# The Impact of Selling Information on Competition[1]


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[2]First draft. Do give comments! Which journal???

**Abstract**

We consider a homogenous good oligoply with identical consumers who learn about prices either by (sequentially) visiting firms or by consulting a price agency who sells information about which firm charges the lowest price. In the sequential equilibrium with maximal trade and minimal search, prices are dispersed and consumers randomize between consulting a price agency and buying at the first firm encountered. Low competition among price agencies induces maximal price dispersion. High competition among firms leads to offers that are either ripoffs or bargains, most consumers visit a price agency and firms' profits are small.

Keywords: intermediary, sequential search, price dispersion.

JEL Classification: ??

# 1 Introduction

As the number of products increases and markets globalize we find an increasing demand for information, in particular, about where products are sold at what price. This has been creating an increasing supply of information. The internet is a growing source of free information; many sites offer information on the location of the firm selling a given product the cheapest (e.g., search routines among internet book stores are very popular). With the expansion of the internet, opportunity costs for finding the appropriate sites have been rising too. Sources that explicitly sell information include newspapers, shopping clubs and price agencies. A *price agency* is a firm who sells information about the cheapest seller of a given product. In recent years, a particular form of price agency has been rapidly spreading from Germany, Austria, Switzerland to the rest of Europe. The consumer is charged a percentage of his savings either relative to the lowest price known to him prior to his visit to the price agency or relative to the producers recommended price.[1]

Consumers can profit two-fold from buying information. Directly, they are guided to the locations of firms who charge lower prices. Indirectly, an increase in the informedness of some consumers creates more competition among the firms to the benefit of all consumers. The service of the price agency is a public good. In particular, when a known set of firms can only be distinguished according to the price they charge for a good then all consumers wishing to purchase this good will not choose to visit a price agency. Market prices would otherwise reflect perfect information and thus eliminate any additional value of visiting a price agency. However, when the goods are valued differently by each consumer (e.g., according to transportation costs from the seller to the buyer) or when it is unknown which firms sell the good then price agencies can have additional value not reflected in increased competition. Here it can pay for each consumer to visit the price agency (cf. Hänchen and von Ungern-Sternberg, 1985).

We aim to explore the impact of costly information providers on price formation, profits and consumer behavior and informedness. We focus on the public good effects of price agencies and assume that their only service is to provide information about prices. We consider a simpler payment scheme of the price agencies than the one described above and assume that a price agency informs its customer about the lowest market price for a fixed fee (or tariff). In the conclusion we discuss more general payment schemes along with some incentive issues of price agencies. The price agency tariff is treated in three

[1](*unverbindliche Preisemfehlung* in German)

treatments: (i) exogenous, (ii) set by a monopoly or by colluding price agencies and (iii) determined through competition among price agencies. Apart from the reference above, this is the first investigation (to our knowledge) in which the price of information in a goods market is endogenous.

The underlying goods market is modelled in the simplest possible way. A homogenous good is supplied by finitely many identical firms without cost or capacity constraints. Finitely many consumers each demand one unit of the good. A finite number of price agencies each charge a flat fee (or tariff) for informing a consumer about which firm is charging the lowest price. In the first part of the paper we consider the strategic interaction between consumers and firms while keeping the tariff of price agencies fixed at an exogenously given commonly known level. The sequence of moves is as follows. First firms simultaneously set prices observable by price agencies but unobservable by consumers. Then each consumer independently and sequentially gathers information about prices, either directly by visiting one of the firms or indirectly by consulting a price agency. Each time a consumer visits a firm he can decide to buy the good at this firm, to visit some other firm, to visit a price agency or to terminate search and to not buy the good at all. It is assumed costly for a consumer to gather information; the first visit to any firm costs $s > 0$.[2]

We choose to solve our model using standard game theory, ruling out uncredible threats by focussing on sequential equilibria. In particular, deviations from equilibrium play can only be learnt from observations. Thus we depart from much of the search literature (e.g., Diamond 1971, Hänchen and von Ungern-Sternberg 1985) that assumes the actual distribution of prices to be known. In addition, we limit our attention to efficient (more precisely, welfare maximizing) equilibria where each consumer buys the good with the minimal number of visits to a firm. In these equilibria, consumers either go directly to a price agency or buy the good at the first firm they visit. Equilibrium conditions ensure that consumers do not prefer to continue their search (or to go to a price agency) after they have visited the first firm.

Typical for such sequential search models (see McMillan and Rothschild 1994), there is no trade if the price agencies' tariff is too large. Without potential customers coming via a price agency, firms have an incentive to raise their price as they take advantage of the additional search costs involved should a consumer choose to visit another firm.

[2] Technically this means sequential search without replacement with free recall.

Notice that this argument is based on the fact that consumers do not learn about a price increase until they actually visit the firm. Thus, if no consumer visits a price agency, firms charge the monopoly price. Consequently, consumers choose not to buy the good and not to visit any firm as they cannot cover search costs of the first visit. A simple corollary is that some consumers will visit the price agency with positive probability whenever trade occurs in equilibrium.

When the services of price agencies are less expensive and some consumers visit a price agency, a firm who raises his price trades off earning more from uninformed consumers and being less likely to be the cheapest in the market and thus lowering the expected number of sales to consumers who visited a price agency. For a sufficiently small price agency tariff efficient equilibria exist and are completely characterized. In these equilibria, firms randomize among prices belonging to a closed interval. Expected market prices lie above the price agencies' tariff as consumers are indifferent between buying directly at the first firm and going to a price agency. When only few (two or three) firms are competing in the market, comparative statics reveal that higher search costs cause expected prices to decrease which can make consumers better off. Although an increase in search costs directly reduces consumer utility it also generates a substitution effect through making price agencies more attractive with higher demand for price agencies' services pushing down expected prices. When there are many firms then most consumers go to a price agency and aggregate firm profits are very small. Firms then mostly either charge a very low (bargain) or a relatively high price (ripoff) (compare to Salop and Stiglitz 1977).

Next we include price agencies as players and assume that they choose their (publicly observable) tariff before firms set prices. Colluding price agencies (such as a price agency monopoly) that maximize their joint profits act as if they do not care about profits and only choose their tariff to maximize price dispersion in the market. When individual search costs are small, then unlike the exogenous tariff setting analyzed above, consumers are always better off under lower search costs. When there are many firms then price agencies obtain almost all surplus.

Finally we briefly consider the effects of competition among price agencies. Should this lead to low tariffs then most consumers will visit the price agency and market prices will be low.

We proceed as follows. In Section 2 we present and analyze the basic model without price agencies. In Section 3 we add price agencies and assume tariffs to be exogenous in Subsection 3.1. In Subsections 3.2 and 3.3 we then analyze collusion and competition.

Section 4 discusses both the literature and the incentive problems of price agencies. The appendix contains the explicit calculations for the special case where there are either two or three firms in the market.

## 2 Individual Search

First we analyze market behavior without price agencies which also results when price agencies' tariffs are too high. Consider the following oligopoly market with search frictions. $n$ identical, risk neutral firms produce a homogenous good at marginal cost 0 (there are neither fix costs nor supply constraints). A finite number of identical, risk neutral consumers each has demand for one unit of the good and a reservation price of 1. To simplify notation, we normalize total demand to 1 which makes all equations appear as if there is only one consumer. Consumers only learn the price offered by a given firm after visiting this firm. We assume that a consumer incurs a cost $s \in (0, 1)$ each time she visits a firm for the first time. $s$ is meant to reflect frictions when gathering information from an unfamiliar source. Later visits of the same firm are thus assumed costless.

