Equilibrium Risk-Matching in Group Lending

Loïc Sadoulet *
ECARES/Université Libre de Bruxelles
sadoulet@ulb.ac.be

June, 1999.

Abstract

This paper examines group formation in group-credit contracts recently popular in credit programs for the poor. The joint-liability in these contracts induces a correlation between the choice of partner and of repayment strategy. We show that this leads to non-monotone matching patterns, which differs with the homogeneous or assortative matching assumptions prevalent in the literature.

The heterogeneity in equilibrium arises from the ability of borrowers to use the joint liability to create credible insurance arrangements among group partners in missing insurance market environments. Beyond a certain risk level, nonetheless, borrowers are unable to remunerate safer partners for the asymmetric insurance, and are hence left to match homogeneously. Distributional measure-consistency requirements can also lead to pockets of homogeneous matching in heterogeneous matching regions. The exact matching pattern depends on the distribution of borrower types, although we show it remains non-monotone for any finite or continuous distribution.

This result challenges the common assumption that joint-liability induces borrowers to form groups with partners with similar risk profiles. In missing market contexts, institutional innovations may spillover to fill other failures. This highlights, in particular, the necessity for empirical analyses to carefully account for the endogeneity of group characteristics, as a failure to so can seriously jeopardize the validity of decisions reached.

1 Introduction

Group lending has been a widely heralded tool in the search for financial mechanisms to extend credit to the poor. Most group lending programs have had extensive outreach to the poor, while maintaining extremely high repayment rates. The salient feature of the group-lending approach is that borrowers organize in credit groups which are made jointly-liable for the entirety of the loans

*I am deeply indebted Patrick Bolton, Isabelle Brocas, Juan Carrillo, Patrick Legros, and Robert Shimer for their comments and help. Any mistakes remain, of course, my own responsability.

taken by members. Default by any member of the group has repercussions on all members of that group. A growing number of papers have examined the role of this joint-liability on repayment rates and borrower welfare, as compared to individual lending contracts.[1] However, most studies assume that this joint-liability leads to borrowers matching homogeneously (as in Stiglitz, 1990, and Armendáriz de Aghion, 1996), or take group composition as given (e.g., Besley and Coate, 1996; Diagne, 1998; and Wydick, 1995). Very few papers have addressed the issue of group formation.[2]

The matching literature has examined exactly this issue: how do individuals form in teams to maximize joint production when there is an indivisible input? Models have extended the "marriage model" (Becker, 1981) to account for search frictions (e.g., Diamond, 1982, and Shimer and Smith, 1998), missing markets (e.g., Fernandez and Galí, 1997, and Legros and Newman, 1998), and varying outside options (e.g., Kremer and Maskin, 1995, and Legros and Newman, 1998).[3] However, matching models take borrowers' actions as given. No model (to our knowledge) has examined how group composition can change individual incentive structures.

This paper presents a model in which group membership is endogenous, and in which a borrower's choice of repayment strategy depends on both his own type and on the type of his partner. Group members choose partners in a context of missing insurance markets and decide whether or not to repay and potentially provide insurance for their partner. The results of our model suggest a non-monotone matching pattern in which: (1) safer borrowers form groups with partners riskier than they in a negative manner to exploit beneficial insurance arrangements; (2) middle-type borrowers match either hetero-

[1] See, for example, Stiglitz (1990); Besley and Coate (1993 and 1995); Armendáriz de Aghion (1995); Ghatak (1995); Wydick (1995); Sadoulet (1997) and Diagne (1997).

[2] Ghatak (1995) is a notable exception. He shows that the joint-liability can counter a "market for lemons" tendency in individual loans where the interest rate drives out the better borrowers, by making groups shoulder most of the cost of the riskier borrowers. Groups then form in a homogeneous manner, with the safer borrowers organizing among themselves. The reason our result will differ is that Ghatak does not model borrowers' incentive to repay.

[3] Legros and Newman (1998) provide a very complete discussion of the issues of limited transferability due to missing markets and varying outside options.

geneously with safer borrowers, or homogeneously with borrowers of identical risk as theirs; and (3) the riskier borrowers form in groups homogeneous in risk, since these borrowers are too risky to be accepted in such insurance schemes. The exact matching pattern in groups depends on the distribution of borrower types, although we show that the matching pattern remains non-monotone for any continuous distribution.

The point we illustrate in this paper is that group characteristics are endogenous and hence analyses that take group composition as given can be misleading. This result is important in light of the emerging empirical work on the performance of credit groups. Empirical studies have often followed their theoretical counterparts in assuming (at least implicitly) that matching is exogenous. Authors then use group characteristics as exogenous explanatory variables in analyses of credit-group performance.[4] However, endogeneity in these variables can lead to serious biases in the analysis, and jeopardizes the reliability of policy conclusions derived from the results. In our case, for example, using risk-heterogeneity to explain performance would find that more heterogeneous groups tend to perform better. Drawing a causal relationship between heterogeneity and performance would, however, be misleading. Special care must be taken in empirical analyses to account for the endogeneity of group membership.

The remainder of the paper is organized as follows. The next section presents a model of group lending in a context of missing insurance markets (Sadoulet, 1997). We use results from that paper to derive borrower repayment strategies and expected returns. Section 3.1 introduces the matching problem and defines the equilibrium concept used in the paper. Section 3.2 characterizes the equilibrium. Section 4 summarizes the results and concludes. The proofs are relegated to the appendices.

[4] Matin (1998), for example, explains default rates by the length of the average membership in a group. Sharma and Zeller (1996) use group size, intra-group variance of land holdings, the proportion of relatives in a group, and the percentage of female members to explain repayment rates. Wenner's (1995) explanatory variables of loan-payments delinquency include the average savings of group members, whether the group reports screening members, and a factor-analysis score measuring the groups' organizational strength.

2 Group Lending

2.1 The Model

Microfinance mechanisms are designed to provide credit to the poor. The model we use tries to capture some of the salient characteristics of poor entrepreneurs that have made lending to them difficult: lack of collateral and of other sources of income, and moral hazard opportunities.

For simplicity, the borrowers in our model are assumed to have no assets and no savings capacity between periods. They want to invest one unit of capital in a productive project. Their project has two states of nature: it can succeed and yield X; or it can fail and yield nothing. Borrowers are characterized by their *type,* P_i, which is the probability of success of their projects:

$$\text{Borrower } i\text{'s project yields } \begin{cases} X & \text{with probability } P_i & (\text{"success"}) \\ 0 & \text{with probability } (1-P_i) & (\text{"failure"}). \end{cases}$$

In each period, individuals need to borrow the required unit of capital from a financial institution at an interest rate r, which they take as exogenous. The financial institution[5] offers two types of loans: individual loans and group loans. Individual loans give borrowers one unit of capital and they must repay $L = 1+r$ units back in order not to be considered in default. To get a group loan, borrowers must choose a partner. Each borrower receives one unit of capital but both borrowers are *jointly-liable* for the entire group loan: if the bank does not receive $2L$ as the group's repayment, then both borrowers are considered in default. The interest rate on both loans is assumed to be equal[6] and borrowers can only participate in one loan at any given time.

Since borrowers have no assets, the financial institution cannot require collateral from the borrowers to back the loans. The incentive for borrowers to repay their loans is maintained by a rule: borrowers are guaranteed access to

[5] I assume that the financial institution is the sole source of "cheap credit" to avoid issues of credibility of institutional rules – see discussion in Sadoulet (1997).

[6] The interest rates in the two different contracts are taken as equal to replicate the policy of *Génesis Empresarial,* the financial institution which hosted the survey in Guatemala. Why the interest rates do not differ is not clear, particularly since these two programs report different repayment rates. A personal conversation with Steven Gross, financial manager of ACCION International, did not resolve the mystery.

future loans upon repayment of the loan; however, any borrower in default – as defined above for each contract – loses access to both individual and group loans forever.[7]

To simplify the notation, we will normalize everything in terms of the repayment L. The borrowers' return when successful will then be

$$x \equiv X/L$$

and borrowers will be thought of as repaying 1 unit. We also restrict groups to be of size 2 as we want to abstract the analysis from the potential trade-offs between group-size and quality of partners.[8]

For convenience, each borrower is assumed to always be *able* (though not necessarily *willing*) to repay the entire group loan when his project is successful, i.e., $x > 2$. Borrowers' alternative sources of credit (typically money-lenders extending individual loans) are assumed to be able to extract all the surplus from borrowers through the use of local information and enforcement mechanisms. A defaulter's fallback value from losing access to future loans from the institution is therefore normalized to zero. Borrowers' projects will be taken as uncorrelated to preclude the problem of trade-off in the choice of partner between the probability of success of his project and its covariance. Borrowers are taken to be infinitely lived and have the same discount factor $\delta \in (0, 1)$. Borrower types P_i are assumed to be (finitely or continuously) distributed according to a distribution F on $\mathcal{B} \subseteq [0, 1]$, and there is a unit mass of borrowers of every type. This is to bypass the question of availability of homogeneous partners, and to insure that all borrowers of a same type match in the same way in equilibrium (*equal treatment property*).[9]

[7] As pointed out in Sadoulet (1997), these lending contracts are obviously not optimal from the financial institution's point of view. In particular, a contract that would dominate these would be a kind of "credit record" as is done with credit cards. Borrowers could use this credit record to insure themselves. However, non-collateralized lending contracts in practice are designed with this "exclusion forever" clause.

[8] If there were no costs of monitoring, groups would be of infinite size to gain in diversification; or contracts between groups would spring up, in essence making them groups of infinite size. The restriction on the size of groups in essence is an assumption that it would be prohibitively expensive for borrowers to monitor more than one party.

