The Value of Public Information in Monopoly[¤]

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Abstract

The logic of the linkage principle of Milgrom and Weber (1982) extends to price discrimination. A non-linear pricing monopolist who sells to a single buyer always prefers to commit to publicly reveal information a¢liated to the valuation of the buyer.

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1. introduction

Consider the standard nonlinear pricing monopoly problem (Mussa and Rosen (1978)). A monopolist offers a price-quantity menu to a single privately-informed buyer with a quasi-linear utility. The traditional framework takes information as exogenously given. In reality, a seller can affect the information of the buyer by adopting certain policies such as relying on an outside certifier of quality or structuring the information system such that data on past buyers become publicly available. Similar policies could be enacted in other monopoly problems, such as franchising or procurement. In this paper, we ask whether committing to reveal an additional signal increases the monopolist's expected profit.

This problem is closely related to the one solved by Milgrom and Weber (1982). They consider an auctioneer selling a single good to several asymmetrically informed bidders and they ask whether the seller gains by adopting a policy of revealing an additional public signal about the good. According to their so-called linkage principle, in an affiliated environment the expected revenue of the seller is increased by such a transparency policy. While Milgrom and Weber's seller operates with an exogenously given mechanism (a certain auction format), our monopolist must change the price-quantity schedule in response to the additional signal.

We show that the logic of the linkage principle extends to the monopoly problem. The monopolist's expected profit cannot decrease if she commits to revealing a signal affiliated to the buyer's private signal. We prove this result for two scenarios according to what happens if the additional signal is not revealed. In the first case, the monopolist does not have access to the signal unless it is made public. In the second and perhaps more realistic case, the monopolist obtains it privately in case it is not revealed to the buyer.

Consider the comparison of public with no information. Revealing a public signal has two effects. Firstly, for any fixed quantity vector a buyer who receives an additional affiliated signal finds local downward deviations less attractive on average. The monopolist can then sell on average at higher prices. Secondly, the monopolist can further increase expected profits by conditioning the quantities offered on the realization of the public signal. Affiliation is crucial for the linkage principle to hold. We report simple examples in which affiliation fails and revealing a public signal hurts the monopolist.

The alternative to public information revelation is often for the monopolist to have private access to the same information. For example, a manufacturer selling to a new retailer has access to data on the final demand for the product which can be publicly disclosed. When does the principal prefer an honesty policy of always revealing directly its own private information? In order to compare public information to private information, we examine the case where the monopolist is privately informed at the contracting stage. This is a particular agency problem with an informed principal (Myerson (1983), Maskin and Tirole (1990) and (1992)). In equilibrium, the buyer may infer (part of) the private information of the monopolist from the menu offered (see e.g. Judd and Riordan (1994)). We show that the monopolist gains by committing to reveal directly the part of information that is revealed indirectly by the menu choice. This is because indirect revelation creates a conflict between the different types of principals which leads to distortions in the choice of menus. With a similar logic, the monopolist is better off committing to 'forget' the part of information that is not revealed in equilibrium. Public full revelation is shown to be the best policy.

Our results are valid also when the buyer's private signal and the public signal provide information also on the seller's cost of production or opportunity cost, as in Akerlof's (1970) market for lemons. We also allow the signals to provide information on the buyer's outside option.

The paper proceeds as follows. Section 2 defines the environment. Section 3 compares public to no information, and Section 4 public to private information. Section 5 discusses the applications and Section 6 concludes.

2. environment

A monopolist wishes to sell to a single buyer. For notational simplicity, the supports of all random variables are taken to be finite: The state of the world, unknown to both the buyer and the seller, is represented by the real random variable S with support $\mathcal{S}$. The private information of the buyer is represented by the real valued random variable T with support $\mathcal{T} = \{t_1, \ldots, t_n\}$, where without loss $t_1 < \ldots < t_n$. The additional signal Z with support $\mathcal{Z}$ is allowed to be multi-dimensional. The random variables S, T, Z are assumed to be affiliated (cf. Milgrom and Weber (1982)). When Z is unidimensional, affiliation means that for any $s', s'' \in \mathcal{S}$, $t', t'' \in \mathcal{T}$ and $z', z'' \in \mathcal{Z}$

$$\Pr\left(\max\langle s', s''\rangle, \max\langle t', t''\rangle, \max\langle z', z''\rangle\right) \Pr\left(\min\langle s', s''\rangle, \min\langle t', t''\rangle, \min\langle z', z''\rangle\right) \geq \Pr(s', t', z') \Pr(s'', t'', z''). \tag{1}$$

Let Q be a finite set of nonnegative real numbers. For concreteness, we interpret $q \in Q$ as quantity, but we could also view it as quality. Both buyer and seller have quasi-linear preferences. In state s, the total profit of the monopolist of providing quantity q for a non-linear price transfer p is $v(q, s) + p$. No assumption is made on the function v. The utility of the buyer is $u(q, s) - p$. Assume that u is strictly supermodular in q and s, i.e.

$$u(q'', s'') - u(q'', s') \geq u(q', s'') - u(q', s') \quad \forall s'' \geq s', \forall q'' \geq q',$$

with strict inequality whenever $s'' > s'$ and $q'' > q'$.

Our framework encompasses a number of monopoly markets. In a more familiar formulation of the non-linear monopoly pricing model, $u(q; s) = qs$ and $v(q; s) = -c(q)$. A buyer with private signal t_j has type $E[S|t_j]$ equal to the marginal willingness to pay for quantity. In the special case with $Q = \{0; 1\}$, we have the classic model of monopoly pricing for a single unit, where the demand function at price $p = E[S|t_j]$ is equal to $\Pr(T \geq t_i)$. More generally, $v(q; s)$ depends on s to allow the buyer to have private information on the opportunity value of the item for the seller. Our model therefore covers monopoly pricing in the lemons market. Clearly, the role of buyer and seller can be reversed, with the caveat that the uninformed party has all the bargaining power, and therefore optimally chooses the mechanism to offer to the informed party. This is also a special case of the procurement problem with endogenous quality of Manelli and Vincent (1995) with only one seller. In Akerlof's (1970) original setup $Q = \{0; 1\}$, we look at the problem of the (monopsonistic) buyer who makes the price offer to a partially informed seller.

The buyer's expected payoff, gross of the price paid, conditional on t_j and z is

$$U(q_i; t_j; z) = E_S[u(q_i; s)|t_j; z] = \sum_{s \in S} \Pr(s|t_j; z)u(q_i; s)$$

when buying quantity q_i. Affiliation and supermodularity interplay nicely. Supermodularity of u in $q; s$ and affiliation of S and T conditional on Z imply that

$$U(q; t; z) \text{ is supermodular in } q; t \tag{2}$$

for any z. This is verified immediately by making use of Milgrom's (1981) Proposition 1. Similarly, supermodularity of u in $q; s$ and affiliation of S and Z conditional on T implies that

$$U(q; t; z) \text{ is supermodular in } q; z \tag{3}$$

for any t.

