Agency Problems and Commitment in Delegated Bargaining*

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Abstract

In the context of (one-sided) delegated bargaining, we analyze how a principal (a seller) should design the delegation contract in order to provide proper incentives for her delegate (an intermediary) *AND* gain strategic advantage against a third party (a buyer). We assume that there are both moral hazard and adverse selection problems in the delegation relationship and every player is risk neutral. In the absence of commitment effect, it is shown that a linear contract is optimal. When delegation contracts have commitment value, the seller can gain substantially by committing the delegate to a minimum price, above which she pays the delegate a commission. We show that the seller's strategic manipulation of the delegation contract may cause bargaining failures between the delegate and the buyer when the seller sets a minimum price exceeding some buyers' valuations. Furthermore, the interaction between commitment (through minimum price) and incentives depends on the nature of the agency problem. We also derive comparative statics of the model. Extensions of the model to multidimensional efforts and unobservable contracts as well as applications to car dealerships are briefly discussed.

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1. INTRODUCTION

In many economic situations, delegates are hired to play games on behalf of their principals. The principal-agent literature has had much success in analyzing how optimal contracts should respond to various types of agency problems (adverse selection, moral hazard and combinations of both) in the delegation relationship. However, the agent in most of this literature does not play a game with other parties, rather, his actions alone determine the principal's payoff subject to perhaps exogenous randomization by nature. Hence, the game the agent is hired to play with other parties is completely suppressed in the studies of optimal agency contracts. On the other hand, since Schelling (1960), it has long been recognized that the principal can gain strategic advantages against a third party by properly designing a contract for the agent. A large amount of subsequent work has investigated when this commitment effect can arise and its implications in various economic situations. But not much attention has been paid to the interactions between agency problems and commitment considerations in the delegation relationship.[1] In this paper, we analyze such interactions in an important class of delegation games, delegated bargaining.

Specifically, we consider the following one-sided delegation game. A seller of one indivisible good hires a delegate (an intermediary) to sell the good for her, perhaps because the delegate has specialized knowledge about selling the good that the seller does not have. They sign a contract, which becomes public knowledge. The cost of the good to the seller is zero. After exerting some "sales efforts", the delegate finds a buyer with valuation of the good $s \in [\underline{s}, \bar{s}]$ given by some distribution $G(S)$, where $0 \leq \underline{s} < \bar{s}$. Once the delegate meets the buyer, he learns the buyer's valuation, and then they bargain over a price, so bargaining is conducted under complete information. If the delegate and the buyer agree on a price, the buyer gets the good and makes the payment, and the delegate delivers the payment to the seller. The seller then pays the delegate a wage according to the delegation contract. The only thing the seller can observe is the sale revenue the delegate brings back to her. We assume that the delegate and the buyer cannot collude and the delegate cannot hide money from the seller. All the players are assumed to be risk-neutral.

We suppose that there are both moral hazard and adverse selection problems in the delegation relationship. That is, the delegate's effort is not observable to the seller; and furthermore, the delegates can differ in their disutility of effort, which is not observable to the seller either. Ignoring the commitment effect of delegation contracts, we can characterize the seller's optimal mechanism. Using the remarkable insights of the earlier literature (e.g., Holmstrom and Milgrom 1987, Laffont and Tirole 1986, and McAfee and McMillan 1987), it can be shown that

[1] One notable exception is Fershtman and Judd (1987b), which will be discussed later.

a contract linear in revenue can implement the seller's optimal mechanism under certain mild conditions. This is done in Section 2.

Of course, the seller would be too foolish not to take advantage of any commitment value the delegation contract may have. For simplicity, we assume that delegation contracts are perfectly observable to potential buyers and cannot be renegotiated.[2] If the seller knew exactly the buyer's valuation, then she could achieve "full commitment" by using a "target contract". A target contract requires the delegate to get a certain price for the good, otherwise he is paid nothing or even faces some penalty. Without uncertainty, the seller can set the price target exactly equal to the buyer's valuation, which commits the delegate to get this price and leave the buyer with no surplus (e.g., Fershtman, Judd and Kalai 1991, Kahenmann 1995).[3] In reality, the seller often does not observe directly what the buyer's valuation is, and the agency problems make it difficult for the agent to communicate his knowledge about the buyer perfectly to the seller. In such cases the target contracts are not feasible anymore, thus the commitment power of delegation contracts is limited and the seller usually cannot achieve full commitment. In Section 3, we show that the seller can still achieve a substantial amount of commitment power by imposing a minimum price with a linear sharing contract. Under fairly general conditions, the seller sets a minimum price that is strictly greater than the lower bound of the buyer's valuations. This means that when the buyer's valuation is below this minimum price, the delegate and the buyer cannot reach a deal despite that there are positive gains from trade.

One implication of our results is that strategic delegation may lead to bargaining failures.[4] In our model, the delegate and the buyer bargain under complete information, yet sometimes they fail to reach agreements because the delegate is pre-committed by the seller to bargain aggressively all the time. In a related paper, Cai (2000) shows that the agency problems in the delegation relationship can cause bargaining inefficiency. Specifically, in Cai's model, a

[2]Several papers, e.g., Katz (1991), Caillaud, Jullien and Picard (1995), Dewatripont (1988), Fershtman and Kalai (1997), Corts and Neher (1998), Kockesen and Ok (1999), have examined whether delegation still has commitment power if delegation contracts are not perfectly observable or can be renegotiated secretly. By and large, these papers show that unobservability and renegotiation of delegation contracts *limit but do not eliminate* the commitment effects of delegation.

[3]Fershtman *et al.* (1991) show that with target contracts, any Pareto optimal outcome in a principals-only game can be achieved when (1) every principal can hire a delegate; (2) contracts are observable and not renegotiable; and (3)there are no agency problems. Kahenmann (1995) reaches similar conclusions in the context of Rubinstein bargaining.

[4]That strategic delegation has welfare implications is not new. For example, Fershtman and Judd (1987) show that strategic delegation leads to lower price, lower profit but greater social surplus if oligopolists compete in Cournot fashion but the opposite is true if they compete in Bertrand fashion (see also Baye, Crocker and Ju 1996, Vickers 1986).

delegate bargains with a third party under complete information but faces reelection after the bargaining outcome is known to his constituency (principals). In this case, delay in reaching agreements can be used by the delegate as a signal to his principals that he is of "good type". In contrast to Cai (2000), the agency problems in the delegation relationship do not directly cause bargaining inefficiency in our model. Rather, bargaining failures are caused by the seller's strategic manipulation of the delegation contract that commits the delegate to bargain aggressively.

Another implication of the model is that the nature of the agency problem affects how the seller should optimally balance commitment and incentives. Specifically, we consider two kinds of moral hazard problems by the delegate. In the first scenario, the delegate exerts "bargaining effort" which increases his bargaining power against the buyer (e.g., doing research about the customers and the product, taking courses to improve bargaining skills). In this case, commitment through minimum prices and incentives for the delegate are *substitutes* for the seller, that is, higher minimum prices are associated with lower incentives for the delegate and hence lower effort by the delegate. In another scenario, the delegate exerts "marketing effort" which increases the chance that he finds a buyer (e.g., doing advertisement, providing good services, having clean showrooms). With "marketing effort", commitment through minimum prices and incentives for the delegate are neither substitutes nor complements. This means that for some exogenous changes in the environment, higher minimum prices are associated with higher incentives for the delegate and hence higher effort by the delegate; but for some other exogenous changes in the environment, minimum prices and incentives move in the opposite directions.

To study in more details how the optimal mechanism responds to exogenous changes in the environment, in Section 4 we derive comparative statics of the model for the case of uniform distributions and quadratic cost functions. We present and compare results for three cases: no commitment effect, commitment effect with bargaining effort, and commitment effect with marketing effort. For concreteness, Section 4 also gives some numerical examples where the model is explicitly computed. In one seemingly reasonable configuration of parameter values, there is a 39% probability that the delegate will not reach a deal with a buyer because of the seller's minimum price policy, resulting in about welfare loss of 16% of the total social surplus. In this case, the seller's expected payoff is more than 65% higher than that if she did not take advantage of the strategic value of delegation contract.

Section 5 discusses two extensions of the model and an application to car dealerships. We first discuss how to extend the model to situations in which the delegate exerts both bargaining and marketing efforts. Our discussion focuses on two polar cases when the two kinds of efforts are perfect complements or substitutes. In a second extension, we consider situations in which the delegation contract is not observable to the buyer.

Finally, Section 6 offers some concluding remarks.

Fershtman and Judd (1987b) is the first (and only) model that study how optimal contracts should respond to both agency problems and commitment considerations. Specifically, they consider a double-sided delegation game in which two managers are hired by their owners to compete with each other in an oligopolistic situation. In their model, like ours, delegation contracts are public information and not renegotiable. Unlike in our model, there is only moral hazard problem in the delegation relationship and the owners are more risk-averse than the managers (so without commitment considerations, the owners should sell the firms to the managers). Fershtman and Judd show that to take advantage of the commitment power of the delegation contracts, the owners "over-compensate" the managers for success and thus bear more risk than efficient risk-sharing. In fact, the incentives for the managers are so strong that an owner is better off if her manager fails.

2. THE BASIC MODEL WITHOUT COMMITMENT EFFECT

The model consists of three *risk-neutral* parties: a seller (P), a delegate (D), and a buyer (B). The seller hires the delegate to sell a good to the buyer. The cost of the good to the seller is normalized to be zero. The delegate's reservation utility is U_0. At the time the seller contracts with the delegate, the valuation of the buyer for the good is unknown to both the seller and her delegate. Their common belief about the valuation is given by a probability distribution $G(s)$ with an everywhere positive density function $g(s)$, where $s \in [\underline{s}, \bar{s}]$ $(0 \leq \underline{s} < \bar{s})$ is the buyer's valuation.

When the delegate meets the buyer, the delegate finds out the buyer's valuation. So they bargain over a price without any information problem. We assume that the bargaining game is some sort of alternating-offer bargaining game such as Rubinstein (1982) or Binmore, Rubinstein and Wolinsky (1986). Equivalently, we can use the cooperative solution concept Nash Bargaining Solution.[5] This type of game has a unique subgame perfect equilibrium in which each bargainer (without delegation) gets a share of the total surplus based on factors such as their relative patience, and/or ability to avoid bargaining breakdown, and/or bargaining costs. For most part of the paper, we will take a reduced-form approach to the bargaining problem and leave out the details of the bargaining game. Specifically, we suppose that if the seller bargains directly with the buyer with a valuation s, then the seller will get $x = r_1 s$, where $r_1 \in (0, 1)$ depends on some exogenous factors in the bargaining game which we do not specify. For example, in the standard Rubinstein game and assuming that the seller moves

[5] See Osborne and Rubinstein (1990) for discussions about the link between non-cooperative alternating bargaining games and the Nash Bargaining Solution.

first, $r_1 = (1 - \delta_2)/(1 - \delta_1\delta_2)$, where δ_1 and δ_2 are the discount factors of the seller and the buyer, respectively. If we use the Nash Bargaining Solution and suppose the seller's relative bargaining power is r_1 while the buyer's is $1 - r_1$, then maximizing $r_1 ln(x) + (1 - r_1) ln(s - x)$ gives $x = r_1 s$.

When the delegate bargains with the buyer on behalf of the seller, how much the delegate will get in equilibrium can be affected by the contract between the seller and the delegate. Our main focus in this paper is on how this commitment consideration affects the design of the delegation contract. To make meaningful comparisons, in this section, we analyze the optimal contract design problem ignoring the commitment effect. Then in the later sections, we will study how the commitment effect changes the design of optimal delegation contracts. So for now, we suppose that for some reason the buyer bargains with the delegate as if the delegate were representing himself. This could happen when the buyer does not know whether the delegate is representing himself or acting as the agent for the seller.[6]

Without any commitment effect on the bargaining process, suppose the delegate's equilibrium share of the total surplus is r. We further assume that before bargaining with the buyer, the delegate can exert efforts to improve his position to get a better deal. We consider two kinds of effort in this paper. The first is "bargaining effort", which increases the delegate's share for any fixed surplus. In this case we write the delegate's share r as a function of his effort e; and we assume that for all e, $r(e) \in (0, 1)$, $r'(e) > 0$ and $r''(e) \leq 0$. Another type of effort is "marketing effort", which increases the probability that the delegate finds a buyer. Conditional on finding a buyer, the delegate will get a fixed share of r_0. We write the probability of finding a buyer p as a function of the delegate's marketing effort; and assume $p(e) \in (0, 1)$, $p'(e) > 0$ and $p''(e) \leq 0$. For a fixed surplus s, the expected price the delegate can get in the case of bargaining effort is $x = r(e)s$ while in the case of marketing effort is $x = r_0 p(e)s$, so there is no real difference in the expected price between these two types of efforts. Indeed, in this section, the two cases are the same in the absence of commitment effect (and we will use the bargaining effort interpretation). But in the next section, when commitment effect is present, the two cases will yield somewhat different results.

