Difusion of countries growth through specialization and trade of intermediate inputs

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Abstract

The purpose of this paper is to contribute from a theoretical point of view to analyze the in‡uence that trade between countries may have to enhance the growing possibilities of the world. We ask ourselves if it is possible to transmit from one country to another its sustained growth rate through trade. The answer that we found is that indeed it is possible when they trade in intermediate goods inputs. Our analysis identify a new element as a potential engine for one country growth, that is trading, which is not related to the total factor productivity of that country, but to some other trading partner's factor productivity. Hence we may need to consider trading relations among countries to explain the in‡uence of some countries growth rates, say the leader countries, on some others countries development, which do not experience productivity gains in their factors of production. We analyze this question in the framework of the Ventura's (1997) model.

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1 Introduction

The purpose of this paper is to contribute from a theoretical point of view to analyze the in‡uence that trade between countries may have to enhance the growing possibilities of the world. We ask ourselves if it is possible to transmit from one country to another its sustained growth rate through trade. The answer that we found is that indeed it is possible, when they trade in intermediate goods inputs. In our paper we analyze the relation between growth and trade in the simple framework of the Ventura'(1997) model. We de…ne an economy with only two countries. Both of them use nontrading factors, capital and labor, to produce respectively the two intermediate goods, which are used in the production of the same …nal non-traded good by using the same technology. The di¤erence between the two countries is that one of them has exogenous positive labor productivity in the production of one intermediate good, which impinges a positive growth rate in autarky; while the other country has zero growth rate in the autarkic steady state.

In a trade situation, since we assume that each country has comparative advantage in the production of one of the intermediate goods, each country completely specializes in the production of one of them, according to the Ricardian theory.

After solving for the steady state we obtain that in this new situation, with trade between the countries, there exist some equilibrium relative prices such that the country without growth in autarky, begin to grow at the same rate as the other country. Hence, we can see that trade can be a vehicle for the transmission of sustained growth rates from one country to another. We also prove by means of some parametric examples that both countries could achieve a higher level of consumption under the trading regime than with respect to the autarkic situation, which puts some rational in the decision process for trading. Even we can show that the less developed country, under the closed economy situation, may overtake the other, in per-capita consumption, as a consequence of trade.

Most of economic growth analysis has been con…ned to a closed economy. The neoclassical growth model which had consider how the …nding of growth will be modi…ed for an open economy, see Ramsey (1928), Solow (1956) and Swan(1956), came up to the conclusion that opening up the neoclassical model has straighforward, and to some extent rather uninteresting, implications.

Also, there is a large empirical literature on the positive relation between growth and trade, see Barro and Sala-i-Martín (1995), but always has proceeded dependently of a closed economy formal theory. Within the neoclassical framework, the sole determinant of long-run growth in income per capita is the rate at which exogenous technological breakthroughs occurs. This suggest that interaction with other countries can have no e¤ect on an economy´s long-run rate of growth. These standard points give too simple a picture how the analysis could be modi…ed for open economies. Other authors that have consider the importance of trade in a dynamic context. They have focused their analysis to answer how countries specialize in the production of goods, and the pattern of trade (see, Uzawa (1965), Stiglitz (1970), Baxter (1992), Stokey (1996)), but this literature do not address the type of question that we do. in this paper.

More recently, in an important paper, Ventura (1997) has shown that recognizing interdependence among countries should change the convergence literature of growth in more fundamental ways. Ventura has emphasized the implication of combining a weak form of the factor price-equalitzation theorem with the Ramsey model. For trading economies, under the assumption of factor price equalization, the law of diminishing returns applies only to world averages, which implies that diminishing returns does not have to be associated with conditional convergence. Hence, interdependence become crucial for explaining the growth experiences of di¤erent countries.

