Entry Deterrence in a Duopoly Market

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Abstract

Can an imperfectly competitive industry prevent entry by controlling access to distribution? To answer this question, this paper considers a capacity competition model with two incumbents and one potential entrant. When the potential entrant can commit to capacity and then auction it to the incumbents, entry is not deterred by control of distribution alone. The incumbents are willing to pay more for the entrant's capacity than they would for new capacity because they realize that if they don't buy it their rival will. If the cost of capacity is sufficiently small, entry deterrence can still be achieved non-cooperatively by committing to more capacity then is optimal absent the threat of entry, though the equilibrium may be asymmetric. If the cost of capacity is large, entry is accommodated.

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1. Introductions

Antitrust agencies typically oppose vertical mergers if they are likely to foreclose a potential entrant's access to distribution. For a monopolist, being able to limit potential rivals' access to distribution is a powerful barrier to entry. In this paper we ask whether duopolists can similarly restrict entry by controlling access to distribution. We study a duopoly model of capacity competition with a potential entrant. While both incumbent duopolists have access to distribution, the potential entrant does not.[1] We argue that limiting access to distribution alone does not prevent entry. When there is more than one incumbent, the incumbents must also be able commit not to distribute the entrants' output.

Foreclosing an entrant is more difficult than it first appears. If the incumbents produce the duopoly output, then neither will have an incentive to unilaterally add output. However, if the entrant offers the firms some additional capacity at cost, then each incumbent firm strictly prefers to buy the entrant's capacity than have their rival buy it. When it sells its capacity to the incumbents, the entrant will exploit a negative externality – each firm is harmed when its rival acquires additional capacity, so each will pay more for the entrant's capacity than they would for other capacity. Since the entrant's capacity will be sold to their rival if they don't buy it, each firm ignores the effect of the additional capacity on the market price of the final good. Consequently, each incumbent may be willing to pay a premium for the entrant's capacity even though they would not have been willing to increase their capacity on their own.

The incumbents may still be able to deter entry by preemptively adding more capacity. We show that when their capacity costs are sufficiently small, the incumbents share the task of entry deterrence equally by symmetrically expanding their capacity. For somewhat larger capacity costs, they deter entry with *asymmetric* capacity commitments – one incumbent is larger than the other. For even larger capacity costs, the incumbents accommodate entry: the firms'

[1] We do not study the incumbents' decisions to vertically integrate and foreclose the market for distribution, but instead focus on the nature of competition that results after vertical integration has taken place. .

capacities are the same as they would have been if the entrant had equal access to distribution. In this last case, the inability of the incumbents to commit not to buy the entrant's capacity causes the entrant to act as if it had equal access to distribution.

The incumbents are harmed by the threat of entry, whether they are able to deter it or not. These losses arise because of firms' inability to commit not to deal with the entrant. If every firm makes such a commitment, or even if all but one do, then the entrant cannot exploit the negative externality among its bidders. In equilibrium firms also fail to coordinate their entry deterrence strategies. Interestingly, this leads to overdeterrence. For some parameter values, entry deterrence can occur in equilibrium when the incumbents' profits would be higher if they did not deterred entry. This happens because the equilibria is asymmetric and the larger firm harms its rival (which it ignores) when expands its capacity to deter entry. If side payments were feasible the firms would instead accommodate entry.

Applications

A few large vertically integrated firms dominate the US motion picture industry. However there are also many independent filmmakers who do no have access to distribution. These independent filmmakers produce films and distribute them through other firms. Existing firms could try to limit the supply of new movies and protect the margin on their other movies by refusing to carry independent film makers products, but instead they compete for this business knowing that if they don't distribute the independents' films someone else will.

The same pattern occurs in the pharmaceutical industry. A few players dominate distribution, but many small pharmaceutical producers count on being able to distribute their drugs through these large competitors. The dominant players compete for the right to distribute the entrants' drugs even though the new drugs will reduce the margin on their existing products because they know that if they don't bring the entrants' drugs to market someone else will.

Literature

Our paper contributes to the literature on entry deterrence begun by Spence (1977) and Dixit (1980). They show that by building extra capacity to deter entry, incumbents can credibly commit to respond aggressively to new entry. Because they sink the costs of their capacity, they make it credible that they will offer a low price when entry occurs. In our paper incumbents make Spence-Dixit style capacity commitments *even though the entrant cannot sell its output directly to consumers.* The incumbents need to make capacity commitments in order to make it credible that neither firm will buy the entrant's capacity.

Rasmusen (1988) extends the Spence-Dixit models to consider buyout. He argues that the Spence-Dixit result is only valid if the incumbent can commit not to acquire the entrant. If they cannot, then the usefulness of capacity commitments to deter entry is diminished. His insight is that when the entrant anticipates buyout it does not care about what his profits will be if it enters, but only how much the incumbent is willing to pay to get rid of him. Rasmusen argues that entry for buyout is less likely in imperfectly competitive markets because buyout becomes a public good. In contrast, in our model the entrant cannot harm a monopoly incumbent, so buyout only occurs in imperfectly competitive markets.

The paper is also related to the literature on vertical contracts, exclusive dealing, and vertical foreclosure. Bernheim and Whinston (1988) examine the incentives to sign exclusive deals to deter entry. Rasmusen () and Segal and Whinston () examine exclusive dealing in the multiple retailer context. Mathewson and Winter (1987) and Besasko and Perry () look at vertical contracts. Ordover, Saloner, and Salop (1990) and Salinger (1988) study vertical mergers. These papers show that a firm may integrate forwards into distribution in order to deter entry or to raise rivals costs.[2]

[2] Chen (1999) argues that if one upstream firm integrates forward, then it still may want to sell to other downstream retailers in order to soften downstream competition. Independent downstream firms prefer to buy at a fixed price

Our model is naturally interpreted as the second stage of a two-stage game in which two duopoly incumbents have each vertically integrated with every potential distributor. Whereas a monopolist clearly forecloses entry through such acquisitions (entry is credibly foreclosed unless the entrant is more efficient), foreclosure is not necessarily credible for duopolists. We argue that some of the lessons of the vertical foreclosure literature may be specific to monopoly incumbent models.

Finally, the paper is related to the literature on the persistence of monopoly. Gilbert and Newbery (1982) showed that new capacity is more valuable to an incumbent than it is to a new entrant, so monopolists will tend to persist. Here, the duopoly is distribution persists by construction. The entrant's value of capacity is equal exactly to what he can get selling it to the incumbents. Nevertheless we show that for sufficiently low capacity cost the incumbents overproduce to preempt entry in production as well. Krishna (1993) extends Gilbert and Newbery to the case where new capacity becomes available in sequentially. She shows the persistence of monopoly depends on the timing of the arrival of new capacity. See also Kamien and Zang (1990), Reinganum (1983), Lewis (1983), and Chen (1999).

