# Self-Selection Consistent Choices*

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**Abstract**

This paper studies collective choice rules whose outcomes consist of a collection of simultaneous decisions, each one of which is the only concern of some group of individuals in society. The need for such rules arises in different contexts, including the establishment of jurisdictions, the location of multiple public facilities, or the election of representative committees. We define a notion of allocation consistency requiring that each partial aspect of the global decision taken by society as a whole should be ratified by the group of agents who are directly concerned with this particular aspect. We investigate the possibility of designing allocation consistent rules which satisfy the Condorcet criterion and respect different notions of voluntarism.

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## 1. Introduction.

Collective choices often involve multiple simultaneous decisions, whose particular aspects may affect different agents to different degrees. If new borders are drawn in a region of the world, I am mainly affected by what my country will look like, although I may also care about the whole map of the region. If a committee is chosen to negotiate on behalf of my union, I am especially interested on those delegates that I am acquainted with, and/or who will more closely represent my interest.

In this paper we concentrate on polar cases, where each agent is solely concerned with one of the components of the global decision, and congestion effects are ignored. For example, several hospitals may be built simultaneously, but if each agent is only allowed to use one of them (and congestion levels are similar), then he will essentially evaluate the overall decision in terms of the particular hospital he is assigned to. Under these circumstances, we discuss the merits of different social choice procedures to determine (1) what set of objects should be chosen, and (2) which agents should benefit from each of the objects.

Since we consider that agents are assigned to specific objects, and that they only care about them, an interesting question arises regarding the overall consistency of the collective procedure. Once a decision is taken, all agents who share the same object emerge naturally as a meaningful group. All those citizens of a new nation after border redrawing, all the trade union members whose opinion will be channeled by a given representative, all users of a new hospital are concerned about the same aspects of the global decision. What if they use the same rule that led society as a whole to make the global choice, and challenge it by suggesting that, as far as they are concerned, the particular object that they have been assigned to should be changed for another one? What if all people who, given the public decision to build hospitals $H_1, H_2$, are assigned to $H_1$, then demand that $H_1'$ be built instead? What if, after talking to one delegation member, the agents he is supposed to represent meet and vote in favor of substituting him for somebody else? In all these cases, there would be some inconsistency between a global decision which turns a group into the major beneficiary of one of its aspects, and the partial decision that these same concerned agents would suggest, regarding this particular aspect. Social choice procedures which avoid these problems will be call allocation consistent.

Many authors have been concerned about the connections among different decisions taken by societies when their members or their resources vary. Different conditions have been imposed requiring that the changes in the social decision associated with changes in the membership of society, or with changes in the set of possible outcomes, respect some notion of consistency (see for example Thomson (1998) for a survey on consistency).

Our concern can also be viewed as one of consistency, but we must qualify the analogy. We want to emphasize the fact that our consistency requirement refers to the connections between global choices and their particular aspects: on this account, our focus is restricted, since we only consider models where this distinction makes sense. A second difference is, we believe, in favor of our notion. We do not look at exogenous changes in the membership of society, which may or may not be reasonably expected. We concentrate on the connections between global decisions, taken by the society at large, and their partial components, as viewed by those agents in the very same society who are affected and concerned by those partial aspects of the decision. For those problems where the structure of the global decision is naturally decomposable, and agents are particularly concerned with only parts of the global picture, we find our notions of allocation and self-selection consistency to be particularly attractive.

Of course, no single criterion is sufficient to determine what rule is most attractive in a given context. In this paper, we focus on rules which are allocation consistent, but also *voluntary* and *respect the Condorcet criterion.* In our context, voluntarism is a normative property requiring that the assignment of agents to objects should be compatible with the will of agents. The object of Section 3 is to discuss this requirement at length, and to propose three attractive properties, each of which can be interpreted as an expression of voluntarism for an appropriate scenario. These properties are *no-envy, Nash stability* and *group Nash stability.* Allocation rules which are allocation consistent and envy-free are called *self-selection consistent,* to emphasize the idea that the concerned agents self-select themselves to play this role, through their voluntary identification with one of the projects, hospitals, representatives or nations.

We also focus on rules which respect the Condorcet criterion. A Condorcet winner is an alternative which defeats every other alternative in majority comparisons. Condorcet winners need not always exist, but when they do, their choice seems quite compelling. We will say that a social choice correspondence respects the Condorcet criterion[1] if it always recommends the choice of Condorcet winners when they exist.

To motivate our further analysis, consider the following example:

*Twenty six agents must choose a delegation of three representatives out of five candidates* $(x, y, z, r, w)$*, over which they have preferences represented in the*

[1]This property is often called Condorcet consistency, but we prefer to reserve the term consistency for properties involving changes in population, resources, or choice possibilities.

*following table,*

| *agents* | $1, ..., 6$ | $7, .., 13$ | $14, .., 17$ | $18, .., 21$ | $22, .., 26$ |
|---|---|---|---|---|---|
| *preferences* | $x$ | $r$ | $w$ | $z$ | $y$ |
| *from* | $y$ | $x$ | $r$ | $w$ | $z$ |
| *better* | $z$ | $y$ | $x$ | $r$ | $w$ |
| *to* | $r$ | $w$ | $z$ | $y$ | $r$ |
| *worse* | $w$ | $z$ | $y$ | $x$ | $x$ |

*If they use an allocation consistent rule that respects the Condorcet criterion, each one of the chosen delegates should be a Condorcet winner for the set of voters that he represents. Who represents whom can be specified in several ways. For example, we could assume that voters only get the chance to communicate with one delegate, and that this is the one we call his "representative". But here we concentrate on the case where, once the delegation is chosen, each voter identifies as his representative the one delegate that he likes most. There are ten possible delegations,* $xyz$, $xyw$, $xyr$, $xzw$, $xrw$, $xzr$, $yzw$, $yzr$, $yrw$, $zrw$. *If* $xyz$ *was chosen, the sets of agents that would feel represented by* $x$, $y$ *and* $z$ *would be respectively,* $U(x) = \{1, ...., 17\}$, $U(y) = \{22, .., 26\}$, $U(z) = \{18, .., 21\}$. *But then, the Condorcet winner for the voters in* $U(x)$ *is* $r$. *If* $xyw$ *was chosen,* $U(x) = \{1, ..13\}$, $U(y) = \{22, .., 26\}$, $U(w) = \{14, .., 21\}$. *But then, the Condorcet winner for voters in* $U(x)$ *is* $r$. The reader may check that a similar inconsistency will appear with any of the remaining possible delegations. This proves that, in the case we just described, no social choice rule can meet our desiderata. This is why, in what follows, we concentrate on a more specific problem: that of choosing $k$ objects on a close interval of the real line, when the preferences of agents over single objects are single peaked.

