The Second Fundamental Theorem of Welfare Economics and the Existence of Competitive Equilibrium in Production Economies over an Infinite Horizon with General Consumption Sets

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Abstract

The purpose of this paper is to prove the second fundamental theorem of welfare economics and the existence of competitive equilibrium in production economies over an infinite horizon with general consumption sets. In the literature the second fundamental theorem of welfare economics has been only approximately proved with uniform properness assumption on preferences. In order to generalize the theorem for a model that allows general comsumption set, the uniform properness assumption should be reduced. We prove the theorem in the exact form not assuming the assumption. The irreducibility of an economy and a joint assumption on consumers' preferences and

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production that makes the sustainable growth of the economy possible play the key role.

1 Introduction

The existence of competitive equilibrium with in¯nite dimensional commodity space has been studied since seminal papers Bewley(1972) and Peleg{Yaari(1970). On the other hand, the second fumdamental theorem of welfare economics in in¯nite dimensional commodity spaces was proved by Debreu(1954) when the production set has non-empty interior by applying the separation theorem as the same way as the one in ¯nite dimensional cases. In in¯nite dimensional commodity spaces the supporting price of Pareto optimal allocations cannot be found by this approach. In order to overcome this di±culty the uniform properness assumption is introduced by Mas-Collel(1986), and there are extensive researches on the equilibrium existence problem along this line.

However there are many economically important commodity spaces where it is inappropriate to assume the uniform properness. Among those spaces here we focus on linear subspaces of s^n, the set of sequences of ¯nite dimensional vectors, which we use as the commodity spaces. They are the class of commodity spaces for economies over an in¯nite horizon. It is inappropriate to assume the uniform properness in this setting, since it is inconsistent with myopia of preferences. For example there is no utility function on s, the set of real sequence, which are strictly monotonic, quasi-concave, product continuous, and at the same time uniformly product proper.[1] It is well known that the product continuity of preference in s expresses the myopia of preferences.

Economies over an in¯nite horizon have been studied by Peleg{Yaari(1970) and Boyd{McKenzie (1993). They established the equilibrium existence theorem with commodity space s^n. Evaluation of the feasible commodity allocations with vectors in s^n which is not the dual of the commodity space is very important for their results. We follow this approach with the broader class of commodity spaces including theirs.

One contribution of this paper is to make clear a su±cient condition for the second fundamental theorem of welfare economics in this setting. In the literature it is shown that a weakly Pareto optimal allocation may fail to be supported by some non-zero linear functional

[1] See, Aliprantis{Brown{Birkinshaw(1989, Example 3.6.9. p. 174).

in the dual of commodity space. Without uniform properness only " ¡ approximate support theorems has been established by Aliplantis-Burkinshaw (1988) and Becker-Bercovici-Foias (1992). With commodity price duality Khan -Vohra(1985), Aliplantis -Burkinshaw(1988) proved " ¡ approximate support property of weak Pareto optimum. The second fundamental theorem of welfare economics in this paper is not " ¡ approximate version. The su±cient condition has its origin in Boyd-McKenzie(1993). However the supportability is shown only at the Edgeworth equilibrium in their paper, since they use the Edgeworth approach to prove the existence theorem. Also the regularity assumption in this paper is weaker than the assumption in Boyd-McKenzie(1993).They impose the regularity assumption to possible net trade set with the technology of each consumer. On the other hand we impose this assumption only to aggregate possible net trade of entire consumers with the technology. By virtue of introducing new price normalization di®erent from the one of Boyd-McKenzie(1993) we can weaken the regularity assumption.

Our regularity assumption is joint condition on preferences, endowments among agents and a production set. The regularity assumption can be interpreted as follows; consumers are su±ciently myopic and the technology is productive in the future so that slight increase of social net trade at the ¯rst period cause some constant net supply in the future far enough and consumers are still well o®.

Another contribution of this paper is to show that even with general consumption sets; they do not have to contain their lower bounds, if the regularity assumption is satis¯ed irreducibility is su±cient for the equilibrium existence theorem. Burke(1988) shows a counter example to the equilibrium existence theorem in an economy over an in¯nite horizon with general consumption sets. Boyd-McKenzie (1993) proves the equilibrium existence theorem with general consumption sets. It, however, must pay a cost of a strong version of irreducibility to assure the equal treatment property in the core allocation. This is crucial for the non-emptiness of the equal treatment core, and so the existence of Edgeworth equilibrium. It says that for any non-trivial partition of consumers, one group of consumers can always spread their gains, if exist, to consumers in the other group, and the resulting allocation is still feasible.[2] This assumption holds when preferences are monotonic and con-

[2] Although they use net trading sets to de¯ne strongly irreducibility, it can be de¯ned in terms of con-

sumption sets are the positive orthant. This is, however, strong in the sense that it assume directly the existence of a special feasible allocation.

We replace strong irreducibility with usual irreducibility and establish the existence of a competitive equilibrium in production economies with general consumption sets over an infinite horizon by Negishi approach appealing to the l_1-price supportability of Pareto optimal allocations with the regularity assumption.

The procedure of this paper is as follows. In section 2, we set up our economy and explain our assumptions. We establish the second fundamental theorem of welfare economics in section 3 and the existence of competitive equilibrium is proved in section 4. Section 5 contains concluding remarks.

2 Economy

We are going to consider a discrete time open ended economy. Commodities are distinguished with their physical properties, their location and the dates on delivery. At each date there is same variation of different commodities. They are indexed with $k = 1, 2, \cdots n$. Thus our commodity space is a subspace of $s^n = R^n \times R^n \times \cdots$. The mathematical description of our commodity space is as follows:

Commodity space E is a subspace of s^n such that there is $W \subset ba$ where $(E; W)$ is a Riesz symmetric dual system and E inherited a natural order from s^n:

There are three important examples of Riesz symmetric dual system which appears in economic literature.

i) $(s^n; c_{00})$ ii) $(l_\infty; l_1)$ iii) $(l_\infty(\beta); l_1(1/\beta))$

Define $\|x\|_\infty = \sup\{|x_k(t)| : t = 0, 1, 2, \cdots; k = 1, 2, \cdots; n\}$. Then $l_\infty = \{x \in s^n : \|x\|_\infty < \infty\}$. Also $c_{00} = \{x \in s^n$: there is T with $x_k(t) = 0$ for any $t > T\}$: Define $\|p\|_1 = \sum_t \sum_k |p_k(t)|$. Then $l_1 = \{p \in s^n : \|p\|_1 < \infty\}$: Also define $\|x\|_\infty(\beta) = \sup\{|x_k(t)|/\beta^t : t = 0, 1, 2, \cdots; k = 1, 2, \cdots; n\}$ and $\|p\|_1(1/\beta) = \sum_t \beta^t \sum_k |p_k(t)|$: Then $l_\infty(\beta) = \{x \in s^n : \|x\|_\infty(\beta) < \infty\}$ and $l_1(1/\beta) = \{p \in s^n : \|p\|_1(1/\beta) < \infty\}$:

sumption sets as well.

Mathematically important property of Riesz symmetric dual system $(E; E')$ is that every order interval of E is weakly compact and if $(E; E')$ is a Riesz symmetric dual system, $(E'; E)$ is likewise a Riesz symmetric dual system. Therefore the weak topology $\sigma(E; E')$ can be considered as weak star topology for dual system $(E'; E)$. This property is convenient to apply Alaoglu's theorem.

Economically l_1 is the space which does not allow growing economy and this is a special case of L_1 which is used in Bewley (1972). s^n and $l_1(\beta)$ may allow growing path of the economy. s^n is used in Peleg–Yaari (1970) and Boyd–Mckenzie (1993). $l_1(\beta)$ is isomorphic to l_1 and can be thought of as having the discount factor $1/\beta$ built in. This space is used by Boyd(1990).

Our description of commodity space E includes all of these cases, thus it is easy to compare the results with others.

There are finite number of consumers indexed with $i = 1, 2, \cdots, H$ who have a consumption set $C^i \subset E$ and a preference P^i which express strict preference over C^i. We can interpret these consumers in two ways: as infinitely lived agents with open ended economy or as finitely lived agents who does not know own terminal date of life and has preference over infinite horizon consumption set.

The market is complete and opens for all commodities at first date. It is possible to consider agents have perfect foresight in the future or there is a market for contingent claim plan over infinite horizon economy.

The production sector is represented with a convex cone technology over E. With constant returns to scale technology we assume perfect competition among producers and there is free entry and exit. Thus the number of producers cannot be set a priori.

Our price space for the market is represented with s^n. This price space is first used by Peleg–Yaari(1970) in exchange economies and later extended into production economies by Boyd–McKenzie (1993). In infinite horizon economy the commodity price duality is often used to represent price system. It evaluates each commodity bundle, but does not necessarily evaluate each commodity itself. In contrast our price system does not necessarily evaluate every commodity bundle, but does evaluate each goods in a commodity bundle in a coodinatewise fashion. These two price space are the same one in finite dimensional commodity space. This is a specific issue to infinite dimensional commodity spaces.