2. The Model

Three firms, A, B and C, produce capacity at constant marginal cost *k*. Each unit of capacity can be costlessly used to bring one unit of the final good to the market. Firms A and B are the "incumbents" and have sole access to distribution, while Firm C is the "entrant." Firm C is equally capable of producing capacity, but is unable to take its product to market. The final market demand is $p(x)=1-x$ where x is the total amount of capacity that is distributed to the market. It will be apparent to the reader that our results can easily be generalized to any linear demand function.

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from integrated suppliers because given that price, the integrated firm will want to raise its downstream price to expand its sales to the independents.

The timing of the game is as follows. First, the incumbent Firms A and B decide simultaneously and non-cooperatively how much capacity to produce, x_A and x_B, at unit cost k. Next, Firm C produces capacity x_C and auctions this block of capacity to Firms A and B in a simultaneous second price auction. Finally, the incumbents decide simultaneously and non-cooperatively how many units of capacity to distribute to the market, and the equilibrium market price is determined. All actions taken by the players are observable to the others in later stages, so there is no incomplete information. For convenience, the timing is shown in Figure 1.

Stage 1:	Firm A and Firm B produce capacity.
Stage 2:	Firm C, the potential entrant, produces capacity.
Stage 3:	Firm A and Firm B bid to acquire C's capacity.
Stage 4:	Firm A and Firm B decide how much of their capacity to take to the market.

Figure 1: The Timing

If there were no threat of entry by Firm C, then Firms A and B would simply choose their capacities Cournot-style. Define each incumbent's reaction function as

$$R(x_{-i}, k) = \arg\max_{x_i} \; x_i p(x_i + x_{-i}) - kx_i$$

So, for example, Firm A would solve $\max_{x_A} x_A p(x_A + x_{-A}) - kx_A$, and given the linear demand specification the first order conditions yield

$$R(x_{-A}, k) = \frac{1 - x_{-A} - k}{2}.$$

So, absent the threat of entry, the Cournot-Nash equilibrium of this game would be symmetric and defined by $x^* = R(x^*, k)$, or $x^* = (1-k)/3$.

Is this outcome with $x^* = R(x^*, k)$ sustainable when Firm C may produce capacity as well? The answer is no. If Firms A and B did indeed produce x^* each, then Firm C could profitably enter and produce an additional $x_C = \Delta$ units of capacity because Firms A and B are willing to pay *more* than *k* per unit for this extra capacity. The reason that they are willing to pay more than their own marginal cost is that there is a *negative externality* in the auction. Formally, each incumbent's valuation for the capacity is its profit when it acquires the capacity *minus* its profit when its rival acquires the capacity, $(x^* + \Delta) p(2x^* + \Delta) - x^* p(2x^* + \Delta)$, which is simply $\Delta \; p(2x^* + \Delta)$. That is, the unit price at auction will be bid up to the market price for the final good! Firm A's profit falls if the capacity is acquired by Firm B, and therefore Firm A is willing to pay a *premium* to keep the capacity out of the hands of Firm B.

Notice that the two incumbents would be better off if they could refuse to deal with the entrant. First, the entrant has forced them to sell beyond their optimal quantities. Second, the auction has forced them to pay a premium. If the two incumbents could collude and jointly commit not to participate in the auction, then the entrant would have no outlet for its capacity and the two incumbents would be jointly better off. Indeed, both incumbents would be better off if even *just one* of them made a unilateral commitment not to buy the entrant's capacity: with a single remaining buyer, the entrant will not receive the price necessary to cover his sunk costs. (If there were three firms, then at least two would have to commit not to by the entrant's capacity.)

Note also that our story relies upon there being at least two incumbent firms. If there were only one incumbent firm, it would produce the monopoly quantity of capacity and would not be willing to pay more than marginal cost *k* for additional capacity. There is no externality because if the monopolist does not buy capacity from the entrant, then that capacity never reaches the market. Firm C would never find it advantageous to enter because its costs would never be covered.

The remainder of the paper solves for a pure strategy subgame perfect equilibrium of this game and characterizes the conditions under which entry is deterred or accommodated. Using backwards reasoning, we first give an intuitive discussion of the distribution game, then solve the auction game, and finally find the firms' capacity decisions.

3. The Distribution Sub-game

We begin with an intuitive discussion of the distribution subgame. In the final stage of the game the incumbents each choose how much capacity to distribute to the final market. Each firm's capacity is fixed; they can distribute up to, but not more than, their capacity. The cost of distribution is zero, however profit may nevertheless be increased by withholding some capacity from the market. In this stage, each firm's "distribution" best response function corresponds to the "zero-production-cost" best response functions, subject to the firms capacity constraints. These best response functions depict the distribution quantity that maximizes each firms' profits given the rivals' distribution quantity *when the firms marginal cost is zero*. Of course each firm cannot distribute more than its capacities, so each firm's will distribute the smaller of its capacity and its optimal distribution given by the best response function.

Figure 2 depicts both these "zero-cost" Cournot best response functions and the best response functions when firms internalize the cost *k* of capacity (and ignore entry). Without the threat of entry the firms would produce at the intersection of $R(x_A, k)$ and $R(x_B, 0)$ well within the zero-cost best response functions. In this case the firms would distribute all their capacity. More generally, whenever both firms' capacity levels lie in region A, i.e., where they are both are less than the Cournot "zero-cost" best response capacities, then in distribution stage both firms will bring all of their capacity to market. In this region the marginal revenue from bringing an additional unit of capacity to market is strictly positive for both firms. Since capacity costs are sunk, both firms are better off operating at their capacity constraints.

However, when the firms' capacity levels lie outside of Region A, i.e., where one or both firms' capacities are greater than their "zero-cost" best response functions, then in the distribution stage each firm will only bring to market enough capacity to reach its "zero-cost" best. Bringing any more capacity to market reduces revenue and profits. If the firms capacities are x_A and x_B, then the firms will distribute $\hat{x}_A = \min\{R(\hat{x}_B, 0), x_A\}$ and $\hat{x}_B = \min\{R(\hat{x}_A, 0), x_B\}$. The arrows in Figure 2 trace mappings from the firms capacities, x_A and x_B, to their equilibrium distribution quantities, $\hat{x}_A$ and $\hat{x}_B$, on the outer envelope of Region A. If both firms' capacities