[9] There are thus the same number of borrowers of each type, and borrowers are more or less densely distributed on $\mathcal{B}$.

Sadoulet (1997) shows that if borrowers are "safe enough," they have an incentive to maintain access to future loans. Borrowers whose expected returns in future periods are larger than the cost of repaying the loan will want to repay loans. Some borrowers will even set up insurance arrangements within their group in which partners will cover each others' loans in case of project failure. The reason is that borrowers operate in risky environments and hence need insurance; group lending allows for insurance arrangements to arise even in environments which are otherwise non-conducive to insurance contracts.[10] What allows these insurance contracts to be sustained is that borrowers cannot renege on their insurance commitment because of the joint-liability: if they do not provide insurance, the whole group – including the borrowers reneging on the insurance commitment – loses access to future loans. In addition, strategic defaults, in which borrowers are *unwilling* to repay when they are *able* to, can be punished by exclusion from future rounds of the loan. Group lending hence creates a punishment technology which allows for the enforcement of insurance contracts. In general, these insurance arrangements require some kind of transfer between members when both are successful, to compensate for asymmetric risks.

We note the difference between *transfers* between borrowers, and *insurance.* Transfers refer to borrowers paying (at least in expected terms) part of their partner's loan when both borrowers' projects are successful. Insurance, on the other hand, is when one borrower cover his partner's loan when his partner's project fails. A borrower providing insurance hence covers the full group loan amount of 2 units.

The general individual rationality conditions for groups to choose mutual-insurance strategies in equilibrium are shown graphically in figure 1.[11] They, in essence, require that groups form within an individual rationality envelope, in

[10] Note that, even though borrowers are risk-neutral, there are gains from insurance since a default makes borrowers lose access to future loans forever.

[11] The alternatives for borrowers are to match with a partner of same risk (homogeneous matching), or to participate in an individual loan. The conditions are detailed in Sadoulet (1997).

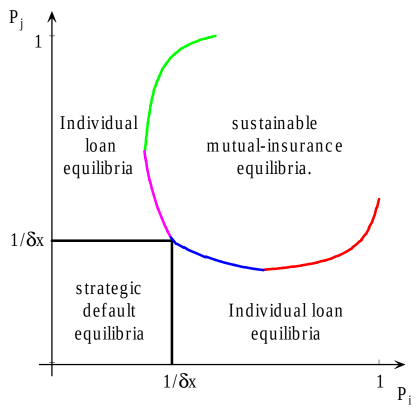

Figure 1: Sustainable mutual-insurance equilibria.

which partners of a group are not "too risky" compared to each other (a borrower who always succeeds would never want to form a group with a partner whose project always fails, for example); and in which borrowers are safe enough that maintaining access to future loans is profitable (a very risky borrower whose project succeeds might not want to repay as there is a high probability that his project will fail in the next round). In this paper, we will concentrate on borrowers who are safe enough to be willing to repay their own (individual) loan. These are borrowers whose discounted expected return next period is greater than the cost of repaying the loan, i.e.,

$$\underline{p} \equiv \min_{P_i \in \mathcal{B}} P_i \geq \frac{1}{\delta x}. \tag{1}$$

We will disregard borrowers riskier than $(\delta x)^{-1}$.[12]

2.2 Expected returns under group lending.

Since borrowers have perfect and complete information about each other's risks, there is no learning or adjustment of repayment strategy from period to period. Borrowers' matching decision is the same in each period. The joint return for

[12] If the set of borrowers $\mathcal{B}$ were to contain borrowers riskier than $(\delta x)^{-1}$, Sadoulet (1997) shows that these borrowers would choose to default in all states of nature in equilibrium and would form groups with other similar borrowers in $\left[0,(\delta x)^{-1}\right]$ or participate in individual loans. As they would match homogeneously in equilibrium, their inclusion would not modify the equilibrium we describe in Section 3.

a group $\{P_i, P_j\}$ with mutual-insurance strategies is then given by the infinite-horizon discounted sum of per-period expected returns:

$$\begin{aligned} W(P_i, P_j) &= [P_i x + P_j x - P_i P_j 2 - P_i (1 - P_j) 2 - P_j (1 - P_i) 2] \\ &\quad + \delta (1 - (1 - P_i)(1 - P_j)) \{W(P_i, P_j)\}. \end{aligned}$$

With probability P_i, borrower i's project is successful and yields x. The same holds for j with probability P_j. When at least one of the projects is successful, both loans get repaid and the group gets access to future loans. If the loan is not repaid, borrowers lose access to future loans and hence get a payoff of zero thereafter.[13] The infinite discounted sum above can be rewritten as:

$$W(P_i, P_j) = \frac{(P_i + P_j) x - (P_i + P_j (1 - P_i)) 2}{1 - (P_i + P_j (1 - P_i)) \delta}. \tag{2}$$

Becker's classic result is that complementarities in the joint-payoff leads to *positive assortative matching* (Becker, 1981, Chapter 4). In our context, this would entail that matched individuals are identical in equilibrium.

Taking cross-partials of the joint payoffs yields:

$$\frac{\partial^2 W(P_i, P_j)}{\partial P_i \partial P_j} = 2 \frac{\delta (1 - P_j - P_i + P_i P_j \delta)(x - 1) + (1 - \delta)(1 - \delta P_i P_j)}{(1 - (P_i + P_j (1 - P_i)) \delta)^3}. \tag{3}$$

We note that if $\delta = 1$, the cross partial is positive:

$$\frac{\partial^2 W(P_i, P_j)}{\partial P_i \partial P_j} = 2 \frac{x - 1}{(1 - (P_i + P_j (1 - P_i)))^2} > 0.$$

There is hence complementarity among borrowers which leads to positive assortative matching: matched partners are identical in equilibrium if $\delta = 1$.

However, if $\delta < 1$, then the cross partial is positive if and only if

$$P_i < \frac{\delta (1 - P_j) x + 1 - \delta (2 - P_j)}{\delta ((1 - \delta P_j) x - (1 - P_j))}.$$

It is easily checked that there exist values of δ and x such that the cross partial in (3) does not have the same sign for all P_i and P_j. There will hence not be complementarities (or substituabilities) over the whole interval of borrowers

[13] As noted above, borrowers benefit from insurance even though they are risk-neutral, because after a failure they lose access to future credit. The expected value of their project is thereafter zero. This, essentially, makes their payoff function concave.

and we cannot refer to Becker's monotone matching result. We hence use an alternative approach, as in Legros and Newman (1998).

2.3 Surplus from matching heterogeneously.

Define a group $\{P_i, P_j\}$'s *surplus* from matching heterogeneously to be the difference between the heterogeneous group's joint-return, and each individual's expected payoffs in separate homogeneous groups[14]:

$$\sigma(P_i, P_j) = W(P_i, P_j) - \frac{1}{2}W(P_i, P_i) - \frac{1}{2}W(P_j, P_j). \tag{4}$$

The joint-return in homogeneous groups is assumed to be split evenly between the two partners.[15] Note that the surplus function is symmetric so that

$$\sigma(P_i, P_j) = \sigma(P_j, P_i),$$

and that the surplus of a homogeneous group is, by definition, zero:

$$\sigma(P_i, P_i) = 0.$$

For future reference, it is also worth noting that $\sigma(P_i, P_j)$ is continuous and differentiable in P_i and P_j.

The following Lemma describes the surplus function $\sigma(p, P_j)$.

Lemma 1 *The surplus function $\sigma(P_i, P_j)$ is such that:*

1. *there exists a unique p' such that $\sigma(p', P_j) < 0$ for all $P_j < p'$ and $\sigma(p', P_j) > 0$ for all $P_j > p'$. Moreover,*

2. *$\forall P_i > p'$, there exists $l(P_i) < p'$ such that*

$$\left| \begin{array}{lll} \sigma(P_i, P_j) < 0 & \forall P_j < l(P_i) & \\ \sigma(P_i, P_j) > 0 & \forall P_j > l(P_i), & P_j \neq P_i \\ \sigma(P_i, P_j) = 0 & \textit{if } P_j = l(P_i) & \textit{or } P_j = P_i. \end{array} \right.$$

[14] *Segregation payoffs* in Legros and Newman (1998).

[15] Homogeneous borrowers have identical preferences and outside option.

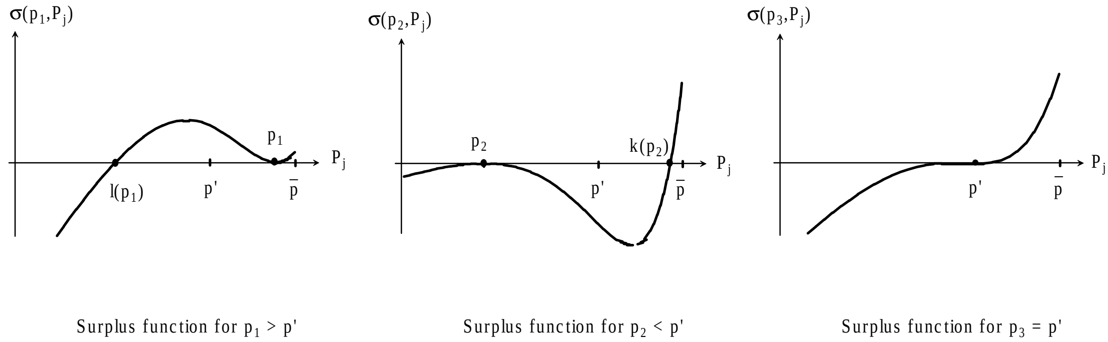

Figure 2: The surplus function $\sigma(p, P_j)$ as a function of partner risk P_j.