Now we are ready to state and discuss our assumptions on economy. Before to do so, let us define some notations for convenience.

a) We denote $x \in s^n$; $x(t) \in R^n$ at time period t, and

$x = (x(0); x(1); \cdots)$

b) Let $x; y \in s^n$: We denote $x \geq y$ if $x_k(t) \geq y_k(t)$ for all $t = 0; 1; \cdots; k = 1; 2; \cdots; n$. As the same way $x > y$ if $x \geq y$ and $x_k(t) > y_k(t)$ for some $t; k$:

c) Let $e = (1; \cdots; 1)$; an unit vector in R^n, then we use

$e(0) = (e; 0; 0; \cdots)$ and $e(t) = (0; \cdots; e; 0; \cdots)$:

d) $x_k^+(t) = \max\{0; x_k(t)\}$; $x(t)^+ = (x_1(t); \cdots; x_n(t))$:

$x_k^-(t) = \min\{0; x_k(t)\}$; $x(t)^- = (x_1(t); \cdots; x_n(t))$:

– Assumptions –

(1) For each consumer i, the consumption set C^i is convex and $\sigma(E; W)$-closed. The net trading set $C^i - \{\omega_i\}$ is bounded below by $b \in l_\infty$ for each i:

(2) For each consumer i; the strongly preferred correspondence P^i is convex and $\sigma(E; W)$-open valued, and has $\sigma(E; W)$-open lower sections relatively in C^i. The preference relation defined from P^i is irreflexive and transitive. The weakly preferred set $R^i(x)$ is the $\sigma(E; W)$-closure of $P^i(x)$ for all $x \in C^i$ unless $P^i(x) = \emptyset$:

(3) Let $x \in C^i$: If $z \geq x$; then $z \in R^i(x)$. (weak monotonicity)

(4) The production set Y is a $\sigma(E; W)$-closed convex cone with vertex at the origin and contains no straight line.

(5) Define $F^i(x_i) = R(x_i) - \{\omega_i\}$ for $x_i \in C_i$. Then for any $v \in \sum_i F^i(x_i) - Y$ the following holds.

For any $\varepsilon > 0$, there exists τ_0 and $\alpha > 0 \in R^n$ such that $\tau > \tau_0$ implies

$$(v(0) + \varepsilon e(0); \cdots; v(\tau); -\alpha; -\alpha; \cdots) \in \sum_i F^i(x_i) - Y: \qquad \text{(regularity assumption)}$$

(6) For all $z \in E$; the set $\{y \in Y : y \geq z\}$ is order bounded.

(7) For all i, there is $\bar{x}_i \in C^i$ and $\bar{y}_i \in Y$ such that $(\bar{x}_i - \omega_i) < \bar{y}_i$ and

$\sum_i (\bar{y}_i - (\bar{x}_i - \omega_i)) = (\gamma, \gamma, \cdots)$ for some $\gamma (\in R^n) \gg 0$. (aggregate adequacy assumption)

(8) The economy is irreducible: whenever I_1 and I_2 is a nontrivial partition of $\{1, \cdots, H\}$ and $\sum_{i \in I}(x_i - \omega_i) \in Y$ with $x_i \in C^i$ for all $i \in I$, there are

$\sum_{i \in I_1}(z_i - \omega_i) + \sum_{i \in I_2} \alpha_i (z_i - \omega_i) \in Y$ with $z_i P^i x_i$ for $i \in I_1$ and,

$z_i \in C^i$ and some $\alpha_i > 0$ for $i \in I_2$. (irreducibility assumption)

It follows that net supply of consumer such as labor is bounded from above by an element of l_∞ from assumption (1). It does not mean that a consumption bundle $x \in C^i$ or ω_i is in l_∞.

In assumption (1) and (2), we induce the topology $\sigma(E, W)$ on the consumption sets. It depends on whether the use of it is economically natural or not. The examples described before we can interpret the continuity of preference with respect to $\sigma(E, W)$ as myopia. Note that $\sigma(s^n, c_{00})$ is product topology and it is well known that the continuity of preferences with respect to the product topology expresses strong myopia of preference. The weak$^{(*)}$ $\sigma(l_\infty, l_1)$ topology has same closed convex sets as the Mackey $\tau(l_\infty, l_1)$ topology has. The Mackey topology is used in Bewley (1972) and it is shown that continuity os preference with respect to the Mackey topology can be interpreted as myopia in the paper. Since $l_\infty(\beta)$ is homeomorphic to l_∞ and $l_1(1/\beta)$ is homeomorphic to l_1, the same interpretation may be possible.

The other assumptions in assumption (1) and (2) are standard in general equilibrium theory. Especially we need assumption (2) for the existence of utility representation. Transitivity plays key role in it. Note that our consumption set C^i is general in the sense it does not necessarily include its lower bound. Thus it allows us for substitution between goods on the boundary and we can consider labor in our commodity space.

Weak monotonicity of preferences is assumed in assumption (3). The weak monotonicity is standard and strong monotonicity in the first period can be interpreted as a result of myopia.

As described before our technology exhibit constant returns to scale. The irreversibility of production process is also assumed in assumption (4). As Boyd–McKenzie (1993) showed this formulation include Malinvaud technology with constant returns to scale.

Assumption (5) is important condition for our result. Consider an aggregate net supply with consumptions weakly preferred to an original consumption. Then any slight increase of aggregate net supply in the first period can produce constant positive aggregate net supply permanently in the future after some period without any change in net supply of other periods and every possible consumptions generated from the new aggregate net supply is still weakly preferred to the original consumption. This is a joint condition on preferences and endowments among consumers and production set. In section 5 we discuss on this condition again.

Assumption (6) is equivalent to that for all time period and goods if net output is larger than some real number $\varepsilon(t,k) > 0$, there exist $\delta(t,k) > 0$ such that net input is larger than $\delta(t,k)$ for all t,k. As the same way if net input is smaller than some real number $\varepsilon(t,k) > 0$ there exists $\delta(t,k) > 0$ such that net output is smaller than $\delta(t,k)$ for each t,k.

Adequacy assumption is same as the one used by Boyd–McKenzie(1993). With this condition, we can show that the aggregate income of the economy is positive. This, however, does not imply that each consumer's income is positive. Only some consumers have positive incomes. Bewley(1972) uses the stronger individual adequacy assumption which is stated as $(y_i - (x_i - \omega_i)) = (-\gamma_i, -\gamma_i, \cdots)$ for some $\gamma_i (\in R^n)$ holds for each i. This individual adequacy assumption trivially implies the aggregate and conclude that every consumer in the economy has a positive income.

The irreducibility assumption (8) is usual one and is not the strong irreducibility assumption used by Boyd–McKenzie(1993). We need this condition to spread the positive incomes of some consumers due to the aggregate adequacy assumption (7) over every consumer, which is necessary in translatating a quasi-equilibrium into a competitive equilibrium. The strong irreducibility of Boyd–McKenzie(1993) is used to establish the non-emptiness of the equal treatment core in their economies with general consumption sets in their Edgeworth equilibrium approach. Since we employ in stead the Negishi approach instead, we do not need this strong irreducibility. The usual irreducibility assumption is enough for our purpose. This is believed to be a contribution of this paper.

3 The Second Fundamental Theorem of Welfare Economics

In this section we prove the second fundamental theorem of welfare economics. In the most of literature this theorem is equivalent to the existence of supporting hyperplanes for the weakly preferred sets of consumers and the production set at every Pareto optimal allocation. The supporting vectors are in the dual of commodity space and determine the value of each allocation. Appealing to the separation theorem based on this duality is powerful for the proof of the theorem if the positive orthant of commodity space has nonempty interior. On the other hand, however, the same argument does not apply to the commodity spaces with empty interior. When we consider economies with ¯nite dimensional commodity spaces the separation theorem can directly apply to prove the second fundamental theorem of welfare economics, since any two convex subsets in ¯nite dimensional commodity spaces which have disjoint interior can be separated at any point which is not interior to either sets. On the other hand in in¯nite dimensional spaces we need to be more careful; the separation theorem requires the existence points which is interior to one of the two sets. As Mas-Collel (1986) shows a weakly preferred set of a consumption bundle in in¯nite dimensional commodity space may fail to have interior points. The uniform properness assures that there exist interior points we actually need.

The class of economies considered in this paper allows the commodity spaces which have no interior points. To determine the values of allocations in commodity spaces, usually the dual space of commodity space and valuation based on the duality are used. Here instead coodinatewise valuation of allocations in Peleg{Yaari(1970) and Boyd{McKenzie(1993) is adapted. In this situation, the support property of Pareto optimal allocations is considered to hold if there is a way of social valuation of allocations satisfying the following; for each consumer if a consumption allocation is not less preferred to the consumption of Pareto optimal allocation, the valuation is also not less than that of the consumption at the Pareto optimal allocation, and any production cannot yield positive pro¯ts and the production at the Pareto optimal allocation has zero pro¯t with the valuation.

We have two versions of second theorem of welfare economics respectively with net trades

(Theorem 3–1) and with consumptions (Corollary 3–1). It is usual to use consumptions to state the theorem, because it should hold independently on distribution of endowments among agents. In finite dimensional commodity space or more generally in the commodity space with price systems as the dual of it, these two versions of second theorems are necessary and sufficient for each other. On the other hand we have our price space as s^n which is not necessarily the dual of commodity space E, these are not equivalent. With the definition of a competitive equilibrium in section 4, the version with net trade is suitable for the interpretation such that any Pareto optimal allocation can be realized as a competitive equilibrium by redistributing endowments properly among consumers.