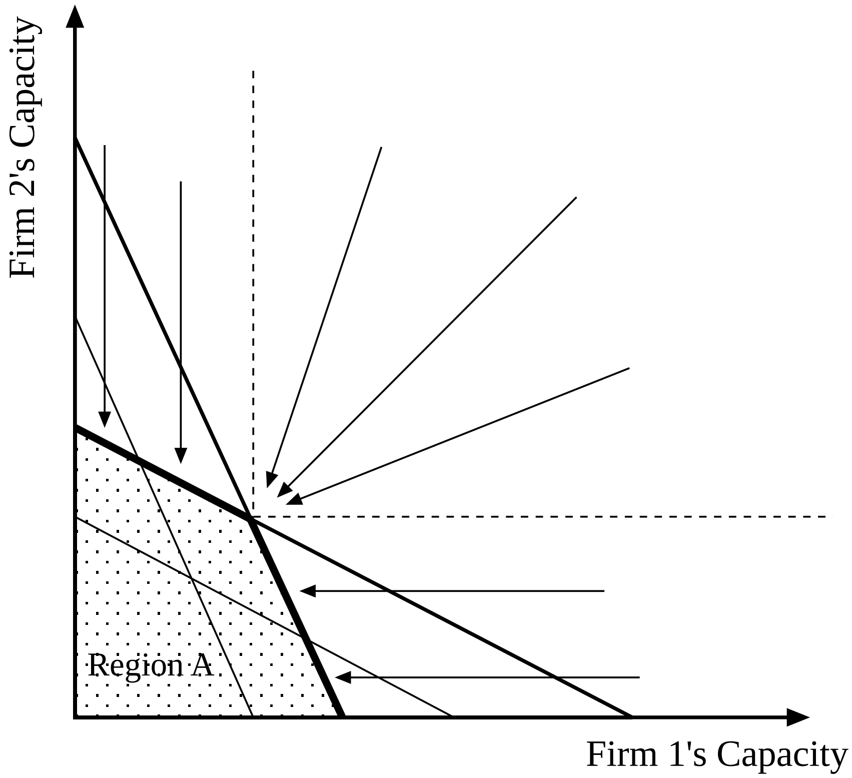

Figure 2: The Distribution Stage

exceed their zero-cost best response functions, they will both dispose of capacity and both distribute $\hat{x}$, where $\hat{x}$ is defined by $\hat{x} = R(\hat{x}, 0)$. If only one firm's capacity exceeds its zero-cost best response function, it alone will dispose of capacity.

4. The Auction Stage

Here we examine the auction of the entrant's capacity to the incumbents in Stage 3 of our game. Each incumbent's willingness to pay for the entrant's capacity is its profit if it acquires the capacity less its profit if its rival acquires the capacity. In the second price auction, the incumbent with the larger willingness to pay will acquire the capacity and pay the valuation of its rival. Note that since there is no asymmetric information, many other auctions besides the second price auction also produce this result.

We first establish that the larger of the two firms, A or B, will always weakly prefer to buy Firm C's capacity. That is, given any capacities for firms A and B, and any capacity put up for auction by Firm C, we can without loss of generality assume that the larger of Firms A and B purchases Firm C's capacity for a price equal to the smaller firm's valuation. The intuition for this result is straightforward. When either of the two firms would bring all of Firm C's capacity to market, then the two firms are willing to pay exactly the same amount for the entrant's capacity. This is because either firm will sell all of the capacity at the market price and this defines each firm's willingness to pay. But when the one or both of the firms have large capacities to begin with, then all the entrant's capacity may not be used in the distribution stage. The entrant's capacity may be large enough to take one or both firms beyond their zero-cost best response functions, in which case some of the capacity would be destroyed. Since the incentive to destroy capacity is always greater for larger firms, the larger firm is always willing to pay at least as much as much as the smaller firm, since he has the option to destroy only as much capacity as the smaller firm would have. But he will be willing to pay more because his profits are higher when he removes more of the capacity from the market. This intuition is formalized in the following lemma.

Lemma 1: *Given any capacities, x_A and x_B, and a capacity x_C available at auction, the incumbent with more capacity values the entrant's capacity x_C weakly higher than the incumbent with less capacity.*

Although it is always an equilibrium of the subgame for the larger incumbent to win the auction, this equilibrium is not necessarily unique. If either incumbent would bring the all of the entrant's capacity to market if they could, then the unit price each is willing to pay is simply the future market price (as illustrated above). In this case, the smaller firm might win the auction and acquire the capacity in equilibrium. However when the smaller firm is willing to pay as much as the larger firm, then it makes no difference to the entrant, incumbents, or consumers which firm actually wins the auction. So for expositional simplicity, we will assume that the larger firm always acquires the entrant's capacity.

Assumption: *When both incumbents bid the same amount, we assume that the larger of the two incumbents wins.*

5. Firm C's Production

In Section 2, we argued that if the incumbents ignored the threat of entry, then the entrant would be able to produce some additional output and sell it to the incumbents. However if the incumbents capacities are sufficiently large, it will no longer be profitable for Firm C to produce. Before considering the incumbents' optimal strategies, in this section we briefly describe the strategies for the incumbents that will successfully deter Firm C's entry. In the next section we characterize when entry deterrence is optimal for the incumbents.

Whether or not Firm C's produces can be characterized using Figure 2. For the reader's convenience we present it again as Figure 3. Let $p_E(x_A, x_B, x_E)$ denote the unit price that the entrant will receive for his capacity when the incumbents' capacities are x_A and x_B. If the incumbents' capacities lies strictly in the interior of Region A, both firms will be will use any additional output they purchase. So, for small ε, if Firm C commits to auction ε additional units of capacity, each firm will be willing to purchase and distribute that output. Moreover each realizes it can do nothing to prevent that capacity from reaching the market. So Firm C can produce and sell its capacity at a unit price $p(x_A + x_B + \varepsilon)$, the market price of capacity. More formally, Firm A's revenues are $(x_A + \varepsilon)p(x_A + x_B + \varepsilon)$ if it wins the auction and $x_A p(x_A + x_B + \varepsilon)$ if it does not (and Firm B does), so Firm A is clearly willing to pay up to $p(x_A + x_B + \varepsilon)$ for the entrant's capacity. By the same argument $p(x_A + x_B + \varepsilon)$ is also the unit price Firm B is willing to pay. So in a second price auction, for small ε, $p_E(x_A, x_B, \varepsilon) = p(x_A + x_B + \varepsilon)$. Since $p(x_A + x_B + \varepsilon) > k$, it follows that if the incumbents' capacities are in the interior of A then Firm C's output will be strictly positive.

In Region C, it is clear that neither firm will use any additional output, so neither Firm is willing to pay for Firm C's capacity. There is no direct value to either firm of having that additional capacity, and more importantly *there is no value in keeping the capacity away from the rival incumbent.* Each firm realizes that if their rival acquires the capacity, it will not increase the output the rival distributes, so their own profits will not be diminished. Each firms' value for Firm C's output is zero. It follows that if the incumbents' capacities are in Region C, then $p_E(x_A, x_B, x_E) = 0$ for all x_E so Firm C's output will be zero.