The delegate incurs effort cost of $C(e, t)$, where t is his "type" that characterizes his disutility of effort. We make the following standard assumptions on $C(e, t)$: (i) $C(e, t)$ is strictly increasing and convex in e, $C_e = \partial C/\partial e > 0$ and $C_{ee} = \partial^2 C/\partial e^2 > 0$; and (ii) higher types have lower effort cost and lower marginal effort cost, that is, $C_t = \partial C/\partial t < 0$ and $C_{et} = \partial^2 C/\partial e \partial t < 0$. The seller does not observe either the effort or the type of the delegate.

[6] Fershtman and Kalai (1997) show that when the third party (here the buyer) either does not know whether or not the delegate is representing himself or simply does not observe the details of the contract, no commitment effect is still a trembling hand sequential equilibrium.

Therefore, there are *both moral hazard and adverse selection* in the delegation relationship. At the time the seller is contracting with the delegate, the seller knows that the delegate's type is drawn from a distribution function $F(t)$ with density function $f(t) > 0$ for every $t \in [\underline{t}, \bar{t}]$, the domain of t.

Throughout the paper, we make the following standard assumption on $F(t)$:

Assumption 1 *The distribution of types $F(t)$ satisfies the monotone hazard rate property, that is, $f(t)/[1 - F(t)]$ is increasing in t.*

This assumption is satisfied by common distributions, such as uniform or log-normal.

For simplicity, we also make the following technical assumptions:

Assumption 2 *(i) C_{et} is a negative constant; (ii) $r'(e)$ and $p'(e)$ are positive constants.*

These two technical assumptions ensure that the agent's expected payoff function is concave. The results of the paper will not be affected if alternatively we make more general but less intuitive assumptions involving C_{eet}, C_{ett}, r'' and p''. By Part (ii), we will write $r(e) = r_0 + r'e$ and $p(e) = p_0 + p'e$, where r' and p' are positive constants.

The timing of the game is as follows. At date 0, the seller (she, henceforth) offers a menu contract to the delegate (he, henceforth). The contract is observable and non-renegotiable. At date 1, nature reveals to the delegate his type t. Then he decides whether to continue the game or quit. If he stays in the game, then at date 2, he chooses an effort level e. At date 3, the delegate meets the buyer, learns the buyer's valuation of the good, and they bargain over a price. Finally, once a deal is reached, the delegate gives the sale revenue to the seller, who then pays the delegate according to their contract. Throughout the game, the seller can only observe the sale revenue. This implicitly assumes that the delegate and the buyer cannot collude, otherwise it would be easy for the buyer to hide some of the revenue. This no-collusion assumption can be justified by the reputation concerns of the delegate or legal constraints.

We assume that all the three players are risk-neutral. Suppose the total surplus is s, and the delegate obtains x (i.e., the price is x) for the seller, and the seller pays the delegate a wage of w. Then the seller's utility is $U_P = x - w$, the delegate gets a utility of $U_D = w - C(e,t)$, and the buyer's utility is $U_B = s - x$.

For future comparisons, let us consider first the case in which both the delegate's effort and type are observable to the seller. For a delegate of type t, the seller asks him to exert effort $e(t)$ and pays him a wage that covers his effort cost and his reservation utility. So $w(t) = C(e(t),t) + U_0$. Then the seller's expected profit is simply $EU_P = \int_{\underline{s}}^{\bar{s}}[r(e)s - w(t)]dG(s) = r(e)E(s) - C(e,t) - U_0$, where $E(s) = \int_{\underline{s}}^{\bar{s}} s dG(s)$. So the optimal effort $e_{FB}(t)$ for the seller satisfies the following condition:

$$r'E(s) = C_e(e_{FB}(t), t) \tag{1}$$

where subscripts are partial derivatives with respect to the corresponding variable (that is, $C_e = \partial C/\partial e$). By our assumptions, the second-order condition is satisfied and the solution to Equation (1) is unique. Also the optimal effort $e_{FB}(t)$ increases in the delegate's type t. Note that since both the delegate's effort and type are observable, there is no need to make wage contingent on sale revenue.

When the seller does not observe the delegate's effort and type, the optimal contract design problem can be analyzed in the mechanism design framework. By the revelation principle, it is without loss of generality to focus on direct revelation mechanisms in which the delegate is provided proper incentives to reveal his type truthfully. In a direct revelation mechanism, a seller's mechanism consists of a wage schedule $w(\hat{t}, x)$ that depends on the delegate's announced type $\hat{t}$ and the sale revenue x he eventually brings back, and a recommendation of effort level $e(\hat{t})$ that depends only on his announced type $\hat{t}$. Given the seller's mechanism, the delegate of type t chooses an announcement of type $\hat{t}$ and an effort level to maximize his expected utility $EU_D = \int_{\underline{s}}^{\bar{s}} w(\hat{t}, x)dG(s) - C(e, t)$.

Formally, the seller's problem is to find a wage schedule $w(\hat{t}, x)$ and a recommendation $e(\hat{t})$ that solves

$$\max_{\{w(t,x),e(t)\}} EU_P = \int_{\underline{t}}^{\bar{t}} \int_{\underline{s}}^{\bar{s}} [x - w(t, x)]dG(s)dF(t) \tag{2}$$

subject to

(i) $(t, e(t)) \in argmax_{\{\hat{t},e)\}}\ EU_D = \int_{\underline{s}}^{\bar{s}} w(x, \hat{t})dG(s) - C(e, t)$

(ii) $U_D(t) = \int_{\underline{s}}^{\bar{s}} w(t, x)dG(s) - C(e(t), t) \geq U_0$, $\forall t$

(iii) $x = r(e)s$, $\forall s$

Condition (i) is the incentive compatible constraint for the delegate. It states that he finds it optimal to report his true type and to choose the recommended level of effort. The interim participation constraint (condition (ii)) requires that the optimal contract has to ensure the delegate at least his reservation utility. Finally, condition (iii) describes the bargaining outcome for every possible buyer's valuation when the commitment effect of delegation contract is ignored.

The mechanism design problem can be solved in two steps. In the first step, we characterize the conditions for an optimal mechanism; and then in the second step we find contracts that implement the optimal mechanism. The results of this section and their derivation closely follow McAfee and McMillan (1987) (see also Laffont and Tirole 1986).

To characterize the conditions for an optimal mechanism, suppose the seller can observe the delegate's effort but not his type and therefore can force upon him an effort schedule $e(\hat{t})$. Then the IC condition (i) is reduced to truth-telling only. Using the Envelope Theorem and integration by parts, one can simplify the mechanism design problem to (technical details in the Appendix):

$$\max_{e(t)} \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E(s) - C(e,t) + C_t(e,t)[\frac{1-F(t)}{f(t)}] \right\} dF(t) - U_0 \tag{3}$$

Let $e^*(t)$ be a solution to Equation (3). Then it has to satisfy the following first-order condition:

$$r'E(s) = C_e(e^*,t) - C_{et}[\frac{1-F(t)}{f(t)}] \tag{4}$$

The following proposition gives the (sufficient) conditions for an optimal mechanism.

Proposition 1 *If a wage contract $w(\hat{t},x)$ can induce the delegate to (i) truthfully reveal his type, and (ii) choose $e^*(t)$, and guarantees him the reservation utility, then the mechanism $\{w(\hat{t},x), e^*(t))\}$ is optimal.*

Proof: See the Appendix.

Comparing Equations (1) and (4), one can see that the optimal effort in the presence of agency problems $e^*(t)$ is lower than that under complete and perfect information ($e_{FB}(t)$) for all types but $\bar{t}$. This is because the term $C_{et}[1 - F(t)]/f(t)$ in Equation (4) is negative for all $t < \bar{t}$. This term has the standard interpretation as the information rent to the delegate. Because of asymmetric information between the seller and the delegate, the economic cost of effort to the seller consists of the direct effort cost to the delegate $C(e,t)$ and the information rent. Equation (4) then simply says that marginal benefit of effort equals marginal cost of effort. Since the information rent increases the marginal cost of effort, the optimal level of effort should be lower.

The next step is to find contracts that satisfy all the conditions in Proposition 1. Consider the following contract that is linear in sale revenue:

$$w(\hat{t},x) = \alpha^*(\hat{t}) + \beta^*(\hat{t})x \tag{5}$$

where $\alpha^*(\hat{t})$ and $\beta^*(\hat{t})$ are

$$\alpha^*(\hat{t}) = C(e^*(\hat{t}),\hat{t}) - \int_{\underline{t}}^{\hat{t}} C_t(e^*(\nu),\nu)d\nu - \frac{C_e(e^*(\hat{t}),\hat{t})}{r'}r(e^*(\hat{t})) + U_0$$

$$\beta^*(\hat{t}) = \frac{C_e(e^*(\hat{t}),\hat{t})}{r'E(s)}$$

The next proposition states that this linear contract actually implements the optimal mechanism.

Proposition 2 *The linear contract presented in Equation (5) implements the optimal recommended effort $e^*(t)$ and induces truthful report of type.*

Proof: See Appendix.

The intuition for the optimality of the linear contract is as follows. The seller needs to provide incentives to the delegate for him to tell the truth and follow the recommended effort. Because of risk-neutrality, these two tasks can be separately accomplished by the linear contract: The slope of the linear contract in Equation (5) provides proper effort incentives while the constant takes care of truth-telling about type.

A simple corollary can be derived from Proposition 2:

Corollary 1 *In the optimal linear contract, the optimal effort $e^*(t)$ and the sharing term $\beta^*(t)$ are non-decreasing in type, and the constant term $\alpha^*(t)$ is non-increasing in type.*

Proof: See Appendix.

This corollary says that with the optimal linear contract, a more able delegate (who dislikes effort less) is provided stronger incentives and hence works harder than a less able one. In particular, it can be checked that the highest type delegate gets all the residual sale revenue ($\beta^*(\bar{t}) = 1$) and exerts the efficient effort ($e^*(\bar{t}) = e_{FB}(\bar{t})$). Since a more able delegate is rewarded a higher proportion of the sale proceeds, the fixed portion of his wage is smaller than that of a less able delegate. In fact, for delegates of sufficiently high types, their fixed portion is negative. The interpretation is that lower types opt for higher fixed wage and smaller commissions, while higher types choose higher commissions and pay fees to get the job (such as franchise fees). Since we assume away any commitment effect by the delegation contract in this section, it does not make a difference whether the fixed portion of the wage contract α is paid before or after the bargaining game. But for the purpose of comparison with later sections, we suppose α is paid up front when the delegate takes the job (accept the contract) but before bargaining with the buyer.

3. LINEAR CONTRACT WITH COMMITMENT EFFECT

In the preceding section, we demonstrate that a linear delegation contract can implement the optimal mechanism for the seller IF delegation contracts have no commitment effect. But as the delegation literature has demonstrated, in general what kind of contracts the seller has for the delegate can affect the bargaining process between the delegate and the buyer. Hence in designing the delegation contract, the seller should take advantage of the contract's potential strategic value. In this section, we study how this commitment effect influences the seller's contract choice and explore its implications. Due to the complexity of the problem, we focus on contracts that are still linear in nature. We first analyze the case with "bargaining effort" and in Subsection 3.2 the "marketing effort" case.

