General Equilibrium with Asymmetric Information: a Dual Approach

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Abstract

We study markets where the characteristics or decisions of certain agents are relevant but not known to their trading partners. Assuming exclusive transactions, the environment is described as a continuum economy with indivisible commodities. We characterize incentive constrained efficient allocations as solutions to linear programming problems and appeal to *duality theory* to demonstrate the generic existence of external effects in these markets. Because under certain conditions such effects may generate non-convexities, randomization emerges as a theoretic possibility. In characterizing market equilibria we show that, consistently with the personalized nature of transactions, prices are generally non-linear in the underlying consumption. On the other hand, external effects may have critical implications for market efficiency. With adverse selection, in fact, *cross-subsidization* across agents with different private information may be necessary for optimality, and so, the market need not even achieve an incentive constrained efficient allocation. In contrast, in the case of a single (type of) commodity, we find that when informational asymmetries arise after the trading period (e.g. moral hazard; ex post hidden types) external effects are fully internalized at a market equilibrium.

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1 Introduction

A company supplying insurance services has a direct concern in the personal risk of each of its customers as well as the prevention measures that they will provide to avoid an accident. The fact that these circumstances are observed privately by the buyers gives rise to adverse selection and moral hazard problems in the insurance market. While it has long been argued that this type of phenomena are typical of competitive markets, attempts to apply standard general equilibrium analysis to model competition under asymmetric information have proven difficult. The purpose of this work is to study the relation between incentive compatibility and pricing from the point of view of *duality theory*, thus providing a new methodology for introducing asymmetric information into general equilibrium theory.

An essential element of the analysis, as the case of insurance illustrates, is the personalized nature of transactions. This is in contrast to the standard model where trade is anonymous. In the case of full information, Makowski's (1979) shows how price discrimination over quantity is characteristic of competitive markets with personalized transactions. Because such instances may be formalized as economies with linear prices and indivisibilities, that result is yet consistent with the basic general equilibrium model; in particular, the standard welfare and existence theorems continue to hold. The personalized environments we are concerned with, on the other hand, display informational asymmetries. In this work we shall restrict to the simplest informational scenario where transactions are completely verifiable and it suffices to consider exclusive trading relations (in which each informed agent deals with a single uninformed partner)[1]. The objects of trade are formalized as relatively complex personalized goods. Insurance, for instance, is sold in indivisible packages which apart from specifying state-contingent payments, include also personal recommendations (e.g. a level of care prevention) as well as information about the customer (e.g. her risk type). The general environment is then described as a continuum economy with indivisible commodities. In this model incentive constraints are critical; we find that, in contrast to Makowski's model, *external effects* arise which the market may fail to internalize.

There are two main parts to the analysis. The first part characterizes incentive constrained efficient allocations as optimal solutions to linear semi-infinite programming (LSIP) problems. As a critical finding, the presence of incentive-related external effects is identified in the dual characterization of these problems. We argue that such

[1]For recent contributions which study non-exclusive transactions see Bisin and Gottardi (1998) and Bisin and Guaitoli (1995).

effects may generate endogenous non-convexities, and so randomization emerges as a theoretic possibility—first introduced by Prescott and Townsend (1984a,b). We also provide conditions under which non-convexities do not arise. With *hidden types*, it is enough that (i) utilities are type-invariant or—as a weaker condition—that (ii) individuals who have an interest in misrepresenting their type are no more risk averse than the individuals they try to impersonate. These conditions in turn ensure the suboptimality of random allocations when preferences and technologies are convex. With *hidden actions*, either (i) the agent's utility is separable in the action (e.g. effort) or (ii) absolute risk aversion is non-decreasing in the action level. Yet moral hazard economies are typically non-convex and random allocations may still be optimal.[2]

The second part of the analysis studies market equilibria. Due to the presence of incentive effects, Walrasian equilibrium prices are generally non-linear in the underlying consumption space, as in Makowski's model. Yet, the internalization of these effects by the market need not take place, leading the economy to a market failure. In particular, adverse selection problems make the price decentralization of incentive constrained efficient allocations problematic. One of our main results is to identify the source of the failure as the existence (prior to the trading period) of benefits from *cross-subsidization* across groups of individuals with different private information. In addition, when such optimal cross-subsidies can be found, market equilibria fail to exist unless some extra restrictions are imposed on the trading possibilities of the uninformed agents. On the other hand, we find that when trading takes place before asymmetries in information arise (e.g. moral hazard; ex post hidden types) market equilibria exist and are incentive constrained efficient; i.e. external effects are fully internalized.