A similar argument works in Region B of Figure 3. Here the additional capacity has a positive direct value to one firm, but not the other. The firm with more capacity would not distribute the entrant's capacity if it acquired it, but is willing to pay to keep it away from his rival. The firm with less capacity would distribute at least some additional capacity. However the smaller firm realizes that the capacity will not be used by its rival, so its profits will not be diminished if its rival wins the auction. Thus the smaller firm's willingness to pay for the entrant's capacity is exactly what it would pay for any new capacity, which in Region B is

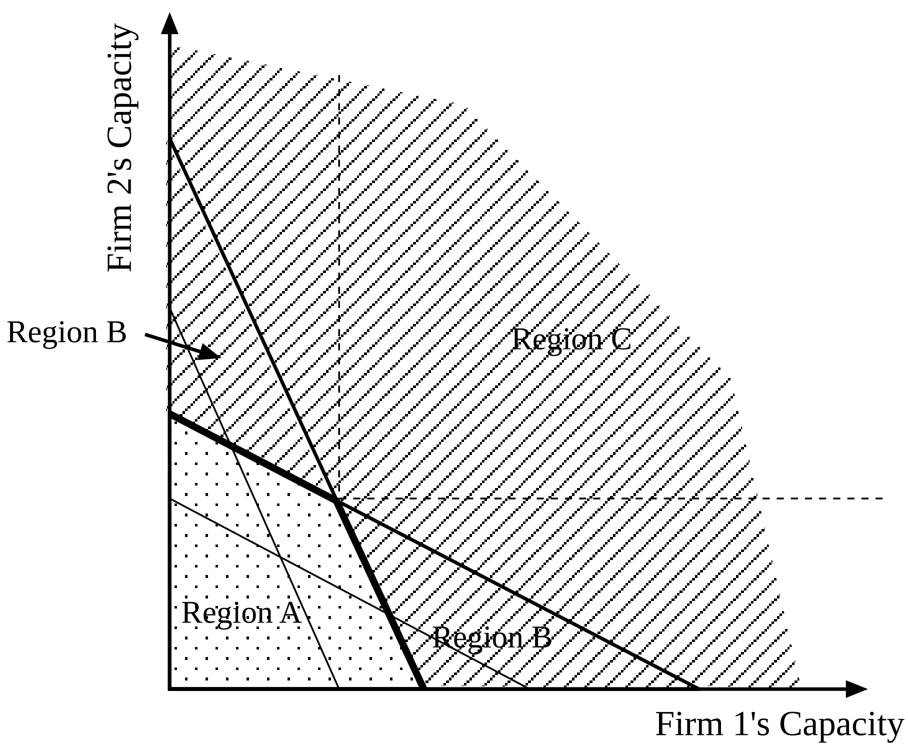

Figure 3: Entry Deterring Capacities

strictly less than *k*. By the lemma, we know the larger firm will buy the capacity at a price equal to the smaller firm's valuation. Since the smaller firm's valuation is less than *k* per unit of capacity, $p_E(x_A, x_B, x_E) < k$ for all x_E, and Firm C's optimal production is zero.

We conclude that if either firm's capacity lies at or beyond the zero-cost reaction function, entry by Firm C is deterred. So Firm C will enter only if the incumbent's capacities are in the interior of Region A.

6. Entry Deterrence

If the cost of capacity is small then in equilibrium entry is deterred non-cooperatively. The incumbents produce more than they would have absent the threat of entry, and production by the entrant is deterred. The incumbents would rather produce a lot of capacity preemptively at marginal cost *k* than buy capacity from the entrant at the market price.

If the cost of capacity is high, then entry will not be deterred in equilibrium. They are better off buying the capacity that the entrant wants to sell (at a high price) then producing the amount of capacity required to deter entry (at cost). In equilibrium, he incumbents produce as if they were (duopolistic) Stackelberg leaders, and the entrant produces as a Stackelberg follower. In other words, the outcome is exactly the same as if the entrant had access to distribution.

We begin with the case where the cost of capacity is small. We show that when the marginal cost of production is less than 1/6, there is a symmetric subgame perfect Nash equilibrium in which entry is deterred. Each incumbent produces $\hat{x} = R(\hat{x}, 0)$, as if they had zero cost. The entrant will not produce neither incumbent would distribute any additional capacity it acquired. The negative externality described earlier is eliminated (whether or not my rival buys the capacity it will have no impact on my profits). Consequently neither firm would be willing to bid anything for C's capacity.

Clearly neither incumbent wants to produce more than $\hat{x}$, but why doesn't either of the incumbents prefer to produce less than $\hat{x}$? Suppose Firm A produced $x_A < \hat{x}$, then since $x + R(x, 0)$ is increasing in *x* and $R(x, 0)$ is decreasing in *x* we know that

$R(\hat{x},0) - x_A > R(x_A,0) - \hat{x} > 0$. So if the entrant subsequently placed a small amount of capacity, Δ, on the market, the incumbent firms would bid the price at auction all the way up to the market price, $p(x_A + x_B + \Delta)$. By producing less, Firm A changed both firms' incentives to use additional capacity. Now neither firm's capacity is at their zero-cost best-response function, so each firm realizes the other will purchase and use Firm C's capacity if they don't. So if Firm A deviates then entry occurs and the firms will pay the market price for the entrant's capacity. At least when the costs of capacity are small, both firms prefer to add capacity at cost *k* and avoid buying more the higher priced capacity from the entrant.

Proposition 1: *When k < 1/6 there exists a symmetric Nash equilibrium of the form* $\{\hat{x}, \hat{x}, 0\}$ *where* $\hat{x} = R(\hat{x},0)$, *or* $\hat{x} = 1/3$. *The incumbents each sell all that they produce and production by the entrant is deterred.*

If capacity costs are slightly larger, then entry is again deterred, but the equilibrium behavior of Firms A and B is asymmetric. Consider when costs are slightly larger than 1/6. It is clear that the symmetric $\hat{x} = R(\hat{x},0)$ cannot be sustained. One firm will lower its output. However entry deterrence can still occur, but the burden of deterring entry falls on only one firm.

The crucial insight is that if in equilibrium Firm A produces less capacity, Firm B can still unilaterally deter entry by producing $R(x_A, 0)$. As long as one firm is producing at least it zero-cost best-response capacity entry is deterred. Firm A might be willing to pay something for the entrant's output, but he knows that if Firm B buys it Firm B will not bring it to market, so the negative externality is gone.

It is more difficult to see why it is in Firm B's interests to deter entry if Firm A has abandoned the task. In equilibrium, the reason firm B will do so is that Firm B can now produce significantly more than half the market. Since Firm A has shifted to being a small producer, Firm B can internalize more of the benefits of entry deterrence.

In equilibrium Firm B will choose a best response to Firm A's capacity as if it had zero costs. Firm A on the other hand acts as a Stackelberg leader. However Firm A is the smaller firm! This paradoxical result is because Firm A acts as Leader with high costs *k*, while Firm B acts as a follower with zero costs. Of course Firm C produces zero. Equilibrium sales are equal to production.

Proposition 2: *When*

$$k \in \left(\frac{1}{6}, \frac{1}{2\sqrt{2}} \right)$$

there is an asymmetric equilibrium of the form $\{\tilde{x}, R(\tilde{x},0),0\}$ *where* $\tilde{x} = \arg\max_{x_A} x_A p(x_A + R(x_A,0)) - kx_A$. *So we have*

$$\{x_A, x_B, x_C\} = \left\{ \frac{1-2k}{2}, \frac{1+2k}{4}, 0 \right\}.$$

When costs of capacity are large entry is accommodated. Firms A and B would rather to buy all the capacity the entrant produces than produce enough capacity to deter entry.

In the accommodated entry equilibrium, Firms A and B produce the same output as they would have if Firm C had access to distribution. Since Firm C produces its capacity after Firms A and B, the equilibrium is not a symmetric oligopoly, but rather a variant on the Stackelberg model with two Stackelberg leaders and one follower. We begin by deriving this equilibrium.