3.1 Bargaining Effort

A first thing to notice is that the linear contract given in Equation (5) (with $\alpha(t)$ being paid up front) does not have any commitment power. The reason is simple. The up-front payment $\alpha(t)$ does not have any impact on the bargaining process since it is sunk before the bargaining game. What matters for the bargaining game is that the delegate gets $w = \beta x$ if the agreed price is x. But the bargaining outcome with this contract is the same as when the delegate is representing himself (in which case his utility is simply x), because a change of scale in the delegate's utility does not affect his behavior. Therefore, the bargaining outcome is still $x = r(e)s$, $\forall s$, and everything is the same as in the previous section.

The seller can do better by modifying the linear contract given in Equation (5) to take advantage of the commitment effect. Consider the following contract.

$$w(\hat{t}, x) = \alpha(\hat{t}) + \beta(\hat{t})(x - z(\hat{t})) \tag{6}$$

where $\alpha(\hat{t})$ is an upfront payment from the seller to the delegate and $z(\hat{t})$ is a minimum price that the seller wants the delegate to obtain. If the delegate brings back more than $z(\hat{t})$, then she pays him a commission β of what the delegate obtains in excess to the minimum price $z(\hat{t})$. Otherwise, the delegate has to pay back money to the seller.[7] Note that the contract considered in the previous section is a special case of the above contract with $z = 0$ for all $\hat{t}$.

Assuming the contract is credible to the buyer, then it will affect the bargaining between the delegate and the buyer. The bargaining outcome under this contract is reported in the following lemma.

[7] Any amount of penalty for a sale price below the minimum price will have the same effect. See Lemma 1.

Lemma 1 *Suppose the delegation contract is given by Equation (6). Then the equilibrium outcome from the bargaining stage is* $x = r(e)(s - z(\hat{t})) + z(\hat{t})$, $\forall s \geq z(\hat{t})$. *When* $s < z(\hat{t})$, *there will be no agreement and everyone gets zero.*

Proof: Suppose $s \geq z(\hat{t})$. Define $\tilde{s} = s - z(\hat{t})$. The delegate has to get at least $z(\hat{t})$ for the seller in order to get paid. So the "real" surplus he and the buyer can bargain over is $\tilde{s}$, of which the delegate should get $r(e)\tilde{s}$. One can easily verify this with a Rubinstein game. So $x = r(e)(s - z(\hat{t})) + z(\hat{t})$. When $s < z(\hat{t})$, there is no way the delegate can get a positive wage from a deal, so there will be no agreement in this case. *Q.E.D.*

From Lemma 1, we can see that when $s \geq z(\hat{t})$, the seller gains an additional amount of surplus $(1 - r(e))z(\hat{t})$ purely from the commitment effect. And this commitment value is larger when the minimum price z is set higher, as long as it is not too high to prevent a deal. Lemma 1 also points out the potential cost of using a minimum price as a commitment device. That is, the seller may go over the board and set a too high price target that prevents the delegate from reaching a deal with the buyer.

If the seller sets a minimum price $z \in [0, \underline{s}]$, then for any possible s the delegate and the buyer will reach a deal. Since commitment comes without cost for $z \in [0, \underline{s}]$, it seems that the seller should seek the maximum amount of commitment in this range. This intuition is verified in the following lemma.

Lemma 2 *For any* $z < \underline{s}$, *the seller can get a greater expected payoff by increasing the minimum price* z. *Therefore, the seller should set the minimum price not less than* $\underline{s}$ *for every* $\hat{t}$.

Proof: See Appendix.

Since the contract analyzed in the previous section corresponds to $z = 0$, Lemma 2 implies that it is not optimal when delegation contracts have commitment power.

Now the central question is whether the seller wants to set a minimum price higher than $\underline{s}$. The seller's mechanism design problem can be stated as

$$\max_{\{\alpha(t),\beta(t),e(t),z(t)\}} EU_P = \int_{\underline{t}}^{\bar{t}} \left\{ \int_{z(t)}^{\bar{s}} \left[x - \beta(t)(x - z(t))\right] dG(s) - \alpha(t) \right\} dF(t) \tag{7}$$

subject to

(i) $(t, e(t)) \in argmax_{\{\hat{t},e\}}\ U_D = \alpha(\hat{t}) + \beta(\hat{t}) \int_{z(\hat{t})}^{\bar{s}} (x - z(\hat{t})) dG(s) - C(e, t)$

(ii) $U_D(t) \geq U_0$, $\forall t$

(iii) $x = r(e(t))(s - z(t)) + z(t)$, for $s \geq z(t)$, and 0 otherwise

(iv) $z(t) \in [\underline{s}, \bar{s}]$ for all t

As before, this problem can be solved in two steps. First we find the conditions for the optimal effort $e^B(t)$ and minimum price $z^B(t)$, (where the superscript B stands for "bargaining effort"). Following similar technical steps as in the proof of Proposition 1, we can rewrite the problem as:

$$\max_{e(t),z(t)} \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E[s - z|s \geq z] + z[1 - G(z)] - C(e,t) + C_t(e,t)[\frac{1 - F(t)}{f(t)}] \right\} dF(t) - U_0 \quad (8)$$

where the argument (t) is suppressed in e and z; $E[s - z|s \geq z] = \int_z^{\bar{s}}[s - z]dG(s)$ and $z \in [\underline{s}, \bar{s}]$.

By point-wise differentiation of Equation (8), and assume interior solutions (i.e., $z^B \in (\underline{s}, \bar{s})$), $e^B(t)$ and $z^B(t)$ must satisfy the following first-order conditions:

$$r' E[s - z^B|s \geq z^B] = C_e(e^B, t) - [\frac{1 - F(t)}{f(t)}]C_{et} \quad (9)$$

$$(1 - r(e^B))(1 - G(z^B)) - z^B g(z^B) = 0 \quad (10)$$

From Equation (10), one can see that z^B must be less than $\bar{s}$, since $\partial EU_P/\partial z = -\bar{s}g(\bar{s}) < 0$ at $z = \bar{s}$.

To implement the optimal mechanism, the next step is to find the optimal α and β that induce the delegate to report his true type and then choose the desired level of effort e^B. Let $\alpha^B(\hat{t})$ and $\beta^B(\hat{t})$ in contract (6) be such that:

$$\begin{aligned} \alpha^B(\hat{t}) &= C(e^B(\hat{t}), \hat{t}) - \int_{\underline{t}}^{\hat{t}} C_t(e^B(\nu), \nu)d\nu - \frac{C_e(e^B(\hat{t}), \hat{t})}{r'} r(e^B(\hat{t})) + U_0 \\ \beta^B(\hat{t}) &= \frac{C_e(e^B(\hat{t}), \hat{t})}{r' E[s - z^B(\hat{t})|s \geq z^B(\hat{t})]} \end{aligned} \quad (11)$$

The next proposition says that the contract (11) implements the optimal level of effort $e^B(t)$.

Proposition 3 *The linear contract (11) with the optimal minimum price $z^B(t)$ implements the recommended effort $e^B(t)$ and induces the delegate to report his true type.*

If the seller's optimal minimum price turns out to be $\underline{s}$, then the first-order condition for the optimal effort, Equation (9), is reduced to

$$r'[E(s) - \underline{s}] = C_e(e^B, t) - [\frac{1 - F(t)}{f(t)}]C_{et} \tag{12}$$

Denote this solution by $\tilde{e}(t)$.

Proposition 4 *Suppose for some $\tilde{t} \in (\underline{t}, \bar{t}]$, $1 - r(\tilde{e}(\tilde{t})) > \underline{s}g(\underline{s})$. Then the seller will set the optimal minimum price $z^B(t)$ above $\underline{s}$ for any delegate of type in $[\underline{t}, \tilde{t}]$. As a result, these low type delegates fail to reach agreements with the buyer with positive probabilities.*

Proof: First note that $\tilde{e}(t)$ is non-decreasing in t. Since r is increasing in e, $1 - r(\tilde{e}(t)) > \underline{s}g(\underline{s})$ for any $t \in [\underline{t}, \tilde{t}]$. Suppose that the seller chooses $z^B = \underline{s}$ and $\tilde{e}(t)$ as in Equation (12) for some $t \in [\underline{t}, \tilde{t}]$. From the first-order condition (10), the seller can increase her expected payoff by choosing a minimum price $z > \underline{s}$. Contradiction. *Q.E.D.*

Proposition 4 points out that the seller's strategic use of delegation contracts may result in bargaining failures. Note that the condition in Proposition 4 is sufficient but not necessary. To understand this condition, let us suppose that the buyer's valuation s is uniformly distributed on $[\underline{s}, \bar{s}]$. If $\underline{s} = 0$ or $\bar{s}$ is very large and $1 - r$ is bounded from below, then for any $t \in [\underline{t}, \bar{t}]$, the seller sets a minimum price above $\underline{s}$. Otherwise, let $r(\tilde{e}(\tilde{t})) = k$ and the condition in Proposition 4 is equivalent to $(1.5 - k)\Delta s > E(s)$ where $\Delta s = \bar{s} - \underline{s}$ and $E(s) = (\bar{s} + \underline{s})/2$. So Proposition 4 roughly says that when the dispersion in the buyer's valuation is large relative to the expected gain from trade, the seller is more likely to set a minimum price higher than the buyer's minimum valuation. Intuitively, the more uncertain the seller is about the buyer's valuation, the more likely she wants to "over-commit" the delegate in order to ensure a relatively high price in most states of the world. On the other hand, if the expected valuation is high relative to the dispersion of valuation, then the seller does not want to risk losing potential profitable deals by over-committing the delegate. To see this last point, consider the converse of Proposition 4. From Equation (10), it is clear that if the valuation distribution satisfies $sg(s)/[1 - G(s)] \geq 1$ for every s, then the seller will always set $z^B = \underline{s}$. For uniformly distributed valuation, this condition simplifies to $2\underline{s} \geq \bar{s}$, or $E(s) \geq 1.5\Delta s$. So when the uncertainty about valuation is relatively small, the seller will set $z^B = \underline{s}$.

The next proposition emphasizes the relationship between the optimal effort and minimum price.

Proposition 5 *In the seller's optimal mechanism, the optimal effort level $e^B(t)$ is non-decreasing in the delegate's type, and the optimal minimum price $z^B(t)$ is non-increasing in the delegate's type. Therefore, higher type delegates are given more chance of success in agreement and work harder than lower types.*

Proof: See the Appendix.

The key to understanding Proposition 5 is that commitment through minimum price and the delegate's effort are *substitutes* for the seller. An easy way to see this is through the bargaining outcome equation $x = r(e)(s - z) + z$. Clearly, the marginal revenue of effort decreases in the minimum price z. More formally, one can see from Equation (8) that the seller's expected payoff function $EU_P(e, -z, t)$ is supermodular in $(e, -z, t)$. By the monotone comparative statics (see Milgrom and Shannon 1994), $e^B(t)$ and $-z^B(t)$ must be non-decreasing in t. Intuitively, Proposition 5 says that since it is relatively easier to induce a more able delegate to work hard and get a good price, the seller will impose a smaller minimum price for him to reduce the chance of no deal.

3.2. Marketing Effort

Now we suppose that the delegate's effort is spent on marketing to attract or find a buyer. The delegate finds a buyer with probability $p(e) \in (0,1)$, where $p(e) = p_0 + p'e$. For simplicity, the delegate's bargaining power relative to the buyer is assumed to be fixed and equals $r_0 \in (0,1)$.

