

NON-UNIQUENESS IN THE FIRST GENERATION BALANCE OF PAYMENTS CRISIS MODELS*

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ABSTRACT

Krugman (1979), building on Salant and Henderson (1978), showed that for a small open economy following policies inconsistent with the fixed exchange rate regime it is on, there is a precise moment when the fixed exchange rate regime would be abandoned. I show that in Krugman's model the timing of collapse may not be unique. The Central Bank may be in possession of a lot of foreign exchange reserves but not an "adequate amount" i.e., not enough to maintain a fixed exchange rate.

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1. INTRODUCTION

Krugman's model of a balance of payments crisis has been one of the most influential ideas in open economy macroeconomics in the last two decades. Krugman (1979), building on Salant and Henderson (1978), showed that for a small open economy following policies inconsistent with the fixed exchange rate regime it is on, there is a precise moment when the fixed exchange rate regime would be abandoned. The unexpected result was that this moment of switch is not the moment when everyone realises that the fixed exchange rate regime will ultimately collapse. Nor is it when the Central Bank of this economy has lost all its foreign exchange reserves and is unable to maintain a fixed exchange rate regime. Krugman showed that the Central Bank will lose its remaining stock of foreign exchange reserves discretely when the domestic credit component of the money supply reaches the level equal to the nominal money stock required under a flexible

exchange rate regime. ¹The exchange rate then depreciates in a continuous manner --i.e., without a discrete jump--from its fixed rate value.²

The exposition of this model has benefited enormously from the notion of a shadow floating exchange rate introduced by Flood and Garber (1984) in the context of a linear model. This is the exchange rate that would prevail on any date if the economy was to move to a flexible exchange rate regime on that date. They showed that since by perfect foresight anticipated jumps in the exchange rate are ruled out, the attack on the Central Bank's reserves occurs when the shadow floating exchange rate just equals the fixed rate.

In these models --often referred to as the "first generation" of crisis models--the Central Bank is a passive player monetizing the deficit. There is another class --the "second generation" --of balance of payments crisis models, surveyed by Flood and Marion (1998), where the Central Bank and the private sector play a game and under certain circumstances a self-fulfilling crisis leads to the abandoning of the fixed exchange rate regime.

An empirical argument is often used to criticize the first generation models, namely that in most cases of a collapse the Central Bank had "a lot" of foreign exchange reserves. In this paper, I shall endeavour to show that Krugman's original model is also riddled with multiple equilibria. It is shown here that the Central Bank may be in possession of a lot of foreign exchange reserves but not an "adequate amount" i.e., not enough to maintain a fixed exchange rate. This could have some serious implications for the literature which continues to use the Krugman model as a basic work-horse.³

In section 2, I outline the basic Krugman model shorn of the inessentials. In section 3, I show that there are an infinity of dates on which the fixed rate could collapse. Some concluding comments are offered in section 4.

2. THE MODEL

The linear version of Kugman's model is well-known territory since the model has been exposted by a number of authors (see e.g., Dornbusch (1987), Flood and Garber (1984), Garber and Svensson (1995), Agenor, Bhandari and Flood (1992), Blackburn and Sola (1993)). My description of it, therefore, will be very brief.

A small open economy produces and consumes a traded good. Hence purchasing-power-parity prevails at all times i.e.,

$$P(t) = E(t).Q(t) \tag{1}$$

¹ On the assumption that the level of reserves that the Central Bank will defend is zero.

² Throughout we will consider bubble-free solutions.

³ See for instance the entire issue of the *Journal of International Economics* (Volume 41, no. 2, 1996) devoted to the Mexican crisis of 1994. Also see the special issue of *Open Economies Review* (1996).

where P (resp. Q) is the domestic currency price (resp. foreign currency price) of the good and E is the exchange rate (the domestic currency price of foreign exchange). Setting the given value of Q to unity, the domestic price level is equal to the exchange rate. The domestic price level remains constant under a fixed exchange rate system.

Uncovered interest parity is assumed to hold between domestic and foreign currency assets i.e.,

$$I(t) = i^* + \lambda(t) \quad (2)$$

where I (resp. i^*) is the domestic interest rate (resp. foreign interest rate) and λ is the expected rate of depreciation of the domestic currency. Under perfect foresight $\lambda(t) = (d \log E(t)/dt)$. This is zero while the fixed rate regime is expected to last.