3. $\forall P_i < p'$, *there exists* $k(P_i) > p'$ *such that*

$$\left| \begin{array}{lll} \sigma(P_i, P_j) > 0 & \forall P_j > k(P_i) & \\ \sigma(P_i, P_j) < 0 & \forall P_j < k(P_i), & P_j \neq P_i \\ \sigma(P_i, P_j) = 0 & \text{if } P_j = k(P_i) & \text{or } P_j = P_i. \end{array} \right.$$

4. The points p', $l(P_i)$ *and* $k(P_i)$ *are independent of the distribution of borrower types* F.

Lemma 1 proves that the surplus is as represented in figure 2. The argument of the proof (in appendix A.1) goes as follows. The surplus function $\sigma(p, P_j)$ as a function of the partner's risk P_j is a cubic function with a double root at $P_j = p$. We show that there exists a unique value of p, which we call p', such that the third root is also equal to p and that the surplus $\sigma(p', P_j)$ is increasing in P_j. This gives the right-hand panel of figure 2. We then establish that: for all $p < p'$, the third root $k(p)$ is greater than p' (as in the middle panel); while for $p > p'$, the third root $l(p)$ is smaller than p'. Finally, knowing the sign of the surplus of matching p' with the riskier partner $\underline{p}$ (always negative) and with the safest partner $\overline{p}$ (always positive, if $\overline{p} > p'$) leads to the shapes drawn in figure 2.

Matching homogeneously yields a surplus of zero, by definition of the surplus function. However, if a relatively safe borrower (i.e., $p_1 > p'$ – left panel of figure 2) matches with someone riskier than he, the surplus from heterogeneous matching can be positive as the net insurance provided by p_1 increases the joint expected return $W(p_1, P_j)$ beyond the sum of each individual returns in separate homogeneous groups. Positive surpluses can be maintained as long as P_j is not too risky:

$$W(p_1, P_j) > \frac{1}{2}W(p_1, p_1) + \frac{1}{2}W(P_j, P_j) \quad \text{if } p_1 > p' \text{ and } P_j > l(p_1).$$

If P_j is riskier than $l(p_1)$, the costs to p_1 of insuring P_j are higher than the benefits P_j gains from the extra insurance. The surplus from those matches become negative.

For relatively risky borrowers (i.e., $p_2 < p'$ – middle panel of figure 2), the only partners that yield a positive surplus are borrowers much safer than they (i.e., with $P_j > k(p_2)$). Matching with any partner P_j riskier than $k(p_2)$ makes the extra cost of insurance for P_j (as compared to a homogeneous P_j group) not worth the benefit to p_2. The surplus of p_2 matching with any partner riskier than $k(p_2)$ therefore yields a negative surplus σ.

The point p' (right panel of figure 2) is such that matching with a partner safer than he yields a positive surplus. However, p' would be better off in a homogeneous group than with a partner riskier than he.

This in particular implies that any two borrowers riskier than p' matched together would yield a negative surplus, unless the partner happens to be of identical risk. We state this as a lemma as we will refer to this result later.[16]

Lemma 2 *$\forall P_i, P_j \leq p'$ with $P_i \neq P_j$, $\quad \sigma(P_i, P_j) < 0$.*

As we will see, this will imply that no borrowers riskier than p' will ever match together in equilibrium unless they are identical.

3 Equilibrium Matching.

3.1 Definitions.

Borrowers are distributed on a set $\mathcal{B}$ according to a (finite or continuous) distribution F with measure μ. We are interested in finding a *matching correspon-*

[16] Proof in Appendix A.2.

dence m which will assign to each $P_i \in \mathcal{B}$ a partner $m(P_i) \in \mathcal{B}$. For m to be an equilibrium matching, it must satisfy the following properties:

Definition 1 *An equilibrium matching correspondance m will be a correspondence from $\mathcal{B}$ onto $\mathcal{B}$ which to each $P_i \in \mathcal{B}$ assigns a partner $m(P_i) \in \mathcal{B}$ such that the matches are:*

1. feasible: *the aggregate of each borrower's expected portion of the total surplus does not exceed the total surplus produced, i.e.,*[17]

$$\frac{1}{2}\int_{\mathcal{B}} [s(P_i) + s(m(P_i))]\ dF(P_i) \leq \int_{\mathcal{B}} \sigma(P_i, m(P_i))\ dF(P_i)$$

where $s(P_i)$ is P_i's expected portion of the total surplus in equilibrium.

2. stable: *no pair of borrowers P_i, P_j (matched or not to each other under m) has an incentive to break their assigned position to form a new group together and share its surplus:*

$$s(P_i) + s(P_j) \geq \sigma(P_i, P_j).$$

In particular, this requires all matched groups to produce a non-negative surplus in equilibrium:

$$\sigma(P_i, m(P_i)) \geq 0 \quad \forall P_i \in \mathcal{B}.$$

3. measure consistent: *every set of borrowers has to match to a set of the same measure, i.e.,*

$$\forall \mathcal{I} \subseteq \mathcal{B}, \quad \mu(\mathcal{I}) = \mu(m(\mathcal{I})).$$

This requires the number of "borrowers" and "partners" to add up.[18]

The definition of equilibrium should also contain an assignment of surplus, as it needs to satisfy feasibility. Nonetheless, the following results from Roth and

[17] The factor $\frac{1}{2}$ is to account for the double counting in the left hand side.

[18] *Measure consistency* is crucial because we are in a one-sided matching problem: the "borrowers" and "partners" in a group come from the same population. This requirement rules out matching rules such as $m(p) = \frac{1}{2}p$ which would match a set $[0,1]$ one-to-one to the smaller set $\left[0, \frac{1}{2}\right]$. Despite being a one-to-one mapping, it is not measure consistent.

Sotomayor (1990) and Legros and Newman (1998) will allow us to characterize the equilibrium without having to consider how the surplus is split between borrowers.

Lemma 3 *There are no transfers between members of different groups in stable outcomes.*[19]

This essentially comes from the assumption of stability which requires individual surpluses to be at least as great as what borrowers would get if they shared all the surplus created by their own group. Hence, there will be no net transfer out of any group.

Lemma 4 *Any equilibrium is optimal in the sense that it maximizes the aggregate surplus (given a group size of* 2*).*[20]

We give no proof of Lemma 4; the interested reader is instead referred to Legros and Newman (1998). In essence the argument is that, if the equilibrium matching m does not maximize total surplus, there exists some other measure consistent matching $\widetilde{m}$ which yields a higher expected surplus. This means that there exists at least one pair of borrowers unmatched under m which, when matched under $\widetilde{m}$, produce a higher surplus than both received under m. This pair would then break from the assigned groups under m and form their own group; m would not be stable.

Lemma 4 insures us that we only have to worry about finding the matching rule that maximizes the aggregate expected surplus without being concerned about the division of surplus within groups in equilibrium.

We also make precise the concept of negative matching, which will be important in the characterization of the equilibrium:

Definition 2 *Borrowers on an interval $\mathcal{I} \subseteq \mathcal{B}$ will be said to <u>match negatively</u> if for any $P_i, P_j \in \mathcal{I}$:*

$$P_i > P_j \Longleftrightarrow m(P_i) < m(P_j).$$

[19] This is an implication of Lemma 8.5 in Roth and Sotomayor (1990).
[20] This is Proposition 1 in Legros and Newman (1998).

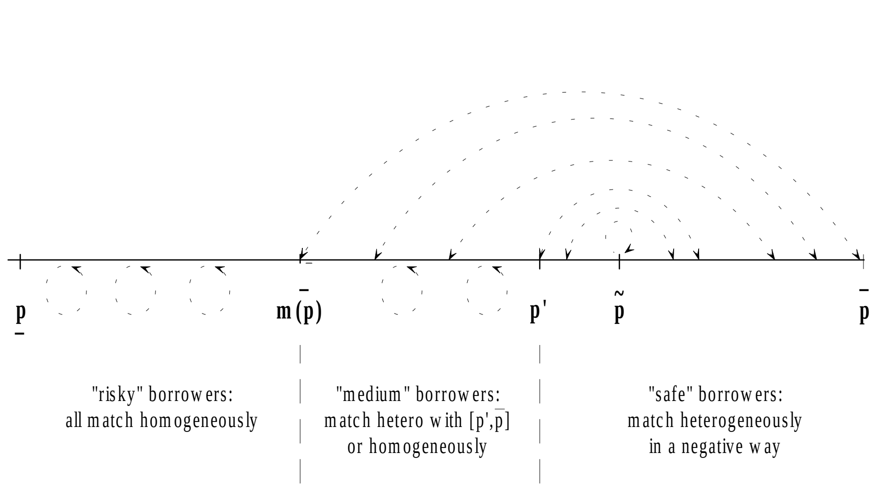

Figure 3: Equilibrium matching pattern.

A borrower P_i will be said to <u>*match homogeneously*</u> *if P_i matches with a partner of identical risk:*

$$m(P_i) = P_i.$$

Note that the definition of *negative matching* does not say anything about the matching pattern in the matching image of $\mathcal{I}$, $m(\mathcal{I})$. This distinguishes negative matching from the concept of *negative assortative matching* which considers matching patterns within a set.[21]

3.2 Non-monotone matching

This section characterizes the equilibrium. The reader might find figure 3 helpful for the discussion.

