Appendix

The proof of Proposition 1.

From equations (3) and (5), we find that those equations are the same except for the second term implying a social cost. Since the second term of (5) is smaller than that of (3) given $\bar{e}$ if l_B is positive, we have $EW^c(l_B, k) < EW^n(l_B)$. On the other hand, if l_B is equal to 0, the second term of (3) is the same as that of (5). Thus, we have $\bar{e}^c(0,k) = \bar{e}^n(0)$ and $EW^c(0,k) = EW^n(0)$ for any k. Since Laffont and Tirole (1991) have shown that the optimal collusion-proof menu of contracts is optimal in all possible menus of contracts, we have the above result. Q.E.D.

The proof of Proposition 3.

From the envelop theorem and simple calculation, $dEW^C / dz = ¶EW^C / ¶z$

$$= (1-q)\{[S-(1+l)(\bar{b}-e^* + y(e^*))] - [S-(1+l)(\bar{b}-e^C + y(e^C))]\} > 0 .$$

If the principal does not employ the regulatory agency, then the principal obtains $EW^S = EW^C|_{z=0}$. Thus, $EW^C > EW^S$ if $z > 0$. Q.E.D.

The proof of Lemma 1.

From simple calculation, we have

$$\frac{dEW^C}{dz} = (1-q)\left[\overline{SW}^* - \overline{SW}^s\right] + (1-(1-q)z)\frac{¶EW^S}{¶z}$$

where $\overline{SW}^*$ is the income from the inefficient type under symmetric information, and $\overline{SW}^s$ is the income from the inefficient type under asymmetric information, and EW^S is the expected social welfare under asymmetric information. It is obvious that $\overline{SW}^*$-$\overline{SW}^s$>0. An increase of z increases the probability of the efficient type conditional on $\hat{s} = \emptyset$ since the regulatory agency reveals the firm's type only if the firm is inefficient. From Laffont and Tirole (1993) ch.1, the expected social welfare under asymmetric information is increasing in the conditional probability of the efficient type. Thus, $¶EW^S / ¶z > 0$. Q.E.D.

The proof of Lemma 2.

From equation (6) and (7), $EW^c(l_B, k) \geq EW_L^n(l_F)$ implies that

$$(1-z)[EW^s(\bar{e}^s, q) - EW^s(\bar{e}^c(l_B,k), q)] \leq qz\left[\frac{1}{1+l_B}\Phi(\bar{e}^s) - \left\{l + \frac{l_B k}{1+l_B}\right\}\Phi(\bar{e}^c(l_B,k))\right].$$

Since $\bar{e}^s$ maximizes $EW^s(\bar{e}, q)$ by definition and $\bar{e}^s > \bar{e}^c(l_B, k)$,

$EW^s(\bar{e}^s, q) > EW^s(\bar{e}^c(l_B,k), q)$. Thus, $\left[\frac{1}{1+l_B}\Phi(\bar{e}^s) - \left\{l + \frac{l_B k}{1+l_B}\right\}\Phi(\bar{e}^c(l_B,k))\right] > 0$.

From equations (6) and (7) and envelop theorem,

$$\frac{\P EW^{c}}{\P z} - \frac{\P EW_{L}^{n}}{\P z} = [EW^{s}(\bar{e}^{s}, q) - EW^{s}(\bar{e}^{c}(\,\boldsymbol{l}_{B}, k), q)]$$

$$+ q\left[\frac{1}{1+\boldsymbol{l}_{B}}\Phi(\bar{e}^{s}) - \left\{\boldsymbol{l} + \frac{\boldsymbol{l}_{B}k}{1+\boldsymbol{l}_{B}}\right\}\Phi(\bar{e}^{c}(\,\boldsymbol{l}_{B}, k))\right].$$

Therefore, $\frac{\P EW^{c}}{\P z} - \frac{\P EW_{L}^{n}}{\P z} > 0$ if $EW^{c} \geq EW_{L}^{n}$. Q.E.D.

The proof of Proposition 4.

First we show that the optimal collusion-proof menu of contracts under full commitment dominates that under delegation weakly. Second, the optimal collusion-proof menu of contracts under full delegation attains the optimal allocation under full commitment.

Delegation case.