Full employment prevails and the level of output is normalized to zero. Real money demand therefore can be written solely as a function of the nominal interest rate I. Money market equilibrium requires that

$$M(t) / E(t) = a - b \cdot I(t) \quad a, b > 0 \quad (3)$$

where M is the nominal money stock. In the absence of commercial banks we have

$$M(t) = R(t) + D(t) \quad (4)$$

where R is the Central Bank's foreign exchange reserves (measured in domestic currency) and D the domestic credit component of the money supply.

In conformity with the first generation models of balance of payments crises, the domestic credit component is assumed to grow at a constant rate μ (e.g.) because the government runs a budget deficit which the monetary authority passively monetizes).

$$d \log D(t) / dt = \mu \cdot D(t) \quad (5)$$

Finally, there is a target level of foreign exchange reserves which the Central Bank will defend i.e., it is to commit the rest of its holding of foreign reserves to the defense of the fixed exchange rate system. Following the literature, we assume that this target \bar{R} level is known.

While the economy is on a fixed exchange rate system (with $E = \bar{E}$ and it is expected to remain on that system

$$I = i^* \quad (6)$$

$$M^F / \bar{E} = a - b \cdot i^* \equiv a \quad (7)$$

where M^F is the nominal money supply and a is the real money demand in the fixed rate regime.⁴

I assume, again this is a standard assumption in most of the literature, that when the fixed rate system breaks down, we move to a freely floating exchange rate regime. Given the structure of the model it is evident that the economy can jump to the flexible exchange rate steady state instantaneously with all the nominal variables growing at μ .

Under a floating rate, we thus have

$$M^s(t) / E(t) = a - b(i^* + \mu) \equiv \beta \quad (8)$$

where $M^s(t)$ is the nominal money supply and β is the real money demand in the floating rate regime.

From equations (7) and (8), note

$$a - \beta = b\mu$$

It is easy to check that in the absence of foreign inflation, a continuing increase in the domestic credit component of the money supply is inconsistent with an everlasting fixed exchange rate regime. In finite time D will become larger than the level of money supply M^F which is consistent with money market equilibrium under a fixed exchange rate regime. Clearly after that time the fixed exchange rate system is no longer viable.

As long as the economy is on a fixed exchange rate, D is growing at a rate μ , and foreign reserves fall by μD since the sum of D and R is M^F i.e.,

$$dD(t) / dt = -dR(t) / dt = \mu D(t) \quad (9)$$

Krugman's insight was to point out that when domestic credit component reaches

$M^s(T)$ say on date T (with the exchange rate at \bar{E}), there will be a discrete run on the Central Bank's reserves and the entire remaining stock of reserves equal to the difference between M^F and $M^s(T)$ --call it $\Delta R(T)$ -- will be cleaned out leaving it with \bar{R} level of reserves (remember that the level which the Central Bank is assumed to defend is \bar{R}).⁵ The exchange rate evolves continuously from \bar{E} i.e., it depreciates at a rate μ from date T onwards (as do all other nominal variables).

⁴ It is evident from equation (7) the nominal money stock which will generate equilibrium in the money market is constant under a fixed exchange rate but is growing at a rate μ under flexible rates.

⁵ In a previous version of this I had assumed, like Krugman, that the level of reserves that the Central Bank will defend is zero. This normalisation turned out to be source of confusion for the point I am trying to make in this paper and hence has been jettisoned here.

This is the bubble-free solution proposed by Krugman. Everyone expects the fixed rate system to collapse on T and it does -- the chronicle of a death foretold.

To see this more clearly look at figure 1. On the horizontal axis we have real money balances and on the vertical axis the nominal interest rate. Demand for real balances is the downward-sloping line LL. The fixed exchange rate equilibrium is shown at point A with real balances at OF^1 and nominal interest rate at OI. This corresponds to the nominal money supply M^F . For convenience assume $\bar{E} = 1$ by a suitable choice of units. This allows us to talk interchangeably between real and nominal money supply as long as the economy is on a fixed exchange rate system. In the diagram we show R measured from right to left (starting vertically below A) while D is measured from left to right starting from the origin. Since D is growing over time, while M^F is constant, R must be falling over time (see equation (9) above). When on date T, the level of D reaches M^S --shown in figure 1 as the horizontal distance OF^1 -or D reaches OU since is the distance UF^1 , agents clean out the rest of the Central Bank's reserves (i.e., OF^1 less OF^1) and the economy moves to a floating rate system with all nominal variables growing at α and the nominal interest rate at $i^* + \mu$ given by the vertical distance ON. This equilibrium is shown at point S in the diagram.