We still focus on linear contracts with minimum prices as in Equation (6). Clearly Lemma 1 from Section 3.1 applies here for a constant r_0. But the seller will get a positive price and pay the delegate only when the delegate finds a buyer. It is also easy to see that Lemma 2 holds for marketing effort as well, that is, the seller will set a minimum price no less than $\underline{s}$. Commitment with a minimum price equal to $\underline{s}$ is costless to the seller, so she should take advantage of it. Again the central question is whether the seller wants to set a minimum price above $\underline{s}$. To answer this question we have to analyze the following optimal mechanism problem:

$$\max_{\{\alpha(t),\beta(t),e(t),z(t)\}} EU_P = \int_{\underline{t}}^{\bar{t}} \left\{ p(e) \int_{z(t)}^{\bar{s}} [x - \beta(t)(x - z(t))]\, dG(s) - \alpha^M(t) \right\} dF(t) \tag{13}$$

subject to

(i) $(t, e(t)) \in argmax_{\{\hat{t},e\}}\ U_D = \alpha(\hat{t}) + p(e)\beta(\hat{t}) \int_{z(\hat{t})}^{\bar{s}} (x - z)(\hat{t}))dG(s) - C(e,t)$

(ii) $U_D(t) \geq U_0$, $\forall t$

(iii) $x = r_0(s - z) + z$, for $s \geq z$, and 0 otherwise

(iv) $z(t) \in [\underline{s}, \bar{s}]$ for all t

Notice that the delegate's expected payoff is the same as in the case of bargaining effort with $r(e)$ being replaced by $r_0 p(e)$. The only difference with problem (7) is how the delegate's effort affects the seller's expected payoff. Bargaining effort increases only the share from the revenue *net* of the minimum price, while marketing effort increases the probability of getting a certain amount of revenue *including the minimum price.*

As before, this problem can be solved in two steps. First we find the conditions for the optimal effort $e^M(t)$ and minimum price $z^M(t)$ (where M stands for "marketing"). Using the same technical steps as in the proof of Proposition 1, we can rewrite the problem as:

$$\max_{e(t),z(t)} \int_{\underline{t}}^{\bar{t}} \left\{ p(e)\Big[rE[s-z|s \geq z] + z[1-G(z)]\Big] - C(e,t) + C_t(e,t)[\frac{1-F(t)}{f(t)}] \right\} dF(t) - U_0 \tag{14}$$

Let $e^M(t)$ and $z^M(t)$ be the level of effort and minimum price that solve this problem. Assume interior solution for z^M. By point-wise differentiation of Equation (14), $e^M(t)$ and $z^M(t)$ must satisfy the following first-order conditions:

$$p'\left\{ r_0 E[s - z^M|s \geq z^M] + z^M[1 - G(z^M)] \right\} = C_e(e^M, t) - [\frac{1-F(t)}{f(t)}]C_{et}(e^M, t) \tag{15}$$

$$p(e^M)\Big[(1 - r_0)(1 - G(z^M)) - z^M g(z^M)\Big] = 0 \tag{16}$$

The second step is to find the contract coefficients α^M and β^M that satisfy the IC and participation constraints that implements the optimal effort e^M. Let $\alpha^M(\hat{t})$ and $\beta^M(\hat{t})$ in contract (6) be such that:

$$\begin{aligned} \alpha^M(\hat{t}) &= C(e^M(\hat{t}), \hat{t}) - \int_{\underline{t}}^{\hat{t}} C_t(e^M(\nu), \nu)d\nu - \frac{C_e(e^M(\hat{t}), \hat{t})}{p'} p(e^M(\hat{t})) + U_0 \\ \beta^M(\hat{t}) &= \frac{C_e(e^M(\hat{t}), \hat{t})}{r_0 p' E[s - z^M(\hat{t})|s \geq z^M(\hat{t})]} \end{aligned} \tag{17}$$

The next proposition states that the contract (17) and the minimum price $z^M(t)$ induce the delegate to exert the recommended effort $e^M(t)$.

Proposition 6 *The linear contract (17) with the optimal minimum price $z^M(t)$ implements the recommended effort $e^M(t)$ and induces the delegate to report his true type.*

Proof: See the Appendix.

Comparing Equations (10) and (16), one can see that the first-order conditions for the optimal minimum price are very similar in the two cases of bargaining and marketing effort. The main difference is that in the case of marketing effort, the minimum price can be solved from Equation (16) alone, and only depends on the distribution of the buyers' valuations and the delegate's relative bargaining power but not on the delegate's effort. The minimum price is also independent of the delegate's type. Similar to Proposition 4, we have the following result:

Proposition 7 *If $1 - r_0 > \underline{s}g(\underline{s})$, then the seller will set a minimum price above $\underline{s}$ for every delegate. Thus, with positive probability, the delegate and the buyer will not make a deal.*

This proposition says that, as in the case of bargaining effort, the seller's strategic manipulation of the delegation contract may cause bargaining failures between the delegate and the buyer. It is easy to see that when the buyer's valuation is disperse relative to the expected gains of trade, then the seller's optimal minimum price will be more likely to exceed the buyer's lowest valuation.

Unlike in the case of bargaining effort, commitment through minimum prices and incentives are *no longer substitutes* with marketing effort. To see this, observe from Equation (15) that $\partial^2 EU_P/\partial e \partial z = p'\Big[(1-r_0)(1-G(z)) - zg(z)\Big]$. This cross partial derivative is negative when z is close to $\bar{s}$. If it is negative for every valuation s, then the minimum price should be set at $\underline{s}$, in which case the minimum price and effort are trivially substitutes (or complements). In more general cases where the cross partial derivative is not negative for every valuation, the minimum price and effort are neither substitutes nor complements.

4. COMPARATIVE STATICS AND NUMERICAL EXAMPLES

In this section we want to derive comparative statics of the model that may be useful in certain applications. In doing so, we need to specify the model a little more further. Specifically, suppose the buyer's valuation is uniformly distributed in $[\underline{s}, \bar{s}]$ and the delegate's type is uniformly distributed in $[\underline{t}, \bar{t}]$. The revenue share the delegate can get is given by $r(e) = r_0 + r'e$ in the case of bargaining effort. The parameter r_0 is the share the delegate can get without extra unobservable effort, and the parameter r' measures how productive the delegate's bargaining effort is (marginal revenue of effort equals $r'E(s)$). To ensure $r \leq 1$, the meaningful range for bargaining effort is constrained to $[0, (1 - r_0)/r']$. On the other hand, in the case of marketing effort, the revenue share the delegate can get is a constant r_0, and the probability of finding a buyer is $p(e) = p_0 + p'e$. In this case, the effort is constrained to $e \leq (1 - p_0)/p'$ to ensure that $p(e) \leq 1$. The delegate's cost function is: $C(e,t) = \gamma_1(\bar{t} - t)e + \gamma_2 e^2$, with γ_1 and γ_2 both positive constants. Finally, we let $U_0 = 0$.

For concreteness, we will solve the model numerically with the following parameter values. The buyer's valuation is uniform on $[10, 950]$, and the delegate's type is uniform on $[0, 1]$. In the bargaining effort case, the delegate's bargaining share is $r(e) = 0.3 + 0.1e$, and $e \in [0, 7]$. In the marketing effort case, the bargaining share is $r_0 = 0.5$. The probability function is $p(e) = 0.3 + 0.1e$, and $e \in [0, 7]$. Under both interpretations, the effort cost function is $C(e,t) = 8(1 - t)e + 12e^2$. In this case, the total expected surplus from trade is 480.

4.1. No Commitment Effect

If delegation contracts do not have any commitment effect, our analysis in Section 2 shows that the seller's optimal effort schedule should maximize

$$(r_0 + r'e)E(s) - 2\gamma_1(\bar{t} - t)e - \gamma_2 e^2$$

where $E(s) = (\bar{s} + \underline{s})/2$. From Equations 4 and 5, the seller's desired level of effort and the commission rate of the delegation contract can be easily found as

$$e^* = \frac{r'E(s)}{2\gamma_2} - \frac{\gamma_1(\bar{t} - t)}{\gamma_2}$$

$$\beta^* = 1 - \frac{\gamma_1(\bar{t} - t)}{r'E(s)}$$

The solution to our numerical model is given in Table A.1.[8] In this case, since the delegate and the buyer will always make a deal, the total expected surplus from trade is 480, which is shared by the seller, the delegate and the buyer. The seller obtains an expected surplus of 169.6, and the buyer gets an expected surplus of 256. The remainder is the delegate's expected wage payment of 54.4, of which 40 is his expected effort cost and 14.4 his expected information rent.

Table 1: Comparative Statics: No Commitment

Increase in	t	$\bar{t}$	r'	$E(s)$	γ_1	γ_2
e^*	↑	↓	↑	↑	↓	↓
β^*	↑	↓	↑	↑	↓	–

The comparative statics are straightforward and are summarized in Table 1.

These results are easy to understand. Since higher type delegates have lower marginal effort costs, optimal effort (and hence incentives through commission rate) should increase in type. The marginal revenue of effort is the product of r' and the expected total surplus $E(s)$. Hence, holding other things fixed, increase in r' or the expected total surplus will lead to higher commission rates and greater effort. The parameter r' measures the importance of effort. When $r' = 0$, the moral hazard problem disappears. In this case, $\beta^* = 0$ and $e^* = 0$, and the seller pays the delegate a fixed wage equal to his reservation utility. On the other hand, the parameter γ_2 measures the difficulty of inducing high effort for any given type of delegate, hence has the opposite effect on the optimal effort as r'. The commission rate β^* is independent of γ_2 because the "physical" effort cost $\gamma_2 e^2$ is compensated by the fixed payment α^*.

Holding other things fixed, increase in $\bar{t}$ means that the degree of adverse selection is greater between the seller and the delegate and hence makes it harder to induce truth-telling from the delegate. Consequently, ceteris paribus, the higher $\bar{t}$, the lower the optimal effort and commission rate. To see this more clearly, consider the extreme case in which $\bar{t}$ collapses to $\underline{t}$ so that there is no adverse selection. Then $t = \bar{t} = \underline{t}$, so $\beta^* = 1$ and $e^* = r'E(s)/(2\gamma_2) = e_{FB}$. This is simply the standard result that the efficient outcome (for the seller) can be achieved with a sell out contract when there is moral hazard and the agent is risk-neutral.

The parameter γ_1 measures the intensity of agency problem between the seller and the delegate. Higher γ_1 means that different delegates differ more in their dislike of effort, which leads to higher information rents. Consequently, other things being equal, the seller would want to set a higher commission rate and induce greater effort from the delegate when γ_1 is lower. When $\gamma_1 = 0$, the delegate's type does not matter, and the seller should sell the good to the delegate.

4.2. Bargaining Effort

[8]The detailed solutions to the numerical model are presented in several Tables at the end of the paper.

Now suppose delegation contracts have commitment power and the delegate exerts bargaining effort. From Section 3.1, for every type t, the seller's desired effort and minimum price should maximize

$$\begin{aligned} U_{P,t} &= (r_0 + r'e)\frac{(\bar{s}-z)^2}{2\Delta s} + z\frac{\bar{s}-z}{\Delta s} - 2\gamma_1(\bar{t}-t)e - \gamma_2 e^2 \\ &= (r_0 + r'e)\frac{(E(s)+\frac{\Delta s}{2}-z)^2}{2\Delta s} + z\frac{(E(s)+\frac{\Delta s}{2}-z)}{\Delta s} - 2\gamma_1(\bar{t}-t)e - \gamma_2 e^2 \end{aligned}$$

We write $\bar{s} = E(s) + \Delta s/2$, where $\Delta s = \bar{s} - \underline{s}$, because we would like to separate out the effects of changes in the expected surplus and changes in the uncertainty (dispersion) of valuations.

The closed form solutions for the optimal effort and minimum price are not readily available from the first-order conditions (which lead to cubic equations of e and z). Table A.2 gives the solution for our numerical example. In this case, the optimal effort is much lower than the case with no commitment. Moreover, the optimal minimum price is set in between [365, 390]. This implies that the chance of bargaining failure is about 39 %. Because of the commitment effect, the seller's expected payoff jumps to 281.33, more than 65 % higher than that in the case of no commitment effect. The delegate's effort cost and information rent are both much lower. The buyer is also screwed, getting an expected payoff of 115, which is less than half of that in the case of no commitment. Bargaining failures cause welfare loss of about 76, about 16 % of the total expected surplus.

We derive comparative statics results for the case of bargaining effort (details in the Appendix), which are summarized in Table 2 below.

Table 2: Comparative Statics: Bargaining Effort

Increase in	t	$\bar{t}$	r'	$E(s)$	Δs	γ_1	γ_2
e^B	↑	↓	↑	↑	↓	↓	↓
z^B	↓	↑	↓	↑/↓	↑	↑	↑
β^B	↑	↓	↑	↑	↓	↓	↓

The cells with two arrows indicate ambiguous comparative statics.

The comparative statics of e^B and β^B with respect to $\{t, \bar{t}, r', E(s), \gamma_1, \gamma_2\}$ are the same as in the case with no commitment effect, and have the same interpretations as given in the previous subsection.