Proposition 1 *Take any continuous distribution F of borrower types on $\mathcal{B} = [\underline{p}, \overline{p}]$. In equilibrium, the set of borrowers $\mathcal{B}$ can be divided into three subsets:*

[21]Negative assortative matching usually refers to a property on a set, say J. Take any two equilibrium groups (a, b) and (c, d) in $\mathcal{J} \times \mathcal{J}$. The equilibrium satisfies negative assortative matching if

$$\max(a, b) \geq \max(c, d) \Longleftrightarrow \min(a, b) \leq \min(c, d).$$

In our definition, we don't look at the matching image of $\mathcal{I}$. In particular, $\mathcal{I}$ would still display negative matching according to our definition if there existed some equilibrium group within the image $m(\mathcal{I})$ of the set $\mathcal{I}$, say $(x, m(x)) \in m(\mathcal{I}) \times m(\mathcal{I})$, such that $x < m(P_i) < m(x) < P_i$ for some $P_i \in \mathcal{I}$ and $m(P_i) \in m(\mathcal{I})$. This would not satisfy negative assortative matching.

1. *'safe' borrowers in $[p', \overline{p}]$ who match heterogeneously and in a negative way;*

2. *'medium' borrowers in $[m(\overline{p}), p')$ who match either heterogeneously with partners in $[p', \overline{p}]$ or homogeneously if no 'safe' borrower is available; and*

3. *'risky' borrowers in $[\underline{p}, m(\overline{p}))$ who match homogeneously.*

The proof of Proposition 1 is provided in Appendix A.4. We only give here the logic of the proof and some interpretation of the result.

Recall that p' is defined as the borrower who can generate a surplus with and only with any partner safer than he. The result of the proposition is that we can classify borrowers in three different categories – 'safe' above p', 'medium' in between $m(\overline{p})$ and p', and 'risky' below p'– and that each category of borrower matches in a different manner in equilibrium.

In equilibrium, the 'safe' borrowers $[p', \overline{p}]$ match heterogeneously in a negative way (Appendix A.4.1). The reason is that for a 'safe' borrower, the marginal effect on the surplus caused by lowering the risk of his partner decreases with his own risk. Therefore, for any two 'safe' borrowers p_1 and p_2 and partners q_1 and q_2 such that

$$q_2 < q_1 < p_2 < p_1,$$

if the matching correspondence prescribed groups $\{p_1, q_1\}$ and $\{p_2, q_2\}$ in equilibrium, then a rearrangement of the partners in the groups would allow to increase the total surplus and make everybody better off.[22] The matches would hence not be stable.

There is one (at most) type of 'safe' borrowers that matches homogeneously in equilibrium, which we call $\widetilde{p}$.[23] All the borrowers above $\widetilde{p}$ match with partners riskier than they in $[m(\overline{p}), \widetilde{p})$, while the rest of the 'safe' borrowers in $[p', \widetilde{p})$ match with partners safer than they in $(\widetilde{p}, p']$. By the property that 'safe' borrower match negatively, as borrowers get closer to $\widetilde{p}$, the heterogeneity in their

[22] By Lemma 4, which states that the aggregate surplus must be maximized in equilibrium, and by Lemma 3, which shows that there are no transfers between groups.

[23] The existence and uniqueness of $\widetilde{p}$ is proved in Corollary 2 in Appendix A.4.1.

group gets smaller. The type $\widetilde{p}$ therefore matches homogeneously in essence as a limit case of heterogeneous matching. The uniqueness of $\widetilde{p}$ stems from the fact that if two different 'safe' types matched homogeneously, they could exchange partners and create two heterogeneous groups with positive surplus.

The borrowers who choose riskier partners do so because they can extract some surplus from their riskier partners to compensate (and possibly remunerate) them for their net insurance provision.[24] The borrowers matching with safer partners have to pay a transfer. However, this transfer buys them higher levels of insurance than under homogeneous matching. How much of the surplus the safer borrowers can extract will depend on the threat points each borrower can use, which in turn depend on F. We note that, unless F is discrete and such that the safest borrower below p' is so risky that a negative *assortative* matching among the 'safe' leads to a higher total payoff, the safest borrower $\overline{p}$ will choose a partner $m(\overline{p})$ no safer than p'. The interval $[m(\overline{p}), p')$ then defines the 'medium' borrowers,[25] and the borrowers below $m(\overline{p})$ are called 'risky'.

The 'risky' borrowers in equilibrium are always too risky to be chosen by 'safe' borrowers to join their group: their expected returns are not sufficient to pay a 'safe' partner for the higher expected cost of insurance. In addition, no borrower riskier than p' will ever match with a partner riskier than p' other than a partner of identical risk as his (by Lemma 2). Therefore, 'risky' borrowers all match homogeneously with partners of identical risk.

The 'medium' borrowers match either with much safer borrowers (i.e., borrowers in $[\widetilde{p}, \overline{p}]$), or homogeneously with other 'medium' borrowers of identical risk. In essence, all 'medium' borrowers would like to match with 'safe' partners, but there might not be enough 'safe' partners for all the 'medium' ones. The alternative of matching with another 'medium' of different risk or with a 'risky' partner would produce a negative surplus (by Lemma 2). The 'medium'

[24] Transfers from the risky to the safe member are necessary to sustain heterogeneous groups as the safe member provides more and receives less insurance than in a homogeneous group. As borrowers always have the option of forming homogeneously, the risky partner must compensate his safer partner for the unbalanced insurance. See Sadoulet 1997.

[25] $m(\overline{p}) \leq m(p')$ as the 'safe' borrowers match negatively and $m(p') \geq p'$.

borrowers who have no 'safe' partners available to them hence match homogeneously.

An interesting feature of the equilibrium is the potential non-assortative matching in the set of 'medium' borrowers. The second point of Proposition 1 stipulates that some 'medium' borrowers can be left to match homogeneously while other 'medium' types around them match with 'safe' partners. We can therefore have pockets of homogeneous matching in a otherwise negative assortative matching pattern. This is due to the fact that all borrowers below $m(\overline{p})$ match homogeneously, and that 'safe' borrowers can always pick a borrower below $m(\overline{p})$ with whom to match, if such at type exists above $l(\overline{p})$.

Take the example of a borrower type-distribution F such that $F(\overline{p}) - F(p') = F(p') - F(p' - \varepsilon)$. By negative matching, it is easy to show that p' matches with p'. In addition, there are sufficiently many 'medium' borrowers for the 'safe' set $[p', \overline{p}]$ to match with the 'medium' set $[p' - \varepsilon, p']$. Define $m^*(\overline{p})$ to be the partner that maximizes the surplus in $\overline{p}$'s group, i.e.,

$$m^*(\overline{p}) \equiv \arg\max_{q \leq \overline{p}} \sigma(\overline{p}, q). \tag{5}$$

If $m^*(\overline{p}) < p' - \varepsilon$, then $\overline{p}$ can increase the surplus in his group (and hence the total surplus) by not matching with $p' - \varepsilon$ and matching with $m^*(\overline{p})$. The 'medium' borrower $p' - \varepsilon$ would be left to match homogeneously. We therefore have the following corollary:

Corollary 1 *If the type distribution F is continuous and such that there exists some $q \in (m^*(\overline{p}), p')$ such that $F(p') - F(q) = F(\overline{p}) - F(p')$, then the equilibrium match will display "holes" in which some 'medium' borrowers match homogeneously while the other 'medium' borrowers match heterogeneously.*

The proof is provided in Appendix A.4.3. Note that there can be no homogeneous matching holes among the 'safe' borrowers because of negative matching.

As illustrated in the example above, the differentiation between 'medium' and 'risky' borrowers, as well as who among the 'medium' segment matches heterogeneously or homogeneously, depends on the distribution of types F. As all borrowers who match homogeneously generate zero surplus, the maximization of total surplus in equilibrium is equivalent to the maximization of the surplus generated by the heterogeneous matches. The points $\widetilde{p}$ and $m(\overline{p})$ are hence jointly determined by the maximization:[26]

$$\max_{\widetilde{p},m(p)} \int_{\widetilde{p}}^{\overline{p}} \sigma(p, m(p))\, dF(p)$$

subject to a measure consistency constraint:

$$F(p) - F(\widetilde{p}) \leq F(\widetilde{p}) - F(m(p)) \qquad \forall p \geq \widetilde{p}.$$

If the measure-consistency constraint never binds for any $p \geq \widetilde{p}$, then the equilibrium is one in which each borrower $P_i \in [\widetilde{p}, \overline{p}]$ chooses a partner $m^*(P_i)$ where m^* is given by:[27]

$$m^*(P_i) \equiv \arg\max_{q \leq P_i} \sigma(P_i, q)$$

and the 'safe' type $\widetilde{p}$ who matches homogeneously is equal simply p'. If in addition F is such that $F(p) - F(p') = F(p') - F(m^*(p))$ for all p, then there will be no "holes" in $[m^*(\overline{p}), p']$ in which 'medium' borrowers are left to match homogeneously. This is because the optimal choices for 'safe' borrowers happen to exactly match the availability of 'medium' borrowers.

To sum up, the equilibrium displays a matching structure in which a non-monotone relationship between individual risk and group risk composition exists. Some borrowers (the 'safe' ones) enter heterogeneous risk-sharing arrangements with intragroup transfers. Others (the 'risky' ones) have projects which fail too often to enable them to pay more extra insurance and hence match homogeneously. And there are potentially some ('medium') borrowers who are in homogeneous group because of the lack of 'safe' partners and yet are surrounded by similar ('medium') borrowers who match heterogeneously. Note

[26] Again, the integrals are sums if F is finite.

[27] Note: the measure consistency not binding does not imply that everbody can match with $P_i = 1$; there are still only one unit of each type. However, there are no binding restrictions on the density of types on $\mathcal{B}$.

that this non-monotone matching within the category of 'medium' borrowers is not due to search or other *matching frictions* which keep borrowers from setting finding their first-best partner,[28] as there is no concept of matching frictions in our model. It is entirely due to a relative dearth of one (endogenous) type of borrowers.[29] The risk composition of credit groups will hence depend not only on borrowers' risk, but on the distribution of borrower types also.