It is easy to show that the regime shift has to occur at T and not earlier or later. For this one needs to remember that since there is no new information the exchange rate E has to move continuously--anticipated jumps in E are ruled out by arbitrage. Given this if the reserves were to be attacked earlier than T then the nominal interest rate would be greater than $i^* + \mu$. This would imply real balances would be falling over time--a process that gets terminated when real balances become zero. Similarly if the attack on the Central Bank's foreign exchange reserves was postponed beyond T, the nominal interest rate would fall continuously over time. An appropriate boundary condition would rule such divergent paths out --in this case the assumption that the new steady state is one with a flexible exchange rate.⁶

To clarify this further, we introduce the shadow floating exchange rate notion due to Flood and Garber. What would happen if the system was to move to a floating rate before T (by cleaning out the foreign exchange reserves in excess of \bar{R})--say when D has reached point V on date τ . Since $D(\tau) +$ is short of M^S , the exchange rate necessary for money market equilibrium and which will take the economy to the floating exchange rate steady-state--the shadow floating exchange rate ($X(\tau)$)--would have to appreciate relative to its fixed rate value so that real balances are equal to β . The earlier the shift to a float the larger the appreciation. This is shown in figure 2. Agents would be making an anticipated capital loss by buying foreign exchange for $t < T$ --they would be buying at \bar{E} but its market price is $X(t)$. Perfect foresight would rule out such an eventuality. Similarly for postponing the float beyond T -- here there are anticipated capital gains with the shadow rate lying above \bar{E} . At time T, the shadow floating rate is just equal to the fixed rate and hence that is the only possible time when the economy can move to a floating rate without a jump in the level of the exchange rate.

Formally

⁶ Alternative post-collapse regimes have been discussed. See e.g., Agenor, Bhandari and Flood (1992) for a discussion of some of these

$$X(t) = (1/\beta) \cdot (D(t) + \bar{R}) = (1/\beta) \cdot (D(0)\exp. \mu t) + \bar{R} \quad (10)$$

and T is determined from from $X(T) = \bar{E}$

$$T = [\log(\beta \bar{E} - \bar{R}) - \log D(0)] / \mu \quad (11)$$

and the loss of reserves at T

$$M^F - M^S = b\mu \quad (12)$$

3. MULTIPLE EQUILIBRIA

I now proceed to show that T is not uniquely determined as is universally accepted. At best, T is the maximum of all possible times of attack if indeed it is associated with the minimum level of reserves that the Central Bank will defend. Suppose that the private agents do not know that the Central Bank will defend level of reserves but believe (with point certainty) that it will defend $R' > \bar{R}$ (given by distance VF^l in figure 1) of reserves. So when D reaches the value $M^S - R'$ (distance OV in figure 1) they will proceed to clean out be $(F^l - F^i$ in figure 1) leaving it with R' . The economy moves to a floating rate regime after this. No discrete jump in the shadow exchange rate is called for! Here the Central Bank has reserves above what it would defend and at the margin it can sell foreign exchange at $\bar{E} \bar{E}$, but it does not have enough reserves to ensure money market equilibrium under fixed exchange rates. This level of M is consistent with a floating rate regime and E moving continuously. The belief about reserves is self-fulfilling. Nothing in the events contradicts their beliefs! For each level R' of reserves that the private sector believes that the Central Bank would defend there is a date of attack to corner $b\mu$ ($F^l F^i$) of foreign exchange reserves.

Now drop the assumption that agents do not know what level of reserves the Central Bank will defend. Suppose that every one knows that the authorities will in fact defend \bar{R} . Is it possible that the private sector can still effect a shift to a flexible exchange rate system before \bar{R} has been reached? Take point V in figure 1. Money supply is backed by OV of D and VF^l plus $F^l F^i$ of R. Agents know the minimum level of foreign exchange reserves that the Central Bank will defend (which we have set equal to \bar{R}). They clean out $F^l F^i$ of the reserves leaving it with VF^l . The economy moves to a floating rate regime after this. No discrete jump in the shadow exchange rate is called for! Here the Central Bank has reserves above what it would defend and at the margin it can sell foreign exchange at \bar{E} , but it does not have enough reserves to ensure money market equilibrium.

Another way of saying this is that for every level of foreign exchange reserves that the private agents leave the monetary authority with, there is a shadow exchange rate path. And on each date t, there is one shadow exchange rate path which does not require a jump in the exchange rate in

moving to a float. The earlier the economy moves to a float the larger the reserves the Central Bank is left with. Using this logic one can shade the entire area above the shadow rate path in figure 2.