Whereas the analysis as well as the main results can be presented in a general set-up with different types of informational asymmetries—as well as many physical goods and contingencies—this paper introduces the basic methodology and presents an intuitive discussion of our results by analyzing two simple economies. Namely, a variation of the adverse selection model of Rothschild and Stiglitz's (1976) and a moral hazard version of the former. The ex post hidden types model proves analytically equivalent to the moral hazard model. This illustrates also how our approach provides a unified framework to study particular economies case by case, bridging the gap with the partial equilibrium literature (Rothschild and Stiglitz (1976), Spence (1973), Stiglitz and Weiss (1981) and Wilson (1977) among others; see Riley (1998) for a comprehensive review). A necessary remark is that our equilibrium characterization describes economies with a single (type of) commodity. In future work we would like

[2]See, for instance, Bennardo and Chiappori (1998).

to extend the analysis to a multi-commodity world.

The paper is organized as follows. Section 2 presents the adverse selection model. First, the linear programming model is developed. Second, incentive constrained efficient allocations are fully characterized. Third, Walrasian equilibria are defined and their efficiency and existence properties are studied. Section 3 presents an analogous analysis for the case of moral hazard. The proofs are gathered in the Appendix.

1.1 Related Literature

This paper related to the seminal work of Prescott and Townsend (1984a) and the methodology that we propose applies to the class of economies which they study. A key modelling assumption in that work—from which we shall deviate—is that the transactions of the informed agents are restricted ex ante to the incentive compatible ones. One implication is that equilibrium prices are always linear in the consumption space, as in the standard full information model. That the uninformed agents should face the incentive compatibility constraints of their informed partners may on the one hand seem more natural. Further, assuming that the informed agents face their own incentive constraints amounts to abstracting from the incentive effects associated to their transactions (by imposing the corresponding shadow costs on the agents generating the externality and, hence, implicitly assuming that such an internalization is possible[3]). On the other hand, our *dual* approach highlights the presence of external effects and focuses on the issue of to what extent these effects will be internalized by competitive markets. This focus connects our work to a quite different line of research pursued by Greenwald and Stiglitz (1986) and Arnott, Greenwald and Stiglitz (1994). An interesting contribution is that we identify the source of the problems encountered by Prescott and Townsend's approach to decentralization with adverse selection. Namely, the need of cross-subsidization. This shows that external effects may be critical and indeed need to be directly analyzed. On the other hand, the efficiency result for economies with a single type of commodity where asymmetries in information arise after the trading period is also established by Prescott and Townsend (see also Kehoe, Levine and Prescott (1998)). The fact that the timing of the generation of asymmetric information with respect to the trading period is critical in these environments is indeed one of the main results in their work. Because for this class of economies we show that external effects are fully internalized by the market, the two approaches are essentially equivalent.

Prescott and Townsend pioneered the introduction of random allocations on the

[3]The property rights (Coasian) approach seems problematic though, given the nature of these effects.

basis of potential non-convexities in the agents' incentive constraint sets. Following their work, Cole (1989) emphasized the separating role of lotteries when the agents' degree of risk aversion depends on their private information (see also Arnott and Stiglitz (1988)); even with convex incentive constrained sets. Kehoe, Levine and Prescott (1998) recently show that, in a class of exchange economies with ex post hidden types and no indivisibilities, lotteries are never used in equilibrium provided the natural assumption of decreasing absolute risk aversion is imposed. Theirs is in fact a general version, for a setting with many goods, of our condition (ii) for the case of ex post hidden types. Intuitively, in their model agents in a high endowment private state may want to claim a low endowment state (which corresponds to a presumably more risk averse agent). Our result is different in that it shows that the idea generalizes to any type of informational asymmetry and, in particular, to the case of *ex ante* hidden types. The corresponding condition for economies with hidden actions is also derived by Arnott and Stiglitz (1998). In addition, our methodology brings to light the theoretical ground underlying this discussion, formally linking the separating role of lotteries and the importance of differences in risk aversion to the presence of non-convexities arising from incentive effects.

The adverse selection analysis is related to Gale (1996). In that model, however, prices are embedded in the traded contracts and equilibrium is achieved through endogenous market rationing. Individual rational expectations about rationing probabilities as well as refinements of out-of-equilibrium beliefs play a central role. A similar equilibrium concept has been used by Perktold (1995) to study the case of heterogeneously informed buyers. The description of equilibrium which we present abstracts from these game theoretic considerations. In the spirit of classical model, agents will optimize taking the prices as given and the latter adjust to clear the market.

As far as the moral hazard literature is concerned, the possibility of non-linear competitive pricing is discussed by Lisboa (1997) for an exchange economy with separable preferences[4]. Our claim is that this feature is characteristic of asymmetric information models with exclusive transactions (in which the uninformed agents face the incentive constraints of their trading partners). Bennardo and Chiappori (1998) have recently proposed a strategic formulation of equilibrium in a simple moral hazard model which clarifies the peculiarities of these competitive environments. Whether a reduced form of that equilibrium can be constructed is an open issue. In fact, the Walrasian equilibrium in Section 3 constitutes a reduced form of the Bennardo-

[4]See Magill and Quinzii's (1998) study of a finance economy when trades are assumed unobservable.

