertemporal model in which agents compare with each other their consumption level. In this paper we analyze in a simple two-period model the effect of relative consumption on saving by assuming that people care at each date about their ordinal rank in the consumption distribution. Saving is affected by the dynamic of the consumption distribution in a non-trivial way. We show by solving a simple version of the model that a rise in consumption inequalities implies a negative impact on saving.

The link between saving and status seeking has been studied by a number of papers. Here we report only some that share similarities with the present model. Corneo and Jeanne (1997) consider a model in which individuals derive utility from their rank in the distribution of wealth. They show that the growth rate of the economy increases with the initial equality of the wealth distribution. Our model essentially departs from theirs by assuming that a higher consumption rather than a higher wealth confers a greater status. This difference is motivated by the fact that consumption is easier to exhibit than wealth, a well-known point noted first by Veblen (1922). We show very different implications on saving. Unlike their model the initial level of wealth inequality does not play a major role. Rather, results crucially depends on the type of preferences postulated which determines how a given level of wealth inequality translates into the dynamics of consumption inequalities. Corneo and Jeanne (1999) propose a second model in which the link between wealth inequalities and saving is more ambiguous. However the same remarks regarding the differences with the present model apply here. Knell (1999) analyzes in an overlapping generation model the effect of relative consumption on saving. There are only two classes of wealth contrary to our model in which a continuum exists. He shows that a concern for relative standing produces a negative link between wealth inequality and growth if two conditions are fulfilled: individuals have a higher concern for their present than for their future relative standing and they refer to people that are wealthier than they are. The first condition is reminiscent of the papers by Franck (1985) or Corneo and Jeanne (1998). In particular Franck (1985) assumes that individuals care about their relative rank in the consumption distribution. In its model saving is depressed because only first period status matters. This very simple mechanism is not reproduced in our model. Indeed, contrary to these three papers we assume that people equally care about today's and tomorrow's status. Yet an impact of inequalities on saving still remains.

The paper proceeds as follows. In section 2 we describe the model and derive the equilibrium conditions. Section 3 analyzes the impact of relative consumption on saving. The model is then fully solved in a simple case (section 4). Section 5 concludes the paper.

2. The model

We consider a single-good economy with two dates: $t = 0, 1$ and a size-one continuum of agents. Agents differ in their first period endowment denoted by $y_0^i \geq 0$. Their second period endowment is zero but they can transfer goods from the first period to the second by means of a linear production function which produces R for each unit invested at date 0.

The endowments are distributed over $\left[y_0^-, y_0^+\right]$ according to the distribution function $F(.)$ Let $f(.)$ denote the corresponding density function. We assume that

$f(.)$ has the following properties[1]:

H1 $f(.)$ is once continuously differentiable over $\left[y_0^-; y_0^+\right]$, left continuous at y_0^+, right continuous at y_0^- and such that $f(y_0^-) = 0$.

Let $(c_0^i; c_1^i)$ be the consumption pattern of an individual endowed with $y_0^i \in \left[y_0^-; y_0^+\right]$ and let $G_t(.)$ and $g_t(.)$ denote respectively the distribution function and the density function of consumptions at date t. $G_t(c)$ is the fraction of the population which consume less than c at date $t = 0; 1$. We assume that people derives utility from social status which is represented by their rank $G_t(c)$ in the consumers' hierarchy. All individuals have identical preferences which depend on consumption and on status:

H2 Let $T_t(c_t^i;) : [0; 1[\to R$ denote the reduced form of the instantaneous utility function at $t = 0; 1$. $T_t(.)$ is defined by:

$$T_t(c_t^i;) = u(c_t^i) + \alpha G_t(c_t^i)$$

where $u(.)$ is an increasing, concave and twice continuously differentiable function.

The coefficient α reflects the strength of the status-seeking motive. We assume that the utility function is linear in the rank term. It amounts to assuming that the utility gain associated with a marginal increase in the rank is the same whatever the initial rank of the person[2].