A new implication from commitment effect is that now the delegate's optimal effort and his incentives (measured by β^B) are lower if uncertainty about the buyer's valuation increases (i.e., Δs increases while holding $E(s)$ fixed). Without commitment effect, uncertainty about the buyer's valuation does not matter because both the seller and the delegate are risk-neutral. With commitment effect, the seller sets a minimum price z^B that can be higher than the buyer's lowest valuation. When $E(s)$ is fixed and the dispersion of valuation Δs increases, the buyer's lowest valuation must decrease. It follows that more likely the buyer's valuation falls below a fixed minimum price. Moreover, the optimal minimum price will increase in this case (see below). Thus, the probability of bargaining failure increases. As a result, the expected return to bargaining effort is reduced, thus leading to lower effort and lower incentives.

Another set of comparative statics results in Table 2 concerns the minimum price. In Section 3.1 we show that incentives and minimum prices are substitutes (Proposition 5) and they move in opposite directions as the delegate's type changes. In fact, this is also true with respect to $\bar{t}$, r', γ_1 and γ_2 (see the Appendix). Basically, when agency problems are more severe and hence it is more costly to induce efforts (higher $\bar{t}$, γ_1 and γ_2), then the seller will substitute incentives for commitment by increasing the minimum price. On the other hand, when effort is more productive (higher r'), then the seller will reduce the minimum price. When the expected surplus $E(s)$ increases (holding Δs fixed), there are two opposite effects on the minimum price. On one hand, effort is more productive, hence the minimum price should go down. On the other hand, since both $\underline{s}$ and $\bar{s}$ increase, the cost of commitment (i.e., no deal) decreases while the benefit of commitment increases, so the minimum price should go up. The net effect of $E(s)$ on z is thus ambiguous. For example, if effort is not very productive (low r') or is costly (high γ_2) or uncertainty about valuation Δs is relatively high, then the second effect dominates so the minimum price increases in $E(s)$. When the dispersion of valuation Δs increases (holding $E(s)$ fixed), the marginal cost of using minimum prices becomes relatively smaller than the marginal benefit. Therefore, minimum price increases in Δs. Moreover, effort will go down, also leading to higher minimum price.

4.3. Marketing Effort

Now we turn to the case of marketing effort. From Section 3.2, for every type t, the seller's desired effort and minimum price should maximize

$$U_{P,t} = (p_0 + p'e)\left[r_0 \frac{(E(s) + \frac{\Delta s}{2} - z)^2}{2\Delta s} + z\frac{(E(s) + \frac{\Delta s}{2} - z)}{\Delta s}\right] - 2\gamma_1(\bar{t} - t)e - \gamma_2 e^2$$

It is easy to check that the optimal effort, the minimum price and the slope of the contract are given by

$$e^M = \frac{p'[E(s) + \frac{\Delta s}{2}]^2}{4\gamma_2(2 - r_0)\Delta s} - \frac{\gamma_1(\bar{t} - t)}{\gamma_2}$$

$$z^M = \frac{1-r_0}{2-r_0}[E(s) + \frac{\Delta s}{2}]$$

$$\beta^M = \frac{2-r_0}{r_0} - \frac{2\gamma_1(\bar{t}-t)(2-r_0)^2\Delta s}{r_0 p'[E(s)+\frac{\Delta s}{2}]^2}$$

The solution to our numerical example is presented in Tables A.3 (no commitment) and A.4 (commitment). Unlike the bargaining effort case, now the optimal effort is higher with commitment than without commitment, that is, incentives and commitment are complements in this example. Consequently, the probability of finding a buyer is higher with commitment, and the commission rate is much higher (greater than 2), which resembles the results in Fershtman and Judd (1987, 1987b) that managers are "over-compensated" on the margin in equilibrium. The seller imposes a minimum price of 316, resulting in a 33% chance of bargaining failure. The welfare loss from bargaining failures is about 11% if holding effort fixed, about 3% if compared to the case under no commitment. The seller's expected utility increases from 74 to 103 (around 40%) due to both the commitment and incentive effects. The buyer is again the victim of the seller's commitment scheme, seeing his expected utility plunge from 88 to 43.

The comparative statics are summarized in Table 3.

Table 3: Comparative Statics: Marketing Effort

Increase in	t	$\bar{t}$	p'	$E(s)$	Δs	γ_1	γ_2
e^M	↑	↓	↑	↑	↓	↓	↓
z^M	–	–	–	↑	↑	–	–
β^M	↑	↓	↑	↑	↓	↓	–

The comparative statics of effort e^M and commission rate β^M are basically the same as in the case of bargaining effort. The minimum price now is independent of all the variables except the buyer's valuations (and the delegate's bargaining power r_0). Furthermore, the minimum price and effort are positively related when the expected valuation changes, but negatively related when the dispersion of valuation increases.

5. EXTENSIONS AND APPLICATIONS

5.1 Multi-dimensional Efforts

In many applications, the delegate's effort is multidimensional, that is, it affects both his chance of finding a buyer and how much he can get from a buyer. For example, the delegate (e.g., a car dealership) may spend time studying the demographics of the local market so that he can market the good more pointedly and also know better about the potential buyer. In such a case, the delegate's marketing and bargaining efforts are complements. In other cases, marketing and bargaining efforts can be substitutes. For instance, a car sales manager may have to allocate his time between taking training classes (or going to trade meetings, things that improve his bargaining position) and maintaining a clean showroom (or other things that attract potential buyers). Our basic model can be straightforwardly extended to situations where the delegate exerts multidimensional efforts.

Suppose the delegate chooses bargaining effort e_B to increase his share in the bargaining stage $r(e_B)$ and marketing effort e_M to increase the probability of finding a buyer $p(e_M)$. The expected revenue for the seller, given a type-t delegate, is

$$p(e_M)\Big\{r(e_B)E[s-z(t)|s\geq z(t)]+z(t)[1-G(z(t))]\Big\}$$

Hence, the optimal mechanism can be stated as choosing $e_B(t)$, $e_M(t)$ and $z(t)$ to maximize

$$\int_{\underline{t}}^{\bar{t}}\Bigg[p(e_M)\Big\{z[1-G(z)]+r(e_B)E[s-z|s\geq z]\Big\}- \\ -C(e_B,e_M,t)+C_t(e_B,e_M,t)[\frac{1-F(t)}{f(t)}]\Bigg]dF(t)-U_0 \tag{18}$$

subject to $z\in[\underline{s},\bar{s}]$.

The solution to the above program is technically quite involved. Here we only discuss briefly two polar cases: i) marketing and bargaining efforts are perfect complement, i.e., $e^B=e^M$; and ii) they are perfect substitutes. In both cases the delegate's effort decision can be reduced to a single-variable choice. Thus, the seller can control the delegate's behavior through a linear contract with a minimum price.

First suppose bargaining and marketing efforts are perfect complements, that is, $e_B=e_M$. The seller's problem (18) simplifies to choosing $e(t)$ and $z(t)$ to maximize

$$\int_{\underline{t}}^{\bar{t}}\Bigg[p(e)\Big\{z[1-G(z)]+r(e)E[s-z|s\geq z]\Big\}-C(e,t)+C_t(e,t)[\frac{1-F(t)}{f(t)}]\Bigg]dF(t)-U_0$$

Under certain technical conditions, the following first-order conditions (where the type is suppressed) characterize the interior solution:

$$\big[r(e)p'(e)+p(e)r'(e)\big]E[s-z|s\geq z]+p'(e)z[1-G(z)]=C_e(e,t)-[\frac{1-F(t)}{f(t)}]C_{et}$$

$$p(e)\Big\{[1-r(e)][1-G(z)]-zg(z)\Big\}=0$$

One can easily see that when $p'=0$, these equations are reduced to Equations (9) and (10) as in Section 3.1 on bargaining effort; and when $r'=0$, they are reduced to Equations (15) and (16) as in Section 3.2 on marketing effort.

It can be shown that the main results from Section 3 still hold under fairly general conditions. Specifically, there is a linear contract with the minimum price $z(t)$ that implements the optimal effort $e(t)$ and induces truth-telling. The seller may set the minimum price above the buyer's lowest valuation, leading to bargaining failures between the delegate and the buyer. The relationship between minimum price and effort is not as clean as in Section 3.1. However, when r' is large relative to p', that is, when the delegate's effort is more important to his bargaining position against the buyer than to the chance of him finding a buyer, then the optimal effort (hence incentives through commission rate) and minimum price are still negatively correlated.

Now we turn to the case in which bargaining and marketing efforts are perfect substitutes. Suppose the delegate has to allocate his total time (normalized to 1) between the two kinds of activities, i.e., $e_B+e_M\leq 1$. When at least one of the two efforts is productive and not very costly, then the delegate should be induced to utilize all available time, hence $e_B+e_M=1$. In such cases, substituting $e_M=1-e_B$ into Equation (18) gives

$$\int_{\underline{t}}^{\bar{t}} p(1-e_B)\left\{r(e_B)E[s-z|s\geq z]+z[1-G]-C(e_B,t)+C_t(e_B,t)[\frac{1-F(t)}{f(t)}]\right\}dF(t)-U_0$$

where $C(e_B,t)=C(e_B,1-e_B,t)$ and $C_t(e_B,t)=C_t(e_B,1-e_B,t)$.

Under certain technical conditions, the following first-order conditions (where all arguments are suppressed) characterize the interior solution:

$$[pr'-rp']\,E[s-z|s\geq z]-p'z[1-G(z)]=[C_{e_B}-C_{e_M}]-[\frac{1-F(t)}{f(t)}]\,[C_{e_Bt}-C_{e_Mt}]$$

$$p(1-e_B)\Big\{[1-r][1-G(z)]-zg(z)\Big\}=0$$

Again, the seller can implement the optimal effort using a linear contract with a minimum price under certain conditions and her manipulation of the delegation contract through the minimum price may result in bargaining failures. Since the bargaining effort and the minimum price are substitutes, so marketing effort and the minimum price will tend to be complements when the delegate's bargaining and marketing efforts are perfect substitutes. This implies that the minimum price will be negatively associated with the bargaining effort and positively with the marketing effort.

5.2 Unobservable Contracts

Another extension is to consider unobservable contracts, since contracts in many applications are not observable to other parties. As mentioned before, Katz (1991), Caillaud, Jullien and Picard (1995), Dewatripont (1988), Fershtman and Kalai (1997), Corts and Neher (1998), Kockesen and Ok (1999), and many others have addressed the issue of whether unobservable contracts are a credible commitment device. Depending on the other party's belief and the equilibrium refinement concept used, these papers find that unobservable contracts still have commitment values at least in some equilibria. This idea should be quite general and should apply to our model as well. However, these papers study quite different games (e.g., oligopolistic competition, take-it-or-leave-it bargaining) and some of them impose strong assumptions on out-of-equilibrium beliefs that may not be appropriate for our model. Hence it is not clear that we can directly apply the existing results to our model, particularly if we have an infinite-horizon Rubinstein bargaining game.

Fortunately, it seems that we can analyze unobservable contracts quite nicely if we employ an infinite-horizon Rubinstein bargaining game. The reason is that in the bargaining stage, all that matters in the delegation contract for the delegate and the buyer is the minimum price. So unobservable delegation contracts are equivalent to unobservable minimum prices. Then with secretive minimum prices, the delegate and the buyer bargain under one-sided asymmetric information (still assume the delegate knows the buyer's valuation). In such a bargaining setting, Gul, Sonnenschein and Wilson (1986) and Gul and Sonnenschein (1988) have shown that under fairly reasonable conditions, the minimum prices can be revealed rather quickly in equilibrium. Applying their results to our model implies that unobservable contracts should not matter that much, that is, results obtained in this paper should hold at least qualitatively if delegation contracts are not observable to the buyers.

5.3 Applications to Car Dealerships

Our model can be applied to car dealerships. On one level, car dealership owners are the sellers, sales managers are the delegates, and customers are the buyers. So the relationship fits nicely to the model: sales managers are hired by car dealers to bargain with buyers. As agents of the car dealerships, sales managers's efforts and skills are critical to the businesses of car dealerships. To provide proper incentives for sales managers, their compensation contracts typically contain both fixed wages and commissions. On the other hand, in the bargaining with customers, a common tactic sales managers use is to try to convince customers that they cannot sell below a certain price, typically dealer's invoice prices plus some "reasonable" markups. Trivially, any bargainer would like to claim that he could not give in too much. But in the context of car dealerships, the use of sales managers seems to make the commitment to a minimum price more credible. Note that our discussion in the previous subsection indicates that the minimum prices do not have to be made public in order to have commitment effects.