Chiappori equilibrium.[5]

We would like to refer to the linear programming description of the standard model by Makowski and Ostroy (1996) as the basic motivation of our work. Also, Myerson (1984) highlights the linear programming structure of principal agent models; an structure which has been exploited by Manelli and Vincent (1995) to characterize optimal procurement mechanisms from a dual perspective. To the best of our knowledge, however, linear programming techniques have not yet been applied to the general equilibrium analysis of asymmetric information.

2 Adverse Selection

2.1 The Economy

Consider an economy with a single consumption good, a continuum of non-atomic households and a finite number of identical firms.

Households. Households are of two types, t_L and t_H, with associated population masses ξ_L and $1-\xi_L$ respectively. Each household faces two private states of nature: in state 1 an accident occurs; in state 2 there is no accident. Whereas agents of type t_L suffer an accident with probability θ_L, the corresponding probability θ_H for an agent of type t_H is strictly higher; i.e. $0 < \theta_L < \theta_H < 1$. Contingent endowments are type-invariant and are denoted by $w = (w_1, w_2)$ where $w_2 > w_1 > 0$; i.e. $w_2 - w_1$ represents the loss in an accident. Households of type t_i are expected utility maximizers with Von-Neumann Morgenstern utility function $U_i : \mathbf{R}_+ \to \mathbf{R}$ $(i = L, H)$.[6] As usual U_i is continuously differentiable and concave. We also assume $\lim_{c \to 0} U_i'(c) = \infty$ and $\lim_{c \to \infty} U_i'(c) = 0$. The model is presented in terms of net trades, the (type-invariant) feasible net trade set Z containing all elements z in $\mathbf{R}^2$ such that $z \geq -w$. We assume there is no aggregate uncertainty[7] and denote the economy's aggregate endownment by $\bar{w}$.

Firms. Insurance companies are large as compared to the non-atomic households. In

[5] Whereas the possibility of aggregate uncertainty is not considered Section 3, the extension of the model in this direction is relatively straightforward. In particular, a non-trivial (non-zero price) Walrasian equilibrium always exists. See Bennardo and Chiappori (1998) for the problems associated to the Prescott-Townsend reduced form with aggregate uncertainty.

[6] The analysis can be easily extended to allow for state-dependent preferences.

[7] The measurability problems associated to the formalization of individual risks as independent random variables with a continuum of agents are well-known. For recent developments in this topic see Hammond and Lisboa (1998) and Sun (1998). Here we circumvent this problem by explicitly assuming underlying processes of individual uncertainty which preclude any macroscopic uncertainty.

insuring a continuum of buyers each the company faces no aggregate risk; thus, the underlying technology displays constant returns to scale.

Time and uncertainty. At time zero households privately learn their type. Then markets open and agents make transactions. As trades are assumed completely verifiable, it suffices to consider exclusive transactions where each household commits to buy insurance from a single company. After the trading period, uncertainty resolves and the final state of each household is publicly observed. Finally, all contractual obligations are enforced[8] and consumption takes place. The structure of uncertainty is common knowledge.

Personalized commodities. The objects of trade can be canonically described following Myerson (1984). Insurance is traded in "packages" (contracts) specifying net payments in each state as well as the *personal* risk type declared by the buyer. Each such contract is a different *indivisible* object of trade. While net payments may in principle be random, only contracts for which no agent has an incentive to misrepresent her type will be traded.

2.2 Allocations

An allocation for the households is a pair of probability measures—one for each type—on the feasible net trade set. The space X of allocations is then the set of pairs (x_L, x_H) of Borel measures on Z satisfying

$$\int_Z dx_i = 1, \quad x_i \geq 0, \quad i = L, H. \tag{2.1}$$

We show that in this model it suffices to consider measures with finite support (c.f. Appendix A). Letting δ_z stand for the mass point measure at z, we may then write any allocation as[9]

$$x_i = \sum_{k=1}^{K_i} \pi_i^k \delta_{z_i^k}, \quad \sum_{k=1}^{K_i} \pi_i^k = 1, \quad \pi_i^k > 0, \quad i = L, H;$$

where K_i is a positive integer and the K_i-dimensional subset of Z

$$\text{supp} x_i = \{z_i^1, \cdots, z_i^{K_i}\}$$

is the support of the measure x_i. In words, x_i is a "lottery" which delivers contingent net payments z_i^k with probability π_i^k to a household of type t_i (there are K_i possible deliveries).

[8]The possibility of default is not considered in this model.

[9]This description is related to Mas-Colell (1975).