We consider the following sequence of moves. At the outset, firms simultaneously choose their price for the good. These prices are not observed by the consumers. Then each consumer either chooses a firm to visit or decides not to buy the good. Once visiting a firm, a consumer can either purchase the good at this firm at the price offered, visit another, possibly previously visited firm, or decide to quit the search and to not buy the good from any firm. The game ends either when the good is purchased or when the consumer decides to terminate his search and to not buy the good at all.

The strategy of a firm $i$ can be identified with a cumulative distribution of prices $F_i$. The strategy of a consumer can be very complicated, although two simple pure strategies are immediate: "do not search and hence do not purchase" and "purchase at any price at firm $i$". Throughout this paper we will analyze symmetric Nash equilibria in which each firm chooses the same c.d.f. $F$ and each consumer uses the same purchasing strategy. Our game has many symmetric Nash equilibria, e.g., each firm charges the price 0 and each consumer visits a random firm and purchases the good there if this firm charges 0 and otherwise does not purchase the good. Notice that consumers' threat in this equilibrium not to buy when a firm charges price $s/2$ is not credible. Given search costs $s$, a consumer would be better off buying at $s/2$ than not purchasing the good at all or purchasing the good at a different firm. In the following we restrict attention to Nash equilibria that are not based on such incredible threats. Formally we will restrict attention to sequential equilibria. Typical to models with sequential search (Diamond 1971, McMillan and Rothschild 1994), we obtain market failure in equilibrium.

**Proposition 1** *There is a unique sequential equilibrium: firms charge the monopoly price 1 and consumers immediately decide not to purchase the good.*

The proof of the inexistence of pure strategy equilibria can be stated verbally. Consumers are willing to pay any price below 1 when they are visiting a firm (since search costs $s$ are sunk). Thus, all firms charge 1; the firm charging the minimum market price strictly below 1 can otherwise raise its price slightly (by less than $s$) without loosing its clients. Consumers anticipate these high prices and decide not to visit any firm (and thus not to purchase the good) as they cannot cover their search cost $s > 0$ of visiting the first firm. The more general proof for mixed strategies is analogous and therefore omitted.

# 3 Search in a Market with Price Agencies

In models of sequential search there is too little competition among firms. Firms exploit consumers visiting their firm which induces consumers not to start to search in the first place. Things change when we add price agencies. A price agency is assumed to know all prices in the market and sells this information to consumers at a publicly known price (or tariff) $b > 0$. Consumers are not able (or allowed) to resell their information to other consumers. When a consumer acquires the services of the price agency, all information necessary for the transaction are included. So we assume that the consumer does not incur any further cost when going to the firm specified by the price agency. On the other hand, the consumer's visit to the price agency also involves individual costs which we assume for simplicity to be equal to $s$.

## 3.1 Exogenous Tariffs

Assume at first that all price agencies charge the same exogenous price $b > 0$ for their services. Notice that the existence of price agencies does not preclude market failure. It is immediate from the analysis in Section 2 that regardless of the size of $b$ there is always a sequential equilibrium in which all firms charge the monopoly price 1 and all consumers decide at the outset to not purchase the good. In order for there to be trade in a sequential equilibrium, some consumers must visit the price agency with positive probability. A visit to a price agency is only profitable when there is uncertainty about the prices in the market. Since consumers know the equilibrium strategy of each firm this means that some firms choose a mixed pricing strategy. However, not all consumers will choose to visit the price agency. Otherwise, Bertrand competition is induced among the firms as only firms offering the lowest price will get customers. Firms will consequently offer price 0 and consumers will purchase directly at firms without going to the price agency first.

In the rest of the paper we restrict attention to symmetric sequential equilibria that are efficient in the sense that each consumer buys the good and the expected number of times the cost $s$ is incurred is minimized. In these equilibria, each consumer randomizes between going directly to a firm to purchase the good and going directly to a price agency to then purchase the good at a firm charging the lowest price.

In the following we will derive necessary properties of these equilibria. Let $\gamma \in (0,1)$ be the probability that a consumer goes directly to a price agency. Equilibrium prices will be contained in $[0,1]$. A firm raising her price from $p$ to $p'$ yields a trade-off between lowering the probability of receiving customers that first went to the price agency (provided $F(p) < F(p')$) and raising profits per sale. Hence, firms will randomize over a closed interval of prices $[\alpha,\beta]$, i.e., $F$ is strictly increasing on $(\alpha,\beta)$ with $[\alpha,\beta] \subseteq [0,1]$. Let $\bar{p} = \int p dF(p)$ be the expected price offered in equilibrium by a firm.

Next we calculate $F$ as a function of $\beta$. In order to give incentives to firms to choose the same pricing strategy, each consumer who decides to go directly to a firm must be equally likely to visit each firm. Hence, with probability $\frac{1}{n}(1-\gamma)$ a consumer visits firm $i$ and purchases the good there. Firm $i$ also sells to all consumers that first went to the price agency if it chose the lowest price in the market. Indifference of firm $i$ over prices $p \in [\alpha,\beta]$ implies

$$\left[\frac{1}{n}(1-\gamma) + \gamma(1-F(p))^{n-1}\right] \cdot p = \frac{1}{n}(1-\gamma) \cdot \beta\ . \qquad (1)$$

Since $F(\alpha) = 0$ we obtain from (1) that

$$\alpha = \frac{1-\gamma}{1-\gamma+n\gamma}\beta$$

so

$$F(p) = F(p,\gamma,\beta) = \begin{cases} 0 & \text{if } 0 \le p < \frac{1-\gamma}{1-\gamma+n\gamma}\beta \\ 1 - \left[\frac{(1-\gamma)(\beta-p)}{np\gamma}\right]^{\frac{1}{n-1}} & \text{if } \frac{1-\gamma}{1-\gamma+n\gamma}\beta \le p \le \beta \\ 1 & \text{if } \beta < p \le 1 \end{cases} \tag{2}$$

where the corresponding density is given by

$$f(p) = \frac{\beta}{n-1}\left(\frac{1-\gamma}{n\gamma}\right)^{\frac{1}{n-1}} \frac{1}{(\beta-p)\,p}\left(\frac{\beta-p}{p}\right)^{\frac{1}{n-1}}$$

for $\alpha \le p < \beta$. In particular, the minimal price $\alpha$ always lies strictly above 0.

Given $\bar{p} = \int pf(p)\,dp$, we obtain

$$\begin{aligned} \bar{p} &= \lim_{z\to\beta}\int_{\frac{1-\gamma}{1-\gamma+n\gamma}\beta}^{z} \tilde{p} f(\tilde{p})\,d\tilde{p} = \frac{\beta}{n-1}\left(\frac{1-\gamma}{n\gamma}\right)^{\frac{1}{n-1}} \lim_{z\to\beta}\int_{\frac{1-\gamma}{1-\gamma+n\gamma}\beta}^{z} \frac{1}{\beta-\tilde{p}}\left(\frac{\beta-\tilde{p}}{\tilde{p}}\right)^{\frac{1}{n-1}} d\tilde{p} \\ &= \frac{\beta}{n-1}\left(\frac{1-\gamma}{n\gamma}\right)^{\frac{1}{n-1}} \lim_{z\to 1}\int_{\frac{1-\gamma}{1-\gamma+n\gamma}}^{z} \frac{1}{1-p}\left(\frac{1-p}{p}\right)^{\frac{1}{n-1}} dp \end{aligned} \tag{3}$$

using change of variables $p = \tilde{p}/\beta$ so $\tilde{p} = \beta p$ and $d\tilde{p} = \beta dp$. Hence, $h(\gamma) := \bar{p}/\beta$ is independent of $\beta$.

