Why are Housing Prices so Volatile? Income Shocks in a Stochastic Heterogeneous-Agents Model*

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First Version: April 1999
Current Version: November 1999

Abstract

Building on a stochastic life-cycle model with credit constraints and heterogeneous agents and dwellings, this paper clarifies the contribution of income fluctuations to housing price volatility. First, housing prices are shown to depend on the current income of young households. Empirical evidence from the UK and the US shows that this income variable is more volatile than aggregate income. Data also suggest that the income of young households affects housing prices independently of aggregate income. Second, credit market imperfections and the implied dependence of demand on recent capital gains amplify price fluctuations. This transmission mechanism is such that a mere slowdown of income growth may trigger a housing price downturn.

*We thank V.V. Chari, Nobu Kiyotaki, Erzo Luttmer, John Moore, Victor Rios-Rull and seminar participants at Duke University, the London School of Economics, Université de Toulouse, MIT Sloan School, the Federal Reserve Bank of Minneapolis, North Carolina State University, University of Pennsylvania for helpful discussions and comments. A preliminary version of this paper was presented at the 1999 International Conference of the American Real Estate and Urban Economics Association (AREUEA). This paper was written in part while the first author visited the Wharton School of the University of Pennsylvania, and the second author visited the Financial Markets Group at the London School of Economics and the Center for Economic Studies at the University of Munich. We are grateful for the hospitality and support of these institutions.

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This paper investigates the transmission of income shocks to housing prices in a world where heterogeneous households face credit constraints in their attempt to climb the property ladder. The first contribution of the paper is to point out the determinant role of the income of young households in short run housing price fluctuations. The second contribution is a characterization of how the credit market amplifies income shocks in a stochastic dynamic framework.

Given that all households have a strong preference for living in a dwelling of their own, the poorest households have a strong demand for the cheapest type of dwellings, starter homes. Through the down-payment constraint, this consumption demand yields a direct relationship between the income of poor households and the price of starter homes. Wealthier households, on the other hand, optimize over types of dwellings on the basis of utility and user cost comparisons. The resulting arbitrage implies that the price of every dwelling can be decomposed into two components: the price of starter homes plus the market value of the utility premium the dwelling provides relative to starter homes. As a consequence, fluctuations in the price of starter homes affect the price of all dwellings. Since the price of starter homes depends on the income of poor credit constrained households, which are primarily the younger ones in our life-cycle economy, the housing price index fluctuates with the income of young households.

The down-payment requirement, combined with the heterogeneity of dwellings, implies that past housing prices and household earnings affect current prices. For households climbing the property ladder, capital gains on their current dwelling speed up the accumulation of net worth toward the down-payment on a more expensive dwelling. We highlight the strength of the capital gains effect with a two-state Markov process for households' earnings. As the prices and earnings of the past period affect the current market, this two-state income process implies a four-state Markov chain for prices. Although households' accumulated earnings are higher after successive periods of high earnings, the capital gains realized when the economy moves from the low to the high earnings state more than compensate for the lower accumulated earnings. As a result, demand and prices are higher following a shift from low to high earnings than after successive periods of high earnings. In the same way, when income shifts from high to low, prices fall to a lower level than if income had been low for two successive periods. Thus, endogenous capital gain fluctuations amplify earnings shocks to generate price overshooting.

To analyze the housing price behavior in a growing environment, we consider an earnings process that increases for a random number of periods, i.e., a boom of uncertain length. Increasing earnings, combined with the expectation of further increases, imply a continuing rise in housing prices. The housing boom is amplified by the continuing capital gains, so much so that housing prices drop when earnings stop growing. The drop is larger than the one due to the overshooting under the Markov process: the realization that the economy has stopped growing adds to the impact of capital gains drying up.[1]

The traditional approach to housing prices views dwellings as a sum of units of housing services. The optimizing behaviour of a representative investor ensures that the equilibrium price of a unit of housing services is forward looking and depends solely on the user cost of capital, rent, and some form of supply cost, as for any other asset. The failings of this approach to account for observed volatility without some departure from rational expectations has spurred the search for alternative theoretical approaches. Focusing on the search and matching feature of the housing market, Wheaton (1990) presents a model where housing prices are determined by the expected time it takes a dissatisfied owner to find a suitable property to move into. In such a world, small fluctuations in demand or supply generate large fluctuations in price through their impact on the equilibrium vacancy rate. Closer to this paper, Stein (1995) develops a model of repeat buyers where a down-payment requirement amplifies the effect of a shift in demand on price. The possibility of multiple equilibria is key to the volatility of prices in this model. The main difference with the present paper is that Stein's model is static. The overshooting result we obtain comes from the analysis of the dynamics of our model in a stochastic environment.

As pointed out in this last paper, there is ample evidence that down-payment constraints affect household purchasing decisions.[2] Survey evidence also indicates that housing equity is one of the two major sources of funds for repeat buyers, the other being own savings. In the years 1984-96, proceeds from the sale of the previous property represented between 30.7 and 57.1 percent of repeat buyers' down-payments in major US metropolitan areas; the larger fraction was observed during market booms.[3]

Graphing both housing prices and aggregate income shows that housing prices are much more volatile (Figure 1).[4] Furthermore, the two series display different trends.

[1]In the context of the stock market, Zeira (1999) obtains boom-bust dynamics similar to ours when market fundamentals change favorably for an unknown period of time. The mechanism at play in his model relies on agent's learning about the underlying changes as the boom goes on.

[2]See for example Linneman and Wachter (1989), Engelhardt (1994, 1996), Haurin, Hendershott and Wachter (1997), Engelhardt and Mayer (1998) and the references in Ortalo-Magné and Rady (1998) .

[3]Source: Annual National Survey of Recent Home Buyers in Major Metropolitan Areas, *The Guarantor*, Chicago Title and Trust Company, 1984-96.

[4]The aggregate income variable here is per capita income, as is common in the literature; e.g. Muellbauer and Murphy (1997), Poterba (1991), Lamont and Stein (1999).

Our model shows, however, that aggregate income is not the only relevant income series. It suggests that the income of young households, who are potential first-time buyers, plays a critical role in housing price fluctuations, through a channel different from the typical income elasticity of housing demand. We will present empirical evidence in support of this prediction in Section 5 below. We will see there that the income of young households displays greater volatility than aggregate income. Moreover, its trend helps reconcile the difference in trends between housing prices and aggregate income.

The overshooting prediction of our model is in agreement with the empirical findings of Lamont and Stein (1999). They report a similar overshooting pattern of housing prices in response to a permanent income shock for cities where a significant fraction of homeowners have high loan-to-value ratios.

The paper is organized as follows. Section 1 introduces the model. Section 2 solves for equilibrium, providing analytical expressions that characterize the role of the income of young households in the formation of housing prices. Section 3 studies a two-state Markov model for household incomes. Section 4 analyses income booms of uncertain length. Section 5 presents some empirical evidence in in support of our findings and offers some concluding remarks. An appendix contains all proofs.

1 The Model

The model must capture the interaction on the housing market of young households eager to climb the property ladder and wealthier ones who trade properties according to preferences and comparisons of user costs. For people to climb the property ladder, they must live at least three periods and be able to choose among at least two types of dwellings. Many options are available for the mathematical representation of agents trading without restrictions imposed by their wealth. Here, we add a fourth period to the life of all households, a period where their wealth is sufficient to afford what they prefer, given market prices. For income shocks to affect the pace at which some agents climb the property ladder, agents must differ in terms of wealth. Similarly, if not all rich agents are to hold the same dwelling, they must have heterogeneous preferences. The challenge is to design a stochastic model that incorporates this double heterogeneity of wealth and preferences and still allows a tractable determination of equilibrium prices from the distribution of dwellings and debt.