Principal offers a menu of contracts, $t_{R}(\hat{s}, \hat{\boldsymbol{b}}, t_{F}, C)$, to the regulatory agency. After the regulatory agency chooses a contract, he offers a menu of contracts, $\{t_{F}(\hat{s}, \hat{\boldsymbol{b}}), C(\hat{s}, \hat{\boldsymbol{b}})\}$, to the firm. Here we assume that (t_{R}, t_{F}, C) is verifiable and that the regulatory agency cannot communicate with the firm before he signs contracts with the principal. Our problem is to find the menu of contract $\{t_{F}(\hat{s}, \hat{\boldsymbol{b}}), C(\hat{s}, \hat{\boldsymbol{b}}), t_{R}(\hat{s}, \hat{\boldsymbol{b}}, t_{F}, C)\}$, which maximizes the principal's objective function under some constraints. Consider the following correspondence, $h: (t_{R}, t_{F}, C) \rightarrow (u_{F}, e, w)$

where $u_{F} = t_{F} - \boldsymbol{y}(\boldsymbol{b} - C)$, $e = \boldsymbol{b} - C$, $w = t_{R}(\hat{s}, \hat{\boldsymbol{b}}, t_{F}, C) - t_{F}(\hat{s}, \hat{\boldsymbol{b}})$.

Then, we can check h is one to one correspondence. Then, we can solve the original problem by changing control variables from (t_{R}, t_{F}, C) to (u_{F}, e, w). That is, we define the principal's and all agents' objective functions and all constraints by (u_{F}, e, w), and then we find the optimal menu of contracts, (u_{F}, e, w). Note that u_{F} represents the level of the firm's benefits, e is the effort level of the firm, and w is the level of the regulatory agency's benefits.

The collusion-proof menu of contracts with delegation

We consider a form of menu of contracts, $\{e(\hat{s}, \hat{\boldsymbol{b}}), u_{F}(\hat{s}, \hat{\boldsymbol{b}}), w(e, u_{F}, \hat{s}, \hat{\boldsymbol{b}})\}$. When it is a collusion-proof menu of contracts and feasible, it must satisfy the following conditions:

$$\{e(\underline{\boldsymbol{b}}, \hat{\boldsymbol{b}}), u_{F}(\underline{\boldsymbol{b}}, \hat{\boldsymbol{b}})\} = \arg\max_{e, u_{F}} w(e, u_{F}, \underline{\boldsymbol{b}}, \hat{\boldsymbol{b}})$$

$$\text{s.t. } u_{F}(\underline{\boldsymbol{b}}, \hat{\boldsymbol{b}}) \geq 0 \text{ for any } \hat{\boldsymbol{b}}. \qquad \text{(a5)}$$

$$\{e(\bar{\boldsymbol{b}}, \hat{\boldsymbol{b}}), u_{F}(\bar{\boldsymbol{b}}, \hat{\boldsymbol{b}})\} = \arg\max_{e, u_{F}} w(e, u_{F}, \bar{\boldsymbol{b}}, \hat{\boldsymbol{b}})$$

$$\text{s.t. } u_{F}(\bar{\boldsymbol{b}}, \hat{\boldsymbol{b}}) \geq 0 \quad \text{for any } \hat{\boldsymbol{b}}. \qquad \text{(a6)}$$

$$\{(e(\boldsymbol{f}, \underline{\boldsymbol{b}}), u_{F}(\boldsymbol{f}, \underline{\boldsymbol{b}})), (e(\boldsymbol{f}, \bar{\boldsymbol{b}}), u_{F}(\boldsymbol{f}, \bar{\boldsymbol{b}}))\} = \arg\max qw(e(\boldsymbol{f}, \underline{\boldsymbol{b}}), u_{F}(\boldsymbol{f}, \underline{\boldsymbol{b}}), \boldsymbol{f}, \underline{\boldsymbol{b}})$$

$$+(1-q)w(e(f,\overline{b}),u_F(f,\overline{b}),f,\overline{b})$$

$$\text{s.t.}\quad u_F(f,\underline{b})\geq 0 \tag{a7}$$

$$u_F(f,\overline{b})\geq 0 \tag{a8}$$

$$u_F(f,\overline{b})\geq u_F(f,\underline{b})-\Phi(e(f,\underline{b})+\Delta b) \tag{a9}$$

$$u_F(f,\underline{b})\geq u_F(f,\overline{b})+\Phi(e(f,\overline{b})) \tag{a10}$$

(IR) of the regulatory agency:

$$w(e(\underline{b},\hat{b}),u_F(\underline{b},\hat{b}),\underline{b},\hat{b})\geq 0 \tag{a11}$$

$$w(e(\overline{b},\hat{b}),u_F(\overline{b},\hat{b}),\overline{b},\hat{b})\geq 0 \tag{a12}$$

$$qw(e(f,\underline{b}),u_F(f,\underline{b}),f,\underline{b}) \ +(1-q)w(e(f,\overline{b}),u_F(f,\overline{b}),f,\overline{b})\geq 0 \tag{a13}$$