An allocation is feasible in terms of resources if the implied ex post aggregate consumption does not exceed the economy's aggregate endowment. Formally, the aggregate net trade is negative if

$$r(x_L, x_H) \;=\; \xi_L r_L(x_L) + (1-\xi_L) r_H(x_H) \;\leq\; 0, \tag{2.2}$$

where for $i = L, H$,

$$r_i(x_i) \;=\; \int_{(z_{i1}, z_{i2}) \in Z} \Big(\theta_i z_{i1} + (1-\theta_i) z_{i2}\Big) dx_i(z_{i1}, z_{i2}).$$

Note that $\xi_i r_i(x_i)$ is the ex post net trade of the population of type t_i when all households in that group are assigned x_i. To emphasize the linear structure of (2.2) we write

$$r_i(x_i) = \langle r_i, x_i \rangle = \int_Z r_i dx_i, \quad i = L, H.$$

Implementable allocations also need to satisfy incentive conditions. For any pair (x_L, x_H) the expected utility of an agent of type t_i who claims to be of type t_j is

$$EU_i(x_j) = \int_{(z_1, z_2) \in Z} \Big(\theta_i U_i(w_1 + z_1) + (1-\theta_i) U_i(w_2 + z_2)\Big)\, dx_j(z_1, z_2).$$

Hence, an allocation is *incentive compatible* if

$$EU_i(x_i) \;\geq\; EU_i(x_j), \quad j \neq i, \quad i = L, H, \tag{2.3}$$

and agents choose not to misrepresent their type. Because (2.3) is linear on X, we write $EU_i(x_j) = \langle EU_i, x_j \rangle = \int_Z EU_i dx_j$.

Finally, an allocation is said to be *feasible* if it is both feasible in terms of resources and incentive compatible.

2.3 Incentive Constrained Efficiency

We proceed to the characterization of incentive constrained efficient allocations. These are feasible allocations for which there exist no other feasible allocation which is weakly preferred by all types and strictly preferred by at least one type. Each of the former corresponds to a solution of the social planner's problem which (for a given choice of utility weights) maximizes the weighted average of agent types' utilities subject to constraints (2.1)–(2.3).

A linear semi-infinite program. Let γ_L be the weight assigned to the low risk type in the social welfare function. The planner's problem is a linear program; specifically, one posed in an infinite dimensional but for which the number of constraints is finite—a linear semi-infinite programming (LSIP) problem.

$$\sup \; \gamma_L \langle EU_L, x_L \rangle \; + \; (1-\gamma_L)\langle EU_H, x_H \rangle$$

subject to

$$\begin{aligned}
&(2.4) & \langle \mathit{1}, x_L \rangle &= 1 \\
&(2.5) & \langle \mathit{1}, x_H \rangle &= 1 \\
&(2.6) & -\langle EU_L, x_L \rangle + \langle EU_L, x_H \rangle &\leq 0 \\
&(2.7) & \langle EU_H, x_L \rangle - \langle EU_H, x_H \rangle &\leq 0 \\
&(2.8) & \xi_L \langle r_L, x_L \rangle + (1-\xi_L)\langle r_H, x_H \rangle &\leq 0 \\
&(2.9) & x_L, x_H &\geq 0
\end{aligned}$$

Remark. In (2.4) and (2.5), *1* stands for the characteristic function on Z; so the former are just the adding-up constraints in (2.1) expressed in terms of the bilinear form $\langle ., . \rangle$.

The primal program (P). According to the theory of linear semi-infinite programming, the above is the *dual* of another LSIP problem: the so-called primal program (c.f. Goberna and López (1998)). Unlike the planner's problem the primal is posed in the Euclidean space. Its feasible set, on the other hand, is characterized by a linear system of infinite-dimensional constraints. Let the shadow prices associated to the adding-up constraints (2.4) and (2.5) be α_L and α_H respectively; the shadow prices associated to the incentive constraints (2.6) and (2.7) are β_L and β_H; finally, q stands for the shadow price of the resource constraint (2.8). Whereas a detailed derivation is provided in Appendix A, here we simply state program (P).[10]

$$\inf \quad \alpha_L \; + \; \alpha_H$$

subject to

$$\begin{aligned}
\alpha_L &\geq \gamma_L EU_L(z_L) + \beta_L EU_L(z_L) - \beta_H EU_H(z_L) - q\xi_L r_L(z_L), \quad \forall z_L \in Z \\
\alpha_H &\geq (1-\gamma_L)EU_H(z_H) - \beta_L EU_L(z_H) + \beta_H EU_H(z_H) - q(1-\xi_L)r_H(z_H), \\
&\quad \forall z_H \in Z
\end{aligned}$$

$$\beta_L, \beta_H, q \geq 0$$

Unlike standard finite dimensional linear programs, neither the existence of optimal solutions nor the equality of the optimal primal and dual values is guaranteed for

[10] Prescott and Townsend (1984a) study constrained efficient allocations through the first order conditions of the planner's problem (a formal difference is that in their framework the consumption space is a finite set, so effectively theirs is a standard finite dimensional LP program). Our purpose is to use duality theory to provide a general characterization of incentive constrained efficiency.

infinite dimensional programs. In Appendix A we appeal to some central results of the Theory of LSIP and demonstrate that the above dual pair is indeed well-behaved.