The problem of choosing several points on a line and having agents cluster around them admits several interpretations. Variants of this problem have provided the basic model for the analysis of local public goods and jurisdictional questions (see Alesina and Spolaore (1997), Greenberg and Weber (1993), Jehiel and Schotchmer (1997), Konishi et al.,(1998), Milchtaich and Winter (1998), Tiebout (1956)). Yet, our model is much more explicit about the connections between the global decision of the whole group and the partial decisions of its different subgroups; our main focus is on allocation consistency. On the other hand, our model explicitly rules out congestion effects, which are important in many contexts, and also takes the number of objects to be chosen as an exogenous parameter (in contrast with models where the number of jurisdictions is an endogenous variable). These two restrictive features of our model are borrowed from a series of recent papers by Miyagawa (1997). His model is very similar to ours, but we have expanded it to encompass the possibility of a variable electorate to chose a variable number of objects: this allows us to stress the issue of consistency and the endogenous character of the groups that share each single object. Even if our models are similar, Miyagawa's analysis and conclusions are very different from ours. His choice of axioms leads him to characterize different rules which tend to select rather extreme outcomes. Moreover, his formal analysis often stops at the case where only two objects are chosen. In contrast, our analysis highlights the importance of rules that extend the median voter principle, and it applies to any fixed number of partial choices.

In Section 2, we present our model in detail. Section 3 presents three definitions expressing the notion of voluntarism under varying scenarios: no envy, Nash and group Nash stability. Section 4 studies the existence of allocation consistent social choice correspondences respecting the Condorcet criterion and leading to envy-free, Nash stable, or group Nash stable allocations. Section 5 concludes.

## 2. The Model.

Let $N = \{1, ..., n\}$ be the set of *agents*. Subsets of $N$ are *coalitions*. For any coalition $S$, $|S|$ denotes the cardinality of $S$.

In order to describe the set of decisions we need a language to describe the number and position of relevant locations, and to denote the sets of agents who are allocated to each location.

Let $\mathbb{N}$ be the set of natural numbers. An element in $\mathbb{N}$ denotes the number of locations.

Given $S \subseteq N$ and $k \in \mathbb{N}$, an $S/k-$*decision* is a $k-$tuple of pairs $d = (x_h, S_h)_{h=1}^{k}$, where $(x_1, .., x_k) \in [0, T]^k$ and $(S_1, .., S_k)$ is a partition of $S$. [2] We shall interpret each $x_h$ as a location and $S_h$ as the set of agents who is assigned to the location $x_{h.}$ Notice that elements in the partition may be empty. This will be the case, necessarily, if $k > |S|$ . We call $d_L = (x_1, .., x_k)$ the vector of *locations*, and $d_A = (S_1, .., S_k)$ the vector of *assignments*.

Given a $S/k-$ decision $d$, and $j \in S$, $S(j, d)$ will denote the set in $d_A$ that contains $j$, and $x(j, d)$ will denote the element in $d_L$ to which agents in $S(j, d)$ are assigned.

We denote by $D(S, k)$ the set of $S/k-$decisions.

The set of $k-$decisions is $D(k) = \bigcup_{S\subseteq N} D(S, k)$.

For each agent $j \in N$, the set of $k-$decisions which concern $j$ is $D_j(k) = \bigcup_{\{S\subseteq N|j\in S\}} D(S, k)$

The set of decisions is $D = \bigcup\limits_{k=1}^{n} D(k)$.

For each agent $j \in N$, *the set of decisions that concern* $j$ is $D_j = \bigcup\limits_{k=1}^{n} D_j(k)$.

Agents are assumed to have complete, reflexive, transitive preferences over

[2] Our analysis does not depend on $I^1$ being a closed, bounded interval of the reals. It could also be the whole real line, or a finite set of integers, just to mention some alternative possibilities. As for choosing $[0, T]$ rather than $[0, 1]$ it is just in order to get nicer numerical examples.

decisions which concern them. That is, *agent $i$'s preferences* are defined on $D_i$, and thus, rank any pair of $S/k$ and $S'/k'$−decisions provided that $i \in S \cap S'$. Denote by $\succcurlyeq_i$ the preferences of agent $i$ on $D_i$.

We shall assume all along that preferences are *singleton-based*. Informally, this means that agents' rankings of decisions only depend on the location they are assigned to, not on the rest of locations or on the assignment of other agents to locations. This assumption is compatible with our interpretation that agents can only use the good provided at one location, and that this is a public good subject to no congestion. Formally, a preference $\succcurlyeq_i$ on $D_i$ is singleton-based if there is a preference $\bar{\succcurlyeq}_i$ on $[0, T]$ such that for all $d$, $d' \in D_i$, $d \succcurlyeq_i d'$ if and only if $x(i, d) \bar{\succcurlyeq}_i x(i, d')$.

In all that follows, we shall assume that for all $i \in N$, $\succcurlyeq_i$ is singleton-based, and in addition, that the order $\bar{\succcurlyeq}_i$ is *single-peaked.*[3] Abusing notation we will use the same symbol $\succcurlyeq_i$ for both orders.

Given $S \subseteq N$, *preference profiles for $S$* are $|S|$-tuples of preferences, and we denote them by $P_S, P'_S, ...$

We denote by $\mathcal{P}$ the set of all preferences described above, and by $\mathcal{P}^S$ the set of preference profiles for $S$ satisfying those requirements.

A *collective choice correspondence* will select a set of $k$−decisions, for each given $k$, on the basis of the preferences of agents in coalition $S$, for any coalition $S \subseteq N$. Formally,

**Definition 1.** *A collective choice correspondence is a correspondence $\varphi : \bigcup_{S \subseteq N} \mathcal{P}^S \times \mathbb{N} \to D$ such that, for all $S \subseteq N$, $P_S \in \mathcal{P}^S$ and $k \in \mathbb{N}$, $\varphi(P_S, k) \subset D(S, k)$.*

[3] That is: for each $\bar{\succcurlyeq}_i$, there is an alternative $p(\ i)$ which is the unique best element for $\bar{\succcurlyeq}_i$; morover, for all $x, y$, if $p(\ i) \geq x > y$, then $x \bar{\succ}_i y$, and if $y > x \geq p(\ i)$, then $x \bar{\succ}_i y$

We now propose three natural and attractive properties that a collective choice correspondence may or may not satisfy. Two of these properties -efficiency and respect for the Condorcet criterion- are well known. The third, allocation consistency, is proposed here for the first time.