(IC) of the regulatory agency:

$$w(e(\underline{b},\hat{b}),u_F(\underline{b},\hat{b}),\underline{b},\hat{b})\geq w(e(f,\underline{b}),u_F(f,\underline{b}),f,\underline{b}) \tag{a14}$$

$$w(e(\overline{b},\hat{b}),u_F(\overline{b},\hat{b}),\overline{b},\hat{b})\geq w(e(f,\overline{b}),u_F(f,\overline{b}),f,\overline{b}) \tag{a15}$$

(CP) of the regulatory agency:

$$w(e(\underline{b},\hat{b}),u_F(\underline{b},\hat{b}),\underline{b},\hat{b})\geq w(e(f,\underline{b}),u_F(f,\underline{b}),f,\underline{b}) \ +u_F(f,\underline{b}) \tag{a16}$$

The optimal collusion-proof menu of contracts under full commitment

The form of menu of contracts is $\{e(\hat{s},\hat{b}),u_F(\hat{s},\hat{b}),w(\hat{s},\hat{b})\}$. If it is collusion-proof and feasible, it must satisfy the following conditions, (a17)-(a26):

$$u_F(\underline{b},\hat{b})\geq 0 \qquad \text{for any } \hat{b}. \tag{a17}$$

$$u_F(\overline{b},\hat{b})\geq 0 \qquad \text{for any } \hat{b}. \tag{a18}$$

$$u_F(f,\underline{b})\geq 0 \tag{a19}$$

$$u_F(f,\overline{b})\geq 0 \tag{a20}$$

$$u_F(f,\overline{b})\geq u_F(f,\underline{b})-\Phi(e(f,\underline{b})+\Delta b) \tag{a21}$$

$$u_F(f,\underline{b})\geq u_F(f,\overline{b})+\Phi(e(f,\overline{b})) \tag{a22}$$

(IR) of the regulatory agency:

$$w(\underline{b},\hat{b})\geq 0 \tag{a23}$$

$$w(\overline{b},\hat{b})\geq 0 \tag{a24}$$

$$w(f,\underline{b})\geq 0 \tag{a25}$$

$$w(f,\overline{b})\geq 0 \tag{a26}$$

(IC) of the regulatory agency:

$$w(\underline{b},\hat{b})\geq w(f,\underline{b}) \tag{a27}$$

$$w(\overline{b},\hat{b})\geq w(f,\overline{b}) \tag{a28}$$

(CP) of the regulatory agency:

$$w(\underline{\beta},\hat{\beta}) \geq w(\varphi,\underline{\beta}) \ \ + u_F(\varphi,\underline{\beta}) \qquad \text{(a29)}$$

Lemma A.1: Delegation is weakly dominated by the optimal collusion-proof menu of contracts under full commitment.

Proof. We will show that there exists a feasible collusion-proof menu of contracts under full commitment which can attain the same allocation as any feasible collusion-proof menu of contracts with delegation.

Suppose $\{e^0(\hat{\sigma},\hat{\beta}), u_F^0(\hat{\sigma},\hat{\beta}), w^0(e,u_F,\hat{\sigma},\hat{\beta})\}$ is feasible under delegation. It means that it satisfies the conditions (a5)-(a16). Consider the following contracts, $\{e^0(\hat{\sigma},\hat{\beta}), u_F^0(\hat{\sigma},\hat{\beta}), w^0(\hat{\sigma},\hat{\beta})\}$ where $w^0(\hat{\sigma},\hat{\beta}) \equiv w^0(e^0(\hat{\sigma},\hat{\beta}), u_F^0(\hat{\sigma},\hat{\beta}), \hat{\sigma},\hat{\beta})$. Since this menu of contracts satisfies (a5)-(a16), it also satisfies (a17)-(a29), and so, it is feasible under full commitment. This implies that full commitment weakly dominates delegation.

Lemma A.2: Under delegation, the principal can attain the same allocation as in the optimal menu of contracts under full commitment.