Theorem 2.1 *The dual is solvable and there is no duality gap.*

Theorem 2.2 *The primal is solvable.*

2.4 Full Information Benchmark

To clarify the economic intuition underlying program (P), consider the case of full information. When types are observable, no incentive constraints arise in the dual program. The constraint system associated to the allocation of type t_i in the primal (P^{FB}) is then

$$\alpha_i \geq \gamma_i EU_i(z_i) - q\xi_i r_i(z_i), \quad \forall z_i \in Z, \quad i = L, H.$$

The first term on the right-hand side of (2.10) is type's contribution to social welfare when allocated a given net trade z_i. Since q measures the shadow price of the consumption good, the second term gives the cost in terms of resources of such an assignment; i.e. the value of the aggregate net trade of the population of type t_i. Equation (2.10) can then be interpreted as defining the set of feasible values for α_i to be the set of upper bounds for the *type's net contribution to welfare*—given by the difference of the previous two terms—for *any* trade assignment.

Let $\alpha_i^*(q)$ be the maximal net contribution of t_i among all possible net trade assignments;

$$\alpha_i^*(q) = \max_{z_i \in Z} \gamma_i EU_i(z_i) - q\xi_i r_i(z_i). \tag{2.10}$$

The complementary slackness theorem (c.f. Krabs (1979)) allows us to characterize first best allocations in terms of maximal net contributions.[11]

Theorem 2.3 *(Complementary slackness) Let γ_L be given in $(0,1)$. Feasible solutions q^* and (x_L^*, x_H^*) for (P^{FB}) and (D^{FB}) respectively are optimal, if and only if*

$$\begin{aligned} 0 &= q^*\Big(\xi_L\langle r_L, x_L^*\rangle + (1-\xi_L)\langle r_H, x_H^*\rangle\Big) && (2.11)\\ \alpha_L^*(q^*) &= \gamma_L EU_L(z_L^*) \; - \; q^*\xi_L r_L(z_L^*), \quad \forall z_L^* \in \text{supp} x_L^* && (2.12)\\ \alpha_H^*(q^*) &= (1-\gamma_L) EU_H(z_H^*) \; - \; q^*(1-\xi_L) r_H(z_H^*), \quad \forall z_H^* \in \text{supp} x_H^* && (2.13) \end{aligned}$$

[11]This full information economy is an example of the general problem studied by Makowski and Ostroy (1996). In particular, $\alpha_i^*(q)$ is the conjugate or indirect utility, redefined in its expected value form for economies with uncertainty. These authors have shown how the fact that the constraints of the primal program (the "pricing problem" in their terminology) can be incorporated into the objective function is characteristic of the LP version of General Equilibrium.

(2.11) is the complementary slackness condition associated to the (dual) resource constraint: the shadow value of the economy's aggregate net trade is zero. More interesting are the complementary slackness conditions (2.12) and (2.13) which correspond to the (primal system of) constraints associated to the low risk and the high risk type allocation. According to these conditions, for each type, only net trades achieving the type's maximal net contribution are assigned with positive probability at an optimum.

Given (2.10) though, any feasible primal solution must satisfy that the net contribution of each type is a bounded function of z_i; so (for unbounded utilities) $q^* > 0$. The first order conditions associated to the concave unconstrained program in (2.10) then yield the standard result for convex economies with no aggregate uncertainty: if households are risk averse and types are observable it is optimal that all agents receive full insurance and all resources are consumed ex post. (In particular, randomization is always suboptimal.)

2.5 Incentive-Related External Effects

Let γ_L be given in the interval $(\bar{\gamma_L}, 1)$ where $\bar{\gamma_L} = \left(1 + \frac{(1-\xi_L)U_L'(\bar{w})}{\xi_L U_L'(\bar{w})}\right)^{-1}$. (For type-invariant utilities, in particular, $\bar{\gamma}_L$ is just ξ_L.) It can be easily shown that for this range the optimal (first best) consumption level is higher for the low risk households. Yet, none of these optimal allocations is implementable when types are privately observed as high risk households have obvious incentives to misrepresent their type. The restriction on γ_L is made for the purpose of the presentation as an identical analysis follows for values of γ_L in $(0, \bar{\gamma}_L)$. For this range, optimal allocations assign a higher consumption level to the high risk agents, violating now the incentive constraint of the low risk type. When $\gamma_L = \bar{\gamma}_L$ all households optimally consume the economy's average endowment regardless of the state and, hence, the allocation is trivially incentive constrained efficient.