Let β denote the psychological discount rate. The optimal consumption path $(c_0^i; c_1^i)$ solves for the following problem (P):

$$(P) \quad \begin{cases} \max_{\{c_0^i; c_1^i\}} T_0(c_0^i) + \beta T_1(c_1^i) \\ s.c.: \quad c_0^i + c_1^i/R = y_0^i \\ c_0^i; c_1^i \geq 0 \quad y_0^i; G_0(.) \text{ and } G_1(.) \text{ given} \end{cases}$$

[1] The hypothesis $f(y_0^-) = 0$ put in H1 will be necessary in the following to ensure that the second order condition is indeed a sufficient condition of the maximization problem stated below. See Appendix B for more details.

[2] Robson (1992) provides arguments in favor of the convex case while Corneo and Jeanne (1997) only consider the concave case. In the latter case the wealth poor has a higher concern for status than the wealth rich (see also the analysis in Corneo and Jeanne (1997)). Note that the present model could be extended in either direction without changing its basic results.

Each individual evaluates its path of consumption by taking as given the evolution of the consumption distribution. The set $\{(c_0^i; c_1^i); \forall y_0^i \in [y_0^-; y_0^+]\}$ is an equilibrium of the economy if $(c_0^i; c_1^i)$ solves for (P) for every possible y_0^i and if $G_0(.)$ and $G_1(.)$ correctly describe the evolution of the resulting consumption distribution.

The associated first order condition is: $T_0'(c_0^i) - \beta R T_1'(c_1^i) = 0$ for all $y^i \in [y_0^-; y_0^+]$ or:

$$u'(c_0^i) - \beta R u'(c_1^i) + \alpha \left[g_0(c_0^i) - \beta R g_1(c_1^i) \right] = 0 \tag{2.1}$$

The consumer benefits from a rank increase of $g_0(c_0^i)$ by marginally increasing today's consumption at cost of a loss of tomorrow's rank of $\beta R g_1(c_1^i)$.

The second order condition of (P) requires for all $y^i \in [y_0^-; y_0^+]$:

$$T_0''(c_0^i) + \beta R^2 T_1''(c_1^i) < 0. \tag{2.2}$$

In the following we restrict our attention to equilibria in which the consumption rank of individuals is their wealth rank that is: $G_0(c_0^i) = G_1(c_1^i) = F(y_0^i)$ for all i. This restriction amounts to assuming a little stronger condition than (2.2) (see Appendix A):

H3 $T_t''(c_t^i) < 0$ for $t = 0, 1$ and for all $y_0^i \in [y_0^-; y_0^+]$.

H3 can be equivalently stated: $u''(c_t^i) + \alpha g_t'(c_t^i) < 0$. The first term is negative by assumption. The second term is relative to the status concern. Wherever the consumption density function is increasing, the concavity of $u(.)$ must be sufficiently strong or the weight α must be sufficiently small for H3 to be satisfied. Indeed, in that case, a marginal increase in consumption implies catching up a greater fraction of individuals, which introduces a convex element in the utility function.

We now analyze how the concern for relative consumption distorts saving behaviors.

3. The effects of relative consumption on saving

The consumption decisions of the whole consumers impose an externality for each of them by shaping the consumption distribution and its evolution through time.

There is however a particular case in which this externality completely disappears as established by proposition 1:

Proposition 1. $c_0^i = c_1^i = \frac{R}{R+1} y_0^i \;\; \forall \alpha \geq 0$ if $R = 1/\beta$.

Proof. The first order condition is: $T_0'(c_0^i) = T_1'(c_1^i)$. A natural guess is therefore: $c_0^i = c_1^i = \frac{R}{R+1} y_0^i$. In this case the consumption distribution is stationary: $G_0(c_0^i) = G_1(c_1^i)$, and so is the instantaneous utility function: $T_0(c_0^i) = T_1(c_1^i)$. Hence the guessed solution does satisfy the first order condition. $\square$

The result can be interpreted as follows: $R = 1/\beta$ implies that if the consumption distribution is time-invariant, then a marginal gain in rank is exactly compensated by the corresponding discounted loss of rank in the other period. Hence each individual is led to consume his permanent income as in the case without relative consumption $\alpha = 0$. Moreover, because they consume their permanent income, the consumption distribution is indeed time-invariant.

This particular case makes apparent that the wealth distribution or the consumption distribution as such does not matter here. Rather the relevant question regarding saving is how this distribution evolves through time as it is now shown.