First, we formulate the condition of Pareto efficiency.

**Definition 2.** *An $S/k-$decision $d$ is efficient if there is no $S/k-$decision $d'$ such that $d' \succcurlyeq_i d$ for every agent $i \in S$ and $d' \succ_j d$ for some $j \in S$.*

Second, we rephrase within our model the classical notions of Condorcet winners and respect for the Condorcet criterion.

**Definition 3.** *An $S/k-$decision $d \in D(S,k)$ is a Condorcet winner for $S$ if*

$$\left|\{i \in S \mid d \succ_i d'\}\right| \geq \left|\{i \in S \mid d' \succ_i d\}\right| \text{ for all } d' \in D(S,k)$$

*Given $S \subseteq N$ and a preference profile $P_S$, let $CW(P_S,k)$ be the set of $S/k-$decisions that are Condorcet winners for $S$.*

Notice that any $S/k-$decision that is a Condorcet winner for $S$ is an efficient decision.

We shall see in Example 1 that for $k > 1$ Condorcet winners may not exist, but when they do, their choice seems quite compelling. Hence, we will demand for a collective choice correspondence to recommend the choice of the Condorcet winners whenever they exist.

**Definition 4.** *A collective choice correspondence $\varphi$ respects the Condorcet criterion if for all $S \subseteq N$ and for all $P_S$ such that $CW(P_S,k) \neq \emptyset$, $\varphi(P_S,k) = CW(P_S,k)$.*

We propose our notion of *allocation consistency* for collective choice correspondences.

**Definition 5.** *A collective choice correspondence $\varphi$ is allocation consistent if for all $S \subseteq N$, $P_S$, $k \in \mathbb{N}$, and $((x_1, S_1), ..., (x_k, S_k)) \in \varphi(P_S, k)$, $(x_h, S_h) \in \varphi(P_{S_h}, 1)$ for all $h$ such that $S_h \neq \emptyset$.*[4]

Before closing this section, we clarify the relationship among some of the requirements on collective choice correspondence that we first described. We show that, whenever a Condorcet winner exists, each of its components is a Condorcet winner for its corresponding group. This guarantees that allocation consistency and the Condorcet criterion are, in principle, compatible requirements. Finally, we also give a necessary condition for an allocation consistent collective choice correspondence to respect the Condorcet criterion.

**Proposition 1.** *Given $S \subseteq N$, $P_S$, and $k \in \mathbb{N}$, if an $S/k-$decision $d = ((x_h, S_h))_{h=1}^{k} \in CW(P_S, k)$, then $d_h = (x_h, S_h) \in CW(P_{S_h}, 1)$ for all $h$ such that $S_h \neq \emptyset$.*

**Proof.** Suppose that there is an $h$ such that $d_h = (x_h, S_h)$ is not a Condorcet winner for $S_h$. Then there is a $d'_h = (x'_h, S_h)$, such that

$$\left|\{i \in S_h \mid d'_h \succ_i d_h\}\right| > \left|\{i \in S_h \mid d_h \succ_i d'_h\}\right|$$

Let $S_{h1} = \{i \in S_h \mid d'_h \succ_i d_h\}$, $S_{h2} = \{i \in S_h \mid d_h \succ_i d'_h\}$, and $d' = (d'_h, d_{-h})$. Then, $\{i \in S \mid d' \succ_i d\} = S_{h1}$, and $\{i \in S \mid d \succ_i d'\} = S_{h2}$. But then, $|\{i \in S \mid d' \succ_i d\}| > |\{i \in S \mid d \succ_i d'\}|$, which contradicts the assumption that $d$ is a Condorcet winner for $S$. ■

[4] Since for $k = 1$ there is a unique assignment of the agents (all of them together), we will use indistinctly $(x_h, S_h) \in \varphi(P_{S_h}, 1)$ or $x_h \in \varphi(P_{S_h}, 1)$

Proposition 1 and efficiency give necessary conditions for an $S/k-$decision to be a Condorcet winner. But they are not sufficient. This is proven in Example 1. In this example there is a unique $S/2-$decision satisfying both necessary conditions, but it is not a Condorcet winner despite our strong restrictions on preferences (which guarantee the existence of Condorcet winners when $k = 1$).

**Example 1.** *Let $N = \{1, 2, ..., 13, 14\}$, and $P = (\succcurlyeq_i)_{i=1}^{14}$ be such that for all $i$, $\succcurlyeq_i$ is euclidean on $[0, 100]$ with the following peaks: $p(i) = i$, for all $i = 1, .., 4$ and $p(5) = 32$, $p(6) = 33$, $p(7) = 34$, $p(8) = 67$, $p(9) = 68$, $p(10) = 69$, $p(11) = 97$, $p(12) = 98$, $p(13) = 99$, $p(14) = 100$. Let's see that $CW(P, N, 2) = \emptyset$. Because of Proposition 1 if $d = (d_1, d_2) \in CW(P, 2)$, $d_h \in CW_x(P_{S_h}, 1)$ for $h \in \{1, 2\}$ and $d$ should be an efficient $N/2-$decision. Let $d = ((4, S_1), (97, S_2))$, with $S_1 = \{1, .., 7\}$, and $S_2 = \{8, .., 14\}$. Notice that $d$ is efficient, $(4, S_1) \in CW(P_{S_1}, 1)$, and $(97, S_2) \in CW(P_{S_2}, 1)$. This is the unique efficient $N/2-$decision such that $d_h \in CW(P_{S_h}, 1)$ for $h \in \{1, 2\}$, and thus, the unique potential candidate. However $d$ is not a Condorcet winner for $N$, since for $d' = ((50, S_1'), (98, S_2'))$, with $S_1' = \{1, .., 10\}$, $S_2' = \{11, .., 14\}$, $\{i \in N \mid d' \succ d\} = \{5, 6, 7, 8, 9, 10, 12, 13, 14\}$, and $\{i \in N \mid d \succ d'\} = \{1, 2, 3, 4, 11\}$.*

