Wealth Composition, Endogenous Fertility and the Dynamics of Income Inequality

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Abstract

This paper analyzes how differences in the composition of wealth between human and physical capital among families affect fertility choices. These in turn influence the dynamics of wealth and income inequality across generations through a tradeoff between quantity and quality of children. Wealth composition affects fertility because physical capital has only a wealth effect on number of children, whereas human capital increases the time cost of child-rearing in addition to the wealth effect. I construct a model combining endogenous fertility with borrowing constraints in human capital investments, in which wealth composition is determined endogenously. The model is calibrated to the PNAD, a Brazilian household survey, and the main findings of the paper can be summarized as follows. First, the model implies that the cross-section relationship between fertility and wealth typically displays a U-shaped pattern, reflecting differences in wealth composition between poor and rich families. Also, the quantity-quality tradeoff implies a concave cross-section relationship between investments per child and wealth. Second, as the economy develops and families overcome their borrowing constraints, the negative effect of wealth on fertility becomes smaller, and persistence of inequality declines accordingly. The empirical evidence presented in this paper is consistent with both implications.

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1. INTRODUCTION

This paper analyzes the implications of the interaction between wealth composition, fertility and investments in children for the dynamic behavior of the income distribution among families. Wealth composition affects fertility because physical capital has only a wealth effect on number of children, whereas human capital increases the time cost of child-rearing in addition to the wealth effect. Fertility choices in turn influence the dynamics of income inequality across generations due to a tradeoff between quantity and quality of children.

Several studies suggest that human and physical capital have different qualitative effects on fertility and investments in children. Becker and Barro (1988), Benhabib and Nishimura (1993) and Alvarez (1994) analyze fertility models in which families are heterogeneous in their physical capital stocks. In these models, rich families dilute their wealth by having more children than poor families, and fertility behavior leads to long-run equality among families. Becker, Murphy and Tamura (1990), Tamura (1994) and Palivos (1995) study the role of heterogeneity in human capital in generating differences in fertility and investment decisions among families. Since child-rearing requires parental time, human capital increases the opportunity cost of children, and fertility is negatively related to income. Because of the tradeoff between quantity and quality of children, endogenous fertility leads to long-run income inequality.

The main contribution of this paper is to analyze a model of fertility and investment decisions in which the wealth and income composition are determined endogenously from the allocation of investments in children between human and physical capital. The endogeneity of wealth composition allows for a characterization of the conditions under which fertility is an equalizing force, and when it creates inequality.

In order to analyze the interaction between wealth composition, fertility and the dynamics of inequality, I combine a borrowing-constraints model of human and physical capital investments with a model of fertility behavior.

I assume that families may invest in children's human and physical capital, but cannot borrow to finance their human capital investments. In any period, families are divided into two groups. Richer families typically are unconstrained, and make efficient human capital investments. Additional investments are made in the form of physical capital. Since the children of unconstrained families are also unconstrained, this group tends to have the same level of (efficient) human capital, and different physical capital stocks.

Poorer families are typically constrained, and make investments in children only in the form of human capital. If the children of poor families are also constrained, they will have the same stock of physical capital, equal to zero, and will be heterogeneous in their human capital stocks.

Since the source of wealth variations differs for constrained and unconstrained families, their fertility behavior differs as well. In particular, the negative effect of wealth on fertility is larger for constrained than for unconstrained families, and the cross-section fertility-wealth profile typically displays a U-shaped pattern at any point in time. Because of the tradeoff between quantity and quality of children, this implies a concave cross-section relationship between investments per child and parental wealth.

The model is calibrated to data from the 1976 Pesquisa Nacional de Amostra Domiciliar (PNAD), a Brazilian household survey. The PNAD is a series of annual representative cross-sections of the Brazilian population collected by the Instituto Brasileiro de Geografia e Estatistica (IBGE). The choice of this data set is motivated by the fact that developing countries in general, and Brazil in particular, are typically characterized by significant cross-section variation in fertility across education and income classes.[1]

Both the theoretical analysis and numerical simulations suggest three qualitatively different long-run patterns of wealth and income distribution. First, if the degree of intergenerational altruism, the time cost of children and the productivity of human capital investments are high relative to the fertility preference parameter, all families eventually overcome their borrowing constraints and there is long-run income equality. The negative effect of wealth on fertility declines across generations, which reduces persistence of inequality.

[1] Kremer and Chen (1998) present evidence that developing countries typically display larger differences in fertility among educated and uneducated families. Lam (1986) provides evidence of a strong negative correlation between fertility and family income in Brazil.

Another pattern generated by the model is one in which the degree of altruism, the time cost of children and the productivity of human capital investments are not high enough relative to preferences for numbers of children to allow families to overcome their borrowing constraints, but are large enough to generate convergence among borrowing-constrained dynasties.

The third possible outcome is a situation in which the degree of altruism, the time cost of children and the productivity of human capital investments are too low relative to preferences for numbers of children, so that numbers of children are sufficiently high to generate long-run inequality.

I provide empirical evidence on the main implications of the model based on an empirical analysis of the PNAD and results from other studies. The evidence shows that the fertility-income cross-section profile typically displays a U-shaped pattern, consistent with the model. The interaction between quantity and quality of children tends in turn to generate a concave investment-income cross-section profile for the PNAD data, even though the evidence is mixed for other studies.

I also explore the implications of the model regarding the dynamic behavior of the degree of intergenerational persistence of inequality. Both the theoretical analysis and the simulations suggest that, given the calibrated parameters, the negative effect of wealth on fertility tends to become smaller as the country develops, reflecting a weaker cross-section association between wealth and labor income over time. Moreover, the quantity-quality tradeoff will tend to reduce the degree of persistence in inequality as the country develops.

In order to verify this implication of the model, I compare the degrees of persistence in earnings and the coefficients obtained from a regression of fertility on income for Brazil and the United States. The results show that a larger negative effect of income on fertility increases the persistence of inequality in Brazil. I also provide evidence that the quantitative importance of fertility as a source of persistence of inequality declined in Brazil between 1976 and 1996, consistent with the model.

This paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 analyzes a special case of the model, in which parents obtain utility from bequests, rather than their children's utility. Section 5 calibrates the model to the 1976 PNAD, and illustrates the possible outcomes with the aid of simulations. Section 6 presents empirical evidence supporting the implications of the model. Section 7 concludes and provides directions for future research.

2. RELATED LITERATURE

Recent research on growth theory has been placing particular interest in the study of fertility and investment in children decisions, and their implications for per capita income growth and income inequality, including Lucas (1998), Galor and Weil (1998), and Jones (1999).[2] This paper is more closely related, however, to two other strands of literature, namely microeconomic-based theories of fertility decisions, and models of the allocation of investments between human and physical capital.

As discussed in the introduction, several fertility studies, including Becker and Barro (1988), Alvarez (1994) and Becker, Murphy and Tamura (1990) show that, depending on the source of wealth, differences in income among families may generate very different fertility and investment patterns, and may accordingly have opposite implications for the dynamics of income distribution across generations.

Empirically, Kuznets (1966, 1971) has documented a secular increase in the aggregate labor income share for several developed countries, as well as significant cyclical fluctuations in the labor share. Both the secular and cyclical patterns of the aggregate labor share suggest the empirical importance of studying the implications of changes in income composition at the aggregate and family level.

In order to study the endogenous determination of wealth composition, I incorporate a model of allocation of investments between human and physical capital along the lines studied in

[2] Hansen and Prescott (1998) analyze the effect of population growth on long-run growth, but they do not attempt to endogeneize fertility decisions.

models of borrowing constraints in human capital investments, including Loury (1981), Laitner (1992), Becker and Tomes (1986) and Mulligan (1997).

This paper is also related to recent papers by Raut (1991), Dahan and Tsiddon (1998), Kremer and Chen (1998,1999), Doepke (1999) and Galor and Moav (1999). These papers analyze the effects of differential fertility between educated and uneducated families on the dynamic behavior of the income distribution. This study differs in two important aspects. First, I analyze the effect of differential fertility on the dynamics of inequality through its effect on investments in future generations, while these studies focus on the effect of differential fertility on factor prices. Second, these papers consider only the role of human capital in generating differences in fertility among families, while a crucial feature of this paper is that I model explicitly the effects of the composition of wealth between human and physical capital on fertility and investments.

This paper is closest in spirit to Galor and Moav (1999). They also analyze how human and physical capital accumulation might have qualitatively different effects on inequality. This paper is fundamentally different, however, in that it attributes a crucial role to endogenous fertility decisions, which are not considered in Galor and Moav (1999).

3. THE MODEL

Consider an economy inhabited by M dynasties, indexed by $i = 1,...,M$. Each dynasty is defined by a parent and her descendants. Each person lives for two periods: childhood and adulthood. Parental variables are indexed by t, while children variables are indexed by $t+1$. Families have two sources of wealth: human capital, h, and physical capital, k.

First-generation parents are heterogeneous in their stocks of human and physical capital, $\left\{h_0^{\,i}, k_0^{\,i}\right\}_{i=1,...,M}$. Parents are assumed to have identical preferences over their consumption, c_t, number of children, n_t, and utility per child, u_{t+1}, described by[3]

[3] I will drop the dynasty superscripts unless in cases in which they are necessary for the understanding of the model.

$$u_t = \alpha \log c_t + \gamma \log n_t + \beta u_{t+1}$$

where $\alpha > 0$, $\gamma > 0$ and $0 < \beta < 1$.

Parents are endowed with one unit of time, and spend λ units of time per child and ϕ units of the consumption good rearing their children. Hence, parents work $1 - \lambda n_t$ units of time, earning a wage rate w_t, described by the following functional specification:[4]

$$w_t = A h_t^{1-\varepsilon}$$

where $A > 0$ and $0 < \varepsilon < 1$.

I will assume that the rental price of capital is determined exogenously, at the constant level r. This is a convenient asumption, since general equilibrium considerations are not crucial in this model.[5]

Parents choose investments in each child's human capital, h_{t+1}, and physical capital, k_{t+1}, but they cannot borrow against their children's earnings in order to finance human capital investments.

The recursive formulation of the decision problem of a typical parent can be described as

$$\begin{aligned} & v(h,k) = \max_{c,n,h',k'} \left\{ \alpha \log c + \gamma \log n + \beta v\left(h', k'\right) \right\} \\ & s.t. \\ & k' \geq 0 \\ & c + n\left(\phi + h' + k'\right) = (1 - \lambda n) A h^{1-\varepsilon} + (1+r) k \end{aligned} \tag{1}$$

where h' and k' denote human and physical capital per child, respectively.

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[4] See Loury (1981), Becker and Tomes (1986) and Mulligan and Song Han (1997) for similar partial equilibrium specifications of this wage function.

[5] The appendix provides a general equilibrium interpretation of the model, in which the wage function is determined endogenously, and conditions are provided under which the assumption that the rental price of capital is exogenous is valid.

The budget constraint captures the interaction between the quantity and quality of children through the term $n_t\left(h_{t+1}+k_{t+1}\right)$. Borrowing constraints are captured by the restriction $k^{'}\geq 0$.

It is convenient to rewrite (1) in terms of full-time wealth, $y\equiv Ah^{1-\varepsilon}+\left(1+r\right)k$, and full-time labor income, $y^L\equiv Ah^{1-\varepsilon}$:

$$
\begin{aligned}
&v\left(h,k\right)=\max_{c,n,h^{'},k^{'}}\left\{\alpha\log c+\gamma\log n+\beta v\left(h^{'},k^{'}\right)\right\}\\
&s.t.\\
&k^{'}\geq 0\\
&c+n\left(\phi+h^{'}+k^{'}+\lambda Ah^{1-\varepsilon}\right)=Ah^{1-\varepsilon}+\left(1+r\right)k
\end{aligned}
\tag{2}
$$

The formulation in (2) makes explicit the fact that human capital affects the price of children through the term $\lambda Ah^{1-\varepsilon}$, while physical capital has only a wealth effect on number of children. This asymmetry between human and physical capital, combined with the interaction between number of children and investments in the budget constraint, is crucial in generating a link between wealth composition and investments per child.