The first order condition (2.1) implicitly provides the consumption optimal rule. The consumers' problems are however not independent and interact through the distributions of consumption described by $g_0(.)$ and $g_1(.)$.

The optimal rule for date 0 consumption is noted: $c_0^i = \Psi_0(y_0^i)$. The optimal date 1 consumption rule is then simply derived from the budget constraint: $c_1^i = Ry_0 - R\Psi_0(y_0^i) = \Psi_1(y_0^i)$. As previously noted, we restrict ourselves to equilibria preserving the wealth rank. This condition can be equivalently stated: $\Psi_0'(y_0^i) > 0$ and $\Psi_1'(y_0^i) > 0$ and is fulfilled in the model:

Lemma 1. $\Psi_0(.)$ and $\Psi_1(.)$ are continuous and increasing functions over $\left[y_0^-; y_0^+\right]$.

Lemma 1 implies that $\Psi_0(.)$ and $\Psi_1(.)$ can be inverted. These functions are respectively denoted $\phi_0(.)$ and $\phi_1(.)$: $y_0^i = \Psi_0^{-1}(c_1^i) = \phi_0(c_0^i)$ and $y_0^i = \Psi_1^{-1}(c_1^i) = \phi_1(c_1^i)$. Let us define $c_1^i = \psi(c_0^i)$ the wealth expansion path which says how the optimal combination of consumptions $(c_0^i; c_1^i)$ evolves when the wealth is increasing. Lemma 1 implies that $\psi(.)$ is continuously increasing over $[\underline{c}_0; \bar{c}_0]$ where $\underline{c}_0$ and $\bar{c}_0$ are respectively the lower bound and the upper bound of the consumption distribution at date 0. By exploiting the rank preserving property of the model: $G_0(c_0^i) = G_1(c_1^i) = F(y_0^i)$ for all i at equilibrium, the probability density functions of consumption can be expressed: $g_0(c_0^i) = \psi'(c_0^i)g_1(c_1^i)$ and $g_1(c_1^i) = \phi_1'(c_1^i)f(y_0^i)$. Hence the first order density function can be rewritten as:

$$u'(c_0^i) - \beta R u'(c_1^i) + \alpha f(y_0^i)\phi_1'(c_1^i)\left[\psi'(c_0^i) - \beta R\right] = 0 \qquad (3.1)$$

The magnitude of the impact of status on saving depends on the wealth density function $f(y_0^i)$. This is intuitive since the gain in term of rank from marginally increasing consumption is proportional to the number of individuals which consume the same level. The sign of the impact is given by the difference between the slope of the wealth expansion path $\psi'(c_0^i)$ and βR, given that $\phi_1'(c_1^i)$ is positive (a consequence of lemma 1). If this gap is positive, the relative consumption hypothesis has a negative impact on saving and the converse is true if the gap is negative.

This result is explained by noting that the slope of the wealth expansion path determines how the consumption distribution evolves through time. If this slope is greater than 1 consumption inequalities are rising (it is evident by recalling that $g_0(c_0^i) = g_1(c_1^i)\psi'(c_0^i)$). They are decreasing if the slope is smaller than one. A high enough slope is then accompanied by less saving because the distance between individuals in terms of their consumption is smaller in first period (or possibly not high enough if $\beta R < \psi'(c_0^i) < 1$) making stronger the contests for status in this period than in the second period. The incentive to catch up other individuals is higher, thereby promoting first period consumption. Notice that this incentive is effective even though individuals eventually fail to improve at equilibrium their rank at both dates compared to their wealth rank. In other words, facing the distorted consumption distributions they are just able to preserve their wealth rank.

A preliminary conclusion is that a negative impact of status on saving is accompanied by a rise in consumption inequalities if the gross interest rate is smaller than $1/\beta$. If R is greater than $1/\beta$ the rise in consumption inequalities must be sufficiently marked for individuals to be deterred from saving. However this conclusion is only partial since the wealth expansion path is endogenous here. In the next section, we completely characterize an equilibrium by posing simple functional forms for the wealth distribution and the utility function.