The valuations of net trade and production are as follows;

Valuation of net trade $\quad \sum_{t=0} p(t)(x_i(t) - \omega_i(t))$ where $x_i \in C^i;\ p \in s^n$:

Valuation of production $\quad \limsup_{\tau} \sum_{t=0} p(t)y(t)$ where $y \in Y;\ p \in s^n$:

It will be shown later in the proof of the theorems that the valuation of net trade has only finite value or $+\infty$ from assumption (1).

Now we define a weakly Pareto optimal allocation and Pareto optimal allocation.

Definition. We call $(x_1, \cdots, x_H, y) \in C^1 \times \cdots \times C^H \times Y$ as an allocation if $\sum_i (x_i - \omega_i) = y$ holds for some $y \in Y$: An allocation $(x_1, \cdots, x_H, y)$ is weakly Pareto optimal, whenever there exists no other allocation $(x'_1, \cdots, x'_H, y')$ satisfying $x'_i \in P^i(x_i)$ for all i: An allocation $(x_1, \cdots, x_H, y)$ is Pareto optimal whenever there exists no other allocation $(x'_1, \cdots, x'_H, y')$ satisfying $x'_i \in R^i(x_i)$ for all i, and there is some j such that $x'_j \in P^j(x_j)$. Obviously a Pareto optimal allocation is weakly Pareto optimal.

In the procedure of our proof we first show that there exists a separating hyperplane between the origin 0 and $(\sum_i F^i - Y) \cap l_1$ where $F^i = F^i(x_i)$ for a weakly Pareto optimal allocation $(x_1, \cdots, x_H, y)$ with supporting vector in $\pi \in ba$: After that we decompose π by Yosida–Hewitt theorem and the l_1 part of it is the candidate of a supporting price of a weakly Pareto optimal allocation. This method is originally developed by Boyd–McKenzie (1993) and they use the idea to prove the supporting property of Edgeworth equilibrium.

To get a desired separating hyperplane, we need to show the $\sigma(E;W)$-closedness of $\sum_i F^i - Y$. The crucial fact for this result is the following Choquet's(1962) theorem. This theorem is used by Boyd–McKenzie(1993) in the case of s^N and by Ali Khan and Vohra(1988) in general locally convex spaces.

Theorem[Choquet(1962)] : If $Z \subset E$ is convex, $\sigma(E;W)$-closed, and contains no straight lines, then for any two convex and $\sigma(E;W)$-closed subsets X and Y in Z, $X+Y$ is $\sigma(E;W)$-closed.[3]

Lemma 3–1 : For any $(x_1,\cdots,x_H) \in C^1 \times \cdots \times C^H$, $\sum_i F^i - Y$ is $\sigma(E;W)$-closed.

Proof) By assumption (1) for all i $C^i - \{\omega_i\}$ is bounded from below by $b \in l_\infty$. This implies $F^i(x_i)$ is bounded below by b for all i. Thus $F^i(x_i) - Y \subset b + E_+ - Y$. We want to show $b + E_+ - Y$ is $\sigma(E;W)$-closed, convex and contains no straight line. Since any finite sum of convex sets is convex, $b + E_+ - Y$ is convex from assumption (4).

Note that if $E_+ - Y$ is $\sigma(E;W)$-closed then $b + E_+ - Y$ is also $\sigma(E;W)$-closed, since any net in $b + E_+ - Y$ has the form $\{b + z^\alpha\}$ where $\{z^\alpha\}$ is a net in $E_+ - Y$. Moreover if $E_+ - Y$ contains no straight line $b + E_+ - Y$ is also contains no straight line since $E_+ - Y$ is convex and b is a single point.

We claim

$$E_+ - Y \text{ is } \sigma(E;W)\text{-closed and contains no straight line.}$$

Suppose $E_+ - Y$ has elements $z, -z$. Then there are $y, y' \in Y$ such that $z \geq -y$ and $-z \geq -y'$. This implies $y + y' \geq 0$. Since Y is a cone, for any $\lambda \geq 0$, $\lambda(y + y') \in Y$. However by assumption (6), $\{y \in Y : y \geq 0\}$ has an upper bound. Therefore $y + y'$ should be 0, and $y = -y'$. From assumption (5), Y contains no straight line. Thus $y = -y' = 0$. It implies $z = 0$ and $E_+ - Y$ contains no straight line.

Next we claim that $E_+ - Y$ is $\sigma(E;W)$-closed. Let $\{z^\alpha\}$ be a net which $z^\alpha \in E_+ - Y$ and $z^\alpha \to z$ in $\sigma(E;W)$ as $\alpha \uparrow$. Fix some order interval such as $z \in [a,c]$. Then there is a converging subnet $\{z^{\alpha(k)}\}$ such that $z^{\alpha(k)} \in [a,c]$ and $z^{\alpha(k)} \to z$ as $k \uparrow$ since $(E;W)$ is

[3] This is a specialization of a theorem in Choquet(1962). The original statement of this theorem uses the $\sigma(E;W)$-completeness of Z. Since $(E;W)$ is Riesz sysmetric dual, indeed, we can replace it with its $\sigma(E;W)$-closedness.

a Riesz symmetric dual system and hence $[a; c]$ is $\sigma(E; W)$-compact. Note that there are $v^{\alpha(k)} \in E_+$; and $y^{\alpha(k)} \in Y$ satisfying $z^{\alpha(k)} = v^{\alpha(k)} - y^{\alpha(k)}$. This implies that $c \geq z^{\alpha(k)} \geq -y^{\alpha(k)}$, so $y^{\alpha(k)} \geq -c$. Thus $\{y^{\alpha(k)}\}$ has uniform lower bound. From assumption (6), $\{y^{\alpha(k)}\}$ has uniform upper bound. Thus $\{y^{\alpha(k)}\}$ is in a compact set, and so has limit $y \in Y$. Since $\sigma(E; W)$ is a Hausdorff topology, the limit of $\{z^{\alpha(k)}\}$ equals to z. Note $z^{\alpha(k)} = v^{\alpha(k)} - y^{\alpha(k)}$ and $z^{\alpha(k)} \geq -y^{\alpha(k)}$ implies that $z \geq -y$ and $v = z + y \geq 0 \in E_+$. Thus $z \in E_+ - Y$, and so $E_+ - Y$ is $\sigma(E; W)$-closed. Thus, (1 – 1) holds.

Now we can apply Choquet's theorem and $\sum_i F^i - Y$ is $\sigma(E; W)$-closed. ■

Lemma 3–2 : For any weakly Pareto optimal allocation $(x_1, \cdots, x_H, y)$ there exist $\pi \in ba$ such that

$\pi \cdot v \geq 0$ holds for all $v \in (\sum_i F^i(x_i) - Y) \cap l_1$; $\pi \cdot v \geq 0$ and,

$\pi \geq 0$, $\pi \neq 0$, and $\|\pi\|_{ba} = 1$.

Proof) Let $F^i = F^i(x_i)$ for a weakly Pareto optimal allocation $(x_1, \cdots, x_H, y)$. We claim that for any $\varepsilon > 0$,

$$-\varepsilon e(0) \notin \sum_i F^i - Y.$$

Suppose not, then there exists $x_i' \in R^i(x_i)$, $y' \in Y$ such that $-\varepsilon e(0) = \sum_i (x_i' - \omega_i) - y'$. Then $\sum_i (x_i' - \omega_i + \varepsilon/H \cdot e(0)) - y' = 0$. From monotonicity assumption $x_i' + \varepsilon/H \cdot e(0) \in P^i(x_i)$ for each i. This contradicts to the weak Pareto optimality of $(x_1, \cdots, x_H, y)$. Thus $-\varepsilon e(0) \notin \sum_i F^i - Y$ for any $\varepsilon > 0$.

From lemma 3–1, $\sum_i F^i - Y$ is $\sigma(E; W)$-closed. Also $\{-\varepsilon e(0)\}$ is trivially $\sigma(E; W)$-compact. Now we can apply the separation theorem(Scheafer(1966) p.65), and there exists $f \in W$ such that $f \cdot v > f \cdot (-\varepsilon e(0))$ for any $v \in \sum_i F^i - Y$. From monotonicity assumption and the separation theorem, $f \geq 0$; $f \neq 0$. Let $u = (e, e, \cdots)$, a unit vector, and $\pi = f / \|f\|_{ba}$. (Recall that $W \subset ba$, so $f \in ba$.) Then $\pi \cdot v > -\varepsilon \pi(0) \cdot e(0)$ and since $\pi \geq 0$, $\pi \cdot u \geq \pi(0) \cdot e(0)$ holds. Thus $\pi \cdot v > -\varepsilon \pi \cdot u$. Clearly $\pi \cdot u = \|\pi\|_{ba} = 1$ by definition of the norm. Consequently $\pi \cdot v > -\varepsilon$ for any $v \in \sum_i F^i - Y$. Define S and $S(\varepsilon)$ as follows.

$S(\varepsilon) = \{\pi \in ba : \|\pi\|_{ba} = 1, \pi \cdot v \geq -\varepsilon \text{ for all } v \in (\sum_i F^i - Y) \cap l_1\}$.