The Euler equation for unconstrained families, that is, families making positive physical capital investments per child, $k_{t+1}>0$, is given by

$$\frac{c_{t+1}}{c_t}=\frac{\beta\left(1+r\right)}{n_t} \tag{3}$$

The Euler equation for constrained families, that is, families for which optimal physical capital investments in the absence of constraints satisfy $k_{t+1}<0$, is given by

$$\frac{c_{t+1}}{c_t}=\frac{\beta A\left(1-\varepsilon\right)\left(1-\lambda n_{t+1}\right)h_{t+1}^{\ \ -\varepsilon}}{n_t} \tag{4}$$

Notice that the rate of return on investments for both constrained and unconstrained families depend on the optimal fertility rate, n_t. Unconstrained families equalize the rate of return between human and physical capital investments, which implies:

$$h_{t+1} = \left[\frac{A(1-\varepsilon)(1-\lambda n_{t+1})}{1+r} \right]^{\frac{1}{\varepsilon}} \tag{5}$$

Since the fertility rate of the next generation affects the amount of time they allocate to the marketplace, optimal human capital investments, h_{t+1}, depend on n_{t+1}, as shown in (5). This dependence of current investments on the fertility behavior of future generations makes it very difficult to characterize the model analytically, and makes it necessary to compute numerical simulations to understand the behavior of the model[6]. I will simulate the model in section 5. First, however, I will analyze in detail a special case which will be useful in motivating and interpreting the numerical simulations.

4. SPECIAL CASE: IMPURE ALTRUISM MODEL

4.1 Policy Functions

This section analyzes in detail a particular case in which parents derive utility from bequests to their children, rather than their children's utility. I use this preference specification because it allows for an analytical characterization of the model, which will prove helpful in motivating and interpreting the simulations conducted in section 5. This special case may be interesting in itself, however, to the extent that similar preference specifications have been used

[6] Kremer and Chen (1998) were faced with the same difficulty, and they dealt with it by assuming that market wages depend on the size of the labor force, rather than the amount of man-hours employed in production.

extensively in several applications under the label of "impure altruism" or "warm-glow" preferences.[7]

I now assume that parental preferences are described by

$$u_t = \alpha \log c_t + \gamma \log n_t + \beta \log y_{t+1}$$

where $y_{t+1} \equiv A h_{t+1}^{1-\varepsilon} + (1+r) k_{t+1}$ denotes the full wealth of the adult child.[8]

The version of the parent's problem described in (2) for the impure altruism case is given by

$$\begin{aligned} &\max_{c_t, n_t, h_{t+1}, k_{t+1}} \left\{ \alpha \log c_t + \gamma \log n_t + \beta \log \left(A h_{t+1}^{1-\varepsilon} + (1+r) k_{t+1} \right) \right\} \\ &s.t. \\ &k_{t+1} \geq 0 \\ &c_t + n_t \left(\phi + h_{t+1} + k_{t+1} + \lambda A h_t^{1-\varepsilon} \right) = A h_t^{1-\varepsilon} + (1+r) k_t \end{aligned} \tag{6}$$

In order to analyze parental decisions with respect to fertility, human and physical capital investments, it will be useful to consider the behavior of the family depending on whether borrowing constraints bind or not.

Unconstrained families

For families characterized by non-binding borrowing constraints, human and physical capital investments are chosen to equalize rates of return. This implies an efficient level of human capital investment, given by

[7] Becker and Tomes (1979) assume that parents obtain utility from the child's income, in a model of income distribution and intergenerational mobility. Banerjee and Newman (1991), Borjas (1992) and Galor and Zeira (1993) construct intergenerational mobility models in which parents derive utility from bequests to their children. These utility functions may be interpreted as capturing imperfections in parental altruism with respect to their children. The "impure altruism" terminology was used in Andreoni (1989).

[8] This model may be viewed as a particular case of the model in section 3 in the sense that the impure altruism case assumes that the value function is given by $v(h,k) = \log\left(A h^{1-\varepsilon} + (1+r) k \right)$.

$$h_{t+1} = h^* = \left[\frac{A(1-\varepsilon)}{1+r} \right]^{\frac{1}{\varepsilon}} \tag{7}$$

Using the definitions $y_t \equiv Ah_t^{1-\varepsilon} + (1+r)k_t$ and $y_t^L \equiv Ah_{t+1}^{1-\varepsilon}$, the fertility policy function for unconstrained families may be expressed as

$$n_t = \frac{\gamma - \beta}{\alpha + \gamma} \left(\frac{y_t}{\phi + \lambda y_t^L + h^* - \dfrac{Ah^{*1-\varepsilon}}{1+r}} \right) \tag{8}$$

where I assume $\gamma > \beta$. Equation (8) makes clear the sense in which fertility depends on the composition of wealth between labor and nonlabor sources.

Using the definition $y_{t+1} \equiv Ah_{t+1}^{1-\varepsilon} + (1+r)k_{t+1}$, we can determine optimal wealth per child:

$$y_{t+1} = \frac{\beta \left[(1+r)(\phi + h^*) - Ah^{*1-\varepsilon} \right]}{\gamma - \beta} + \frac{\beta (1+r) \lambda}{\gamma - \beta} y_t^L \tag{9}$$

Notice that, from (9), y_{t+1} depends only on the labor income component of parental wealth. Wealth composition affects wealth per child through two channels. First, it affects the price of children through its effect on the value of household time. Second, the effect on fertility is transmitted to physical capital investments through the interaction between quantity and quality of children.

Constrained Families

Borrowing-constrained families are the ones for which $k_{t+1} < 0$ in the absence of constraints. This condition implies a threshold level of human capital, $\hat{h}$, such that parents with $h_t < \hat{h}$ are subject to binding borrowing constraints, where $\hat{h}$ satisfies:[9]

$$\hat{h} = \left[\frac{\gamma A h^{*1-\varepsilon} - \beta(1+r)(\phi + h^*)}{\lambda A \beta (1+r)} \right]^{\frac{1}{1-\varepsilon}} \tag{10}$$

Even though the model shares the usual property that the level of resources is crucial to determine whether borrowing constraints are binding, (10) shows that fertility behavior will also determine the extent to which families are constrained in their human capital investments. Specifically, the higher is the cost of children, as captured by the term $\phi + h^*$, the lower is fertility, and the less likely the family is to be constrained.

The fertility policy function for constrained families is described by

$$n_t = \frac{\gamma - \beta(1-\varepsilon)}{\alpha + \gamma} \left(\frac{y_t}{\phi + \lambda {y_t}^L} \right) \tag{11}$$

As observed for the fertility policy function for unconstrained families (see (8)), number of children of constrained families depend on the composition of wealth among labor and nonlabor sources.

Since $k_{t+1} = 0$ for constrained families, adult child's labor income and total wealth are equal and given by

$$ {y_{t+1}}^L = y_{t+1} = \left[\frac{A^{\frac{1}{1-\varepsilon}} \beta(1-\varepsilon)}{\gamma - \beta(1-\varepsilon)} \right]^{1-\varepsilon} \left(\phi + \lambda {y_t}^L\right)^{1-\varepsilon} \tag{12}$$

[9] The result that the threshold can be expressed only in terms of human capital, rather than the pair human-physical capital, is particular to log preferences. Even though this is clearly a simplification, it is convenient in this case because it allows for a sharper analytical characterization of the model.

4.2. Long-run implications

In order to analyze the evolution over time of the cross-section of dynasties, it will be useful to consider a log-linear least-squares fit of the cross-section data, for each variable of interest. This procedure can be viewed as a convenient way to summarize the cross-section patterns. Also, it will be relevant for an empirical analysis of the model, since most data sets that are suitable for an analysis of persistence of inequality do not have a number of observations large enough for a nonlinear study.

The cross-section analysis is based on the following procedure. First, I log-linearize the fertility policy functions around the efficient level of human capital for both constrained and unconstrained groups. Then I fit the cross-section relationship between fertility and parental wealth by ordinary-least squares (OLS), omitting $\log y_t^L$. The estimated fertility regression equation is described by:

$$\log n_t = cons + \left\{ \begin{array}{l} p_t \left[1 - \left(\dfrac{\lambda y^{*L}}{\phi + \lambda y^{*L}} \right) b_t^C \right] + \\ +(1-p_t)\left[1 - \left(\dfrac{\lambda y^{*L}}{\phi + \lambda y^{*L} + h^* - \dfrac{y^*}{1+r}} \right) b_t^U \right] \end{array} \right\} \log y_t \qquad (13)$$

where p_t is the fraction of families that are constrained at time t, $y^* \equiv Ah^{*1-\varepsilon}$ is the labor income associated with the efficient human capital level, h^*, $b_t^U \equiv \left[\dfrac{\text{cov}\left(\log y_t, \log y_t^L\right)}{\text{var}\left(\log y_t\right)} \right]^U$ is the regression coefficient obtained from a least-squares regression of $\log y_t^L$ on $\log y_t$ among unconstrained families, and $b_t^C \equiv \left[\frac{\text{cov}\left(\log y_t, \log y_t^L\right)}{\text{var}\left(\log y_t\right)}\right]^C$ is the regression coefficient obtained from a least-squares regression of $\log y_t^L$ on $\log y_t$ among constrained families.

The coefficient on $\log y_t$ in (13) is a weighted average of the coefficients of the two groups. The negative terms inside brackets in (13) capture the bias arising due to the omission of $\log y_t^L$. This bias tends to reduce the effect of $\log y_t$ on fertility for both groups, since $\log y_t^L$ increases the cost of fertility and is correlated with $\log y_t$.

The negative coefficient on $\log y_t$ for unconstrained families is not as large as the one for constrained families, for two reasons. First, since unconstrained families make efficient human capital investments in their children, the time cost of fertility is a smaller fraction of total fertility cost for these families. Second, since the human capital stock of all unconstrained families is the same, equal to the efficient level, h^*, the correlation between labor income and total wealth is smaller for unconstrained families. This implies that b_t^U is smaller than b_t^C.[10]

If we pool the wealth data for the whole cross-section of dynasties, a least squares fit of the cross-section relationship between adult child's and parental full wealth leads to:

$$\log y_{t+1} = cons + \left\{ \begin{array}{l} p_t \left[(1-\varepsilon) \left(\dfrac{\lambda y^{*L}}{\phi + \lambda y^{*L}} \right) b_t^C \right] + \\ + (1-p_t) \left[\left(\dfrac{\lambda y^{*L}}{\phi + \lambda y^{*L} + h^* - \dfrac{y^*}{1+r}} \right) b_t^U \right] \end{array} \right\} \log y_t \qquad (14)$$

The coefficient on $\log y_t$ in (14) captures the degree of persistence in wealth inequality across generations. It is a weighted average of the coefficients of the constrained and

[10] In fact, if the variance of total income is larger among constrained families, b_t^C may be smaller than b_t^U. However, since the difference in the covariance of labor and total income between the two groups is likely to be large and increasing, b_t^C in general be larger than b_t^U. This will be clear from the simulations in section 5.

unconstrained groups. The effect of the interaction between quantity and quality of children on the degree of persistence of inequality can be observed by comparing the coefficient on $\log y_t$ across the fertility and wealth equations, (13) and (14), respectively. This comparison shows that the shares of time costs in total fertility costs for both constrained and unconstrained families appear in (13) and (14) with opposite signs, reflecting the quantity-quality tradeoff.