3.3 Example

We illustrate the qualitative features of the equilibrium with the following example. Take the parameters of the model to be the following:

$$\begin{cases} x = 2 \\ \delta = .9 \end{cases}$$

so that projects yield a return of $2L$ when successful,[30] and that agents discount at a rate of 10% between periods. Using equations (8), (10), and (5), we can calculate the values of p', which distinguishes 'safe' and 'medium' borrowers; the point $m^*(1)$, which maximizes the surplus in $p = 1$'s group; and the point $l(1)$, below which $p = 1$ would never choose a partner as it would lead to a negative surplus. The values of these variables are the following:

$$\begin{cases} p' & = .8333 \\ m^*(1) & = .75037 \\ l(1) & = .5555 \end{cases}$$

[28] *Matching frictions* refer to any characteristics of borrowers or the lending environment which impede borrower's ability of matching with their (constrained) first-best partner. Such frictions may include problems of availability of partners, informational problems which restrict borrowers' monitoring ability, social codes restricting enforcement sanctions, or characteristics which impede borrowers' credibility in promising or requiring transfers.

[29] Note that "dearth" refers to the fact that certain types are distributed more "densely" than others, i.e. there can exist p_1 and p_2 such that

$$\int_{p_1}^{p_1+a} dF(p) < \int_{p_2}^{p_2+a} dF(p).$$

Not all borrowers in the interval $[p_2, p_2 + a]$ will be able to match with borrowers in $[p_1, p_1 + a]$, despite there being a unit mass of each type.

[30] As mentioned above, $x = 2$ implies returns of 200% which seems unlikely in practice, particularly considering that microfinance loans are often short term loans. This assumption was necessary to insure that borrowers were always *able* to provide insurance when their partner's project failed. Nonetheless, it is also unlikely that projects fail completely, yielding zero returns, in practice. The assumption that $x \geq 2$ should be interpreted as an assumption that borrowers are always able to provide the partial-payment insurance their partners need.

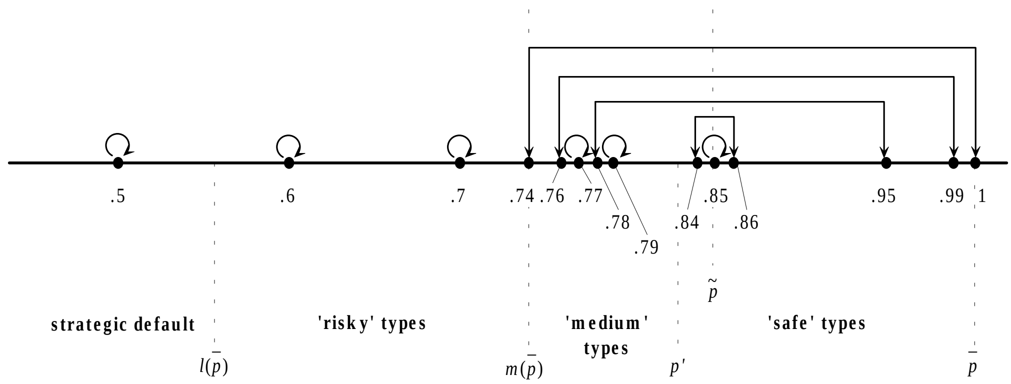

Figure 4: Example 1 - non-assortative matching.

Suppose that there are 14 borrower types given by:[31]

$$\mathcal{B} = \{.5, .6, .7, .74, .76, .77, .78, .79, .84, .85, .86, .95, .99, 1\}.$$

Taking all the possible combinations of types in groups of two, and using Lemma 4 which stipulates that the equilibrium matching will be the one which maximizes the aggregate surplus, we find the equilibrium groups to be the following:[32]

$\{.5, .5\}$	$\{.6, .6\}$	$\{.7, .7\}$	$\{.74, 1\}$	$\{.76, .99\}$	$\{.77, .77\}$	$\{.78, .95\}$
$\{.79, .79\}$	$\{.84, .86\}$	$\{.85, .85\}$	$\{.86, .84\}$	$\{.95, .78\}$	$\{.99, .76\}$	$\{1, .75\}$

We depict the equilibrium matching in Figure 4.

The 'safe' borrowers are the types above $p' = .8333$. These borrower match negatively: take any two 'safe' borrowers; the safer of the two has a partner riskier than the other 'safe' type's partner.[33] The only 'safe' type who matches homogeneously is $\widetilde{p} = .85$.

The 'medium' types are in the interval $[m(1), p'] = [.74, .8333]$. These borrowers match either with 'safe' types, or they match homogeneously. The fact that $p = .77$ and $p = .79$ match homogeneously provides an illustration of the

[31] Practically, we take 2 borrowers of each type to simulate the unit mass of each type: it allows borrowers to match homogeneously, and insures an *equal treatment* property (all borrowers of each type match similarly).

[32] The simulations were programmed in Stata with the help of Vince Wiggins from Stata Corp.

[33] Take p_1 and p_2 to be two "safe" borrowers, with $p_1 > p_2$ (i.e., p_1 is safer than p_2). Then p_2's partner $m(p_2)$ is safer than p_1's partner $m(p_1)$, as the "safe" borrowers match negatively.

homogeneous matching pockets among 'medium' types described in Proposition 1. There is a dearth of "safe" types which leads some "medium" types to match homogeneously.

The 'risky' types below $m(1) = .74$ match homogeneously. Matching in homogeneous groups gives them higher expected returns than individual loans, but no safer borrower will accept them in a heterogeneous group. With the parameters chosen, borrowers below $(\delta x)^{-1} = .556$ will always default strategically and hence are indifferent between individual loans and group loans.[34]

Two interesting properties of this equilibrium are the difference in matching patterns between the 'risky' types and the others, and the existence of non-assortative matching within the 'medium' region. In particular, it is interesting to note how these properties adjust with the parameters of the model and with the distribution of borrower types. We examine these in turn.

3.3.1 The effect of the type distribution F

If we modulate the example by changing the types slightly, we find that the equilibrium changes. Say that borrower types are the following:

$$\mathcal{B} = \{.5, .6, .7, .74, .76, .77, .78, .79, .84, .85, .90, .95, .97, 1\}$$

maintaining the preceding values for x and δ. We changed only two types as compared to the first example: .99 and .86 were replaced by .97 and .9, respectively. The equilibrium then becomes one in which there are no homogeneous groups among the 'medium' and the 'safe' borrowers, as shown in Figure 5. In particular, there is no 'safe' type $\widetilde{p}$ who matches homogeneously.

3.3.2 The effect of project returns x and discount rate δ

Increasing the returns x of projects when they are successful increases the benefit of maintaining access to future loans. One could then expect that surpluses from heterogeneous matching would increase, and that this could lead to more

[34] Note that this implies that no borrower wanting to repay loans would ever choose borrowers below $(\delta x)^{-1}$ as partners. The search could hence have been limited to the 13 types above $p = .556$.

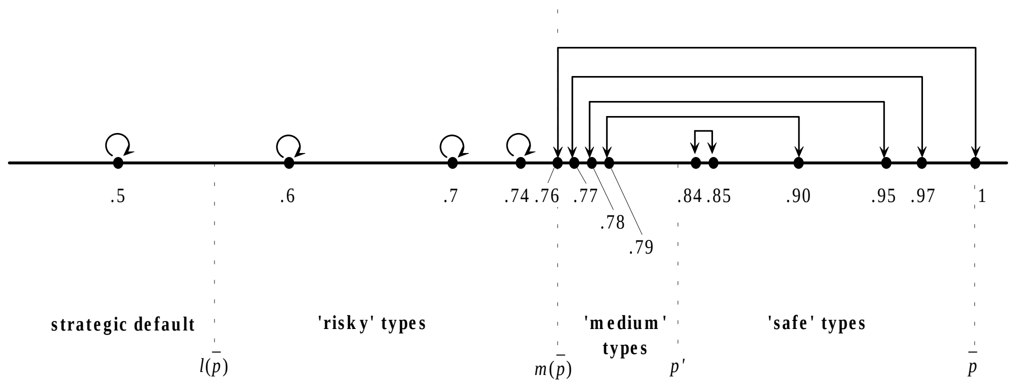

Figure 5: Example 2 - no homog. matching among 'medium' and 'safe'.

heterogeneous matching. Using the fact that the risk of the optimal partner is increasing in x, i.e.,

$$\frac{\partial m^*(p)}{\partial x} < 0 \quad \forall p > p',$$

we have that the maximum risk-heterogeneity in heterogeneous groups is non-decreasing in x, even for discrete F. The argument for continuous types distributions stems directly from the fact that $m(p) \leq m^*(p)$ in equilibrium and that p will try to match as close to $m^*(p)$ as possible.[35] As $m^*(p)$ falls for all p as x increases, $m(p)$ cannot increase or measure consistency would be violated. In discrete distributions, it is possible for $m(p)$ to be strictly greater than $m^*(p)$ if $\sigma(p, m(p)) > \sigma(p, q)$ for all $q < m^*(p)$ and $m^*(p) \notin \mathcal{B}$. However, as x increases, $m^*(p)$ shifts left for all p, and hence the available (formerly 'risky') borrowers get closer to $m^*(p)$ which makes them more appealing as partners.

In our example, increasing x has exactly that effect as shown in the table below.