Of course, whether or not commitment effects are an important consideration for car dealerships is an empirical question. If data on compensations and performances of car sales managers is available, it may be feasible to test the implications of our model using the comparative statics derived in Section 4. Such empirical work is badly needed for the delegation literature, because, to our best knowledge, there has been no empirical study providing evidence on the existence of strategic delegation despite a large number of theoretical works.

Our model may also be applied to car dealerships on another level, albeit not as clear-cut as before. That is, we can think of car manufacturers as the sellers, car dealerships as the delegates, and customers as the buyers. One may object to this application by arguing that car dealerships are not "hired" by car manufacturers to sell the cars; rather, car dealers buy the cars from the manufacturers and then sell the cars to the buyers. But the picture is in fact more complicated. Sometimes car dealers have to do special orders from car makers for the customers after they agree on prices, in which cases car dealers are not legal owners of the cars. Even for cars in the dealers' parking lots, the contractual relationships between car makers and car dealers are not over at the moment dealers get cars from car makers. Instead, their contractual relationships last until cars are transferred to the buyers. For example, one of the common provisions is dealer holdbacks, whereby car makers promise to pay the dealers a certain percentage (usually 2-3 %) of the invoice prices when the cars are sold. A natural question is why not car makers just simply sell the cars to the car dealers at a lower price and leave the car dealers to sell at a higher price. One explanation for dealer holdbacks is that car makers try to convince the buyers that the dealers' minimum prices are the invoice prices. In a world where customers differ greatly in their time costs, some of them might be "cheated" to believe so. In addition to dealer holdbacks, car makers offer many different kinds of incentives (dealer rebates, volume discounts, credit discounts, etc.) from time to time, which makes customers hard to find out. In sum, the contractual relationships between car makers and car dealers are quite complicated and may involve some considerations of commitment effects.[9] More detailed analysis of the contractual relationships is thus highly desirable.

[9]Contracts between car makers and dealers are franchise contracts, and have to take into account many other important considerations (e.g., competition among dealers) in franchise relationships, see, e.g., Klein and Murphy (1988), Klein (1995) and Tirole (1988) for an excellent textbook treatment. See also Bresnahan and Reiss (1985) for an early empirical work on pricing practices between car manufacturers and dealers.

6. CONCLUSION

In this paper we develop a framework that can be used to analyze the interactions between agency problems and commitment effect in delegated bargaining situations. Among other things, we find that the seller's strategic manipulation of the delegation contracts can cause bargaining failures between her delegate and the buyer. Furthermore, the interactions between incentives and commitment depend on the nature of the agency problem: they are substitutes in the case of bargaining effort but not in the case of marketing effort. We also derive comparative statics and apply the model to car dealerships.

APPENDIX

Proof of Proposition 1:

Let EU_P be the seller's expected utility when she pays the delegate a wage $w(\hat{t}, x)$, that is,

$$EU_P = \int_{\underline{t}}^{\bar{t}} E[x - w(\hat{t}, x)]dF(t) = \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E(x) - E[w(\hat{t}, x)] \right\} dF(t) \tag{19}$$

and let $U_D(\hat{t}, t)$ be the type-t delegate's utility when he announces type $\hat{t}$, which is

$$U_D(\hat{t}, t) = E[w(\hat{t}, x)] - C(e, t) \tag{20}$$

where the expectation $E[.]$ in these two equations is taken over the random variable s.

Consider a seller's effort recommendation $e(\hat{t})$. Suppose the delegate follows it. The IC condition reduces to truth-telling only. The first-order condition with respect to the delegate's type announcement is

$$\frac{\partial U_D(\hat{t}, t)}{\partial \hat{t}}|_{\hat{t}=t} = 0$$

Let $U_D(t) = U_D(t, t)$ be the delegate's utility when he reports his true type. The total derivative of $U_D(t)$ with respect to his type report can be obtained from the Envelope Theorem as follows

$$\frac{dU_D(\hat{t}, t)}{dt}|_{\hat{t}=t} = \frac{\partial U_D(\hat{t}, t)}{\partial \hat{t}}|_{\hat{t}=t} + \frac{\partial U_D(\hat{t}, t)}{\partial t}|_{\hat{t}=t} = \frac{\partial U_D(\hat{t}, t)}{\partial t}|_{\hat{t}=t} = -C_t(e, t)$$

where the last equality comes from Equation (20). Since this is a total derivative the delegate's utility can be reconstructed by integrating this equation with respect to his type.

$$U_D(t) = U_D(\underline{t}) - \int_{\underline{t}}^{t} C_t(e, \nu)dF(\nu) \tag{21}$$

So, from Equations (20) (evaluated at the delegate's true type) and (21) we can solve for the wage schedule as follows

$$E[w(t, x)] = U_D(t) + C(e, t) = U_D(\underline{t}) - \int_{\underline{t}}^{t} C_t(e, \nu)dF(\nu) + C(e, t)$$

Plugging the wage schedule into the seller's expected utility function (Equation (19)) gives

$$EU_P = \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E(s) - C(e, t) + \int_{\underline{t}}^{t} C_t(e, \nu)dF(\nu) \right\} dF(t) - U_D(\underline{t}) \tag{22}$$

Next, integrating by parts the second term of the integral yields

$$\int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t} C_t(e,\nu)dF(\nu)dF(t) =$$

$$= \left[-(1-F(t)) \int_{\underline{t}}^{t} C_t(e,\nu)dF(\nu)\right]_{\underline{t}}^{\bar{t}} + \int_{\underline{t}}^{\bar{t}} [\frac{1-F(t)}{f(t)}] C_t(e,t)dF(t) =$$

$$= \int_{\underline{t}}^{\bar{t}} [\frac{1-F(t)}{f(t)}] C_t(e,t)dF(t) \tag{23}$$

Note that if the seller ensures a type-$\underline{t}$ delegate a utility $U_D(\underline{t}) = U_0$, the interim participation constraint is satisfied for all types. The reason is that the delegate's expected utility function (Equation (21)) is increasing in t since C_t is negative. Hence the seller should set $U_D(\underline{t}) = U_0$.

Using Equation (23) and $U_D(\underline{t}) = U_0$, one can rewrite Equation (22) as

$$EU_P = \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E(s) - C(e,t) + C_t(e,t)\frac{1-F(t)}{f(t)} \right\} dF(t) - U_0$$

This is Equation (3). Note that the seller has to pay the delegate his effort cost, his reservation utility and some information rent. The seller will choose an effort recommendation that maximizes her expected payoff. Differentiating point-wise with respect to effort, we get the following first-order condition for $e^*(t)$:

$$r'E(s) - C_e(e^*,t) + \frac{1-F(t)}{f(t)} C_{et} = 0$$

This is Equation (4). The second-order condition is clearly satisfied because the integrand in Equation (3) is concave in e: r' is a constant, $C_{ee}(.,t) > 0$, and C_{et} is a constant. *Q.E.D.*

Proof of Proposition 2:

For later reference, notice that effort $e^*(t)$ is non-decreasing in type. From Equation (4), it is clear that the "total" marginal cost of effort decreases with type (the inverse of the hazard rate decreases with type and C_{et} is negative). By the monotone comparative statics (Milgrom and Shannon 1994), effort must be non-decreasing in t.

If the seller offers the delegate the contract (5), the delegate's utility when he exerts effort e and reports $\hat{t}$ is

$$\begin{aligned} U_D(\hat{t},e,t) &= C(e^*(\hat{t}),\hat{t}) + \frac{C_e(e^*(\hat{t}),\hat{t})}{r'}[r(e) - r(e^*(\hat{t}))] - \\ &\quad - \int_{\underline{t}}^{\hat{t}} C_t(e^*(\nu),\nu)d\nu - C(e,t) + U_0 \end{aligned} \tag{24}$$

The first-order conditions are the following:

$$\frac{\partial U_D(\hat{t},e,t)}{\partial e} = C_e(e^*(\hat{t}),\hat{t}) - C_e(e,t) = 0$$

$$\frac{\partial U_D(\hat{t},e,t)}{\partial \hat{t}} = \frac{[r(e) - r(e^*(\hat{t}))]}{r'} \frac{d}{d\hat{t}} C_e(e^*(\hat{t}),\hat{t}) = 0$$

They are satisfied at $\hat{t} = t$ and $e = e^*(t)$. The second-order conditions for a maximum are also satisfied since the delegate's profit is concave in effort and the determinant of the second-order matrix is positive.

$$\frac{\partial^2 U_D(t,e,t)}{\partial e^2} \frac{\partial^2 U_D(t,e,t)}{\partial t^2} - \left(\frac{\partial^2 U_D(t,e,t)}{\partial e \partial t}\right)^2 = -C_{et} \frac{d}{dt} C_e(e^*(t),t) \geq 0$$

This last inequality holds because of the following equation (derived from Equation (4)):

$$\frac{d}{dt} C_e(e^*(t),t) = C_{et} \frac{\partial}{\partial t} [\frac{1-F(t)}{f(t)}] \geq 0 \tag{25}$$

Q.E.D.

Proof of Corollary 1:

That $\beta^*(t)$ is non-decreasing in type can be checked from Equation (25). In the beginning of the proof of Proposition 2 we showed that the effort is non-decreasing in type. From the definition of $\alpha^*(t)$ (Equation (5)),

$$\frac{\partial \alpha^*(t)}{\partial t} = -\frac{r(e^*(t))}{r'} \frac{d}{dt} C_e(e^*(t),t) \leq 0$$

Q.E.D.

Proof of Lemma 2:

Consider any direct revelation mechanism ($\alpha(\hat{t})$, $\beta(\hat{t})$, $z(\hat{t})$, $e(\hat{t})$), where $z(\hat{t}) < \underline{s}$. This mechanism gives the seller a revenue of $r(e(t))[E(s) - z(t)] + z(t)$. But the seller can do better with another mechanism which also implement the same effort recommendation $e(\hat{t})$ but imposes the minimum price equal to $\underline{s}$. Consider the following mechanism ($\tilde{\alpha}(\hat{t})$, $\beta(\hat{t})$, $\underline{s}$, $e(\hat{t})$), where $\tilde{\alpha}(\hat{t}) = \alpha(\hat{t}) + \beta(\hat{t}) r(e(\hat{t}))(\underline{s} - z(\hat{t}))$. The expected wage is the same since

$$\begin{aligned} E[w(x,\hat{t})] &= \tilde{\alpha}(\hat{t}) + \beta(\hat{t})E[x - \underline{s}] = \\ &= \tilde{\alpha}(\hat{t}) + \beta(\hat{t})r(e(\hat{t}))[E(s) - \underline{s}] = \\ &= \alpha(\hat{t}) + \beta(\hat{t})r(e(\hat{t}))(\underline{s} - z(\hat{t})) + \beta(\hat{t})r(e(\hat{t}))[E(s) - \underline{s}] = \\ &= \alpha(\hat{t}) + \beta(\hat{t})r(e(\hat{t}))[E(s) - z(\hat{t})] \end{aligned}$$

All the (IC) and (IR) conditions must be satisfied as they are in the old mechanism ($\alpha(\hat{t})$, $\beta(\hat{t})$, $z(\hat{t})$, $e(\hat{t})$). The cost to the seller is also the same, but her expected revenue increases since

$$\begin{aligned} E(x) &= r(e(t))E[s - \underline{s}] + \underline{s} = r(e(t))E(s) + [1 - r(e(t))]\underline{s} > \\ &> r(e(t))E(s) + [1 - r(e(t))]z(t) = r(e(t))[E(s) - z(t)] + z(t) \end{aligned}$$

Q.E.D.