Timing of events is as follows: First, an information regime is chosen. We consider three information regimes. While the buyer always observes the private signal T, observation of Z depends on the information regime: (a) No Information: neither the monopolist nor the buyer observes Z; (b) Public Information: both parties observe Z; (c) Private Information: only the monopolist observes Z. Second, observation of the signals take place, according to the information regime. Third, the monopolist proposes a menu of quantity-price pairs to the buyer. Fourth, the buyer selects a quantity-price pair within the menu offered by the seller or takes the outside option ($p = 0$, $q = 0$).

3. public information versus no information

This section compares public information to no information. Before making the pricing decision, the monopolist decides whether the signal Z should be made available to both himself and the buyer. The alternative to revelation is that no one observes the signal. Which policy results in higher profits?

The change in profits due to the addition of public information can be decomposed in two effects. First, holding constant the quantities sold to each type of buyer and optimizing only on price transfers, expected profits can either increase or decrease when the public signal is available. Second, the monopolist can further increase the expected profits by conditioning quantities on the realization of the public signal. In order to obtain an unambiguous comparison of profits, we identify a condition under which the first comparison is unambiguous.

With public information, the monopolist is allowed to offer a different menu of contracts $\langle q(z), p(z)\rangle = \langle q_i(z), p_i(z)\rangle_{i=0}^{n}$ depending on the realization z of the random variable Z. Consider the choice of the buyer who is offered such a price-quantity schedule in state z. The expected payoff (conditional on z) from $q_i(z), p_i(z)$ for the buyer who observes realization t_j of the private signal T and z of the public signal Z, is $U(q_i(z), t_j, z) - p_i(z)$.

The monopolist's maximal expected profit is denoted by $\pi(Z; \{T, Z\})$ (to indicate that the monopolist's information set is Z and the buyer's is $\{T, Z\}$) and is given by

$$\pi(Z; \{T, Z\}) = \max_{\langle q(z), p(z)\rangle} \sum_{s\in S}\sum_{z\in Z}\sum_{i=1} \Pr(t_i, z, s)(v(q_i(z), s) + p_i(z)) \tag{4}$$

subject to the individual rationality and incentive compatibility constraints

$$U(q_i(z), t_i, z) - p_i(z) \geq U(0, t_i, z) \quad \forall i$$
$$U(q_i(z), t_i, z) - p_i(z) \geq U(q_k(z), t_i, z) - p_k(z) \quad \forall i, \forall k.$$

Each non-excluded type t_i selects the contract $q_i(z), p_i(z)$ designed for that type. We deal with the individual rationality constraints with the convention that the menu of contracts offered by the monopolist must always include the null contract $q_0(z) \equiv p_0(z) \equiv 0$, which provides the outside option to the buyer. A vector $q = (q_0, \ldots, q_n)$ is said to be implementable if there exists a vector $p = (p_0, \ldots, p_n)$ such that $\langle q, p\rangle$ satisfies for all i and k

$$U(q_i(z), t_i, z) - p_i(z) \geq U(q_k(z), t_i, z) - p_k(z). \tag{$IC_{i,k}$}$$

We now report the characterization of the solution of the monopolist problem, restating well-known results (e.g. Maskin and Riley (1984)) in our setting. Affiliation of T and S

and supermodularity of $u(q;s)$ allow us to restrict attention to menus for which the local downward constraints are always binding:

Proposition 1 Let $U(q;t;z)$ be strictly supermodular in q and t for any z. Then: (i) q is implementable if and only if it is monotonic, $q_0 \leq \cdots \leq q_n$; (ii) Given an implementable q, at a profit maximizing price vector p, the local downward incentive compatibility constraints are binding,

$$p_i = p_{i-1} + U(q_i;t_i;z) - U(q_{i-1};t_i;z) \quad \forall i; \tag{5}$$

with $p_0 \equiv 0$.

Proof. See the Appendix. ∎

The monopolist's problem is not finite because price is a continuous variable. Nevertheless, Proposition 1 guarantees that, for every z, the problem has a solution because each q yields a unique optimal price vector, and Q is finite.

The problem without public information is identical to that with a completely uninformative public signal $Z = \emptyset$. Abusing notation, the buyer's expected payoff conditional on t_j is

$$U(q_i;t_j) = E_S[u(q_i;s)|t_j] = \sum_{s \in S} \Pr(s|t_j)u(q_i;s);$$

where we have dropped the functional dependence on the uniformative realizations of $\emptyset$. The maximal expected profit for the monopolist is

$$\pi(\emptyset;T) = \max_{\langle q;p \rangle} \sum_{s \in S} \sum_{i=1} \Pr(t_i;z)(v(q_i(z);s) + p_i) \tag{6}$$

subject to $\langle q;p \rangle$ being implementable.

The monopolist achieves higher expected profits in the presence of public affiliated information:

Theorem 1 If $S;T;Z$ are affiliated random variables, the monopolist achieves higher expected profits by publicly revealing Z, $\pi(Z;\{T;Z\}) \geq \pi(\emptyset;T)$.

We prove this result by showing that there is a suboptimal but feasible strategy which allows the monopolist to achieve higher expected profits once the additional affiliated signal is publicly reavealed. Suppose that the monopolist continues to offer the quantity vector which was optimal in the absence of information and appropriately modifies the prices in response to the realization of the public information. It is shown that this possibly suboptimal strategy results in higher expected profits under the affiliation assumption.

Let $\langle \hat{q}, \hat{p} \rangle$ be a menu of contracts which solves the seller's problem with no public information. Because $\hat{q}$ is implementable with no information, the necessity part of Proposition 1 (i) implies that $\hat{q}$ is nondecreasing. Since $\hat{q}$ is nondecreasing and $U(q;t;z)$ is supermodular in q,t given any z, the sufficiency part of Proposition 1 (i) guarantees that $\hat{q}$ is implementable for any realization z of the public signal.

Next, consider the case with public information and suppose that, for each z the seller offers menu $\langle \hat{q}, \tilde{p}(z) \rangle$, where $\tilde{p}(z)$ is defined by

$$\tilde{p}_i(z) = \tilde{p}_{i-1}(z) + U(\hat{q}_i; t_i; z) - U(\hat{q}_{i-1}; t_i; z) \qquad \forall i \tag{7}$$

with $\tilde{p}_0(z) \equiv 0$. Obviously, while $\langle \hat{q}, \hat{p} \rangle$ is optimal for the seller in the case with no information, $\langle \hat{q}, \tilde{p}(z) \rangle$ need not be optimal in the case with public information because, in general, the monopolist can do better by letting q depend on z.