	$m^*(1)$	$m(1)$	max heterogeneity $1 - m(1)$
$x = 2$	.75037	.74	.26
$x = 3$	.67925	.7	.3
$x = 4$	.64864	.6	.4

As benefits of keeping access to further loans increase, borrowers are willing to pay higher transfers to their safer partners, which leads to more heterogeneity

[35] By equation (17) in the proof of Corollary 1, and the fact that $\sigma_2(p, q) > 0$ for all $q < p$.

in equilibrium (conditional on the availability of types). Nothing, however, can be said about the effect of x on the qualitative importance of the homogeneous matching pockets as p' depends on x too. The effect of x on the relative size of the sets of 'medium' and 'safe' types is hence not characterizable in a general setting.[36]

Similarly we can examine the role of δ. As noted in Section 2.2, the "joint-production" function $W(P_i, P_j)$ is supermodular when $\delta = 1$. The matching is hence homogeneous in equilibrium. We note, nonetheless, that even for values of δ as high as .99999, the equilibrium in our example has $p = 1$ match with $m(1) = .95$, while $p = .99$ matches homogeneously. There is hence still potential for some heterogeneous matches in which borrowers exploit risk-sharing benefits, even at extremely high values of δ. This equilibrium also demonstrates the potential for non-assortative matching among 'medium' types: the delimiter between 'safe' and 'medium' types p' is equal to .9968 in this case; $p = .99$ is therefore a 'medium' type who matches homogeneously.

4 Conclusion.

This paper challenges the assumptions that matching in credit groups is exogenous, and that joint-liability in group lending contracts induces groups to form homogeneously. We provide a model in which risk heterogeneity in groups emerges as a rational response to missing insurance markets, and the risk-heterogeneity of groups and insurance arrangements depend non-monotonically on borrower types and on the distribution of those types.

Econometrically, this poses certain difficulties for studies trying to evaluate the impact of group composition on group performance. Within the current "Microfinance Revolution,"[37] there is an acute interest in understanding which types of groups "perform" better, where performance is gauged by both outreach (ability to reach the poor) and repayment rates. Empirical studies have sought

[36] Note that the quantitative importance of the level of non-assortative matching among 'medium' types can be made arbitrarily large, by concentrating the type distribution on 'medium' types matching homogeneously in equilibrium.

[37] Morduch, 1998.

to explain differences in performance by using group characteristics such as the percentage of group members of the same sex and/or ethnicity, the proportion of family members within a group, differences in activities between members of a same group, the potential for social sanctions, the existence of a "behavior code" in groups, group size, etc. Such factors can be related to the severity of adverse selection and moral hazard behavior in credit groups (see, for example, Matin, 1998; Sharma and Zeller, 1997; Wenner, 1995; Wydick, 1998; and Zeller, 1998). However, as shown by Proposition 1, group characteristics are endogenous: borrowers choose their partners. Additionally, there can exist non-monotonicities in the matching process so that group composition does not necessarily match back one-to-one with a measure of borrower quality. Treating these group characteristics as exogenous explanatory variables biases the empirical results and jeopardizes the reliability of the conclusions reached.

In a companion empirical paper (Sadoulet and Carpenter, 1999), we attempt to solve the first problem by estimating the relationship between individual risk preferences and group risk-heterogeneity using data from a survey conducted in Guatemala. We find evidence that risk-matching is not homogeneous, even when accounting for possible matching frictions. The next step would be to examine determinants of credit group performance taking into account the matching process which determines group characteristics.[38]

This analysis also reinforces the importance for microfinance institutions to allow borrowers to freely choose their group composition. Borrowers need to be able to assess potential partners' riskiness, monitor their partners, and enforce sanctions (such as exclusion from the group) for credit groups to operate successfully.

In addition, while there certainly are other justifications for credit groups to exist besides the insurance motive presented in this paper, this analysis allows us to reconsider the role of "peer pressure" in group lending programs. "Peer pressure" has often been offered as an explanation for the better repay-

[38] Ackerberg and Botticini (1998) is another example that controls for matching effects in empirical estimation.

ment performance of group lending programs as compared to individual lending programs. Group members can monitor partners' actions and sanction moral hazard behavior, hence leading to higher repayment rates (e.g., Stiglitz, 1990; Besley and Coate, 1995; and Arméndariz de Aghion, 1994). However, if borrowers can choose which group they will join, "peer pressure" cannot *force* a borrower who would intentionally default on an individual loan to repay a group loan. That borrower would not choose to join a group which would force him to repay in the first place; he would simply match with other borrowers who intend to default on their group loan, or take an individual loan were one available. Problematic borrowers *self-select* out of groups which would punish them. This "peer-pressure" can lead to higher repayment rates if it improves the effectiveness of financial institutions screening technology. Since joint-liability leads to the worst borrowers matching homogeneously, the detection of one "bad" borrower in a group signals the whole group's intention of defaulting.

When groups form homogeneously, the benefit of joint-liability is that it provides a framework which allows borrowers to set up insurance arrangements in environments which might otherwise not be conducive to such arrangements. This insurance increases aggregate repayment rates compared to individual loans. In addition, the heterogeneous matching provides 'medium' borrowers even better insurance than homogeneous matching as they receive from 'safe' borrowers (for a payment, but which leaves them still better off). Borrowers unwilling to repay match amongst themselves and hence do not affect repayment rates as compared to individual loans. The increase in repayment rates is not only beneficial from an accounting perspective, it also allows a greater proportion of borrowers to maintain access to future loans, thereby increasing the outreach of lending programs. It is hence important to recognize the role of endogeneity of group formation when analyzing the justifications for the establishment of credit-group programs.

References

[1] Ackerberg, D.A. and M. Botticini (1998). "Endogenous Matching and the Empirical Determinants of Contract Form," mimeo, Boston University.

[2] Armendáriz de Aghion, B. (1995) "On the Design of a Credit Agreement with Peer Monitoring", typescript, February 1995.

[3] Armendáriz de Aghion, B. and C. Gollier (1996). "Peer Grouping in an Adverse Selection Model", typescript, March 1996.

[4] Becker, G. (1981) . *A Treatise on the Family,* Harvard University Press, Cambridge: MA.

[5] Besley, T. and S. Coate (1995). "Group Lending, Repayment Incentives and Social Collateral", *Journal of Development Economics*, Vol. 46, pp.1-18.

[6] Diagne, A. (1997). "Default Incentives, Peer Pressure, and Equilibrium Outcomes in Group-based Lending Programs", mimeo, November 1997.

[7] Diamond, P. (1992). "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, (Oct), pp. 881-894.

[8] Fernandez, R. and J. Galí (1997). "To Each According To...? Markets, Tournaments, and the Matching Problem with Borrowing Constraints," mimeo, NYU.

[9] Ghatak, M. (1995). "Group Lending and the Peer Selection Effect", typescript.

[10] Kremer, M. and Maskin (1995). "Segregation by Skill and the Rise in Inequality," NBER Working Paper 5718.

[11] Legros, P. and A. Newman (1998). "Monotone Matching in Perfect and Imperfect Worlds", typescript, March 1998.

[12] Matin, I. (1998). "Too Much Too Easily: Loan Default of Grameen Bank Borrowers in Madhupur", typescript.

[13] Roth, A. and M. Sotomayor (1990). *Two-sided matching: a study in game-theoretic modeling and analysis*, Cambridge University Press.

[14] Sadoulet, L. (1997). "The Role of Mutual Insurance in Group Lending", typescript, December 97.

[15] Sadoulet, L. (1998) "Equilibrium Risk-Matching in Group Lending," typescript, October 98.

[16] Sadoulet, L. and S. Carpenter (1999). "Risk-Matching in Credit Groups: Evidence from Guatemala," typescript, March 99.

[17] Sharma, M. and M. Zeller (1997). "Repayment Performance in Group-based Credit Programs in Bangladesh: An Empirical Analysis", *World Development*, October 1997.

[18] Shimer R. and L. Smith (1998). "Assortative Matching and Search," forthcoming in *Econometrica.*

[19] Stiglitz, J. (1990). "Peer Monitoring and Credit Markets", *World Bank Economic Review*, 4, pp. 351-366.

[20] Varian, H. (1990). "Monitoring Agents with Other Agents", *Zeitschrift für die gesamte Staatswissenschaft,* 146, pp. 153-174.

[21] Wenner, M. (1995). "Group Credit: A Means to Improve Information Transfer and Loan Repayment Performance", *Journal of Development Studies,* Vol.32, No.2, December 1995, pp.263-281.

[22] Wydick, W.B. (1995). "Group Lending as a Credit Delivery Mechanism in Guatemala", mimeo, October 1995.

[23] Wydick, W.B. (1998). "Can Social Cohesion be Harnessed to Repair Market Failures," forthcoming in *Economic Journal.*

[24] Zeller, M. (1998). "Determinants of Repayment Performance in Credit Groups: The Role of Program Design, Intra-group Risk Pooling, and Social Cohesion in Madagascar," *Journal of Economic Development and Cultural Change,* January.

A Proofs

For notational ease, will denote the partials of σ with respect to its first and second arguments by σ_1 and σ_2, and the cross partial by σ_{12}.

A.1 Lemma 1

We prove each claim of Lemma 1 in turn.

Lemma 1.1: *There exists a unique* $p' \in B$ *such that*

$$\begin{array}{lll} \sigma(p', P_j) < 0 & \forall P_j < p', & \text{and} \\ \sigma(p', P_j) > 0 & \forall P_j > p'. & \end{array} \tag{6}$$

Proof. We proceed in 3 steps.

