1. Introduction

The classical issue in traditional oligopoly theory dealing with the appropriateness of the equilibrium concept, Cournot-Nash or Stackelberg, in various imperfectly competitive settings has enjoyed a major revival over the last decade. Through the infusion of modern game theory, it is now widely recognized that the Stackelberg solution concept is not well-defined for one-shot games. Rather, it corresponds precisely to the notion of subgame-perfect (Nash) equilibrium of a two-stage game with sequential moves, perfect information and exogenously defined first and second movers: See [Friedman (1977), p. 78-84] for a detailed discussion of this and related points.

This revival consists of two main strands of research that are somewhat related and sometimes overlap. The first deals with the issue of endogenous timing. Its guiding premise is that in duopoly models, the determination of simultaneity versus sequentiality of moves, as well as of the assignment of roles to the players in the latter case, should be completely endogenous. In other words, t he order of play in a given two-player game ought to reflect the players' own intrinsic incentives, in the absence of any natural exogenously-determined timing structure. Studies pursuing this view include among many others: Dowrick (1986), Boyer and Moreaux (1987), Robson (1990), Anderson and Engers (1994), Amir and Grilo (1999), and van Damme and Hurkens (1999).

The second strand of research deals with the determination of first- and second-mover advantages in given subclasses of the general class of duopoly games characterized by monotone best-responses (upward or downward-sloping), and monotone profits in rival's actions. In other words, this strand compares the equilibrium payoffs of the two firms in the two sequential games (of perfect information) obtained by considering both orders of moves. This strand comprises, among others, Gal-Or (1985), Mailath (1993), Daughety and Reinganum

(1994), and Reinganum (1985).

The present paper belongs mostly to the latter strand, and deals with duopoly price competition with differentiated products and constant marginal costs. It is widely believed that price competition is typically characterized by a second-mover advantage, in the sense that a firm's most preferred situation is to have its rival commit to a price, and then optimally react by appropriately undercutting the observed rival's price. As Bertrand's classical critique of Cournot's work shows, this intuition certainly holds in an extreme form in the case of homogeneous products, owing to the totally discontinuous nature of each firm's demand along the price diagonal. This intuition has also been proven right in the differentiated-products case when firms are identical: Gal-Or (1985) and Dowrick (1986).

The present paper has three main objectives. First, we generalize the well-known results in the literature by removing the common (and sometimes tacit) assumptions of concavity of profits in own action (or continuity and single-valuedness of the reaction curve), and of uniqueness of the Bertrand-Nash equilibria. To do so, we invoke the recent results of supermodular optimization/ games.

Second, we clarify the crucial role played by the strategic complementarity or substitutability of prices in determining timing advantage with asymmetric firms. To this end, we consider all three possible cases making assumptions on primitives leading to each case. We prove that when both optimal reactions slope upwards, at least one firm has a second-mover advantage. When both optimal reactions slope downwards, both firms have a first-mover advantage. Finally, in the mixed case, the firm with a downward-sloping reaction always has a first-mover advantage.

Last, we further investigate the scope of the second-mover advantage under strategic complementarity of prices, asymmetric firms and differentiated products. We show that under the latter three assumptions, the second-mover advantage property fails even when demand is linear and symmetric and unit costs are constant and unequal. More precisely, we show that for some parameter values, the low-cost firm can have a first-mover advantage. On the other hand, the high-cost firm always has a second-mover advantage in this case, thus partly confirming the conventional intuition.

The paper is organized as follows. Section 2 describes the model, the equilibrium concepts, and the results. Section 3 has all the proofs. Finally, an appendix, with a brief and simple outline of the lattice-theoretic notions and results used here, is given.

2. Model and Results

Consider the standard model of duopolistic price competition with differentiated goods. Firm i charges price p_i, faces demand $D_i(p_{1,}p_2)$ and is assumed to have linear production costs with marginal cost c_i, $i = 1, 2$. The profit of firm i is then given by

$$\Pi_i(p_1, p_2) = (p_i - c_i)D_i(p_1, p_2). \tag{1}$$

We consider three different games of price competition that are distinguished only by their timing structure: a simultaneous-move game G and two games with sequential moves and perfect information, G_1 and G_2.

In the simultaneous-move game G, firms act simultaneously. So, a pure strategy for firm i in G is to choose an element of its price set P_i , which is a compact real interval. In game G_i, firm i (the leader) moves first, choosing a pure strategy $p_i \in P_i$, and the other firm (the follower) moves after observing the price of the rival, choosing its pure strategy $\gamma(p_i)$, where $\gamma(p_i)$ is a mapping from P_i to P_j, $j \neq i$.