Having said this, we let $\beta_L = 0$ and focus on the incentives of the high risk agents. Consider first the system of primal constraints associated to t_L restated as

$$\alpha_L \;\geq\; \alpha_L^*(\beta_H, q) = \max_{z_L \in Z} \gamma_L EU_L(z_L) - q\xi_L r_L(z_L) - \beta_H EU_H(z_L)$$

The net social contribution of the low risk type. When types are privately observed, feasible values of α_L are upper bounds for an "adjusted version" of the net contribution function of the low risk type. Note that, apart from the direct contribution to social welfare and the cost in terms of resources, a *third* term arises in the above constraint which has its origin in the incentive constraint of the high risk type. This term represents the negative effect of assigning low risk households a given net trade in terms of the potential "envy" generated upon high risk individuals. In other words, the better the assignment of a low risk household in the eyes of high risk agents, the higher the amount of resources that will need to be transferred to the latter to prevent them from misrepresentation. The emergence of this extra term in the first constraint system shows that this *incentive effect* must be explicitly considered in order to evaluate the net contribution of low risk allocations.

A natural question is, what is the counterpart (if any) of this negative externality for the high risk group.

The net social contribution of the high risk type. The second system of constraints in (P) may be restated as

$$\alpha_H \geq \alpha_H^*(\beta_H, q) = \max_{z_H \in Z} (1-\gamma_L)EU_H(z_H) - q(1-\xi_L)r_H(z_H) + \beta_H EU_H(z_H).$$

In addition to the direct contribution to welfare and the corresponding cost, a *third* (in this case, positive) term arises which gives the incentive effect of any net trade assignment for this group. Intuitively, the higher the utility that high risk households derive from the assignment the stronger their incentives to truthfully reveal their information. Hence, the right-hand side of the second constraint system in (P), given by the combination of these three terms, gives the net social contribution of the type's consumption.

Because (for $i = L, H$) feasible values of α_i are upper bounds for the net social contribution function of t_i, they are greater than or equal to the corresponding maximal net contribution (given the price q of the consumption good and the price β_H of incentive effects), $\alpha_i^*(\beta_i, q)$.

The Modified Primal. Given the minimization nature of the problem, we may redefine the primal program in terms of maximal net contributions as,

$$\min_{\beta_H, q \geq 0} \alpha_L^*(\beta_H, q) + \alpha_H^*(\beta_H, q) \qquad (P')$$

That is, the primal is equivalent to the unconstrained convex problem which chooses the price of resources and the price of incentive constraint of the high risk type to minimize the sum of the type's maximal net social contributions. In what follows β_H^* and q^* will denote the optimal prices.

2.6 Randomization

Theorem 2.3 can be directly generalized to allow for incentive constraints. First, any constrained optimal allocation satisfies that the shadow value of the economy's aggregate net trade is zero. That is, (since –for unbounded utilities– feasible primal solutions satisfy $q > 0$) all resources must be consumed. Second, any element in the support of each type's allocation necessarily achieves the type's maximal net social contribution.

Therefore, for high risk households

$$\alpha_H^*(\beta_H^*, q^*) \; = \; (1-\gamma_L)EU_H(z_H^*) - q^*(1-\xi_L)r_H(z_H^*) + \beta_H^* EU_H(z_H^*) \quad \forall z_H^* \in \text{supp} x_H^*.$$

Note, however, that the net contribution function of the high risk agents is strictly concave when the latter are risk averse; so the support of x_H^* is a singleton. In addition, the associated first order conditions show that it is always optimal for this group to receive the same consumption level regardless of the state.[12]

Proposition 2.1 *Let $\gamma_L \in (\bar{\gamma}_L, 1)$. In this part of the (constrained) Pareto frontier high risk households are fully insured. In particular, lotteries are suboptimal for this group.*

A similar analysis applies to the low risk households. In this case,

$$\alpha_L^*(\beta_H^*, q^*) \; = \; \gamma_L EU_L(z_L^*) - q^*\xi_L r_L(z_L^*) - \beta_H^* EU_H(z_L^*) \quad \forall z_L^* \in \text{supp} x_L^*.$$

Yet, note that the net contribution function of the low risk type need not be concave. As special case of strict concavity can be found in original Rothschild-Stiglitz (1976) screening model (see also Wilson (1977)).