4. An example

To completely characterize the equilibrium we need to find the optimal policy rule $\Psi_0(.)$ which solves for the first order condition together with the budgetary constraint. To keep the problem tractable, we assume that the wealth density function takes a linear form: $f(y_0^i) = ay_0^i + b \ \forall y \in [y_0^i; y_0^+]$. Since the hypothesis H1: $f(y_0^i) = 0$ must hold, it follows that the wealth distribution has a triangular form with a positive slope $a > 0$ and $f(y_0^+) > 0$ [3]. The direct utility function is assumed to be quadratic [4]:

$$T(c_t) = c_t - \frac{\theta}{2}(c_t)^2 + \alpha G_t(c_t).$$

By exploiting the rank preserving property of the model, the first order condition (2.1) can be expressed as:

$$u'(c_0^i) - \beta R u'(c_1^i) + \alpha f(y_0^i)\left[\phi_0'(c_0^i) - \beta R \phi_1'(c_1^i)\right] = 0 \qquad (4.1)$$

We solve for the policy rule $\Psi_0(y_0^i)$ by the method of undetermined coefficients. We guess that $\Psi_0(y_0^i) = \delta + \gamma y_0^i$ where δ and γ are unknown parameters. We have then $\Psi_1(y_0^i) = R[-\delta + (1-\gamma)y_0^i]$. The first order condition (4.1) can be simplified to:

$$1 - \theta c_0^i - \beta R[1 - \theta(Ry_0^i - Rc_0^i)] + \alpha(a + by_0^i)\left(\frac{1}{\gamma} - \frac{\beta}{1-\gamma}\right) = 0$$

We obtain a linear function of c_0^i in term of y_0^i. As a result the parameters δ and γ can readily be identified:

Proposition 2. the optimal rule for date 0 consumption takes the following form: $c_0^i = \delta + \gamma y_0^i$ in which δ and γ satisfy:

$$(1 + \beta R^2)\theta\gamma^3 - (2\beta R^2 + 1)\theta\gamma^2 + (\theta\beta R^2 - \alpha b(1 + \beta))\gamma + \alpha b = 0$$

$$\delta = \frac{1 - \beta R + \alpha a((1/\gamma) - \beta/(1 - \gamma))}{\theta(1 + \beta R^2)}$$

[3] The distribution of wealth postulated here does not resemble real distributions. This does not affect however the conclusions about the effects on aggregate saving, which is the focus of the paper.

[4] We shall assume throughout that the marginal utility is always positive. This is the case if $1/\theta > c_t^i \ \forall y^i \in [y_0^i; y_0^+]$ and $\forall t = 0, 1$ which is a sufficient condition here.

The slope γ is the solution of a polynomial of degree 3. We are led to a numerical determination of η and γ. As an illustration, let the parameters of the economy be: $(\theta; \beta; a; b; \alpha) = (0.01; 0.1615; 4.5; 0.15; 0.038)$ and $R = 1.1/\beta$[5]. There are three real roots for γ. However two roots are rejected since they do not satisfy the assumption H3 which assures the rank preserving property of the model and the second order condition of the problem. We are left with a single value for γ. It follows that the policy rule as well as the equilibrium are unique. This result is preserved when we modify the parameters of the economy in a way that preserves H3.

Figure 1 plots saving as a function of the first period consumption. The solid line represents the wealth expansion path of this economy in the (c_0-c_1) space. The density function of the first period consumption is plotted over its support $[\underline{c}_0; \bar{c}_0]$. The corresponding density function of the second period consumption is then readily shown on the c_1-axis given the wealth expansion path. The dashed line represents the budget expansionary path without status-seeking ($\alpha = 0$).

Two observations can be drawn from this figure. First, the departure of saving from the case without status is greater the richer the individuals. This is explained by the fact that the effect of the status on saving is proportional to the density function as shown by the first order condition (4.1) together with the triangular form of the wealth density function.