In this model, persistence of wealth inequality among unconstrained families tends to be smaller than for constrained families because the negative effect of wealth on fertility is smaller within this group, and this is transmitted to investments through the quantity-quality tradeoff.

The long-run behavior of the model is determined by whether constrained families eventually overcome their borrowing constraints. Since I did not introduce stochastic shocks into the model, families that are not constrained at time t, that is, parents for which $h_t > \hat{h}$, where $\hat{h}$ is given by (10), will never be constrained in the future, that is, $h^* \geq \hat{h}$.[11]

Using the definition $y_t^L \equiv A h_t^{1-\varepsilon}$, the dynamic behavior of constrained families can be described by the following difference equation:

$$h_{t+1} = \frac{\beta(1-\varepsilon)}{\gamma - \beta(1-\varepsilon)}\left(\phi + \lambda A h_t^{1-\varepsilon}\right) \qquad (15)$$

The appendix shows that the dynamic system described by (15) converges locally to $\tilde{h}$, where $\tilde{h}$ is a fixed point of (15). If $\phi = 0$, a sufficient condition for constrained families to eventually become unconstrained is given by

$$1 < \frac{\gamma}{\beta} \leq \lambda(1+r) + 1 - \varepsilon \qquad (16)$$

[11] If this were not the case, we would have $h^* < \hat{h} \leq h_t$, which would imply that parental human capital stocks were greater than the efficient level. Even though this might happen in principle, the relevant case (both theoretically and empirically) is clearly the one in which $h_t \leq h^*$, which will be assumed in the text.

Equation (16) imposes bounds on the magnitude of the quantity-quality interaction that have to be satisfied in order for constrained families to eventually become unconstrained. Preferences for numbers of children relative to adult child's welfare, $\frac{\gamma}{\beta}$, need to be high enough for parents to have nonzero fertility, but it should be not too high so that it precludes constrained dynasties from eventually overcoming their borrowing constraints.

From the previous analysis, we can distinguish three qualitatively different long-run patterns. One possibility is a situation in which (16) does not hold, so that the degree of altruism, the time cost of children and the productivity of human capital investments are not high enough relative to preferences for numbers of children to allow families to overcome their borrowing constraints, but are large enough to generate convergence among borrowing-constrained dynasties.

Another possibility is a situation in which (16) does not hold, and the degree of altruism, the time cost of children and the productivity of human capital investments are too low relative to preferences for numbers of children, so that numbers of children are sufficiently high to generate long-run inequality.[12]

Finally, we can have a situation in which (16) holds, and then we derive the following implications. First, families that are unconstrained at some period t will always be unconstrained. Second, families that are constrained at some period t will eventually become unconstrained.

In this case, we conclude that eventually all families will become unconstrained. All parents will have the same amount of human capital, h^*, which in turn implies that they will have the same amount of full labor income, y^{*L}. From (9), this implies that in the following generation all families will have the same amount of full wealth, y_t.

The long-run implications of the model may be interpreted as a combination of results previously obtained in the fertility literature. Specifically, they may be viewed as associating the role of fertility as a destabilizing force, as in Becker, Murphy and Tamura (1990), with the

[12] Notice that the model exhibits local convergence to $\tilde{h}$ among constrained families, but not necessarily global convergence.

behavior of constrained families. On the other hand, the role of fertility as a stabilizing force, as in Becker and Barro (1988), is associated with the behavior of unconstrained families. Moreover, the model characterizes the conditions under which the Becker-Barro fertility pattern will prevail in the long run.

From (13), we can see that if (16) holds, and all families overcome their borrowing constraints over time, the negative OLS coefficient of the fertility regression on wealth will become smaller over time, reflecting the change in the cross-section correlation between labor income and wealth associated with the increase in the fraction of unconstrained families in total population. The interaction between quantity and quality of children in turn implies that persistence of wealth inequality tends to decline over time.

The cross-section relationships summarized by (13) and (14) will be later used to interpret the simulation results in section 5. I will also use (16) to analyze how parameter changes affect the dynamic behavior of income inequality.

5. SIMULATION OF THE MODEL WITH DYNASTIC PREFERENCES

5.1. Calibration

The baseline parameters will be calibrated to data from the 1976 Pesquisa Nacional de Amostra Domiciliar (PNAD), a Brazilian household survey. The PNAD is a series of representative cross-sections of the Brazilian population which have been collected annually (except for 1980, 1990 and 1991) since 1973 by the Instituto Brasileiro de Geografia e Estatistica (IBGE).[13]

I assume that a generation takes 25 years. The annual rate of return on physical capital is chosen to match the average rate of return on 30-year U.S. government bonds, which is about

[13] The PNAD is close to a nationally representative sample, though it is not fully representative of rural areas, especially in the remote frontier regions.

5.7%. This implies that the net rate of return on physical capital over a generation, r, is chosen to be $r=3$. These numbers were taken from Mulligan and Song Han (1997).

I require that $\beta(1+r)=1$, under the assumption that r corresponds to the steady state rate of return for the U.S.[14] Hence $\beta=0.25$, which corresponds to an annual altruism rate of 0.946.

The elasticity of wages with respect to human capital is $1-\varepsilon$. The appendix shows that parental and adult child's schooling, h_t and h_{t+1}, respectively, are related by the folowing equation:

$$\log h_{t+1} = cons + \left[\left(\frac{\lambda \bar{y}^L}{\phi + \lambda \bar{y}^L}\right)(1-\varepsilon)\right]\log h_t \tag{17}$$

where $\frac{\lambda \bar{y}^L}{\phi + \lambda \bar{y}^L}$ is the sample average of the share of time costs in total fertility costs and $cons$ denotes a constant.

I was not able to compute this share from the Brazilian data, so I used the share of time costs for American women with only elementary education, obtained from Espenshade (1977). This implies a share of fifty-percent. Since the average woman in Brazil in 1976 had on average four years of schooling, this may provide a reasonable approximation for Brazil. I then estimated (17) using years of schooling of father and the oldest adult child, and obtained a coefficient on $\log h_t$ equal to 0.4. This implies that $1-\varepsilon=0.8$, so I set the baseline value of ε to 0.2.[15]

In order to provide an estimate of the fraction of time allocated to child rearing, λ, I use the restriction imposed by the model on the maximum fertility rate:

$$n_{\max} = \frac{1}{\lambda}$$

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[14] Laitner (1992) and Navarro-Zermeno (1993) suggest that consumption does not grow across generations among non-borrowing-constrained families. I thank Casey Mulligan for this observation.

[15] Mulligan and Song Han (1997) estimate a value of $\varepsilon=0.4$ for the U.S.

where $n_{\max}$ is the maximum number of children and the total time endowment is normalized to 1. This restriction has to be satisfied in order for the head of household to supply positive labor hours.

I computed $n_{\max}$ by choosing the highest number of children currently alive for married women aged 35-55 currently employed, and then used the restriction above to estimate λ. I considered only women because there is considerable evidence that women allocate a higher fraction of their time to child-rearing than men.[16] I used number of children currently alive as a measure of fertility, rather than number of children-ever-born, to reduce the effect of child mortality on the empirical measure of fertility. The chosen age bracket for married women was chosen in order to obtain a measure of completed fertility. For this sample, $n_{\max}$ was found to equal 7, which implies $\lambda = 0.14$.[17]

I order to estimate the parameter A in the wage function, I used the estimated values of ε, λ and r and the expression for the optimal human capital per child given by (5). To compute the optimal human capital investment per child, I computed the average schooling among children aged 15-25 still living with their parents. Moreover, this measure was calculated for parents in the highest income decile, because they are more likely to be making optimal human capital investments in children. The restriction that children live with their parents is necessary for one to be able to link data on parents and their children. I used the number of children currently alive of the highest income decile in 1996 as the measure of n_{t+1}. These calculations lead to an estimate of A equal to 5.

In order to estimate the preference parameters α and γ, I assume that $\alpha + \gamma = 1$, and use the fact that α and γ are the shares of parental consumption and total expenditures in children on family income, respectively. These shares were obtained from the 1987/88 Pesquisa de Orçamento Familiar (POF), a Brazilian household survey that provides detailed data on family

[16] See Leibowitz (1974).
[17] Leibowitz (1974) provides evidence that $\lambda = 0.18$ for the U.S.

expenditures for 9 metropolitan regions. The POF data set implies a share of total expenditures in children of approximately one third, which implies that $\alpha = 0.67$ and $\gamma = 0.33$.[18]

I estimated the goods cost of fertility per child, ϕ, by using $\frac{\lambda \bar{y}^L}{\phi + \lambda \bar{y}^L} = 0.5$, the estimated value of λ and the average value of labor income (computed in model units, see below), which yields $\phi = 0.5$.[19]

I also chose the range and the initial distribution of the state variables, human and physical capital, in order to match the corresponding values for the PNAD 76. In particular, the initial distribution consists of 10 pairs of human and physical capital combinations, chosen to fit the distribution of schooling and financial wealth across deciles among male household heads aged 45 years old and over.

These human-physical capital initial pairs were constructed as follows. I used average schooling of the household head by deciles as a measure of the distribution of human capital among families. I then used the wage function of the model, calibrated as described above, to compute the lifetime labor income of the household head. From the PNAD 76, I computed the average ratio between capital income (interest, dividends and rent) and labor income for each decile. Finally, I applied these ratios to the constructed measures of lifetime labor income, and used the calibrated interest rate, r, to construct measures of the capital stock for each decile. The resulting pairs of human and physical capital were used to compute all time series simulated from the model.

Table 1 presents the baseline parameter values. They will be used in all simulations, unless noted otherwise.

5.2. Simulations

[18] Espenshade (1984) estimates that expenditures on children account for between 30 and 50 percent of total family expenditures in the U.S.

[19] Tamura uses $\phi = 0.25$.

In this subsection, I will simulate the model presented in section 3 for three different parametrizations, intended to illustrate the different long-run patterns that may arise. In section 6, I will estimate the empirical counterparts to the simulations and will assess to what extent the model is able to match the Brazilian data.

One possibility is that the degree of altruism, the time cost of children and the productivity of human capital investments are not high enough relative to preferences for numbers of children, but are large enough to generate convergence among borrowing-constrained families. This possibility will be analyzed in the first simulation.

A second possibility corresponds to the case in which (16) is satisfied, so all families eventually overcome their borrowing constraints and there is long-run wealth and income equality among families. This possibility will be analyzed in the second simulation.

A third possibility is that the degree of altruism, the time cost of children and the productivity of human capital investments are too low relative to preferences for numbers of children, so that the number of children is sufficiently high to generate long-run inequality. This possibility will be considered in the third simulation.

In addition to the parameter restriction described in (16), I will use the cross-section relationships between fertility and adult child's wealth, on the one hand, and parental wealth, on the other, described by (13) and (14), both to motivate and interpret the simulations.

a) Baseline case

Figures 1 and 2 present simulation results for the baseline parameterization. Figure 1 displays the policy function for fertility as a function of the state variables, human and physical capital. The policy function for fertility shows that fertility decreases with human capital, and increases with physical capital. This asymmetry of fertility behavior with respect to the source of wealth is particularly pronounced at low human and physical capital levels.

The policy function for physical capital investments for this parameter configuration is trivial, since all families are borrowing-constrained and thus do not leave any bequests in the form of physical capital. This is consistent with the 1976 PNAD data for Brazil, which shows that for a large majority of the population the share of asset income in total income is very small.

The first step in the analysis is to study how the cross-section relationship between fertility and parental wealth affects the cross-section investment profile through the quantity-quality tradeoff, and how both cross-section relationships are affected by the cross-section correlation between parental wealth and labor income. All simulated cross-section relationships were obtained by simulating (2) using the baseline parameter values (unless noted otherwise) and the initial distribution of pairs of human and physical capital, $\{h_0{}^i, k_0{}^i, i = 1, ..., N\}$, constructed in the way described in the previous subsection.