The following statistical property will be useful:

Lemma 1 Take any $q_{j-1} \leq q_j$. The expected marginal utility of type t_j from buying q_j rather than q_{j-1} for all $j \leq i$ in the absence of public information is (weakly) lower than its expectation with respect to the affiliated signal Z conditional on information t_i:

$$\sum_{z \in Z} \Pr(z|t_i) \left(U(q_j; t_j; z) - U(q_{j-1}; t_j; z) \right) \geq U(q_j; t_j) - U(q_{j-1}; t_j) \quad \forall j \leq i. \tag{8}$$

Proof. By supermodularity (3), $U(q_j; t_j; z) - U(q_{j-1}; t_j; z)$ is non-decreasing in z. Affiliation of T and Z implies that $Z|t_i$ first-order stochastically dominates $Z|t_j$ if $i \geq j$. Then

$$\sum_{z \in Z} \Pr(z|t_i) \left(U(q_j; t_j; z) - U(q_{j-1}; t_j; z) \right) \geq \sum_{z \in Z} \Pr(z|t_j) \left(U(q_j; t_j; z) - U(q_{j-1}; t_j; z) \right) \tag{9}$$

for all $j \leq i$. Using the definitions and the law of total probabilities, we have

$$\begin{aligned}
\sum_{z \in Z} \Pr(z|t_j) \left(U(q_j; t_j; z) - U(q_{j-1}; t_j; z) \right) &= \sum_{z \in Z} \Pr(z|t_j) \sum_{s \in S} \Pr(s|t_j; z)(u(q_j; s) - u(q_{j-1}; s)) \\
&= \sum_{s \in S} \sum_{z \in Z} \Pr(z|t_j) \Pr(s|t_j; z)(u(q_j; s) - u(q_{j-1}; s)) \\
&= U(q_j; t_j) - U(q_{j-1}; t_j).
\end{aligned}$$

The result follows. $\square$

The following thought experiment is useful to interpret this result. Suppose that all the different quantities were sold at the same price. Consider a local downward deviation for the buyer with type $j = i$. Type t_i's utility loss when buying the quantity designed for the type immediately below is equal to $U(q_j; t_j; z) - U(q_{j-1}; t_j; z)$. As shown in the proof of the lemma, the expected cost of this deviation in the presence of public information is the same as the cost in the absence of public information. The lemma shows that in the eyes of type i the expected cost of a local deviation by all lower types $j \leq i$ is higher with public information than without.

Applying Lemma 1 to $\hat{q}$ and substituting (5) and (7) into (8) we obtain

$$\sum_{z \in Z} \Pr(z|t_i)\left(\tilde{p}_j(z) - \tilde{p}_{j-1}(z)\right) \geq \hat{p}_j - \hat{p}_{j-1} \quad \forall j \leq i. \tag{10}$$

Now, fix i and sum (10) from $j = 1$ to $j = i$. As $\tilde{p}_0(z) = \hat{p}_0 = 0$, we have

$$\sum_{z \in Z} \Pr(z|t_i)\,\tilde{p}_i(z) \geq \hat{p}_i \quad \forall i. \tag{11}$$

When selling the same quantities to the same buyer types, the monopolist can charge on average higher prices for each type once public information is revealed. Higher prices are incentive compatible because the expected cost of a local deviation is higher with public information than without, as guaranteed by Lemma 1.

We are now ready for the proof of Theorem 1:

Proof of Theorem 1. By (6),

$$\pi(\emptyset, T) = \sum_{s \in S} \sum_{i=1} \Pr(t_i; s)(v(\hat{q}_i; s) + \hat{p}_i)$$

and

$$\pi(Z; \{T; Z\}) \geq \sum_{s \in S} \sum_{z \in Z} \sum_{i=1} \Pr(t_i; z; s)(v(\hat{q}_i; s) + \tilde{p}_i(z))$$

Because the expected opportunity cost to the seller is the same in both cases

$$\sum_{s \in S} \sum_{z \in Z} \sum_{i=1} \Pr(t_i; z; s) v(\hat{q}_i; s) = \sum_{s \in S} \sum_{i=1} \Pr(t_i; s) v(\hat{q}_i; s);$$

the inequality $\pi(Z; \{T; Z\}) \geq \pi(\emptyset, T)$ reduces to

$$\sum_{s \in S} \sum_{z \in Z} \sum_{i=1} \Pr(t_i; z; s)\tilde{p}_i(z) \geq \sum_{s \in S} \sum_{i=1} \Pr(t_i; s)\hat{p}_i$$

That is,

$$\sum_{z \in Z} \sum_{i=1} \Pr(t_i; z)\tilde{p}_i(z) \geq \sum_{i=1} \Pr(t_i)\hat{p}_i$$

or

$$\sum_{i=1} \Pr(t_i) \sum_{z \in Z} \Pr(z|t_i)\, \bar{p}_i(z) \geq \sum_{i=1} \Pr(t_i) \hat{p}_i$$

(11) guarantees that this last inequality holds. ¤

If the monopolist does not alter the quantity vector, the expected value of the social welfare $u(q; s) + v(q; s)$ remains constant. However, the presence of public information makes local downward deviations more costly and allows the monopolist to charge higher prices. The cake is the same, but the monopolist gets a larger slice under the affiliation assumption.

Our result can be strengthened by showing that the monopolist cannot do better with any other policy of partial information disclosure. A policy of partial information disclosure corresponds to revelation of an experiment W, which is Blackwell less informative than Z. If Z is a more informative experiment than (or Blackwell sufficient for) W, the conditional distribution of S and T given Z and W is identical to the conditional distribution of S and T given Z only. We establish the following simple result on affiliation:

Lemma 2 Assume that S, T, and Z are affiliated random variables, and W is Blackwell less informative than Z. Then S, T, and Z are affiliated conditional on W.

Proof. We have

$$\Pr(s; t; z|w) = \Pr(s; t|z; w) \Pr(z|w) = \Pr(s; t|z) \Pr(z|w) = \frac{\Pr(s; t; z) \Pr(z|w)}{\Pr(z)}; \qquad (12)$$

where the first and third equalities are due to the definition of conditional probability and the second to the above-mentioned sufficiency property. Next, substitute (12) in the definition (1) of affiliation of S, T, and Z conditional on W, and notice that

$$\frac{\Pr(\max \langle z', z'' \rangle |w)}{\Pr(\max \langle z', z'' \rangle)} \frac{\Pr(\min \langle z', z'' \rangle |w)}{\Pr(\min \langle z', z'' \rangle)} = \frac{\Pr(z'|w)}{\Pr(z')} \frac{\Pr(z''|w)}{\Pr(z'')}. \qquad (13)$$

The result then follows from the assumption that S, T, and Z are affiliated. ¤

Theorem 1 can be applied repeatedly, once part W of the information contained in Z has become public. Regardless of the information W which has already become public, making public the remaining information contained in Z cannot hurt the seller. As this holds for any possible realization, it holds also ex ante:

Theorem 2 The monopolist achieves a higher expected profit by publicly revealing signal Z than by publicly revealing signal W Blackwell less informative than Z, $\pi(Z; \{T; Z\}) \geq \pi(W; \{T; W\})$.