$S = \{\pi \in ba : \|\pi\|_{ba} \leq 1, \pi \cdot v \geq -1 \text{ for all } v \in (\sum_i F^i - Y) \cap l_1$.

Clearly for any $\varepsilon > 0$, $S(\varepsilon)$ is non-empty and $S(\varepsilon)$ forms a nested set sequences, and for any $\varepsilon \le 1$ $S(\varepsilon) \subset S(1) \subset S$. Note $S(\varepsilon)$ is a subset of the unit sphere of ba-norm equals to 1 and S is the subset of unit ball of ba-norm equals to1 and the fact that the evaluation function is continuous with respect to $\sigma(ba; l_1)$ and inequality is weak. Since the closed unit ball in ba with respect to$(ba; l_1)$ is compact by Alaoglu's theorem, S is compact. The non-emptiness of $S(\varepsilon)$ allow us to pick up $\pi^{\varepsilon} \in S(\varepsilon)$ for all $(0 <)\varepsilon \le 1$. Let $[0,1]$ be a directed set with direction $\varepsilon \preceq \varepsilon'$ if and only if $\varepsilon \ge \varepsilon'$. Then $\{\pi^{\varepsilon}\}$ forms a net in S. From the compactness of S; there exists a converging subnet $\{\pi^{\varepsilon(\lambda)}\}$ in S such that $\pi^{\varepsilon(\lambda)} \to \pi$ in the weak $\sigma(ba; l_1)$ topology. Note that $1 = \left\|\pi^{\varepsilon(\lambda)}\right\|_{ba} = \pi^{\varepsilon(\lambda)} \cdot u$ and the evaluation function is $\sigma(ba; l_1)$ continuous. Thus $\pi^{\varepsilon(\lambda)} \cdot u \to \pi \cdot u$. Consequently $\pi \cdot u = \|\pi\|_{ba} = 1$. Also for any $\delta > 0$ there exists $\varepsilon_o \in [0,1]$ such that $\varepsilon_o \preceq \varepsilon(\lambda)$ implies $\left|\pi^{\varepsilon(\lambda)} \cdot v - \pi \cdot v\right| < \delta$. Suppose $\pi \cdot v < 0$ and take δ small enough. Then there exists $\varepsilon(\lambda)$ which is close enough to 0 and $\varepsilon_o \preceq \varepsilon(\lambda)$ and $\left|\pi^{\varepsilon(\lambda)} \cdot v - \pi \cdot v\right| > \delta$. Thus $\pi \cdot v \ge 0$. ■

Now we have a candidate for the supporting price of a weakly Pareto optimal allocation. We are going to extend the supportability not only in l_1 but over whole space.

Theorem 3–1 : For every weakly Pareto optimal allocation $(x_1, \cdots, x_H, y)$, there exists a supporting price $\pi_c \in l_1(\subset s^n)$ such that;

(1) $\sum_{t=0}^{\infty} \pi_c(t) \cdot (x_i'(t) - \omega_i(t)) \ge \sum_{t=0}^{\infty} \pi_c(t) \cdot (x_i(t) - \omega_i(t))$ for all $x_i' \in R^i(x_i)$,

(2) $\sum_{t=0}^{\infty} \pi_c(t) \cdot y(t) \ge \limsup_{\tau \to \infty} \sum_{t=0}^{\tau} \pi_c(t) \cdot y'(t)$ for all $y' \in Y$ and

$\sum_{t=0}^{\infty} \pi_c(t) \cdot y(t) = 0$,

(3) $\pi_c \ge 0$, $\pi_c \ne 0$.

Proof) First notice that by assumption (5)(the regularity assumption) and assumption (3), we know whenever $v \in \sum_i F^i(x_i) - Y$ for any $\varepsilon > 0$, there exists $\overline{\tau}$ such that $\tau > \overline{\tau}$ implies

$$(v(0), \cdots, v(\tau), 0, 0, \cdots) + \varepsilon e(0) \in \sum_i F^i(x_i) - Y. \tag{1}$$

For any $x'_i \in R^i(x_i)$, $x'_i - x_i = x'_i - \omega_i + \sum_{j\neq i}(x_j - \omega_j) - y$ holds since the feasibility of the weakly Pareto optimal allocation $(x_1, \cdots, x_H; y)$ implies $0 = \sum_i (x_i - \omega_i) - y$. Thus $x'_i - x_i \in \sum_i F^i(x_i) - Y$.

Therefore from (1) we have $(x'_i(0) - x_i(0), \cdots, x'_i(\tau) - x_i(\tau), 0, 0, \cdots) + \varepsilon e(0) \in \sum_i F^i(x_i) - Y$. Then from lemma 3–1, there is $\pi \in ba$ such that

$$\pi \cdot (x'_i(0) - x_i(0), \cdots, x'_i(\tau) - x_i(\tau), 0, 0, \cdots) + \pi \cdot \varepsilon e(0) \geq 0. \qquad (2)$$

Note that $(x'_i(0) - x_i(0), \cdots, x'_i(\tau) - x_i(\tau), 0, 0, \cdots) + \varepsilon e(0)$ has only finite nonzero elements and so it is in l_1. From the Yosida–Hewitt theorem for this $\pi (\geq 0)$ there is $\pi_c \in l_1^+$ such that $\pi = \pi_c + \pi_f$ where $\pi_f\ (\geq 0)$ is the purely finitely additive part. Since π_f has zero values over c_{00} and $(x'_i(0) - x_i(0), \cdots, x'_i(\tau) - x_i(\tau), 0, 0, \cdots) + \varepsilon e(0)$ is in c_{00}, we indeed have

$$\sum_{t=0}^{\tau} \pi_c(t) \cdot (x'_i(t) - \omega_i(t)) \geq \sum_{t=0}^{\tau} \pi_c(t) \cdot (x_i(t) - \omega_i(t)) - \varepsilon \pi_c(0) \cdot e(0) \qquad (3)$$

We claim that for any $z \in C^i - \{\omega_i\}$,

$$\lim_{\tau \to \infty} \sum_{t=0}^{\tau} \pi_c(t) \cdot z(t) \text{ exists and } \pi_c \cdot z = \sum_{t=0}^{\infty} \pi_c(t) \cdot z(t) \text{ is a finite value or } +\infty. \qquad (4)$$

Define z^- for $z (\in s)$ by $z^- = 0$ when $z \geq 0$ and $z^- = z$ when $z < 0$. When $z \geq 0$, (4) holds from the non-negativity of π_c. When $z < 0$, $\sum_{t=0}^{\tau} \pi_c(t) \cdot z(t) = \sum_{t=0}^{\tau} \pi_c(t) \cdot z^-(t)$ and $\lim_{\tau \to \infty} \sum_{t=0}^{\tau} \pi_c(t) \cdot z^-(t)$ exists and has a finite value or $+\infty$ due to $z \geq b$ for some $b \in l_1$. Thus (4) holds and hence $\sum_{t=0}^{\infty} \pi_c(t) \cdot z(t)$ is well-defined for any $z \in C^i - \{\omega_i\}$.

From (3), we have $\sum_{t=0}^{\tau} \pi_c(t) \cdot (x'_i(t) - \omega_i(t)) \geq \sum_{t=0}^{\tau} \pi_c(t) \cdot (x_i(t) - \omega_i(t)) - \varepsilon \pi_c(0) \cdot e(0)$. Letting $\tau \to \infty$ for given $\epsilon > 0$ and then letting $\varepsilon \to 0$, we have

$$\sum_{t=0}^{\infty} \pi_c(t) \cdot (x'_i(t) - \omega_i(t)) \geq \sum_{t=0}^{\infty} \pi_c(t) \cdot (x_i(t) - \omega_i(t)) \text{ for all } x'_i \in R^i(x_i). \qquad (5)$$

Let $y' \in Y$. Then we have $y - y' = \sum_i (x_i - \omega_i) - y \in \sum_i F^i(x_i) - Y$ from the feasibility of the weak Pareto allocation $(x_1, \cdots, x_H; y)$, $\sum_i (x_i - \omega_i) - y = 0$. By the same argument as before, we can show that for any $\varepsilon > 0$ there is τ_0 such that $\tau > \tau_0$ implies

$$\sum_{t=0}^{\tau} \pi_c(t) \cdot y(t) \geq \sum_{t=0}^{\tau} \pi_c(t) \cdot y'(t) - \varepsilon \pi_c(0) \cdot e(0). \qquad (6)$$

By the feasibility of weakly Pareto optimal allocations, $y = \sum_i (x_i - \omega_i) \in C^i - \{\omega_i\}$ holds. Thus (4) implies that $\pi_c \cdot y = \sum_{t=0}^{\infty} \pi_c(t) \cdot y(t)$ is a finite value or $+\infty$. By letting $\tau \to \infty$ and then taking $\varepsilon \to 0$, we obtain

$$\sum_{t=0}^{\infty} \pi_c(t) \cdot y(t) \geq \limsup_{\tau} \sum_{t=0}^{\tau} \pi_c(t) \cdot y'(t) \text{ for all } y' \in Y. \tag{7}$$

Next we claim

$$\pi_c \cdot y = \sum_{t=0}^{\infty} \pi_c(t) \cdot y(t) = 0. \tag{8}$$

Since $0 \in Y$ is assumed, (7) implies $\sum_{t=0}^{\infty} \pi_c(t) \cdot y(t) \geq 0$. Note that $-y = \sum_i (x_i - \omega_i) - 2y \in \sum_i F^i(x_i) - Y$ from the feasibility of the weak Pareto allocation $(x_1, \cdots, x_H; y)$, $\sum_i (x_i - \omega_i) - y = 0$. By using the argument similar to that for getting (7), we have $\sum_{t=0}^{\infty} \pi_c(t) \cdot (-y(t)) \geq 0$ and so $\sum_{t=0}^{\infty} \pi_c(t) \cdot (y(t)) \leq 0$. Thus we get the other part of the inequality in (8) and hence (8).