Proposition 2.2 *When utilities are type-invariant, lotteries are always suboptimal.*

Proof: It U_i is type-invariant the second derivative of the net contribution of t_H in each state never changes sign. Further, if this derivative is not strictly negative the net contribution is both negative and strictly decreasing, and so the maximum is achieved at a zero consumption level; a contradiction.[13] $\square$

In general, however, the presence of incentive effects may give rise to *non-convexities* in the net social contribution of the low risk type. To understand the intuition underlying the beneficial role of randomization, note that net trades $z_L^* = (z_{L1}^*, z_{L2}^*)$ in the support of x_L^* satisfy

$$(2.14) \qquad z_{L1}^* \in \arg \max_{z_{L1} \geq -w_1} U_L(w_1 + z_{L1}) - \frac{\beta_H^* \theta_H}{\gamma_L \theta_L} U_H(w_1 + z_{L1}) - \frac{q^* \xi_L}{\gamma_L} z_{L1}$$

[12] Specifically, $z_{Hs}^* = c_H^* - w_s$ for $s = 1, 2$, where $c_H^* = {U_H'}^{-1}\left(\frac{q^*(1-\xi_L)}{1-\gamma_L+\beta_H^*}\right)$.

[13] Proposition 2.2 holds also when $\lim_{c\to 0} U'(c)$ is bounded. The difference is that the solution need not be interior in this case. This conclusion is also established in Prescott-Townsend (1984a).

$$(2.15) \qquad z_{L2}^* \in \arg \max_{z_{L2} \geq -w_2} U_L(w_2 + z_{L2}) - \frac{\beta_H^*(1-\theta_H)}{\gamma_L(1-\theta_L)} U_H(w_2 + z_{L2}) - \frac{q^* \xi_L}{\gamma_L} z_{L2}$$

Clearly, if the degree of risk aversion of the high risk type is high enough as compared to that of the low risk type, the objective functions in (2.14) and (2.15) may have more than one global maximum. To understand why differences in risk aversion may lead to gains from randomization, take the extreme case in which the low risk households are risk neutral and the high risk households are risk averse[14]. There, one can easily devise an allocation which is in fact first best optimal. Agents announcing a high risk type are assigned their first best deterministic allocation. Agents announcing low risk, on the other hand, receive a lottery. Whereas the implied expected net trade (and, hence, the utility) of these agents is the optimal (first best) one, the certainty equivalent that high risk agents assign to it is exactly (below) their own deterministic consumption, preventing any misrepresentation.

The idea that random allocations can be used to separate agents on the basis of their attitude towards risk was discussed by Prescott and Townsend (1984b) and further investigated by Cole (1989) and Arnott and Stiglitz (1988). Whenever the agent who has incentives to misrepresent his information is more risk averse than the type which he is trying to misrepresent, lotteries may lead to a Pareto improvement by helping relax the incentive constraints. The bite of the LSIP methodology is that it brings to light the theoretical ground underlying this discussion by establishing a formal link between the separating role of lotteries and the presence of non-convexities arising from incentives effects

We now give sufficient conditions for randomization to be suboptimal. Intuitively, when the low risk type is at least as risk averse as the high risk type, it is always suboptimal to assign the former a random allocation. Let the coefficient of absolute risk aversion of type t_i be A_i. The result is as follows.

Proposition 2.3 *Let $\gamma_L \in (\bar{\gamma}_L, 1)$. Then, if $A_L \geq A_H$, assigning the low risk households a random allocation is suboptimal.*

Proof: Denote the objective functions in (2.14) and (2.15) by f_{L1} and f_{L2} respectively. If type t_H (but not t_L) is risk neutral, the result is straightforward. Assume t_H is strictly risk averse. Because $f''_{L1} > f''_{L2}$, it suffices to show that $f''_{L1} < 0$. To do so, we write $f'_{L1} = (g_1 + g_2)g_3$, with

$$g_1(z_{L1}) = \frac{U'_L(z_{L1})}{U'_H(z_{L1})}, \quad g_2(z_{L1}) = -\frac{\beta_H \theta_H \xi_L q^*}{\theta_L \gamma_L^2 U'_H(z_{L1})}, \quad g_3(z_{L1}) = U'_H(z_{L1})$$

Clearly $g'_2, g'_3 < 0$. Finally, defining $A_i = -\left(\frac{U''_i}{U'_i}\right)$ and assuming $A_L \geq A_H$ yields

[14] This example is essentially that in Prescott and Townsend (1984b) and Cole (1989); theirs is a case of ex post hidden types.

$$g_1{}' = \frac{U_L''U_H' - U_L'U_H''}{(U_H')^2} = \frac{\left(\frac{U_L''U_H'}{U_L'U_H''} - 1\right)U_L'U_H''}{(U_H')^2} = \frac{\left(\frac{A_L}{A_H} - 1\right)U_L'U_H''}{(U_H')^2} \leq 0$$

□

As it has been already mentioned, a similar analysis goes through for the part of the (constrained) Pareto frontier where the aggregate consumption of the high risk group if higher (i.e. for $\gamma_L \in (0, \bar{\gamma}_L)$). In this case there may be benefits from assigning a lottery to households claiming a high risk type provided that they are sufficiently less averse to risk than low risk agents. The latter, however, will always receive full insurance. We summarize the results for this case in the following proposition.