Second, aggregate saving is promoted. This feature comes from the particular value of the saving return in the numerical example: $R = 1.1/\beta$. Indeed, numerical experiments show that agents save more whenever $R > 1/\beta$ and save less when $R < 1/\beta$. Figure 2 shows the case in which $R = 0.9/\beta$. The explanation is directly related to the sign of $\psi'(c_0^i) - \beta R$ as stressed in the previous section. Let us first consider the evolution of the consumption distribution without social status ($\alpha = 0$). In his case the wealth expansion path denoted by $\tilde{\psi}(.)$ is:

$$c_1^i = \frac{1}{\beta R} c_0^i - \frac{\beta R - 1}{\theta \beta R} R$$

The slope $\tilde{\psi}'(c_0^i)$ is then $(\beta R)^{-1}$. Hence $\tilde{\psi}'(c_0^i) - \beta R$ is negative when $R > 1/\beta$. Moreover $\tilde{\psi}'(c_0^i)$ is smaller than 1 implying that the distribution of the second period consumption is more concentrated than the distribution of the first

[5] Given the slope a of the wealth density, the weight α is chosen small enough such that H3 is verified. Notice that the density function does not sum to one in the example. This is without consequence since individuals are concerned about their ordinal rank in the distribution.

period consumption. Now, when agents care about their consumption rank, this initial asymmetry provides an additional incentive to save and to consume more in second period. This comes from the fact that a higher consumption in the second period entails a gain of rank greater than the corresponding loss in the first period. Hence the contest for status is stronger in this period as explained in the previous section. Moreover, since the incentive is proportional to the wealth density function and since the latter is upward sloping, the slope of the wealth expansion path with status-seeking ($\tilde{A}'(c_0^i)$) is greater than the one without status. However, the equilibrium slope remains inferior to βR in order to keep the saving incentive at equilibrium. The converse case in which $R < 1/\beta$ leads to a symmetric reasoning. The economy without status displays a more concentrated first period consumption distribution, a property which deters saving when agents care about their consumption rank.

5. Conclusion

In this paper, we have shown how a concern for the rank in the consumption distribution affects saving. The consumption decisions are interrelated in a complex manner and turn out to depend on how the distribution of consumption evolves through time. In a simple version of the model we have shown a negative impact on saving must be accompanied by a rise in consumption inequalities. The latter fact seems indeed the case in most developed countries.

The model has other potential implications not investigated in the present paper. Posing a more realistic wealth distribution would allow to examine which class of people according to their wealth are the most sensitive to the status effect. Second, the qualitative impact on saving depends on how the wealth distribution translates into the dynamics of the consumption distribution. This mechanism turns out to be primarily determined by the form of the utility function. Therefore it could be interesting to generalize the model along this dimension. The model could also be extended by assuming long-lived agents. All these extensions would require the problem to be numerically solved however.

Appendix

A Proof of Lemma 1

Let us demonstrate that the hypothesis H3: $T_t''(c_t^i) < 0$ for $t = 0; 1$ and for all $y_0^i \in \left[y_0^i; y_0^+\right]$ ensures $\varphi_0'(y_0^i) > 0$ and $\varphi_1'(y_0^i) > 0$.

The optimal rule $c_0^i = \varphi_0(y_0^i)$ is implicitly given by: $T_0'(c_0^i) - \beta R T_1'(Ry_0^i - Rc_0^i) = 0$. The slope $\varphi_0'(y_0^i)$ is given by the implicit function theorem:

$$\varphi_0'(y_0^i) = \frac{\beta R^2 T_1''(Ry_0^i - Rc_0^i)}{T_0''(c_0^i) + \beta R^2 T_1''(Ry_0^i - Rc_0^i)}$$

Since the second order condition ensures that the denominator is negative, the slope is positive following H3: $T_1''(c_t) < 0$. The optimal second period consumption is given by: $c_1^i = \varphi_1(y_0^i) = R(y_0^i - \varphi_0(y_0^i))$. The slope of $\varphi_1(y_0^i)$ is then:

$$\varphi_1'(y_0^i) = 1 - \frac{\beta R^2 T_1''(Ry_0^i - Rc_0^i)}{T_0''(c_0^i) + \beta R^2 T_1''(Ry_0^i - Rc_0^i)}$$

which is positive if $T_0''(c_t) < 0$. ∎

B The first order condition is a sufficient condition under H3.

First, notice that the rank preserving property of the model holds under H3 (see Appendix A).