Now we claim

$$\pi_c \neq 0. \tag{9}$$

Suppose contrary that $\pi = \pi_f$. From assumption (5), $v \in \sum_i F^i(x_i) - Y$ implies that for any $\epsilon > 0$ there is τ' such that $(v(0), \cdots, v(\tau), -\alpha, -\alpha, \cdots) + \varepsilon e(0) \in \sum_i F^i(x_i) - Y$ holds for any $\tau > \tau'$. Since $\alpha > 0$ in R^n, $(v(0), \cdots, v(\tau), -\alpha, -\alpha, \cdots) + \varepsilon e(0)$ is in l_∞. Then we have

$$\begin{aligned} & \pi \cdot [(v(0), \cdots, v(\tau), -\alpha, -\alpha, \cdots) + \varepsilon e(0)] \\ = \; & \pi_f \cdot [(v(0), \cdots, v(\tau), -\alpha, -\alpha, \cdots) + \varepsilon e(0)] \\ = \; & \pi_f \cdot (v(0), \cdots, v(\tau), 0, 0, \cdots) + \pi_f \cdot \varepsilon e(0) + \pi_f \cdot (0, \cdots 0 \cdots, 0, -\alpha, -\alpha, \cdots) \\ = \; & \pi_f \cdot (-\alpha, -\alpha, \cdots) \geq 0 \end{aligned} \tag{10}$$

since π_f has only zero values over c_{00}. On the other hand, $\pi = \pi_f \geq 0$ and $\alpha > 0$ imply

$$\pi_f \cdot (-\alpha, -\alpha, \cdots) \leq 0. \tag{11}$$

Thus, $\pi_f \cdot (-\alpha, -\alpha, \cdots) = 0$ and hence $\pi = \pi_f = 0$ holds from (10) and (11). This is, however, a contradiction to $\pi \neq 0$. Therefore we establish (9). ∎

Theorem 3–1 is a form of the second fundamental theorem of welfare economics with net trades. This theorem holds whenever the net trading sets are convex, closed, bounded from below by $b \in l_\infty$, even if the endowments cannot be expressed with points in E. We will revisit this point in section 5. We indeed obtain the usual second theorem which is independent of the distribution of endowments for consumers.

Corollary 3–1 : For every Pareto optimal allocation $(x_1, \cdots, x_H; y)$, there exists $\pi_c \in l_1$ such that

$$\sum_{t=0}^{\infty} \pi_c(t) \cdot x_i'(t) \geq \sum_{t=0}^{\infty} \pi_c(t) \cdot x_i(t) \text{ for all } x_i' \in R^i(x_i) \text{ and}$$

$$\limsup_{\tau} \sum_{t=0}^{\tau} \pi_c(t) \cdot y'(t) \leq 0 \text{ for all } y' \in Y.$$

Proof) From the same way as that in the proof of Theorem 3–1, we have the inequality (3 - 1) with letting π_c a supporting price in theorem 3–1 since a Pareto optimal allocation is weakly optimal. Thus, we have

$$\sum_{t=0}^{\tau} \pi_c(t) \cdot x_i'(t) \geq \sum_{t=0}^{\tau} \pi_c(t) \cdot x_i(t) - \varepsilon \pi_c(0) \cdot e(0).$$

Since it is easy to see C^i is bounded from below by $b \in l_\infty$ from assumption (1), $\sum_{t=0}^{\infty} \pi_c(t) \cdot x_i'(t)$ and $\sum_{t=0}^{\infty} \pi_c(t) \cdot x_i(t)$ has only finite value or $+\infty$ as the same way as before. By letting $\tau \to \infty$, and then taking $\varepsilon \to 0$, we have

$$\sum_{t=0}^{\infty} \pi_c(t) \cdot x_i'(t) \geq \sum_{t=0}^{\infty} \pi_c(t) \cdot x_i(t) \text{ for any } x_i' \in R^i(x_i).$$

From theorem 3–1 we already have

$$\limsup_{\tau} \sum_{t=0}^{\tau} \pi_c(t) \cdot y'(t) \leq 0 \text{ for all } y' \in Y$$

and $\pi_c \geq 0$, $\pi_c \neq 0$. ■

Both theorem 3–1 and corollary 3–1 do not exclude the case where $\sum_{t=0}^{\infty} \pi_c(t) \cdot (x_i(t) - \omega_i(t)) = +\infty$ and $\sum_{t=0}^{\infty} \pi_c(t) \cdot x_i(t) = +\infty$. If $\sum_{t=0}^{\infty} \pi_c(t) \cdot (x_i(t) - \omega_i(t)) = +\infty$, then such allocation x_i is neither a quasi-equilibrium nor a competitive equilibrium. On the other hand if $\sum_{t=0}^{\infty} \pi_c(t) \cdot x_i(t) = +\infty$, then such allocation cannot be a valuation equilibrium. It is easy to see that $\sum_{t=0}^{\infty} \pi_c(t) \cdot (\sum_i x_i(t)) = \sum_{t=0}^{\infty} \pi_c(t) \cdot (\sum_i \omega_i(t))$ holds for every Pareto optimal allocation by assumption (4). Thus if there is some i with $\sum_{t=0}^{\infty} \pi_c(t) \cdot x_i(t) = +\infty$, then the valuation of aggregate endowments equals to $+\infty$, and so it is possible to distribute any amount of income among consumers. Thus it is impossible for this allocation to be a valuation equilibrium.

4 The Existence of a Competitive Equilibrium

In this section we prove the existence of a competitive equilibrium. As discussed in the previous section we take price system in s^n which is not the dual of the commodity space. With this price system there is commodity bundle which does not have the value.

We define a competitive equilibrium as follows:

Definition: A pair of an allocation and a price system $((x_1, \cdots, x_H; y); p) \in C^1 \times \cdots \times C^H \times Y \times s^n$ is a competitive equilibrium if ;

1. For each i; $x_i \in B^i(p) = \{x \in C^i : \sum_{t=0}^{\infty} p(t) \cdot (x(t) - \omega_i(t)) \le 0\}$ and $x' \in P^i(x_i)$ implies $\sum_{t=0}^{\infty} p(t) \cdot (x'(t) - \omega_i(t)) > 0$.
2. $y \in Y$, $p \cdot y = 0$, and $\limsup_{\tau} \sum_{t=0}^{\tau} p(t) \cdot y'(t) \le 0$ for $y' \in Y$.
3. $\sum_i (x_i - \omega_i) = y$

The positive part of valuation is net expenditure and the negative part of it is net income from trade. As usual net expenditure cannot exceed the net income in the budget set B^i. Definition 1 means that the allocation in the budget set is not strictly preferred to the equilibrium allocation for each consumer.

Definition 2 is a form of the profit maximization condition with constant returns to scale technology. It is not necessary for every production plan to be evaluated by the equilibrium price system. Thus definition 2 requires that no production plan in the technology set can get strictly positive profit in the long run. Definition 3 expresses the feasibility of a competitive equilibrium allocation.

The step of our proof is the following: we use Negishi approach to find a quasi-equilibrium by exploiting theorem 3–1. After that we show the quasi-equilibrium is actually a competitive equilibrium 8by using monotonicity and adequacy assumption. In order to apply Negishi approach we must show the utility possibility set is compact in the first place.

Lemma 4-1 : Define $\overline{F} = \{(z_1, \cdots, z_H) : z_i \in C^i$ for all i and $(z_1, \cdots, z_H) \leq (x_1, \cdots, x_H)$ where $\sum_i (x_i - \omega_i) \in Y;\ x_i \in C^i$ for all $i\}$. We call $\overline{F}$ a feasible set. $\overline{F}$ is non-empty, convex, and compact in the product $\prod_H E$ with respect to the product $\prod_H \sigma(E, W)$ topology.

Proof) From assumption (7) (adequacy assumption), there is $\overline{x}_i \in C^i$ and $\overline{y}_i \in Y$ such that $\overline{x}_i - \omega_i < \overline{y}_i$. Let $\overline{x'}_i = \overline{x}_i + (\overline{y}_i - \overline{x}_i + \omega_i) = \overline{y}_i + \omega_i$. By the monotonicity assumption, $\overline{x'}_i \in C^i$ holds. Since $\sum_i (\overline{x}_i - \omega_i) = \sum_i \overline{y}_i \in Y$, $\overline{F}$ is non-empty. Clearly $\overline{F}$ is convex by the convexity of C^i and Y.