Step 1: defining p'**.**

The derivation of σ_2 is straightforward, but the expression is lengthy so we do not report it. Consider the partial $\sigma_2(P_i, P_j)$ as a function of P_j. It is equal to zero for four values of P_j, only two of which are real. One of the real roots is at $P_j = P_i$; denote the other real root by $r_2(P_i)$ and the two complex ones by $z_1(P_i)$ and $z_2(P_i)$. The partial σ_2 can then be rewritten as:

$$\sigma_2(P_i, P_j) = (P_j - P_i)(P_j - r_2)(P_j - z_1)(P_j - z_2) \cdot \alpha(P_i, P_j) \tag{7}$$

where $\alpha(P_i, P_j)$ is a term continuous in P_i and P_j which never goes to zero (since σ_2 has only four roots). Both real roots of σ_2 are equal when:

$$P_i = r_2 = \frac{1}{\delta(\delta x - 1)} \left(\delta(x-1) - \sqrt{\delta(1-\delta)(-2x\delta + x^2\delta + 1)} \right) \equiv p'. \tag{8}$$

We note that p' is unique and does not depend on the distribution of types F.

Step 2: matching any borrower P_j **with** p'**.**

Following the decomposition in (7), and using the fact that the two complex roots z_1 and z_2 are conjugates $a \pm bi$, the partial $\sigma_2(p', P_j)$ can be rewritten as:

$$\begin{aligned} \sigma_2(p', P_j) &= (P_j - p')^2 (P_j - (a + bi))(P_j - (a - bi)) \cdot \alpha(p', P_j) \\ &= (P_j - p')^2 \left((P_j - a)^2 + b^2 \right) \cdot \alpha(p', P_j). \end{aligned}$$

$\sigma_2(p', P_j)$ therefore has the sign of $\alpha(p', P_j)$. Calculating $\sigma_2(p', 0)$ gives a positive sign so that $\alpha(p', P_j)$ is positive, and hence $\sigma_2(p', P_j) > 0$. Therefore

$\sigma(p', P_j)$ is monotonically increasing in P_j, except at $P_j = p'$:

$$\sigma_2(p', P_j) > 0 \quad \forall P_j \neq p'. \tag{9}$$

As $\sigma(p', p') = 0$, we hence have that

$$\sigma(p', P_j) < 0 \quad \forall P_j < p'$$

and

$$\sigma(p', P_j) > 0 \quad \forall P_j > p'.$$

which proves the claim. ■

Lemma 1.2:

$$\forall P_i > p', \quad \exists l(P_i) \text{ such that } \begin{array}{ll} \sigma(P_i, P_j) < 0 & \forall P_j < l(P_i) \quad \text{and} \\ \sigma(P_i, P_j) \geq 0 & \text{otherwise.} \end{array}$$

Proof. We proceed in 2 steps

Step 1: for all $P_i > p'$, there exists a unique $l(P_i) < p'$ such that $\sigma(P_i, l(P_i)) = 0$.

Solving $\sigma(P_i, q) = 0$ for q yields three roots: a double root at $q = P_i$, and another at $q = \dfrac{(x-1)\delta P_i + \delta(2-x) - 1}{\delta((\delta x - 1)P_i + 1 - x)}$.

Define $l(P_i)$ to be this last root:

$$l(P_i) \equiv \frac{(x-1)\delta P_i + \delta(2-x) - 1}{\delta((\delta x - 1)P_i + 1 - x)}. \tag{10}$$

To prove $l(P_i) \in \left(\underline{p}, p'\right)$, we use the Intermediate Value Theorem:

- the surplus of matching with the riskiest borrower in $\mathcal{B}$ – namely $\underline{p} = 1/(x\delta)$ – is negative:

$$\sigma\left(P_i, \frac{1}{x\delta}\right) = \frac{(-1+P_i)(-P_i\delta x + 1)^2}{\delta\left[1 - (P_i + P_i(1-P_i))\delta\right]\left[(x-1)(1-P_i) + x(1-\delta)P_i\right]} < 0. \tag{11}$$

- The surplus of matching P_i with p' is positive by equation (6).

- $\sigma(P_i, q)$ is continuous in q.

Hence, by the Intermediate Value Theorem, there exists some $q \in (\underline{p}, p')$ such that $\sigma(P_i, q) = 0$. Since $P_i > p'$, the only candidate root is $q = l(P_i)$.

Step 2: establishing the sign of $\sigma(P_i, q)$.

Since $\sigma(P_i, q)$ is continuous in q, it can only change sign at its roots $q = l(P_i)$ and $q = P_i$, where $l(P_i) < P_i$ for $P_i > p'$. The sign of σ in each of the intervals defined by these roots – $(\underline{p}, l(P_i))$, $(l(P_i), P_i)$ and $(P_i, 1)$ – can hence be established by examining the sign at one point of each interval.

- The surplus of matching with $\underline{p}$ is negative by equation (11), and $\underline{p} < l(P_i)$. Therefore, $\sigma(P_i, q) < 0$ for all $q < l(P_i)$.

- The point p' lies in $[l(P_i), P_i)$ as

$$l(p') = \frac{1}{\delta(\delta x - 1)}\left(\delta(x-1) - \sqrt{\delta(1-\delta)(-2x\delta + x^2\delta + 1)}\right) = p'$$

and

$$\frac{\partial l(q)}{\partial q} = -(1-\delta)\frac{(x-2)\delta x + 1}{\delta((x\delta - 1)q + 1 - x)^2} < 0. \tag{12}$$

By equation (6) (and by symmetry of σ), the surplus is strictly positive when P_i matches with p' :

$$\sigma(P_i, p') > 0 \text{ as } P_i > p'.$$

The surplus $\sigma(P_j, q)$ is hence strictly positive for all q in $(l(P_j), P_j)$.

- The surplus of matching with $q = 1$ yields:

$$\sigma(P_i, 1) = (P_i - 1)^2 \frac{x\delta P_i - 1}{(1-\delta)(1 - \delta P_i(2 - P_i))} \tag{13}$$

which is positive as long as $P_i > \dfrac{1}{\delta x} \equiv \underline{p}$. Therefore, all points $q > P_i$ yield a positive surplus when matching with P_i: $\sigma(P_i, q) > 0 \quad \forall q > P_i$.

Therefore, for any $P_i > p'$, $\sigma(P_i, P_j) < 0$ for any $P_j < l(P_i)$ and $\sigma(P_i, P_j) \geq 0$ otherwise. ■

Lemma 1.3:

$$\forall P_i < p', \quad \exists k(P_i) \text{ such that } \begin{array}{ll} \sigma(P_i, P_j) > 0 & \forall P_j > k(P_i) \quad \text{and} \\ \sigma(P_i, P_j) \leq 0 & \text{otherwise.} \end{array}$$

Proof. By equation (13), $\sigma(P_i, 1) > 0$. By equation (6) and symmetry of σ, $\sigma(P_i, p') < 0$ as $P_i < p'$. Since $\sigma(P_i, q)$ is continuous in q, there exists some $k(P_i) \in (p', 1)$ such that $\sigma(P_i, k(P_i)) = 0$.

The surplus $\sigma(P_i, q)$ has three roots in q as shown in Step 1 of the proof of Claim 2. Two of those roots are equal to P_i. Hence $k(P_i)$ is the unique root greater than p'. This implies that $\sigma(P_i, q) > 0$ for all $q > k(P_i)$. ∎

This completes the proof of Lemma 1.

A.2 Lemma 2

The claim is that two borrowers riskier than p' that match together yield a negative surplus:

$$\forall P_i, P_j < p', \quad \sigma(P_i, P_j) < 0.$$

Proof. The proof follows immediately from Lemma 1.1 and Lemma 1.3. If $P_i < p'$, then $k(P_i) > p'$ since $\sigma(P_i, p') < 0$ by Lemma 1.1. Hence, as $P_j < p'$, $\sigma(P_i, P_j) < 0$ by Lemma 1.3. ∎

A.3 Lemma 3

This result is from Roth and Sotomayor (1990): There are no transfers between members of different groups in stable outcomes.

Proof. Let $s(P_i)$ be borrower P_i's expected surplus in equilibrium. By the feasibility of this equilibrium, the sum of borrower expected returns cannot be greater than the total expected surplus:

$$\frac{1}{2} \int_{\mathcal{B}} \left[s(P_i) + s(m(P_i)) \right] \, dF(P_i) \leq \int_{\mathcal{B}} \sigma(P_i, m(P_i)) \, dF(P_i) \tag{14}$$

For the equilibrium to be stable, it must be that the individual surpluses of any P_i and P_j are at least as great as if they shared the surplus created by their own group:

$$s(P_i) + s(P_j) \geq \sigma(P_i, P_j). \tag{15}$$

As this holds for all P_i and P_j, we can integrate (15) over P_i and P_j, reorganize the terms, and combine with (14) to get that

$$\frac{1}{2}\int\limits_{\mathcal{B}} [s(P_i) + s(m(P_i))] \; dF(P_i) = \int\limits_{\mathcal{B}} \sigma(P_i, m(P_i)) \; dF(P_i)$$

in equilibrium. In addition, both members must get at least their expected returns in homogeneous groups, i.e.,

$$s(P_i) \geq 0 \quad \text{and} \quad s(P_j) \geq 0 \quad \forall P_i, P_j \in \mathcal{B}$$

so that

$$s(P_i) + s(m(P_i)) = \sigma(P_i, m(P_i)) \quad \forall P_i \in \mathcal{B}$$

A group's surplus is entirely divided among its members and there are no transfers to members of other groups. ■

A.4 Proposition 1

Proposition 1 claims that the set of borrowers $\mathcal{B}$ can be split into three intervals, according to the matching patterns. We prove the claims slightly out of order to enable the use of Proposition 1.3 in the proof of Proposition 1.2.

A.4.1 The 'safe' match negatively.

Proposition 1.1: *Borrowers in* $[p', \overline{p}]$ *match negatively:*

$$\forall P_i, P_j \in [p', \overline{p}], \; P_i > P_j \Longleftrightarrow m(P_i) < m(P_j)$$

Proof. Assume without loss of generality that $P_i > P_j$. We construct the proof in 3 steps.

