DETERMINANTS OF INTER-TRADE DURATIONS USING PROPORTIONAL HAZARD ARMA MODELS

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Abstract. This paper disseminates the survivor function of inter-trade durations as a key feature of the intraday trading process. It sheds light on the time varying trading intensity and, thus, liquidity of a traded asset and the information channels which propagate price signals among asymmetrically informed market participants. To obtain a consistent estimate of the baseline survivor function and capture well-known serial dependency in the trade intensity process as well we use a semiparametric proportional hazard model wich is augmented by an ARMA structure very similar to the obiquous ACD model.

Based on transaction data from the DTB, Frankfurt, we find evidence that past sequences of prices and volumes have a significant impact on the trading intensity in accordance with theoretical models on the basis of rational expectations equilibria. However, we cannot find any evidence in favour of strategic behaviour with respect to the chosen transaction volume by informed traders. From an inspection of conditional failure probabilities we find weak evidence for the use of non-trading intervals as an indication for the absence of price information among market participants. However, this information content seems to be diluted by a high liquidity base level, particularly with respect to large inflow of traders of the U.S. market.

1. Introduction

The survivor function plays a key role to model trade frequency and thus the liquidity of financial markets. Recently, this was demonstrated e.g. by the analysis of Gourieroux, Jasiak, and LeFol (1999). We use a hazard-rate model for the inter-trade duration process which permits us to model the conditional probability to observe the next transaction as a function of the elapsed time since the last trade occurred, conditional on explanatory variables capturing the

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state of the market. The temporal shape of this conditional probability provides a characterization of the underlying stochastic duration process and gives us insights in the duration dependence. This duration dependence allows us to assess in which way traders infer information from no-trade-time-intervals, i.e. the elapsed time since the last transaction and shed some light on the dynamics of market activities.

In market microstructure theory the timing of trades plays an important role in the learning mechanism of market participants who draw inferences from the trading process. In this context inter-trade durations are regarded as means to aggregate information on price signals available to individual traders in an asymmetric information environment, complementary to other information channels like the sequence of price changes and traded volumes.

Empirical studies on the transaction intensity under the light of market microstructure hypotheses rest on the crucial assumption that the arrival rate of non informed traders is constant while informed traders only enter the market if information is present, see the seminal contribution by Easley and O'Hara (1992). In this framework the timing of trades depends only on the occurrence of information events.

The goal of our study is to get more insights into the market microstructure and the behaviour of market participants by explicitly modelling these inter-trade durations. We want whether certain behaviour of informed and uninformed market participants can be identified by investigating the impact of present and past volumes and price changes on the expected waiting time until the next trade [1].

Our study is based on the central results of noisy expectation equilibrium models (see e.g. Hellwig (1982)) that market participants learn from past market sequences. A key assumption in our setting is that inter-trade durations reflect the decisions of market agents which depend on the state of the market. We want to investigate whether traders which learn from past market activities tend

[1] A study with a related focus has been done by Grammig and Wellner (1999) who investigated similar economic questions by analyzing interdependencies between durations and volatility.

to exploit this informational advantage by increasing the trading intensity and, thus, the liquidity. If not only the informational content of past inter-trade durations but also of past sequences of prices and volumes is reflected in the present waiting time then one would expect that the inclusion of such variables might improve the prediction of these inter-trade durations.

A further scope of our paper is to analyze strategic behaviour of market participants. Kyle (1985) shows that informed traders have preferences to camouflage their information. Barclay and Warner (1993) find evidence that informed tend to trade medium trading-sizes. We investigate whether such strategic behaviour is reflected in inter-trade durations.

Furthermore, we want to shed some light on the influence of the heterogeneity of information on the trading intensities. In this context we compare survivor functions based on market phases before and after the opening of the American trading. The resulting survivor functions indicate whether an increasing of the heterogeneity of information is reflected in market dynamics and temporal dependence.

A specific feature of inter-trade durations is the occurrence of clustering. In econometric literature exist two central approaches accounting for clustered duration data: Engle and Russel (1995) introduce the Autoregressive Conditional Duration (ACD) model for intertemporally correlated inter-trade durations, which is based on a parametric autoregressive specification for the conditional mean of the duration and is the counterpart of the GARCH model for price processes. Ghysels, Gourieroux, and Jasiak (1998) propose a class of two factor models for duration data, where the first factor accommodates dynamics in the conditional mean and the second factor in the conditional variance. Because of its strong relation to stochastic volatility models they call it the Stochastic Volatility Duration (SVD) model. They show that the SVD model captures interactive dynamics of the conditional mean and variance and models clustering and persistence effects in both conditional moments.

A drawback of both types of models is the requirement of parametric specifications for the assumed distribution of the durations. Grammig and Maurer (1999) provide Monte Carlo studies and investigate therewith whether the estimations of the parameters in the ACD framework are affected by a misspecification in the conditional hazard function. They show that the ML estimators of the basic ACD model tend to be biased and inefficient when the true data generating process requires non-monotonic hazards. For this reason Bauwens and Veredas (1999) introduce the Stochastic Conditional Duration (SCD) model where the durations are generated by a latent stochastic factor allowing an autoregressive process. The main innovation of this class of models is to allow for a wider range of shapes of hazard functions.

In order to account on the one hand for non-monotonic distributions of the inter-trade durations of an unknown form and on the other hand for serial dependencies in the duration process we use the proportional hazards ARMA model proposed by Gerhard and Hautsch (2000) that extends the traditional semiparametric proportional hazard model by allowing for serial dependencies in the durations. The advantage of this type of duration model is on the one hand that it does not require to assume a parametric specification of the durations but provides a nonparametric estimation of the baseline hazard while it allows on the other hand to estimate ARMA-structures in the inter-trade duration process. Explanatory variables can be included dynamically, corresponding to an ARMAX specification, but also statically, i.e. without any lag structure.

By analyzing Bund Future transaction data of the Deutsche Terminbörse (DTB) in Frankfurt we show that the PHARMA model does a good job capturing the serial dependencies in the inter-trade duration process while it allows to semiparametrically asses the shape of the baseline survivor function. By using past absolute price changes and volumes per transaction as explanatory variables we can show that these variables have a significant impact on the expected waiting time until the next transaction, confirming our hypotheses. By obtaining a nonlinear relationship between the contemporaneous volume and the expected waiting time until the next transaction we find evidence for the informational content of certain volume sizes that might be caused by strategic behaviour of informed traders. Furthermore we show that liquidity effects caused by the opening of American trading have a significant impact on the duration dependence indicated by survivor functions and conditional failure probabilities.