We consider a life-cycle economy with a numeraire consumption good and two types

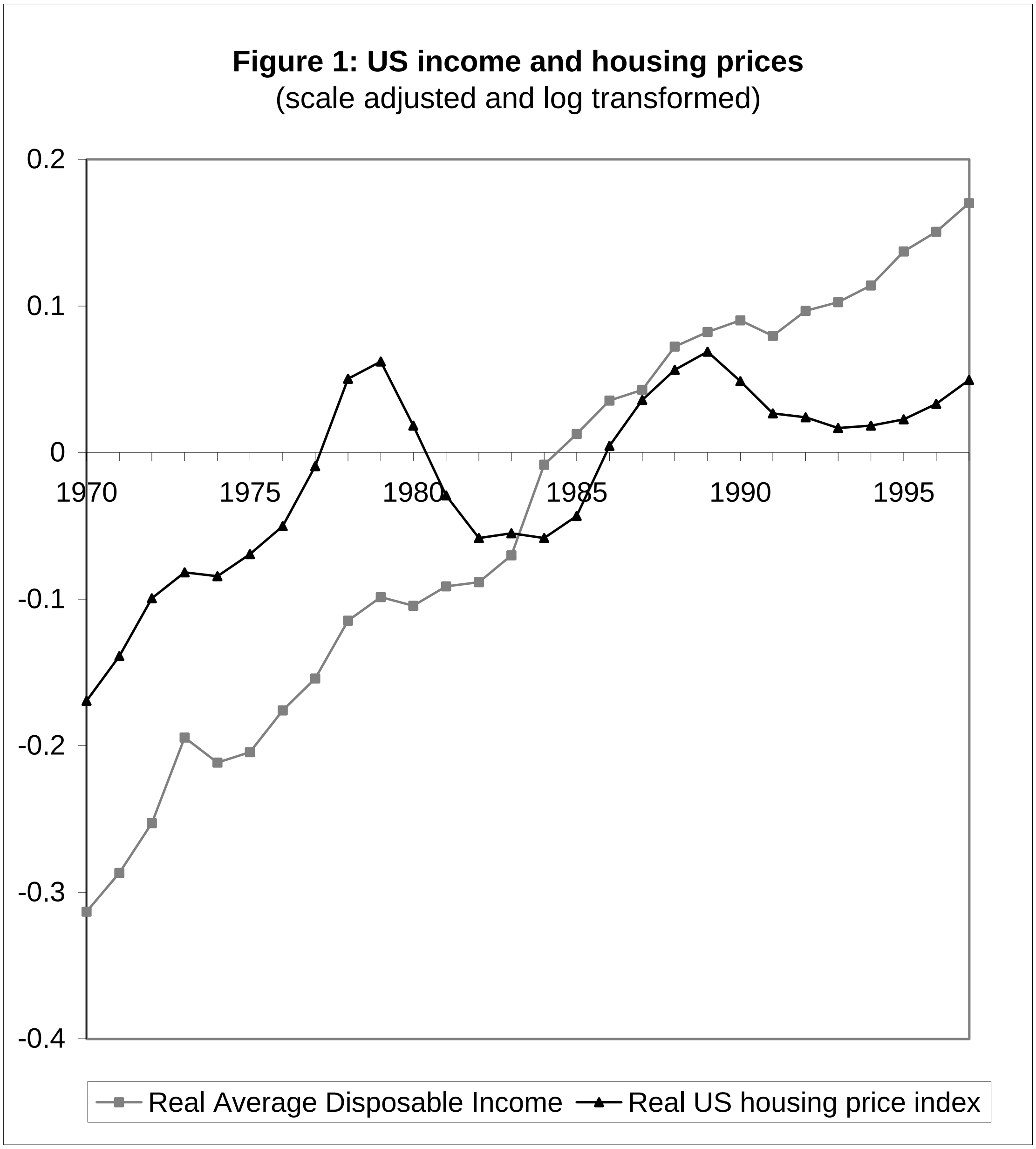


Figure 1: US income and housing prices
(scale adjusted and log transformed)
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
1970
1975
1980
1985
1990
1995
Real Average Disposable Income
Real US housing price index

of dwellings: starter homes ("flats" hereafter) and larger dwellings ("houses" hereafter). Both types of dwelling are available in fixed quantities, S^F and S^H, respectively.[5] In addition to living in a flat or house, agents may remain with a parent at no cost to the parent. This is where all agents are born and spend the first period of their life. Transactions take place at the end of each period but before consumption of the numeraire good. Housing is consumed at the beginning of the period.

Lenders do not allow agents to hold a debt higher than a fixed proportion, γ, of the value of the dwelling purchased, the only collateralizable asset. So an agent's net savings s_t are required to satisfy $s_t \geq -\gamma q_t^h$, where q_t^h denotes the time t price of a dwelling of type $h \in \{F, H\}$. Thus, the minimal required down-payment at the time of the purchase is $(1-\gamma)\, q_t^h$.

Agents live for four periods. In each period, a continuum of measure one is born with no assets. Each agent is identified by an index $i \in [0,1]$ determining her endowment stream of the numeraire good at ages 1 to 3. The endowment at date t for constrained agent i of age $j = 1, 2, 3$ is of the form $(1 + \alpha_j\, i)\, w_t(j) > 0$ with a random variable $w_t(j)$ and a constant $\alpha_j > 0$ that provides a measure of income dispersion: the richest agent of age j has $1 + \alpha_j$ times the income of the poorest agent in this cohort. At age 1 and age 2, individual wealth will affect investment. At age 3, we want preferences and user costs to determine the choice of dwelling. This is achieved by providing every age 3 agent with a large endowment, i.e., by specifying a sufficiently large $w_t(3)$. The wealth with which age 3 agents come into the period is then irrelevant for their housing market behaviour. Differences in housing purchases at age 3 will instead arise from the heterogeneity of preferences.

These preferences are described by a time-separable utility function over bundles of the numeraire good and the type of housing, $h \in \{P, F, H\}$, where P, F, and H stand for parents, flat and house, respectively. The instantaneous utility is assumed additively separable in the numeraire and housing, and linear in the numeraire. All agents strictly prefer a house to a flat to living with a parent. We assume that an agent's utility premia for a house over a flat, and for a flat over staying with a parent, increase in i.[6]

[5] This assumption will not be critical to our result. Of course, the more responsive the supply to price changes, the smaller the response of prices. However, the effect we are analyzing will work as long as supply is not perfectly elastic, which will hold given an upward sloping supply curve for land.

[6] This assumption will enable us later to discuss how the predictions of our model would differ if there were no credit constraint. It is not required for our analysis of the model with credit constraint. What we need there is that agents' purchases at age 1 and 2 are restricted by their wealth, so that no agent consumes less housing than she can afford. We obtain this by making houses sufficiently

The heterogeneity of preferences across agents is most relevant for the age 3 purchase, which is enjoyed at age 4. We assume that at age 3, agents draw a new name, $m \in [0, 1]$, which determines how strongly they prefer a house to a flat one period later. (A flat is still strongly preferred to the option of not occupying a dwelling of one's own.) We model the heterogeneity of preferences by specifying agent m's utility premium for a house over a flat in the simple form $\underline{u}^H + \eta\, m$ with $\eta > 0$. The parameter η thus measures the dispersion of agents' housing preferences. Given that all agents have sufficient wealth to afford the dwelling of their choice at age 3, they all invest in either a flat or a house.

This construction means that an agent ends up with two names, i and m, one describing the income stream she will receive, the other her tastes at age 4. We will not need to know precisely how these two dimensions of heterogeneity are related because only one dimension will matter at any given time in the life of the individual.

Individuals discount utility from future consumption by a factor β exceeding $1/(1+r)$ where r is the exogenously given interest rate. This assumption, combined with linear utility and a non-negativity constraint, yields a convenient optimal plan for consumption of the numeraire good: agents postpone all such consumption until the end of their lives.

Given an initial distribution of dwellings and net savings across agents, an equilibrium in this economy is fully determined by stochastic processes of flat prices q_t^F, house prices q_t^H, and processes of dwellings $h_t(\cdot, j)$ $(j = 2, 3, 4)$ for all agents[7] such that at all t, these allocations solve each agent's constrained utility maximization problem, and the flat and house markets clear.

For tractability, we assume preference parameters and income profiles such that, in equilibrium, all agents move out of their parents' home as soon as they can afford to do so. Moreover, the young agents' utility premium for a house is such that they buy a house as soon as they can afford the required down-payment. With such parameters, the relationship between an agent's name i and the type of dwelling purchased is monotonic within each cohort. As to purchases at age 3, the high incomes received in this period and our specification of preferences yield a monotonic relationship between an agent's name m and the type pf dwelling.

attractive relative to flats, and flats sufficiently attractive relative to the option of not owning one's accommodation.

[7]The dot in $h_t(\cdot, j)$ stands for the individual's characteristics. We will use either i or m, whichever is relevant.

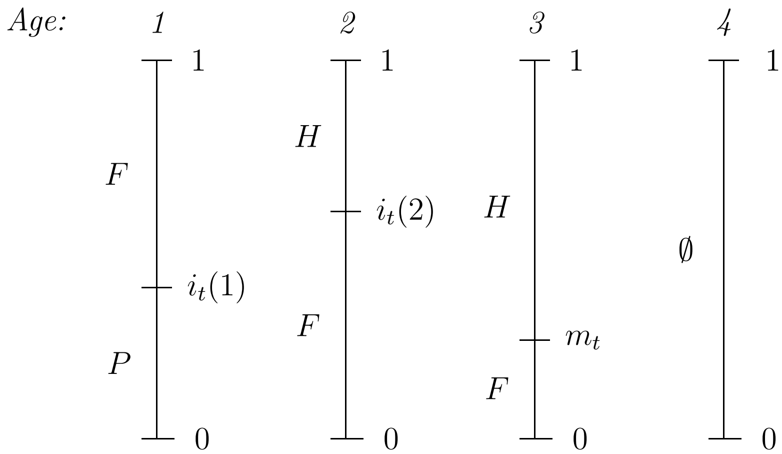


Figure 2: End of period housing choices by agent age and name

Note that our assumptions eliminate any trade-off between the numeraire and housing consumption for young agents. While this feature of our model is not attractive in itself, we accept it as a necessary evil to keep the model analytically tractable, in particular with respect to the equilibrium law of motion of the distribution of dwellings and savings. In fact, our assumptions imply that the relevant statistics of this distribution of agents are few and simple to characterize: only the names of the agents at the margins between the different housing option are relevant. This overcomes one of the main difficulties in modeling dynamic economies with hetereogeneous agents.