The three plots in Figure 2 illustrate the relationships between wealth composition, fertility behavior, and investments per child at $t = 0$. Labor income-share, $\frac{Ah_t^{1-\varepsilon}}{Ah_t^{1-\varepsilon} + (1+r)k_t}$ increases slightly at low-wealth levels, $Ah_t^{1-\varepsilon} + (1+r)k_t$, and declines for higher levels of wealth. Fertility, n_t, declines at low levels of wealth, then becomes constant, and increases slightly at the highest end of the wealth distribution.

The reduction in fertility among poor and middle-income families tends to increase persistence of inequality, as captured by the slope of the graph that relates adult child's wealth, $Ah_{t+1}{}^{1-\varepsilon} + (1+r)k_{t+1}$, to parental wealth, $Ah_t^{1-\varepsilon} + (1+r)k_t$. Further increases in parental wealth are associated with constant and rising fertility, which reduces persistence of inequality among richer families. These patterns are consistent with the ones described in (13) and (14).

Table 2 displays the dynamic behavior of the fertility-wealth (labor income) relationship. These estimates, denoted θ_t and $\theta_t{}^L$, respectively, are ordinary least-squares coefficients obtained by regressing fertility, n_t, on parental log wealth (labor income), $\log y_t \left(\log y_t{}^L\right)$, where the observations were generated by simulating time series of wealth and labor income for all families.

Table 2 shows that the negative effect of wealth (labor income) on fertility in the cross-section of families declines across generations. At $t = 4$, all families have the same fertility rate

($\theta_4 = 0$), and in the long-run the fertility regression coefficient cannot be calculated, since there is long-run equality ($\theta_\infty = .$).

Table 3 displays the dynamic behavior of the degrees of intergenerational persistence of inequality of wealth and labor income, denoted ρ_t and $\rho_t^{\,L}$, respectively. The time subscripts denote the parents' generation. These estimates are ordinary least-squares coefficients obtained by regressing total adult child's log wealth (labor income), $\log y_{t+1}\left(\log y_{t+1}^{\,L}\right)$, on parental log wealth (labor income), $\log y_t\left(\log y_t^{\,L}\right)$, where the observations were generated by simulating time series of wealth and labor income for all families[20].

Three features of Table 3 deserve comment. First, both degrees of persistence of inequality are always less than one, which implies convergence. Second, both persistence coefficients decline monotonically over time. Third, the decline in persistence of inequality is very slow. In fact, as can be observed from Table 3, only at $t = 8$ persistence is zero for both wealth and labor income, implying convergence after nine generations.

The baseline parameterization leads to policy functions and time series that resemble the equilibrium family behavior in Becker, Murphy and Tamura (1990). Families derive all their income from labor, and fertility is negatively related to income, at least at low-income levels. The dynamic behavior of income distribution for the baseline parametrization differs from the one implied by Becker, Murphy and Tamura (1990, BMT), however, in two important aspects. First, the human capital technology in the model presented in this paper is concave, and not linear, as in B-M-T. Second, because of the concavity assumption, the model tends to generate long-run equality in labor shares across families, which in turn tends to reduce the negative correlation between fertility and parental wealth, which reinforces the underlying tendency for long-run equality.

b) Increase in time spent in child-rearing $(\lambda = 0.4)$

[20] Mulligan (1997) analyzes how the OLS coefficient obtained from a regression of adult child's log income on parental log income may be interpreted as capturing the degree of intergenerational inequality persistence.

Figures 3 and 4 present simulation results for the case in which $\lambda = 0.4$ (as opposed to $\lambda = 0.14$ in the baseline case). The parameter λ is the fraction of time devoted to child-rearing. An increase in λ may be interpreted as a reduction in the productivity of child numbers or, conversely, as an increase in the cost of fertility. All other parameters and the initial distribution of state variables remain the same.

Figure 3 displays the policy functions for fertility and physical capital investments, respectively, as functions of the state variables, human and physical capital. As observed in the previous example, for any given level of physical capital, fertility declines when human capital increases. Also, for any given level of human capital, fertility increases with physical capital.

One interesting feature of Figure 3 is that, as opposed to the baseline case, a significant fraction of the population is not borrowing-constrained, as can be observed from the fact that many families leave positive bequests to their children in the form of physical capital. This result illustrates the fact that the extent to which borrowing constraints are binding may be significantly affected by fertility decisions, since the only difference between the policy functions displayed in Figures 1 (baseline case) and 3 is that the latter are computed for a higher λ.

The three plots in Figure 4 illustrate the relationship between wealth composition, fertility behavior, and investments per child at $t = 0$. The cross-section relationship between fertility and parental wealth displays a U-shaped pattern, which mirrors the inverse U-shaped relationship between labor share and parental wealth.[21] The positive relationship between number of children and parental wealth at higher levels of wealth contributes significantly for the very low degree of persistence in wealth inequality among rich families. These patterns are consistent with the ones described in (13) and (14).

Table 4 shows that the effect of wealth (labor income) on fertility in the cross-section of families becomes positive at $t = 1$. At $t = 4$, all families have the same fertility rate ($\theta_4 = 0$), and in the long-run the fertility regression coefficient cannot be calculated, since there is long-run equality ($\theta_\infty = .$).

[21] See Becker and Tomes (1976) for a theoretical analysis that also generates a U-shaped fertility-income cross-section profile.

Table 5 displays the dynamic behavior of the degrees of intergenerational persistence of inequality of wealth and labor income. From $t = 5$ on, there is wealth and income equality among families. This exercise corresponds to the case in which (16) is satisfied. The degree of persistence of labor income declines monotonically, as families overcome their borrowing constraints across generations. At $t = 2$ all families are unconstrained, and this is reflected in a persistence coefficient for labor income equal to zero. There is still some persistence in wealth, however, as parents make investments in children in the form of physical capital. At $t = 3$, all families have the same efficient human capital levels, and this leads to equality of total investments in children, as in Becker and Barro (1988).

The case in which $\lambda = 0.4$ leads to a dynamic behavior that combines elements from Becker and Barro (1988) and Becker, Murphy and Tamura (1990). Initially, most families derive all their income from labor, and fertility is negatively related to income at low-income levels. At high-income levels, fertility is positively related to income, as these families derive most of their income from physical capital. As the economy develops, constrained families eventually become unconstrained, and the relationship between labor share and wealth becomes negative. Because of the change in the cross-section wealth composition, the relationship between fertility and wealth becomes positive, which leads to equality in investments across families through the quantity-quality tradeoff.

c) Reduction in the rate of return to human capital $(\varepsilon = 0.5)$.

Figures 5 and 6 present simulation results for the case in which $\varepsilon = 0.5$ (as opposed to $\varepsilon = 0.2$ in the baseline case). The parameter ε affects the rate of return of human capital investments. It can be interpreted as a technological parameter, or as capturing the effect of institutions and government policy on the efficiency of investments in education.[22] All other parameters and the initial distribution of state variables remain the same.

[22] Hall and Jones (1999) interpret productivity parameters as functions of the economic infrastructure, including institutions and government policy.

Figure 5 displays the policy function for fertility. For any given level of physical capital, fertility declines when human capital increases. For any given level of human capital, fertility increases with physical capital. This asymmetry of fertility behavior with respect to the source of wealth is qualitatively similar to the one observed for the higher productivity case (Figure 1), but is considerably sharper.

The policy function for physical capital investments for this parameter configuration is trivial, since all families are borrowing-constrained and thus do not leave any bequests in the form of physical capital.

The three plots in Figure 6 illustrate the relationship between wealth composition, fertility behavior, and investments per child at $t = 0$. The qualitative patterns are similar to the ones observed for the higher rate of return case (Figure 2). The main difference is that total investments in children are smaller when $\varepsilon = 0.5$. It should also be noted that, despite the lower productivity, fertility levels at $t = 0$ are higher than the ones observed for $\varepsilon = 0.2$.

Table 6 shows that the negative effect of wealth (labor income) on fertility persists for several generations. If we compare Table 6 to Table 2, we can observe that the negative effect of wealth on fertility is larger in every period when the rate of return is smaller.

Table 7 displays the dynamic behavior of the degrees of intergenerational persistence of inequality of wealth and labor income. The dynamic pattern of the coefficients is considerably different from the one displayed when the rate of return of human capital investments is higher (Table 2). When the rate of return is low, both degrees of persistence increase over time, and from $t = 4$ on the relative distance in wealth and labor income among dynasties remains constant (degree of persistence equals 1). . If we compare Table 7 to Table 2, we can observe that persistence of inequality is higher in every period when the rate of return is smaller, which is consistent with the fertility pattern displayed in Table 6.

This exercise suggests that persistence of inequality is affected not only by the differences in fertility rates across income classes, but also by fertility levels. Long-run inequality in this example thus arises because of endogenous fertility decisions and, particularly, by the fact that poor families tend to have a number of children excessively high relative to the level of technology.

6. EMPIRICAL EVIDENCE

6.1. Evidence on U-shaped fertility-income cross-section profile.

Both the theoretical analysis and the simulations generate a few interesting implications, which I will assess in this section using data from the PNADs 76 and 96 and by providing evidence collected by other researchers. One implication of the model is that the cross-section fertility-income profile tends to produce a U-shaped pattern, reflecting differences in wealth composition between constrained and unconstrained families.

The interaction between quantity and quality of children tends in turn to generate a concave quality-income cross-section profile, where quality is captured by either schooling or income of the adult child.

Table 8 presents ordinary least-squares (OLS) regressions of fertility, adult child's schooling and adult child's log income on parental log income and log income squared, using data from the PNAD 76.[23] Parental and adult child's income are used as proxies for parental and child's wealth.[24] I also tried to fit polynomials of higher order to the data, but only the coefficients on father's log income and log income squared were found to be statistically significant.

The regressions control for mother's education, since mother's education may affect both the desired and actual number of children through several channels, including knowledge about

[23] Fertility is defined as number of childrer-ever-born to married women aged 40-55. The results are similar if we use number of children currently alive as the measure of fertility. Only families with nonzero fertility are included in the regressions, since childless adults cannot affect intergenerational mobility. See Mulligan (1997) for a discussion of differences in the fertility-income cross-section relationship when childless families are included.

[24] I measure family income as the average family income for men who are household heads and who work 40 hours per week on average. Different income averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area, and assigned to all men in each category.

contraceptive methods and child-mortality related investments.[25] All regressions use sample weights provided by IBGE to produce a representative sample of individuals for the Brazilian population. The appendix discusses the data and empirical counterparts to the model variables in more detail.

Table 8 shows that the coefficient on log parental income in the fertility regression is negative, while the coefficient on log income squared is positive. The estimated fertility-income profile thus displays a U-shaped pattern, consistent with the model.

Also, in the quality regressions (adult child's schooling and income), the coefficient on log parental income is positive, while the coefficient on log income squared is negative. Thus, the adult child's schooling and income cross-section profiles display a concave pattern, which is also consistent with the model.[26]

Figure 7 uses the regression coefficients displayed in Table 8 to plot the estimated relationship between fertility and parental income, computed for sample means of the other explanatory variables. The range of parental income in Figure 7 is the same as the one used in the simulations, to make the estimated and simulated plots comparable.

The cross-section relationship between fertility and parental income simulated by the model for the baseline parametrization (Figure 2) has a pattern similar to the one estimated from the PNAD 76 data based on the same income range (Figure 7). The main difference is that the simulated fertility-income profile underestimates the fertility differential found in the 76 PNAD data.