**Proposition 2.** *Let $\varphi$ be an allocation consistent collective choice correspondence. If $\varphi$ respects the Condorcet criterion, then for all $S \subseteq N$, for all $P_S$, for all $k \in \mathbb{N}$, and for all $S/k-$decision $d \in \varphi(P_S, k)$, $d_h \in CW(P_{S_h}, 1)$, for all $h \in \{1, .., k\}$ such that $S_h \neq \emptyset$.*

**Proof.** This is because, since $\varphi$ is allocation consistent, $(x_h, S_h) \in \varphi(P_{S_h}, 1)$. Since $\varphi$ respects the Condorcet criterion, $\varphi(P_{S_h}, 1) = CW(P_{S_h}, 1)$. ■

## 3. Voluntary assignments.

We have modeled social choice correspondences as the conjunction of rules which (a) decide the *location* of each object, and (b) *assign* each agent to one object. This formulation gives no room for individual behavior other than voting or revealing preferences. Therefore, our notions of voluntarism are not associated with the actual behavior of agents within the model, since the agents do not actually choose where to go: once their preferences are known, they are assigned to one location. But we are still interested in normative properties of the assignment rule related to the following question. If agents were given the chance to join a group other than the one they are actually assigned to, would they want to use this privilege? If not, we say that the present assignment is voluntary. Otherwise, it is an imposition from the rule. Hence, voluntarism is a normative requirement in our analysis, not the description of any allocative process. In order to be precise about this normative requirement, we must be explicit about the consequences that agents may expect under the hypothetical statement that they are "given the chance to join another group". Our notions of No-envy, Nash and group Nash stability correspond to three different specifications of what agents might expect under this hypothesis. If agents envisage the possibility of joining another group, but not of changing the location of any object, then voluntarism is equal to No-envy. If agents envisage the possibility of joining another group, and consider that the object assigned to this new group may be reallocated accordingly, then voluntarism is Nash stability. Group Nash stability would be similar, with the added possibility that agents might coordinate with others when deciding whether or not to change groups.

We now proceed to define these notions of voluntarism, and to study their compatibility with other desirable features of the collective choice rules.

**Definition 6.** *An $S/k-$decision $d \in D(S,k)$ is envy-free if for all $i \in S$, $x(i,d) \succcurlyeq_i x_h$ for all $x_h \in d_L$.*

**Remark 1.** *Notice that any efficient $S/k-$decision is envy-free, therefore, Condorcet winners, whenever they exist, satisfy this voluntarism property.*

**Definition 7.** *A choice correspondence $\varphi$ is envy-free if for all $S \subseteq N$ and $P_S$, $\varphi(P_S,k)$ selects envy-free $S/k-$decisions.*

We now present the definition which appears in the title of the paper.

**Definition 8.** *A collective choice correspondence is self-selection consistent if it is envy-free and satisfies allocation consistency.*

We emphasize the conjunction of the two properties which give rise to self-selection consistency, because the groups of agents whose partial decisions must match with the global decision are self selected as the set of people who would attach themselves to each location, out of a voluntary choice.

Identifying self-selection consistent collective choice correspondences respecting the Condorcet criterion will be the object of the next section. As a first step in that direction, we study here the correspondence that selects for each $S \subseteq N$, and for each $k$, all the $S/k-$decisions that are envy-free and satisfy the necessary condition of Proposition 2, and prove that such correspondence is well defined.

**Definition 9.** *Let $\varphi^E$ be the collective choice correspondence such that for each $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$,*

$$\varphi^E(P_S,k) = \{d \in D(S,k) \mid d \text{ is envy-free and } d_h \in CW(P_{S_h},1) \text{ for all } h \text{ s.t.} S_h \neq \emptyset\}$$

**Proposition 3.** *The correspondence $\varphi^E$ is well defined. That is, for each $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$, $\varphi^E(P_S,k) \neq \emptyset$.*

**Proof.** We offer the proof for $k = 1, 2, 3$. For any other $t$ the proof is similar. For $k = 1$, $\varphi^E(P_S, 1) = CW(P_S, 1)$. Since preferences are restricted to be single-peaked on $[0, T]$, $CW(P_S, 1) \neq \emptyset$.

Before considering the cases $k = 2, 3,$ let us fix some notation. The lower median of a finite collection $K$ of real numbers is denoted by $lmed(K)$. It stands for the median when the cardinality of $K$ is odd, and for the lowest value of the median if the cardinality is even[5].

For $k = 2$, let us order the agents by increasing order of their peaks. Let $S_1^0 = \{i \in S \mid p(i) = p(1)\}$, and $S_2^0 = \{i \in S \mid p(i) > p(1)\}$. Let $x^1 = (x_1^1, x_2^1)$ be such that $x_h^1 = lmed(p(i))_{i \in S_h^0}$, for $h \in \{1, 2\}$. Let $S_1^1 = S_1^0 \cup \{i \in S_2^0 \mid x_1^1 \succcurlyeq_i x_2^1\}$, and $S_2^1 = S_2^0 \backslash \{i \in S_2^0 \mid x_1^1 \succcurlyeq_i x_2^1\}$. If $S_h^1 = S_h^0$ for all $h \in \{1, 2\}$, then $((x_1^1, S_1^1), (x_2^1, S_2^1)) \in \varphi^E(P_S, 2)$. Otherwise we compute the lower median of the peaks of the agents in $S_1^1$, and $S_2^1$, and let $x_h^2 = lmed(p(i))_{i \in S_h^1}$ for $h \in \{1, 2\}$, $x^2 = (x_1^2, x_2^2)$, $S_1^2 = S_1^1 \cup \{i \in S_2^1 \mid x_1^2 \succcurlyeq_i x_2^2\}$,and $S_2^2 = S_2^1 \backslash \{i \in S_2^1 \mid x_1^2 \succcurlyeq_i x_2^2\}$. If $S_h^2 = S_h^1$ for all $h \in \{1, 2\}$, then $((x_1^2, S_1^2), (x_2^2, S_2^2)) \in \varphi^E(P_S, 2)$. Otherwise we repeat the same process, which is finite because there is a finite set of agents, at each step $x_h^j \geq x_h^{j-1}$ for all $h \in \{1, 2\}$, where $x_h^j = lmed(p(i))_{i \in S_h^{j-1}}$, and furthermore, $S_1^{j-1} \subseteq S_1^j$ and $S_2^j \subseteq S_2^{j-1}$. Hence, for some $j$ we will get $S_h^{j-1} = S_h^j$ for all $h$, and $d = (x_h^j, S_h^j)_{h=1}^2 \in \varphi^E(P_S, 2)$.