In the existent literature we can also …nd some work about how the growth of one country di¤uses to others. Mostly it is through the transmission of technology what have been analyzed. The idea that products are …rst produced in the developed economy, after which their production is relocated to the less developed economy , was the basis for Vernon's (1966) product-cycle theory of international trade. This was …rst formalized by Krugman (1979) in a model which the rate of innovation and imitation are exogenous, and then subsequently extended in a Schumpeterian context by Grossman and Helpman(1990; 1991a) , and by Segerstrom (1991).From the empirical perspective it is interesting to consider the work by Coe and Helpman(1995), keller(1998) and Bayoumi et al. (1999). But in our case, it is the simple trade of intermediate goods that transfer the growth rate of one country to another, neather the imitation nor the innovation process.

The paper is organized as follows: Section 2 develops the model and we o¤er some general results. Section 3 is devoted to a parametric example to obtain some interesting particular results. Finally, section 4 is devoted to the conclusions.An appendix concludes.

2 The Model

2.1 The Environment

Our model is a continuous time world economy with only two countries; $j = 1,2$. The countries´population grow at the same rate n, and the population at each time, $l_j(t)$; can be identify with the labor resources existing in the economy. There is one …nal good, $y_j(t)$; not tradable, and two potentially tradable intermediate goods, x^i_{kj}; $k = 1;2$; where $x^i_{kj}(t) \in \Re_{++}$ is the amount of intermediate good k produced by the representative …rm of country $i (i = 1,2)$, and used as an input to produce the …nal good at date t in country j. Each country has many competitive firms producing the same final good with the same production function $y_j = F(x_{1j}; x_{2j})$, that combine the two intermediate goods as inputs, and can be produced by both countries. We assume that the production function is continuously differentiable, has constant return to scale, satisfies the Inada conditions, and exhibit positive and diminishing marginal product with respect each input. Each country has a different technology to produce intermediate goods by using labor, $l_j(t)$;and capital, $k_j(t)$. In country 1, one effective unit of labor produces one unit of intermediate good 1 according to $x_1^1 = A_1^1 e^{\gamma t} l^1$, where $A_1^1 \in \Re_{++}$ is a measure of labor productivity and γ is the labor productivity growth rate, $\gamma > 0$: The intermediate good 2 is produced by using only capital according to $x_2^1 = A_2 k^1$. In country 2; the intermediate good technologies are given by $x_1^2 = A_1^2 l^2$; and $x_{22}^2 = A_3 k^2$; where $A_1^2 \in \Re_{++}$ is a measure of labor productivity in country 2, and $A_3 \in \Re_{++}$ is a measure of capital productivity in country 2.[1] That is:

$$\text{Country } 1: \ x_1^1 = A_1^1 e^{\gamma t} l^1; \ x_2^1 = A_2 k^1:$$

$$\text{Country } 2: \ x_1^2 = A_1^2 l^2; \ x_2^2 = A_3 k^2:$$

Where k_j^1 is the total amount of capital used in country 1 to produce the intermediate good used as an input by country j; and l_j^1 is the fraction of the total amount of labor used in country 1 to produce the intermediate good used as an input by country j. That is:

$$l_1^1 + l_2^1 = 1; \ k_1^1 + k_2^1 = k^1:$$

$$l_1^2 + l_2^2 = 1; \ k_1^2 + k_2^2 = k^2:$$

Each country has the same initial endowments: $k^i(0) = \bar{k} > 0$; $l^i(0) = \bar{l} > 0$, and a representative consumer with an instantaneous CES utility function:

$$U(c_j) = \int_0^{\infty} e^{-(\rho_j - n)t} \frac{c_j^{1-\theta}}{1-\theta} dt; \quad \theta > 0$$

with $\rho_j > n > 0$, where ρ_j is the rate of impatience for country j, and $c_j(t) \in \Re_{++}$ is the per capita consumption rate of the consumer of country j at date t.

From the previous description of the economy, we see that there are the following competitive markets: the factor markets, which include capital and labor; the two intermediate goods markets, and the final good market. WeWe assume that capital does not depreciate and international factor movements are not permitted.