The outline of the paper is as follows: In section 2 testable hypothesis are discussed originating in market microstructure models. The econometric model is described in section 3. Section 4 gives a description of the data set, while Section 5 presents the empirical results. Section 6 concludes.

2. Market Microstructure

The theoretical background for an empirical study on the determinants of inter-trade durations is based on Diamond and Verrecchia (1987) and Easley and O'Hara (1992) which explicitly account for the time between particular trades. In the Diamond and Verrecchia (1987) setting, at any point in time either good or bad news do exist. Because traders are short-sale constrained no-trading intervals, i.e. long inter-trade durations are associated with bad news.

In the Easley and O'Hara (1992) framework time is correlated with any factor related to the value of the asset that can arise from properties of the information structure in the market. Easley and O'Hara (1992) demonstrate that market participants learn from both trades and the lack of trades at any point in time, thus, from the length of inter-trade waiting times. The key element in this framework is that the information content of the inter-trade durations arises by their correlation with different aspects of information.

A central assumption in our study is that the timing of trades is not only driven by the occurrence of information but also reflects the individuals decisions of traders. This implies that an agents learning from past sequences of market activities is also reflected in the expected waiting times until the next transaction. The assumption of the informativeness of past price sequences is based on the noisy rational expectation equilibrium models of Hellwig (1982) and Diamond and Verrecchia (1981) which analyze rational expectation equilibria in a market where investors learn from past prices. If a traders' preference for immediacy of transactions increases if past market activities provide information to him then past price sequences have also an impact on the expected inter-trade duration. By assuming that the information content of a price process is correlated with its volatility leads to the following testable hypothesis

Hypothesis H1a: *Large absolute price changes in the past imply a decreased expected waiting time until the next trade.*

Blume, Easly, and O'Hara (1994) extend this theoretical framework and analyze the informational role of volume. They resolve how the statistical properties of volume relate to the behaviour of market prices and show that traders can also learn from sequences of volume. The crucial result is that volume provides information that cannot be deduced from the price statistic. In our framework we want to investigate whether this informational content of trading volumes is also reflected in inter-trade durations. Based on this theoretical setting we formulate the hypothesis H1b:

Hypothesis H1b: *Past sequences of volumes are informative for expected inter-trade durations even if past price sequences are accounted for.*

Furthermore, we want to get insights into the impact of the heterogeneity of information on the trade-to-trade waiting times. Lang, Litzenberger, and Madrigal (1992) show that the dispersion of private information across the agents influences the trading volume, but not the price [2]. This divergence of beliefs arising from asymmetric information plays an important role in generating activity. According to this theoretical literature one would expect that an increase of the heterogeneity of information has a significant impact on the speed of market activities. The influence of this dispersion of private information on the inter-trade durations can be empirically tested by analyzing the trade intensity at the DTB before and after the beginning of the American trading. We base this investigation on the conditional survivor functions and the conditional hazard function, given past and present market activities, that indicate changes in the temporal dependence of the timing of trades. Based on this framework we formulate the

[2] They illustrate that this result is completely consistent with the noisy rational expectation hypothesis and that prices depend only on aggregate and fundamental questions.

following testable hypothesis:

Hypothesis H2: *The inflow of additional market participants form the U.S. causes changes in the temporal dependence of inter-trade durations.*

The last hypothesis we want to check concerns strategic behaviour of market participants. The theoretical literature analyzing the strategic behaviour of agents is heavily influenced by Kyle (1985). He shows that profit-maximizing informed investors attempt to camouflage their information, e.g. by spreading trades over time. Admati and Pfleiderer (1988) assume two types of uninformed traders, "discretionary" liquidity traders, who have some choice over the time at which they transact, and "nondiscretionary" liquidity traders whose orders are assumed to arrive in a random fashion. They show that it is optimal for liquidity traders and also for insiders to trade together leading to concentrations of trading in particular time periods. While both studies ignore the choice of the trade size Barclay and Warner (1993) examine the proportion of cumulative price changes that occur in certain volume categories. Based on an empirical study they find evidence that most of the cumulative price change is due to medium-size trades. This result is consistent with the hypothesis that informed traders tend to use medium volume sizes [3]. The empirical framework for testing the evidence of such implications consist in analyzing the impact of the contemporaneous volume per transaction on the expected waiting time until the next trade. If informed investors trade medium sizes and want to exploit their informational advantage by executing a transaction as soon as possible then one would expect a nonlinear relationship between inter-trade durations and the present trading volume. These implications are summarized in the following hypothesis:

Hypothesis H3: *A nonlinear relationship can be observed between the contemporaneous volume and the expected time between trades.*

[3]A paper with a related focus is Kempf and Korn (1999) who empirically analyze the relation between unexpected net order flow and price changes and find highly nonlinear relationships.

3. The Proportional Hazard ARMA model

We use an extended semiparametric proportional hazard model with a discretized dependent variable. This particular quantal response model introduced by Han and Hausman (1990) has two salient features worth discussing in the context of inter-trade duration analysis. First, it allows the direct semiparametric estimation of the hazard rate which is a key feature in the analysis of conditional trade frequency, i.e. liquidity. Second, in some floor markets, the discretization of the dependent variable can compensate for some irregularities inflicted on the time series by the fact that it is collected by price reporters. The model is augmented by ARMA structures to account for clustering effects in the durations (see Gerhard and Hautsch (2000)). This kind of model is related to the ACD model and can be seen as a combination of ACD type models and hazard rate models.

Consider the sequence of arrival times $t_0, t_2, \ldots, t_n$ with $t_0 < t_1 < \ldots < t_n$ as a stochastic process. Associated with this process for the arrival times is a process for the waiting times between the trades, $\tau_i = t_i - t_{i-1}$, $i = 1, \ldots, n$, the inter-trade durations.

By assuming an ARMA(p, q) process for the durations we can express them by

$$\tau_t = \sum_{j=1}^{p} \phi_j \tau_{t-j} - \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t, \tag{1}$$

where ϵ_t follows an unknown distribution.

The ACD model proposed by Engle and Russel (1995) can also be formulated as an ARMA(p, q) model (see Engle and Russel (1998)

$$\tau_t = \sum_{j=1}^{max(p,q)} (\phi_j + \theta_j) \tau_{t-j} - \sum_{j=1}^{q} \theta_j \eta_{t-j} + \eta_t, \tag{2}$$

where $\eta_t = \tau_t - \psi_t$ and $\psi_t = E[\tau_t | \bar{\tau}_{t-1}]$ denotes the conditional mean of τ_t given $\bar{\tau}_{t-1} = [\tau_{t-1}, \tau_{t-2}, \ldots, \tau_1]$, the sequence of past durations.