For $t = 3$, let $p(1)$ be the smallest peak of the agents, $p(2)$ the second smallest peak. Let $S_1^0 = \{i \in S \mid p(i) = p(1)\}$, $S_2^0 = \{i \in S \mid p(i) = p(2)\}$, and $S_3^0 = \{i \in S \mid p(i) > p(2)\}$. Let $x^1 = (x_1^1, x_2^1, x_3^1)$ be such that $x_k^1 = lmed(p(i))_{i \in S_h^0}$. Notice that $x_1^1 = p(1)$, $x_2^1 = p(2)$, for all $i \in S_1^0$, $x_1^1 \succ_i x_2^1 \succ_i x_3^1$, and for all

[5] We cannot simplify our analysis by assuming an odd number of voters because the nature of our questions require the size of the electorate to be variable and endogenously given.

$i \in S_2^0$, $x_2^1 \succ_i x_1^1$, $x_2^1 \succ_i x_3^1$. If in addition for all $i \in S_3^0$, $x_3^1 \succcurlyeq_i x_2^1 \succcurlyeq_i x_1^1$, then $x_h^1 \in CW(P_{S_h^0}, 1)$ for all $h \in \{1,2,3\}$, and $((x_h^1, S_h^0))_{h=1}^3 \in \varphi^E(P_S, 3)$. Otherwise, let $S_1^1 = S_1^0$, $S_2^1 = S_2^0 \cup \{i \in S_3^0 \mid x_2^1 \succcurlyeq_i x_3^1\}$, and $S_3^1 = S_3^0 \backslash \{i \in S_3^0 \mid x_2^1 \succcurlyeq_i x_3^1\}$. For each of these sets, we compute the lower median of the peaks of the agents in those sets, formally: $x_h^2 = lmed(p(i))_{i \in S_h^1}$, for $h \in \{1,2,3\}$. Notice that $x_h^2 \geq x_h^1$ for all $h \in \{1,2,3\}$. Let $S_1^2 = S_1^1 \cup \{i \in S_2^1 \mid x_1^2 \succcurlyeq_i x_2^2\}$, $S_2^2 = [S_2^1 \backslash \{i \in S_2^1 \mid x_1^2 \succcurlyeq_i x_2^2\}] \cup \{i \in S_3^1 \mid x_2^2 \succcurlyeq_i x_3^2\}$, and $S_3^2 = S_3^1 \backslash \{i \in S_3^1 \mid x_2^2 \succcurlyeq_i x_3^2\}$. If $S_h^2 = S_h^1$ for all $h \in \{1,2,3\}$, then $x_h^2 \in CW(P_{S_h^2}, 1)$, and $((x_h^2, S_h^2))_{h=1}^3 \in \varphi^E(P_S, 2)$. Otherwise we repeat the same process, which is finite because there is a finite set of agents, at each step $x_h^j \geq x_h^{j-1}$, $h \in \{1,2,3\}$, $S_1^{j-1} \subseteq S_1^j$ and $S_3^j \subseteq S_3^{j-1}$. Hence, for some $j$ we will get $S_h^j = S_h^{j-1}$, $x_h^j = lmed(p(i))_{i \in S_h^{j-1}}$, for all $h \in \{1,2,3\}$, and $d = (x_h^j, S_h^j)_{h=1}^3 \in \varphi^E(P_S, 3)$. ■

The necessary condition of Proposition 2 tells us that if we want a self-selection consistent collective choice correspondence $\varphi$ which respects the Condorcet criterion, this correspondence should be a selection from $\varphi^E$. Unfortunately, the following example shows that the correspondence $\varphi^E$ itself is not a solution to our question. This natural correspondence is “too large”, because it may select $S/k$-decisions which violate the Condorcet criterion.

**Example 2.** *Let* $N = \{1, 2, ..., 7, 8\}$, *and* $P = (\succcurlyeq_i)_{i=1}^8$ *be such that for all* $i$, $\succcurlyeq_i$ *is euclidean on* $[0, 15]$ *with the following peaks:* $p(i) = i$, *for all* $i \in \{1,2\}$ *and* $p(3) = 4$, $p(4) = 5$, $p(5) = 6$, $p(6) = 9$, $p(7) = 10$, $p(8) = 11$. *Let* $d = ((2, S_1), (9, S_2))$, *with* $S_1 = \{1, .., 4\}$ *and* $S_2 = \{5, .., 8\}$, $d \in \varphi^E(P, 2)$, *because for all* $i \in S_1$, $2 \succ_i 12$, *and for all* $i \in S_2$, $12 \succ_i 2$, *and* $d_1 \in CW(P_{S_1}, 1)$, $d_2 \in CW(P_{S_2}, 1)$. *However* $d \notin CW(P, 2)$, *because for* $d' = ((4, S_1'), (10, S_2'))$, *with* $S_1' = \{1, .., 5\}$, $S_2' = \{6, .., 8\}$, $\{i \in N \mid d' \succ_i d\} = \{3, 4, 5, 7, 8\}$, *while* $\{i \in N \mid d \succ_i d'\} = \{1, 2, 6\}$, *and* $d'$ *is a Condorcet winner for* $N$.

We now turn to our next definitions relating to voluntarism. Both the notion of Nash and group Nash stability are based on the assumption that agents may compare different sets of decisions, resulting from their use of alternative strategies (this is because we are studying social choice correspondences). Since agents' preferences have been defined up to now on single decisions, we must be precise on the kind of set comparisons that we allow for. We do that by proposing an extension rule, which generates a partial order on sets. Given our extension, the rest of definitions are standard.

Given a preference relation $\succcurlyeq$ defined on $\mathbb{R}$, let $\sqsupset (\succcurlyeq)$ denote the *extension* of $\succcurlyeq$ over subsets of $\mathbb{R}$ defined by: for all $A_1, A_2 \subset \mathbb{R}$, $A_1 \sqsupset (\succcurlyeq) A_2$ if for all $x \in A_1$, and for all $y \in A_2$, $x \succ y$.