For each of these games, let's define the associated equilibrium concept. A pair (p_1^*, p_2^*)

constitute a Nash (or Bertrand) equilibrium in game G if

$$\Pi_1(p_1^*, p_2^*) \geq \Pi_1(p_1, p_2^*), \text{ for all } p_1 \in P_1, \text{ and}$$

$$\Pi_2(p_1^*, p_2^*) \geq \Pi_2(p_1^*, p_2), \text{ for all } p_2 \in P_2.$$

For games $G_1$and G_2, the equilibrium concept is subgame-perfect equilibrium or SPE (also known as Stackelberg equilibrium in traditional oligopoly theory), which is defined as follows, say for game G_1. A pair $(p_1^*,\ \gamma^*(\cdot))$ is a SPE equilibrium for game G_1 if

$$\Pi_2(p_1, \gamma^*(p_1)) \geq \Pi_2(p_1, p_2), \text{ for all } p_1 \in P_1 \text{and } p_2 \in P_2, \text{ and}$$

$$\Pi_1(p_1^*, \gamma^*(p_1^*)) \geq \Pi_1(p_1, \gamma^*(p_1)), \text{ for all } p_1 \in P_1.$$

In other words, a SPE imposes the following restrictions on players' behavior:

(i) the second-mover must be using as strategy a (single-valued) selection from his best-response correspondence, defined as usual as

$$r_2(p_1) \triangleq \arg\max \left\{ \Pi_2(p_1, p_2) : p_2 \in P_2 \right\},$$

and

(ii) the first-mover must choose a price that maximizes his payoff given the anticipation of a rational reaction (according to the strategy $\gamma^*(\cdot)$) by the rival.

Thus, a SPE for the sequential game of perfect information G_1 formalizes in precise game-theoretic terms the classical concept of Stackelberg equilibrium for the duopoly price game. Subgame perfection requires that the second-mover react optimally (here, according to the strategy $\gamma^*(\cdot)$) for *any price* that the first-mover might choose, optimal or not.

The following standard assumption is made throughout the paper:

(A1) The demand function $D_i(p_1, p_2)$ is assumed to be twice continuously differentiable, and to satisfy

$$\frac{\partial\ D_i(\ p_1, p_2)}{\partial\ p_i} < 0 \text{ and } \frac{\partial\ D_i(\ p_1, p_2)}{\partial\ p_j} > 0 \text{ for all } (p_1, p_2) \in P_1 \times P_2.$$

The first inequality says that demand for good i is downward sloping in its own price and the second that goods are substitutes, i.e., the demand for a good increases with the price of the competitor's good.

A well known result is that in case of a symmetric duopoly game, there is a second-mover (first-mover) advantage for both players when each profit function is strictly concave in own action and strictly increasing (decreasing) in rival's action, and reaction curves are upward (downward) sloping: See Gal-Or (1985). Dowrick (1986) extends this result to asymmetric duopoly under more general assumptions.

Before formulating our results, let us extend the definition of the notions of "first- and second-mover advantage" to asymmetric games. We say that firm i has a first (second) mover advantage if its equilibrium payoff in $G_i(G_j)$ is higher than in $G_j(G_i)$. While game G may have multiple Bertrand-Nash equilibria, the games $G_1$and G_2 will (essentially) not have multiple SPEs, as we now argue. Multiple SPEs for (say) game G_1 can arise in two different ways. The first is that, given the follower's strategy γ^*, the leader's payoff $\Pi_1(p_1, \gamma^*(p_1))$ may have more than one argmax'. While possible, this situation is generically removable, in that the smallest perturbation of any of the game parameters or primitives will result in a unique argmax. The second source of nonuniqueness of SPEs is that at the leader's optimal choice p_1^*, $r_2(\cdot)$ is multi-valued. In this case, Amir, Grilo and Jin (1999) show that there is a unique SPE, which is $(p_1^*, \overline{r}_2(\cdot))$, where $\overline{r}_2$ is the maximal selection of r_2, due to the fact that the two goods are substitutes. In view of these arguments, we will henceforth assume uniqueness of the SPE of games $G_1$and G_2, but not of the game G.

Our first proposition requires the following assumption on the demand functions:

(A2) $D_i(p_1, p_2)$ is strictly log-supermodular on $P_1 \times P_2$.