One important reason for this discrepancy between the simulated and actual fertility differential may be the omission of controls for child mortality in the regressions displayed in

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[25] Mother's education may also affect fertility by increasing the mother's value of time. However, that would not be a reason to control for mother's education in this context, since the U-shaped fertility pattern is expected to arise precisely because of the omission of the time cost of fertility. In any case, Veloso (1999) shows that, for this Brazilian data, mother's education has significant effects on children in addition to any possible effects through changes in the value of time.

[26] The same patterns emerge if we use data from the PNAD 96. In fact, the fertility-income U-shaped profile for 1996 is very robust to changes in the set of explanatory variables. In particular, the qualitative results for 1996 do not change if mother's education is not controlled for, whereas if one does not control for mother's education in 1976 a positive relationship between fertility and income is observed at low-income levels for some specifications of the regression equation. These results are available from the author upon request.

Table 8. Richer families typically face lower child mortality, which tends to reduce the number of children necessary to achieve a desired fertility rate, and increase the fertility differential between rich and poor families.[27]

Figure 7 also presents analogous results for the cross-section relationship between adult child's income and parental income, which may be compared to the simulated plot in Figure 2. As in the model, the empirical investment cross-section profile displays a concave pattern. However, since the model underestimates the fertility differential in the data, it also underestimates the investment differential between poor and rich families arising from the quantity-quality tradeoff.

In addition to the evidence for Brazil provided in this paper, there is some evidence of a U-shaped fertility-income profile for other countries. Mulligan (1997) presents evidence for the United States, using data from the 1990 U.S. Census and the 1989 Survey of Consumer Finances.

Figure 8 displays the fertility-income cross-section relationship estimated in Mulligan (1997) from the 1989 Survey of Consumer Finances. Mulligan uses number of children of household heads between the ages of forty-one and eighty-four as his fertility measure. Both plots displayed in Figure 8 were computed by fitting a fifth-order polynomial in log annual family income to the number of children. The top figure excludes childless families, as is preferable in a study of persistence of inequality, and corresponds to the Brazilian fertility-income relationship presented in Figure 7. The bottom figure includes childless families.

Fertility declines with income for family incomes up to US$ 300,000, and rises with income for incomes between US$ 300,000 and US$ 3,000,000. Fertility declines with income after US$ 3,000,000, but this is based on only thirty-three observations.

In addition to Mulligan (1997), Willis (1973) also provides evidence for the U.S., using data from the 1960 U.S. Census. Ben-Porath (1973) presents evidence for Israel, using an empirical model similar to Willis (1973).

[27] See Meltzer (1992) for a discussion of the effects of child mortality on fertility.

The evidence on a concave relationship between adult child's income and father's income, however, is mixed. In addition to the evidence provided in this paper, Mulligan (1993) also found evidence of an inverse U-shaped relationship, but the regression coefficient on the quadratic term is not significant. Both Behrman and Taubman (1990) and Solon (1992) found evidence in the opposite direction, indicating a U-shaped relationship between adult child's and parental income, but the regression coefficients also tend to be insignificant.

The mixed evidence on the shape of the cross-section investment-income profile may be attributed to at least two reasons. First, with the exception of this paper, all studies cited above use small samples, which are not particularly suitable for a study of income nonlinearities. Second, from a theoretical standpoint the implication that the investment-income profile has a concave pattern is less robust than the analogous implication for the fertility profile, because investment is affected by other variables in addition to fertility.

The empirical evidence thus strongly supports the implications of the model regarding the shape of the cross-section relationship between fertility and parental income, but is not conclusive with respect the shape of the investment-income cross-section profile.

6.2. Evidence on the dynamic relationship between fertility and persistence of inequality.

Another implication of the model is related to the dynamic behavior of the fertility-wealth relationship, and its effect on the degree of intergenerational persistence of inequality. Both the theoretical analysis and the simulations suggest that, given the calibrated parameters, the negative effect of wealth on fertility tends to become smaller as the country develops, reflecting a weaker cross-section association between wealth and labor income over time. Moreover, the quantity-quality tradeoff will tend to reduce the degree of persistence in inequality as the country develops.

In order to verify this implication of the model, I compare the degrees of persistence in full-time wages and the coefficients obtained from a regression of fertility on wages for the 1976

PNAD and the Panel Study of Income Dynamics (PSID), where the U.S. regressions are taken from Mulligan (1993).[28] Mulligan uses information on father's wage rate and fertility taken from the period 1967-72, which makes the U.S. and Brazilian samples approximately contemporary.

Table 9 presents simple OLS regressions of fertility and adult child's wage on father's wage, for both Brazil and the U.S. Both the fertility and persistence coefficients obtained from the PNAD 76 are roughly the double (in absolute value) of their PSID counterparts. The results for the 1996 PNAD are not shown in Table 9, but they confirm the differences between Brazil and the U.S. For the 1996 sample, the coefficient on father's wage in the fertility regression is -2.10, whereas the wage coefficient in the mobility regression is 0.67.

Table 10 uses a procedure similar to the one used in Mulligan (1993) to estimate the quantitative importance of fertility decisions for the the persistence of inequality across generations. These regressions include as an explanatory variable the fitted value obtained from the regression of mother's schooling on father's fertility, father's wage, age variables, a gender dummy and agriculture's share of personal income in the county where the son grew up.[29] The idea is that the fitted value of mother's schooling controls for variations in the price of fertility among families. The change in the coefficient on father's wage when the cost of children is controlled for may then be interpreted as the contribution of fertility behavior for inequality persistence.

If we compare Tables 9 and 10, we can observe that the coefficient on father's wage in 1976 declines fifty-percent (from 0.76 to 0.38) when we control for the cost of children. The corresponding decline in 1996 is of thirty-four percent (from 0.67 to 0.44). This is consistent with the prediction that fertility becomes relatively less important as a source of inequality as the country develops.

[28] Becker and Tomes (1986) present evidence from a dozen samples for the period 1960 through 1982 drawn from five countries (U.S., England, Sweden, Switzerland, and Norway). They generally found low intergenerational persistence of inequality, averaging about 0.25. These low estimates may be downward biased, due to sample homogeneity and measurement error in parental income, but revised estimates for the U.S. earnings estimates are in general around 0.4, which is considerably smaller than the estimates for Brazil presented in the text.

[29] Because of data limitations, I use mother's labor income instead of parental fertility as an instrument. Also, for the same reason, I use a dummy variable indicating whether the father lives in a urban area, rather than agriculture share of personal income in the county where the son grew up.

Table 10 also shows that the coefficient on father's wage for the U.S. declines forty-four percent (from 0.36 to 0.20) when one controls for the cost of children. The relative importance of fertility for persistence of inequality in the U.S. (around 1967-71) is thus slightly smaller then the contemporaneous figure for Brazil. Since it is likely that other sources of inequality persistence differ between Brazil and the U.S., it is probably more accurate to compare the absolute values of the effect of fertility on inequality, rather than the percentage change. In this case, the impact of fertility on persistence is much higher for Brazil (0.38) than the U.S. (0.16), consistent with the model.

6.3. Evidence on differences in the relative importance of education for inequality in Brazil and the U.S..

One additional piece of evidence provides support for the implication of the model which states that differences in the degree of persistence of inequality and the fertility-income relationship among developed and developing countries are related to differences in the correlation of labor and total income.

Because of lack of detailed information about current and lifetime capital income, especially for developing countries, it is difficult to compare the relationship between income composition and income for Brazil and the United States. However, we can explore a different but closely related implication of the model. The implication that the negative effect of parental income on fertility tends to decline with development is associated with the result that human capital tends to be equalized across families, which in turn tends to generate equality in labor income among dynasties. The latter is key in reducing the cross-section variation of the time cost of fertility across generations.

In the data, labor income depends not only on education, but on several other variables, including experience. In this case, the model predicts that schooling will become relatively less important as a source of variations in earnings as the economy develops. One way to provide quantitative evidence on this mechanism is to run a regression of log earnings on schooling for both Brazil and the United States, and compare the rates of return to schooling and the coeffficients of determination $\left(R^2\right)$ among the two regressions.

Mincer (1993) surveys several empirical studies which have estimated earnings functions using data from the U.S. Census between 1960 and 1976. Mincer reports that the average rate of return to schooling varies between 7 per cent and 11 per cent, depending on the particular data set used. The simple coefficient of determination between earnings and years of schooling averages 7 per cent. Standardizing for effects of age doubles the coefficient in age groups 35-44, but the coefficients decline below and above these ages.

Table 11 presents OLS regressions of log earnings on schooling, both with and without age controls, using data from the PNAD 76 (columns 1 and 2) and U.S. data (columns 3 and 4) from Mincer (1993). The results taken from Mincer (1993) do not include standard errors and number of observations.

Table 11 displays sharp differences in rates of return to schooling and the relative importance of schooling for the cross-section variation in earnings between Brazil and the United States. The return to schooling is 0.17 in Brazil and is considerably smaller in the U.S., averaging 0.09. The differences in the coefficients of determination of the schooling regressions are even more remarkable, varying between 0.45 in Brazil and 0.07 in the U.S. in a simple regression of log earnings on schooling (columns 1 and 3), and 0.47 in Brazil and 0.14 in the U.S. when age controls are included (columns 2 and 4).[30] The larger coefficients of determination for Brazil are consistent with both the theoretical model and the simulations.

The evidence provided by Tables 9-11 taken together supports the implications of the model regarding differences in the relationship between fertility and persistence of inequality between developing and developed countries. Moreover, the results suggest that these differences may be related to a higher relative importance of schooling for the cross-section variation of earnings in developing countries.

[30] Reis and Barros (1991) discuss differences in the coefficient of determination in schooling regressions among Brazil and developed countries. In particular, they argue that in Brazil, as opposed to developed countries, schooling is considerably more important than age as a source of inequality in wages and earnings.

7. CONCLUSION

This paper analyzed the interaction between wealth composition, endogenous fertility and the dynamics of wealth and income inequality, combining theoretical analysis, numerical simulations and empirical evidence. The major findings of the paper may be summarized as follows.

The first set of findings relates cross-section relationships between parental wealth, on the one hand, and wealth composition, fertility and investments per child, on the other. The cross-section relationship between fertility and wealth tends to display a U-shaped pattern, reflecting differences in the correlation between wealth and labor income among constrained and unconstrained families. The interaction between quantity and quality of children implies a concave cross-section relationship between investments per child and parental wealth.

The second set of findings is related to the dynamic behavior of the wealth and income distributions. If the degree of intergenerational altruism, the time cost of children and the productivity of human capital investments are high relative to preferences for numbers of children, all families eventually overcome their borrowing constraints and there is long-run equality among families. The negative effect of wealth on fertility declines as physical capital becomes relatively more important as a source of wealth variations among families. The quantity-quality tradeoff implies that the degree of persistence in income inequality tends to decrease as the economy develops and families become unconstrained.

I provided empirical evidence on these two sets of implications using data from the PNADs 76 and 96 and evidence taken from other studies. First, the evidence shows that the fertility-income cross-section relationship tends to display a U-shaped pattern, as implied by the model. The cross-section relationship between adult child's income and parental income displays a concave pattern, reflecting the interaction between quantity and quality of children.

Second, I provided evidence on the dynamic behavior of persistence of inequality by comparing fertility behavior and persistence of income inequality between Brazil and the United States. I found that the negative effect of income on fertility is larger in Brazil, which raises persistence of inequality in Brazil relative to the U.S. Moreover, I provided evidence that the differences in fertility and persistence patterns in Brazil and the United States may be associated with significant differences in the cross-section correlations between schooling and earnings in both countries. I also provided evidence that the quantitative importance of fertility as a source of persistence of inequality declined in Brazil between 1976 and 1996, consistent with the model.