<u>Stackelberg with 2 leaders and 1 follower</u>

Consider the Stackelberg problem where Firms A and B choose capacity first and then Firm C chooses capacity, and assume all three firms have costless access to production. As the Stackelberg follower, given Firm A and B's output, Firm C produces $R(x_A + x_B, k)$. So we can write Firm A's production problem (and the symmetric problem for Firm B) as

$$\max_{x_A} x_A p\left(x_A + x_B + R(x_A + x_B, k)\right) - kx_A,$$

or

$$\max_{x_A} x_A \left(1 - x_A - x_B - \left(\frac{1 - x_A - x_B - k}{2} \right) \right) - kx_A,$$

or

$$\max_{x_A} x_A\left(\frac{1-x_A-x_B+k}{2}\right)-kx_A.$$

The first order condition for Firm A

$$\frac{1-2x_A-x_B+k}{2}-k=0$$

which by symmetry yields

$$3x=1-k\text{, or}$$
$$x_A=x_B=\frac{1}{3}(1-k).$$

So Firm C's output is

$$x_C=R\big(2(1-k)/3,k\big)=\frac{1-2\left(\frac{1-k}{3}\right)-k}{2}=\frac{1}{6}(1-k).$$

Total industry output is

$$x_A+x_B+x_C=\frac{5}{6}(1-k),$$

and the market price is

$$p=\frac{(1-k)}{6}.$$

Proposition 3 shows that when capacity costs are sufficiently large, entry is accommodated, and each of the three firms produce as if the entrant does not have access to distribution, even though the entrant must auction its output to one of the other firms.

Proposition 3: *When k >.261204 there exists a symmetric equilibrium of the entry deterrence game in which entry is accommodated and the firms' capacities are*

$$\{x_A,x_B,x_C\}=\left\{\frac{1-k}{3},\frac{1-k}{3},\frac{1-k}{6}\right\},$$

the same as in the game in which C has access to production.

5. Conclusion (to be written)

Appendix

Proof of Lemma 1:

Suppose $x_A \geq x_B$. Let $TR_i(x_A, x_B)$ denote Firm i's reduced form total revenue as a function of the capacity endowments of the firms. (Note that a Firm i may choose to utilize less than its full endowment of capacity and the function $TR_i(x_A, x_B)$ expresses revenue as a function of the endowments, not what the firm actually utilizes.) Firm A's willingness to pay for capacity x_C is $TR_A(x_A + x_C, x_B) - TR_A(x_A, x_B + x_C)$, his total revenue if he acquires the capacity minus his total revenue if his rival acquires the capacity. Similarly, Firm B's willingness to pay is $TR_B(x_A, x_B + x_C) - TR_B(x_A + x_C, x_B)$. So Firm A will win the auction when his willingness to pay is higher, or

$$TR_A(x_A + x_C, x_B) + TR_B(x_A + x_C, x_B) \geq TR_A(x_A, x_B + x_C) + TR_B(x_A, x_B + x_C)$$

In other words, Firm A will win the auction for x_C if total industry revenues are weakly higher when Firm A acquires x_C that when Firm B acquires x_C. We can rewrite this expression as:

$$\int_0^{x_C} \left[\frac{\partial TR_A(x_A + s, x_B)}{\partial x_A} + \frac{\partial TR_B(x_A + s, x_B)}{\partial x_A} \right] ds \geq \int_0^{x_C} \left[\frac{\partial TR_A(x_A, x_B + s)}{\partial x_B} + \frac{\partial TR_B(x_A, x_B + s)}{\partial x_B} \right] ds$$

A sufficient condition for this to be true (and the lemma to be proved) is that for all s,

$$\frac{\partial TR_A(x_A + s, x_B)}{\partial x_A} + \frac{\partial TR_B(x_A + s, x_B)}{\partial x_A} \geq \frac{\partial TR_A(x_A, x_B + s)}{\partial x_B} + \frac{\partial TR_B(x_A, x_B + s)}{\partial x_B}. \qquad (*)$$

We now show that the sufficient condition holds for all s. Since $x + R(x,0)$ is increasing in x, $x_A + R(x_A,0) \geq x_B + R(x_B,0)$ which implies $R(x_B,0) - x_A \leq R(x_A,0) - x_B$. So there are three cases to consider.

Case 1. First suppose that $R(x_B, 0) - x_A \leq R(x_A, 0) - x_B < s$. Rearranging terms, $R(x_B, 0) < s + x_A$. That is, Firm A's best response to Firm B's endowment x_B is to utilize *less* than its total endowment, $s + x_A$. Since Firm A would not use all of its available capacity we have

$$\frac{\partial TR_A(x_A+s,x_B)}{\partial x_A}+\frac{\partial TR_B(x_A+s,x_B)}{\partial x_A}=0\,.$$

Similarly since $R(x_A,0)\le s+x_B$, Firm B would not use all of its available capacity either, so

$$\frac{\partial TR_A(x_A,x_B+s)}{\partial x_B}+\frac{\partial TR_B(x_A,x_B+s)}{\partial x_B}=0\,.$$

Both sides of (*) are zero.

Case 2. Next, suppose that $s<R(x_B,0)-x_A\le R(x_A,0)-x_B$. Rearranging terms, $R(x_B,0)\ge s+x_A$ and $R(x_A,0)\ge s+x_B$ so both firms will use all their capacity and both sides of (*) are equal (industry profits are the same regardless of which firm does the producing).

Case 3. Finally, suppose $R(x_B,0)-x_A\le s\le R(x_A,0)-x_B$ then the left-hand side of (*) is zero because Firm A would not use all available capacity. Firm B, on the other hand, would use all available capacity because $s>R(x_B,0)-x_A$. Since $x+R(x,0)$ is increasing in x it follows that $s+x_A+x_B>R(x_B,0)+x_B>R(0,0)$, where $R(0,0)$ is the level of sales that maximizes *total* industry revenues. Since the total capacity utilized, $s+x_A+x_B$, exceeds the level of capacity that maximizes total industry revenues we conclude that

$$\frac{\partial TR_A(x_A,x_B+s)}{\partial x_B}+\frac{\partial TR_B(x_A,x_B+s)}{\partial x_B}<0\,.$$

Q.E.D.

Proof of Proposition 1:

First, we will show that if $x_A=x_B=\hat{x}$ then the best response for Firm C is $x_C=0$. In this case, A and B's marginal value of one more unit of output is exactly 0 because neither would sell that unit if they acquired it. No matter what they acquire from Firm C, sales for both will be $\hat{x}$. Since neither Firm A nor Firm B is willing to pay anything to acquire $x_C>0$, Firm C will not produce.

Now consider deviations by firm A (equivalent arguments can be made for firm B). It is easy to see that Firm A will want to not increase its output. If Firm A did increase its output and

$x_C = 0$, then Firm A would destroy its additional output and equilibrium sales would be $x_A = x_B = \hat{x}$. So, as before, A and B's marginal value of one more unit of output is exactly 0, so it is optimal for Firm C to produce nothing, $x_C = 0$.