Each consumer buys the good and is indifferent between going directly to a firm and going first to a price agency. Hence,

$$s + b + p_{Bn} = s + \bar{p} \le 1 \ , \tag{4}$$

where $p_{Bk}$ denotes the expected minimum price in the market when $k$ firms randomize independently according to $F$. Moreover, a consumer who decides to go directly to a firm will not purchase the good if the price is above $s + \bar{p}$ since he always has the option to visit a second firm. Hence, $\beta \le s + \bar{p}$.

In the following we will derive additional properties of $F$ when $\beta = s + \bar{p}$ as we shall show below that $\beta = s + \bar{p}$ holds in these equilibria. Notice that this means that any consumer who goes directly to a firm where the good is offered at the maximal price $\beta$ is indifferent between buying the good at this firm and visiting another firm. Since $\beta = s + \bar{p} = s + \beta h(\gamma)$ we obtain

$$\beta = \frac{s}{1-h} \ .$$

The profit of a firm equals what it earns by charging the highest price $\beta = s + \bar{p}$. It must also earn an equal share of what consumers pay directly at firms and what they pay for the good after visiting the price agency. Hence,

$$(s+\bar{p})(1-\gamma)\frac{1}{n}=(1-\gamma)\frac{1}{n}\bar{p}+\gamma\frac{1}{n}p_{Bn}\ ,$$

which holds if and only if

$$p_{Bn}=\left(\frac{1}{\gamma}-1\right)s\ . \tag{5}$$

Thus,

$$b=\bar{p}-\left(\frac{1}{\gamma}-1\right)s=\left(\frac{1}{1-h}-\frac{1}{\gamma}\right)s.$$

This leads us to the first main result.

**Proposition 2** *An efficient symmetric sequential equilibrium exists if and only if there exists* $\gamma\in(0,1)$ *that satisfies*

$$\beta=s+\bar{p}=\frac{s}{1-h}\leq 1\ \textit{and}\ b=\bar{p}-p_{Bn}=\beta-\frac{s}{\gamma}=\left(\frac{1}{1-h}-\frac{1}{\gamma}\right)s\ . \tag{6}$$

*In such an equilibrium, each firm chooses prices according to* $F$ *satisfying (2). Each consumer goes directly to the price agency with probability* $\gamma$ *and goes equally likely to one of the* $n$ *firms with probability* $1-\gamma$*. A consumer buys the good at any firm she visits if this price is less or equal to* $s+\bar{p}$*.*

The expected price $\pi_C$ paid by a consumer, the expected profit of a firm $\pi_f$ and the expected profit of the price agency $\pi_P$ in a given efficient symmetric sequential equilbrium are given by

$$\begin{aligned}\pi_C&=\bar{p}=\beta-s=\frac{h}{1-h}s\ ,\\ \pi_f&=\frac{1}{n}(1-\gamma)\beta=\frac{1-\gamma}{n(1-h)}s\ ,\\ \pi_P&=\gamma\beta-s=\left(\frac{\gamma}{1-h}-1\right)s.\end{aligned}$$

**Proof.** All we are left to show is $\beta=s+\bar{p}$ is necessary and that $\gamma\in(0,1)$ satisfying (6) actually induces a symmetric efficient sequential equilibrium. Notice that a firm has no incentive to offer a price below $\alpha$ since a firm setting $\alpha$ already almost surely attracts all consumers who visit a price agency.

First we show that a consumer will never choose to go to the price agency after visiting $k$ firms. Let $x$ be the smallest observed price. Assume that $x=\beta$. Then the expenditure including search cost of going to the price agency equals $s+b+p_{B(n-k)}$. Since $s+b+p_{B(n-k)} > s+b+p_{Bn} = s+\bar{p} \geq \beta$ a consumer offered the good at price $\beta$ strictly prefers buying over going to the price agency. Now consider the case where $x < \beta$. Then the disutility of going to the price agency equals

$$s+b+\int_0^x p \cdot (n-k) f(p) [1-F(p)]^{n-k-1} dp + [1-F(x)]^{n-k} x \ .$$

Using the fact that the derivative of the above expression with respect to $x$ equals

$$[1-F(x)]^{n-k}$$

we obtain that the consumer strictly prefers buying at a firm who charges $x$ over going to the price agency.

Finally, we show that each consumer buys at the first firm visited. Consider a consumer's behavior after searching $n-1$ firms with $x$ being the lowest price offered by these firms. Expenditure including search cost of going to the $n$-th firm equals

$$s+\int_0^x pf(p) \, dp + (1-F(x)) \, x \ ,$$

expenditure when not going to the $n$-th firm is $x$. We will show that the individual never strictly prefers to visit the $n$-th firm, i.e., that

$$x \leq s+\int_0^x pf(p) \, dp + (1-F(x)) \, x \qquad (7)$$

given $\beta \leq s+\bar{p}$. Notice that (7) is true when $x=\beta$. Moreover, the derivative of the right hand side with respect to $x$ for $x < \beta$ equals $1-F(x) < 1$and hence (7) holds for all $x \in [\alpha, \beta]$.

Now consider the choice of whether to visit the $(n-1)$-th firm. Since the $n$-th firm will never be visited it is as if there is not an $n$-th firm. By induction on the number of firms it then follows that each consumer buys at the first firm visited as long as $\beta \leq s+\bar{p}$. In particular, this implies that a firm will choose $\beta = s+\bar{p}$ since sales at price $\beta$ are only made to consumers who do not consult the price agency and consumers will buy if $p < s+\bar{p}$. This completes the proof. ■

Next we discuss when such equilibria exist together with some of their properties. It is easily verified that $h(0)=1$, $h(1)=0$ and that $h$ is continuous and decreasing in $\gamma$ as

$$h'(\gamma) = \frac{1}{(n-1)\gamma(1-\gamma)} \left[ -h + \frac{1-\gamma}{1-\gamma+n\gamma} \right] < 0 \ .$$

Given $0 < s < 1$ let $\tilde{\gamma} = \tilde{\gamma}(s) \in (0,1)$ be the unique solution to $h(\tilde{\gamma}) = 1 - s$. Then it follows from (6) that the probability that a consumer visits a price agency in one of our equilibria is at least $\tilde{\gamma}$. Now consider some $\gamma \in [\tilde{\gamma}, 1)$. Then the price distribution $F$ is well defined which implies that $\bar{p} - p_{Bn} > 0$. So if

$$b = \bar{p} - p_{Bn} = \left( \frac{1}{1 - h(\gamma)} - \frac{1}{\gamma} \right) s$$

then (6) is satisfied. Hence, any $\gamma \in [\tilde{\gamma}, 1)$ (and only these) can be supported by an efficient sequential equilibrium if the tariff $b$ is chosen appropriately. In particular, $\gamma = \tilde{\gamma}$ and $b = 1 - s/\tilde{\gamma} > 0$ can be supported in which case the largest price offered equals the monopoly price 1 (i.e., $\beta = 1$), expected price $\bar{p}$ is maximal and consumers are indifferent between not buying and buying the good (either at a random firm or via the price agency). When $\gamma > \tilde{\gamma}$ then all prices are bounded away from 1 and consumers strictly prefer to purchase the good. For $\gamma$ close to 1, the tariff $b$ supporting this equilibrium as well as the expected price is close to 0 (while the maximal price $\beta$ is close to $s$) as $\bar{p} \geq p_{Bn}$, $\bar{p} - p_{Bn}$ is continuous in $\gamma$ and $\bar{p}$ gets small as $\gamma$ gets close to 1. In particular,