The model constructed in this paper abstracts from several issues, in order to focus on the mechanism relating wealth composition, fertility and investments per child. In particular, the analysis does not consider the relationship between differences in fertility among skilled and unskilled workers and the dynamic evolution of the wage premium, as in Kremer and Chen (1998) and Doepke (1999). One interesting extension of the model would be to consider the general equilibrium interaction between wealth composition, fertility and factor prices. The extended model would necessarily require numerical simulations, and the model presented in this paper can provide a benchmark against which these simulations may be compared.

Another possible extension of the model would be to assume that investments per child also require parental time, in addition to expenditures in goods. In this case, wealth composition would affect investments both directly and through changes in fertility.

Also, Veloso (1999) provides evidence that differences in the allocation of time to child-rearing between husbands and wives is quantitatively important for fertility and schooling investments in children. This suggests that, in addition to the composition of income between labor and nonlabor sources, the composition of household income between mother's and father's income has important effects on fertility and investments in children.

Finally, the model presented in this paper abstracts from sources of heterogeneity among families other than differences in human and physical capital. This implies a unique level of human capital which equalizes rates of return between human and physical capital. The introduction of differences in ability among individuals would generate differences in human capital even among unconstrained families. This could potentially explain why fertility seems to rise with income only for the very rich, but I do not expect it to modify the main qualitative predictions of the model regarding differences in fertility and investment behavior between poor and rich families.

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APPENDIX

A. General equilibrium interpretation of the model.

Let the production technology be described by the following production function:

$$F(K,\tilde{H}) = BK^{\alpha}\tilde{H}^{1-\alpha}$$

where K denotes physical capital and $\tilde{H} \equiv LH^{1-\varepsilon}$ denotes effective labor, assumed to be a function of labor, L, and human capital, H, and $B > 0, 0 < \alpha < 1, 0 < \varepsilon < 1$.

I assume a competitive economy, so that individuals and firms take prices as given. Firms maximize profits, subject to the restriction that the rental rate on capital is fixed at r. The assumption of constant returns to scale implies that production in this economy is described by an aggregate production function given by

$$F(\sum_{i=1}^{M} N^i k^i, \sum_{i=1}^{M} N^i l^i \left(h^i\right)^{1-\varepsilon}) = B\left(\sum_{i=1}^{M} N^i k^i\right)^{\alpha}\left(\sum_{i=1}^{M} N^i l^i \left(h^i\right)^{1-\varepsilon}\right)^{1-\alpha}$$

where N^i denotes the number currently alive members of dynasty i.

The equilibrium wage rate received by each member of dynasty i at time t is given by

$$w_t^i = \frac{\partial F}{\partial l_i} = A\left(h_t^i\right)^{1-\varepsilon}$$

where $A \equiv B(1-\alpha)\left(\frac{\sum_{i=1}^{M} N^i k^i}{\sum_{i=1}^{M} N^i l^i \left(h^i\right)^{1-\varepsilon}}\right)^{\alpha}$ is constant, since the constant rental rate of capital implies that the aggregate capital-effective labor ratio (term in brackets) is constant. This justifies the wage function formulation used in the text.

B. Local convergence among borrowing-constrained families

Borrowing-constrained families converge locally to $\tilde{h}$, where $\tilde{h}$ satisfies

$$\tilde{h} = \frac{\beta(1-\varepsilon)}{\gamma - \beta(1-\varepsilon)}\left(\phi + \lambda A \tilde{h}^{1-\varepsilon}\right) \tag{1}$$

This convergence result may be established by taking logs in (1) and log-linearizing the resulting equation around $\tilde{h}$, to obtain

$$\log h_{t+1} \approx cons + \left[\left(\frac{\lambda \tilde{y}^L}{\phi + \lambda \tilde{y}^L}\right)(1-\varepsilon)\right]\log h_t \tag{2}$$

where $\tilde{y}^L \equiv A\tilde{h}^{1-\varepsilon}$ and $cons$ denotes a constant, which depends on steady state values.

The coefficient on $\log h_t$ in (2) may be interpreted as the degree of persistence of inequality among constrained families. The fact that this coefficient is less than one implies that human capital converges locally to $\tilde{h}$. Also, from (2) we can observe that there are two forces affecting the degree of persistence of inequality among constrained families.

First, the term $1-\varepsilon$ captures the effect of diminishing returns in human capital accumulation, and is standard in the intergenerational mobility literature (see, for example, Loury (1981), Becker and Tomes (1986) and Mulligan and Song Han (1997)).

Second, the term $\frac{\lambda \tilde{y}^L}{\phi + \lambda \tilde{y}^L}$ is the steady state fraction of total child costs accounted for by the time cost of fertility, and results from the interaction between fertility and human capital investments per child. This term captures the extent to which the interaction between quantity and quality of children creates a relationship between wealth composition and human capital investments per child.

We can characterize the convergence mechanism further, by considering the special case in which $\phi = 0$. In this case, $\tilde{h}$ is given by

$$\tilde{h} = \left[\frac{\lambda \beta A (1-\varepsilon)}{\gamma - \beta (1-\varepsilon)} \right]^{\frac{1}{\varepsilon}} \qquad (3)$$

In order for constrained families to eventually join the group of families that is not subject to borrowing constraints, a sufficient condition is that $\tilde{h} \geq h^*$, where $\tilde{h}$ is given in (3), and the efficient level of human capital, h^*, is described by (4). For the special case in which $\phi = 0$, this condition amounts to

$$1 < \frac{\gamma}{\beta} \leq \lambda (1+r) + 1 - \varepsilon \qquad (4)$$

where the first inequality follows from the assumption $\gamma > \beta$.

C. PNAD data and variable description

In this section, I describe the sample from the PNAD data set used in the empirical analysis, and the choice of empirical counterparts to the model variables. The empirical analysis will be conducted on the 1976 and 1996 PNAD surveys of Brazilian households.

The Pesquisa Nacional de Amostra Domiciliar (PNAD) is a series of cross-sections that have been collected annually since 1973 (except for 1980, 1990 and 1991) by the Instituto Brasileiro de Geografia e Estatistica (IBGE). Each cross-section contains over 100000 observations on Brazilian households, and over 300000 observations on individuals. The PNAD is close to a nationally representative sample, though it is not fully representative of rural areas, especially in the remote frontier regions.

The PNAD is mainly concerned about labor market outcomes, but it also has information on some individual demographic variables, including fertility for some specific years. The empirical analysis uses two subsamples, for fertility and for schooling (income) of the adult child. The fertility sample consists of about 18000 married women aged 45 years and over in 1976.

Since information for both the child's and parental schooling in 1976 is available only for families in which children live with their parents, I used information from the PNAD 96 to construct measures of the adult child's schooling and income. This is possible because the 1996 PNAD has information on schooling of parents for household heads and spouses of household heads. The schooling (income) sample consists of about 18000 respondents aged 41-50 in 1996. The rationale for the choice of the latter age range is that it corresponds to the cohort aged 21-30 in 1976, for which I want to gather education and income information.

The measure of fertility used in the paper is the number-of-children-ever born to married women aged 45 years and older. This age range is chosen to have a measure of completed fertility.

The measure of full income (labor income) for fathers used in this paper is the log of average income (labor income) for individuals working on average 40 hours per week during the year. I compute different averages of full-time income (labor income) for male household heads grouped according to some characteristics, such as education, region of residence, and whether they live in an urban or rural area. This measure is computed for individuals working full-time, and is assigned to all individuals sharing the same characteristics.

Years of schooling of adults aged 41-50 in 1996 is used as the measure of the adult child's human capital. I construct the adult child's income (labor income) measure by first computing different averages of income (labor income) for male household heads working on average 40 hours per week during the year, grouped according to their education, region of residence, and whether they live in an urban or rural area. These full-time income (labor income) averages are then assigned to men and women aged 41-50, according to their characteristics.

I use several control variables, including age and age squared of the husband, wife and oldest child, and the sex of the oldest child. I also use dummy variables for region residence and for whether the family lives in an urban or rural area. The latter may be viewed a proxy for the goods cost of fertility, ϕ, since the cost of living is usually higher in urban than rural areas.

All regressions use sampling weights available from the PNAD.

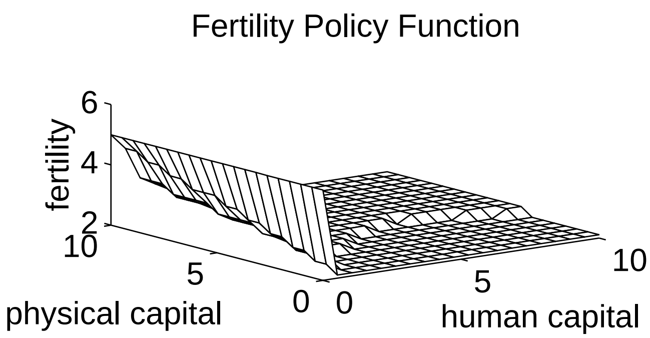
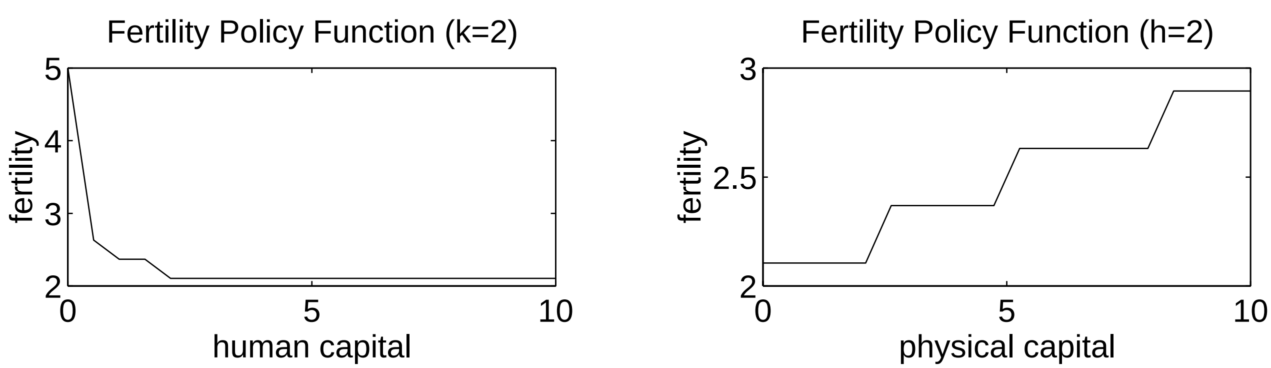


Figure 1-Fertility Policy Functions (baseline parameters)