It is more subtle to show that Firm A cannot profitably deviate and *decrease* its output to $x_A < x_B = \hat{x}$. Since $x + R(x,0)$ is increasing in x we know that $R(\hat{x},0) - x_A > R(x_A,0) - \hat{x} > 0$. An implication of this is that if C produced a small amount, then the price at auction would be bid up to the market price. First, we will prove that Firm C would respond to this deviation in the subsequent subgame by producing capacity $x_C \geq R(x_A,0) - \hat{x} > 0$.

Claim: If $x_A < x_B = \hat{x}$ then it will be optimal for Firm C to produce $x_C \geq R(x_A,0) - \hat{x} > 0$.

Proof: Suppose not: $x_C < R(x_A,0) - \hat{x}$. So if Firm B acquires x_C then it will sell $\hat{x} + x_C$ because $\hat{x} + x_C < R(x_A,0)$. If Firm A acquires x_C, on the other hand, then $x_A + x_C < x_A + R(x_A,0) - \hat{x} = x_A + R(x_A,0) - R(\hat{x},0)$. When $x_A = \hat{x}$ the right hand side of this expression reduces to $R(\hat{x},0)$, but since $x_A < \hat{x}$ and

$$\frac{\partial}{\partial x_A}[x_A + R(x_A,0) - R(\hat{x},0)] > 0$$

we conclude that $x_A + x_C < R(\hat{x},0)$. So if Firm A acquires the additional capacity it will sell all of it as well. Since both firms would sell capacity x_C if they acquired it, the auction yields a price per unit for x_C of $p(x_A + x_B + x_C)$.

Using our linear demand specification gives us $x_B = \hat{x} = 1/3$, so we can write Firm C's profits as $\pi_C = x_C(1 - x_A - 1/3 - x_C) - kx_C$. The derivative of this profit function, $2/3 - x_A - 2x_C - k$, is positive when $k < 1/3$ because $2/3 - x_A - k$ > $1/3 - x_A = 2[R(x_A,0) - \hat{x}] > 2x_C$. This is a contradiction! So it must be the case that $x_C \geq R(x_A,0) - \hat{x}$.

Since $x_A < x_B$ we know from Lemma 1 that it is an equilibrium for Firm B to acquire x_C at auction. Since $x_B + x_C = \hat{x} + x_C > R(x_A, 0)$ (by the previous claim) it is not an equilibrium for B to actually sell all of the acquired capacity. Rather, Firm B will restrict its sales to $R(x_A, 0)$. So when Firm A chooses to deviate to $x_A < \hat{x}$, he knows that the total industry sales will ultimately be $x_A + R(x_A, 0)$ and the industry price will be $p(x_A + R(x_A, 0))$. In other words, when Firm A is considering an optimal deviation to $x_A < \hat{x} = 1/3$, it is as if Firm A is a Stackelberg leader with costs k and Firm B is a Stackelberg follower with zero costs! The best deviation form Firm A is:

$$x_A = \arg\max_{x_A} x_A p(x_A + R(x_A, 0)) - kx_A$$

Since $R(x_A, 0) = (1 - x_A)/2$ and $p(x) = 1 - x$, we can write the problem as

$$\max_{x_A} x_A \left(\frac{1 - x_A}{2} \right) - kx_A .$$

The derivative of this expression is $1/2 - x_A - k$. Since $k < 1/6$, this is positive for all deviations $x_A < 1/3$, and therefore a profitable deviation does not exist. Q.E.D.

Proof of Proposition 2:

Note that in the specified equilibrium $x_B - R(x_A, 0) = 0 > x_A - R(x_B, 0)$.

First, consider Firm C. We claim $x_C = 0$ is a best response to $\{\tilde{x}, R(\tilde{x}, 0)\}$. Suppose not. If B acquires x_C then B would destroy the additional capacity because his optimal response to $\tilde{x}$ is simply $R(\tilde{x}, 0)$. So there is no negative externality in the auction from A's perspective. If A acquires x_C, on the other hand, then A will sell $\tilde{x} + x_C$ because $x_A - R(x_B, 0) < 0$. In this case, in the distribution stage B will destroy some of its capacity and sell $R(\tilde{x} + x_C, 0)$ instead of $R(\tilde{x}, 0)$. Since there is no negative externality, A's marginal value of one more unit of output of capacity is less than *k* because A chose it own capacity optimally. Since B is larger than A, it buys all of C's capacity for A's valuation keeps the extra capacity off of the market. So the price B pays is less than *k*, and Firm C produces zero.

Second, consider Firm A. If Firm A *increases* its output beyond $\tilde{x}$ then C will produce nothing. As above, Firm B would destroy some of his capacity and sell $R(x_A,0) < R(\tilde{x},0)$. Firm A's payoff is $x_A p(x_A + R(x_A,0)) - kx_A$ which is maximized at $\tilde{x}$.

Now suppose instead that Firm A *decreases* its output to $x_A < \tilde{x}$.

Claim: If $x_A < \tilde{x}$ and $x_B = R(\tilde{x},0)$ then $x_C \geq R(x_A,0) - x_B > 0$.

Proof of Claim: Suppose instead that $x_C < R(x_A,0) - x_B$. If Firm B acquired the additional capacity then he would sell it because $x_C < R(x_A,0) - x_B$. Moreover, if Firm A acquired the additional capacity then Firm A would sell it to. This follows from the fact that $x + R(x,0)$ is increasing in x, and so $x_A + R(x_A,0) < x_B + R(x_B,0)$, which implies $R(x_A,0) - x_B < R(x_B,0) - x_A$. Since $x_C < R(x_A,0) - x_B$ it must also be the case that $x_C < R(x_B,0) - x_A$. Since both Firm A and Firm B would sell capacity x_C if they acquired it, the auction yields a price per unit for x_C of $p(x_A + x_B + x_C)$. Using our linear demand specification we can write Firm C's profits as $\pi_C = x_C(1 - x_A - x_B - x_C) - kx_C$, and the derivative is $1 - x_A - x_B - 2x_C - k$ so the best choice for Firm C is

$$x_C = \frac{1 - x_A - x_B - k}{2}.$$

But by assumption $x_C < R(x_A,0) - x_B = (1 - x_A)/2 - x_B$, so together these imply that $x_B < k$. But $x_B = (1 + 2k)/4$ so this is a contradiction for all $k < 1/2$. So it must be the case that $x_C \geq R(x_A, 0) - x_B$. This completes the proof of the claim.

Since $x_A < x_B$ we know that Firm B will purchase the entrant's capacity x_C, but since $x_B + x_C \geq R(x_A,0)$ Firm B may destroy some of that capacity capacity. Instead, Firm B will sell only $R(x_A,0)$. So when Firm A chooses to deviate to $x_A < \tilde{x}$, he realizes that the total industry sales will ultimately be $x_A + R(x_A,0)$ and the industry price will be $p(x_A + R(x_A,0))$. Therefore when Firm A is considering an optimal deviation it is as if Firm A is a Stackelberg leader with costs k and Firm B is a Stackelberg follower with zero costs. Formally, Firm A maximizes $x_A p(x_A + R(x_A,0)) - kx_A$, which yields an optimal capacity $\tilde{x}$ as in the proposition. So Firm A will not decrease it capacity either.