Let $V(c_0^i)$ be the objective of the individual i endowed with y_0^i:

$$V(c_0^i) = T_0(c_0^i) + \beta T_1(Ry_0^i - Rc_0^i)$$

The individuals maximize $V(c_0^i)$ by choosing c_0^i over $[0; y_0^i]$ where $G_t(.)$ are given, $t = 0; 1$. Let us define $\underline{c}_0$, $\bar{c}_0$, $\underline{c}_1$ and $\bar{c}_1$ which are respectively the lower bound and the upper bound of the first period consumption distribution and the lower bound and the upper bound of the second period consumption distribution. Let $\mathbf{b}_0$ and $\mathbf{e}_0$ be the first period consumption implying respectively $c_1^i = \underline{c}_1$ and $c_1^i = \bar{c}_1$ via the budget constraint: $\mathbf{b}_0 = y_0^i - \underline{c}_1/R$ and $\mathbf{e}_0 = y_0^i - \bar{c}_1/R$.

Let us consider some wealth y_0^i and a given consumption distribution at both dates. There exist several cases: the amount the individual may consume in first period belongs or do not belong to the current consumption distribution; the second period consumption belongs or do not belong to the current consumption distribution [6].

[6] Notice that an agent is not necessarily concerned by all the cases, depending on his wealth y_0^i.

Case 1. $c_0^i \in]\underline{c}_0; \overline{c}_0[\cap]\tilde{c}_0; \hat{c}_0[$.

In this case the first period consumption and the second period consumption belongs to the current distribution. $V(.)$ is concave if the condition (2.2) is verified:

$$u''(c_0^i) + \alpha g_0'(c_0^i) + \beta R^2 (u''(c_1^i) + \alpha g_1'(c_1^i)) < 0$$

which is true under H3.

Case 2. $c_0^i \in]\underline{c}_0; \overline{c}_0[\cap [0; \tilde{c}_0[\cap]\hat{c}_0; y_0^i]$.

The first period consumption is interior to the distribution, contrary to the second period consumption. This implies $g_1(c_1^i) = 0$ in the neighborhood of c_1^i. $V(.)$ is concave if:

$$u''(c_0^i) + \alpha g_0'(c_0^i) + \beta R^2 u''(c_1^i) < 0$$

which holds under H3.

Case 3. $c_0^i \in [0; \underline{c}_0[\cap]\overline{c}_0; y_0^i] \cap]\tilde{c}_0; \hat{c}_0[$.

The second period consumption is interior to the distribution, contrary to the first period consumption. This implies $g_0(c_1^i) = 0$ in the neighborhood of c_0^i. Then $V(.)$ is concave if:

$$u''(c_0^i) + \beta R^2 (u''(c_1^i) + \alpha g_1'(c_1^i)) < 0$$

which is verified under H3.

Case 4. $c_0^i \in [0; y_0^i] -]\underline{c}_0; \overline{c}_0[-]\tilde{c}_0; \hat{c}_0[$.

The consumption is outside the consumption distribution at both periods. As a result $g_0(c_0^i) = 0$ and $g_1(c_1^i) = 0$. $V(.)$ is concave by assumption in this case: $u''(c_0^i) + \beta R^2 u''(c_1^i) < 0$.

It remains to verify that $V(.)$ is also concave in the neighborhood of $\underline{c}_0$, $\underline{c}_1$, $\overline{c}_0$, $\overline{c}_1$. To do so, we have to take account that $G_0(.)$ and $G_1(.)$ are not twice continuously differentiable at these points. Here, it is however sufficient to show that the first derivative $V'(c_0)$ is decreasing in the neighborhood of these points. We shall consider four cases: (I) $c_0^i = \underline{c}_0$, (II) $c_0^i = \hat{c}_0$ (or equivalently $c_1^i = \underline{c}_1$), (III) $c_0^i = \overline{c}_0$ and (IV) $c_0^i = \tilde{c}_0$ (or $c_1^i = \overline{c}_1$).