More specifically, the remainder of the paper focuses on a configuration of our model such that the pattern of housing choices in equilibrium is as in Figure 2. Our results are robust to a wider class of configurations as long as the marginal young house buyer, $i_t(2)$, is a flat owner. This is necessary if capital gains on flats are to influence the demand for houses.[8]

Figure 2 shows the results of the end-of-period trade in dwellings for each group of agents, ordered by age, i.e., the distribution of agents across dwellings at the end of the period. Each vertical line represents a cohort with the agents ordered by name. In steady state, the same figure depicts the sequence of investments each agent makes over her lifetime. For example, Figure 2 shows that agent $i = 0$ cannot afford a flat after one period of life, but moves into one after two periods. Agent $i = 1$ can afford a flat after one period and a house after two periods. For old agents, the deciding factor is the comparison between the utility premium and the user cost premium of a house

[8]Once the equilibrium prices for this configuration are derived, it is straightforward to state explicitly what conditions the primitives of the model must satisfy in order to produce this configuration.

relative to a flat. In Figure 2, agent $m = 0$ prefers a flat at the equilibrium prices, whereas agent $m = 1$ prefers a house. At the end of their lives (age 4), all agents sell their dwellings and consume their wealth.

For our analysis, we are interested in characterizing the dynamic process that determines how many agents are in houses and how many are in flats. To this end, we must identify the agents at the margin between parents and flats, and between flats and houses. The name of the poorest age 1 agent who buys a flat at the end of the current period and moves into it at the beginning of the next, is $i_t(1)$. Similarly, $i_t(2)$ is the name of the poorest age 2 agent who buys a house at the end of the current period. Finally, m_t is the name of the age 3 agent who is just indifferent between acquiring a flat and acquiring a house, given the current prices of dwellings, the expected prices next period and her utility premium for houses next period.

2 Price Fundamentals

Solving for equilibrium prices first requires characterising the relevant cutoff indices. By definition, agent $i_t(1)$ has an endowment at age 1 just sufficient to pay the downpayment on a flat; i.e.,

$$(1 + \alpha_1\, i_t(1))\, w_t(1) = (1 - \gamma) q_t^F. \tag{1}$$

In other words, the equilibrium price of flats is a multiple of the current income earned by the marginal constrained agent of age 1.

Having bought a flat at $t - 1$ and earned income for two periods, agent $i_t(2)$ has just enough net worth at time t to afford the downpayment on a house after selling her flat:

$$(1 + r) \left[(1 + \alpha_1\, i_t(2))\, w_{t-1}(1) - q_{t-1}^F\right] + (1 + \alpha_2\, i_t(2))\, w_t(2) + q_t^F = (1 - \gamma) q_t^H. \tag{2}$$

Writing $W_t(2) = (1 + r)\, w_{t-1}(1) + w_t(2)$ for the wealth (gross of any property transactions) of agent $i = 0$, in the age 2 cohort, and $G_t(2) = (1 + r)\, \alpha_1 w_{t-1}(1) + \alpha_2 w_t(2)$ for the corresponding wealth gradient, we obtain a more convenient equation for $i_t(2)$:

$$W_t(2) + i_t(2)\, G_t(2) - (1 + r)\, q_{t-1}^F + q_t^F = (1 - \gamma) q_t^H. \tag{3}$$

While the cutoff indices $i_t(1)$ and $i_t(2)$ for young constrained agents are backward looking, i.e., depend on previous and current incomes and prices, the cutoff index m_t for old unconstrained agents is forward looking. This index depends on preferences and anticipated user costs. It is the name of the age 3 individual at time t for whom the utility premium of living in a house at time $t+1$ is equal to the difference between the house and flat user costs, expressed in time $t+1$ terms. So the equation for m_t is

$$\underline{u}^H + \eta\, m_t = \left[(1+r)\, q_t^H - \mathrm{E}_t q_{t+1}^H\right] - \left[(1+r)\, q_t^F - \mathrm{E}_t q_{t+1}^F\right] \tag{4}$$

with E_t denoting the conditional expectation operator given the information available at time t.

Solving for $i_t(2)$ and m_t and inserting them in the market clearing condition for houses, $(1 - i_t(2)) + (1 - m_t) = S^H$, we obtain the following equation for the evolution of the house price in equilibrium:

$$\begin{aligned}&\left(\frac{1-\gamma}{G_t(2)} + \frac{1+r}{\eta}\right) q_t^H - \frac{1}{\eta}\, \mathrm{E}_t q_{t+1}^H \\ &= \; K + \frac{W_t(2)}{G_t(2)} + \frac{q_t^F - (1+r) q_{t-1}^F}{G_t(2)} - \frac{\mathrm{E}_t q_{t+1}^F - (1+r) q_t^F}{\eta}\end{aligned} \tag{5}$$

with $K = 2 - S^H + \underline{u}^H/\eta$.

Market clearing for both houses and flats requires that the number of agents living with a parent, $1 + i_t(1)$, be equal to the total number of agents minus the number of dwellings, $4 - S^F - S^H$, so the marginal constrained agent of age 1 is

$$i_t(1) = 3 - S^F - S^H. \tag{6}$$

Inserting this into equation (1), we find the equilibrium price of flats:

$$q_t^F = \frac{1 + \alpha_1 \left(3 - S^F - S^H\right)}{1-\gamma}\, w_t(1). \tag{7}$$

Equations (7) and (4) provide a simple intuition for the *fundamental determinants* of housing prices in this framework. The first states that the equilibrium price of flats depends on both the average income of age one agents, as well as the distribution of income within this cohort, as represented by $w_t(1)$ and α_1, respectively. Since the endowment of first-time buyers is independent of housing price changes, there cannot be any bubble here. Hence, $\lim_{s\to\infty}(1+r)^{-s}\mathrm{E}_t q_{t+s}^h = 0$ for $h = F, H$ and all t. Equation (4) then yields the following expression for the price of houses:

$$q_t^H = q_t^F + \sum_{s=0}^{\infty}(1+r)^{-s-1}\mathrm{E}_t[\underline{u}^H + \eta\, m_{t+s}(1)]. \tag{8}$$

The price of houses is the price of flats plus the present discounted value of the marginal unconstrained agents' future utility premia.

For constant endowment parameters $w_t(j) = w(j)$ $(j = 1, 2, 3)$, equations (5) and (7) imply unique steady state flat and house prices. Given these prices and equations (3)–(7), it is straightforward to formulate conditions on the model parameters that guarantee a steady state configuration as in Figure 2.

3 A Markov Model for Incomes

Suppose that for $j = 1, 2$, the income variable $w_t(j)$ for constrained agents of age j follows a Markov chain with two states, labelled $+$ and $-$, associated income levels $w_+(j) > w_-(j) > 0$, and nondegenerate continuation probabilities ρ_+ and ρ_-. Equation (7) implies a two-state Markov chain for the equilibrium flat price, with levels $q^F_+ > q^F_-$.