Proof. Lemma 2 guarantees that S, T, and Z are affiliated conditionally on any realization of W. For any given realization w, all the conditions of Theorem 1 are verified, so that committing to reveal Z is profitable. Then, this is also true taking expectation over W. Therefore, publicly revealing both W and Z is more profitable than revealing only W, $\pi(\{Z; W\}; \{T; Z; W\}) \geq \pi(W; \{T; W\})$. Finally, $\pi(\{Z; W\}; \{T; Z; W\}) = \pi(Z; \{T; Z\})$ because revealing both W and Z is equivalent to revealing only Z, Blackwell sufficient for W. ∎

3.1 Welfare of the Buyer

Revelation of affiliated public information has an ambiguous effect on the expected payoff of the buyer. Clearly, when a perfectly informative signal is revealed publicly, the buyer is necessarily (weakly) worse off, being deprived of all informational rent. When the quantity vector is held fixed, public information results in a reduction of the rent of each type of buyer. Nevertheless, the buyer may benefit from the introduction of affiliated information, once the quantity vector offered is optimally re-adjusted by the monopolist in response to the affiliated public signal. This is illustrated in the single unit problem ($Q = \{0; 1\}$) with $u(q; s) = sq$, $v(q; s) = -cq$, with two ex-ante equally likely states $s_1 = 0$ and $s_2 = 1$, and symmetric binary signals ($\Pr(t_1|s_1) = \Pr(t_2|s_2) = \tau$ and $\Pr(z_1|s_1) = \Pr(z_2|s_2) = \zeta$). Set $c = 7/10$, $\tau = 6/10$, and $\zeta = 9/10$. Without public information no sale occurs ($\hat{q}_1 = \hat{q}_2 = 0$), resulting in zero rent for the buyer. With public information, $\hat{q}_1(z_2) = \hat{q}_2(z_2) = 1$ and $\hat{p}_1(z_2) = \hat{p}_2(z_2) = E[S|t_1; z_2] = 6/7$, as $E[S|t_1; z_2] - c = 11/70 > 67/500 = \Pr(t_2|z_2)[E[S|t_2; z_2] - c]$. Type t_2 buyer enjoys the positive rent $E[S|t_2; z_2] - E[S|t_1; z_2] = 15/203$ when z_2 is realized, and zero rent otherwise.

3.2 Social Welfare

Similarly, the effect of affiliated public information on the expected value of the sum of the payoffs of the buyer and the seller is ambiguous. Clearly, a perfectly informative public signal cannot decrease total welfare. However, a partially informative signal may decrease it by inducing the seller to choose a more distortive quantity schedule, in order to extract more rent from the buyer. Consider the binary one unit example of the previous subsection, with $c = 0$, $\tau = 5/8$, and $\zeta = 19/20$. Without public information, the monopolist does not exclude the low type ($\hat{q}_1 = \hat{q}_2 = 1$ and $\hat{p}_1 = \hat{p}_2 = 3/8$), thereby implementing the socially optimal allocation with expected social welfare $1/2$. With public information, exclusion of the low type ($\hat{q}_1(z_1) = \hat{p}_1(z_1) = 0$) is optimal for the monopolist when z_1 is realized, as $E[S|t_1; z_1] = 3/98 < 1/32 = \Pr(t_2|z_1) E[S|t_2; z_1]$. The resulting social welfare is $\Pr(z_1)\Pr(t_2|z_1) E[S|t_2; z_1] + \Pr(z_2) E[S|z_2] = 157/320 < 1/2$.

3.3 When Affiliation Fails

One might think that the monopolist would always profit from revealing a public signal, because she erodes the buyer's rent by reducing the informational asymmetry. We now show that this is not the case. A public signal which is not affiliated to the valuation can actually result in lower profits for the monopolist.

We have used the following four implications of affiliation of S, T, and Z: (i) $U(\hat{q}_j; t) - U(\hat{q}_{j-1}; t)$ is a nondecreasing function of t, due to affiliation of S and T; (ii) $U(\hat{q}_j; t; z) - U(\hat{q}_{j-1}; t; z)$ is a nondecreasing function of t for given z, due to affiliation of S and T conditional on any z; (iii) $U(\hat{q}_j; t; z) - U(\hat{q}_{j-1}; t; z)$ is an increasing function of z for given t, due to affiliation of Z and S conditional on any t; and (iv) T is affiliated to Z. Fact (i) guarantees monotonicity of the optimal quantity schedule $\hat{q}$. Fact (ii) combined with part (ii) of Proposition 1 guarantees that $\hat{q}$ is implementable for any realization of the public signal z. Facts (iii) and (iv) allow us to establish our comparison by means of (9). Indeed, if either of these three affiliation assumptions (the one needed for (i) is sufficient for (ii)) is violated, the monopolist may lose from committing to reveal public information. We give three counterexample to Theorem 1, where we relax one at a time each of the these three crucial affiliation conditions. In all these examples the monopolist can sell zero or one unit at no cost: $Q = \{0; 1\}$, $v(q; s) \equiv 0$, $u(q; s) = qs$.

Example 1. Relaxing affiliation of Z and S. Consider three equally likely states, $\{s_1 = 10; s_2 = 11; s_3 = 12\}$, a binary private signal T affiliated to S with $\Pr(t_1|s_1) = \Pr(t_1|s_2) = 1$ and $\Pr(t_2|s_3) = 1$, and a binary public signal Z not affiliated to S with $\Pr(z_1|s_1) = \Pr(z_1|s_3) = 1$ and $\Pr(z_2|s_2) = 1$. Furthermore, Z and T are independent conditionally on S, and therefore affiliated conditionally on S and unconditionally. All the affiliation conditions used in the proof of Theorem 1 are satisfied, other than affiliation of Z and S conditional on some t. With no public information, the monopolist has three implementable quantity choices ($q_1 = q_2 = 0$), ($q_1 = 0; q_2 = 1$), and ($q_1 = q_2 = 1$). The first yields maximal expected profit zero, the second 12/3, and the third 21/2 (by setting $p_1 = p_2 = E[S|t_1] = 21/2$). With public information the expected profit if ($q_1 = q_2 = 1$) (which is clearly always optimal) is equal to $E[S|t_1; z_1] = 10$ with probability 2/3 and

$E[S|t_2; z_2] = 11$ with probability 1/3. Hence, the expected profit is lower than 21/2 and the monopolist is worse off with revelation of public information.

Example 2. Relaxing affiliation of T and S. The monopolist is worse off by committing to reveal the public signal Z' affiliated to the valuation S, when the private signal T' of the buyer is not affiliated to S. Take $Z' = T$ and $T' = Z$ of the previous example. Now, the only assumption not satisfied is affiliation of T and S conditional on some z. In this case, profit without public information are $E[S|t'_1] = E[S|t'_2] = 11$. With public information, expected profits are $\Pr(z'_1)\, E[S|t'_1; z'_1] + \Pr(z'_2)\, E[S|t'_1; z'_2] = 20/3 + 12/3 < 11$.

Example 3. Relaxing affiliation of Z and T In this example Z and S are affiliated conditional on t, T and S are affiliated conditional on z, but Z and T are not affiliated. Unlike the previous examples, we need Z and T not independent conditional on S. Consider two equally likely states, $\{s_1 = 10; s_2 = 11\}$. The private signal T alone is uninformative about S: there are two possible realizations, with $\Pr(t|s_1) = \Pr(t|s_2) = 1/2$ for $t = 0; 1$. In the absence of public information, maximal profits are equal to $21/2$. Consider the effect of the public signal $Z = S - T$. By observing both Z and T, the buyer can infer the state perfectly. With revelation of Z, the expected profit for the monopolist is $(3/4)\, 10 + (1/4)\, 11 < 21/2$.