Let $F = \{(x_1, \cdots, x_H) : x_i \in C^i$ for all i and $\sum_i (x_i - \omega_i) \in Y\}$. We claim that F is a closed subset of the topological product of $\prod_H E$. Let $\{x^\alpha = (x_1^\alpha, \cdots, x_H^\alpha)\}$ a converging net in F with the limit $\hat{x}_i$ with respect to $\sigma(E, W)$ for each i. Since C^i is $\sigma(E, W)$-closed, $\hat{x}_i \in C^i$ for each i. Consider the topological sum of E, $\sum_H E$. Then an open set V in $\sum_H E$ can be represented as $V = \sum_i V_i$ where V_i is an open set in E for each i. Since the sum of open sets is open (Scheafer(1966) p.13) , V is $\sigma(E, W)$-open. Since Y is closed it follows that $\sum_i (\hat{x}_i - \omega_i) \in Y$. Thus $(\hat{x}_1, \cdots, \hat{x}_H) \in F$, and hence F is closed.

Note that $(\sum_i (C^i - \{\omega_i\})) \cap Y$ is bounded from assumptions (1) and (6). Indeed $\sum_i (C^i - \{\omega_i\})$ is bounded below by Hb. Then from assumption (6), $(\sum_i (C^i - \{\omega_i\})) \cap Y$ has an upper bound $a \in E$. Then if $(x_1, \cdots, x_H) \in F$, we can see that $b \leq x_i \leq a - \sum_{j \neq i} x_j \leq a - (H - 1)b$ for all i. It follows that F is a compact set due to $F \subset \prod_H [b, a - (H - 1)b]$. Recall that any order interval of Riesz symmetric dual space is $\sigma(E, W)$-compact. Then by Tychonoff's theorem $\prod_H [b, a - (H - 1)b]$ is compact in the product topology $\prod_H \sigma(E, W)$. Therefore F is compact since any closed set in a compact set is compact.

Let $\{z^\alpha = (z_1^\alpha, \cdots, z_H^\alpha)\}$ in $\overline{F}$ which is converging net with the limit $\hat{z} = (\hat{z}_1, \cdots, \hat{z}_H)$. Then we can take $(x_1^\alpha, \cdots, x_H^\alpha)$ such that $x_i^\alpha \in C^i$ for all i and $\sum_i (x_i^\alpha - \omega_i) \in Y$ (thus $(x_1^\alpha, \cdots, x_H^\alpha) \in F$) and $(z_1^\alpha, \cdots, z_H^\alpha) \leq (x_1^\alpha, \cdots, x_H^\alpha)$. Since F is compact there exist a converging subnet of $\{x^{\alpha(k)}\}$ such that $x_i^{\alpha(k)} \to \hat{x}_i$ and $(\hat{x}_1, \cdots, \hat{x}_H) \in F$. Then $(z_1^{\alpha(k)}, \cdots, z_H^{\alpha(k)}) \leq (x_1^{\alpha(k)}, \cdots, x_H^{\alpha(k)})$ and a subnet $\{z_i^{\alpha(k)}\}$ converges to $\hat{z}_i$ since $z_i^\alpha \to \hat{z}_i$ and $\sigma(E, W)$ is Hausdorff topology. Thus $(\hat{z}_1, \cdots, \hat{z}_H) \leq (\hat{x}_1, \cdots, \hat{x}_H)$ holds. Also $\hat{z}_i \in C^i$ from the $\sigma(E, W)$-closedness of C^i for each i. Thus $\overline{F}$ is closed. As the same way as before, $\overline{F} \subset \prod_H [b, a - (H - 1)b]$ holds, and so $\overline{F}$ is compact. ■

Next lemma shows that there is a utility representation for our preference. We need this because we use Negishi approach to prove the existence of a competitive equilibrium.

Lemma 4–2 : Define $G_i = \pi_i(\overline{F})$: the projection of $\overline{F}$ into C^i. Then for all i there exists a $\sigma(E,W)$-continuous function $u_i : G_i \to \mathbb{R}$ such that $x_i \in P^i(z_i)$ if and only if $u_i(x_i) > u_i(z_i)$.

Proof) From lemma 4–1, $\overline{F}$ is compact. Since π_i is continuous, G_i is compact. Moreover, the preference P^i is continuous, transitive, irreflexive, and convex, and $R^i(x)$ is the closure of $P^i(x)$ for all $x \in C^i$. Thus, we can apply Proposition 1 in Boyd–McKenzie (1993).[4] It assures the existence of a desired continuous function u_i. ■

Define $\Delta = \{s = (s_1, \cdots, s_H) \in \mathbb{R}^H_+ : s_1 + \cdots + s_H = 1\}$ and $U = \{(u_1(x_1), \cdots, u_H(x_H)) : (x_1, \cdots, x_H) \in \overline{F}\}$, and $\rho(s) = \sup\{\alpha > 0 : \alpha s \in U\}$.

Lemma 4–3 : $\rho(s)$ is well defined for $s \in \Delta$ and $\rho : \Delta \to \mathbb{R}$ is a continuous function.

Proof) Since G_i is compact and $u_i : G_i \to \mathbb{R}$ is $\sigma(E,W)$-continuous, Weiersraus's theorem implies that there exists $a_i, b_i \in G_i$ such that $u_i(a_i) \le u_i(x) \le u_i(b_i)$ for $x \in G_i$. Thus without loss of generality we can assume $u_i(a_i) = 0$ for each i. From the adequacy assumption there exists $\overline{x}_i \in C^i$ and $\overline{y}_i \in Y$ such that $\overline{x}_i - \omega_i < \overline{y}_i$. Let $\overline{x'}_i = \overline{y}_i + \omega_i$. Then $\overline{x'}_i \in G_i$ and $u_i(\overline{x'}_i) > u_i(\overline{x}_i) \ge 0$ hold for each i. Thus $\{(z_1, \cdots z_H) : 0 \le (z_1, \cdots, z_H) \le (u_1(\overline{x'}_1), \cdots, u_H(\overline{x'}_H))\} \subset U$ and hence $\rho(s)$ is well defined.

We claim that $\rho(s)$ is continuous. Let $\alpha > 0$ satisfying $\alpha s \in U$ and let $0 < \beta < \alpha$. Pick $(x_1, \cdots, x_H) \in \overline{F}$ such that $\alpha s = (u_1(x_1), \cdots, u_H(x_H))$. Then by continuity of the function u_i there exists some $0 < \delta < 1$ such that $(u_1(\delta x_1), \cdots, u_H(\delta x_H)) > \beta s$. let $s_n \to s$, then we know that $(u_1(\delta x_1), \cdots, u_H(\delta x_H)) > \beta s_n$ holds for sufficiently large n. Note that $0 \le (z_1, \cdots, z_H) \le (z^*_1, \cdots, z^*_H)$ and $(z^*_1, \cdots, z^*_H) \in U$ implies that $(z_1, \cdots, z_H) \in U$ from the construction. Therefore $\beta s_n \in U$ and so $\beta \le \rho(s_n)$ holds for sufficiently large n. Thus $\beta \le \liminf_{n \to \infty} \rho(s_n)$ holds for all $0 < \beta < \alpha$. Consequently $\alpha \le \liminf_{n \to \infty} \rho(s_n)$ for all $\alpha > 0$ with $\alpha s \in U$. Therefore $\rho(s) \le \liminf_{n \to \infty} \rho(s_n)$ holds.

[4] Although they use the product topology on s, the argument same as theirs still applies to our setting with $\sigma(E,W)$ as well. The crucial fact in their argument is the connectedness of unit interval $[0,1]$.

Next let $\rho(s) < \beta$. Fix r with $\rho(s) < r < \beta$. Since $s_n \to s$ and $rs < \beta s$, $rs < \beta s_n$ holds for sufficiently large n. Suppose $\beta s_n \in U$, then $rs \in U$. This, however, contradicts to $\rho(s) < r$. Therefore $\beta s_n \notin U$ holds for sufficiently large n. It follows that $\limsup_{n\to\infty} \rho(s_n) \leq \beta$ for all β with $\rho(s) < \beta$ since $\rho(s_n) \leq \beta$ holds for sufficiently large n. Therefore $\limsup_{n\to\infty} \rho(s_n) \leq \rho(s)$ holds. Together with the previous results we have $\lim \rho(s_n) = \rho(s)$ and so $\rho(s)$ is continuous. ■

Define a quasi-equilibrium as follows:

Definition: The pair of an allocation and price system $((x_1, \cdots, x_H, y), p) \in C^1 \times \cdots \times C^H \times Y \times s^n$ is a quasi equilibrium if :

1. For each i, $\sum_{t=0} p(t) \cdot (x(t) - \omega_i(t)) \leq 0$ and $x \in R^i(x_i)$ implies $\sum_{t=0} p(t) \cdot (x(t) - \omega_i(t)) \geq 0$.
2. $y \in Y$, $\sum_{t=0} p(t) \cdot y^0(t) = 0$, and $y^0 \in Y$ implies $\limsup_{\tau} \sum_{t=0} p(t) \cdot y^0(t) \leq 0$.
3. $\sum_i (x_i - \omega_i) = y$.

Lemma 4–4 : There is a quasi-equilibrium $((x_1, \cdots, x_H, y), \pi_c)$ with a price system $\pi_c \in l_\infty^+ \setminus \{0\}(\subset s^n)$.