Engle and Russel provide strong evidence for duration clustering for IBM stock (Engle and Russel (1995) and Engle and Russel (1998)) and foreign exchange market data (Engle and Russel (1997)). They show that this class of duration models does a good job of capturing the dynamics of the data. They proofed that the quasi maximum likelihood estimators for GARCH(1,1) models developed by Lee and Hansen (1994) can actually be applied to EACD(1,1) models [4]. Hence their model provides consistent estimations of the parameters even when the underlying distribution for the duration process is misspecified. However, several empirical studies (Engle and Russel (1995), Engle and Russel (1997), Engle and Russel (1998), Grammig and Maurer (1999)) provide evidence that the assumption of exponential distributed inter-trade durations is not appropriate.

Grammig and Maurer introduced a more flexible ACD model by assuming a Burr-distribution for the standardized durations[5] and provide evidence in form of Monte Carlo simulations in favour to this ML estimator.

Furthermore, in the ACD framework a nonparametric baseline hazard rate cannot be estimated directly. Viewing the EACD as a QMLE a baseline hazard can only be computed in a further step based on the empirical distribution of the standardized residuals.

In order to provide a nonparametric baseline hazard rate and to account for clustering structures in the inter-trade duration process we use the semiparametric proportional hazard model proposed by Cox (1972) as a starting point

$$\lambda(\tau_t|m_t) = \lambda_0(\tau_t)exp(-m_t), \quad t = 1, \dots, n, \tag{3}$$

where $\lambda_0(\tau_t)$ is an unspecified baseline hazard and $m_t = m(x_t, \beta)$ a mean function depending on a vector of covariates x_t and a vector of coefficients β.

Originally, the proportional hazard model admits an interpretation as a linear regression model. This relationship is obtained by parameterizing $m(x_t, \beta) = x_t'\beta$ and using the fact that

$$\tilde{\tau}_t = m(x_t'\beta) + \epsilon_t, \tag{4}$$

[4]The EACD (Exponential Autoregressive Duration) model corresponds to the ACD model where the standardized durations follow an exponential distribution.

[5]The Burr-ACD (BACD) nests the standard Exponential-ACD and Weibull-ACD models as special cases.

where $\tilde{\tau}_t \equiv -ln \int\limits_0^{\tau_t} \lambda_0(s)ds$, $m_t = E[\tilde{\tau}_t|x_t]$ its conditional expectation, and ϵ_t is an extreme value distributed error term [6]. (See e.g. Kiefer (1988) or Han and Hausman (1990)).

We employ the model suggested by Gerhard and Hautsch (2000) based on the work of Gerhard (2000) where an ARMA structure is included in the mean function in order to account for serial dependence in the duration process. In this context

$$\tilde{\tau}_t = x_t'\beta + \zeta + e_t, \tag{5}$$

where

$$e_t = \sum_{j=1}^{p} \phi_j e_{t-j} + \sum_{j=0}^{q} \theta_j \eta_{t-j} + w_t'\gamma. \tag{6}$$

The random variable η_t is defined by $\eta_t = \epsilon_t - c$ where $c = E[\epsilon_t]$ is the mean of the extreme value distributed error term ϵ. The substitution of e_t by η_t is necessary to obtain an error term with zero mean in the ARMA specification and is compensated by the inclusion of the term $\left(\zeta \equiv c \sum\limits_{j=0}^{q} \theta_j\right) / \left(1 - \sum\limits_{j=1}^{p} \phi_j\right)$ [7].

A vector of explanatory variables w_t is included in the dynamic structure with γ as the corresponding vector of coefficients.

Using the state space form, the model can be rewritten as

$$\tilde{\tau}_t = H \cdot \xi_t + x_t'\beta + \zeta \tag{7}$$

$$\xi_t = F \cdot \xi_{t-1} + w_t'\gamma + e_1\eta_t, \tag{8}$$

where $H = \begin{bmatrix} 1 & \theta_1 & \dots & \theta_q \end{bmatrix}$, $F = \begin{bmatrix} \phi_1 & \cdots & \phi_p \\ I_{p-1} & & 0 \end{bmatrix}$ and $e_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}$.

By including this dynamic specification the mean function m_t takes the form $m_t = E[\tilde{\tau}_t|\bar{\tau}_{t-1}]$.

The discretization of $\tau_t \in \mathbb{R}^+$ which is used to estimate a semiparametric baseline hazard yields a count variable $\tau_t^* \in \{1, 2, \dots, K\}$ depending on the category

[6]The model can be extended by accounting for unobserved heterogeneity. Such effects are included by specifying a compounder ω acting multiplicatively with the hazard function. By analyzing LIFFE Bund Future data, Hautsch (1999) shows that unobservable effects captured by the compounder are only very weak. For this reason we ignore the impact of such effects.

[7]Note that resolving this substitution leads to an ARMA model with an extreme value distributed error term, hence the constant ζ is identified.

in which the observable, continuous τ_t falls, using μ_k, $k = 1, 2, \ldots, K-1$ as the thresholds between the categories. We use the estimation procedure proposed by Gerhard and Hautsch (2000) by considering eq. (5) as a latent process.

We observe the inter-trade duration category $\tau_t^* = k$ if the latent variable $\tilde{\tau}_t$ lies between the two thresholds $(\tilde{\mu}_k; \tilde{\mu}_{k-1}]$, i.e. the conditional probability for observing $\tau_t^* = k$ is

$$Pr(\tau_t^* = k | x_t, \bar{\tau}_{t-1}) = \int_{\tilde{\mu}_k - m_t}^{\tilde{\mu}_{k-1} - m_t} f_\eta(s) ds \tag{9}$$

where $f_\eta(s)$ denotes the density function of η.