From now on, when it is obvious that $\sqsupset (\succcurlyeq)$ is the extension of $\succcurlyeq$, we shall just write $\sqsupset$; likewise, $\sqsupset_i$ will refer to the extension $\sqsupset (\succcurlyeq_i)$ when $\succcurlyeq_i$ is an obvious reference, etc.

Notice again that $\sqsupset$ is a criterion for extending $\succcurlyeq$ which allows for $\sqsupset (\succcurlyeq)$ to be highly incomplete. Our next definitions of Nash and group Nash stability are natural, but of course relative to $\sqsupset$ .

**Definition 10.** *A collective choice correspondence $\varphi$ is Nash stable if there is no $d \in \varphi(P_S, k)$, $i \in S$, and $d_j = (x_j, S_j)$ with $i \notin S_j$ such that $\varphi(P_{S_j \cup \{i\}}, 1) \sqsupset_i x(i, d)$.*

**Definition 11.** *A collective choice correspondence $\varphi$ is group Nash stable if there is no $d \in \varphi(P_S, k)$, $d_j = (x_j, S_j)$ and $I \subset S_h, h \neq j$, such that $\varphi(P_{S_j \cup I}, 1) \sqsupset_i x(i, d)$ for all $i \in I$.*

Condorcet winners, whenever they exist, behave well with respect to group Nash stability, and consequently, with respect to Nash stability.

**Proposition 4.** *Given $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$ such that $CW(P_S, k) \neq \emptyset$, there is no $d \in CW(P_S, k)$, $d_j = (x_j, S_j)$ and $I \subset S_h, h \neq j$, such that $CW(P_{S_j \cup I}, 1) \sqsupset_i x(i, d)$ for all $i \in I$.*

**Proof.** Suppose there is $d \in CW(P_S, k)$, $d_j = (x_j, S_j)$ and $I \subset S_h, h \neq j$, such that $CW(P_{S_j \cup I}, 1) \sqsupset_i x(i, d)$ for all $i \in I$. Without loss of generality suppose that $h = j + 1$. Let $[y_1, y_2] = CW(P_{S_j \cup I}, 1)$. Let $d'$ be such that $d'_l = d_l$ for all $l \notin \{j, h\}$, $d'_j = (y_1, S_j \cup I)$, $d'_h = (x_h, S_h \backslash I)$. Let's see that $|\{i \in S \mid d' \succ_i d\}| > |\{i \in S \mid d \succ_i d'\}|$. Notice that $\{i \in S \mid d' \succ_i d\} = \{i \in S_j \cup I \mid y_1 \succ_i x_j\}$, and since $y_1 \in CW(P_{S_j \cup I}, 1)$, $|\{i \in S_j \cup I \mid y_1 \succ_i x_j\}| \geq |\{i \in S_j \cup I \mid x_j \succ_i y_1\}|$. Furthermore, since $d \in CW(P_S, k)$, $d$ is envy-free, then $x(i, d) \succcurlyeq_i x_j$ for all $i \in I$, therefore $x_j \notin CW(P_{S_j \cup I}, 1)$, and since $x_j < y_1 \leq y$ for all $y \in CW(P_{S_j \cup I}, 1)$, then $|\{i \in S_j \cup I \mid y_1 \succ_i x_j\}| > |\{i \in S_j \cup I \mid x_j \succ_i y_1\}|$, and trivially, $|\{i \in S_j \cup I \mid x_j \succ_i y_1\}| = |\{i \in S \mid d \succ_i d'\}|$, which contradicts the fact that $d \in CW(P_S, k)$. ■

Again, identifying self-selection consistent collective choice correspondence satisfying (Group) Nash stability and respecting the Condorcet criterion will be the object of the next section. As a first step in that direction, we define the maximal subcorrespondence of $\varphi^E$ which is (Group) Nash stable, and we prove that such correspondence is well defined.

**Definition 12.** *Let $\varphi^N$ be the subcorrespondence of $\varphi^E$ such that for each $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$, $\varphi^N(P_S, k) = \varphi^E(P_S, k) \backslash NN(P_S, k)$, where $NN(P_S, k)$ is the set of all $d \in \varphi^E(P_S, k)$, such that there is an $i \in S$, and $d_j = (x_j, S_j)$, $i \notin S_j$, such that $CW(P_{S_j \cup \{i\}}, 1) \sqsupset_i x(i, d)$.*

**Definition 13.** *Let $\varphi^G$ be the subcorrespondence of $\varphi^E$ such that for each $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$, $\varphi^G(P_S, k) = \varphi^E(P_S, k) \backslash NG(P_S, k)$, where $NG(P_S, k)$ is the* *set of all $d \in \varphi^E(P_S, k)$, such that there is $d_j = (x_j, S_j)$ and $I \subset S_h, h \neq j$, such that $CW(P_{S_j \cup I}, 1) \sqsupset_i x(i, d)$ for all $i \in I$.*

**Remark 2.** *Notice that for all $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$, $\varphi^G(P_S, k) \subseteq \varphi^N(P_S, k)$, and not necessarily identical as the following example shows.*

**Example 3.** *Let $N = \{1, 2, .., 16\}$, and $\mathrm{P} = (\succcurlyeq_i)_{i=1}^{16}$ be such that for all $i$, $\succcurlyeq_i$ is euclidean on $[0, 14]$ with the following peaks: $p(i) = i$, for all $i = 1, 2, 3$, $p(4) = 5$, $p(5) = 6$, $p(8) = 8$, for all $j = 6, 7, 8, 9$, $p(k) = k$, for all $k = 10, 11, 12, 13$, $p(h) = 14$, for all $h = 14, 15, 16$. Let $d = ((3, S_1), (11, S_2))$, $S_1 = \{1, .., 5\}$, and $S_2 = \{6, .., 16\}$. Notice that $d \in \varphi^N(P, 2)$, but $d \notin \varphi^G(P, 2)$, because all agents in the set $I = \{6, 7, 8, 9\}$ strictly prefer $CW(P_{S_1 \cup I}, 1) = \{6\}$ to the location that they are actually assigned to.*

**Proposition 5.** *The correspondence $\varphi^G$ is well defined. That is, for each $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$, $\varphi^G(P_S, k) \neq \emptyset$.*