4. public versus private information

In the previous section we have compared regime $G(Z; \{T; Z\})$ where both the seller and the buyer observe Z to regime $G(\emptyset; T)$ where neither the seller nor the buyer observe Z. In this section we compare the equilibria and profits in $G(Z; \{T; Z\})$ and $G(Z; T)$. While in both $G(\emptyset; T)$ and $G(Z; \{T; Z\})$, the monopolist has no informational advantage over the buyer when she proposes the contract, $G(Z; T)$ is a principal-agent problem with an informed principal and an informed agent. When both privately informed, the monopolist and the buyer are playing a principal-agent game with an informed principal (Myerson (1983) and Maskin and Tirole (1990, 1992)).

Note that, if Z were a verifiable signal, a standard unraveling argument (e.g. Milgrom (1981)) would guarantee that in equilibrium Z is fully revealed to the buyer. Hence, a verifiable private signal is equivalent to a public signal, which we have already analyzed. For the remaining of the section, we assume that Z is not verifiable.

A menu of contracts M is a collection of quantity-price pairs $(q; p)$ with $q \in Q$ and $p \in [0; \infty)$, containing the null contract $(0; 0)$. Let $\mathcal{M}$ be the collection of all possible M's. The monopolist's action consists of selecting $M \in \mathcal{M}$. Given M, the buyer's action is a choice $(q; p) \in M$.

By choosing a menu, the monopolist may reveal some of her information to the buyer. Following Maskin and Tirole (1990), we focus on perfect Bayesian equilibria. As will become obvious later, our results hold a fortiori if we impose refinements.

In order to define perfect Bayesian equilibria, we introduce mixed strategies and beliefs. Given the monopolist's private information z, let $\mu(M|z)$ be the probability that the monopolist offers menu M. Given that the buyer observes t and is offered menu M, let $\sigma((q,p)|M,t)$ be the probability that he selects the quantity-price pair (q,p). To avoid unnecessary complications, we restrict the monopolist to randomize between only a finite number of menus for each realization of z, and to offer menus containing only a finite number of price-quantity pairs.[1] The buyer also uses the information he has to form a belief on the monopolist's 'type'. Given menu M and signal t, let $\beta(z|M,t)$ be the probability that the buyer assigns on the monopolist having observed signal z.

Similarly to the definition of U, let $V(q,t,z) = \sum_{s\in S}\Pr(s|t,z)v(q,s)$. A perfect Bayesian equilibrium (PBE) e of $G(Z,T)$ is a triple (μ^*,σ^*,β^*) satisfying: (i) monopolist's best reply

$$\sum_{M\in M^*(z)}\sum_{t\in T}\sum_{(q,p)\in M}\mu^*(M|z)\Pr(t|z)\sigma^*((q,p)|M,t)[V(q,t,z)+p] \ge \sum_{t\in T}\sum_{(q,p)\in M'}\Pr(t|z)\sigma^*((q,p)|M',t)[V(q,t,z)+p] \quad \forall z\in Z, \forall M'\in \mathcal{M} \tag{14}$$

where $M^*(z) = \{M\in\mathcal{M} \mid \mu^*(M|z)>0\}$; (ii) buyer's best-reply

$$\sum_{z\in Z}\sum_{(q,p)\in M}\beta^*(z|M,t)\sigma^*((q,p)|M,t)[U(q,t,z)-p] \ge \sum_{z\in Z}\beta^*(z|M,t)[U(q',t,z)-p'] \quad \forall t\in T, \forall M\in\mathcal{M}, \forall (q',p')\in M, \tag{15}$$

and (iii) consistency of buyer's belief following actions of the monopolist taken in equilibrium

$$\beta^*(z|M,t) = \frac{\mu^*(M|z)\Pr(z|t)}{\sum_{\tilde{z}\in Z}\mu^*(M|\tilde{z})\Pr(\tilde{z}|t)} \quad \forall z, \forall M\in M^*(z), \forall t. \tag{16}$$

Let $\pi_e(Z,T)$ be the expected profit of the monopolist in equilibrium e.[2]

[1] Formally, let K and L be two positive natural numbers. A menu M is defined as a collection of less than K quantity-price pairs (q,p). The mixed strategy μ of the monopolist must be such that, for each z, at most L menus are played with positive probability.

[2] We choose to define (MBR) in pure strategies and (BBR) in mixed strategies for notational convenience. The restriction to pure-strategy deviations in (MBR) is without loss of generality. If there exists a profitable deviation in mixed strategies, then there exists a profitable deviation in one of the pure strategies in the support.

In a perfect Bayesian equilibrium of $G(Z;T)$ the buyer may infer information about Z from the monopolist's choice of menu. We show that the monopolist is better off by committing to reveal directly the information inferred by the buyer and to forget the information that she does not use in equilibrium. For this purpose, it is useful to define the implicit signal W_e revealed by e. The (finite) set of menus offered with positive probability in e is $\mathcal{M}^* = \bigcup_{z \in Z} \mathcal{M}^*(z)$. Assign to each element of $\mathcal{M}^*$ a different index $w_e \in \mathcal{W}_e$, where $\mathcal{W}_e$ is a set with the same cardinality of $\mathcal{M}^*$. Hence, $M^*(w_e)$ denotes a menu of contracts which is chosen in equilibrium with positive probability and is indexed with w_e. Define the random variable W_e with support $\mathcal{W}_e$ and conditional probability

$$\Pr(W_e = w_e|z) = \mu^*(M^*(w_e)|z) \quad \forall z; \forall w_e.$$

The random variable W_e is the implicit signal on Z that the monopolist produces through her choice of menu. Clearly, W_e cannot be more informative than Z (on S and T) in the sense of Blackwell. If the PBE under consideration is separating, then W_e is sufficient statistics for Z. If the e is pooling, then W_e is uninformative. In partially revealing equilibria, W_e is informative but less so than Z.

Given a PBE e of $G(Z;T)$, consider $G(W_e;\{T;W_e\})$, that is, the game in which the monopolist observes W_e and the buyer observes both W_e and T. As the monopolist has no private information, this is a straightforward principal-agent problem with an uninformed principal. Because the agent forms no beliefs, the conditions for a PBE are the same as the conditions for a subgame-perfect equilibrium. The set of expected profits that the monopolist can reach in a PBE is equal to the set of expected profits that she can reach in a subgame perfect equilibrium. In turn, as is well known from the principal-agent theory, the maximum expected profit of this set is equal to the value of a maximization problem in which the monopolist chooses both her strategy and the agent's strategy, subject to the agent's strategy being a best response. There is no loss of generality in assuming that both parties play pure strategies. However, it is convenient for our proofs to allow the buyer to use a mixed strategy. A pure strategy for a monopolist is the choice of a (finite) menu $\gamma(w) \in \mathcal{M}$ given for each realization of signal w. A mixed strategy for the buyer consists of assigning probability $\tau((q;p)|M;t;w)$ of selecting the price-quantity pair $(q;p)$ from the offered menu M, given that he has observed t and w. The monopolist problem is

$$\pi(W_e;\{T;W_e\}) = \max_{\gamma;\tau} \sum_{w \in \mathcal{W}_e} \sum_{t \in T} \sum_{(q;p) \in \gamma(w)} \Pr(t;w)\,\tau((q;p)|M;t;w)[V(q;t;w) + p] \qquad (17)$$

subject to buyer's best response

$$\sum_{(q;p) \in M} \tau((q;p)|M;t;w)[U(q;t;w) - p] \geq U(q;t;w) - p \quad \forall t; \forall w; \forall M; \forall (q;p) \in M \qquad (18)$$

As we saw in the previous section, this problem can be reduced to a finite problem and thus has a solution. Moreover, $\pi(W_e, \{T, W_e\})$ is equal to the maximal expected profit the monopolist can obtain in a PBE of $G(W_e, \{T, W_e\})$. We now show that this value cannot be lower than $\pi_e(Z, T)$.