(I) $c_0^i = \underline{c}_0$. We restrict our analysis to the individuals $y_0^i > y_0^-$. A similar reasoning would however apply to individuals endowed with y_0^-. Suppose $c_0^i = \underline{c}_0 + \varepsilon$ with ε a small positive real number. We have in this case:

$$V'(c_0^i) = u'(c_0^i) - \beta R u'(c_1^i) + \alpha(g_0(c_0^i) - \beta R g_1(c_1^i))$$

If $c_0^i = \underline{c}_0 - \varepsilon$:

$$V'(c_0^i) = u'(c_0^i) - \beta R u'(c_1^i) - \alpha\beta R g_1(c_1^i))$$

since the rank in the distribution of the first period consumption is unchanged in the left-neighborhood of c_0^i. The difference between the two expressions taken at the limit:

$$\lim_{c_0 \to \underline{c}_0^+} V'(c_0) - \lim_{c_0 \to \underline{c}_0^-} V'(c_0) = \alpha g_0(\underline{c}_0) = 0$$

because $g_0(\underline{c}_0) = f(y_0^-)\varphi_0'(\underline{c}_0) = 0$ following $f(y_0^-) = 0$. We have therefore:

$$\lim_{c_0 \to \underline{c}_0^+} V'(c_0) = \lim_{c_0 \to \underline{c}_0^-} V'(c_0) = V'(\underline{c}_0).$$

As a result $V'(c_0)$ is continuous and decreasing in the neighborhood of $\underline{c}_0$.

(II) $c_0^i = \mathbf{b}_0$ (implying $c_1^i = \underline{c}_1$). We limit our attention to $y_0^i > y_0^-$ without loss of generality. If $c_0^i = \mathbf{b}_0 + \varepsilon$ with ε a small positive real number:

$$V'(c_0^i) = u'(c_0^i) - \beta R u'(c_1^i) + \alpha g_0(c_0^i)$$

since the rank in the distribution of the second period consumption is unchanged in the right-neighborhood of c_0^i. The difference between the two expressions taken at the limit is:

$$\lim_{c_0^i \to \mathbf{b}_0^+} V'(c_0^i) - \lim_{c_0^i \to \mathbf{b}_0^-} V'(c_0^i) = \alpha g_1(\underline{c}_1) = 0$$

because $g_1(\underline{c}_1) = f(y_0^-)\varphi_1'(\underline{c}_1) = 0$ due to $f(y_0^-) = 0$. Consequently:

$$\lim_{c_0^i \to \mathbf{b}_0^+} V'(c_0^i) = \lim_{c_0^i \to \mathbf{b}_0^-} V'(c_0^i) = V'(\mathbf{b}_0).$$

$V'(c_0^i)$ is therefore continuously decreasing in the neighborhood of $\mathbf{b}_0$.

(III) $c_0^i = \bar{c}_0$. We focus on individuals endowed with $y_0^i < y_0^+$. Suppose $c_0^i = \bar{c}_0 + \varepsilon$ with ε a small positive real number. We have:

$$V'(c_0^i) = u'(c_0^i) - \beta R u'(c_1^i) - \alpha\beta R g_1(c_1^i)$$

since the rank in the distribution of the first period consumption is constant in the right-neighborhood of c_0^i. The difference between the two expressions is:

$$\lim_{c_0^i \to \bar{c}_0^+} V'(c_0^i) - \lim_{c_0^i \to \bar{c}_0^-} V'(c_0^i) = -\alpha g_0(\bar{c}_0)$$

which is negative. Consequently $V'(c_0^i)$ is not continuous but remains decreasing in the neighborhood of $\bar{c}_0$.

(IV) $c_0^i = \mathbf{e}_0$ (or $c_1^i = \bar{c}_1$). Again we restrict our attention to $y_0^i < y_0^+$. Suppose $c_0^i = \bar{c}_0 - \varepsilon$ with ε a small positive real number. In this case:

$$V'(c_0^i) = u'(c_0^i) - \beta R u'(c_1^i) + \alpha g_0(c_0^i)$$

since the rank in the distribution of the first period consumption is constant in the left-neighborhood of c_0^i. The difference between the two limits is:

$$\lim_{c_0^i \to \mathbf{e}_0^+} V'(c_0^i) - \lim_{c_0^i \to \mathbf{e}_0^-} V'(c_0^i) = -\alpha g_1(\bar{c}_1)$$

which is negative. As a consequence $V'(c_0^i)$ is decreasing in the neighborhood of $\mathbf{e}_0$. ∎