By equation (5), the house price q^H_t depends on time $t-1$ income variables through the flat price q^F_{t-1}, the basic wealth level $W_t(2)$ and the wealth gradient $G_t(2)$. To make the house price process Markovian, therefore, we extend the state space to four states, $++$ (two high incomes in a row), $+-$ (high income followed by low income), $-+$ (low income followed by high income), and $--$ (two low incomes in a row). We define age 2 wealth levels and gradients for each state in the obvious way; for example, $W_{++}(2) = (1+r)\, w_+(1) + w_+(2)$ and $G_{++}(2) = (1+r)\, \alpha_1 w_+(1) + \alpha_2 w_+(2)$. We can now solve for the unique stochastic equilibrium in which the house price process assumes only four different values, one for each state.[9] The vector of equilibrium house prices $(q^H_{++}, q^H_{+-}, q^H_{-+}, q^H_{--})$ is the unique solution of the following linear $4{\times}4$ system, obtained from equation (5):

$$\begin{aligned}
&\left(\frac{1-\gamma}{G_{++}(2)} + \frac{1+r-\rho_+}{\eta}\right) q^H_{++} - \frac{1-\rho_+}{\eta}\, q^H_{+-} \\
&= \quad K + \frac{W_{++}(2)}{G_{++}(2)} - \frac{r q^F_+}{G_{++}(2)} + \frac{(1+r-\rho_+)q^F_+ - (1-\rho_+)q^F_-}{\eta}\,,
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\left(\frac{1-\gamma}{G_{+-}(2)} + \frac{1+r}{\eta}\right) q^H_{+-} - \frac{\rho_-}{\eta}\, q^H_{--} - \frac{1-\rho_-}{\eta}\, q^H_{-+} \\
&= \quad K + \frac{W_{+-}(2)}{G_{+-}(2)} + \frac{q^F_- - (1+r)q^F_+}{G_{+-}(2)} + \frac{(1+r-\rho_-)q^F_- - (1-\rho_-)q^F_+}{\eta}\,,
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\left(\frac{1-\gamma}{G_{-+}(2)} + \frac{1+r}{\eta}\right) q^H_{-+} - \frac{\rho_+}{\eta}\, q^H_{++} - \frac{1-\rho_+}{\eta}\, q^H_{+-} \\
&= \quad K + \frac{W_{-+}(2)}{G_{-+}(2)} + \frac{q^F_+ - (1+r)q^F_-}{G_{-+}(2)} + \frac{(1+r-\rho_+)q^F_+ - (1-\rho_+)q^F_-}{\eta}\,,
\end{aligned} \tag{11}$$

[9] In other words, if two consecutive house prices are in this set of four values, then the house price will always stay there. Of course, other initial conditions imply equilibrium paths outside this set.

$$
\begin{aligned}
&\left(\frac{1-\gamma}{G_{--}(2)}+\frac{1+r-\rho_-}{\eta}\right) q^H_{--} - \frac{1-\rho_-}{\eta}\, q^H_{-+} \\
&= \quad K + \frac{W_{--}(2)}{G_{--}(2)} - \frac{r q^F_-}{G_{--}(2)} + \frac{(1+r-\rho_-)q^F_- - (1-\rho_-)q^F_+}{\eta}\ . \qquad (12)
\end{aligned}
$$

Previous incomes and realised capital gains on flats are crucial when we compare q^H_{++} with q^H_{-+}, and q^H_{--} with q^H_{+-}. Income expectations do not play any role for this comparison since they are identical in states $++$ and $-+$, and in states $--$ and $+-$. To gain some qualitative insight into this aspect of the equilibrium, we start from a steady state as in Figure 2 with endowment parameters $w(j)$ $(j = 1, 2, 3)$, and parameterise the stochastic income process by setting $w_\pm(j) = (1 \pm \sigma)\, w(j)$ with $\sigma > 0$. That is, σ measures the percentage income deviation from the benchmark steady state level. Let q^F denote the benchmark flat price, and $i(2)$ the steady state index of the marginal constrained agent who moves from a flat into a house at the end of the second period of life.

Proposition 3.1 *If*

$$
q^F > (1 + \alpha_1\, i(2))\, w(1), \qquad (13)
$$

then the equilibrium house price process satisfies

$$
q^H_{+-} < q^H_{--} \text{ and } q^H_{++} < q^H_{-+} \qquad (14)
$$

for sufficiently small σ.

Condition (13) is not in terms of model parameters directly, but has the advantage of relaying the right intuition. It states that the marginal constrained house buyer (in the benchmark steady state, and hence also for small σ) borrowed when buying a flat at the end of the previous period. The key determinant of whether capital gains imply overshooting is whether an increase in flat price and income by the same proportion yields a flat price increase greater than that of income in *levels*. In state $-+$, for example, flat owners enjoy capital gains as the income level rises. On the other hand, the capitalized value of age 2 agents' earnings is lower in state $-+$ than in state $++$; in the former they have had one period of low earnings and one period of high earnings, in the latter two periods of high earnings. If the price of flats is larger than the first period earnings, then the capital gains on the flat in state $-+$ more than compensate for the endowment disadvantage. Therefore, the total wealth of age 2 agents and hence their demand for houses is greater in state $-+$, implying a higher price in equilibrium.

The proof of the proposition shows that condition (13) is tight in the sense that the converse inequality $q^F < (1 + \alpha_1\, i(2))\, w(1)$ implies $q^H_{+-} > q^H_{--}$ and $q^H_{++} > q^H_{-+}$ for small σ. A violation of condition (13) means that the equilibrium price of flats is extremely small relative to the price of houses: the same agent who can just afford the down-payment on a house was able to pay cash for a flat in the previous period. Hence, the case where (13) does not hold is of no interest.

The condition that σ be sufficiently small is imposed for two reasons, one substantial, the other technical. First, σ must be small enough so that the equilibrium configuration remains that of Figure 2. Second, the proof uses the linearisation of the system (9)–(12) with respect to σ; so σ must be small enough that the linear approximation gives the correct ranking of prices. Numerical simulations suggest that the second restriction on σ is not binding: Proposition 3.1 appears to hold for all σ that are compatible with the configuration of Figure 2. Moreover, deviations from the configuration in Figure 2 will not change the nature of this result as long as the age 2 marginal house buyer remains an indebted flat owner.

The effect of capital gains is illustrated in Table 1 and Figure 3, where we report equilibrium house prices for a parameter constellation that conforms to Figure 2 and satisfies condition (13).

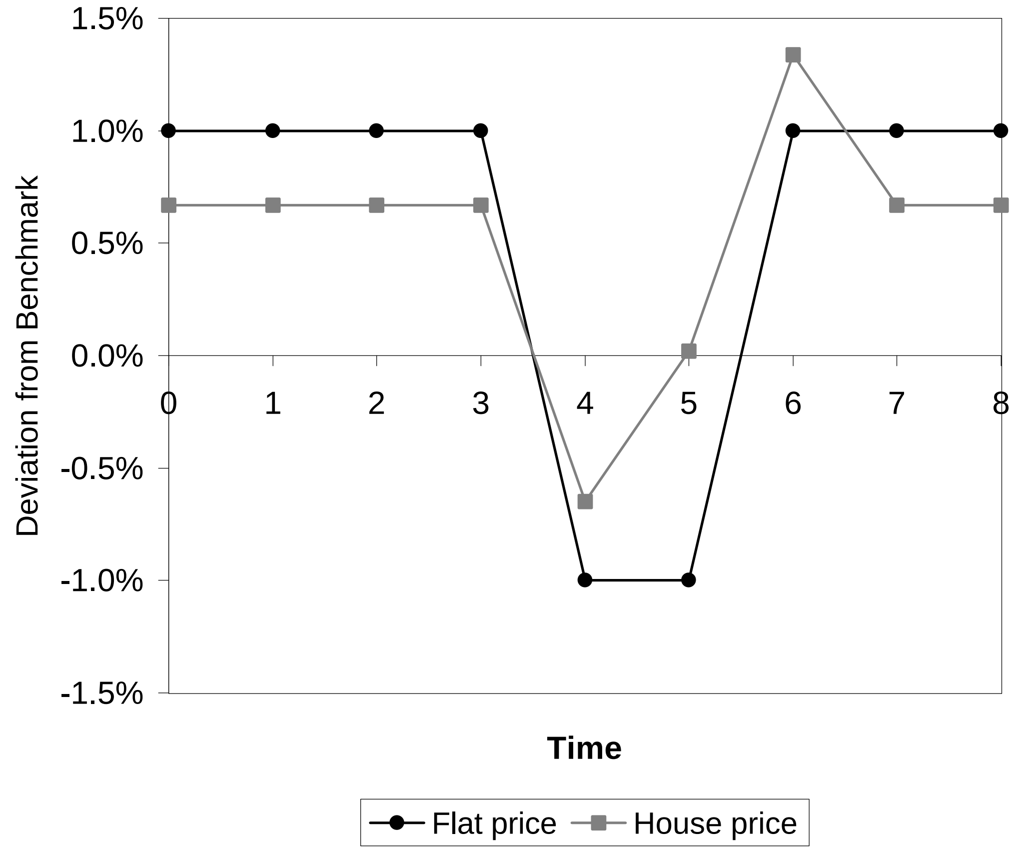
Table 1: Equilibrium house prices in the two-state Markov model

State	House price
$-+$	86.2137
$++$	85.6434
$--$	85.0914
$+-$	84.5232

The underlying model parameters are $r = 0.05, \gamma = 0.8, w_+(1) = w_+(2) = 8.08, w_-(1) = w_-(2) = 7.92, \alpha_1 = \alpha_2 = 1, S^F = 1.7, S^H = 1.2, M = 1, \eta = 16, \underline{u}^H = -8, \rho_+ = 0.8, \rho_- = 0.2$.