**Corollary 3** *An efficient equilibrium exists when* $0 < b \leq 1 - s/\tilde{\gamma}$.

It remains unclear whether efficient equilibria only exist if $b \leq 1 - s/\tilde{\gamma}$ and whether equilibrium demand for price agencies is decreasing in the tariff (although both statements are easily verified for $n \in \{2, 3\}$, see appendix). Of course, efficient equilibria (as well as any other sequential equilibria with trade) fail to exist if $s$ is sufficiently small for given $b$ since a consumer strictly prefers visiting each firm over going to the price agency whenever $(n-1)s < b$. Similarly, equilibria with trade fail to exist when $s$ is sufficiently large for given $b$ since consumers will not participate in search whenever $b \geq 1 - s$ as this implies that $\bar{p} + s > 1$. When there are only three firms then the set of $(s, b)$ constellations where efficient equilibria exist can be calculated explicitly (calculations given in the appendix, regions illustrated in Figure 1).

When price agencies are sufficiently attractive (i.e., their tariff $b$ is sufficiently small) for given $s$ then efficient equilibria exist. Demand $\gamma$ for their services will be arbitrarily close to 1 and the expected market price will be arbitrarily close to 0. Almost all surplus will go to the consumers. When there are either two or three firms, then comparative statics for intermediate values of $b$ are also available as the demand for price agencies is decreasing in their price. Lowering $b$ decreases the maximal, the minimal and the expected

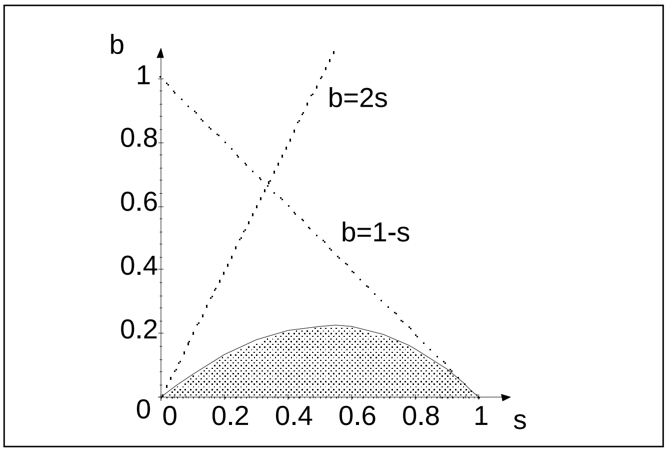


Figure 1: Shaded area: Parameter range where an efficient sequential equlbirium exists.

price of the good. This makes consumers better off and firms worse off (changes in profits of the price agency will be analyzed in the next section).

Efficient equilibria do not exist for extreme values of the individual search cost $s$ given $b$. Again, comparative statics can be obtained for intermediate values only when $n \in \{2,3\}$. Increasing $s$ leads to two opposite effects. Since $\bar{p}$ is increasing in $s$ for given $\gamma$ we find that a higher individual search cost has a tendency to raise prices. On the other hand, there is a substitution effect between individual search and going to the price agency. Increasing $s$ is like decreasing $b$. For given $b$, as $s$ increases, the demand for services of the price agencies increases. This causes a decrease in normalized prices $h = \bar{p}/\beta$. In the appendix we show that the latter effect dominates the former as we find that expected prices are decreasing in the search costs $s$. Of course, expected prices remain above $b$ so that eventually, when $s$ is too large then an efficient equilibrium fails to exist as $b = \bar{p} - p_{Bn}$ can no longer be satisfied. Whether or not consumers are in fact better off when search costs $s$ increase depends on the change of $s + \bar{p}$. The substitution effect is strongest when search costs are small (i.e., when equilibrium demand is small). For $n = 2$ we find that consumers are better off if and only if the equilibrium demand is below 0.635, for $n = 3$ this is the case when $\gamma$ is below 0.786. The next two figures show the graph of $\bar{p}$ as a function of $s$ where $b = 0.05$ and $n = 2$ or $n = 3$. In particular, we find that there is only a relatively small range of search costs $s$ where an increase in $s$ makes the consumers better off ($s \in (0.17, 0.274)$ for $n = 2$, $s \in (0.066, 0.14)$ for $n = 3$).

The calculations of the equilibrium strategies when there are either two or three firms (see appendix) also reveal the explicit pricing strategy of a firm. While the price density is decreasing for $n = 2$, it is u-shaped for $n = 3$; there is a higher concentration of prices at the extreme ends of the price interval $[\alpha, \beta]$ with the density at $\beta$ being unbounded (the later remains true for $n > 3$).

Equilibrium characteristics can also be derived when the market is very competitive. In particular, our next proposition reveals that efficient equilibria exist when $n$ is sufficiently large for given $b$ and $s$ with $b < 1 - s$. Price agencies become very attractive and each consumer consults one with an arbitrarily high probability. Firms either compete for consumers who visit the price agency and charge a price close to 0 (bargain) or they exploit the uninformed and charge a high price (ripoff) slightly above the price agencies' tariff (actually a price close to $b + s$). Asymptotically, bargains and ripoffs are offered with probabilities $s/(b+s)$ and $b/(b+s)$ and the expected market price equals $b$. All surplus is divided between price agencies and consumers.

**Proposition 4** *For $b < 1 - s$ and $\varepsilon \in (0, b+s)$ there exists $n_0$ such that $n > n_0$ implies there exists $\hat{\gamma}_n \in (1-\varepsilon, 1)$ with*

$$h_n(\hat{\gamma}_n) \leq 1 - s$$

$$b = \left( \frac{1}{1 - h_n(\hat{\gamma}_n)} - \frac{1}{\hat{\gamma}_n} \right) s$$

$$\max \{ |F_n(\varepsilon) - s/(b+s)|, |F_n(b+s-\varepsilon) - s/(b+s)| \} < \varepsilon \ .$$

**Proof.** Following Corollary 3 the proof of the existence of an efficient equilibrium for sufficiently large $n$ and given $b$ is complete once we show that $\liminf_{n\to\infty} \tilde{\gamma}_n = 1$ where $h_n(\tilde{\gamma}_n) = 1 - s$. Assume that $\liminf \tilde{\gamma}_n = 1 - \delta < 1$. Setting $\gamma = \tilde{\gamma}_n$ and $\beta_n = s/(1 - h(\tilde{\gamma}_n)) = 1$,

$$\limsup [1 - F_n(p)] = \limsup \left[ \left(\frac{1}{n}\right)^{\frac{1}{n-1}} \left(\frac{\delta}{1-\delta}\right)^{\frac{1}{n-1}} \left(\frac{1-p}{p}\right)^{\frac{1}{n-1}} \right] = 1$$

for any given $p < 1$. Thus, $h_n(\tilde{\gamma}_n) > 1 - s$ holds once $n$ is sufficiently large. This however contradicts the fact that $1 = s + h_n(\tilde{\gamma}_n)$ holds for all $n$. Hence, $\liminf \tilde{\gamma}_n = 1$.