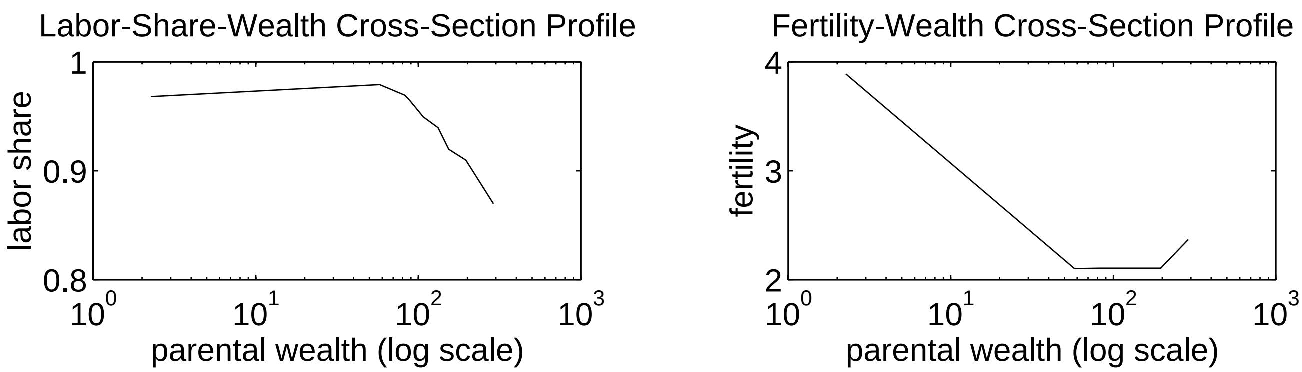
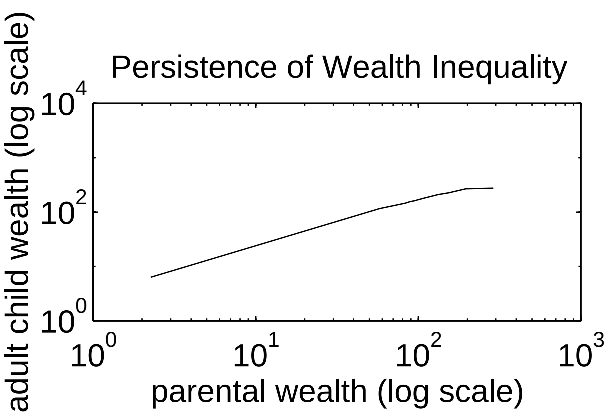


Figure 2-Cross-Section Profiles at t=0 (baseline parameters)



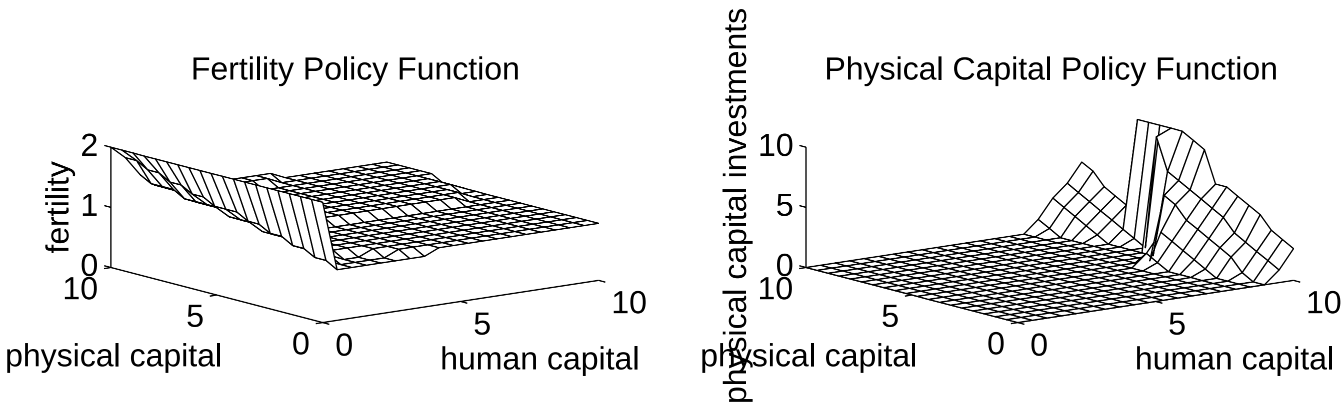
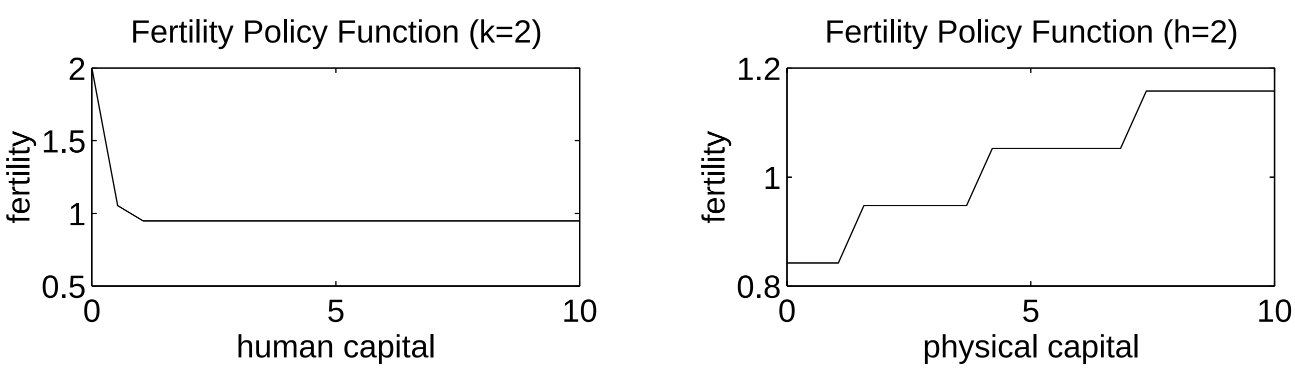
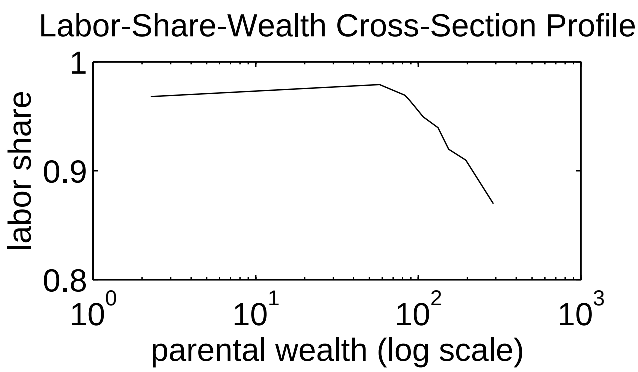


Figure 3-Fertility Policy Functions (lambda=0.4)





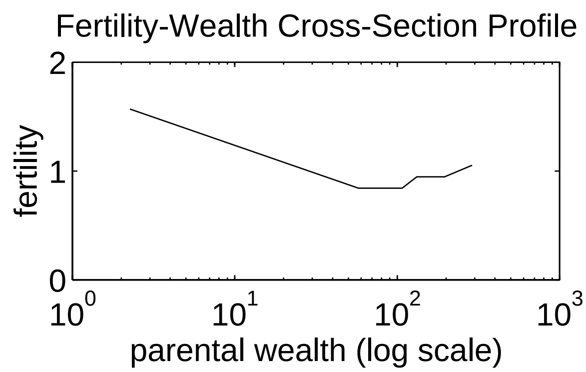
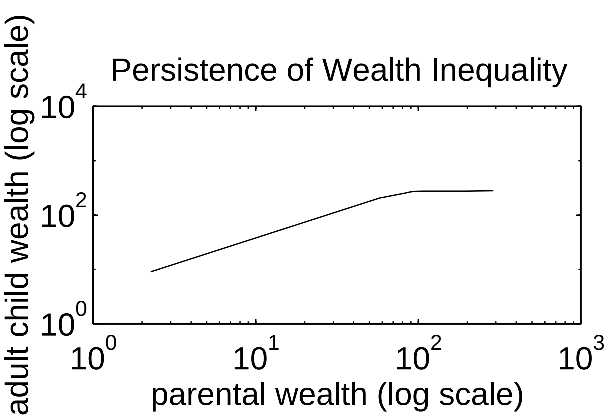


Figure 4-Cross-Section Profiles at t=0 (lambda=0.4)



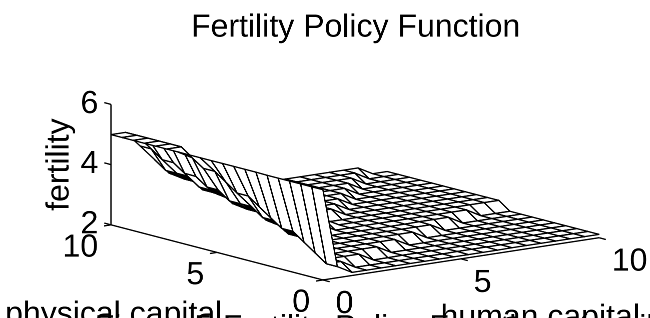
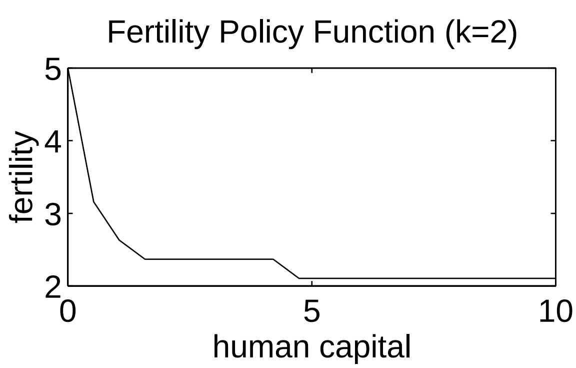
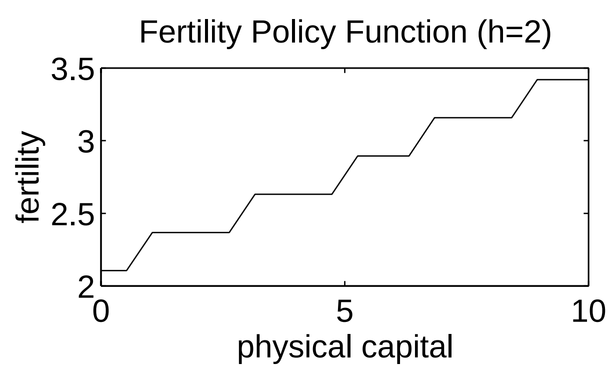


Figure 5-Fertility Policy Functions (epsilon=0.5)





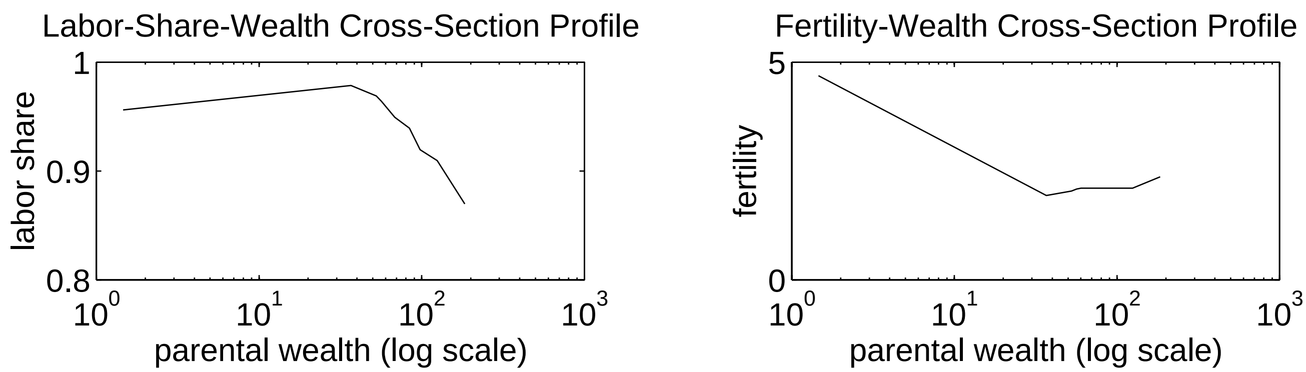
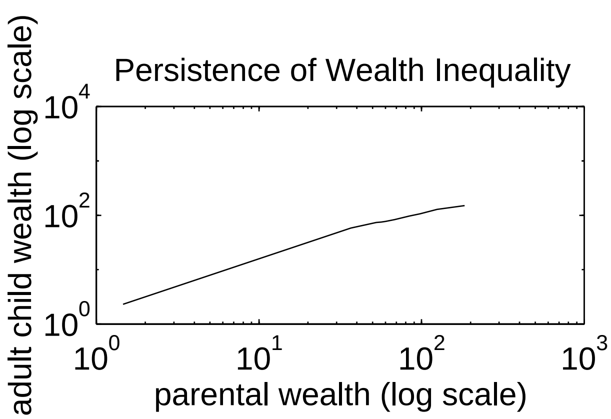