Proof of Proposition 3:

The type-t delegate's utility when he reports $\hat{t}$, chooses e and is paid according to contract (11) is

$$\begin{aligned} U_D(\hat{t}, e, t) &= \alpha^B(\hat{t}) + \beta^B(\hat{t})r(e)E[s - z^B(\hat{t})|s \geq z^B(\hat{t})] - C(e,t) \\ &= C(e^B(\hat{t}), \hat{t}) + \frac{C_e(e^B(\hat{t}), \hat{t})}{r'}[r(e) - r(e^B(\hat{t}))] - \int_{\underline{t}}^{\hat{t}} C_t(e^B(\nu), \nu)d\nu - C(e,t) + U_0 \end{aligned}$$

Notice the similarity between this utility and that of Equation (24). The proof is similar to that of Proposition 2 with a change of the superscript "$*$" to the superscript "B" and a change of $E(s)$ to $E[s - z(\hat{t})|s \geq z(\hat{t})]$. The second-order condition is satisfied because $C_e(e^B(t), t)$ is non-decreasing in type. From Equation (9),

$$\begin{aligned} \frac{d}{dt}C_e(e^B(t), t) &= r'\frac{\partial E[s - z^B|s \geq z^B]}{\partial z}\frac{\partial z^B}{\partial t} + C_{et}\frac{d}{dt}\frac{1 - F(t)}{f(t)} = \\ &= -r'[1 - G(z)]\frac{\partial z}{\partial t} + C_{et}\frac{\partial}{\partial t}\frac{1 - F(t)}{f(t)} \end{aligned}$$

Proposition 5 shows that z is non-increasing with type. Hence the first term of the equality is non-negative. From Equation (25), the second term is also non-negative. *Q.E.D.*

Proof of Proposition 5:

From the seller's expected payoff function in Equation (8), we can show that

$$\frac{\partial^2 EU_P}{\partial e \partial(-z)} = r' \frac{\partial E[s - z | s \geq z]}{\partial(-z)} = r'[1 - G(z)] \geq 0$$

$$\frac{\partial^2 EU_P}{\partial e \partial t} = -C_{et}\left(1 - \frac{\partial}{\partial t}\frac{1 - F(t)}{f(t)}\right) \geq 0$$

$$\frac{\partial^2 EU_P}{\partial(-z)\partial t} = 0$$

Therefore, $EU_P(e, -z, t)$ is supermodular, and by the monotone comparative statics, $e(t)$ is non-decreasing in t and z is non-increasing in t. *Q.E.D.*

Proof of Proposition 6:

Recall that $p(e)$ in Section 3.2 has the same interpretation as $r(e)$ in Sections 2 and 3.1. The delegate's utility under contract (17) is

$$\begin{aligned} U_D(\hat{t}, e, t) &= \alpha^M(\hat{t}) + \beta^M(\hat{t}) r_0 p(e) E[s - z^M(\hat{t}) | s \geq z^M(\hat{t})] - C(e, t) = \\ &= C(e^M(\hat{t}), \hat{t}) + \frac{C_e(e^M(\hat{t}), \hat{t})}{p'}[p(e) - p(e^M(\hat{t}))] - \int_{\underline{t}}^{\hat{t}} C_t(e^M(\nu), \nu) d\nu - C(e, t) + U_0 \end{aligned}$$

Next compare the delegate's utility under this contract with his utility in the Proof of Proposition 2 (see Equation (24)). The rest of the proof is similar to that of Proposition 2 with a change of the superscript "*" to the superscript "M". The second-order condition is satisfied because $C_e(e^M(t), t)$ is non-decreasing with type. Using Equation (15), and taking into account that z^M does not change with type (from Equation (16)),

$$\frac{d}{dt} C_e(e^M(t), t) = C_{et} \frac{\partial}{\partial t}[\frac{1 - F(t)}{f(t)}] \geq 0$$

Q.E.D.

Comparative statics: Bargaining Effort. Table 2

The seller's utility for a given type t, assuming that the parameters are such that $z^B \in (\underline{s}, \bar{s})$, is

$$U_{P,t} = (r_0 + r'e)\frac{(E(s) + \frac{\Delta s}{2} - z)^2}{2\Delta s} + z\frac{(E(s) + \frac{\Delta s}{2} - z)}{\Delta s} - 2\gamma_1(\bar{t} - t)e - \gamma_2 e^2$$

This function, written as $U_{P,t}(e, -z, t, -\bar{t}, r', -\gamma_1, -\gamma_2)$, is supermodular since

$$\frac{\partial^2 EU_{P,t}}{\partial e \partial z} = -r'\frac{(E(s) + \frac{\Delta s}{2} - z)}{\Delta s} < 0 \qquad \frac{\partial^2 EU_{P,t}}{\partial e \partial t} = 2\gamma_1 > 0 \qquad \frac{\partial^2 EU_P}{\partial e \partial \bar{t}} = -2\gamma_1 < 0$$

$$\frac{\partial^2 EU_P}{\partial e \partial r'} = \frac{(E(s) + \frac{\Delta s}{2} - z)^2}{2\Delta s} > 0 \qquad \frac{\partial^2 EU_P}{\partial e \partial \gamma_1} = -2(\bar{t} - t) \leq 0 \qquad \frac{\partial^2 EU_P}{\partial e \partial \gamma_2} = -2e < 0$$

$$\frac{\partial^2 EU_P}{\partial z \partial r'} = -e\frac{(E(s) + \frac{\Delta s}{2} - z)}{\Delta s} < 0 \qquad \frac{\partial^2 EU_{P,t}}{\partial z \partial t} = \frac{\partial^2 EU_P}{\partial z \partial \bar{t}} = \frac{\partial^2 EU_P}{\partial z \partial \gamma_1} = \frac{\partial^2 EU_P}{\partial z \partial \gamma_2} = 0$$

By the monotone comparative statics, e and $-z$ are non-decreasing in t and r' and non-increasing in $\bar{t}$, γ_1 and γ_2.

The equation for the commission β^B is given by

$$\beta^B = 1 - \frac{2\gamma_1(\bar{t} - t)(2 - r_0 - r'e)^2 \Delta s}{r'[E(s) + \frac{\Delta s}{2}]^2} \tag{26}$$

This commission increases in t and r', and decreases in $\bar{t}$, γ_1 and γ_2.

The response of effort and minimum price to changes in $E(s)$ and Δs is not straightforward, but we can get some results from the first-order conditions. Combining those two conditions ((9) and (10)) we obtain the following equations (they are displayed in Figures 1 and 2):

$$\frac{r'[E(s) + \frac{\Delta s}{2}]^2}{2\Delta s(2 - r_0 - r'e)^2} = 2\gamma_1(\bar{t} - t) + 2\gamma_2 e \tag{27}$$

$$z = \left[1 - r_0 - r'\left\{\frac{r'(E(s) + \frac{\Delta s}{2} - z)^2}{4\gamma_2 \Delta s} - \frac{\gamma_1(\bar{t} - t)}{\gamma_2}\right\}\right](E(s) + \frac{\Delta s}{2} - z) \tag{28}$$

We can see that the left-hand side of Equation (27) increases in $E(s)$ for every effort level. Hence, e^B increases in $E(s)$. On the other hand, the change in the right-hand side of Equation (28) is undetermined since

$$\frac{\partial RHS}{\partial E(s)} = 1 - r_0 - \frac{r'^2[E(s) + \frac{\Delta s}{2} - z]^2}{2\gamma_2 \Delta s} + \frac{r'\gamma_1(\bar{t} - t)}{\gamma_2} \lesseqgtr 0$$

This is so because two opposite forces work here: effort increases in $E(s)$ (minimum price should decrease), and the net benefit of commitment increases (minimum price should increase). So we cannot say much more unless we put some additional restrictions on parameters.

From condition (10) we can show that $z = [E(s) + \Delta s/2](1 - r(e))/(2 - r(e))$. Hence, $[E(s) + \Delta s/2 - z] = [E(s) + \Delta s/2]/(2 - r(e))$. Taking into account that effort increases in $E(s)$, this term also increases in $E(s)$. Therefore β^B increases in $E(s)$.

The left-hand side of Equation (27) decreases in Δs for every effort level because

$$\frac{\partial LHS}{\partial \Delta s} = \frac{-r'(E(s) + \frac{\Delta s}{2})(E(s) - \frac{\Delta s}{2})}{2(2 - r_0 - r'e)^2 \Delta s^2} = \frac{-r'\bar{s}\underline{s}}{2(2 - r_0 - r'e)^2 \Delta s^2} < 0$$

Hence, e^B decreases in Δs, which indicates that the fraction $(1-r(e))/(2-r(e))$ increases. This effect together with the initial increase in Δs implies that the minimum price increases.

The second term of the equation for β^B (26) is inversely proportional to the left-hand side of equation (27), so increases in Δs. Moreover, effort decreases in Δs, which causes the term β^B to decrease in Δs.

Q.E.D.

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Table A.1: A numerical example of bargaining effort (no commitment, z=0).

t	e*	r(e*)	β(e*)	EU_P	EU_D	C(e*)	EU_B
0	1.33	0.43	0.833	176.0	0.0	32.0	272.0
0.1	1.40	0.44	0.850	175.4	2.2	33.6	268.8
0.2	1.47	0.45	0.867	174.5	4.7	35.2	265.6
0.3	1.53	0.45	0.883	173.4	7.4	36.8	262.4
0.4	1.60	0.46	0.900	172.2	10.2	38.4	259.2
0.5	1.67	0.47	0.917	170.7	13.3	40.0	256.0
0.6	1.73	0.47	0.933	169.0	16.6	41.6	252.8
0.7	1.80	0.48	0.950	167.0	20.2	43.2	249.6
0.8	1.87	0.49	0.967	164.9	23.9	44.8	246.4
0.9	1.93	0.49	0.983	162.6	27.8	46.4	243.2
1	2.00	0.50	1.000	160.0	32.0	48.0	240.0
Ex-ante value	**1.67**	**0.47**	**0.917**	**169.6**	**14.4**	**40.0**	**256.0**

Basic Information:

s(max)= 950, $\underline{s}$=10, E(s)=480, Δs=940, γ_1=8, γ_2=12, r'=0.1, r_0=0.3, $U_D(\underline{t})$=0.

Revenue Function: $r(e) = r_0 + r'\,e$, Cost Function: $C(e,t) = \gamma_1\,(1-t)\,e + \gamma_2\,e^2$

Notation: t = type; e*= effort; r(e*) = bargaining share; EU_P = seller's utility; EU_D = delegate's utility; C(e*) = effort cost; EU_B= buyer's utility.

Table A.2: A numerical example of bargaining effort (commitment effect).

t	e^B	$r(e^B)$	z^B	β^B	EU_P	EU_D	$C(e^B)$	EU_B	Surplus (*)	Welfare Loss (**)
0	0.03	0.303	390.3	0.520	282.6	0.0	0.2	116.2	399.0	81.0
0.1	0.10	0.310	387.9	0.572	283.1	0.2	0.8	116.0	400.0	80.0
0.2	0.17	0.317	385.4	0.623	283.3	0.6	1.5	115.7	401.0	79.0
0.3	0.25	0.325	383.0	0.673	283.2	1.2	2.1	115.5	402.0	78.0
0.4	0.32	0.332	380.5	0.722	283.0	2.0	2.8	115.3	403.0	77.0
0.5	0.39	0.339	378.0	0.770	282.5	3.1	3.4	115.0	404.1	75.9
0.6	0.46	0.346	375.5	0.818	281.8	4.5	4.1	114.7	405.1	74.9
0.7	0.54	0.354	372.9	0.865	280.8	6.0	4.8	114.5	406.1	73.9
0.8	0.61	0.361	370.3	0.910	279.6	7.8	5.5	114.2	407.1	72.9
0.9	0.68	0.368	367.7	0.956	278.2	9.9	6.2	113.9	408.1	71.9
1	0.76	0.376	365.1	1.000	276.5	12.1	6.9	113.6	409.2	70.8
Ex-ante value	**0.39**	**0.339**	**377.9**	**0.766**	**281.3**	**4.3**	**3.5**	**115.0**	**404.1**	**75.9**

Probability of bargaining failure	**39.1%**
Increase in Seller's Utility with respect to no commitment	**65.9%**
Welfare Loss/Expected Surplus	**15.8%**

Basic Information:

s(max)= 950, $\underline{s}$=10, E(s)=480, Δs=940, γ_1=8, γ_2=12, r'=0.1, r_0=0.3, $U_D(\underline{t})$=0.

Revenue Function: $r(e) = r_0 + r' e$, Cost Function: $C(e,t) = \gamma_1 (1-t) e + \gamma_2 e^2$

Notation: t = type; e^B= effort; $r(e^B)$ = bargaining share; z^B = minimum price; EU_P = seller's utility;

EU_D = delegate's utility; $C(e^B)$ = effort cost; EU_B= buyer's utility.