[1] The specification of the previous production functions to obtain the intermediate goods, is chosen because it drastically simplifies the mathematics of the paper. Ventura (1997) interprets this technology as a limiting case of a more general description of a technology in which each intermediate industry uses both labor and capital as inputs, and shows that the main results of his paper also holds for the generalized model. In our case, we can also extend the analysis in the same way.

2.2 The Autarky Situation

In the autarky situation,we assume that there is no trade between the two countries and perfect competition prevails in each market of each country. From the standard theory we will find that the growth rate in each country in an stationary equilibrium will be given by the exogenous rate of growth of the labor productivity. Since in country 1 the labor productivity grows at rate $\gamma > 0$; and for country 2 we assume no productivity growth at all. Hence we find that in an stationary state the main endogenous variables of each country will be growing at its own exogenous factor productivity growth rate.

2.3 The Trading Situation

In a trading situation, we now assume that each country can only trade in the intermediate goods market. Since the intermediate good 1 produced in country 1 can also be used as an input in the production of the final good in country 2, and the same for the other country and the other intermediate good.

We solve for the competitive equilibrium of the world economy through the corresponding planner problem for each country.

A competitive equilibrium is a sequences of per capita consumptions plans: $\overline{c_j}(t)$;per capita capital stock: $\overline{k}^j(t)$ and labor: $\overline{l}^j(t)$; intermediate per capita goods:$\overline{x}^1_{j1}(t), \overline{x}^1_{j2}(t), \overline{x}^2_{j1}(t), \overline{x}^2_{j2}(t)$; final per capita good $\overline{y}_j$ and a sequences of prices: $\overline{p}_1(t), \overline{p}_2(t), \overline{p}_3(t), \overline{p}_4(t)$ such that:

i)-Given prices and the demand for intermediate good exports: $\overline{x}^1_{12}, \overline{x}^1_{22}$, we have that: $\overline{x}^1_{11}, \overline{x}^2_{11}, \overline{x}^1_{21}, \overline{x}^2_{21}, \overline{c}_1, \overline{k}^1, \overline{l}^1$ solves the problem for the representative consumer in country 1:

$$\text{Max} \int_0^{\infty} e^{-(\rho_1 - n)t} \frac{c_1^{1-\theta}}{1-\theta} dt$$

$$c_1 + \dot{k}_1 + nk_1 \le F(x_{11}, x_{21}) + \overline{p}_1 \overline{x}^1_{12} + \overline{p}_2 \overline{x}^1_{22} - \overline{p}_3 x^2_{11} - \overline{p}_4 x^2_{21}$$

$$x_{11} \le g_1(x^1_{11}, x^2_{11})$$

$$x_{21} \le g_2(x^1_{21}, x^2_{21})$$

$$x^1_{11} + \overline{x}^1_{12} \le A^1_1 e^{\gamma t} l^1$$

$$x^1_{21} + \overline{x}^1_{22} \le A_2 k^1$$

$$x^1_{11}, x^2_{11}, x^1_{21}, x^1_{21}, c_1, k^1, l^1 \ge 0$$

where $\overline{p}_1 \overline{x}^1_{12} + \overline{p}_2 \overline{x}^1_{22} - \overline{p}_3 x^2_{11} - \overline{p}_4 x^2_{21}$ is the trade balance of country 1.

ii) Given prices and the demand for exports $\overline{x}^2_{11}, \overline{x}^2_{21}$; we have that: $\overline{x}^1_{12}, \overline{x}^2_{12}, \overline{x}^1_{22}, \overline{x}^1_{22}, \overline{c}_2, \overline{k}^2, \overline{l}^2$ solves the problem for the representative consumer in country 2, analogous that we do for country 1.

2.4 Result on Specialization

Here we show that countries specializes in the production of one intermediate input due to the fact that we assume that each country has a different technology to produce those intermediate products.