A novel feature of this model is that it accounts for clustering effects in the data but also provides a nonparametric baseline survivor function which is obtained directly by the estimated thresholds. It can be calculated at the k discrete points by

$$S_0(\mu_k) = exp(-exp(\tilde{\mu}_k)), \quad k = 1, \ldots, K-1. \tag{10}$$

Because the dynamic structure is based on the unobservable underlying variable $\tilde{\tau}_t$, we calculate it by using the concept of generalized residuals proposed by Gourieroux, Monfort, and Trognon (1985). If $\tau_t^* = k$, the conditional expectation of the residual η_t is given by

$$E[\eta_t | \tau_t^* = k] = \frac{1}{F_\eta(\tilde{\mu}_k - m_t) - F_\eta(\tilde{\mu}_{k-1} - m_t)} \cdot \int_{\tilde{\mu}_{k-1} - m_t}^{\tilde{\mu}_k - m_t} s f_\eta(s) ds, \tag{11}$$

where $F_\eta(.)$ denote the distribution function of η. For the mean function a recursion is proposed in Gerhard (2000) based on the state space form given in eq. (7). The recursion takes the following form [8]:

$$E[\xi_t | \bar{\tau}_{t-1}] = F\left(E[\xi_{t-1} | \bar{\tau}_{t-1}] + E[\eta_{t-1} | \bar{\tau}_{t-1}]\right). \tag{12}$$

From this, m_t is directly evaluated as

$$m_t = E[\tilde{\tau}_t | \bar{\tau}_{t-1}] = H' E[\xi_t | \bar{\tau}_{t-1}]. \tag{13}$$

[8] For ease of notation the regressors x_t and w_t are omitted.

The recursion is initialized with the unconditional expectation of the latent variable

$$E[\xi_1|\bar{\tau}_0] = E[\xi_t] = 0. \tag{14}$$

The log likelihood function takes the usual form

$$l(\beta, \gamma, \mu, \phi, \theta) = \sum_{t=1}^{N} \sum_{k=1}^{K} y_{tk}\, ln \int_{\tilde{\mu}_k - m_t}^{\tilde{\mu}_{k-1} - m_t} f_\eta(s) ds, \tag{15}$$

where the indicator variable y_{tk} is defined by

$$y_{tk} = \begin{cases} 1 & , \quad if \quad \tau_t^* = k \\ 0 & , \quad else. \end{cases}$$

Under the usual regularity conditions we can show consistency and asymptotic normality for this maximum likelihood estimator (for more details see in the appendix).

4. The data

The sample contains intra-day transaction data of the Bund Future trading at the screen based trading system of the Deutsche Terminbörse (DTB), Frankfurt, from 01/30/95 to 02/24/95, corresponding to 20 trading days. Within this period the Bund-Future was one of the most liquid futures in Europe and corresponded to a 6% German government bond of DEM 250.000 face value. The Bund Future had a maturity of 8.5 years and four contract maturities per year, March, June, September and December. Prices were denoted in basis points of face value, thus, one tick was equivalent to a value of DEM 25.

The dataset contains time stamped prices and volumes and consists of 44810 observations, where the overnight durations are omitted. Furthermore, we do not use the first 10 minutes of a trading day to avoid the opening phase that shows erratic price changes within the first trading minutes which are due to the price finding process at the market opening based on the occurrence of information overnight. Because these erratic price changes even out after a few transactions we eliminate these observations from the dataset.

To use the estimation procedure of the proportional hazard ARMA model we have to categorize the trade-to-trade durations. We use a categorization that ensures, on the one hand, satisfactory frequencies of the observations in the categories but allows us, on the other hand, to derive the temporal dependence of the inter-trade durations for longer time intervals. Because the distribution of the waiting times is extremely skewed to the right (see table 1) we use smaller categories for the lower durations and a larger categorization (30 second intervals) for higher waiting times. This categorization based on 30 second intervals is reasonable because we want to asses in which way agents learn from no-trade-intervals, e.g. the last 30 seconds.

Several empirical studies (Wood, McInish, and Ord (1985), Engle and Russel (1995), Engle and Russel (1997), Guillaume, Dacorogna, Dave, Müller, Olsen, and Pictet (1996) or Dacorogna, Morgenegg, Müller, Olsen, Pictet, and Schwarz (1990)) found evidence for highly significant seasonality patterns. Thus, consistent estimations of the impact of covariates and temporal dependencies on inter-trade durations require the inclusion of seasonality effects. To account for intraday seasonalities we use the flexible Fourier form proposed by Andersen and Bollerslev (1998) based on Gallant (1981) which is given by

$$s(\delta, t^*, P) = \delta_1 \cdot t^* + \sum_{p=1}^{P} \left(\delta_{c,p} \cos(t^* \cdot 2\pi p) + \delta_{s,p} \sin(t^* \cdot 2\pi p)\right), \tag{16}$$

where p is identical with the order of the term, $t^* \in [0,1]$ defined by

$$t_1^* = \frac{\text{seconds since 8:40}}{\text{seconds between 8:40 and 17:15}} \tag{17}$$

and $\delta_{c,p}$, $\delta_{s,p}$ and δ denote the corresponding coefficients.

To check hypothesis H2, concerning the impact of an increase of information heterogeneity, we define two dummy variables indicating trading after 14:30, the opening of U.S. trading and the 02/20/95, the 'President's Day', American holiday. To investigate the further market microstructure hypotheses we include log-volume and absolute price changes as explanatory variables.

5. Empirical Results

5.1. **Persistence and Intraday-Seasonalities.** The proportional hazard ARMA model is proposed to model both the underlying baseline hazard rate and the clustering structures in the inter-trade durations. To investigate the autoregressive structures in the data we start our analysis by calculating the autocorrelation (acf) and partial autocorrelation functions (pacf) in the trade-to-trade waiting times. Table 2 shows the acf and pacf which indicate highly significant autocorrelations. They show the typical slow rate of decay of a long memory process, a feature of this data which is well documented in the recent literature [9].

To simplify the model selection we, first, run several ARMA models on the raw and also seasonally adjusted inter-trade durations. Table 3 shows the results of four ARMA specifications based on the raw data. The high values of the ARMA parameters indicate a high persistence of the duration-process and are comparable to the results found by Engle and Russel (1998) who investigated price intensities by running ACD models. Table 4 presents the corresponding results based on seasonal adjusted durations [10]. The ARMA parameters are nearly unaffected which indicates that the persistence is not captured by the inclusion of seasonality parameters. On the other hand, table 2 (column B) shows that the autocorrelation and partial autocorrelations in the data are significantly reduced by accounting for seasonality patterns. Hence, the strength of the intertemporal correlations is weakened while the degree of the persistence is nearly unaffected.

The model selection is based on the Bayesian information criterion (BIC) leading to an ARMA(2,2) for the raw durations respectively an ARMA(1,2) for the seasonal adjusted waiting times as the best specification. We used these results as a starting point for the model selection in the class of proportional hazard ARMA (PHARMA) models and also obtained a PHARMA(1,2) as the best specification.