<u>Finally consider firm B.</u> B clearly won't produce *more* than $R(\tilde{x},0)$. Firm C's production is already zero and any more output that Firm B produces would be destroyed. However, we must check that Firm B will not produce *less* than $R(\tilde{x},0)$. By lowering its production Firm B may change which firm is the largest, so we consider deviations to $x_B \in \left[\tilde{x}, R(\tilde{x},0)\right)$ and $x_B \in [0,\tilde{x}]$ separately.

<u>Case #1:</u> $x_B \in \left[\tilde{x}, R(\tilde{x},0)\right)$.

Notice that in this case $x_A \le x_B$, so Firm B will buy any capacity produced by the entrant. We divide Firm C's possible responses into two ranges. First, suppose that Firm C's optimal response to a deviation by Firm B to x_B satisfies $x_C \ge R(\tilde{x},0) - x_B$, so Firm C produces enough capacity to make Firm B's final sales equal to $R(\tilde{x},0)$. This is profitable for Firm C, so Firm C must be receiving a price at least equal to *k*. Therefore Firm B is better off staying at $x_B = R(\tilde{x},0)$ where he pays a lower price and buys no more capacity than $R(\tilde{x},0)$. So Firm C's optimal response satisfies $x_C \ge R(\tilde{x},0) - x_B > 0$ for some deviation x_B, it will not be profitable for Firm B to make that deviate.

Suppose instead that Firm C's best response to a deviation by Firm B to x_B satisfies $x_C < R(\tilde{x},0) - x_B$. (Note that this implies that $x_C < R(x_B,0) - \tilde{x}$ as well, since $x_A \le x_B$ and $x + R(x,0)$ is increasing in x.) From the lemma we know Firm B will acquire the entrant's additional capacity, x_C, and will pay a price $p(x_A + x_B + x_C)$. Using our linear demand specification we can write Firm C's profits as $\pi_C = x_C(1 - x_A - x_B - x_C) - kx_C$, and the first order condition is $1 - \tilde{x} - x_B - 2x_C - k = 0$, and so a local interior optimum, $x_C = (1 - \tilde{x} - x_B - k)/2$. If Firm C's best response satisfies $x_C < R(\tilde{x},0) - x_B$, then $x_C = (1 - \tilde{x} - x_B - k)/2$.

If Firm B anticipates that Firm C will respond to his deviation in this way, what is the best deviation for Firm B? Firm B would choose his deviation to solve $\max_{x_B} x_B(1 - \tilde{x} - x_B - x_C - k)$, and using the expression for x_C this becomes

$$\max_{x_B} x_B\left(\frac{1 - \tilde{x} - x_B - k}{2}\right).$$

Using the fact that $\tilde{x} = 1/2 - k$ gives us the (unconstrained) solution, $x_B = 1/4$. However we are considering only the case where Firm B deviates to the interval $x_B \in [\tilde{x}, R(\tilde{x},0))$, and so Firm B's constrained solution is $x_B = \min\{1/4, \tilde{x}\}$. In other words, if Firm C responds with $x_C = (1 - \tilde{x} - x_B - k)/2$ to some deviation, the profits to Firm B from deviating can be no more than they are at $x_B = \min\{1/4, \tilde{x}\}$ when firm C responds in this way

First, suppose that $k > 1/4$. In this case the most profitable deviation is $x_B = 1/4$ since $x_B = 1/4 > 1/2 - k = \tilde{x}$. So Firm B's profits from this deviation are

$$x_B \left(\frac{\frac{1}{2} - x_B}{2} \right)$$

or 1/32. Firm B's profits from not deviating, on the other hand, are $1/4(1/4 - k^2)$. So deviating is not profitable as long as $k < 1/(2\sqrt{2})$.

Now suppose instead that $k \le 1/4$. Now $1/4 \le 1/2 - k = \tilde{x}$, so the best deviation for Firm B from the set $x_B \in [\tilde{x}, R(\tilde{x},0))$ is $\tilde{x}$, and this clearly yields profits for Firm B that are lower than 1/32. We conclude that for

$$k \in \left(\frac{1}{6}, \frac{1}{2\sqrt{2}} \right)$$

Firm B has no profitable deviation in the set $x_B \in [\tilde{x}, R(\tilde{x},0))$.

<u>Case #2:</u> $x_B \in [0, \tilde{x}]$.

Note that in this case $x_A \ge x_B$, so Firm A will buy the entrant's capacity.

As before, we divide Firm C's possible responses into two ranges. First suppose that $x_C \ge R(x_B,0) - \tilde{x}$. Then Firm A's final sales are exactly $R(x_B,0)$. Firm B's payoff from a deviation to this intreval resembles that of a Stackelberg leader with a zero cost rival, and he solves:

$$\max_{x_B} x_B p(x_B + R(x_B,0)) - kx_B \, .$$

This gives the (unconstrained) optimum $x_B = 1/2 - k = \tilde{x}$ (however notice that $\tilde{x} \in [0, \tilde{x}]$ so the constraint is not binding). Firm B's profits from this deviation are $(1/2)[1/2 - k]^2$.[3] The profits from not deviating on the other hand are $(1/4)[1/2 - k][1/2 + k]$. It is straightforward to check that Firm B will not find it profitable to deviate for all $k > 1/6$.

Now suppose instead that $x_C < R(x_B, 0) - \tilde{x}$. As before, this implies $x_C < R(\tilde{x}, 0) - x_B$. Firm A will acquire the entrant's capacity, x_C, and will pay a price $p(x_A + x_B + x_C)$. Proceeding exactly as before, using our linear demand specification we can write Firm C's profits as $\pi_C = x_C(1 - x_A - x_B - x_C) - kx_C$, and the first order condition as $1 - \tilde{x} - x_B - 2x_C - k$, and so a local interior optimum, $x_C = (1 - \tilde{x} - x_B - k)/2$.

If Firm B anticipates that Firm C will respond to a deviation in this way, what is the best deviation for Firm B? Firm B will maximize $\max_{x_B} x_B(1 - \tilde{x} - x_B - x_C - k)$, and as before this yields the solution $x_B = 1/4$. So if any deviation exists for which Firm C responds with $x_C < R(x_B, 0) - \tilde{x}$ it yields a profit to firm B of no more than Firm B earns at $x_B = 1/4$ when Firm C responds this way

If $k < 1/4$ then $x_B = 1/4 > 1/2 - k = \tilde{x}$. So the best deviation for Firm B in the range $x_B \in [0, \tilde{x})$ is $\tilde{x}$, which we have already seen is suboptimal. If $k > 1/4$ then $x_B = 1/4 < \tilde{x}$. As before, it is straightforward to calculate that Firm B's profits from this deviation are 1/32. Firm B's profits from not deviating, on the other hand, are $1/4(1/4 - k^2)$. So his profits from deviating are smaller so long as $k < 1/(2\sqrt{2})$. Q.E.D.