Theorem 3 Let e be a PBE of $G(Z, T)$ and W_e the corresponding implicit signal. The expected profit that the monopolist can achieve if W_e is revealed directly is higher than the expected profit in e: $\pi(W_e, \{T, W_e\}) \geq \pi_e(Z, T)$.

Proof. Three preliminary results will prove useful:

Claim 1: $\beta^*(z|M^*(w), t) = \Pr(z|w, t) \quad \forall t, \forall w, \forall z.$

Check: By (16),

$$\begin{aligned}
\beta^*(z|M^*(w), t) &= \frac{\mu^*(M^*(w)|z)\Pr(z|t)}{\sum_{\tilde{z}\in Z}\mu^*(M^*(w)|\tilde{z})\Pr(\tilde{z}|t)} = \frac{\Pr(w|z)\Pr(z|t)}{\sum_{\tilde{z}\in Z}\Pr(w|\tilde{z})\Pr(\tilde{z}|t)} \\
&= \frac{\Pr(w|t, z)\Pr(z|t)}{\sum_{\tilde{z}\in Z}\Pr(w|t, \tilde{z})\Pr(\tilde{z}|t)} = \frac{\Pr(w, z|t)}{\sum_{\tilde{z}\in Z}\Pr(w, \tilde{z}|t)} = \frac{\Pr(w, z|t)}{\Pr(w|t)} = \Pr(z|w, t).
\end{aligned}$$

where the third equality is because W is less informative than Z.

Claim 2: $\sum_{z\in Z}\Pr(z|w, t)[U(q, t, z) - p] = U(q, t, w) - p \quad \forall t, \forall w, \forall z.$

Check: By the definition of U,

$$\begin{aligned}
\sum_{z\in Z}\Pr(z|w, t)[U(q, t, z) - p] &= \sum_{z\in Z}\sum_{s\in S}\Pr(z|t, w)\Pr(s|t, z)[u(q, s) - p] \\
&= \sum_{z\in Z}\sum_{s\in S}\Pr(z|t, w)\Pr(s|t, w, z)[u(q, s) - p] \\
&= \sum_{z\in Z}\sum_{s\in S}\Pr(s, z|t, w)[u(q, s) - p] \\
&= \sum_{s\in S}\Pr(s|t, w)[u(q, s) - p] = U(q, t, w) - p
\end{aligned}$$

Claim 3: $\sum_{z\in Z}\Pr(w|t, z)\Pr(t, z)[V(q, t, z) + p] = \Pr(t, w)[V(q, t, w) + p] \quad \forall t, \forall w.$

Check: By the definition of V,

$$
\begin{aligned}
\sum_{z\in Z} \Pr(w|t;z)\Pr(t;z)\,[V(q;t;z)+p] &= \sum_{z\in Z}\sum_{s\in S} \Pr(t;w;z)\Pr(s|t;z)[v(q;s)-p] \\
&= \sum_{z\in Z}\sum_{s\in S} \Pr(t;w;z)\Pr(s|t;w;z)[v(q;s)-p] \\
&= \sum_{z\in Z}\sum_{s\in S} \Pr(s;t;w;z)[v(q;s)-p] \\
&= \sum_{s\in S} \Pr(s;t;w)[v(q;s)-p] \\
&= \sum_{s\in S} \Pr(t;w)\Pr(s|t;w)[v(q;s)-p] \\
&= \Pr(t;w)\,[V(q;t;w)+p]
\end{aligned}
$$

Let $(\nu^{\circ};\tau^{\circ})$ be a solution of the monopolist problem (17). Define $(\hat{\nu};\hat{\tau})$ as follows:

$$\hat{\nu}(w) = M^{*}(w) \quad \forall w$$

$$\hat{\tau}((q;p)|M;t;w) = \begin{cases} \sigma^{*}((q;p)|M;t) & \text{if } M = M^{*}(w) \\ \tau^{\circ}((q;p)|M;t;w) & \text{otherwise} \end{cases} \qquad \forall w;\forall t;\forall M$$

Let $\hat{\Pi}$ indicate the expected profit the monopolist gets if she plays $\hat{\nu}$ and the buyer plays $\hat{\tau}$. We prove the theorem by showing that $\pi_e(Z;T) = \hat{\Pi} \le \pi(W_e;\{T\};W_e)$.

To show that $\hat{\Pi} \le \pi(W_e;\{T\};W_e)$, it is sufficient to prove that $(\hat{\nu};\hat{\tau})$ satisfies (18). To see this, note that, for a every w and t, (15) implies:

$$\sum_{z\in Z}\sum_{(q;p)\in M^{*}(w)} \beta^{*}(z|M^{*}(w);t)\sigma^{*}((q;p)|M^{*}(w);t)[U(q;t;z)-p] \ge \sum_{z\in Z}\beta^{*}(z|M^{*}(w);t)[U(q;t;z)-p] \quad \forall (q;p)\in M^{*}(w)$$

or, by Claim 1,

$$\sum_{z\in Z}\sum_{(q;p)\in M^{*}(w)} \Pr(z|w;t)\sigma^{*}((q;p)|M^{*}(w);t)[U(q;t;z)-p] \ge \sum_{z\in Z}\Pr(z|w;t)[U(q;t;z)-p] \quad \forall (q;p)\in M^{*}(w)$$

which, by the definition of $(\hat{\nu};\hat{\tau})$, rewrites as

$$\sum_{z\in Z}\sum_{(q;p)\in \hat{\nu}(w)} \Pr(z|w;t)\hat{\tau}((q;p)|\hat{\nu}(w);t;w)[U(q;t;z)-p] \ge \sum_{z\in Z}\Pr(z|w;t)[U(q;t;z)-p] \quad \forall (q;p)\in \hat{\nu}(w)$$

By Claim 2, the latter reduces to

$$\sum_{(q;p)\in \hat{M}(w)} \hat{\tau}((q;p)|\hat{M}(w);t;w)[U(q;t;w)-p] \ge U(q;t;w)-p \quad \forall (q;p)\in \hat{M}(w);$$

which means that, given t and w, if $M=\hat{M}(w)$, then $\hat{\tau}$ satisfies (18). If $M \neq \hat{M}(w)$, then $\hat{\tau}$ satisfies (18) because $\tau^{\pm}$ satisfies it by definition.