This assumption implies that

$$D_i(p_1, p_2)\,\frac{\partial^2 D_i(\,p_1,\,p_2)}{\partial p_1 \partial p_2} - \frac{\partial D_i(\,p_1,\,p_2)}{\partial p_i}\frac{\partial D_i(\,p_1,\,p_2)}{\partial p_j} \geq 0$$

and, conversely, the strict version of this inequality implies (A2). The main implication of (A2) is that it leads to reaction correspondences that are nondecreasing (in the sense that each selection is nondecreasing).

Proposition 1 *Under assumptions* $(A_1), (A_2)$*, at least one of the firms has a second-mover advantage.*

This result is closely related to a result of Dowrick (1986). We have a different sufficient condition for increasing best-responses, and our proof makes it clear that the continuity or the single-valuedness of the optimal reactions, as well as the uniqueness of the Bertrand equilibrium, are not needed for the result to hold.

Another result relating the same three games has appeared in the literature under some extra assumptions (Gal-Or, 1985, Dowrick, 1986) It says that each firm prefers games G_1 and G_2 to game G. In other words, each firm prefers to be a Stackelberg player (whether leader or follower) to playing simultaneously. Amir, Grilo and Jin (1999) prove this result precisely under our assumptions here.

The next result deals with the case where the reaction correspondences of both firms are downward sloping. This requires the following assumption on demands:

(A3) $D_i(p_1, p_2)$ is strictly log-submodular in $P_1 \times P_2$, $i = 1, 2$.

This assumption implies that

$$D_i(p_1, p_2)\,\frac{\partial^2 D_i(\,p_1,\,p_2)}{\partial p_1 \partial p_2} - \frac{\partial D_i(\,p_1,\,p_2)}{\partial p_i}\frac{\partial D_i(\,p_1,\,p_2)}{\partial p_j} \leq 0$$

and, conversely, the strict version of this inequality implies (A3). The main implication of (A3) is that it leads to reaction correspondences that are nonincreasing (in the sense that each selection is nonincreasing).

Proposition 2 *Under assumptions* (A_1) *and* (A_3)*, each firm has a first mover advantage.*

The next result deals with the mixed case, when one of the firms (say firm 2) has log-supermodular demand function, and hence an upward-sloping reaction, and firm 1 has a log-submodular demand function, and hence a downward-sloping reaction. Here, the added quasi-concavity assumption is needed only to guarantee existence of a Bertrand equilibrium (as Tarski's theorem clearly does not apply when the two optimal reactions are monotone in different directions).

Proposition 3 *Under conditions* (A_1) *for both firms, assumption* (A_2) *for firm* 2*, assumption* (A_3) *for firm* 1*, and under the additional assumption that each firm's profit function is strictly quasi-concave in own price, firm* 1 *has a first mover advantage.*

The rest of the paper deals with a possible extension of Proposition 1. The argument used in its proof here (as well as in Dowrick's proof) cannot be extended to establish or to exclude a second-mover advantage for the other firm. Furthermore, at an intuitive level, one typically thinks of price competition as being characterized by a second-mover advantage. Indeed, the option to undercut the rival's price seems rather appealing. With homogeneous products, price undercutting allows a firm to capture the entire market, and thus an extreme advantage. With differentiated products and identical firms, a second-mover advantage always prevails when prices are strategic complements. It is thus a very natural question to ask whether a second-mover advantage would survive high degrees of firm asymmetry and of product differentiation (still under strategic complementarity of prices.) To this end, the case of linear demand and costs provides a convenient framework of analysis.

Henceforth, assume symmetric linear demands of the form

$$D_i(p_1, p_2) = a - p_i + bp_j, \text{ where } 0 < b < 1 \ , \tag{2}$$

with the profit of the firm i still given by (1). Furthermore, w.l.o.g., we assume that $c_1 > c_2$.

To guarantee interiority of solutions for all three games at hand, the following assumption on demand and unit costs is needed for firm 1 (the high-cost firm):

(B) The demand function is given by (2) and satisfies: $a > M_1$, where $M_1 \triangleq \frac{c_1 \ (2-b^2)-bc_2}{2+b}$.

Assumption (B) guarantees that quantities are positive and prices are above marginal cost in each of the three games at hand.