**Proof.** Given $S \subseteq N$, $P_S$, and $k \in \mathbb{N}$, we start with the first $S/k$-decision in $\varphi^E(P_S, k)$ that we find in the process described in Proposition 3.Suppose that this decision was obtained at step $j$. Let $d^j = ((x_h^j, S_h^j))_{h=1}^k$ be such decision. Suppose that $d^j \notin \varphi^G(P_S, k)$, then, there is a $d_l^j = (x_l^j, S_l^j)$, and $I \subset S_t^j$, $t \neq l$, such that $CW(P_{S_l^j \cup I}, 1) \sqsupset_i x(i, d)$ for all $i \in I$. Because preferences are single peaked on $[0, T]$, $S_t^j$ must be $S_{l+1}^j$ or $S_{l-1}^j$. Let's prove that, given the process described in Proposition 3, $I \subset S_{l+1}^j$. That is, the movements of agents go always in the same direction; at any step, if agents move at all, they move from groups $S_{l+1}$ to $S_l$. Suppose that $I \subset S_{l-1}^j$, and let $y_l \in CW(P_{S_l^j \cup I}, 1)$. Notice that $y_l < x_l^j$. Previous to step $j$ there was a step $q$ such that $I$ was in $S_l^q$, and $x_l^q = lmed(p(i))_{i \in S_l^q} \leq y_l$, but they move, probably not all of them at the same time. Let $q$ be such that

$I \subset S_l^q$, and at $q+1$, there was $J \subseteq I$ such that $J \subset S_{l-1}^{q+1}$ because $x_{l-1}^q \succcurlyeq_i x_l^q$ for all $i \in J$. At step $j$, we get the following relation, $x_{l-1}^j \geq x_{l-1}^{q+1} \geq x_{l-1}^q$, which implies that $x_{l-1}^j \succcurlyeq_i x_l^q \succcurlyeq_i y_l$ for all $i \in J$. Therefore, $I$ must be a subgroup of $S_{l+1}^j$. Let $x_l^{j+1} = lmed(p(i))_{i \in S_l^j \cup I}$, $x_{l+1}^{j+1} = lmed(p(i))_{i \in S_{l-1}^j \backslash I}$ and $x_h^{j+1} = x_h^j$ for all $h \neq l, l+1$. Let $S_1^{j+1} = S_1^j \cup \{i \in S_2^j \mid x_1^{j+1} \succcurlyeq_i x_2^{j+1}\}$, $S_h^{j+1} = (S_h^j \backslash \{i \in S_h^j \mid x_{h-1}^{j+1} \succcurlyeq_i x_h^{j+1}\}) \cup \{i \in S_{h+1}^j \mid x_h^{j+1} \succcurlyeq_i x_{h+1}^{j+1}\}$ for all $h \in \{2, .., k-1\}$ and $S_k^{j+1} = S_k^j \backslash \{i \in S_k^j \mid x_{k-1}^{j+1} \succcurlyeq_i x_k^{j+1}\}$. If $S_h^{j+1} = S_h^j$ for all $h \in \{1, .., k\}$, then $d^{j+1} = ((x_h^{j+1}, S_h^{j+1}))_{h=1}^k \in \varphi^E(P_S, k)$, if this is no the case, we proceed as in Proposition 3 until we get a decision in $\varphi^E(P_S, k)$. Let $d^m$ be such decision. If $d^m \in \varphi^G(P_S, k)$, we are done. Otherwise, we proceed as in the beginning.. The process is finite for the same reason that it was finite in Proposition 3, and because, at any step, the movements of the agents go always in the same direction, form $S_{l+1}$ to $S_l$. ■

A self-selection consistent and group Nash stable collective choice correspondence which respects the Condorcet criterion should be a selection from $\varphi^G$. Unfortunately, the following example shows that the correspondence $\varphi^G$ itself is not a solution to our question, because it may select $S/k$-decisions which violate the Condorcet criterion.

**Example 4.** *Let* $N = \{1, 2, .., 8\}$, *and* $\mathrm{P} = (\succcurlyeq_i)_{i=1}^8$ *be such that for all* $i$, $\succcurlyeq_i$ *is euclidean on* $[0, 14]$ *with the following peaks:* $p(i) = i$, *for* $i = 1, 2$, $p(3) = 5$, $p(4) = 6$, $p(5) = 9$, $p(6) = 12$, $p(7) = 13$, $p(8) = 14$. *For this profile the set of Condorcet winners for* $k = 2$ *is not empty. For example,* $((5, S_1), (13, S_2))$ *with* $S_1 = \{1, 2, 3, 4, 5\}$ *and* $S_2 = \{6, 7, 8\}$ *is a Condorcet winner. Let* $d = ((2, S_1), (12, S_2))$ *with* $S_1 = \{1, .., 4\}$, $S_2 = \{5, .., 8\}$. *Notice that* $d \in \varphi^G(P, 2)$, *but this is not a Condorcet winner. It is dominated by* $((7, S_1'), (13, S_2'))$ *with* $S_1' = \{1, .., 5\}$, $S_2' = \{6, .., 8\}$.

## 4. Self-selection Consistent and (Group) Nash stable correspondences respecting the Condorcet criterion.

In the previous section we have noted that self-selection consistent correspondences respecting the Condorcet criterion must be strict subcorrespondences of another, that we have called $\varphi^E$, which we have proven to be well defined but will not always respect the Condorcet criterion. Similarly, allocation consistent correspondences satisfying (Group) Nash stability and respecting the Condorcet criterion must be subcorrespondences of $\varphi^N$ and $\varphi^G$, which again do not always respect the Condorcet criterion. Hence, while there is still hope to identify some adequate subcorrespondence meeting all desiderata, some methods to select from the above correspondences must be suggested.

An alternative route toward finding satisfactory rules could have been to first identify some correspondence that guarantees respect for Condorcet, and then check for its respect of consistency and voluntarism. We now introduce the Simpson correspondence (Moulin (1988)), which indeed respects the Condorcet criterion and it will be shown not to be allocation consistent. Yet, its introduction is useful because an appropriate modification of the Simpson rule, when combined with our results in the previous section, will allow us to construct correspondences satisfying all our requirements.

We first define the Simpson rule and show that it does not directly define an allocation consistent choice correspondence.

**Definition 14.** *Given a preference profile $P$, and a set of agents $S$, for any $d, d' \in D(S,k)$, let $N(d,d') = |\{i \in S \mid d \succcurlyeq_i d'\}|$. Given $d \in D(S,k)$, the Simpson score of $d$, denoted $SC(d)$, is the minimum of $N(d,d')$ over all $d' \in D(S,k)$, $d' \neq d$. An $S/\ k$-decision $d$ is a Simpson winner for $S$ if $SC(d) \geq SC(d')$ for all $d' \in$*

$D(S,k)$.