With a fixed exchange rate and uncovered interest parity neither open market operations nor borrowing of foreign exchange reserves would help, since money supply is demand determined. To see this assume the Central Bank conducts an expansionary open market operation by buying domestic bonds from the public to try and restore the money supply to the fixed exchange rate level. It would lose foreign exchange reserves of an equivalent amount. If it persists in carrying out these open market operations it would lose all the foreign exchange reserves until . Thus reserves would be driven down to the minimum level that the Central Bank would defend as in Krugman's example. But even here the attack could occur on any date. Multiple equilibria remain even when the Central Bank tries to play Canute.

Let us do a formal analysis similar to that in the previous section for this argument

$$X^M(t) = (1/\beta) \cdot [D(t) + R'(j)] = (1/\beta) \cdot [D(0) \exp(\mu t) + R'(j)] \quad (13)$$

with the index $j \in J$ such that $R(0) > R(0) - \Delta R(T^M) \geq R'(j) > 0$

where R' is the amount of reserves the Central Bank is left with following the shift to a floating rate regime--which in section 2 above was assumed to be zero and X^M is the shadow floating rate when there are multiple equilibria.

Again setting $X^M(T^M) = \bar{E}$, we can solve for T^M

$$T^M = [(\log \bar{E} - (1/\beta) R'(j)) - \log D(0) + \log \beta] / \mu \quad (14)$$

Comparing (14) with (11), we find that $T^M < T$ since $R'(j) \geq \bar{R} \geq 0$ and $\beta > 0$. Also the larger is j and $R'(j)$, the smaller is T^M .

Note one implication of the above analysis, namely that if the fixed rate system is eventually going to break down there is no notion of "sufficient reserves" or borrowing to postpone the day of reckoning. The large holdings of foreign exchange reserves by countries whose Central Banks suffered attacks as reported in Table 8.1 (p. 566) of Obstfeld and Rogoff (1996) should also be seen to have little bearing on the issue at hand. Since only the amount of reserves that the private sector needs to corner is the difference between the fixed rate money supply and the flexible rate money supply (i.e., $\Delta R(T^M)$), the actual holding of foreign exchange reserves by the Central Bank is irrelevant to the ability of the private sector to engineer the collapse.

What determines the timing of the attack in this section? We are unable to answer this question within the framework of this model⁷. Extraneous considerations like sunspots or model specific concerns of credibility could be introduced. These features place the analysis closer to a second generation model but I emphasize that the model itself is still a first generation model because of the postulated behaviour of the Central Bank. As in Krugman (1979) and Flood and Garber (1984), the Central Bank continues to "monetize" the deficit at a rate μ . The only new twist here is that the timing of the switch to a flexible exchange rate regime does not depend on the private sector cornering all of the Central Bank's reserves .

Before concluding, I want to raise a point of interest about the genealogy of this literature. Salant and Henderson (1978) tried to explain a risk premium in the price of gold due to possibility of a sudden sale of gold by Central Banks. The path of price of gold was based on the Central Banks selling all their reserves if they sold any at all. In their model presumably there are multiple paths for the price of gold for each quantity of gold sold by Central Banks.⁸

4. CONCLUSIONS

In Krugman's model of a speculative attack which results in a collapse of the fixed exchange rate system, I showed the time of attack is not uniquely determined. A continuum of paths to a floating regime exist i.e., the model has multiple equilibria. The balance of payments crisis can occur at any time. The only difference across periods (in which the attack can occur) is the amount of foreign exchange reserves left with the Central Bank and hence the proportion of foreign exchange reserves that will back the money supply under a floating rate regime. How much reserves the Central Bank holds or is able to borrow is not germane to the issue of collapse (as long as the fixed exchange rate is expected to collapse eventually). The Central Bank may hold "large" stocks of reserves but not "enough" to maintain a fixed rate regime. Its "large" reserves and domestic credit component of the money supply do not add upto the fixed exchange rate money supply, which is demand determined.

What determines which of the infinitely many moments for the collapse of the pegged exchange rate system is not answered in this paper.

The analysis blurs the clear distinction between a first and a second generation model because now issues like credibility of a policy or co-ordination among various private agents become crucial in choosing when exactly the crisis would occur. The behaviour of the Central Bank, however, is exactly as postulated in a first generation model.

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⁷ See e.g. Obstfeld (1996) or Morris and Shin (1998) for models within a second generation framework.