In an attempt to capture the empirical observation that phases of income growth tend to be longer than recessions, we have chosen the continuation probabilities such that the high income state is more persistent than the low income state ($\rho_+ = 0.8$ and $\rho_- = 0.2$). A transition from state $++$ to state $+-$ is therefore quite likely to be followed by a transition to $-+$ and back to $++$. In this event, the house price first falls by 1.31 percent while the income of the youngest constrained agents drops by 1.98 percent. Then, as this income returns to its original level, the house price rises by 2.00 percent before falling back by 0.66 percent to state $++$.

Figure 3: Two State Income Process



It is instructive to compare this equilibrium to that of the same model without the down-payment requirement. The equilibrium allocation of properties in this unconstrained world is entirely determined by the relative preferences the agents have for one type of accommodation relative to another. Within each cohort, the agents with low utility for a dwelling of their own would stay with parents. Those with high utility for houses would be willing to pay the price to occupy one, and the remaining agents would be in flats. At present, income does not affect the demand of unconstrained agents. This is because we have abstracted from the usual income elasticity of demand. A positive elasticity can be introduced without giving up the model's tractability by making the agents' instantaneous utility from housing consumption depend on their current income, with a higher income implying a higher utility premium for a house over a flat and for a flat over the option of no dwelling. Still, for reasonable income elasticity parameters, income shocks would only have relatively small effects on prices. With transition probabilities as in the example above ($\rho_+ = 1 - \rho_-$), prices would be completely insensitive to income shocks in the unconstrained model; prices would become somewhat sensitive to current incomes for transition probabilities $\rho_+ \neq 1 - \rho_-$, because now different current incomes imply different expectations about *future* incomes and prices.

4 An Income Growth Path of Uncertain Length

The analysis of the dynamic response of our model to shocks in growth rates is complicated by the fact that the model does not admit a balanced growth path. As the economy grows, the distribution of agents across dwellings shifts progressively. This shift is key to the model's response to shocks. The alternative we can study is an income process that grows deterministically up to some random time $\tau > 0$ and stays constant thereafter. This experiment combines the findings of the previous section with a more typical expectation effect.

Given a positive growth rate σ, we set

$$\begin{aligned} w_t(j) &= (1+\sigma)\, w_{t-1}(j) \quad \text{for all } t < \tau \text{ and } j = 1, 2, 3; \\ w_t(j) &= w_t(j) \quad \text{for all } t \geq \tau \text{ and } j = 1, 2, 3. \end{aligned}$$

Thus, $\tau = \min\{t > 0 : w_t(j) = w_{t-1}(j) \text{ for all } j\}$, the first time that income is seen to stagnate since the previous period. Note that households learn about the end of growth only after the fact: if income grows for the last time between dates $t-1$ and t, then agents will realize this at time $\tau = t + 1$.

Let $\rho = \text{Prob}\,(\tau > t+1 | \tau > t)$ be the probability that growth will continue for another period. That is, conditional on $w_t(j) > w_{t-1}(j)$, we will have $w_{t+1}(j) = (1+\sigma_t)\, w_{t-1}(j)$ with probability ρ, and $w_{t+1}(j) = w_t(j)$ with probability $1-\rho$.

The price of flats is directly tied to current endowments $w_t(1)$, so stays constant from time $\tau - 1$ on. The time t house price depends on time $t-1$ incomes through the wealth level $W_t(2)$ and wealth gradient $G_t(2)$. These wealth parameters increase from period $\tau - 1$ to τ, and are constant for $t \geq \tau$. Under a no-bubbles condition, the time τ house price in any rational expectations equilibrium equals the steady state price corresponding to the income levels at which the growth path stopped. By equation (5), this price, which we denote by $\bar{q}^H_\tau$, solves

$$\left(\frac{1-\gamma}{G_\tau(2)} + \frac{r}{\eta}\right) \bar{q}^H_\tau \;\; = \;\; K + \frac{W_\tau(2)}{G_\tau(2)} - \left(\frac{1}{G_\tau(2)} - \frac{1}{\eta}\right) r q^F_{\tau-1}. \tag{15}$$

In periods $t < \tau$, there is still the possibility of further growth. With probability ρ, income will increase, the price of flats will rise to $(1+\sigma)\, q^F_t$, and the house price will attain some level q^H_{t+1}.[10] With the complementary probability, income and the price of flats will stagnate, and the house price will be at the new steady state level $\bar{q}^H_{t+1}$. So equation (5) becomes

$$\begin{aligned} &\left(\frac{1-\gamma}{G_t(2)} + \frac{1+r}{\eta}\right) q^H_t - \frac{\rho}{\eta}\, q^H_{t+1} - \frac{1-\rho}{\eta}\, \bar{q}^H_{t+1} \\ &\quad = \;\; K + \frac{W_t(2)}{G_t(2)} + \frac{\sigma - r}{G_t(2)}\, q^F_{t-1} - \frac{\rho\sigma - r}{\eta}\, q^F_t . \end{aligned} \tag{16}$$

If the length of the growth path can exceed every finite bound with positive probability, households may become rich enough so that the configuration of Figure 2 eventually breaks down. To preserve tractability of the model, therefore, we impose a finite, deterministic time horizon on the growth path: $\tau \leq T+1$. We assume that the endowments after the longest possible growth path are such that the corresponding steady state prices give rise to the configuration of Figure 2. For small growth rates, this will then also be the configuration over some range of earlier times. So we can solve for equilibrium by first evaluating equation (15) for $\tau = T+1$ and then iterating equation (16) back in time.

Let $w(j)$ $(j = 1,2,3)$ denote the time T income parameters after the longest possible growth path, and q^F and q^H the corresponding steady state prices of flats and houses, respectively. Define

$$\bar{W}(2) \;\; = \;\; (1+r)\, w(1) + w(2), \tag{17}$$

[10] As a convention, we will write q^H_t for the house price when $t < \tau$, and $\bar{q}^H_\tau$ when $t \geq \tau$.

$$
\begin{aligned}
\bar{G}(2) &= (1+r)\,\alpha_1 w(1) + \alpha_2 w(2), && (18)\\
W(2) &= (1+r)\,w(1)/(1+\sigma) + w(2), && (19)\\
G(2) &= (1+r)\,\alpha_1 w(1)/(1+\sigma) + \alpha_2 w(2). && (20)
\end{aligned}
$$

Then the age 2 wealth parameters in equation (15) are

$$
\begin{aligned}
W_\tau(2) &= \bar{W}(2)/(1+\sigma)^{T+1-\tau}, && (21)\\
G_\tau(2) &= \bar{G}(2)/(1+\sigma)^{T+1-\tau}, && (22)
\end{aligned}
$$

so the steady state house price $\bar{q}^H_\tau$ at the end of growth is given by

$$
\begin{aligned}
&\left(\frac{(1-\gamma)(1+\sigma)^{T+1-\tau}}{\bar{G}(2)} + \frac{r}{\eta}\right) \bar{q}^H_\tau \\
&= K + \frac{\bar{W}(2)}{\bar{G}(2)} - \left(\frac{1}{\bar{G}(2)} - \frac{1}{(1+\sigma)^{T+1-\tau}\eta}\right) r q^F. \qquad (23)
\end{aligned}
$$

As expected, the terminal house price increases with the length of the growth path. In fact, the coefficient of $\bar{q}^H_\tau$ in (23) is decreasing in τ, while the right-hand side is increasing. So $\bar{q}^H_\tau$ increases with τ, the maximum steady state price being $\bar{q}_{T+1} = q^H$.

Turning to dates where growth has not stopped yet, we see that the age 2 wealth parameters in equation (16) are

$$
\begin{aligned}
W_t(2) &= W(2)/(1+\sigma)^{T-t}, && (24)\\
G_t(2) &= G(2)/(1+\sigma)^{T-t}. && (25)
\end{aligned}
$$

If income grows throughout time T, therefore, the house price q^H_T satisfies

$$
\left(\frac{1-\gamma}{G(2)} + \frac{1+r}{\eta}\right) q^H_T - \frac{1}{\eta}\,\bar{q}^H_{T+1} \;=\; K + \frac{W(2)}{G(2)} + \left(\frac{\sigma - r}{(1+\sigma)\,G(2)} + \frac{r}{\eta}\right) q^F. \qquad (26)
$$

At any other time $t < \tau$ we have the house price equation

$$
\begin{aligned}
&\left(\frac{(1-\gamma)(1+\sigma)^{T-t}}{G(2)} + \frac{1+r}{\eta}\right) q^H_t - \frac{\rho}{\eta}\, q^H_{t+1} - \frac{1-\rho}{\eta}\, \bar{q}^H_{t+1} \\
&= K + \frac{W(2)}{G(2)} + \left(\frac{\sigma - r}{(1+\sigma)\,G(2)} + \frac{r - \rho\sigma}{(1+\sigma)^{T-t}\eta}\right) q^F. \qquad (27)
\end{aligned}
$$

As in Section 3, we want to characterise the equilibrium qualitatively. Again, the house price process depends crucially on whether marginal constrained house buyers must borrow when buying a flat one period earlier. Let $i(2)$ be the steady state index of the marginal constrained agent who moves from a flat into a house at the end of the second period of life, given that the prices of flats and houses are at the steady state levels q^F and q^H. Thus, $i(2)$ is the marginal house buyer after the longest possible growth path.