We now show that $\hat{\pi}=\pi_e(Z;T)$. By the definition of perfect Bayesian equilibrium e,

$$\begin{aligned}
\pi_e(Z;T) &= \sum_{z\in Z}\sum_{M\in M^*(z)}\sum_{t\in T}\sum_{(q;p)\in M} \mu^*(M|z)\Pr(t;z)\,\sigma^*((q;p)|M;t)[V(q;t;z)+p] \\
&= \sum_{z\in Z}\sum_{w\in W_e}\sum_{t\in T}\sum_{(q;p)\in M^*(w)} \Pr(w|z)\Pr(t;z)\,\sigma^*((q;p)|M^*(w);t)[V(q;t;z)+p] \\
&= \sum_{z\in Z}\sum_{w\in W_e}\sum_{t\in T}\sum_{(q;p)\in M^*(w)} \Pr(w|t;z)\Pr(t;z)\,\sigma^*((q;p)|M^*(w);t)[V(q;t;z)+p] \\
&= \sum_{z\in Z}\sum_{w\in W_e}\sum_{t\in T}\sum_{(q;p)\in \hat{M}(w)} \Pr(w|t;z)\Pr(t;z)\,\hat{\tau}((q;p)|\hat{M}(w);t;w)[V(q;t;z)+p] \\
&= \sum_{w\in W_e}\sum_{t\in T}\sum_{(q;p)\in M} \Pr(t;w)\,\hat{\tau}((q;p)|\hat{M}(w);t;w)[V(q;t;w)+p] = \hat{\pi}
\end{aligned}$$

where the second equality is due to the definition of w, the third is due to the fact that W_e is less informative than Z, the fourth comes from the definition of $(\hat{M};\hat{\tau})$, and the fifth is due to Claim 3. $\blacksquare$

Intuitively, revealing information directly imposes less constraints on the monopolist than letting the buyer infer it. In the latter case, there are constraints across types of monopolists, corresponding to all possible realizations w of W. In order to differentiate herself from other types, a certain type of monopolist may have to choose a menu which is suboptimal given the implicit signal sent. This cannot occur if the implicit signal is revealed directly and the monopolist has no residual information.

It is useful to examine the two extreme cases of completely informative and completely uninformative equilibria. If e is a pooling equilibrium, Theorem 3 says that the monopolist is better off not knowing something rather than knowing it and not using it in equilibrium. The unused knowledge does not help her extract rent from the buyer but may constrain her choice of menu. If e is a separating equilibrium, the monopolist's choice of menu reveals all her information to the buyer. However, if this information were already publicly available, it would still be used and the incentive-compatibility constraints across monopolist types would not need to be satisfied.

Note that if e is a separating equilibrium, the expected profit in e is lower than $\pi(Z;\{T;Z\})$, in which case the monopolist is better off revealing her private signal (whether or not the signal is affiliated). If e is not a fully separating equilibrium, we still need to show that $\pi(Z;\{T;Z\}) \geq \pi(W_e;\{T;W_e\})$, which, however, is an immediate consequence of Theorem 2 (but requires affiliation). The main result of this section is:

Theorem 4 *In an affiliated environment, the monopolist increases her expected profits by committing to reveal her private information,* $\pi(Z;\{T;Z\}) \geq \pi(Z;T)$.

Proof. For $e \in E(Z;T)$, $\pi_e(Z;T) \leq \pi(W_e;\{T;W_e\})$ by Theorem 3. Theorem 2 then implies $\pi(W_e;\{T;W_e\}) \leq \pi(Z;\{T;Z\})$, as W_e is a garbling of Z. We conclude that for all $e \in E(Z;T)$, $\pi_e(Z;T) \leq \pi(Z;\{T;Z\})$. ∎

4.1 Discussion

We have assumed that the payments required by the monopolist from the buyer cannot depend on the ex-post realizations of the monopolist's private information. When contracts with payment contingent on the realization of such information are allowed, the seller might be better off not making public such information. In the spirit of the example contained in Section 7 of Myerson (1981), the monopolist could exploit the correlation of this information with that of the buyer in order to achieve higher profits than in the absence of information or with information ex-ante publicly available also to the buyer. While the literature on rent extraction typically considers the case where correlated information is possessed by other agents (see Crémer and McLean (1985) and McAfee and Reny (1992) for important developments), here the problem is complicated by the fact that this information is possessed by the monopolist herself. An important difference with the case considered in this literature on rent extraction in the presence of correlated information is that the seller observes the realization of Z before offering the menu of contracts to the seller.

A privately informed principal might then signal such information by its choice of mechanism. According to the inscrutability principle of Myerson (1983), with general mechanisms the principal should never need to communicate any information to the agent(s) by her choice of mechanism, because such communication can always be built into the mechanism itself. This is not the case in the restricted class of mechanisms we allow. (?)

Maskin and Tirole (1990, 1992) further analyzed the principal-agent problem with privately informed principal. They distinguish between common values (the general case) and private values (in which the information of the principal does not directly affect the agent's payoff). They also distinguish between the generic case and the quasi-linear case, in which the payoffs of both the principal and the agent are separable in money. The present model can be seen as a Maskin-Tirole problem with common values and quasi-linear payoffs.

Maskin and Tirole (1990, Proposition 1) have a result that appears to go in the opposite direction from ours. They …nd that, with private values and generic payo¤s, the principal is better o¤ if she has a private signal rather than if the same signal is publicly revealed. The intuition for that result is that, when the principal is privately informed, there could be a pooling or semi-pooling equilibrium, in which the di¤erent types of principal provide insurance to each other against the uncertainty created by the private information of the agent. This statement does not hold in the (nongeneric) case of quasi-linear payo¤s. There, the insurance motive disappears and the principal is indi¤erent between being privately informed or committing to reveal her signal (Proposition 11).

As Maskin and Tirole point out, things change in common values. When the principal is privately informed, there is a con‡ict of interest between the di¤erent types of principal, which creates negative externalities. Usually, this occurs when low types have an incentive to pretend to be high types and therefore high types must choose ine¢cient actions in order to di¤erentiate themselves. Seen in this framework, our result does not appear surprising. By focusing on the quasi-linear case, we kill the insurance motive. To summarize, combining Maskin and Tirole's work with ours, we can tackle the question of whether committing to reveal private information bene…ts the principal: In the quasi-linear private value case, the principal is indi¤erent; In the generic private value case, the answer is negative; In the quasi-linear common value case, the answer is positive; and …nally in the generic case with common values, the answer is ambiguous.

There is a di¤erence between our model and Maskin and Tirole's, which is worth pointing out. They allow for mechanisms in which the both the principal and the agent send a message. Their timing is as follows: (1) Principal and Agent observe private information; (2) Principal chooses mechanism; (3) Agent accepts or rejects; (4) Principal and Agent simultaneously choose messages. In our model, the principal cannot send a message in stage 4. Her only action lies in the choice of the mechanism. For our purpose, this is a realistic mechanism space. However, it would be interesting to investigate how our results are modi…ed by using a larger mechanism space.