[9]See e.g. Jasiak (1999) who accounts for this high persistence by modelling a fractional integrated ACD-model.

[10]In this context we regress the durations on the variables of an flexible Fourier form of order 4, inclusive two dummies indicating the opening of American trading at 14:30 and the American 'President's Day' at 02/20/95, by OLS. Based on these consistent (but inefficient) estimates of the covariates we calculate the residuals and use these as seasonal adjusted durations.

Table 5 shows the estimation results of three regressions based on the PHARMA(1,2) model [11]. Column A presents the results of a regression without any explanatory variables, based only on the thresholds and the ARMA coefficients, while columns B and C contain the corresponding results with included seasonal covariates [12]. The similarity of the ARMA parameters to the simple ARMA(1,2) regressions on the raw respectively the seasonal adjusted durations (see tables 3 and 4) indicate the robustness of the results. Furthermore, by running several regressions with different categorizations of the durations we find evidence that the estimations of the ARMA parameters and also the coefficients of the covariates are not affected by the choice of the categories. This result is in accordance with the property of the semiparametric proportional hazard model that it allows consistent estimations of the coefficients of the explanatory variables even when the form of the baseline hazard is unknown (see Meyer (1990)).

To investigate the impact of intraday-seasonalities we calculate the impact of these variables of the latent variable $\tilde{\tau}_t$. Figure 1 shows the typical intradaily seasonality pattern with high market activities in the morning, a significant dip at the lunch time and the shortest inter-trade durations after the opening of the American trading at 14:30. It is an interesting result that the inclusion of the two dummies indicating trading after 14:30 and the American 'President's Day' at 02/20/95 (see Column C) decrease the significance of the most of the trigonometric terms. Hence, the main impact of the estimated intraday seasonalities seem to be captured by the 14:30-dummy. The high significance of this dummy indicates that the opening of the American trading has a strong impact on the speed of market activities and increases the liquidity according to lower trade-to-trade waiting times.

Figure 2 shows the estimated baseline survivor functions conditional on trading before and after 14:30 and at the American holiday day [13]. The graphs show

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[11] The maximum likelihood estimation of the model is performed using the BFGS algorithm with numerical derivatives in GAUSS.

[12] We also run regressions by including dummy variables that accounted for day-of-the-weak-effects but didn't find any significance concerning such effects.

[13] The functions are conditioned on mean values for the explanatory and the dynamic variables.

that the pattern of the baseline survivor functions change significantly indicating that the American trading plays an important role for the market dynamics. While the probability for observing inter-trade time intervals longer than 30 seconds is approximatively 0.37 at U.S. holiday, this probability is approximatively 0.09 at 'normal' days before 14:30 and nearly zero after 14:30. These significant changes of the patterns of the survivor function and, thus, the underlying duration distribution [14], may economically be attributed to an increased heterogeneity in traders' price signals caused by a major inflow of potential traders from the U.S. market that would confirm hypothesis H2. An interesting feature in this context is the fact that, while the mean inter-trade duration is at the U.S. holiday three times as large as at 'normal days' (see table 1), the transaction volume is nearly unaffected. Hence, liquidity effects are only caused by the increase of the trading intensity, not by trading volumes.

Figure 3 illustrates the conditional probability for the end of a spell in the next duration category given the time it lasted already, given by

$$\tilde{\lambda}_0(\mu_k) = \frac{S_0(\mu_k) - S_0(\mu_{k+1})}{S_0(\mu_k)}, \qquad k = 1, \dots, K-1. \tag{18}$$

These conditional failure probabilities allow us to characterize the duration dependence for longer time intervals that gives us insights in which way traders infer information from no-trade-time intervals. By virtue of the chosen categorisation it is straightforward to interpret the conditional failure probabilities depicted in figure 3. There we observe the conditional probabilities for a transaction to occurr within the next 30 seconds, given that we have just observed the last transaction and given that the last transaction happened before 30, 60, 90, 120, and 150 seconds. We note that the conditional failure probability is decreasing slowly. The fact that no trading took place would support the uninformed trader who infers that there is indeed no information in the market and thus it is quite plausible in the light of Easley and O'Hara (1992) that the conditional probability to observe a transaction decreases. From the fact that the conditional probability decreases slowly, with an almost linear pattern, one could conclude that the non-trading

[14] This feature is also reflected in the descriptive statistics (see table 1).

is indeed informative but has not a very weak impact on the decisions of market participants. One reason might be seen in the high base level of liquidity, i.e. in the abundance of market participants with exogeneous motives, usually termed noise traders in the theoretical literature. If one is willing to accept this hypothesis one could argue that the inflow of additional traders from the U.S. has a particularly high share in noise traders as the decrease in the conditional probability increases from 7 to 11 to 14 percentage points comparing the sample after 14:30 to the sample before 14:30 and to the U.S. holiday.

5.2. **Testing the market microstructure hypotheses.** In order to check the empirical evidence of the further market microstructure hypotheses proposed in section 2, we run two regressions with included market microstructure covariates (see table 6). Because we showed that the main part of the impact of deterministic intraday seasonalities is captured by the 14:30-dummy we omit the trigonometric seasonality terms. Regression D (table 6) presents a regression based on a nonlinear function of the actual log-volume. While the linear term ($lvol$) is insignificant the significance of the quadratic ($lvol^2$) and the cubic term ($lvol^3$) indicate a highly nonlinear impact of the actual volume on the expected waiting time until the next transaction. To illustrate this result we plot the aggregated impact of these volume-covariates on the latent variable (see figure 5). The graph depicts a non-monotonic function with a global minimum at a traded volume of approximatively 100. By considering the descriptive statitics (see table 1) we note, however, that transactions of this size are not very common. As a matter of fact in the volume range observed in 75% of all cases, i.e. smaller than 20 shares per transaction, the marginal influence of volume on the latent variable is well approximated by a linear function. Thus, we find no empirical evidence for a strategic behaviour of informed traders given the modelling strategy we used.

In order to test our hypothesis H1 we run a regession where we include the first lags of the log volume and the absolute price change dynamically (see regression E, table 6). For both variables we find highly significant negative coefficients. Hence, the higher past volumes and the more volatile the past price sequence the lower the expected trading intensity which confirmes hypotheses 1a and 1b. The calculation of the explicit influence of past volumes and past absolute price changes on the present trade frequency depends not only on the coefficient of the regressor but obviously also on the ARMA parameters. It yields a slowly decaying lag structure with the median lag at 13 for both regressors. Taking also into account that the mean time between transactions is about 14 seconds (see table 1), we can - cum grano salis - invoke the intuition that the weighted volume and the weighted absolute price changes of the last 3 minutes before a transaction make up about 50% of this regressors impact. The coefficients of the lag structure are all negative. Thus, these results can be interpreted as empirical evidence for the fact that investors seem to increase their preference for immediacy of further transactions if past market activities provide information to them.