Proposition 4.1 *If*

$$q^F > (1 + \alpha_1 \, i(2)) \, w(1), \tag{28}$$

then the equilibrium house price process satisfies

$$q^H_{\tau-1} > \bar{q}^H_\tau \tag{29}$$

for σ sufficiently small and τ sufficiently close to $T+1$.

The condition that σ be sufficiently small is imposed for exactly the same reasons as in Proposition 3.1. The difference $T+1-\tau$ is only required to be small enough that the equilibrium configuration at time τ remains that of Figure 2.

Condition (28) is of course just a restatement of (13), and plays the same role, guaranteeing that the marginal constrained house buyer was a borrower at the time he purchased a flat. For $\tau = T+1$, the above result has exactly the same explanation as Proposition 3.1: capital gains on flats are sufficiently amplified by leverage to push the house price q^H_T above the final steady state level $\bar{q}^H_{T+1}$. For $\tau < T+1$, this leverage effect is reinforced by an expectations effect: the realization that income has stopped growing means a further downward adjustment in house prices.

As an illustration, Table 2 shows equilibrium house prices for an income process that can grow for another ten periods at most ($T = 10$).[11] Figure 4 visualizes an example where income grows for four periods exactly, so that $\tau = 5$. ¿From its base level $q^H_0 = 79.0351$, the house price rises by 3.58 percent to $q^H_4 = 81.8677$ before dropping by 1.53 percent to the steady state $\bar{q}^H_5 = 80.6070$, which is 1.98 percent higher than q^H_0. Thus, when growth stops, almost half the preceeding increase in the house price is wiped out.

Intuition suggests that the expectations effect discussed above should be increasing with the persistence of growth as measured by ρ. This is formalised in the next proposition.

Proposition 4.2 *Under the conditions of Proposition 4.1, the size of the downward jump $q^H_{\tau-1} - \bar{q}^H_\tau$ for $\tau < T+1$ increases strictly in ρ.*

Finally, Table 1 shows that for sufficiently persistent growth, the house price can overshoot its terminal steady state more than one period ahead. In fact, we have $q^H_{\tau-2} > \bar{q}^H_\tau$ for all stopping times τ except those very close to the upper bound $T+1$.

[11] The parameter constellation used to calculate this table again conforms to Figure 2 and satisfies condition (28).

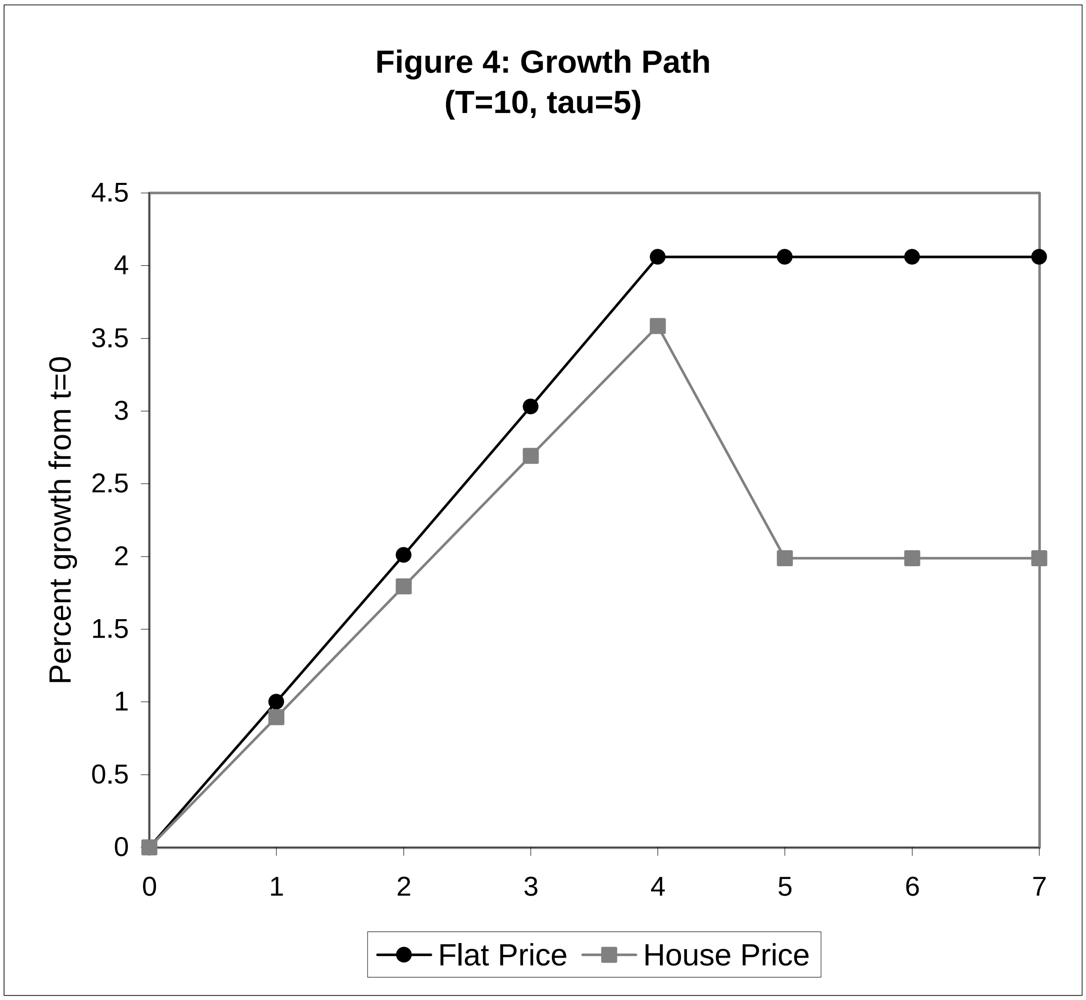


Figure 4: Growth Path
(T=10, tau=5)
Percent growth from t=0
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
Flat Price
House Price

Table 2: Equilibrium house prices for a growth path of uncertain length

t	q_t^H	$\bar{q}_t^H$
0	79.0351	
1	79.7430	77.7495
2	80.4528	78.4550
3	81.1622	79.1664
4	81.8677	79.8837
5	82.5635	80.6070
6	83.2411	81.3364
7	83.8868	82.0717
8	84.4797	82.8132
9	84.9869	83.5608
10	85.3579	84.3146
11		85.0746

The underlying model parameters are $r = 0.05, \gamma = 0.8, w(1) = w(2) = 8, \alpha_1 = \alpha_2 = 1, S^F = 1.7, S^H = 1.2, M = 1, \eta = 16, \underline{u}^H = -8, \sigma = 0.01, \rho = 0.8$.

5 Concluding Remarks

In an environment where current net worth affects the discrete housing choices of heterogeneous agents, we show that the pricing of dwellings is subject to arbitrage across types of dwelling but not with non-residential investments. Forward-looking investors ensure that the relative price of all dwellings reflects the different utilities and returns that each provides. However, the consumption demand for housing services by credit constrained households links the price of the cheapest type of dwelling to their earnings. Hence, the price of all dwellings depends on the current income of young first-time buyers, a prediction which seems in agreement with the empirical evidence, as we shall argue below. With stochastic earnings, the model yields price overshooting in response to income shocks, and a price fall in response to a slowdown in income growth.

Our model suggests that the income of young households, who are potential first-time buyers, plays a critical role in housing price fluctuations. We first want to point out that the income of young households is more volatile than per capita GDP in both the US and the UK. In the UK, the first-time buyers are the 20 to 29 year old. Their average income decreased by 11.9 percent during the 1989-93 bust when housing prices fell by 25 percent and GDP never more than 2.1 percent. During the preceding boom, when housing prices increased by 88 percent in 7 years, the income of the young also fluctuated more than GDP: it grew by 27.5 percent, whereas GDP grew by 21.9 percent.[12,13] The same holds true in the US, as Figure 5 below shows.

Second, the key question raised by the theoretical work in this paper is whether the income of young households affects housing price fluctuations independently of the per capita income variable usually incorporated in housing price regressions (e.g., Muellbauer and Murphy 1997, Poterba 1991, Lamont and Stein 1999). Here, we argue that beyond the effect of per capita income through the income elasticity of housing demand, changes in the income of young households affect their ability to pay for a down payment on a starter home. The resulting changes in the price of starter homes yield capital gains for their owners. This capital gain effect ripples through the market, affecting the demand and hence the price of other properties higher up the ladder.