Now consider the equilibria induced by tariff $b$ and $\gamma = \hat{\gamma}_n$. Since $\hat{\gamma}_n \geq \tilde{\gamma}_n$, we obtain $\lim_{n\to\infty} \hat{\gamma}_n = 1$. Hence, $\beta = b + s/\hat{\gamma}_n$ tends to $b + s$ as $n \to \infty$. (5) implies that $p_{Bn}$ tends to 0 and hence $\lim_{n\to\infty} \bar{p}(n) = \lim_{n\to\infty} b + p_{Bn} = b$.

Let

$$\nu := 1 - \liminf_{n\to\infty} \left( \frac{1-\hat{\gamma}_n}{\hat{\gamma}_n} \right)^{\frac{1}{n-1}} .$$

For any fixed $p \in (0, b+s)$, we obtain $\liminf_{n\to\infty} F_n(p) = \nu$. Hence, $\liminf_{n\to\infty} \bar{p}(n) = (1-\nu)(b+s)$. Comparing this to the fact that $\lim_{n\to\infty} \bar{p}(n) = b$ we obtain $\nu = s/(b+s)$. Repeating the same argument for lim sup then completes the proof. ■

## 3.2 Monopolistic Price Agencies and Collusion

While the price agencies' tariff was assumed exogenous above, we now include price agencies as players. For simplicity, assume zero cost for price agencies to obtain information about prices charged. The results below are readily generalized to include fix and constant marginal costs. At first we consider a single monopolistic price agency who publicly determines its tariff $b$ before prices are set and consumers search. This price agency aims to maximize its expected profits $\pi_P = \gamma \cdot b$.

In this enlarged game, each possible choice $b$ of the price agency induces a subgame. In each of these subgames there are multiple sequential equilibria including one that induces market failure (see the beginning of Section 3.1). In the following, we focus on efficient sequential equilibria in which trade is efficient in each subgame in which this is possible; when $b$ is chosen such that there exists $\gamma \in [\tilde{\gamma}, 1]$ with

$$b = \left( \frac{1}{1-h(\gamma)} - \frac{1}{\gamma} \right) s$$

then firms and consumers follow an efficient sequential equilibrium of this subgame as characterized in Proposition 2. Since $b$ is chosen to maximize expected profits, an alternative interpretation of the analysis of this section is that we are searching for the efficient sequential equilibrium that maximizes the joint profits attainable by colluding price agencies.

In our next result we measure *price dispersion* by the normalized difference between average price charged by firms and the minimal price charged in the market; prices $p$ are normalized by dividing by $\beta$. So expected price dispersion in an efficient equilibrium equals $(\bar{p} - p_{Bn})/\beta$. Our next results shows that maximum profits are obtained precisely when expected price dispersion is maximal.

**Proposition 5** *Consider the equilibria characterized in Proposition 2. Then*

1. *The profit $\pi_P$ of a price agency is maximal if and only if the expected price dispersion $(\bar{p} - p_{Bn})/\beta$ is maximal.*

2. *There exist $s^\circ, \gamma^* \in (0,1)$ such that:*

   *(a) If $s < s^\circ$ then the profit maximimum is attained when demand equals $\gamma^*$; in this case all consumers strictly prefer to purchase the good and $\alpha$, $\beta$, $\bar{p}$, $\pi_f$, $b$ and $\pi_P$ are proportional to $s$.*

   *(b) If $s \geq s^\circ$ then profit maximizing demand is greater than $\gamma^*$. If in addition $n \in \{2,3\}$ then consumers are indifferent between buying and not buying and expected prices are decreasing in $s$.*

**Proof.** The proof of part 1 is complete once we show

$$\arg\max_{\gamma\in(0,1)}\left\{\frac{\gamma}{1-h(\gamma)} - 1|h(\gamma) \leq 1-s\right\} = \arg\max_{\gamma\in(0,1)}\left\{h(\gamma) - g(\gamma)|h(\gamma) \leq 1-s\right\}$$

where $g = p_{Bn}/\beta$. Following (5), $g = g(\gamma) = (1-\gamma)(1-h)/\gamma$ and hence, $h - g = 1 - (1-h)/\gamma$ is maximized if and only if $(1-h)/\gamma$ minimized if and only if $\gamma/(1-h)$ is maximized. Thus the claim is proven.

Notice that $h(\gamma) \leq 1-s$ if and only if $\beta(\gamma) = s/(1-h(\gamma)) \leq 1$. Let $\gamma^*$ be the largest maximizer of the heterogeneity $h-g$, i.e.,

$$\gamma^* = \max\left\{\arg\max_{\gamma}\left\{h(\gamma) - g(\gamma) : 0 \leq \gamma \leq 1\right\}\right\}.$$

Then $\gamma^* \in (0,1)$. Let $s^\circ = 1 - h(\gamma^*)$. Then $s \leq s^\circ$ implies $h(\gamma^*) \leq 1-s$ and hence,

$$\gamma^* \in \arg\max_{\gamma}\left\{\gamma b(\gamma)|h(\gamma) \leq 1-s\right\}.$$

The fact that $\alpha$, $\beta$, $\bar{p}$, $\pi_f$, $b$ and $\pi_P$ are all proportional to $s$ follows from (6) and the fact that $\gamma^*$ does not depend on $s$.

If $s \geq s^{\circ}$ then $h(\gamma^*) \geq 1 - s$ so $h(\gamma) \leq 1 - s$ implies $\gamma \geq \gamma^*$.

For $n \in \{2, 3\}$, $b$ is decreasing in $\gamma$ and price heterogeneity is concave in $\gamma$. Consequently, $s > s^{\circ}$ implies that demand equals $\tilde{\gamma}$ and $b = 1 - s/\tilde{\gamma}$ in the profit maximizing efficient equilibrium. In this case, $\bar{p} = 1 \cdot h(\tilde{\gamma}) = 1 - s$. ■

Table 8 gives the explicit values of the equilibrium parameters from Proposition 5 when $n \in \{2, 3\}$ and $s < s^{\circ}$ (calculations contained in the appedix).

| $n$ | $s^{\circ}$ | $b$ | $\gamma^*$ | $\bar{p}$ | $\pi_f$ | $\pi_P$ |
|---|---|---|---|---|---|---|
| 2 | 0.569 | $0.183s$ | 0.635 | $0.758s$ | $0.321s$ | $0.116s$ |
| 3 | 0.528 | $0.426s$ | 0.681 | $0.89s$ | $0.2s$ | $0.29s$ |

(8)

Notice how the comparative statics in search cost $s$ have changed in comparison to our analysis in Section 3.1 for a fixed tariff $b$. When $s$ is small, the price agency chooses a small tariff which makes consumers strictly prefer to buy the good. Increasing $s$, the price agency offsets increased demand for its services due to increase in individual search cost $s$ by increasing its tariff $b$. Demand $\gamma$ in fact remains unchanged and hence expected prices increase as firms take advantage of higher search costs. When $s$ is above $s^{\circ}$, the price agency charges the highest tariff under which consumers are still willing to participate in the search for the good. Increases in $s$ are now compensated by decreases in $b$ as the price agency has to make itself more attractive to compensate for higher individual search costs. Expected price decreases to keep total consumer utility unchanged.