6. Conclusions

In this sudy we use the proportional hazard ARMA model proposed by Gerhard and Hautsch (2000) to estimate inter-trade durations of the Bund-Future trading at the DTB, Frankfurt. The advantage of this class of models is that it accounts for clustering effects in inter-trade durations but also provides a nonparametric shape of the underlying baseline hazard rate. Futhermore, covariates can be included statically and dynamically.

The goal of this paper is to get more insights into the market microstructure by modelling the waiting times between particular transactions. We investigate whether the timing of trades reflects the decisions of traders which learn from past market activities.

By including past volumes and absolute price changes as dynamic covariates we find evidence for the fact that these variables have a significantly negative impact on the expected trading intensity. These results are in accordance with market microstructure hypotheses which imply that the informativeness of past sequences of market activities is reflected in a traders' preference for immediacy of transactions, i.e. in lower inter-trade durations. We find that past sequences of volume and absolute price changes yield a slowly decaying lag structure with a median lag at 13 for both regressors.

Furthermore, we investigate the impact of an increase of the heterogeneity of information on the trading intensity by estimating survivor and hazard functions based on market phases before and after the opening of the American trading and at a U.S. bank holiday. We obtain significant changes in market dynamics indicated by differences in the estimated survivor functions, and, thus the duration distribution.

The last hypothesis which is tested empirically concerns strategic behaviour of market participants. By including a nonlinear function of the contemporaneous trading volume we find a nonlinear relationship between present volume and trading intensity. As a linear approximation for the influence of traded volume per transaction on the latent variable seems appropriate over a sensible range we cannot find evidence for a camouflaging behaviour of informed traders.

The estimated ARMA paramaters indicate a high persistence of the duration-process which is a wellknown property of this kind of data. The obtained coefficients are comparable to the results already found by Engle and Russel (1998) on the basis of ACD models. Furthermore, we find a high robustness of all estimated paramaters, especially against the choice of the categories.

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7. Appendix

7.1. Asymptotic Properties of the Proportional Hazard ARMA model.

The following assumptions will be needed for consistency of the estimator and its asymptotic normality

Assumption (0): The DGP is of the form (5) and (6).
Assumption (1): The $\{\eta_t\}$ are i.i.d. with $\mathrm{E}\eta_t = 0$ and $\mathrm{E}\eta_t^2 = \sigma^2$.
Assumption (2): Stationarity of the AR component, the roots of the characteristic equation of $\phi(L)$ are outside the unit circle.
Assumption (3): Invertibility of the MA component, the roots of the characteristic equation of $\theta(L)$ are outside the unit circle.
Assumption (4): The characteristic polynomials of $\phi(L)$ and $\theta(L)$ have no roots in common and $\phi_p \neq 0$ $\theta_q \neq 0$
Assumption (5): θ_0 is in the interior of Θ, a compact subset of R^m. For ease of exposition we assume that we have a reasonably sized compact subset *within* the stationary and invertible region of our process.[15]
Assumption (6): The conditional mean function m_t has no additive components, which are constant over t, i.e. $m_t(\theta) \neq m_t(\theta') + c$ for all t and $\theta \neq \theta'$, and $c \neq 0$.[16]
Assumption (7): The strict inequality $-\infty < \mu_1 < \mu_2 < \ldots < \mu_{K-1} < \infty$ holds for the thresholds of the quantal response model.

The asymptotic properties of an ARMA process are somewhat lengthy and tedious to derive but nevertheless well documented in literature, see e.g. Deistler (1985). The assumption (5), which is stronger than the usual assumptions (2) and (3) in the forementioned literature allows us to concentrate on the pecularities of the relationship between the observable and the latent model and its implications for the asymptotic properties, so that we can raise the following

Proposition: *Under assumptions (0), (1), and (4)-(7) the estimator derived from* (15) *is*

1. *consistent and*
2. *asymptotically normal.*

Proof: It is possible to invoke generic theorems on consistency and asymptotic normality, e.g. theorems 5.1 and 5.2 of Wooldridge (1994) to assess this problem, inspite of the ARMA nature of the latent process because of assumption (5), which allows a simplified analysis. See e.g. the discussion in Pötscher and Prucha (1997, chapter 4.5).

Given assumption (5) or (2) and (3), the DGP is stationary and ergodic. From the identifiability assumption (4) follows that $m_t(\theta) \neq m_t(\theta')$ for $\theta \neq \theta'$. Then we have also $F_\eta(\mu_k - m_t(\theta)) \neq F_\eta(\mu_k' - m_t(\theta'))$ from $\mu_k \neq \mu_k'$ and assumption (6). Furthermore it is true that $F_\eta(\mu_k - m_t(\theta)) - F_\eta(\mu_{k-1} - m_t(\theta)) \neq F_\eta(\mu_k' - m_t(\theta')) - F_\eta(\mu_{k-1}' - m_t(\theta'))$ in conjunction with assumption (7).

[15] For an AR(1) we would thus impose that $\phi \in [l, u] \subset (-1, 1)$.

[16] This amounts to the usual identifying assumption on the level of the latent variable. The scale of the latent variable is identified through the constant variance of the extreme value distribuion.

The conditional expectation m_t can be seen as a one-period ahead forecast based on past observations, or more concisely $m_t = \sum_{j=1}^{\infty} \psi_j \tau_{t-j}$ where the ψ_j are a function of the autocovariances of τ_t. The restriction of the parameter space imposed by assumption (5) translates to restrictions on the autocovariances and thus on the ψ_j so that it is possible to give two sequences of constants c_j and d_j so that $b_l < m_t < b_u$ with $b_l = \sum_{j=1}^{\infty} d_j \tau_{t-j}$ and $b_u = \sum_{j=1}^{\infty} c_j \tau_{t-j}$ for which it is true that $\mathrm{E}|\mathrm{b_l}| < \infty$ and $\mathrm{E}|\mathrm{b_u}| < \infty$. From assumption (1) and the information inequality, see e.g. Newey and McFadden (1994), lemma 2.2, we can deduce a unique maximum of the likelihood.