Proof. Consider the following contract for the regulatory agency, $w(e,u_F,\hat{\sigma},\hat{\beta})$:

$$w(e,u_F,\underline{\beta},\hat{\beta}) = S - (1+\lambda)(\underline{\beta} - e + \psi(e)) - \lambda u_F - I(\underline{\beta},\hat{\beta})$$

$$w(e,u_F,\bar{\beta},\hat{\beta}) = S - (1+\lambda)(\bar{\beta} - e + \psi(e)) - \lambda u_F - I(\bar{\beta},\hat{\beta})$$

$$w(e(\varphi,\underline{\beta}), u_F(\varphi,\underline{\beta}), \varphi,\underline{\beta})$$

$$= -\zeta u_F(\varphi,\underline{\beta}) + (1-\zeta)\,[S - (1+\lambda)(\underline{\beta} - e(\varphi,\underline{\beta}) + \psi(e(\varphi,\underline{\beta}))) - \lambda u_F(\varphi,\underline{\beta})] - I(\varphi,\underline{\beta})$$

$$w(e(\varphi,\bar{\beta}), u_F(\varphi,\bar{\beta}), \varphi,\bar{\beta}) = (1-\zeta)[S - (1+\lambda)(\bar{\beta} - e(\varphi,\bar{\beta}) + \psi(e(\varphi,\bar{\beta}))) - \lambda u_F(\varphi,\bar{\beta})]$$
$$-I(\varphi,\bar{\beta})$$

where

$$I(\underline{\beta},\hat{\beta}) = S - (1+\lambda)(\underline{\beta} - e^* + \psi(e^*)) - \Phi(\bar{e}^c)$$

$$I(\bar{\beta},\hat{\beta}) = S - (1+\lambda)(\underline{\beta} - e^* + \psi(e^*))$$

$$I(\varphi,\underline{\beta}) = -\zeta\Phi(\bar{e}^c) \ +(1-\zeta)[S - (1+\lambda)(\underline{\beta} - e^* + \psi(e^*)) - \Phi(\bar{e}^c)]$$

$$I(\varphi,\bar{\beta}) = (1-\zeta)[S - (1+\lambda)(\underline{\beta} - \bar{e}^c + \psi(\bar{e}^c))].$$

Then, the regulatory agency will choose the following allocation:

$$(e(\underline{\beta},\hat{\beta}), u_F(\underline{\beta},\hat{\beta})) = (e(\bar{\beta},\hat{\beta}), u_F(\bar{\beta},\hat{\beta})) = (e^*, 0).$$

$$(e(\varphi,\underline{\beta}), u_F(\varphi,\underline{\beta})) = (e^*, \Phi(\bar{e}^c))$$

$$(e(\varphi,\bar{\beta}), u_F(\varphi,\bar{\beta})) = (\bar{e}^c, 0).$$

Then the regulatory agency receives the following benefits:

$$w(e,u_F,\underline{\beta},\hat{\beta}) = \Phi(\bar{e}^c)$$

$$w(e,u_F,\bar{\beta},\hat{\beta}) = w(e(\varphi,\underline{\beta}), u_F(\varphi,\underline{\beta}), \varphi,\underline{\beta}) \ = w(e(\varphi,\bar{\beta}), u_F(\varphi,\bar{\beta}), \varphi,\bar{\beta}) = 0.$$

This allocation is exactly the same as the optimal allocation under full commitment.

QED.

The proof of Proposition 6.

By definition, $\Delta EW_{ILA}(\Delta \boldsymbol{l}_B, 0) = \Delta EW_{IL}(\Delta \boldsymbol{l}_B, 0)$. Since administrative reform $\Delta \boldsymbol{z} > 0$ pushes up EW_L^n (lemma 2), we have $\partial \Delta EW_{ILA}(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z}) / \partial \Delta \boldsymbol{z} > 0$ for any $\Delta \boldsymbol{l}_B$. Since $\lim_{\Delta l_F \to 0} C_L'(\Delta \boldsymbol{l}_B) = 0$, we have $\partial \{\Delta EW_{ILA}(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z}) - C_A(\Delta \boldsymbol{z})\} / \partial \Delta \boldsymbol{z} > 0$ at $\Delta \boldsymbol{l}_B = 0$. Then it is obvious that $\Delta NEW_{ILA}{*} > \Delta NEW_{IL}{*}$. Q.E.D.

The proof of Proposition 8.

By definition, $\Delta EW_{DILA}(0,0) = \Delta EW_{DI}(0,0)$. By **DI**, the principal can push up expected social welfare from EW^c to EW^n. Since administrative reform increases EW^n and $\lim_{\Delta \boldsymbol{z} \to 0} C_L'(\Delta \boldsymbol{z}) = 0$, $\partial \Delta NEW_{DILA}(0,0) / \partial \Delta \boldsymbol{z} > 0$. Thus, $\Delta NEW_{DILA}^* > \Delta NEW_{DI}^*$. Similarly, we can also prove $\Delta NEW_{DILA}^* > \Delta NEW_{DIL}^*$. Q.E.D.

The proof of Proposition 9.