Figure 6-Cross-Section Profiles at t=0 (epsilon=0.5)



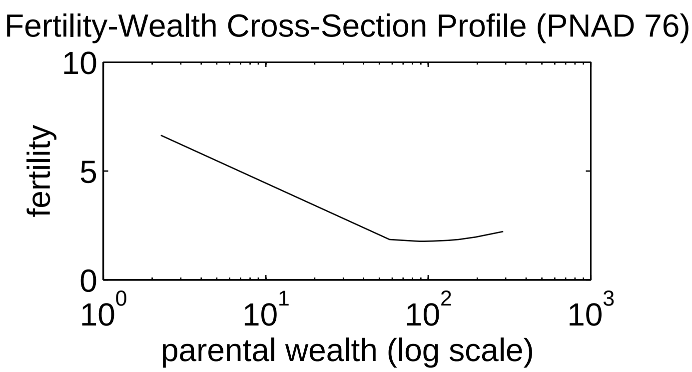
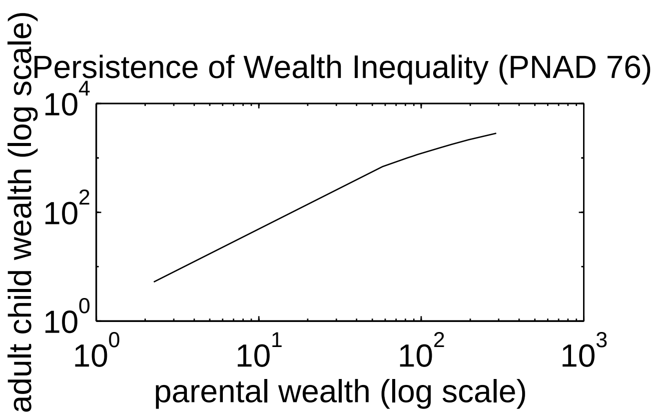


Figure 7-Cross-Section Profiles (PNAD 76)



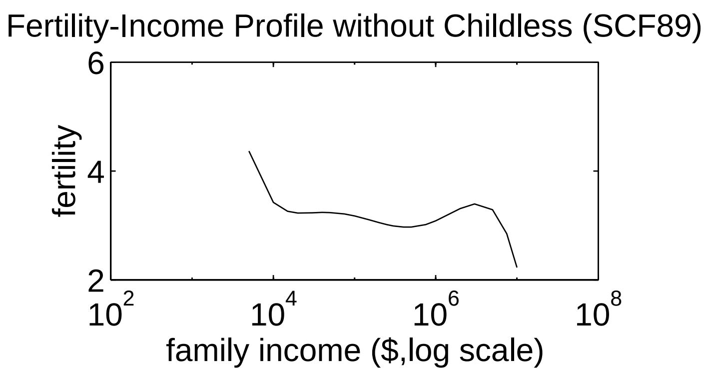


Figure 8-Fertility-Income Cross-Section Profiles (SCF 89)

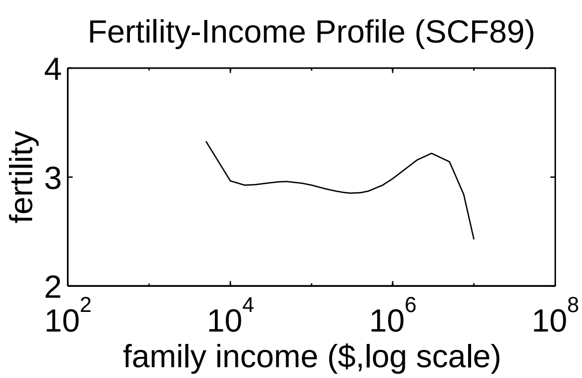


Table1:baseline parameters
$\beta = 0.25$
$r = 3$
$\varepsilon = 0.2$
$A = 5$
$\lambda = 0.14$
$\alpha = 0.67$
$\gamma = 0.33$
$\phi = 0.5$

Table 2: Fertility regression coefficients (baseline parameters)	
wealth	labor income
$\theta_0 = -0.7$	$\theta_0^{\ L} = -0.61$
$\theta_1 = -0.79$	$\theta_1^{\ L} = -0.79$
$\theta_2 = -0.8$	$\theta_2^{\ L} = -0.8$
$\theta_3 = -0.58$	$\theta_3^{\ L} = -0.58$
$\theta_4 = 0$	$\theta_4^{\ L} = 0$
$\theta_\infty = .$	$\theta_\infty^{\ L} = .$

Table 3: Persistence of inequality (baseline parameters)	
persistence of wealth	persistence of labor income
$\rho_0 = 0.94$	$\rho_0^{\ L} = 0.90$
$\rho_1 = 0.88$	$\rho_1^{\ L} = 0.88$
$\rho_2 = 0.81$	$\rho_2^{\ L} = 0.81$
$\rho_3 = 0.71$	$\rho_3^{\ L} = 0.71$
$\rho_4 = 0.62$	$\rho_4^{\ L} = 0.62$
$\rho_5 = 0.55$	$\rho_5^{\ L} = 0.55$
$\rho_6 = 0.44$	$\rho_6^{\ L} = 0.44$
$\rho_7 = 0.13$	$\rho_7^{\ L} = 0.13$
$\rho_8 = 0$	$\rho_8^{\ L} = 0$
$\rho_\infty = .$	$\rho_\infty^{\ L} = .$

Table 4: Fertility regression coefficients ($\lambda = 0.4$)	
wealth	labor income
$\theta_0 = -0.25$	$\theta_0{}^L = -0.22$
$\theta_1 = 0.01$	$\theta_1{}^L = 0.01$
$\theta_2 = 0.22$	$\theta_2{}^L = 0.22$
$\theta_3 = 0.89$	$\theta_3{}^L = .$
$\theta_4 = 0$	$\theta_4{}^L = .$
$\theta_\infty = .$	$\theta_\infty{}^L = .$

Table 5: Persistence of inequality ($\lambda = 0.4$)	
persistence of wealth	persistence of labor income
$\rho_0 = 0.52$	$\rho_0{}^L = 0.45$
$\rho_1 = 0.35$	$\rho_1{}^L = 0.28$
$\rho_2 = 0.19$	$\rho_2{}^L = 0$
$\rho_3 = 0.16$	$\rho_3{}^L = .$
$\rho_4 = 0$	$\rho_4{}^L = .$
$\rho_\infty = .$	$\rho_\infty{}^L = .$

Table 6: Fertility regression coefficients ($\varepsilon = 0.5$)	
wealth	labor income
$\theta_0 = -0.76$	$\theta_0^L = -0.69$
$\theta_1 = -0.92$	$\theta_1^L = -0.92$
$\theta_2 = -0.95$	$\theta_2^L = -0.95$
$\theta_3 = -0.88$	$\theta_3^L = -0.88$
$\theta_4 = 0$	$\theta_4^L = 0$
$\theta_\infty = 0$	$\theta_\infty^L = 0$

Table 7: Persistence of inequality ($\varepsilon = 0.5$)	
persistence of wealth	persistence of labor income
$\rho_0 = 0.93$	$\rho_0^L = 0.91$
$\rho_1 = 0.92$	$\rho_1^L = 0.92$
$\rho_2 = 0.95$	$\rho_2^L = 0.95$
$\rho_3 = 0.98$	$\rho_3^L = 0.98$
$\rho_4 = 1$	$\rho_4^L = 1$
$\rho_\infty = 1$	$\rho_\infty^L = 1$

Table 8: OLS Regression of fertility, adult child's schooling and adult child's income on family income (full-time)- PNAD 76

independent variables	number of children-ever-born	adult child's schooling	adult child's income
parental income	-3.18 * (0.76)	10.17 * (0.58)	2.14 * (0.09)
parental income squared	0.17 * (0.05)	-0.86 * (0.05)	-0.13 * (0.007)
adjusted R squared	0.17	0.48	0.57
N	18227	17658	17658

Notes: (a) All income variables are measured in logs. Standard errors in parentheses. The regressions use sample weights provided by IBGE. N refers to the unweighted number of observations.
(b) The full-time concept of full income is defined as average family income for men who are household heads and work 40 hours per week on average. Different income averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area.
(c) A constant, the mother's age, its age squared, the father's age, its age squared, the oldest child age, its age squared, the oldest child's sex, a dummy variable for urban areas, a dummy variable for the state in which the family resides and mother's schooling are included in each regression.
(d) * significant at the one-percent level

Table 9: OLS Regression of fertility and adult child's wage on father's wage (full-time)- PNAD 76 and Mulligan (1993)

independent variables	number of children-ever-born (PNAD 76)	adult child's wage (PNAD 76)	number of children-ever-born (Mulligan (1993))	adult child's wage (Mulligan (1993))
father's wage	-2.18 * (0.04)	0.76 * (0.006)	-1.17 * (0.16)	0.36 * (0.04)
adjusted R squared	0.18	0.46	0.1	0.21
N	13384	20408	648	648

Notes: (a) All wage variables are measured in logs. Standard errors in parentheses. The PNAD regressions use sample weights provided by IBGE. N refers to the unweighted number of observations.
(b) The full-time wage measure for the PNAD is defined as average hourly wage for men who are household heads and work 40 hours per week on average. Different wage averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area.
(c) A constant, the father's age, its age squared, the child's age, its age squared and the child's sex are included in each regression.
(d) * significant at the one-percent level

Table 10: 2SLS Regression of fertility and adult child's wage on father's wage (full-time)- PNAD 76, PNAD 96 and Mulligan (1993)

independent variables	adult child's wage (PNAD 76)	adult child's wage (PNAD 96)	adult child's wage (Mulligan (1993)
father's wage	0.38 * (0.01)	0.44* (0.01)	0.20 ** (0.11)
mother's schooling	0.07 * (0.003)	0.03* (0.001)	0.06 (0.05)
adjusted R squared	0.46	0.45	0.21
N	16380	34889	648

Notes: (a) All wage variables are measured in logs. Standard errors in parentheses. The PNAD regressions use sample weights provided by IBGE. N refers to the unweighted number of observations.
(b) The full-time wage measure for the PNAD is defined as average hourly wage for men who are household heads and work 40 hours per week on average. Different wage averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area.
(c) For the Mulligan (1993) PSID data mother's schooling is the fitted value from a regression of mother's schooling on the age variables, father's wage, fertility, a gender dummy and agriculture's share of personal income in the county where the son grew up. For the PNAD, mother's schooling is the fitted value from a regression of mother's schooling on the age variables, father's wage, mother's full-time labor income, a gender dummy , and a dummy variable indicating whether the father lives in a urban area.
(d) A constant, the father's age, its age squared, the child's age, its age squared and th
(e) * significant at the one-percent level
** significant at the five-percent level

Table 11: OLS Regression of log earnings on schooling (with and without age controls) - PNAD 76 and Mincer (1993)

independent variables	log earnings (PNAD 76)	log earnings (PNAD 76)- age controls	log earnings (Mincer (1993))	log earnings (Mincer)-age controls
schooling	0.17 * (0.0007)	0.18 * (0.0007)	0.09*	0.09 *
adjusted R squared	0.45	0.47	0.07	0.14
N	62875	62860		

Notes: (a) Standard errors in parentheses. All regressions include a constant term. The PNAD regressions use sample weights provided by IBGE. N refers to the unweighted number of observations.
(b) The respondent's age and his age squared are used in the regressions displayed in columns 2 and 4.
(d) * significant at the one-percent level