⁸ This is true of the analysis presented in the text of their paper. In the working paper version they had discussed partial sales of gold stocks by Central Banks

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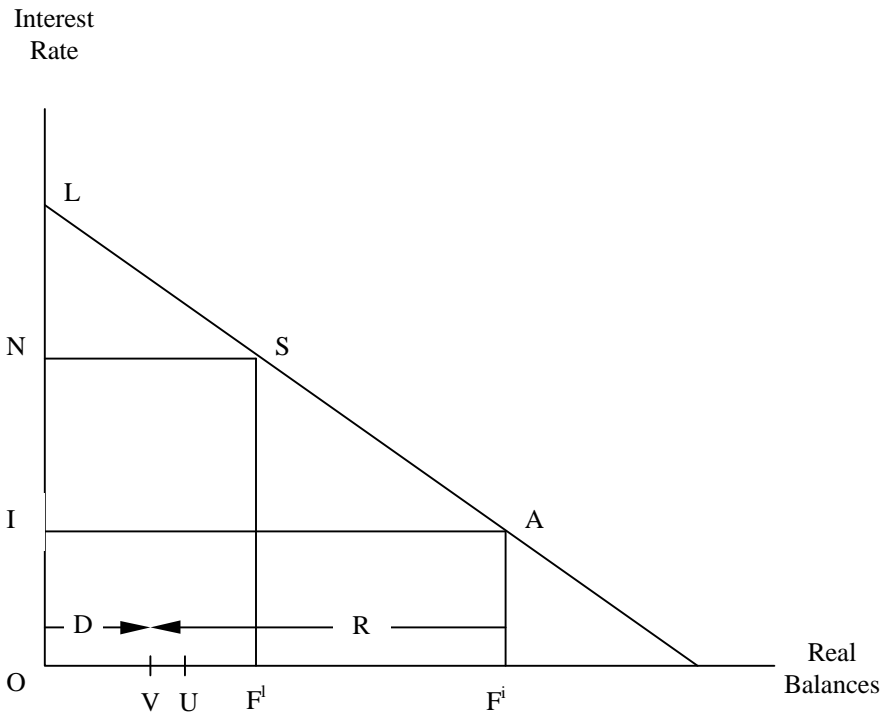


Figure 1

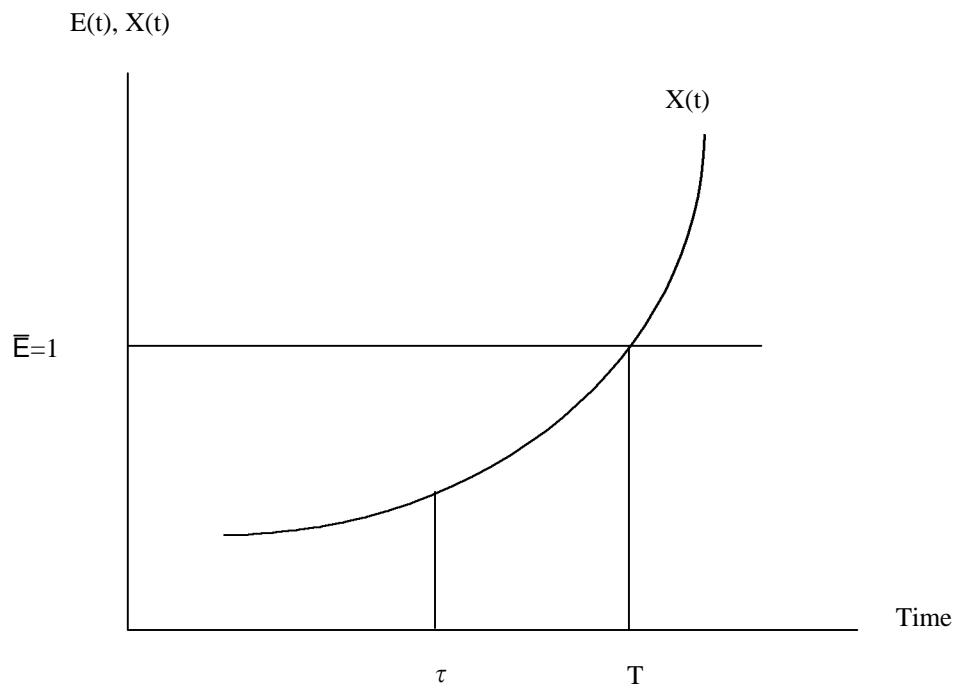


Figure 2