Lemma 4 *Under Assumption (2), the unique Bertrand-Nash equilibrium (for game G) is given by (p_1^N, p_2^N) where (with $i, j = 1, 2 \ , \ i \neq j$.) :*

$$p_i^N = \frac{a(2+b) + bc_j + 2c_i}{4-b^2} \ . \tag{3}$$

The Stackelberg equilibrium prices and profits of the game G_i under Assumption (B) and linear demand and cost functions is given by (here[1] $i, j = 1, 2, \ i \neq j$):

$$p_i^L = \frac{a(2+b) + bc_j}{2(2-b^2)} + \frac{c_i}{2}, \tag{4a}$$

$$p_j^F = \frac{(4-b^2)(a+c_j) + 2ab}{4(2-b^2)} + \frac{c_i b}{4}, \tag{4b}$$

[1] *The superscripts L and F stand for Stackelberg leader and follower, respectively. Thus p_i^L and p_i^F are firm i's prices in game G_i and G_j, respectively. This notation is meant to avoid (confusing) superscripts for games.*

$$\Pi_i^L = \frac{(a(2+b)+(b^2-2)c_i+bc_j)^2}{8(2-b^2)}, \quad (4c)$$

$$\Pi_i^F = \frac{(a(b^2-2b-4)+b(b^2-2)c_j+(4-3b^2)c_i)^2}{16(b^2-2)^2}\ . \quad (4d)$$

To state our next result, we need to define

$$M_2 \triangleq \frac{((2-b^2)c_1-(2b^2+b-2)c_2+\sqrt{2(2-b^2)}(1+b)(c_1-c_2))}{4+3b}. \quad (5)$$

As will be seen in the Appendix, M_2 is the unique feasible value of a for which firm i is indifferent between leading and following, i.e., $a = M_2 \Leftrightarrow \Pi_i^L = \Pi_i^F$.

Our last result provides a complete characterization of the comparison between the two-stage games G_1 and G_2 for our linear specification.

Proposition 5 *Under linear demand and cost functions and Assumption (B), firm* 1 *(high-cost) has a second-mover advantage for all feasible a. Firm* 2 *(low cost) has a first-mover advantage when* $a \in [M_1, M_2]$*, but a second-mover advantage when* $a \in [M_2, \infty)$*,where* M_1 *is given in assumption* (B) *and* M_2 *is defined by (5). Furthermore, the scope for first-mover advantage increases with the cost difference* $c_1 - c_2$*.*

This result establishes that the conjecture that price undercutting is always a favorable option in Bertrand competition is generally false. It fails to hold globally (i.e., for all feasible parameter values) even in the most common specification of linear symmetric demand and constant unit costs. Thus, this conjecture relies crucially on the discontinuity of the single firms' demand functions along the price diagonal inherent in homogeneous-product Bertrand competition, or on the symmetry between the two firms in the differentiated-products case. Furthermore, as confirmed by Propositions 2 and 3, the strategic complementarity of prices is also needed for the second-mover advantage to prevail, even in the case of symmetric firms with differentiated products.

3. Proofs.

Proof of Proposition 1.

Since $\log \Pi_i(p_1, p_2) = \log(p_i - c_i) + \log D_i(p_1, p_2)$ it follows that $\log \Pi_i$ is supermodular in $(p_1,\, p_2)$. Applying Topkis's Theorem to this transformed payoff function, we get that every selection from each firm's reaction correspondence is a nondecreasing function of the rival's price.

As shown in [Amir, Grilo and Jin (1999), Proposition 2.4], Stackelberg prices are higher than Nash prices. This follows from the following argument, say for Game G_1. Let (p_1^S, p_2^S) be the price pair associated with an arbitrary Stackelberg equilibrium of game G_1 and (p_1^N, p_2^N) be the largest Bertrand equilibrium of game G, which is the Pareto-preferred Bertrand equilibrium [Theorem 7 of Milgrom and Roberts, 1990]. Then

$$\begin{aligned}(p_1^S - c_1)D_1(p_1^S, p_2^S) &\geq (p_1^N - c_1)D_1(p_1^N, p_2^N) \\ &\geq (p_1^S - c_1)D_1(p_1^S, p_2^N), \qquad (6)\end{aligned}$$

where the first inequality follows from the fact that a leader's payoff cannot be worse than his Nash payoff[2] , and the second from the Nash property. Since $D_1(p_1^S, .)$ is increasing, (6) implies that $p_2^S \geq p_2^N$. Since both (p_1^S, p_2^S) and (p_1^N, p_2^N) lie on r_2 and every selection of r_2 is nondecreasing, $p_2^S \geq p_2^N$ implies that $p_1^S \geq p_1^N$.