Assume that trade balance is in equilibrium, and there exist perfect substitution between the products produced in both countries; this means in particular that in the previous maximization problem we assume: $g_1(x^1_{11}; x^2_{11}) = x^1_{11} + x^2_{11}$ and $g_2(x^1_{21}; x^2_{21}) = x^1_{21} + x^2_{21}$. Furthermore we specialize the production function $F(x_{11}; x_{21}) = (x_{11})^{\alpha}(x_{21})^{1-\alpha}$; and similarly for country two.

After substituting the last four constraints of the problem of country one into the first one we have the Hamiltonian function for the problem of country 1:

$$
\begin{aligned}
H \;=\; & e^{-(\rho_1)t}\frac{c_1^{1-\theta}}{1-\theta} + \lambda[(x^1_{11} + x^2_{11})^{\alpha}(x^1_{21} + x^2_{21})^{1-\alpha} + \\
& + p_1(A^1_1 e^{\gamma t} l^1 - x^1_{11}) + p_2(A_2 k^1 - x^1_{21}) - \\
& - p_3 x^2_{11} - p_4 x^2_{21} - c_1 - nk^1]
\end{aligned}
$$

The first order neccesary and sufficient Conditions are:

$$\frac{\partial H}{\partial c_1} = 0 \;\rightarrow\; \frac{1}{c_1^{\theta}} e^{-(\rho_1 - n)t} - \lambda = 0 \tag{1}$$

$$x^1_{11} \cdot \frac{\partial H}{\partial x^1_{11}} = 0 \;\rightarrow \tag{2}$$

$$\lambda[\alpha(x^1_{11} + x^2_{11})^{\alpha-1}(x^1_{21} + x^2_{21})^{1-\alpha} - p_1] \le 0;\, x^1_{11} \ge 0$$

$$x^2_{11} \cdot \frac{\partial H}{\partial x^2_{11}} = 0 \;\rightarrow \tag{3}$$

$$\lambda[\alpha(x^1_{11} + x^2_{11})^{\alpha-1}(x^1_{21} + x^2_{21})^{1-\alpha} - p_3] \le 0;\, x^2_{11} \ge 0$$

$$x^1_{21} \cdot \frac{\partial H}{\partial x^1_{21}} = 0 \;\rightarrow \tag{4}$$

$$\lambda[(1-\alpha)(x^1_{11} + x^2_{11})^{\alpha}(x^1_{21} + x^2_{21})^{-\alpha} - p_2] \le 0;\, x^1_{21} \ge 0$$

$$x^2_{21} \cdot \frac{\partial H}{\partial x^2_{21}} = 0 \;\rightarrow \tag{5}$$

$$\lambda[(1-\alpha)(x^1_{11} + x^2_{11})^{\alpha}(x^1_{21} + x^2_{21})^{-\alpha} - p_4] \le 0;\, x^2_{21} \ge 0$$

$$
\begin{aligned}
c_1 + \dot{k}_1 + nk_1 \;\le\; & (x^1_{11} + x^2_{11})^{\alpha}(x^1_{21} + x^2_{21})^{1-\alpha} + \\
& + p_1(A^1_1 e^{\gamma t} l^1 - x^1_{11}) \\
& + p_2(A_2 k^1 - x^1_{21}) - p_3 x^2_{11} - p_4 x^2_{21}
\end{aligned}
\tag{6}
$$

$$\dot{\lambda} = \lambda(n - p_2 A_2) \tag{7}$$

$$\lim_{t \to \infty} \lambda_t k_t = 0 \quad (8)$$

Suppose that conditions (1-8) holds with equality, then in the steady state equilibrium we find:

$$p_1 = p_3 \text{ and } p_2 = p_4;$$

also:

$$p_2 = \frac{\mathbf{b} + n + \rho_1}{A_2} \text{ in country 1,}$$

$$p_2 = \frac{\gamma^* + n + \rho_2}{A_3} \text{ in country 2,}$$

and we known that p_2 need to be equal for both countries, so:

$$\frac{\mathbf{b} + n + \rho_1}{A_2} = \frac{\gamma^* + n + \rho_2}{A_3} \quad (9)$$

but at the same time we find from the FOC, that:

$$k^1 = \frac{A_1^1 e^{\gamma t} l^1}{[(n + \mathbf{b}) + \frac{c_1}{k^1}]\frac{\alpha}{(1-\alpha)p_1} + (n + \mathbf{b}) + \frac{c_1}{k^1} - \text{-} A_2}$$

$$k^2 = \frac{A_1^2 l^2}{[(n + \gamma^*) + \frac{c_2}{k^2}]\frac{\alpha}{(1-\alpha)p_1} + (n + \gamma^*) + \frac{c_2}{k^2} - \text{-}^* A_3}:$$

In the steady state $\frac{\dot{k}_1}{k_1} = \gamma$; and $\frac{\dot{k}_2}{k_2} = 0$; then $\mathbf{b} = \gamma$, $\gamma^* = 0$; so if we substitute in (9) we have a contradiction.

So we conclude that:

$$x_1^1 > 0; x_2^1 = 0;$$
$$x_2^2 > 0; x_1^2 = 0;$$

and so specialization takes place.

2.5 Results on Growth

Once the specialization takes place in each country in the production of the intermediate good, we just solve the problem for the steady state equilibrium.

After solving for the equilibrium in an stationary state, were $\frac{\dot{c}_2}{c_2} = \frac{\dot{k}_2}{k_2} = \mathbf{b}^*$, we obtain the unique value of $\overline{p}_2 = \frac{\theta \mathbf{b}^* + \rho_2}{A_3}$ which is the world interest rate; also from the first order necessary conditions we have $\overline{p}_1$; then we know the relative price $\frac{\overline{p}_1}{\overline{p}_2} = \overline{p}$: Since F_2' is homogeneous of degree 0 and $F_2'' < 0$, we obtain the value of $\frac{A_3 k_2^2}{A_1 e^{\gamma t} l_1^2} \equiv g(\overline{p_2})$. Then we can get the rest of the variables in the steady state:

$$\bar{l}_1^2 = \frac{g(\overline{p_2})}{\bar{p} + g(\overline{p_2})};\ \bar{l}_1^1 = 1 - \bar{l}_1^2.$$

$$\bar{k}_2 = \frac{A_1 \cdot g(\overline{p_2})}{A_3} e^{\gamma t};\ \bar{k}_2^1 = \frac{A_1}{A_3}\frac{g(\overline{p_2}) \cdot \bar{p}}{\bar{p} + g(\overline{p_2})} e^{\gamma t};\ \bar{k}_2^2 = \bar{k}_2 - \bar{k}_2^1.$$

$$\bar{c}_1 = F(A_1 e^{\gamma t} \bar{l}_1^1; A_3 \bar{k}_2^1).$$

$$\bar{c}_2 = F_2(A_1 e^{\gamma t} \bar{l}_{12}; A_3 \bar{k}_{22}) - \dot{k}_2 - n\bar{k}_2.$$

Hence, the level of all variables depends on the parameters: $\theta; A_3; \rho_2; \delta^*$. At the same time, as we see, it follows from the previous expressions $\gamma = \delta^*$; then we can conclude that both countries will grow at the same rate $\gamma > 0$ in the steady state.

3 An Example

Using the same Cobb-Douglas production function for each country, $y_j = (x_{1j}^{\alpha} \cdot x_{2j}^{1-\alpha})$; $0 < \alpha < 1$, and assuming the trade balance is in equilibrium ($VExp_j = 0$);. then solving the equilibrium for each country we will have:

$$\bar{p}_2 = \frac{\theta\delta^* + \rho_2}{A_3};$$

$$\bar{p}_1 = \alpha\left[\frac{A_3(1-\alpha)}{\rho_2 + \theta\delta^*}\right]^{\frac{1-\alpha}{\alpha}}.$$

$$\bar{l}_1^1 = \alpha;$$

$$\bar{l}_1^2 = 1 - \alpha.$$

$$\bar{k}_2^1 = \left[\frac{A_3(1-\alpha)}{\rho_2 + \theta\delta^*}\right]^{1/\alpha} \frac{A_1 e^{\gamma t}\alpha}{A_3};$$