(*) Surplus is the sum of the delegate's cost and the seller, delegate, buyer's utility.

(**) Welfare loss is equal to Expected Surplus (480) minus Surplus. It can also be calculated as the expected surplus between s and z.

Table A.3: A numerical example of marketing effort (no commitment, z=0).

t	e*	p(e*)	β(e*)	EU_P	EU_D	C(e*)	EU_B	Surplus (+)
0	0.33	0.333	0.667	76.0	0.0	4.0	80.0	160.0
0.1	0.40	0.340	0.700	76.2	0.6	4.8	81.6	163.2
0.2	0.47	0.347	0.733	76.1	1.5	5.6	83.2	166.4
0.3	0.53	0.353	0.767	75.8	2.6	6.4	84.8	169.6
0.4	0.60	0.360	0.800	75.4	3.8	7.2	86.4	172.8
0.5	0.67	0.367	0.833	74.7	5.3	8.0	88.0	176.0
0.6	0.73	0.373	0.867	73.8	7.0	8.8	89.6	179.2
0.7	0.80	0.380	0.900	72.6	9.0	9.6	91.2	182.4
0.8	0.87	0.387	0.933	71.3	11.1	10.4	92.8	185.6
0.9	0.93	0.393	0.967	69.8	13.4	11.2	94.4	188.8
1	1.00	0.400	1.000	68.0	16.0	12.0	96.0	192.0
Ex-ante value	**0.67**	**0.367**	**0.833**	**73.6**	**6.4**	**8.0**	**88.0**	**176.0**

Basic Information:

s(max)= 950, $\underline{s}$=10, E(s)=480, Δs=940, γ_1=8, γ_2=12, p'=0.1, p_0=0.3, r_0=0.5, $U_D(\underline{t})$=0.

Probability Function: $p(e) = p_0 + p' e$, Cost Function: $C(e,t) = \gamma_1 (1-t) e + \gamma_2 e^2$

Colums 2 to 4 are calculated as those in Table 1, but replacing r_0p' for r'.

Notation: t = type; e*= effort; p(e*) = probability of finding a buyer; EU_P = seller's utility;

EU_D = delegate's utility; C(e*) = effort cost; EU_B= buyer's utility.

(+) Surplus is the sum of the delegate's cost and the seller, delegate, buyer's utility.
The ex ante revenue in Expected Utilities and Surplus is the corresponding revenue times the pobability of the delegate finding a buyer.

Table A.4: A numerical example of marketing effort (commitment effect).

t	e^M	$p(e^M)$	z^M	β^M	EU_P	EU_D	$C(e^M)$	EU_B	Surplus (*)	$p(e^M)E(s)$	Loss from failure(**)	Welfare Loss vs. no commitment (***)
0	0.67	0.367	316.7	2.250	106.7	0.0	10.7	39.1	156.5	176.0	19.5	3.5
0.1	0.73	0.373	316.7	2.325	106.6	1.2	11.7	39.8	159.3	179.2	19.9	3.9
0.2	0.80	0.380	316.7	2.400	106.3	2.6	12.8	40.5	162.2	182.4	20.2	4.2
0.3	0.87	0.387	316.7	2.475	105.7	4.2	13.9	41.3	165.0	185.6	20.6	4.6
0.4	0.93	0.393	316.7	2.550	105.0	6.0	14.9	42.0	167.8	188.8	21.0	5.0
0.5	1.00	0.400	316.7	2.625	104.0	8.0	16.0	42.7	170.7	192.0	21.3	5.3
0.6	1.07	0.407	316.7	2.700	102.8	10.2	17.1	43.4	173.5	195.2	21.7	5.7
0.7	1.13	0.413	316.7	2.775	101.5	12.7	18.1	44.1	176.4	198.4	22.0	6.0
0.8	1.20	0.420	316.7	2.850	99.9	15.4	19.2	44.8	179.2	201.6	22.4	6.4
0.9	1.27	0.427	316.7	2.925	98.0	18.2	20.3	45.5	182.1	204.8	22.7	6.7
1	1.33	0.433	316.7	3.000	96.0	21.3	21.3	46.2	184.9	208.0	23.1	7.1
Ex-ante value	**1.00**	**0.400**	**316.7**	**2.625**	**102.9**	**9.1**	**16.0**	**42.7**	**170.7**	**192.0**	**21.3**	**5.3**

Probability of bargaining failure	**32.6%**
Loss from bargaining failure (**)	**11.1%**
Increase in Seller's Utility with respect to no commitment	**39.9%**
Welfare Loss/Expected Surplus	**3.0%**

Basic Information:

s(max)= 950, $\underline{s}$=10, E(s)=480, Δ s=940, γ_1=8, γ_2=12, p'=0.1, p_0=0.3, r_0=0.5, $U_D(\underline{t})$=0.

Probability Function: $p(e) = p_0 + p'e$, Cost Function: $C(e,t) = \gamma_1 (1-t) e + \gamma_2 e^2$

Notation: t = type; e^M= effort; $p(e^M)$ = probability of finding a buyer; z^M = minimum price; EU_P = seller's utility;

EU_D = delegate's utility; $C(e^M)$ = effort cost; EU_B= buyer's utility.

(*) Surplus is the sum of the delegate's cost and the seller, delegate, buyer's utility.

(**) Loss from bargaining failure is equal to Expected Surplus (p E(s)) minus Surplus. It can also be calculated as the expected surplus between s and z.

(***) Welfare Loss against no commitment is the difference between Surplus under no commitment and Surplus under commitment.

Figure 1: First-Order Condition for Bargaining Effort

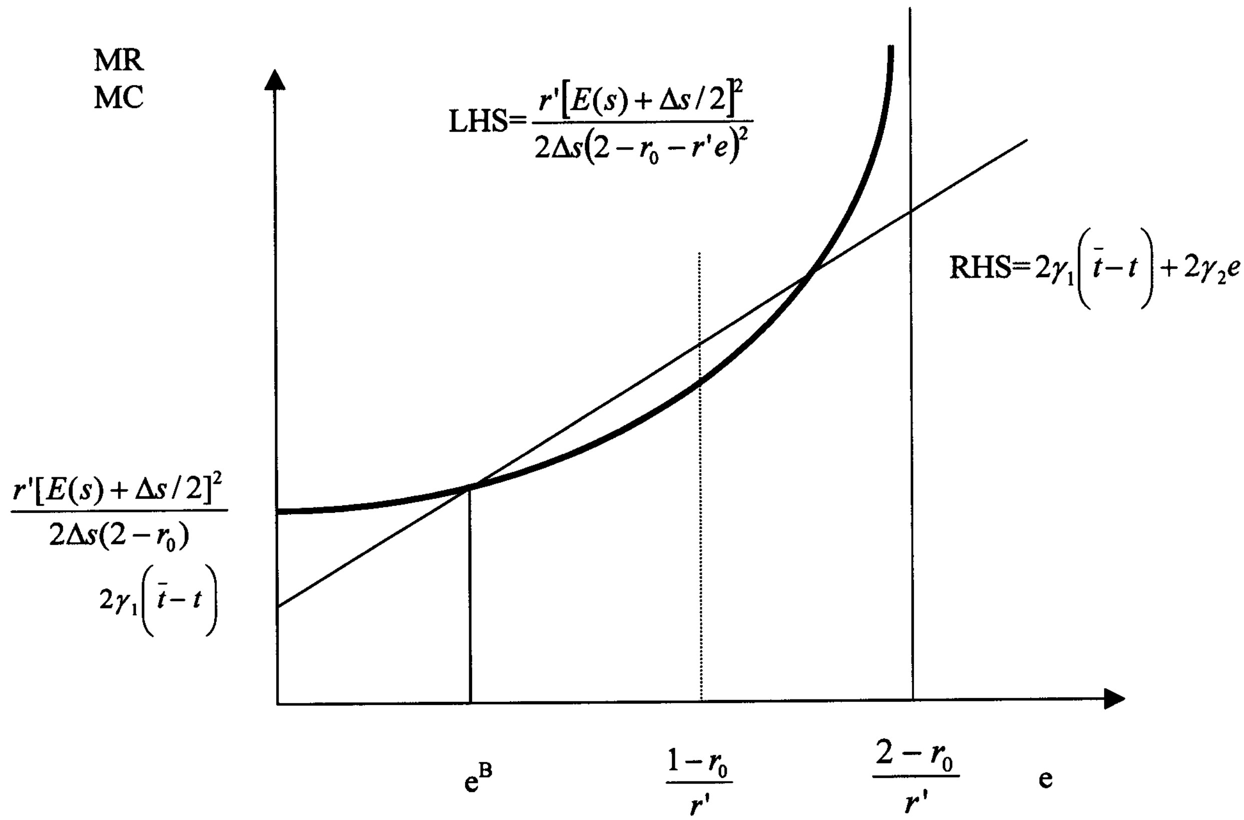
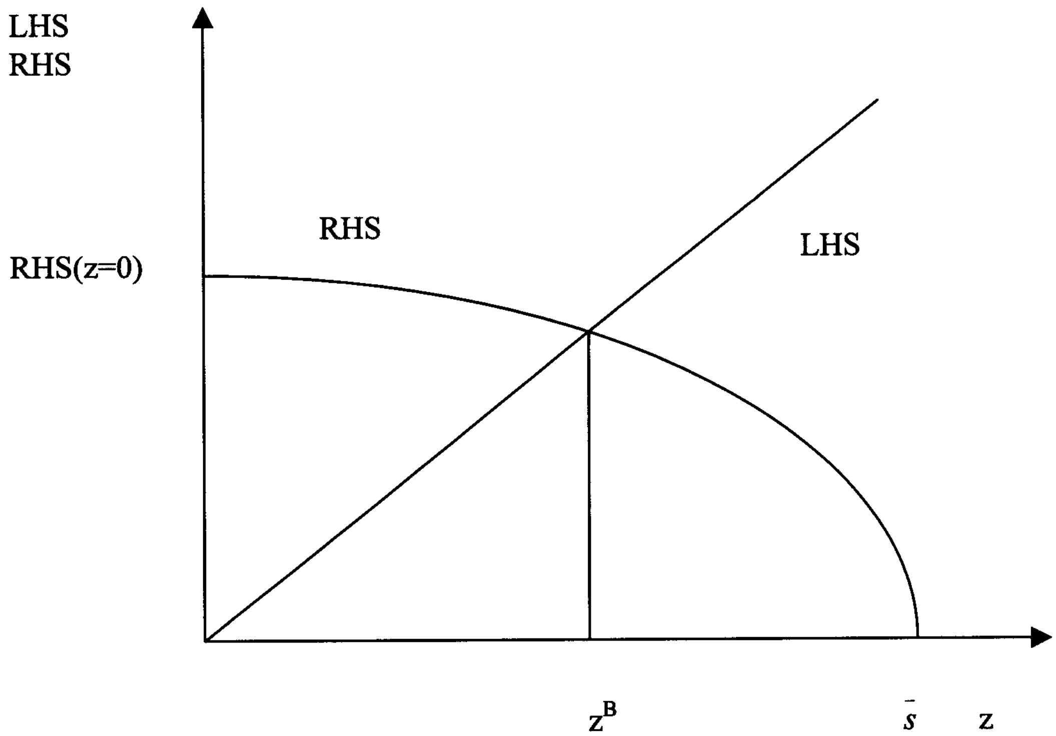


Figure 2: First-Order Conditions for Minimum Price (Bargaining Effort Case).



$$RHS = \left\{1 - r_0 - \frac{r'^2 \left(E(s) + \Delta s / 2 - z\right)^2}{4\gamma_2 \Delta s} + \frac{r'\gamma_1\left(\bar{t} - t\right)}{\gamma_2}\right\}\left(E(s) + \Delta s / 2 - z\right)$$

$$RHS(z = 0) = 1 - r_0 - \frac{r'^2 \left[E(s) + \Delta s / 2\right]^3}{4\gamma_2 \Delta s} + \frac{r'\gamma_1\left(\bar{t} - t\right)\left[E(s) + \Delta s / 2\right]}{\gamma_2}$$