Next we consider the profit maximizing sequential equilibrium found in Proposition 5 when there are many firms. As we already saw in the case where tariffs were fixed (Proposition 4), price agencies become very attractive when there are many firms. Thus, it is not astonishing that we find below that the monopolistic price agency sets its tariff close to the maximal level $1 - s$ and still attracts most consumers. The rest of the results for fixed tariffs carry over. Most prices are either bargains close to 0 or ripoffs which are now close to the monopoly level 1. Approximately a fraction $1 - s$ of the firms offer a ripoff resulting in an expected market price close to the maximal level $1 - s$. Almost all consumers go to the price agency, they are nearly indifferent to not participating in the search. In particular, this means that the cutoff level $s^{\circ}$ from Proposition 5 tends to 0 as $n$ tends to infinity.

**Proposition 6** *For given $s > 0$, consider a sequence $n = 1, 2, ..$ of efficient sequential equilibria indexed by the number of firms in the market where efficient trade occurs in* *subgames whenever possible. Then the equilibrium parameters satisfy*

$$\lim_{n\to\infty} \alpha_n = 0, \quad \lim_{n\to\infty} \gamma_n = \lim_{n\to\infty} \beta_n = 1, \lim_{n\to\infty} \bar{p}_n = \lim_{n\to\infty} b_n = 1 - s$$
$$\text{and} \lim_{n\to\infty} F_n(p, \gamma_n, \beta_n) = s \text{ for any } 0 < p < 1.$$

**Proof.** Write equilibrium parameters as a function of the equilbrium value of demand $\gamma$. Let $\gamma_n^*$ be the profit maximizing level and let $\tilde{\gamma}_n$ solve $h_n(\tilde{\gamma}_n) = 1 - s$. Following the proof of Proposition 4, $\lim_{n\to\infty} \tilde{\gamma}_n = 1$. Since

$$\beta_n(\gamma_n^*) \cdot \gamma_n^* - s \geq \beta_n(\tilde{\gamma}_n) \cdot \tilde{\gamma}_n - s = \tilde{\gamma}_n - s$$

we obtain $\lim_{n\to\infty} \beta_n(\gamma_n^*) = \lim_{n\to\infty} \gamma_n^* = 1$ which implies $\lim_{n\to\infty} \alpha_n(\gamma_n^*) = 0$ and $\lim_{n\to\infty} b_n(\gamma_n^*) = 1 - s$. Using the fact that $\bar{p}_n(\gamma_n) = \beta_n(\gamma_n^*) - s$ we also obtain $\lim_{n\to\infty} \bar{p}_n(\gamma_n^*) = 1 - s$.

Finally, let

$$\nu = 1 - \liminf_{n\to\infty} \left( \frac{1 - \gamma_n^*}{\gamma_n^*} \right)^{\frac{1}{n-1}} .$$

Then $\liminf_{n\to\infty} F_n(p, \gamma_n^*, \beta(\gamma_n^*)) = \nu$ for $0 < p < 1$ which implies $\liminf_{n\to\infty} \bar{p}_n(\gamma_n^*) = 1 - \nu$. Since $\lim_{n\to\infty} \bar{p}_n(\gamma_n^*) = 1 - s$ we obtain $\nu = s$. Completing this argument using lim sup instead completes the proof. ■

## 3.3 Competing Price Agencies

Up to now we assumed that price agencies colluded, or alternatively, that there is a single price agency who acts as a monopolist. Consider now at least two price agencies competing against each other. Instead of explicitly specifying demand and calculating equilibria we discuss the impact of competition with the help of comparative statics in the tariff $b$. Consider competition among price agencies that leads to all price agencies charging the same tariff where more competition leads to a lower value of the market tariff $b$. The discussion in Section 3.1 on page 10 reveals that demand for price agencies will be arbitrarily close to 1 and expected market prices arbitrarily close to 0 if competition among the price agencies reduces tariffs close to 0. When either two or three firms are supplying the good then any increase in competition leads to a higher demand for services of the price agencies and lower expected market prices, making consumers better off. However, increased competition need not result in lower price dispersion in the goods market as we observed above that price dispersion increases initially when tariffs lie above the joint profit maximizing level.

# 4 Concluding Remarks

Information intermediaries such as price agencies facilitate trade and increase the informedness of consumers. They profit from price dispersion and the degree of information in the market, parameters they influence when they fix the price for their services. To obtain a more detailed understanding about the impact of price agenicies on competition, we set up a simple model where price agencies provide information about prices in a homogenous good oligopoly. We consider sequential search so that consumers are always free to visit one more firm if they are not pleased with the prices previous encountered. Consumers remember which firms they have visited in the past and may go back to any of these at no cost (formally, sampling is without replacement and with free recall). This last assumption complicates the analysis substantially so we rest ambitions to find all equilibria and focus on efficient equilibria.[3]

Summarizing our main results, we find price dispersion where the expected market price inducing trade is always greater than the price agency tariff. Maximal price dispersion obtains when competition among price agencies is weak or when there are many firms. Colluding price agencies deliberately set their tariffs to induce maximal price dispersion as this maximizes their profits. When there are many firms, price agencies become very attractive to consumers and most consumers visit a price agency. Firms consequently face fierce competition for business attracted via a price agency. This leads to extreme price dispersion as firms either offer bargains or ripoffs.

The equilibrium price dispersion uncovered in the seminal paper of Salop and Stiglitz (1977) is driven by heterogeneous consumers. Consumers with low search costs get the bargains while those with high search costs are left with a random draw among the bargains and the ripoffs. Only pure strategy equilibria are considered. Whether or not there is price dispersion depends on the relative search costs. In our model, consumers are identical, differences in their informedness is endogenous. Given many firms we find an analogous result, the informed (which are the customers of the price agencies) get the bargains whereas the others get a random draw. Similar to Burdett and Judd (1983) and Shiloney (1977), we find that ex ante consumer heterogeneity is not necessary to obtain price dispersion. Price dispersion obtains in our model whenever there is trade in a sequential equilibrium (see the beginning of Section 2), existence requires that firms

[3]It is a simple excersize to deduce that the equilibrium we characterize is the only sequential equilibrium in which trade occurs given sampling with replacement.

choose a mixed pricing strategy. The price distributions generalize the findings of Gale (1988) who considers two firms and an exogenous proportion of informed consumers. In particular, each firm randomizes among an infinite number of different prices. Since we consider only a finite number of firms, our efficient equilibria thus *cannot* be reinterpreted in a setting in which firms do not randomize and consumers only know the distribution of prices in the market.

In the model of Salop and Stiglitz (1977) (see also Burdett and Judd 1983), consumers are not allowed to search again once they visit a firm charging a high price. Thus, some firms charge the monopoly price whenever there is price dispersion. In our model of sequential search, consumers are never forced to buy the good. Consequently, in an efficient equilibrium, the maximal price charged by a firm never differs from expected price by more than the search cost $s$. When the price agency tariff is small then the maximal market price is just above $s$.