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Figure 1: Saving in the case beta*R=1.1

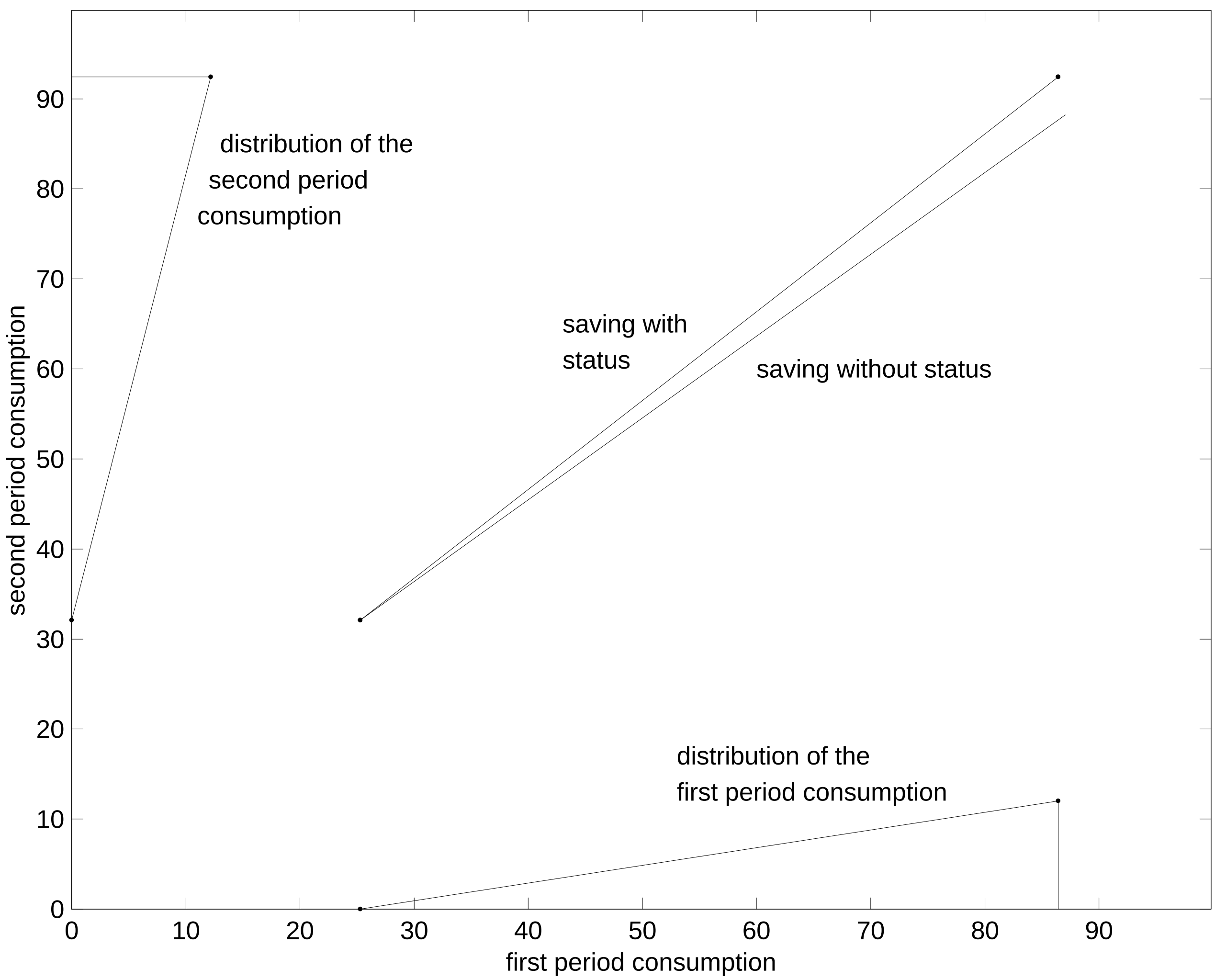

Figure 2: Saving in the case beta*R=0.9