Due to the facts that every selection from r_1 and r_2 is a nondecreasing function of the rival's price, and that Stackelberg prices are higher than Nash prices, there are only 3 possible ways to relate equilibrium prices in G_1 and G_2 : (i) $p_2^L \geq p_2^F$ and $p_1^L \geq p_1^F$, (ii) $p_2^L \geq p_2^F$ and $p_1^L \leq p_1^F$, and (iii) $p_2^L \leq p_2^F$ and $p_1^L \geq p_1^F$. We now analyse each case separately:

[2] A formal proof of this is given in [Amir, Grilo and Jin (1999), Lemma 4.1].

Case (i). Consider the following inequalities in games G_1 and G_2:

$$\begin{aligned}(p_1^F - c_1)D_1(p_1^F, p_2^L) &\geq (p_1^L - c_1)D_1(p_1^L, p_2^L) \\ &\geq (p_1^L - c_1)D_1(p_1^L, p_2^F).\end{aligned}$$

where the first inequality for firm 1 follows from the definition of Stackelberg equilibrium, and the second from the facts that $p_2^L \leq p_2^F$ and $D_1(p_1^S, .)$ is increasing. This says that the profit of firm 1 in higher in game G_2 than in game G_1, so that firm 1 prefers game G_2 to game G_1. A similar argument shows that firm 2 prefers game G_1 to game G_2.

Case (ii). Here, it can be seen that the argument of Case (i) can be applied only for firm 1, so we can conclude only that firm 1 prefers to be a follower.

Case (iii). Here, it can be seen that the argument of Case (i) can be applied only for firm 2, so we can conclude only that firm 2 prefers to be a follower.

Overall then, this establishes that there is always at least one firm that prefers to follow.

Straightforward calculations shows that

$$\begin{aligned}\frac{\partial(M_2 - M_1)}{\partial c_1} &= \frac{(1+b)(2b^2 - 4 + (2+b)\sqrt{4 - 2b^2})}{(2+b)(4+3b)} \\ \frac{\partial(M_2 - M_1)}{\partial c_2} &= -\frac{(1+b)(2b^2 - 4 + (2+b)\sqrt{4 - 2b^2})}{(2+b)(4+3b)}\end{aligned}$$

It can be shown that $\frac{\partial(M_2 - M_1)}{\partial c_1} > 0$ while $\frac{\partial(M_2 - M_1)}{\partial c_2} < 0$. ■

Proof of Proposition 2.

Here, we know from Topkis's Theorem that all the selections from both firms' reaction correspondences are nonincreasing. [Amir,Grilo and Jin, (1999), Lemma 4.1] proves that, under either Assumption (A2) or (A3), both firms prefer their Stackelberg leader payoff to their largest Bertrand equilibrium payoff. Likewise, [Amir, Grilo and Jin (1999), Proposition 2.5] shows that both firms prefer their worst Bertrand equilibrium payoffs (the one with lowest prices) to their Stackelberg follower payoffs. Putting these two results together, it follows that both players have a first-mover advantage. ■

Proof of Proposition 3.

Here, we know from Topkis's Theorem that all the selections from r_1 are nonincreasing while all the selections from r_2 are nondecreasing. [Amir, Grilo and Jin (1999), Lemma 4.1] shows that firm 1 prefers its Stackelberg leader payoff to its Bertrand equilibrium payoff. [Amir, Grilo and Jin (1999), Proposition 2.6] shows that firm 1 prefers its Bertrand equilibrium payoff to its Stackelberg follower payoff. It follows that firm 1 has a first-mover advantage. ■

Proof of Lemma 4.

From firm i's best-response problem $\max_{p_i} (p_i - c_i)(a - p_i + bp_j)$, i, $j = 1, 2$, $i \neq j$, we obtain its reaction curve $r_i(p_j) = \frac{a+bp_j+c_i}{2}$. The pair of Bertrand-Nash equilibrium prices (3) is the unique point where the two reaction curves intersect.

To find Stackelberg point in game G_i, firm i maximizes its profit along r_j, i.e. its objective function is $\Pi_i[p_i, r_j(p_i)] = (p_i - c_i)[a - p_i + b\frac{a+bp_i+c_j}{2}]$. This yields p_i^L as given in (4a). The Stackelberg equilibrium price of the follower p_j^F (see 4b) was found by substitution p_i^L into r_j.

To be sure that the solutions of all three games are interior, Assumption (B) is easily seen to be what is needed. ■

Proof of Proposition 5.