$$\bar{k}_2^2 = \left[\frac{A_3(1-\alpha)}{\rho_2 + \theta\delta^*}\right]^{1/\alpha} \frac{A_1 e^{\gamma t}(1-\alpha)}{A_3}.$$

$$\bar{c}_1^T = \alpha A_1 e^{\gamma t}\left[\frac{A_3(1-\alpha)}{\rho_2 + \theta\delta^*}\right]^{\frac{1-\alpha}{\alpha}}.$$

$$\bar{c}_2^T = (\rho_2 - n + \delta^*(\theta - 1))\left[\frac{A_3(1-\alpha)}{\rho_2 + \theta\delta^*}\right]^{1/\alpha} \frac{A_1 e^{\gamma t}}{A_3}.$$

From the previous expressions,we can compare the per capita consumption levels in country 1 in an autarkic situation and in a free trade situation: We obtain $\bar{c}_1^T > \bar{c}_1^A$ if

$$A_3^{\frac{1}{\alpha}} \geq \frac{[\rho_1 + \delta^*(1-\theta) + \alpha(n + \delta^*) - n]\rho_2^{\frac{1-\alpha}{\alpha}}}{\alpha \cdot \rho_1^{1/\alpha} \cdot \theta \cdot \delta^*} > 1.$$

But this condition is always satisfy by assumption on A_3; which is the one allow us to say that country 2 specializes in the production of intermediate good 2.

Also we can compare the per capita consumption levels between countries, and we will have $\bar{c}_2^T > \bar{c}_1^T$ if:

$$\alpha \leq \frac{\mathbf{b}^*(\theta - 1) + \rho_2 - n}{2\rho_2 + \theta \mathbf{b}^* - n} < 1.$$

Hence, the consumption level of country 2 could be higher than the consumption level of country 1 if α is sufficiently small. That means that the output elasticity of the intermediate good 1 should be small. This is an important result because it shows that the less developed country,before trade take place, may overtake the other growing country, in per-capita consumption, as a consequence of trade.

4 Conclusions

In this model we consider a simple two country model within the framework of the Ventura´s (1997) paper. We analyze the role that trade in intermediate products plays in the transmission of exogenous growth rates from one country to another.

We conclude that, under some technological assumptions in the production function of the intermediate goods,that leads to specialization (as the Ricardian doctrine would suggest) in the production of those intermediate inputs, trade of such products can in fact set up a positive sustained growth in one country which would be otherwise impossible under an autarkic situation.

One of the implications of this kind of result is that convergence results in countries growth rates need to be revised as the Ventura work would also suggest... Hence, we want to point out that some further research need to be done in the empirical arena in order to see the implications for growth of trade relationships between countries. If we ignore this interdependence, the results that we may have will not be a good picture of the real engine of a country wealth.

It would be also interesting to compare our result with those that may come from the analysis of several different ways to integrate both countries. Some transitional dynamics analysis need also to be done as an additional result to the ones we present in this paper.

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6 APPENDIX

The corresponding planner problem for country 1:

$$Max \int_0^{1} \frac{c_1^{1-\theta}}{1-\theta} \cdot e^{-(\rho_1 - n)t} dt$$

$$c_1 + \dot{k}_1 + nk_1 \leq F(x_{11}^1, x_{22}^1) + p_1 x_{11}^2 - p_2 x_{22}^1$$

$$x_{11}^1 \leq A_1 e^{\gamma t} l_1^1$$

$$x_{11}^2 \leq A_1 e^{\gamma t} l_1^2$$

$$x_{22}^1 \leq A_3 k_2^1$$

$$l_1^1 + l_1^2 \leq 1$$

$$x_{11}^1, x_{12}^1, c_1, k_2^1, l_1^1 \geq 0$$

The Hamiltonian function for this problem after substituting the restrictions of the intermediate goods in the resource constrain, will be:

$$\begin{aligned} H &= \frac{c_1^{1-\theta}}{1-\theta} \cdot e^{-(\rho_1 - n)t} + \lambda[F(A_1 e^{\gamma t} l_1^1, A_3 k_2^1) - c_1 + \\ &\quad + p_1 A_1 e^{\gamma t}(1 - l_1^1) - p_2 A_3 k_2^1] \end{aligned}$$

the first order necessary, and sufficient, conditions for any interior solution to this problem are:

$$\frac{\partial H}{\partial c_1} = 0 \Leftrightarrow \frac{1}{c_1^{\theta}} \cdot e^{-(\rho_1 - n)t} - \lambda = 0 \tag{0}$$

$$\frac{\partial H}{\partial (A_1 e^{\gamma t} l_1^1)} = 0 \Leftrightarrow \lambda[F_1'(A_1 e^{\gamma t} l_1^1, A_3 k_2^1) - p_1] = 0 \tag{1}$$

$$\frac{\partial H}{\partial (A_3 k_2^1)} = 0 \Leftrightarrow \lambda[F_2'(A_1 e^{\gamma t} l_1^1, A_3 k_2^1) - p_2] = 0 \tag{2}$$

$$c_1 = F(A_1 e^{\gamma t} l_1^1, A_3 k_2^1) - p_2 k_2^1 + p_1(A_1 e^{\gamma t}(1 - l_1^1)) \tag{3}$$

Where, F_k' denotes the partial derivative of the function with respect to the k variable. From (2) and (3) we obtain that the relative prices are equal to the marginal productivity of factors:

$$\frac{p_1}{p_2} = \frac{F_1'(A_1 e^{\gamma t} l_1^1, A_3 k_2^1)}{F_2'(A_1 e^{\gamma t} l_1^1, A_3 k_2^1)} \tag{4}$$

The planner problem for country 2 is:

$$Max \int_0^{1} \frac{c_2^{1-\theta}}{1-\theta} \cdot e^{-(\rho_2 - n)t} dt$$

$$c_2 + \dot{k}_2 + nk_2 \le F(x_{11}^2; x_{22}^2) + p_2A_3(k_2 - k_2^2) - p_1A_1e^{\gamma t}l_1^2$$
$$x_{11}^2 \le A_1e^{\gamma t}l_1^2$$
$$x_{22}^1 \le A_3k_2^1$$
$$x_{22}^2 \le A_3k_2^2$$
$$k_2 \le k_2^1 + k_2^2$$
$$x_{11}^2; x_{22}^2; c_2; k_2^2; k_2^1 \ge 0$$

By substituting the restrictions of the intermediate goods in the resource constrain, we obtain the Hamiltonian function of this problem:

$$H = \frac{c_2^{1-\theta}}{1-\theta} \cdot e^{-(\rho_2 - n)t} + \lambda[F(A_1e^{\gamma t}l_1^2; A_3k_2^2) - c_2 - nk_2 - p_1A_1e^{\gamma t}l_1^2 + p_2A_3(k_2 - k_2^2)]$$

and the first order necessary, and sufficient conditions for any interior solution are:

$$\frac{\partial H}{\partial c_2} = 0 \;,\; \frac{1}{c_2^{\theta}} \cdot e^{-(\rho_2 - n)t} - \lambda = 0 \quad (6)$$

$$\frac{\partial H}{\partial (A_1e^{\gamma t}l_1^2)} = 0 \;,\; \lambda[F_1'(A_1e^{\gamma t}l_1^2; A_3k_2^2) - p_1] = 0 \quad (7)$$

$$\frac{\partial H}{\partial (A_3k_2^2)} = 0 \;,\; \lambda[F_2'(A_1e^{\gamma t}l_1^2; A_3k_2^2) - p_2] = 0 \quad (8)$$

$$\frac{\partial H}{\partial k_2} = -\dot{\lambda} \;,\; \lambda(-n + A_3p_2) = -\dot{\lambda} \quad (9)$$