To get a sense of whether income of young households matters independently of per capita income, we first graph housing prices, per capita disposable income and the median income of 25-34 year old households (Figure 5).[14] From the model, we derive that the relevant young income variable is the income of households who are potential first-time buyers. In the model, this is the income of a fixed percentile of the youngest cohort. The median income of 25-34 year is the only available percentile of the income distribution among potential first-time buyers. The graph suggests that housing prices embody the fluctuations in both per capita income and income of the young. In particular, the volatility of housing prices is closer to that of the income of young households. The trend of housing prices is somewhere in between the trends of the two income series. A simple linear regression using yearly data from 1975 to 1997 confirms the visual impression. Together, the variables explain 67 percent of housing price variation. Both are highly significant. This evidence is consistent with our theoretical findings.[15]

To be consistent with our theoretical approach, we should compare the contribution of young households' income to housing price fluctuations not with that of per capita income, but with that of average *household* income. This is shown in Figure 6.[16] The main impression remains: both the income of the young and the income of

[12] Source: Family Expenditure Survey and UK Economic Accounts, as compiled by Simons (1996).

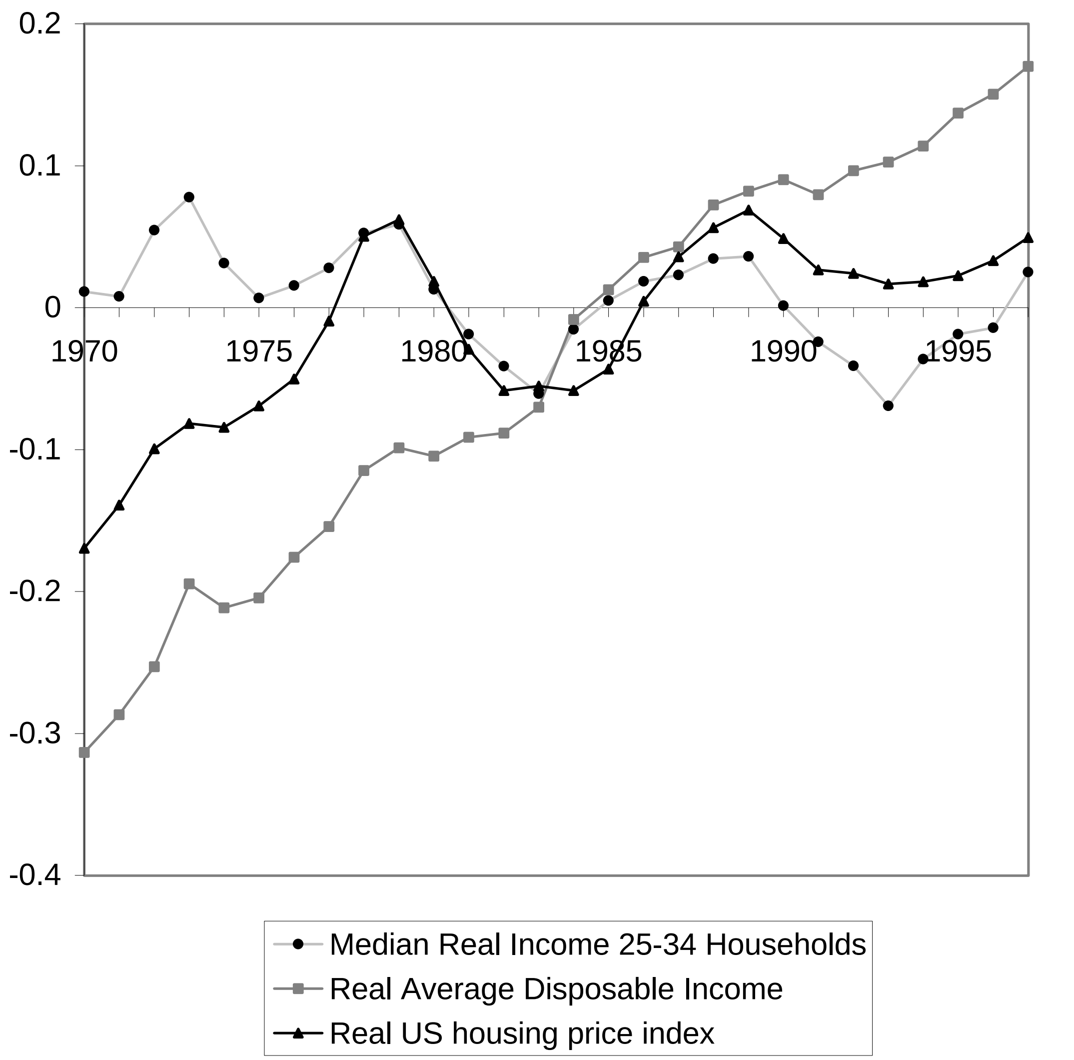
[13] Ortalo-Magné and Rady (1999) argue that the liberalisation of the UK mortgage market in the early Eighties also contributed significantly to the magnitude of this housing boom.

[14] Data sources: Freddie Mac national housing price index (1975-1997) extrapolated to 1970 using the Census Bureau median price of existing home index, real average disposable income from the Economic Report of the President, and median 25-34 year-old household income from the Census Bureau.

[15] The estimated coefficients for income per capita and the median income of 25-34 year old households are 0.28 (0.05)and 0.78 (0.17), respectively; standard errors are in parentheses.

[16] Data source for average income of all households: Census Bureau.

Figure 5: US income and housing prices
(scale adjusted and log transformed)



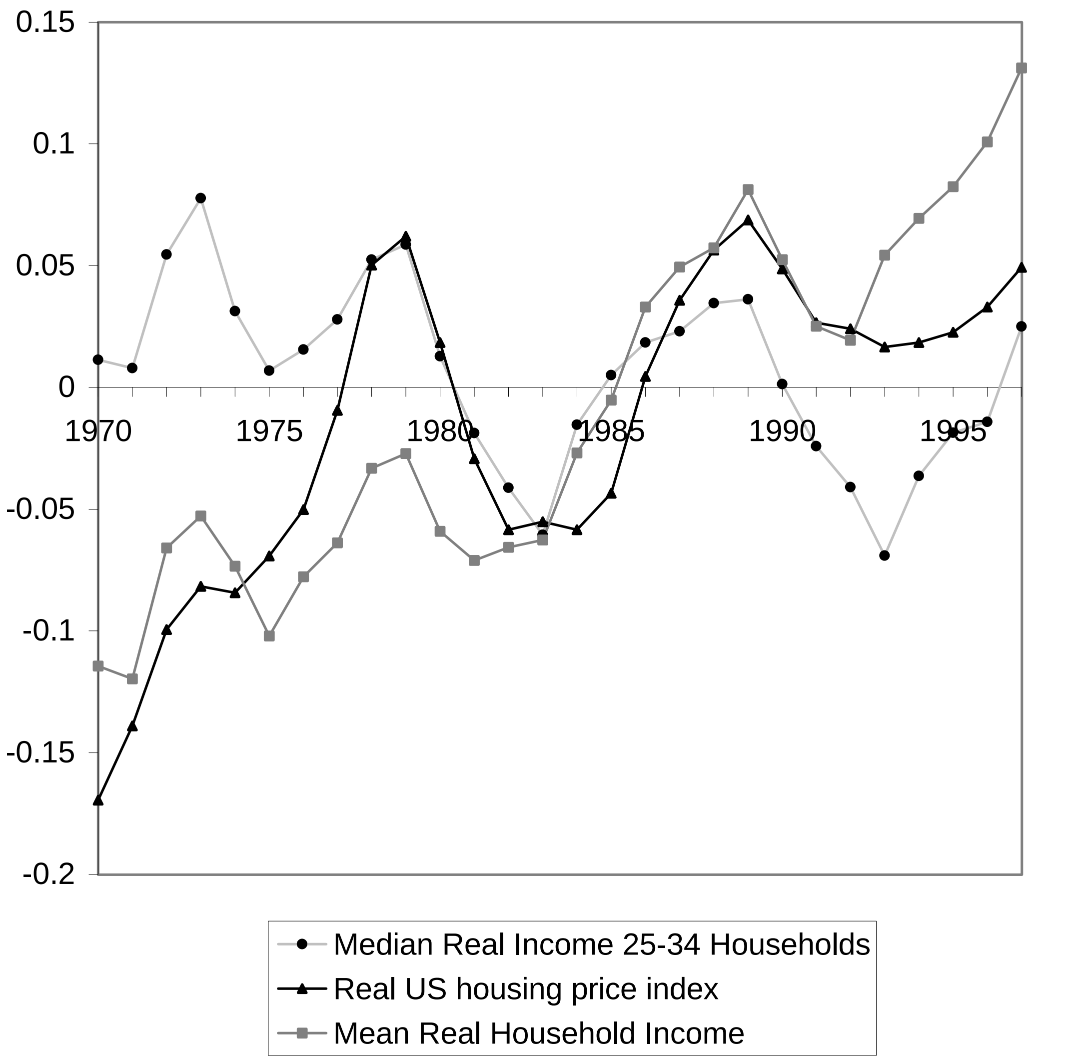
all households contribute independently to housing price fluctuations. Again, this is confirmed by a simple regression of log housing prices on both log income variables.[17]

These findings are robust to the introduction of a rental market for first-time buyer type properties as in Ortalo-Magné and Rady (1998, 1999). Allowing households to rent flats has a quantitative effect whose strength depends on the ease with which flats can be converted from rental to owner occupancy and back. The more rigidities there are in this conversion, the closer we approach the results presented here.