When there are only few firms ($n = 2$ or 3) then we can derive more specific results as we find here that demand for price agency services is decreasing in the tariff $b$, in particular efficient equilbria are unique. Both the expected and the maximal market price are then decreasing in the tariff of the price agencies. However, the opposite happens when individual search costs become small. We find a result similar to Samuelson and Zhang (1992) that both expected and maximal price increase when the individual search cost $s$ decreases. When $s$ decreases, substitution effects cause the services of the price agency to become relatively more expensive. This leads to a decrease in the demand for price agencies, consumers are less informed and competition among firms declines. Of course, when search costs $s$ and the tariff $b$ decline by the same percentage then it is immediate that prices decrease too (see (6)).

The only other model (we know) involving market competion with endogenous costs of price intermediation is due to Hänchen and von Ungern-Sternberg (1985). They explicitly model production costs of the information intermediary who decides how many firms to sample (with replacement). In their "circular road" market goods are extremely differentiated as no two firms provide the same good for some consumer. Given this lack of competition, with or without an intermediary, all firms charge the same price in equilibrium.[4] When production costs of the information intermediary are sufficiently low then all consumers visit an intermediary although they are indifferent to searching on

[4]Their equilibrium y only consider equilibria with credible threats are considered.

their own. As all firms charge the same price, the information intermediary only provides information on which product is closest to a consumer. As in our model, consumers are better off when intermediaries are present.

In our model, as in Hänchen and von Ungern-Sternberg (1985), we assume that price agencies actually investigate all prices and reveal the lowest price. However, as consumers cannot verify their information, a price agency with production costs per firm investigated would choose to save costs of gathering information and to only check the prices of a few (or one) firms. Here public monotoring would be needed to guarantee the truthful revelation of the price agencies and thus to support our equilibrium.

The incentives of a price agency to reveal the lowest market price can also be straightened out in the following decentralized mechanism. It is plausible that a price agency as a firm with many employees has a technology that makes it profitable to sample prices simultaneously instead of in sequence. Given a sufficiently large number of consumers and a tariff system in which the payment to the price agency is decreasing in the price revealed to the consumer, price agencies will choose to learn about all prices and to reveal the lowest price to its customers. Such variable tariff systems can be found among price agencies acting in Germany where a customer without a price quote researched on his own pays a percentage of the savings relative to the manufacturer's recommended price.[5] To see how such a variable tariff system affects our model, assume that the tariff of the price agency equals a percentage of the savings relative to the largest possible market price, i.e. it charges $r(\beta - p_{Bn})$ where $r \in [0,1]$ is the parameter chosen by the price agency. The indifference condition of consumers analogous to (4) becomes $s + r(\beta - p_{Bn}) + p_{Bn} = s + \bar{p} \leq 1$ and hence

$$r = \frac{\bar{p} - p_{Bn}}{\beta - p_{Bn}} = \frac{\gamma + h - 1}{\gamma - (1-\gamma)(1-h)} .$$

All results then extend directly to this alternative tariff structure, the only changes being that $b$ has to be replaced by $r$ in the statements. The values of $r$ needed to complete table (8) are $r_2 = 0.155$ and $r_3 = 0.297$.

In the most popular tariff employed in Germany, price agencies charge a proportion (typically 30-33%) of the savings relative to a price the consumer has researched on his own. Notice that our calculations cannot be used to rationalize the use of such tariffs as we focussed on equilibria where consumers who visit a price agency have no information

[5] At the same time, some price agencies charge a tariff that is increasing in the price they find.

about prices charged in the market. Moreover, it is an easy calculation to see that only these equilibria emerge under the common simplification that consumers forget which firms they have visited (sampling with replacement and without recall). Thus, it remains an intriguing topic for future research to analyze circumstances where these tariffs are used in equilibrium.

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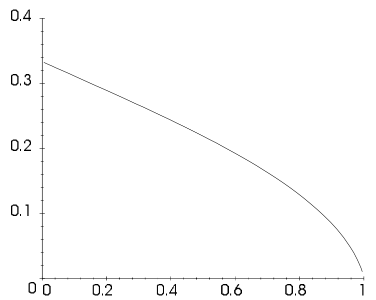
# A Two or Three firms

In the following we calculate the efficient sequential equilibrium characterized in Proposition 5 when there are either two or three firms in the market. When there are only two firms (i.e., $n = 2$) then solving the integral in (3) we obtain

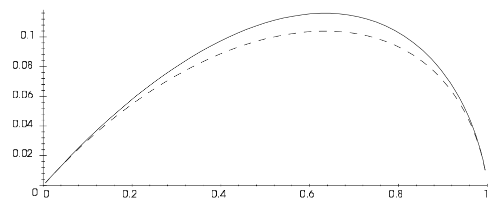
$$F_2(p) = 1 - \frac{1}{2}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{\beta_2 - p}{p}\right) \text{ for } \frac{1-\gamma}{1+\gamma}\beta_2 \leq p \leq \beta_2$$

$$
\begin{aligned}
f_2(p) &= \beta_2 \frac{1-\gamma}{2\gamma p^2} \\
\bar{p}_2 &= \beta_2 h_2(\gamma) = \beta_2 \frac{1-\gamma}{2\gamma} \ln \frac{1+\gamma}{1-\gamma} \\
\beta_2(\gamma) &= \frac{2\gamma}{2\gamma - (1-\gamma)\ln \frac{1+\gamma}{1-\gamma}} \cdot s \\
b_2 &= \beta_2 - \frac{s}{\gamma} = \left[ \frac{2\gamma}{2\gamma - (1-\gamma)\ln \frac{1+\gamma}{1-\gamma}} - \frac{1}{\gamma} \right] \cdot s \\
p_{B2}(\gamma) &= \beta_2 g_2(\gamma) = \beta_2 \frac{1-\gamma}{\gamma} - \frac{\beta_2}{2} \left( \frac{1-\gamma}{\gamma} \right)^2 \ln \frac{1+\gamma}{1-\gamma}
\end{aligned}
$$

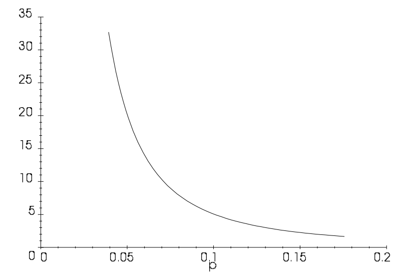
Here we find the equilibrium tariff $b_2(\gamma)$ to be decreasing in the proportion of consumers going to the price agency (the figure below shows $b_2(\gamma)/s$ as a function of $\gamma$).

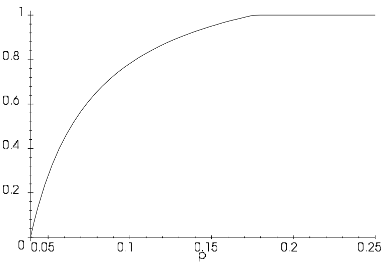



It is easily verified that $\gamma b_2(\gamma)$ is convex in $\gamma$ and attains its maximum $0.1159s$ when $\gamma_2^* \approx 0.635$ (the next figure shows $\gamma b_2(\gamma)/s$ together with the price dispersion $(\bar{p} - p_{B2})/\beta$ as functions of $\gamma$).