$$c_2 + \dot{k}_2 + nk_2 = F_2(A_1e^{\gamma t}l_{12}; A_3k_{22}) + p_2(k_2 - k_2^2) - p_1A_1e^{\gamma t}l_1^2 \quad (10)$$

then we have from the previous expressions (7) and (8), the relative prices for country 2:

$$\frac{p_1}{p_2} = \frac{F_1'(A_1e^{\gamma t}l_1^2; A_3k_2^2)}{F_2'(A_1e^{\gamma t}l_1^2; A_3k_2^2)} \quad (11)$$

By definition, if the steady state exist, for some $\psi^* \ge 0$; we must have:

$$\frac{\dot{c}_2}{c_2} = \frac{\dot{k}_2}{k_2} = \psi^*.$$

From equation (6) and previous expression we have:

$$\overline{p}_2 = \frac{\theta\psi^* + \rho_2}{A_3}. \quad (12)$$

By substituting (12) in (8) we obtain:

$$F_2'(A_1e^{\gamma t}l_1^2; A_3k_2^2) = \frac{\theta\psi^* + \rho_2}{A_3}. \quad (13)$$

Hence, since $F_2^{\prime}$ is homogeneous of degree 0 and $F_2^{\prime\prime} < 0$, we know the value of $\frac{A_3 k_2^2}{A_1 e^{\gamma t} l_1^2}$;say $g(\overline{p_2})$ such that:

$$g(\overline{p_2}) \equiv \frac{A_3 k_2^2}{A_1 e^{\gamma t} l_1^2}$$

which substituted in the expression(7) yields the value of $\overline{p}_1$ in the steady state. We can see that independently of the trade balance, the relative prices in equilibrium are unique, say $\overline{p}$. Since those equilibrium prices are the same for both countries, and by assumption the production functions for the final good are the same for both countries, and from the previous expressions (0), (1), (7) and (8) we obtain:

$$\frac{A_3 k_2^2}{A_1 e^{\gamma t} l_1^2} = \frac{A_3 k_2^1}{A_1 e^{\gamma t} l_1^1}$$

or

$$\frac{k_2^2}{k_2^1} = \frac{l_1^2}{l_1^1}. \qquad (14)$$

If we assume that the trade balance is in equilibrium we have:

$$p_1 A_1 e^{\gamma t} l_1^2 = p_2 A_3 k_2^1$$

then we obtain, substituting the equilibrium relative prices:

$$\overline{p} = \frac{A_3 k_2^1}{A_1 e^{\gamma t} l_1^2} \qquad (15)$$

Using expressions (14), (15) and $g(\overline{p_2})$;we find:

$$l_1^2 = \frac{g(\overline{p_2})}{\overline{p} + g(\overline{p_2})}$$

$$l_1^1 = 1 - \frac{g(\overline{p_2})}{\overline{p} + g(\overline{p_2})}$$

$$k_2 = \frac{A_1 e^{\gamma t} \cdot g(\overline{p_2})}{A_3}$$

$$k_2^1 = \frac{A_1}{A_3} e^{\gamma t} \frac{g(\overline{p_2}) \cdot \overline{p}}{\overline{p} + g(\overline{p_2})}$$

$$k_2^2 = A_1 e^{\gamma t} \cdot g(\overline{p_2}) [\frac{1}{A_3} - \frac{\overline{p}}{A_3 \overline{p} + g(\overline{p_2})}]$$

$$c_1 = F(A_1 e^{\gamma t} l_1^1; A_3 k_2^1)$$

$$c_2 = F_2(A_1 e^{\gamma t} l_{12}; A_3 k_{22}) - \dot{k}_2 - n k_2$$

The level of all variables depends of $\bar{p}_2$;we know that

$$\bar{p}_2 = \frac{\mu °© + ½_2}{A_3};$$

since all variables depends of the parameters: μ; A_3; $½_2$; °©:At the same time, we can conclude that both countries will grow at the same rate °