*The Simpson correspondence, SW, is defined so that for each $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$,*

$$SW(P_S,k) = \{d \in D(S,k) \mid SC(d) \geq SC(d') \text{ for all } d' \in D(S,k)\}$$

While clearly the $SW$ correspondence respects the Condorcet criterion and is envy-free, it may violate allocation consistency, as shown by the following example.

**Example 5.** *Let's consider Example 1 again. If $SW$ were allocation consistent, since it is envy-free and respects the Condorcet criterion, then it should be a selection of $\varphi^E$. Notice that in this example, $\varphi^E(P,N,2) = \{((4,S_1),(97,S_2))\}$, with $S_1 = \{1,..,7\}$, and $S_2 = \{8,...,14\}$. However, $d = ((4,S_1),(97,S_2))$ is not a Simpson winner for $N$. To see that let $d' = ((x'_1,S'_1),(x'_2,S'_2))$ be such that $41 < x'_1 < 60$, $97 < x'_2 \leq 98$, $S'_1 = \{1,..,10\}$, and $S'_2 = \{11,..,14\}$. This $N/2$-decision is such that $d' \succ_i d$ for all $i \in \{5,6,7,8,9,10,12,13,14\}$, and $d \succ_i d'$ for all $i \in \{1,2,3,4,11\}$. Therefore the number of voters preferring $d$ to $d'$ is five, and this is the minimum over all $\bar{d}$. So, the Simpson score of $d$ is five. For $\hat{d} = ((3,S_1),(98,S_2))$, it is easy to see that the Simpson score of $\hat{d}$ is six. Then $d$ can not be a Simpson winner. Therefore, $SW$ is not allocation consistent.*

Even if the Simpson correspondence is not consistent, we can use the Simpson scores in order to define subcorrespondences of any given correspondence. We provide a general definition, and then use it for our specific purposes.

**Definition 15.** *Given a subset of $S/k-$decisions, $H \subset D(S,k)$, the set of $H-$Simpson winners is the set of decisions in $H$ whose Simpson score ( still computed on pairwise comparisons over all $D(S,k)$) is maximal on $H$.*

Hence, the $H-$Simpson winners are those elements in $H$ which have the maximal Simpson scores. But these scores are computed not only in pairwise comparisons among elements in $H$, but on pairwise comparisons among all elements of the larger set $D(S, k)$.

This method can now be applied to define subcorrespondences of any given correspondence.

**Definition 16.** *Given a correspondence $\varphi$, its Simpson subcorrespondence, $\varphi^*$, is defined so that for each $S \subseteq N$, $P_S$ and $k \in \mathbb{N}$,*

$$\varphi^*(P_S, k) = \{d \in \varphi(P_S, k) \mid d \text{ is } \varphi(P_S, k) - \text{Simpson winner}\}$$

Finally, apply this general procedure to $\varphi^E$, and define its Simpson subcorrespondence, $\varphi^{*E}$. It is straightforward to check that $\varphi^{*E}$ will satisfy all our requirements.

**Proposition 6.** *Let $\varphi^{*E}$ be the Simpson subcorrespondence of $\varphi^E$. This correspondence is self-selection consistent and respects the Condorcet criterion.*

For identical reasons, we can defined satisfactory Simpson subcorrespondences of $\varphi^N$, and $\varphi^G$.

**Proposition 7.** *The Simpson subcorrespondence $\varphi^{*N}$ is self-selection consistent, satisfies Nash stability and respects the Condorcet criterion.*
*The Simpson subcorrespondence $\varphi^{*G}$ is self-selection consistent, satisfies group Nash stability and respects the Condorcet criterion.*

Propositions 6 and 7 provide us with specific and natural examples of collective choice correspondences satisfying combinations of all our desiderata.

Our last example shows that the rules defined in Propositions 6 and 7 need not be identical. We do not know yet whether they are always nested.

**Example 6.** *Let* $N = \{1, 2, .., 16\}$, *and* $\mathrm{P} = (\succcurlyeq_i)_{i=1}^{16}$ *be such that for all* $i$, $\succcurlyeq_i$ *is euclidean on* $[0, 200]$ *with the following peaks:* $p(1) = 10$, $p(2) = 29$, $p(3) = 30$, $p(4) = 40$, $p(5) = 50$, $p(6) = 80$, $p(7) = 82$, $p(8) = 90$, $p(9) = 105$, $p(10) = p(11) = 135$, $p(12) = 170$, $p(13) = p(14) = 175$, $p(15) = 190$, $p(16) = 200$. *Let* $d = ((40, S_1), (17, S_2))$, *with* $S_1 = \{1, .., 8\}$, $S_2 = \{9, .., 16\}$, *and* $d' = ((50, S_1'), (175, S_2'))$, *with* $S_1' = \{1, ..., 9\}$, $S_2' = \{10, .., 16\}$. *These are the unique envy-free* $N/2-$*decisions such that* $d_h \in CW(P_{S_h}, 1)$, $d_h' \in CW(P_{S_h'}, 1)$, *for all* $h \in \{1, 2\}$. *Therefore,* $\varphi^E(P, 2) = \{d, d'\}$. *It is easy to see that* $SC(d) = SC(d') = 6$. *Then* $\varphi^{*E}(P, 2) = \{d, d'\}$, *but* $\varphi^N(P, 2) = \{d'\}$, *then,* $\varphi^{*N}(P, 2) = \{d'\}$.

## 5. Final remarks.

We have proven that social choice correspondences satisfying our quite demanding requirements do exist. We have not provided characterization results, but just examples involving specific rules. Their construction combines a first choice of decisions satisfying basic necessary conditions, followed by a further choice among them (through the Simpson rule) in order to guarantee respect for the Condorcet criterion. Our use of the Simpson rule is not essential; other extensions of majority, also respecting the Condorcet criterion, could have been used. However, not all extensions would have been appropriate. The Simpson rule is handy because it is easy to extend to the choice out of an infinity of alternatives (unlike the Copeland rule) and because it is simply based on pairwise comparisons between alternatives.

The above remarks suggest two directions for further research. One would be to find additional social choice correspondences satisfying our properties. The other would involve the axiomatic characterization of the rules we have proposed, or of larger classes containing them.

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