Proof) For each $s \in \Delta$ there exists an allocation $(x_1^s, \cdots, x_H^s, y^s)$ satisfying $\rho(s)s = (u_1(x_1^s), \cdots, u_H(x_H^s))$ and $\sum_i (x_i^s - \omega_i) = y^s$. Note that any allocation which satisfies the above equalities is weakly Pareto optimal.

From lemma 3–2, we can well define the following set :

$$P(s) = \{\pi \in ba : \|\pi\|_{ba} = 1, \pi \cdot v \geq 0 \text{ for all } v \in (\sum_i F^i(x_i^s) - Y) \cap l_\infty, \pi \neq 0, \pi \geq 0\}.$$

$P(s)$ is nonempty and convex. Now for each $s \in \Delta$ we define the set :

$$\Phi(s) = \{(z_1(s), \cdots, z_H(s)) \in R^H : z_i(s) = \sum_{t=0}^{\infty} \pi_c(t)(x_i^s(t) - \omega_i^s(t)) \text{ for all } i,$$

$$\text{where } \pi_c \text{ satisfies } \pi = \pi_c + \pi_f \text{ for some } \pi \in P(s).\}$$

Since $P(s)$ is nonempty from lemma 3–2 and it is convex, $\Phi(s)$ is nonempty and convex. We claim that $\Phi(s)$ is uniformly bounded in R^H independent of s and Φ has a closed graph.

First we show that $\Phi(s)$ is uniformly bounded. Since $P(s) \subset P = \{\pi \in ba^+ : \|\pi\|_{ba} \le 1\}$. By Alaoglu's theorem (Dunford and Schwartz(1958) p. 424), P is $\sigma(ba; l_1)$-compact. By the $\sigma(ba; l_1)$-continuity of $\pi \cdot f$ for $f \in l_1$, we can apply Weierstraus' theorem and conclude that there exists $\bar{\pi} \in P$ such that $\bar{\pi} \cdot b \le \pi \cdot b$ for all $\pi \in P$. Remember $b \in l_1$ is a lower bound of the net trading sets and $b \le x_i^s - \omega_i$ for any $s \in \Delta$. Since $\pi \ge 0$ implies $z_i(s) = \sum_{t=0}^{\infty} \pi_c(t) \cdot (x_i^s(t) - \omega_i(t)) \ge \sum_{t=0}^{\infty} \pi_c(t) \cdot b(t)$, $z_i(s)$ has a uniform lower bound. Note that $\sum_i z_i(s) = \sum_{t=0}^{\infty} \pi_c(t) \cdot (\sum_i (x_i^s(t) - \omega_i(t)) = \sum_{t=0}^{\infty} \pi_c(t) \cdot y^s(t) \le \limsup \sum_{t=0}^{T} \pi_c(t) \cdot y^s(t) \le 0$ holds. Therefore

$$z_i(s) = -\sum_{i \ne j} z_j(s) \le -(H-1) \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot b(t)$$

follows. Let $\delta = H \left| \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot b(t) \right|$. Then $z(s) \in \Phi(s)$ implies $|z_i(s)| \le \delta$ for each i and $s \in \Delta$. Thus $\Phi(s)$ is uniformly bounded independent of s in R^H.

Next we define a nonempty, compact, and convex subset of R^H:

$$T = \{t = (t_1, \cdots, t_H) \in R^H : \|t\|_1 = \sum_i |t_i| \le H\delta\}.$$

From the uniform boundedness of $\Phi(s)$, $\Phi(s) \subset T$ holds for every $s \in \Delta$. Recall that $U(\overline{F})$ is compact by the compactness of $\overline{F}$ and the continuity of u_i for all i. Thus $U(\overline{F}) \subset R^H$ has upper bound $\alpha = (\alpha_1, \cdots, \alpha_H) \in R^H_+$. Fix some $\eta > H^2 \delta A$ where $A = \sum_i \alpha_i$. Let $r(s) = \sum_i u_i(x_i^s)$ and define the function $f : \Delta \times T \to \Delta$ by

$$\begin{aligned} f(s;t) = \Big(& [s_1 + \eta^{-1} t_1 r(s)]^+ / \sum_i [s_i + \eta^{-1} t_i r(s)]^+, \cdots, \\ & [s_H + \eta^{-1} t_H r(s)]^+ / \sum_i [s_i + \eta^{-1} t_i r(s)]^+ \Big), \end{aligned}$$

where $x^+ = \max\{0, x\}$ for $x \in R$. We claim f is well defined. Indeed $\sum_i [s_i + \eta^{-1} t_i r(s)]^+ \ge \sum_i (s_i + \eta^{-1} t_i r(s)) = 1 + \eta^{-1} \sum_i t_i r(s)$ holds. We know that $-H\delta \le t_i \le H\delta$ and $0 \le \sum_i u_i(x_i^s) = r(s) \le A$. Then $-H\delta A \le t_i r(s) \le H\delta A$ holds for any $s \in \Delta$. Therefore

$$1 + \eta^{-1} \sum_i t_i r(s) \ge 1 - \eta^{-1} A \delta H^2 > 0$$

holds. Consequently f is well defined and continuous over $\Delta \times T$.

Finally we define the nonempty correspondence $\Psi : \Delta \times T \to 2^{\Delta \times T} \setminus \{\emptyset\}$ by

$$\Psi(s;t) = \{f(s;t)\} \times \Phi(s).$$

Ψ is convex valued. The fact Φ has a closed graph together with the continuity of f implies that Ψ has also closed graph. Thus we can apply Kakutani's fixed point theorem and the correspondence Ψ has a fixed point $(\bar{s}, \bar{t}) \in \Delta \times T$ such that $\bar{s} = f(\bar{s}, \bar{t})$ and $\bar{t} \in \Phi(\bar{s})$.

Pick some $\bar{\pi} \in P(\bar{s})$ such that $\bar{t}_i = \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot (\omega_i(t) - x_i^{s}(t))$. We claim $\bar{t}_i = 0$ for each i. Suppose $\bar{s}_i = 0$. Then $[\bar{s}_i + \delta^{-1}\bar{t}_i r(\bar{s})]^+ = [\delta^{-1}\bar{t}_i r(\bar{s})]^+ = 0$. Since $\delta^{-1} > 0$ and $r(\bar{s}) \geq 0$, $\delta^{-1}\bar{t}_i r(\bar{s}) = 0$ or $\bar{t}_i < 0$ must hold. On the other hand,

$$\sum_i \bar{t}_i = \sum_i \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot (\omega_i(t) - x_i^{\bar{s}}(t)) = \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot \Big(\sum_i (\omega_i(t) - x_i^{\bar{s}}(t))\Big)$$
$$= \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot y^{\bar{s}}(t) = 0.$$

Thus there must exists some j with $\bar{t}_j > 0$, and $[\bar{s}_j + \delta^{-1}\bar{t}_j r(\bar{s})]^+ = \bar{s}_j + \delta^{-1}\bar{t}_j r(\bar{s}) = \bar{s}_j$ follows. Then $\bar{t}_j r(\bar{s}) = 0$ and $\bar{t}_j = 0$ or $r(\bar{s}) = \sum_i u_i(x_i^{\bar{s}}) = 0$ holds. But $\bar{t}_j > 0$ implies $\sum_i u_i(x_i^{\bar{s}}) = 0$. Since for each i, $x_i \in C^i$ implies $u_i(x_i) \geq 0$, if $\sum_i u_i(x_i^{\bar{s}}) = 0$, then $u_i(x_i^{\bar{s}}) = 0$ holds for each i. Also by the adequacy assumption there exist $\bar{x}_i \in C^i$, $\bar{y}_i \in Y$ such that $\bar{x}_i - \omega_i < \bar{y}_i$ for each i. Let $x_i^0 = \bar{y}_i + \omega_i > \bar{x}_i$. Then by the monotonicity assumption, $u_i(x_i^0) > u(x_i^{\bar{s}})$ and $\sum_i (x_i^0 - \omega_i - \bar{y}_i) = 0$ hold. This, however. contradicts to the weakly Pareto optimality of $(x_1^{\bar{s}}, \cdots, x_H^{\bar{s}}, y^{\bar{s}})$. Thus $r(\bar{s}) \neq 0$ holds and it implies $\bar{s}_i > 0$ for all i.

Therefore $[\bar{s}_i + \delta^{-1}\bar{t}_i r(\bar{s})]^+ > 0$ holds and this implies $\bar{s}_i + \delta^{-1}\bar{t}_i r(\bar{s}) = \bar{s}_i$, or $\bar{t}_i r(\bar{s}) = 0$ for all i. Since $r(\bar{s}) \neq 0$ holds for all i, thus $\bar{t}_i = \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot (\omega_i(t) - x_i^{\bar{s}}(t)) = 0$ must hold for all i. Since theorem 3–1 implies $\sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot (x_i^0(t) - \omega_i(t)) \geq \sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot ((t) - \omega_i(t))$ for all $x_i^0 \in R^i(x_i^{\bar{s}})$, therefore

$$\sum_{t=0}^{\infty} \bar{\pi}_c(t) \cdot (x_i^0(t) - \omega_i(t)) \geq 0 \text{ holds for all } x_i^0 \in R^i(x_i^{\bar{s}}).$$

The profit maximization condition and the feasibility condition are already established in theorem 3–1. Therefore we obtain a quasi-equilibrium $(x_i^{\bar{s}}, \cdots, x_H^{\bar{s}}, y^{\bar{s}}, \bar{\pi}_c)$. ■

Theorem 4–1: There exists a competitive equilibrium $((x_1, \cdots, x_H, y), p)$ with a price system $p(\geq 0) \in l_1$, $p \neq 0$.