Proof of Proposition 3:

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[3] Because

$$x_B\left(1 - \left(x_B + \frac{1 - x_B}{2}\right) - k\right) = x_B\left(\frac{1 - x_B}{2} - k\right) = \frac{1}{2}\left(\frac{1}{2} - k\right)^2$$

First consider firm C. To describe C's profit function we must consider three distinct intervals.

1) If $x_C \le R(x_A,0) - x_B$ and $x_C \le R(x_B,0) - x_A$ then Firm C's profits are

$$x_C\left(p(x_A + x_B + x_C) - k\right)$$

since A and B would each pay up to $x_C\, p(x_A + x_B + x_C)$ to keep C's output out the other's hands.

2) If $x_C > R(x_B,0) - x_A$ and $x_C < R(x_A,0) - x_B$ and $x_A > x_B$ (or equivalently $x_C < R(x_B,0) - x_A$ and $x_C > R(x_A,0) - x_B$ and $x_B > x_A$) then Firm C's profits are

$$(x_B + x_C)p(x_A + x_B + x_C) - x_B\left(x_B + R(x_B,0)\right) - x_C k$$

since A will buy C's output for B's maximum willingness to pay.

3) If $x_C > R(x_B,0) - x_A$ and $x_C > R(x_A,0) - x_B$ and without loss of generality $x_A \ge x_B$, then Firm C's profits are

$$R(x_A,0)p\left(x_A + R(x_A,0)\right) - x_B p\left(x_B + R(x_B,0)\right) - x_C k$$

since again A will buy C's output for B's maximum willingness to pay.

Let $x_A = x_B = (1-k)/3$. Then on interval one C's profits are $x_C\left(p(x_A + x_B + x_C) - k\right)$, and C's profit maximizing output (on interval one) is

$$x_C^* = R(x_A + x_B, k) = \frac{1}{6}(1-k).$$

Note that this is strictly less than

$$R(x_B,0) - x_A = \frac{1-x_B}{2} - x_A = \frac{1}{2} - \frac{1}{6}(1-k) - \frac{1}{3}(1-k) = \frac{k}{2}$$

when $k > 1/4$. It is useful for later to note that if $x_B = (1-k)/3$ and $x_A < (1-k)/3$ then $x_C^* = R(x_A + x_B, k)$ which is also strictly less than $R(x_B,0) - x_A$ because

$$\frac{\partial R(x,k)}{\partial x} < 1.$$

We still have to check that C's profit is not higher in three. Note that for $x_A = x_B$ interval 2 does not exist. Suppose $x_C > R(x_B,0) - x_A$. Inspection of the profit function in interval three reveals that firm C will want to minimize its output; this implies C can raise profits by lowering its output until $x_C^* = R(x_B, 0) - x_A$ which contradictions the assumption that the profit maximizing output satisfies $x_C^* > R(x_B, 0) - x_A$. So C's best response to $x_A = x_B = (1-k)/3$ is $x_C^* = (1-k)/6$.

Now consider Firm A (or equivalently firm B): Suppose that $x_A = (1-k)/3$ is not optimal. In particular, let $x_A^* \neq \frac{1}{3}(1-k)$ be firm A's optimal strategy. Suppose that Firm C's optimal response satisfies $x_C < R(x_A, 0) - x_B$ and $x_C < R(x_B, 0) - x_A$; we have already shown it will whenever $x_A < \frac{1}{3}(1-k)$. Then Firm C's optimal response is $R(x_A + x_B, k)$. So firm A's profit is

$$x_A p\left(x_A + x_B + R(x_A + x_B, k)\right) - kx_A$$

and $x_A^* = (1-k)/3$, which is a contradiction. So Firm C's optimal response must violate $x_C < R(x_B, 0) - x_A$ (if it violates either $x_C < R(x_A, 0) - x_B$ and $x_C < R(x_B, 0) - x_A$ then it clearly violates the latter). It follows that $x_A^* > (1-k)/3$.

Suppose now that Firm C's optimal response satisfies $x_C \geq R(x_B, 0) - x_A$. We claim that it again follows that $x_A^* = R(x_B, 0)$.

First, it cannot be the case that $x_A > R(x_B, 0)$. Suppose $x_A \geq R(x_B, 0)$. Then the most A will pay for C's output is the value of C's output to Firm B, which in this case is

$$(x_B + x_C)p\left(R(x_B + x_C, 0) + x_B + x_C\right) - kx_B - x_B\left(p\left(R(x_B, 0) + x_B\right) - k\right)$$

which is less than kx_C because x_B was chosen optimally, so $x_C = 0$. Note a similar argument holds if $x_B + x_C > R(x^*, 0)$. In that case firm C still is paid less than kx_C. Producing any more than $R(x_B, 0)$ has no effect on firm C's output and no effect on firm A's sales (it will be destroyed), but is costly and lowers profits. It follows that $x_A \leq R(x_B, 0)$.

If $x_A < R(x_B, 0)$ and $x_C \geq R(x_B, 0) - x_A > 0$, then Firm A buys C's output for $\hat{p} > k$, and firm A's profits are

$$R(x_B, 0)p\left(x_B + R(x_B, 0)\right) - kx_A - \hat{p}x_C$$

while if $x_A = R(x_B, 0)$ and $x_C \geq R(x_B, 0) - x_A > 0$, then firm C produces nothing and firm A's profits are

$$R(x_B, 0)p\left(x_B + R(x_B, 0)\right) - kR(x_B, 0).$$

So clearly firm A will choose $x_A = R(x_B, 0)$.

It follows that depending on Firm C's optimal response, Firm A is either better off producing $R(x_B, 0)$ or $(1-k)/3$. So it is sufficient to consider the change in firm A's profits from deviating to $R(x_B, 0)$ to determine whether $(1-k)/3$ is firm A's optimal strategy.

First, A's profits at $(1-k)/3$ are

$$\frac{1}{3}(1-k)\left(1-\frac{5}{6}(1-k)-k\right)=\frac{1}{18}(1-k)^2=\frac{1}{18}-\frac{1}{9}k+\frac{1}{18}k^2$$

Firm A's profits at $R(x_B, 0)$ are

$$\frac{1}{3}(1+k)\left(1-\frac{1}{3}(1-k)-\frac{1}{3}(1+k)-k\right)$$

$$=\frac{1}{3}(1+k)\left(\frac{1}{3}-k\right)=\frac{1}{9}-\frac{2}{9}k-\frac{1}{3}k^2$$

So A will want to deviate to $R(x_B, 0)$ if

$$\frac{1}{9}-\frac{2}{9}k-\frac{1}{3}k^2>\frac{1}{18}-\frac{1}{9}k+\frac{1}{18}k^2$$

or

$$2-4k-6k^2>1-2k+k^2$$

$$1-2k-7k^2>0$$

$$k<.261204.$$

Q.E.D.

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