Our arguments here are based on simple but tedious (closed-form) computations. Consider the indifference relations

$$\Pi_2^L - \Pi_2^F = 0 \qquad (7)$$

and

$$\Pi_1^L - \Pi_1^F = 0, \tag{8}$$

where Π_2^L, Π_2^F (Π_1^L, Π_1^F) are the equilibrium profits of firm 2 (firm 1) in games G_2, G_1 (G_1, G_2), respectively. Equations (4c), (4d) provide expressions for all four profit levels. It can be easily seen that (7) is a quadratic equation with respect to the parameter a. That means the equation has two solutions - two values of parameter a. But one of the roots (the smaller one) does not belong to the interval of feasible parameters a, and is therefore not considered. Only the largest root, which is given in (5) and denoted by M_2 is a valid solution of (7). Since (7) is a quadratic equation w.r.t. a and the term a^2 has a negative coefficient, it follows that $\Pi_2^L - \Pi_2^F \geq 0$ if and only if $a \in [M_1, M_2]$.

Concerning (8), it can be easily shown using arguments analogous to the above that, given $c_1 > c_2$ and our parameter restrictions (B), we always have $\Pi_1^L \leq \Pi_1^F$, so that firm 1 always has a second-mover advantage here. ■

4. Appendix A

Here, we provide a brief but self-contained summary of all the lattice-theoretic notions and results used in the present paper, in the simple framework of real action and parameter spaces: Every result presented here is a special case of the indicated original result.

A function $F : [0, \infty)^2 \to I\!R$ is strictly supermodular (strictly submodular) if

$$F(x_1, y_1) - F(x_2, y_1) > (<) F(x_1, y_2) - F(x_2, y_2) \ \text{ for all } x_1 > x_2, y_1 > y_2. \tag{A.1}$$

$F : [0, \infty)^2 \to I\!R$ has the strict single-crossing property or SSCP (dual SSCP) in (x; y) if

$$F(x_1, y_2) \geq (\leq) F(x_2, y_2) \Rightarrow F(x_1, y_1) > (<) F(x_2, y_1) \text{ for all } x_1 > x_2, y_1 > y_2. \tag{A.2}$$

Note that F strictly supermodular (strictly submodular) $\Rightarrow$ F has the SSCP (dual SSCP), while the converse is generally not true. Furthermore, supermodularity is a cardinal property while the SSCP is ordinal. If $F : [0, \infty)^2 \to I\!R$ and h is a strictly increasing function from $I\!R$ to $I\!R$ such that $h \circ F$ is strictly supermodular, then F has the SSCP. In this paper, we use only a special case of the SSCP (the dual SSCP), arising from the profit function being log-supermodular (submodular).

Supermodularity and submodularity have complete characterizations in terms of the sign of cross-partial derivatives in case of smooth objective functions (Topkis, 1978). On the other hand, the strict versions of these notions can be given separate (related) necessary and sufficient conditions. Let F be twice continuously differentiable. If $\frac{\partial^2 F}{\partial x \partial y} > (<) 0, \forall\, x, y$, then F is strictly supermodular (strictly submodular). Conversely, if F is strictly supermodular (strictly submodular), then we have $\frac{\partial^2 F}{\partial x \partial y} \geq (\leq) 0, \forall\, x, y$. The latter inequality is equivalent to supermodularity (submodularity) of F defined by (A.1) with a nonstrict inequality.

The monotonicity theorem repeatedly used in this paper is due to Topkis (1978).

Theorem A.1. *Every function $x^*(y) \in argmax_{x \geq 0} F(x, y)$ is nondecreasing (nonincreasing) in y if F is strictly supermodular (strictly submodular) in (x, y).*

This result has been generalized by Milgrom and Shannon (1994) who showed the conclusion of the theorem still holds if the assumption that F is strictly supermodular (submodular) is replaced by the assumption that F satisfies the SCCP (dual SSCP) in $(x; y)$. In this paper, Topkis's Theorem is applied to profits and to log-profits, under different assumptions.

We close with a statement of (a special case of) the associated fixed-point theorem, due to Tarski (1955):

Theorem A.2. *Let K_1, K_2 be compact intervals in $[0, \infty)$, and $f : K_1 \times K_2 \to K_1 \times K_2$ be nondecreasing. Then the set of fixed-points of f is nonempty and contains a smallest and a largest element.*

A normal-form game is supermodular (ordinally supermodular) if the payoff functions are supermodular (have the SSCP). In both cases, we also say that the game has strategic complementarities. A two-player game with payoffs satisfying submodularity (resp., the dual SSCP) becomes a supermodular (resp., ordinally supermodular) game once we reverse the order on one of the players' action set.

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