5. applications

An independent agency which certi…es the quality of the product of a monopolist o¤ers a valuable service to the monopolist, provided the reports are a¢liated to the quality of the good . Similarly, a monopolist pro…ts by committing to reveal the level of satisfaction of other consumers. Our …ndings have important implications for experimentation and price dynamics in models of social learning. See Ottaviani (1996) for a model of monopoly pricing with social learning by the buyers. In this situation, the monopolist's current pricing strategy a¤ects the amount of information publicly revealed to future buyers. The consumers are learning from each other's observable purchasing behavior, as in the models of Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992). The seller a¤ects this social learning process by its choice of a dynamic pricing strategy. In general, a patient monopolist deviates from the myopically optimal price in order to increase the amount of public information revealed.

The possibility of trade in Akerlof's (1970) lemons market can be a¤ected in a non-monotonic fashion as the information asymmetry decreases because of the public revelation of a¢liated information. This can be easily seen in simple examples, analogous to those constructed by Levin (1998) to compare the possibility of trade as the private information of the informed party improves. Despite this non-monotonicity in public information of the set of prices supporting trade, our general result guarantees that the uninformed price-maker is necessarily better o¤ when a¢liated information is revealed publicly.

6. conclusion

Since its discovery by Milgrom and Weber (1982), the linkage principle is acquiring a central role in models of pricing. We have shown here that its logic extends to the classic environment of a price-discriminating monopolist selling multiple units (or single units of heterogenous quality) of a good to a single buyer. While the principle generalizes in some interesting directions, two negative results have been recently provided. Perry and Reny (1999) have recently shown that the linkage principle does not generalize to multi-unit Vickrey auctions with more than one buyer who each demands more than one unit.[3] Moscarini and Ottaviani (1998) show that the linkage principle does not hold when competing principals sell to a the buyer with private information on the relative value of goods.

In the monopoly problem we have studied here the buyer's type has one dimension only. Our proofs rely on the structure of the monopoly solution for the unidimensional case and do not readily extend to the characterizations provided by the recent literature on multidimensional monopoly (Armstrong (1996) and Rochet and Choné (1998)). It is an open question whether our results extend to the multidimensional case.

In this paper we do not discuss the value for the monopolist of the private information of the buyer. By selecting trial and return policies, the seller can often controls the

[3] In a Vickrey auction equilibrium, losing bids are based on underestimates of the signals of the competing bidders, while winning bids on overestimates. Revelation of a¢liated public information not only results on average in an increase of losing bids but also in a decrease of winning bids. When bidders demands multiple units, the second e¤ect may dominate the …rst one.

amount of private information available to the buyer when purchasing the product. Lewis and Sappington (1995) o¤er a series of interesting examples to illustrate how the seller's pro…ts change as the buyer becomes better informed about the quality of the product. In contrast to the case of public information, no general principle has yet emerged in the comparison of situations with di¤erent buyer's private information.

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APPENDIX

Proof of Proposition 1

We provide the details of a revisitation in our setting of some standard results on the reduction of the self selection (see Section 3 of Maskin and Riley's (1984)). As the public signal z is held fixed throughout this Appendix, we can lighten notation by omitting it.

Proposition 1 Let $U(q;t)$ be strictly supermodular in q and t. Then: (i) q is implementable if and only if it is monotonic, $q_0 \leq \cdots \leq q_n$; (ii) Given an implementable q, at a profit maximizing price vector p the local downward incentive compatibility constraints are binding,

$$p_i = p_{i-1} + U(q_i;t_i) - U(q_{i-1};t_i) \quad \forall i;$$

with $p_0 \equiv 0$.

This is proved by the following four results.

Lemma 3 q is implementable only if $q_0 \leq \cdots \leq q_n$.

Proof. Suppose not, i.e. $q_i < q_k$ for an $i > k$. Supermodularity of U (with $y_i \geq y_k$ and $q_i < q_k$) implies

$$U(q_i;y_i) + U(q_k;y_k) < U(q_k;y_i) + U(q_i;y_k). \tag{19}$$

Implementability of q implies

$$U(q_i;y_i) + U(q_k;y_k) \geq U(q_k;y_i) + U(q_i;y_k);$$

obtained by summing $(IC_{i;k})$ and $(IC_{k;i})$, in contradiction with (19). ¤

Lemma 4 Suppose $q_0 \leq \cdots \leq q_n$. Consider $(q;p)$. If all the adjacent downward incentive compatibility constraints $IC_{i;i-1}$ hold as equalities, then all other IC's are satisfied.

Proof. The statement is proven in two steps. First, we show that all upward constraints are satisfied, then we show that all downward constraints are satisfied.

Proof. Step 1: $(IC_{i;i-1}); \forall i \rightarrow (IC_{k;i}); \forall k < i$. To see this,

$$\begin{aligned}
& p_i - p_k \\
= \; & (p_i - p_{i-1}) + (p_{i-1} - p_{i-2}) + \ldots + (p_{k+1} - p_k) \\
= \; & (U(q_i;y_i) - U(q_{i-1};y_i)) + (U(q_{i-1};y_{i-1}) - U(q_{i-2};y_{i-1})) + \ldots + (U(q_{k+1};y_{k+1}) - U(q_k;y_{k+1})) \\
\geq \; & (U(q_i;y_k) - U(q_{i-1};y_k)) + (U(q_{i-1};y_k) - U(q_{i-2};y_k)) + \ldots + (U(q_{k+1};y_k) - U(q_k;y_k)) \\
= \; & U(q_i;y_k) - U(q_k;y_k);
\end{aligned}$$

where the second equality is $(IC_{i,i-1})$, the strict inequality comes from supermodularity (and the assumption that q is nondecreasing), and the last equality is an immediate simplification. ■

Step 2: $(IC_{i,i-1}), \forall i \rightarrow (IC_{i,k}), \forall k < i$. To see this,

$$
\begin{aligned}
& p_i - p_k \\
= \; & (U(q_i, y_i) - U(q_{i-1}, y_i)) + (U(q_{i-1}, y_{i-1}) - U(q_{i-2}, y_{i-1})) + \ldots + (U(q_{k+1}, y_{k+1}) - U(q_k, y_{k+1})) \\
\leq \; & (U(q_i, y_i) - U(q_{i-1}, y_i)) + (U(q_{i-1}, y_i) - U(q_{i-2}, y_i)) + \ldots + (U(q_{k+1}, y_i) - U(q_k, y_i)) \\
= \; & U(q_i, y_i) - U(q_k, y_i),
\end{aligned}
$$

where the argument is analogous to Step 1. □

Corollary 1 If $q_0 \leq \cdots \leq q_n$, then q is implementable.

Proof. For any q, it is always possible to construct a p such that all $(IC_{i,i-1})$ hold as equalities. If $q_1 \leq \cdots \leq q_n$, Lemma 4 guarantees that $\langle q, p \rangle$ also satisfy the other IC's. □

Corollary 2 For any implementable q, the monopolist maximizes profits by making $(IC_{i,i-1})$ binding.

Proof. Immediate from Lemmas 3 and 4. The monopolist can always increase profits by eliminating slack from $(IC_{i,i-1})$. □