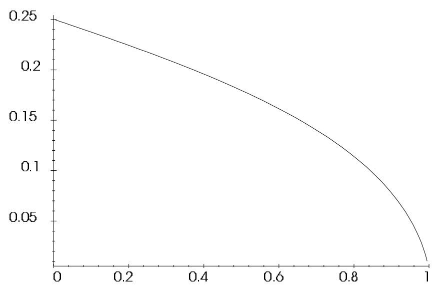



Thus, $s^\circ = 1 - h_2(\gamma_2^*) \approx 0.569$ and we obtain for $s \leq s^\circ$ the profit maximizing level $\gamma_2^*$ with $\pi_C = \bar{p}_2 \approx 0.758s$, $\pi_f \approx 0.321s$, $b_2 = 0.183s$ and $\pi_P \approx 0.116s$. For $s > s^\circ$, the profit maximizing demand $\hat{\gamma}_2$ solves $h(\hat{\gamma}_2) = 1 - s$ and hence $\hat{\gamma}_2 > \gamma_2^*$. The density and c.d.f. of the price distribution for $s = 0.1$ corresponding to $\gamma_2^*$ is shown in the next two figures ($[\alpha, \beta] \approx [0.393s, 1.757s]$).



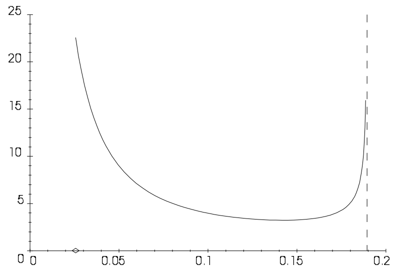

For the alternative linear tariff system presented in Section 4 the inverse demand $r$ as a function of $\gamma$ for $n = 2$ is shown in the next figure.

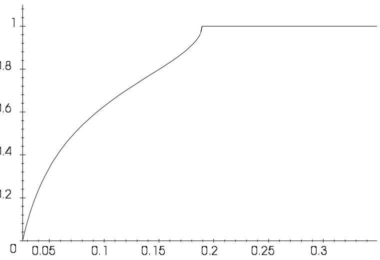



Similar calculations for the case of three firms (i.e., $n = 3$) yields

$$\begin{aligned}
F(p) &= 1 - \sqrt{\frac{(1-\gamma)(\beta_3 - p)}{3\gamma p}} \text{ for } \frac{1-\gamma}{1+2\gamma}\beta_3 \leq p \leq \beta_3 \\
f(p) &= \frac{\beta_3}{2}\sqrt{\frac{1-\gamma}{3\gamma}}\frac{1}{p^{\frac{3}{2}}\sqrt{\beta_3 - p}} \\
\bar{p} &= \frac{\beta_3}{2}\frac{\sqrt{1-\gamma}}{\sqrt{3\gamma}}\left[\frac{\pi}{2} - \arcsin\left(\frac{1-4\gamma}{1+2\gamma}\right)\right] \\
\beta_3 &= \frac{2\sqrt{3\gamma}}{2\sqrt{3\gamma} - \sqrt{1-\gamma}\left[\frac{\pi}{2} - \arcsin\left(\frac{1-4\gamma}{1+2\gamma}\right)\right]} \cdot s \\
b_3 &= \frac{-4(1-\gamma)\sqrt{3\gamma} + \sqrt{1-\gamma}\pi - 2\sqrt{1-\gamma}\arcsin\left(\frac{1-4\gamma}{1+2\gamma}\right)}{4\gamma\sqrt{3\gamma} - \gamma\sqrt{1-\gamma}\pi + 2\gamma\sqrt{1-\gamma}\arcsin\left(\frac{1-4\gamma}{1+2\gamma}\right)} \cdot s
\end{aligned}$$

Again, we find that $b_3$ is decreasing in $\gamma$ and that $b_3\gamma$ is concave. The profit of the price agency $b_3\gamma$ attains is maximum $0.29s$ at $\gamma \approx 0.681$. Thus we obtain for $s < s^\circ = 1 - h_3(\gamma_3^*) \approx 0.528$ that the profit maximizing level $\gamma_3^* \approx 0.681$ which yields $\bar{p}_3 \approx 0.89s$, $\pi_f \approx 0.2s$, $b_2 \approx 0.426s$ and $\pi_P \approx 0.29s$. The corresponding density and c.d.f. for $s = 0.1$ are shown below ($[\alpha, \beta] = [0.256s, 1.894s]$).




Finally, we provide the main calculations used to show how $\bar{p}$ changes in $s$ for given $b$. Given

$$w(\gamma) := \left(\frac{1}{1-h} - \frac{1}{\gamma}\right) = b/s \ ,$$

differentiating $b = w(\gamma(s))\, s$ with respect to $s$ we obtain

$$\begin{aligned} 0 &= w'(\gamma) \cdot \gamma'(s) \cdot s + w(\gamma) \\ \gamma'(s) &= \frac{-w}{s \cdot w'(\gamma)} > 0 \end{aligned}$$

and hence,

$$\frac{\partial}{\partial s}\bar{p} = \frac{h}{1-h} + \frac{h'(\gamma)}{(1-h)^2}\gamma'(s)\, s = \frac{1}{1-h}\left[h - \frac{h'(\gamma)}{1-h}\frac{w}{w'(\gamma)}\right] \ .$$

For $n = 2$ this implies

$$\begin{aligned} h'(\gamma) &= \frac{d}{d\gamma}\left[\frac{1-\gamma}{2\gamma}\ln\frac{1+\gamma}{1-\gamma}\right] = -\frac{1}{2\gamma^2}\ln\frac{1+\gamma}{1-\gamma} + \frac{1}{\gamma(1+\gamma)} \\ w'(\gamma) &= 2\frac{\frac{2\gamma}{1+\gamma} - \ln\frac{1+\gamma}{1-\gamma}}{\left(2\gamma - (1-\gamma)\ln\frac{1+\gamma}{1-\gamma}\right)^2} + \frac{1}{\gamma^2} \\ \frac{\partial}{\partial s}\bar{p} &= \frac{1}{1 - \frac{1-\gamma}{2\gamma}\ln\frac{1+\gamma}{1-\gamma}}\left(\frac{1-\gamma}{2\gamma}\ln\frac{1+\gamma}{1-\gamma} - \frac{\left(-\frac{1}{2\gamma^2}\ln\frac{1+\gamma}{1-\gamma} + \frac{1}{\gamma(1+\gamma)}\right)\left(\frac{2\gamma}{2\gamma-(1-\gamma)\ln\frac{1+\gamma}{1-\gamma}} - \frac{1}{\gamma}\right)}{\left(1 - \frac{1-\gamma}{2\gamma}\ln\frac{1+\gamma}{1-\gamma}\right)\left(2\frac{\frac{2\gamma}{1+\gamma} - \ln\frac{1+\gamma}{1-\gamma}}{\left(2\gamma-(1-\gamma)\ln\frac{1+\gamma}{1-\gamma}\right)^2} + \frac{1}{\gamma^2}\right)}\right) \\ &< 0 \end{aligned}$$

where numerical calculations reveal that $\frac{\partial}{\partial s}\bar{p} > -1$ holds if and only if $\gamma > \arg\max\{(\bar{p} - p_{Bn})/\beta\} \approx 0.635$.

For $n = 3$ we obtain

$$h'(\gamma) = -\frac{\pi\sqrt{\gamma}}{8\gamma^2\sqrt{3(1-\gamma)}} + \frac{1}{2\gamma(1+2\gamma)} \ .$$

Given $w := b/s$ similar though more length calculations show that $\frac{\partial}{\partial s}\bar{p} < 0$ where numerical calculations reveal that $\frac{\partial}{\partial s}\bar{p} > -1$ holds if and only if $\gamma > 0.786$.