It is obvious from figure 10 that $\Delta EW_{DILA}(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z}) > \Delta EW_{ILA}(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z})$ *for all* $(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z})$. *Thus, if* $C_D = 0$, $\Delta NEW_{DILA}(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z}) > \Delta NEW_{ILA}(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z})$ *for all* $(\Delta \boldsymbol{l}_B, \Delta \boldsymbol{z})$, *and so,* $\Delta NEW^*_{DILA} > \Delta NEW^*_{ILA}$. *It is obvious from figure 10 that* $\Delta EW_{DILA}(0, \Delta \boldsymbol{z}) \geq \Delta EW_A(\Delta \boldsymbol{z})$ *for all* $\Delta \boldsymbol{z} \geq 0$, *and so,* $\Delta NEW_{DILA}(0, \Delta \boldsymbol{z}) \geq \Delta NEW_A(\Delta \boldsymbol{z})$. *From* $¶EW^{cp} / ¶\Delta \boldsymbol{l}_B > 0$ *and figure 11,* $¶\Delta EW_{DILA}(0, \Delta \boldsymbol{z}) / ¶\Delta \boldsymbol{l}_B > 0$. *Thus,* $¶\Delta NEW_{DILA}(0, \Delta \boldsymbol{z}) / ¶\Delta \boldsymbol{l}_B > 0$. *This implies that* $\Delta NEW^*_{DILA} > \Delta NEW^*_A$. *Q.E.D.*

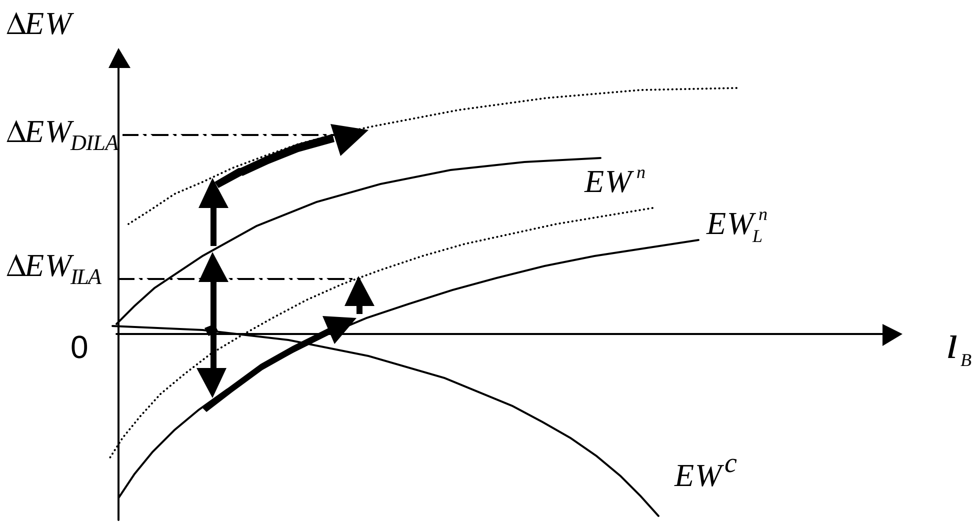

Figure 10

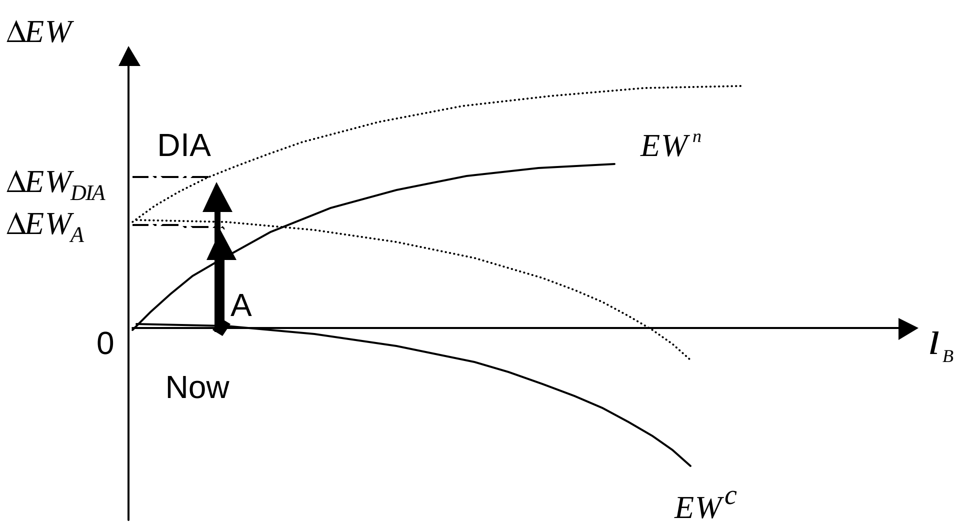

Figure 11