Note that under assumption (5) the likelihood of the latent process and its second derivative satisfy the uniform weak law of large numbers, as well as the score at the true parameters satifies the central limit theorem. Invoking lemmata 1, 4, and 3 from Gourieroux, Monfort, Renault, and Trognon (1987) which are not limited to the linear exponential family completes the proof.

7.2. **Empirical Results.**

7.2.1. *Descriptive Statistics.*

TABLE 1. Descriptive Statistics of inter-trade durations and volume per transaction. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations.
A: After 14:30, no U.S. holiday.
B: Before 14:30, no U.S. holiday.
C: U.S. bank holiday, (President's Day (02/20/95)).
D: Over all observations.

	A	B	C	D
Inter-trade durations				
0.25-quantile	1	3	5	2
0.5-quantile	5	7	17	6
0.75-quantile	12	20	57	16
Mean	10.22	17.68	49.52	14.16
Std. Dev.	17.74	32.07	84.29	26.56
Volume per transaction				
0.25-quantile	5	4	3	5
0.5-quantile	11	10	10	10
0.75-quantile	23	20	20	21
Mean	19.77	18.67	16.07	19.19
Std. Dev	25.49	25.85	24.25	25.68

TABLE 2. acf and pacf functions of intra-trade durations. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations.
Column A: Raw durations.
Column B: Seasonal adjusted durations (based on FFF of order $p = 4$, 14:30- and holiday-dummies).

	A		B	
	acf	pacf	acf	pacf
lag1	0.2171	0.2171	0.1539	0.1539
lag2	0.2013	0.1617	0.1368	0.1159
lag3	0.1859	0.1230	0.1203	0.0870
lag4	0.1874	0.1122	0.1216	0.0824
lag5	0.1967	0.1113	0.1317	0.0870
lag6	0.1896	0.0915	0.1240	0.0711
lag7	0.1745	0.0670	0.1076	0.0492
lag8	0.1867	0.0785	0.1209	0.0630
lag9	0.1735	0.0571	0.1065	0.0433
lag10	0.1722	0.0532	0.1052	0.0408
lag11	0.1522	0.0286	0.0836	0.0169
lag12	0.1584	0.0385	0.0904	0.0276
lag13	0.1672	0.0480	0.0999	0.0378
lag14	0.1621	0.0391	0.0944	0.0298
lag15	0.1684	0.0461	0.1016	0.0379

TABLE 3. Estimation of ARMA models for raw inter-trade durations. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

	ARMA(1,1)		ARMA(1,2)		ARMA(2,2)		ARMA(3,3)	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
$AR1$	0.9860	0.0000	0.9873	0.0000	1.9066	0.0000	0.9114	0.0000
$AR2$					−0.9072	0.0000	−0.5307	0.0036
$AR3$							0.5975	0.0000
$MA1$	0.9180	0.0000	0.8978	0.0000	1.8289	0.0000	0.8244	0.0000
$MA2$			0.0249	0.0000	−0.8335	0.0000	−0.5248	0.0021
$MA3$							0.5691	0.0000
$Mean$	14.1803	0.0000	14.1820	0.0000	14.1861	0.0000	14.18168	0.0000
BIC	414183		414167		414158		414184	
AR and MA Roots								
$AR1$	1.0141		1.0128		1.0938		1.2852	
$AR2$					1.0076		1.2852	
$AR3$							1.0131	
$MA1$	1.0892		37.0964		1.1586		1.2738	
$MA2$			1.0813		1.0354		1.2738	
$MA3$							1.0829	

Table 4. Estimation of ARMA models for seasonal adjusted inter-trade durations. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

	ARMA(1,1)		ARMA(1,2)		ARMA(2,2)		ARMA(3,3)	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
$AR1$	0.9742	0.0000	0.9767	0.0000	1.3219	0.000	0.9251	0.0000
$AR2$					−0.3363	0.0448	−0.5914	0.0007
$AR3$							0.6261	0.0000
$MA1$	0.9084	0.0000	0.8920	0.0000	1.2368	0.0000	0.8421	0.0000
$MA2$			0.02226	0.0000	−0.2906	0.0661	−0.5835	0.0003
$MA3$							0.5948	0.0000
$Mean$	0.0104	0.9797	0.0081	0.9846	0.0106	0.9803	0.0087	0.9834
BIC	413961		413951		413958		413967	
			AR and MA Roots					
$AR1$	1.0237		1.0128		2.9077		1.2486	
$AR2$					1.0224		1.2486	
$AR3$							1.0242	
$MA1$	1.1008		40.5258		3.1712		1.2402	
$MA2$			1.0813		1.0851		1.2402	
$MA3$							1.0929	

7.2.2. *Regression Results.*

TABLE 5. Estimation of Proportional Hazard ARMA(1,2) models for grouped durations. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

	A		B		C	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Thresholds						
μ_1 $(\tau_t = 1)$	−4.0915	0.0000	−3.2266	0.0000	−2.8096	0.0000
μ_2 $(\tau_t = 5$	−3.0255	0.0000	−2.1591	0.0000	−1.7407	0.0000
μ_3 $(\tau_t = 10)$	−2.5136	0.0000	−1.6472	0.0000	−1.2282	0.0000
μ_4 $(\tau_t = 30)$	−1.7431	0.0000	−0.8769	0.0000	−0.4571	0.0001
μ_5 $(\tau_t = 60)$	−1.2342	0.0000	−0.3671	0.0016	0.0538	0.3346
μ_6 $(\tau_t = 90)$	−0.9456	0.0000	−0.0772	0.2678	0.3455	0.0030
μ_7 $(\tau_t = 120)$	−0.7634	0.0000	0.1062	0.1969	0.5308	0.0000
μ_8 $(\tau_t = 150)$	−0.6038	0.0003	0.2669	0.0160	0.6940	0.0000
μ_9 $(\tau_t = 180)$	−0.4948	0.0025	0.3766	0.0012	0.8061	0.0000
Intraday Seasonalities						
δ_1			0.2228	0.0000	1.0286	0.0000
$\delta_{s,1}$			0.1946	0.0000	0.0314	0.2759
$\delta_{s,2}$			0.0785	0.0045	0.0348	0.1387
$\delta_{s,3}$			−0.1099	0.0000	−0.0328	0.1035
$\delta_{s,4}$			0.0744	0.0009	0.0275	0.1584
$\delta_{c,1}$			−0.2183	0.0000	−0.0140	0.3886
$\delta_{c,2}$			0.1811	0.0000	0.0676	0.0169
$\delta_{c,3}$			−0.0724	0.0018	−0.0542	0.0081
$\delta_{c,4}$			0.0310	0.0988	0.0767	0.0003
14:30h and Bank Dummy						
After 14:30					−0.8264	0.0000
Bank Holiday					0.8684	0.0000
ARMA Parameters						
AR1	0.9877	0.0000	0.9815	0.0000	0.9779	0.0000
MA1	0.8879	0.0000	0.8836	0.0000	0.8815	0.0000
MA2	0.0443	0.0000	0.0411	0.0000	0.0402	0.0000
BIC and Mean Log Likelihood						
Mean Log Likelihood	−1.6229		−1.6225		−1.62093	
BIC	−72742.3563		−72721.1864		−72649.9385	

TABLE 6. Estimation of Proportional Hazard ARMA(1,2) models for grouped durations and BIC. Based on BUND futures trading at DTB, Frankfurt, from 01/30/95 to 02/24/95. 44810 observations. P-values based on asymptotic t-statistics.