The model presented here highlights how demographics may affect housing prices. Both the size of each cohort and the distribution of wealth within each cohort matter. In our framework, any change in the wealth of the marginal buyers caused by changes in the composition or the size of the young cohorts could have a dramatic effect on housing prices. An increase in the first-time buyer population would not only raise the price of starter homes but also the price of larger properties due to the consequent capital gains. We intend to explore this further in future work.

[17]Using yearly data from 1975-1997, we estimate coefficients of 0.46 (0.09) and 0.55 (0.16) for mean household income and median income of 25-34 year old households, respectively, with the standard error in parentheses. The two income series account again for 67 percent of housing price variability.

Figure 6: US income and housing prices
(scale adjusted and log transformed)



Appendix

Proof of Proposition 3.1

Subtracting equation (9) from equation (11) yields

$$\begin{aligned}
&\left(\frac{1-\gamma}{G_{-+}(2)}+\frac{1+r}{\eta}\right) q^H_{-+} - \left(\frac{1-\gamma}{G_{++}(2)}+\frac{1+r}{\eta}\right) q^H_{++} \\
&= \frac{W_{-+}(2)}{G_{-+}(2)} + \frac{q^F_+ - (1+r)q^F_-}{G_{-+}(2)} - \frac{W_{++}(2)}{G_{++}(2)} + \frac{rq^F_+}{G_{++}(2)}. \qquad (30)
\end{aligned}$$

Let $W(2)$ and $G(2)$ denote the age 2 income level and gradient in the benchmark steady state, and q^F and q^H the benchmark prices. Clearly, $q^F_\pm = (1 \pm \sigma)q^F$. For each state $S \in \{++, +-, -+, --\}$, we define

$$\delta^H_S = \left.\frac{dq^H_S}{d\sigma}\right|_{\sigma=0}.$$

Differentiating (30) with respect to σ and evaluating the result at $\sigma = 0$ yields

$$\begin{aligned}
&\frac{G(2)^2}{2(1+r)}\left(\frac{1-\gamma}{G(2)}+\frac{1+r}{\eta}\right)\left(\delta^H_{-+}-\delta^H_{++}\right) \\
&= (\alpha_1 W(2) - G(2))w(1) - rq^F\alpha_1 w(1) + q^F G(2) - (1-\gamma)q^H\alpha_1 w(1). \qquad (31)
\end{aligned}$$

Using equation (2) to replace $(1-\gamma)q^H$, we can simplify the right-hand side of this equation to $G(2)\left[q^F - (1+\alpha_1 i(2))\, w(1)\right]$. If this is positive, then $\delta^H_{-+} > \delta^H_{++}$ and hence $q^H_{-+} > q^H_{++}$ for small σ.

Repeating the above steps for the difference between equations (12) and (10) shows that the condition $q^F > (1+\alpha_1 i(2))\, w(1)$ also implies $q^H_{--} > q^H_{+-}$ for small σ.

Proof of Propositions 4.1–4.2

We use the following notation:

$$\delta^H_t = \left.\frac{dq^H_t}{d\sigma}\right|_{\sigma=0}, \quad \bar{\delta}^H_\tau = \left.\frac{d\bar{q}^H_\tau}{d\sigma}\right|_{\sigma=0}.$$

Differentiating equation (23) with respect to σ and evaluating the result at $\sigma = 0$ yields

$$\left(\frac{1-\gamma}{\bar{G}(2)}+\frac{r}{\eta}\right)\bar{\delta}^H_\tau = -(T+1-\tau)\left(\frac{(1-\gamma)q^H}{\bar{G}(2)}+\frac{rq^F}{\eta}\right). \qquad (32)$$

Of course, $\bar{\delta}^H_{T+1} = 0$ since $\bar{q}_{T+1} = q^H$.

From equation (26), we obtain

$$\begin{aligned}
&\left(\frac{1-\gamma}{\bar{G}(2)} + \frac{1+r}{\eta}\right) \delta^H_T \qquad (33)\\
&= \left(-(1+r)w(1)[\alpha_1 (1-\gamma)q^H + \alpha_1 \bar{W}(2) + \bar{G}(2)] + [\bar{G}(2) + r\alpha_2 w(2)]q^F\right) \Big/ \bar{G}(2)^2.
\end{aligned}$$

Using (2) to replace $(1-\gamma)q^H$ on the right-hand side of (33), we can rewrite this equation as

$$\left(\frac{1-\gamma}{\bar{G}(2)} + \frac{1+r}{\eta}\right) \delta^H_T = \frac{1+r}{\bar{G}(2)} \left[q^F - (1+\alpha_1 i(2))\, w(1)\right]. \qquad (34)$$

Under condition (28), therefore, we have $\delta^H_T > 0 = \bar{\delta}^H_{T+1}$, which proves Proposition 4.1 for $\tau = T+1$.

Repeating the above steps for equation (27), we find

$$\begin{aligned}
\left(\frac{1-\gamma}{\bar{G}(2)} + \frac{1+r}{\eta}\right) \delta^H_t &= \frac{\rho}{\eta}\, \delta^H_{t+1} + \frac{1-\rho}{\eta}\, \bar{\delta}^H_{t+1} - (T-t) \left(\frac{(1-\gamma)q^H}{\bar{G}(2)} + \frac{rq^F}{\eta}\right)\\
&\quad + \frac{1+r}{\bar{G}(2)} \left[q^F - (1+\alpha_1 i(2))\, w(1)\right] - \frac{\rho}{\eta}\, q^F \qquad (35)
\end{aligned}$$

for $t < T$. Subtracting equation (32) for $\tau = t+1 < T+1$ from equation (35) yields

$$\begin{aligned}
&\left(\frac{1-\gamma}{\bar{G}(2)} + \frac{1+r}{\eta}\right) \left(\delta^H_t - \bar{\delta}^H_{t+1}\right) \qquad (36)\\
&= \frac{\rho}{\eta} \left[\delta^H_{t+1} - \bar{\delta}^H_{t+1} - q^F\right] + \frac{1+r}{\bar{G}(2)} \left[q^F - (1+\alpha_1 i(2))\, w(1)\right].\\
&= \frac{\rho}{\eta} \left[(\delta^H_{t+1} - \bar{\delta}^H_{t+2}) + (\bar{\delta}^H_{t+2} - \bar{\delta}^H_{t+1}) - q^F\right] + \frac{1+r}{\bar{G}(2)} \left[q^F - (1+\alpha_1 i(2))\, w(1)\right].
\end{aligned}$$

By (32), the difference $\bar{\delta}^H_{t+2} - \bar{\delta}^H_{t+1}$ is a weighted average of the prices q^H and q^F, and since $q^H > q^F$, this average exceeds q^F:

$$\bar{\delta}^H_{t+2} - \bar{\delta}^H_{t+1} = \frac{\frac{1-\gamma}{\bar{G}(2)}\, q^H + \frac{r}{\eta}\, q^F}{\frac{1-\gamma}{\bar{G}(2)} + \frac{r}{\eta}} > q^F. \qquad (37)$$

Under condition (28), therefore, a simple induction argument yields Proposition 4.1 for all $\tau < T+1$.

Turning to Proposition 4.2, we differentiate equation (35) with respect to the continuation probability ρ:

$$\left(\frac{1-\gamma}{\bar{G}(2)} + \frac{1+r}{\eta}\right) \frac{d\delta^H_t}{d\rho} = \frac{\rho}{\eta}\, \frac{d\delta^H_{t+1}}{d\rho} + \frac{1}{\eta} \left[\delta^H_{t+1} - \bar{\delta}^H_{t+1} - q^F\right]. \qquad (38)$$

For $t < T$, Proposition 4.1 and the arguments in the previous paragraph imply that $\delta_{t+1}^H - \bar{\delta}_{t+1}^H > \bar{\delta}_{t+2}^H - \bar{\delta}_{t+1}^H > q^F$. Using (38) inductively starting from $d\delta_T^H/d\rho = 0$, therefore, we see that $d\delta_t^H/d\rho > 0$ for all $t < T$. Since $\bar{\delta}_{t+1}$ is independent of ρ, Proposition 4.2 follows.

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