Proof) From lemma 4–4 we have a quasi-equilibrium $((x_1, \cdots, x_H, y), \pi_c)$. We claim this is actually a competitive equilibrium. Since the feasibility (condition 3. in the definition of a competitive-equilibrium) and profit maximization condition (condition 2. in the definition) are already met, we only have to show condition 1. holds at quasi-equilibrium.

By assumption (7)(the aggregate adequacy assumption), there exists $\bar{x}_i \in C^i$ and $\bar{y}_i \in Y$ such that $\bar{x}_i - \omega_i < \bar{y}_i$ and $\sum_i(\bar{y}_i - (\bar{x}_i - \omega_i)) = (\gamma; \gamma; \cdots)$ for some $\gamma (\in R^n) > 0$. Then we have $\bar{x}_i \in \overline{F}$,

$$\sum_{t=0}^{\infty} \pi_c(t) \cdot (\bar{x}_i(t) - \omega_i(t)) \le \limsup \sum_{t=0}^{\infty} \pi_c(t) \cdot \bar{y}_i(t) \le 0$$

for all i, and

$$\begin{aligned} \sum_i \Big(\sum_{t=0}^{\infty} \pi_c(t) \cdot (\bar{x}_i(t) - \omega_i(t))\Big) &= \sum_{t=0}^{\infty} \pi_c(t) \cdot \Big(\sum_i (\bar{x}_i(t) - \omega_i(t))\Big) \\ &< \limsup \sum_{t=0}^{\infty} \Big(\pi_c(t) \cdot \sum_i \bar{y}_i(t)\Big) \le 0. \end{aligned}$$

Thus there exists at least for some j satisfying

$$\sum_{t=0}^{\infty} \pi_c(t) \cdot (\bar{x}_j(t) - \omega_j(t)) < 0. \qquad (12)$$

Then from lemma 4–4 we know $\bar{x}_j \notin R^j(x_j)$ for $j \in I_1$. For $x_j^0 \in P^j(x_j)$, define $z_\theta = \theta x_j^0 + (1 - \theta)\bar{x}_j$ for each $\theta \in (0;1)$. Note that from lemma 4 - 4, $\sum_{t=0}^{\infty} \pi_c(t) \cdot (x_i^0(t) - \omega_i(t)) \ge 0$ holds. Now since P^j is $\sigma(E;W)$-open valued in C^j from assumption (2) and x_j^0 is in $P^j(x_j)$, there exists $\theta_j \in (0;1)$ such that $z_{\theta_j} \in P^j(x_j)$. Moreover,

$$\begin{aligned} & \sum_{t=0}^{\infty} \pi_c(t) \cdot (z_{\theta_j}(t) - \omega_j(t)) \\ = & \sum_{t=0}^{\infty} \pi_c(t) \cdot [\theta_j(x_j^0(t) - \omega_j(t)) + (1 - \theta_j)(\bar{x}_j(t) - \omega_j(t))] \\ = & \ \theta_j \sum_{t=0}^{\infty} \pi_c(t) \cdot ((x_j^0(t) - \omega_j(t)) + (1 - \theta_j) \sum_{t=0}^{\infty} \pi_c(t) \cdot (\bar{x}_j(t) - \omega_j(t)) \end{aligned}$$

holds since $\sum_{t=0}^{\infty} \pi_c(t) \cdot (\bar{x}_i(t) - \omega_i(t)) < 0$ holds and has a only finite value (or $+\infty$). Thus, if $\sum_{t=0}^{\infty} \pi_c(t) \cdot (x_j^0(t) - \omega_j(t)) = 0$ holds, then we have $\sum_{t=0}^{\infty} \pi_c(t) \cdot (z_{\theta_j}(t) - \omega_j(t)) < 0$. This implies $z_{\theta_j} \notin R^j(x_j)$ from lemma 4–4. This is, however, a contradiction. Thus $\sum_{t=0}^{\infty} \pi_c(t) \cdot (x_j^0(t) - \omega_j(t)) \neq 0$ and hence we have for every $j \in I_1$ that

$$x_j^0 \in P^j(x_j) \text{ implies } \sum_{t=0}^{\infty} \pi_c(t) \cdot (x_j^0(t) - \omega_j(t)) > 0. \qquad (13)$$

Condition 3. of competitive equilibrium holds for the consumers in I_1.

Denote I_1 for the set of consumers satisfying $\pi_c \cdot (x_i^0 - \omega_i) < 0$ for some $x_i^0 \in C_i$. From (12) we know $I_1 \neq \emptyset$. Let I_2 be its complementary set in I. From the definition, for any $i \in I_2$,

$$x_i^0 \in C_i \text{ implies } \pi_c \cdot (x_i^0 - \omega_i) \geq 0. \tag{14}$$

By using the argument similar to the one yielding (13) from (12), we can show that (13) holds for the consumers in I_1. Thus, it is enough to show $I = I_1$ to prove that $((x_1, \cdots, x_H, y), \pi_c)$ is a competitive equilibrium.

Suppose I_2 is non-empty. From the irreducibility assumption, we know that there are $\alpha_i > 0$ and x^i with $x^i \in C_i$ for $i \in I_2$, and $x^j \in P^j(x_j)$ for $j \in I_1$ such that $y^0 = \sum_{j \in I_1}(x^j - \omega_j) + \sum_{i \in I_2} \alpha_i (x^i - \omega_i) \in Y$. Since $\pi_c \cdot (x^i - \omega_i) \geq 0$ holds for any $i \in I$, we have

$$\pi_c \cdot y^0 = \pi_c \cdot \sum_{j \in I_1} (x^j - \omega_j) + \pi_c \cdot \sum_{i \in I_2} \alpha_i (x^i - \omega_i) \leq 0 \tag{15}$$

from the profit maximization condition in lemma 4–4. Since (13) holds for any $j \in I_1$, $\pi_c \cdot (x^j - \omega_j) > 0$, and hence

$$\sum_{j \in I_1} \pi_c \cdot (x^j - \omega_j) > 0 \tag{16}$$

holds.

Now consider $i \in I_2$. Then, the fact that $x^i \in C_i - \{\omega_i\}$ for some $\sigma_i > 0$ yields

$$\pi_c \cdot \alpha_i (x^i - \omega_i) \geq 0 \text{ for any } i \in I_2,$$

and so

$$\sum_{i \in I_2} \pi_c \cdot \alpha_i (x^i - \omega_i) \geq 0 \tag{17}$$

holds. (16) and (17) are, however, a contradiction to (15). Thus, this contradiction implies $I_2 = \emptyset$ or $I_1 = I$. Therefore condition 3. of competitive equilibrium holds for each consumer i at $((x_1, \cdots, x_H, y), \pi_c)$, and $((x_1, \cdots, x_H, y), \pi_c)$ is a desired competitive equilibrium. ■

5 Conclusion

It has been shown that the regularity assumption is sufficient condition for the second fundamental theorem of welfare economics with general consumption sets in economies over an discrete time infinite horizon. There is a combination of the separate conditions on preferences, consumption sets and the production set which implies the regularity assumption. If preferences are $\sigma(E; W)$- continuous, consumption sets contain the lower bounds in l_∞ and the aggregate adequacy assumption is satisfied then the exclusion assumption on the production set implies the regularity assumption. In other words if consumers are myopic enough to supply some constant positive net supply in the far future and a production can be stopped at some period then the regularity assumption is satisfied. In order to obtain an equilibrium with price system in l_1, Bewley(1972) assumed the exclusion assumption , consumption set is positive orthant of l_∞; and ω_i is in the interior of positive orthant of l_∞: Our theorem assures that equilibrium price system is actually in l_1 with general consumption set without interiority assumption for endowment when $E = l_\infty$ and $W = l_1$: Thus our result is a generalization of Bewley's result in the case of l_∞.

Assumption 7 in Boyd–McKenzie(1993) implies that both of the regularity assumption and the aggregate adequacy assumption holds. They use their assumption 7 in order to translate Edgeworth equilibrium into a competitive equilibrium. Our separation argument shows that we can substitute the regularity assumption of Boyd–McKenzie(1993) with our regularity assumption for the same purpose. This is possible by virtue of our new price normalization instead of theirs. Thus in the setting of this paper Edgeworth equilibrium can be translated into a competitive equilibrium based on our regularity assumption. It implies the equivalence of a competitive equilibrium allocation and Edgeworth euilibrium allocation under our regularity assumption and irreducibility. The equivalence holds with stronger regularity assumption and the strong irreducibility assumptiom in Boyd–McKenzie(1993) .

Our results might be extended to economies with general convex production set drawing on the regularity assumption and the same approach used here.

Comparing the uniform properness assumption with the regularity assumption would be interesting. Both assumptions are closely relevant to the marginal rate of substitutions of preferences and production.

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