	D		E	
Variable	Coeff.	p-value	Coeff.	p-value
Thresholds				
μ_1 $(\tau_t = 1)$	-2.9277	0.0000	-3.9468	0.0000
μ_2 $(\tau_t = 5$	-1.8577	0.0000	-2.8766	0.0000
μ_3 $(\tau_t = 10)$	-1.3441	0.0000	-2.3629	0.0000
μ_4 $(\tau_t = 30)$	-0.5710	0.0000	-1.5899	0.0000
μ_5 $(\tau_t = 60)$	-0.0589	0.2397	-1.0776	0.0000
μ_6 $(\tau_t = 90)$	0.2332	0.0025	-0.7850	0.0000
μ_7 $(\tau_t = 120)$	0.4188	0.0000	-0.5993	0.0000
μ_8 $(\tau_t = 150)$	0.5827	0.0000	-0.4360	0.0028
μ_9 $(\tau_t = 180)$	0.6958	0.0000	-0.3239	0.0238
Static Covariates				
lvol	0.0241	0.1647		
$lvol^2$	-0.3863	0.0009		
$lvol^3$	0.5034	0.0021		
Dynamic Covariates				
lvol lag1			-0.0126	0.000
dp lag1			-0.0214	0.0284
14:30h and Bank Dummy				
After 14:30	-0.8858	0.0000	-0.8113	0.0000
Bank Holiday	0.8530	0.0000	0.7578	0.0000
ARMA Parameters				
AR1	0.9781	0.0000	0.9729	0.0000
MA1	0.8820	0.0000	0.8770	0.0000
MA2	0.0373	0.0000	0.0383	0.0000
BIC and Mean Log Likelihood				
Mean Log Likelihood	-1.6191		-1.62011	
BIC	-72569.2805		-72613.1943	

7.3. **Figures.**

Figure 1: Intraday Seasonality Pattern

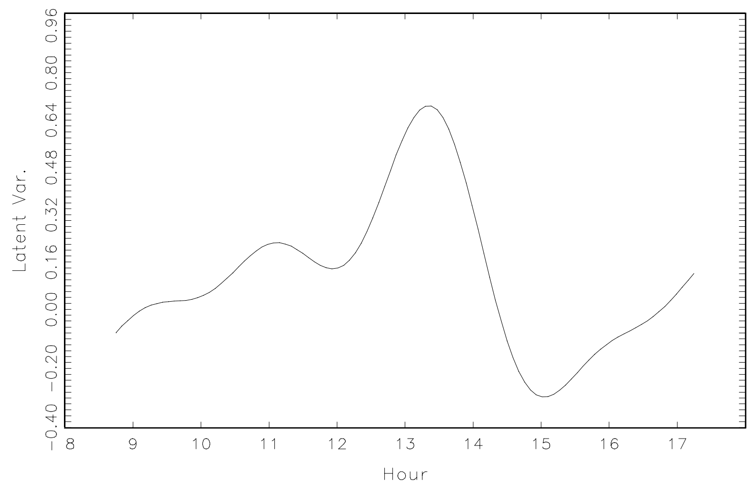

Figure 2: Survivor Function,
(above: U.S. Holiday, 10.00; middle: 10:00; below: 15:00)

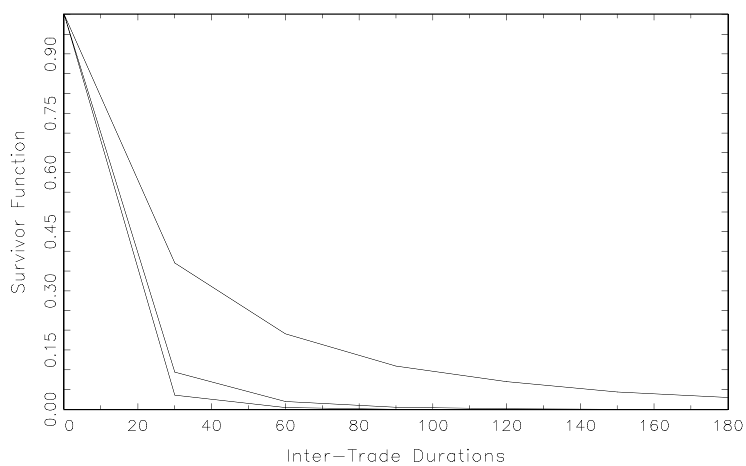

Figure 3: Conditional Failure Probabilities,
(above: 15:00; middle: 10:00; below: U.S. Holiday, 10.00)

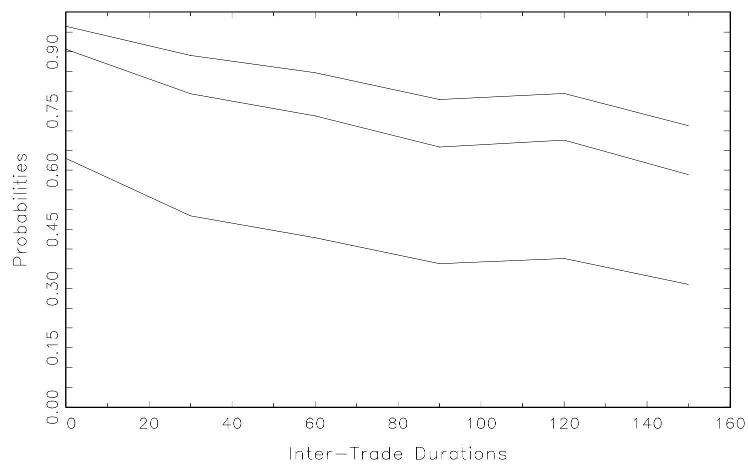

Figure 4: Latent variable vs. volume