Step 1: the surplus $\sigma(p, q)$ is strictly positive for all groups (p, q) such that $l(P_j) \leq q \leq P_j < p$.

By equation (12), we have that $l(p) < l(P_j)$ since $p > P_j > p'$. By Lemma 1, the surplus $\sigma(p,q)$ is strictly positive for all $q > l(p)$, $q \neq p$. Therefore, the surplus $\sigma(p,q)$ is strictly positive for all q in $(l(P_j), P_j)$.

Step 2: the cross partial $\sigma_{12}(p,q)$ is strictly negative for all p and q such that $p > p'$ and $l(p) < q \leq \overline{p}$.

Setting the cross-partial σ_{12} to zero:

$$\frac{\partial^2}{\partial p \partial q}\sigma(p,q) = 2\frac{\delta\left(\left(\delta x - 1\right)p + 1 - x\right)q - \delta\left(x-1\right)p + \delta\left(2-x\right) - 1}{\left(1 - \left(p + q\left(1-p\right)\right)\delta\right)^3} = 0 \quad (16)$$

yields a *unique* solution for q at $q = l(p)$.

In addition, the cross-partial evaluated at $q = 1$ is negative:

$$\sigma_{12}(p,1) = 2\frac{\delta x\left(\delta - 1\right)p - 1 + \delta\left(3 - 2x\right)}{\left(1-\delta\right)^3} < 0$$

as $\delta < 1$ and $x > 2$.

As $\sigma_{12}(p,q)$ is continuous in q on $[l(p), 1]$ and the solution to equation (16) is unique, we have that

$$\sigma_{12}(p,q) < 0 \quad \forall q \in [l(p), 1]$$

by the Intermediate Value Theorem. This is in particular true for all q in $[l(p), \overline{p}]$ as $\overline{p} \leq 1$.

Step 3: $\forall P_i, P_j$ with $P_i > P_j \geq p'$, $\quad m(P_i) < m(P_j)$ for the matching to be stable.

Step 2 implies that for all $P_i > P_j > p'$, and for all q_1 and q_2 in $(l(P_j), \overline{p})$ with $q_1 > q_2$:

$$\sigma(P_j, q_1) - \sigma(P_j, q_2) > \sigma(P_i, q_1) - \sigma(P_i, q_2)$$

This means that for all P_i and P_j in $[p', \overline{p}]$ such that $P_i > P_j$, the partner P_i matches with in equilibrium – namely $m(P_i)$ – has to be riskier than the partner P_j matches with for the equilibrium to be stable:

$$\forall P_i, P_j \quad \text{with } P_i > P_j \geq p', \quad m(P_i) < m(P_j)$$

If this result didn't hold, a rearrangement of partners – namely P_j with $m(P_i)$ and P_i with $m(P_j)$ – could make P_j, $m(P_j)$, P_i and $m(P_i)$ better off without making anybody else worse off. The matching m would hence not be stable. ■

Corollary 2 *There exists at most one* $\widetilde{p} \in [p', \overline{p}]$ *such that* $m(\widetilde{p}) = \widetilde{p}$ *in equilibrium.*

Proof. By Lemma 2, p' is such that $\sigma(p', P_j) < 0$ for all $P_j < p'$. Therefore, to satisfy stability, the equilibrium matching must satisfy that p' matches with a partner at least as safe as himself:

$$m(p') \geq p' \text{ in equilibrium.}$$

By the negative matching of 'safe' borrowers in Proposition 1.1:

- if $m(p') = p'$, then for all $P_j > p'$, $m(P_j) < p'$. The unique $\widetilde{p}$ is hence p'.
- if $m(p') > p'$: if there existed q_1 and q_2 such that $q_1 > q_2 > p'$ and $m(q_1) = q_1$ and $m(q_2) = q_2$, then the matching among safe borrowers would not be negative, which contradicts Proposition 1.1.

We hence have that there is at most one $\widetilde{p} \in [p', \overline{p}]$ that matches homogeneously. ■

Remark 1 *The points* $\widetilde{p}$ *and* $m(\overline{p})$ *are jointly determined by the maximization of the aggregate surplus*

$$\int_{\widetilde{p}}^{\overline{p}} \sigma(p, m(p))\, dF(p)$$

subject to the measure consistency constraint

$$F(p) - F(\widetilde{p}) \leq F(\widetilde{p}) - F(m(p)) \quad \forall p > \widetilde{p}.$$

Note that there will hence only be heterogeneous matches in equilibrium provided that the distribution of types is such that there are some borrowers above p'. If $[p', \overline{p}]$ is not a proper interval, then there will only be homogeneous matching in equilibrium.

For $[p', \overline{p}]$ to be a proper interval, the parameters x, δ and $\overline{p}$ must be such that

$$x \geq \frac{2\delta - 1 - \overline{p}\delta\left(2 - \overline{p}\right)}{\delta\left(1 - \overline{p}\left(2 - \overline{p}\delta\right)\right)}.$$

Low values of $\overline{p}$ require x to be very large ($x \geq 10$ for any δ when $\overline{p} = .65$): the benefit of maintaining access to future loans has to be high enough for borrowers to set up heterogeneous groups when probabilities of success are low.

A.4.2 The 'risky' match homogeneously

Proposition 1.3: *Borrowers in* $\left[\underline{p}, m\left(\overline{p}\right)\right)$ *match homogeneously:*

$$\forall P_i < m\left(\overline{p}\right), \quad m\left(P_i\right) = P_i.$$

Proof.

- By Proposition 1.1, the safest borrower $\overline{p}$ matches with p' or a borrower riskier than p', i.e.,

$$m\left(\overline{p}\right) \leq p'.$$

- By Proposition 1.1, all borrowers in $[p', \overline{p}]$ match with borrowers safer than $m\left(\overline{p}\right)$, so that no borrower in $[p', \overline{p}]$ are available to match with a borrower $P_j < m\left(\overline{p}\right).$

- By lemma 2, for all $P_j < p'$, matching with a partner less than p' yields a negative surplus if that partner is not a twin:

$$\sigma\left(P_j, q\right) < 0 \qquad \forall P_j < p' \text{ and } \forall q < p' \text{ with } q \neq P_j.$$

Therefore, borrowers with $P_j < m\left(\overline{p}\right)$ will match homogeneously. ■

A.4.3 The 'medium' match heterogeneously, or homogeneously.

Proposition 1.2 *Borrowers in* $\left[m\left(\overline{p}\right), p'\right]$ *match either heterogeneously with partners in* $(p', \overline{p}]$, *or homogeneously:*

$$\forall P_i \in \left[m\left(\overline{p}\right), p'\right), \quad m\left(P_i\right) \in [p', \overline{p}] \text{ or } m\left(P_i\right) = P_i.$$

Proof. By Lemma 2, any $P_i < p'$ will never match in equilibrium with another borrower $P_j < p'$ unless $P_j = P_i$, as this produces negative surplus which would violate the equilibrium stability requirement. So a 'medium' P_i will match either with a partner identical to himself, or with a partner $P_j > k(P_i)$, where $k(P_i)$ was defined in Lemma 1.3 and shown to be safer than p'. ■

Corollary 1 *If F is continuous and such that there exists some $q \in (m^*(\overline{p}), p')$ such that $F(p') - F(q) = F(\overline{p}) - F(p')$, then the equilibrium match will display "holes" in which some 'medium' borrowers match homogeneously while the other 'medium' borrowers match heterogeneously.*

Proof. We prove the Corollary in two steps. First we show that for any continuous F, $m(\overline{p}) < m^*(\overline{p})$ where $m^*(\overline{p})$ was defined as the partner who maximizes the surplus created by $\overline{p}$'s group, i.e.,

$$\forall \text{ continuous } F, \quad m(\overline{p}) \leq m^*(\overline{p}) \text{ where } m^*(\overline{p}) \equiv \arg\max_{q \leq \overline{p}} \sigma(\overline{p}, q). \tag{17}$$

Assume not. By Proposition 1.3, any $q < m(\overline{p})$ matches homogeneously and hence produces a surplus of zero. Define $\widetilde{m}$ to be a matching correspondence from $\mathcal{B}$ onto $\mathcal{B}$ such that:

$$\begin{array}{ll} \widetilde{m}(\overline{p}) = m^*(\overline{p}) & \\ \widetilde{m}(m(\overline{p})) = m(\overline{p}) & \\ \widetilde{m}(p) = m(p) & \text{for all other } p \in \mathcal{B} \end{array}$$

Then the total surplus under $\widetilde{m}$ would be greater than the total surplus under m. The matching m could hence not be an equilibrium.

Secondly, we show that if F is such that there exists $q > m^*(\overline{p})$ such that

$$F(p') - F(q) = F(\overline{p}) - F(p') \tag{18}$$

then some 'medium' borrowers in $[m(\overline{p}), p']$ will match homogeneously in equilibrium.

By Lemma 2, there are at most $F(\overline{p}) - F(p')$ 'safe' borrowers matching with borrowers below p'. The measure of 'medium' borrowers can be written as

$$F(p') - F(m(\overline{p})) = \{F(p') - F(q)\} + \{F(q) - m(\overline{p})\}$$

By the first step of the proof, $q > m^* (\overline{p}) \geq m(\overline{p})$ and hence $F(q) - m(\overline{p})$ is positive. There are hence more 'medium' borrowers than 'safe' borrowers. As 'medium' borrowers match homogeneously if they cannot match with 'safe' borrowers, if F is such that the q defined in the equation (18) exists, there will be "holes" in which 'medium' borrowers match homogeneously in equilibrium. ■