Proposition 2.4 *Let $\gamma_L \in (0, \bar{\gamma}_L)$. In this part of the Pareto frontier low risk households are fully insured. Further, if $A_H \geq A_L$, assigning the high risk households a random allocation is suboptimal.*

2.7 The Insurance Market

Consider a competitive market in which insurance companies offer their services to the households. Since firms have access to identical constant returns technologies, we restrict to a single firm.

2.7.1 Prices

Let P denote the vector space $C(Z) \times C(Z)$, where $C(Z)$ is the set of continuous linear functions on Z. The space X of allocations shall be endowed with the weak topology associated to the dual pair $\langle X, P\rangle$, denoted by $\sigma(X, P)$ (c.f. Anderson and Nash (1987)). Under this topology, P is the set of continuous linear functionals on X – the natural price space.

A price functional is a pair $p = (p_L, p_H) \in P$. Note that prices need not be anonymous, as for given net payments $z \in Z$, the price charged to low and high risk households may differ ($p_L(z)$ need not equal $p_H(z)$). Second, prices need not be linear in the underlying net trade space either (as, say, $p_L(z)$ need not take the form $p_L \cdot z$ for some vector $p_L \in \mathbf{R}^2_+$). Even when this may seem inconsistent with general equilibrium analysis, such an inconsistency is only apparent for, in this model (just as in the standard framework) prices are linear on the space X of traded objects. Given a price system $p \in P$, the cost associated to bundles $x \in X$ is given by the linear functional

$$\langle p, x\rangle = \sum_{i=L,H} \langle p_i, x_i\rangle = \sum_{i=L,H} \int_Z p_i(z) dx_i(z)$$

In fact, the crucial deviation from the benchmark model is rather different: in the presence of incentive constraints and exclusive transactions, X will always be a space different from the space of consumption (in particular one of much larger dimension.)

2.7.2 Market Equilibrium

Assuming all traders take prices as given, an equilibrium is defined in the standard way.

A Walrasian equilibrium is an allocation for the economy $(\bar{x}_L^h, \bar{x}_H^h, \bar{x}^f)$ *and a price system* $\bar{p} \in P$ *such that the following conditions hold.*

(i) Optimality for households:

$$\begin{aligned} \bar{x}_i^h &= \arg \max_{x_i^h \in X^h} \quad \langle EU_i, x_i^h \rangle \\ & \quad s.t. \quad \langle \bar{p}_i, x_i^h \rangle \leq 0, \qquad i = L, H \end{aligned}$$

where X^h *is household's trading possibilities set; that is, the set of finitely supported measures* x^h *on* Z *which satisfy* $\langle 1, x^h \rangle = 1$ *and* $x^h \geq 0$.

(ii) Optimality for the firm:

$$\bar{x}^f = \arg \min_{x^f \in X^f} \langle \bar{p}, x^f \rangle$$

where X^f *is the set of technologically feasible and incentive compatible allocations for the firm. Formally,* $x^f \in X$ *belongs to* X^f *if and only if* $x^f = 0$ *or*[15]

$$\begin{aligned} \langle r_L, x_L^f \rangle + \langle r_H, x_H^f \rangle &\geq 0 \\ -\frac{1}{||x_L^f||}\langle EU_L, x_L^f \rangle + \frac{1}{||x_H^f||}\langle EU_L, x_H^f \rangle &\geq 0 \\ \frac{1}{||x_L^f||}\langle EU_H, x_L^f \rangle - \frac{1}{||x_H^f||}\langle EU_H, x_H^f \rangle &\geq 0 \\ x_L^f, x_H^f &< 0 \end{aligned}$$

(iii) Market clearing:

$$\bar{x}_i^f + \xi_i \bar{x}_i^h = 0, \quad i = L, H.$$

Since the set X^f is a pointed cone, (ii) yields the standard zero profit result for constant return production technologies.

[15] $||x_i^f||$ stands for the norm of any non-zero measure x_i^f; i.e. $||x_i^f|| = \sup_{f \in C(Z), ||f|| \leq 1} \quad |\langle f, x_i^f \rangle|$.

Lemma 2.1 *The firm makes zero profits in equilibrium; i.e.* $\langle \bar{p}, \bar{x}^f \rangle = 0$.

A market equilibrium satisfies also a critical *no arbitrage* property.

Lemma 2.2 *Prices of traded lotteries measure the value of the resources used by those lotteries;*

$$\langle \bar{p}_i, \bar{x}_i^h \rangle = \bar{y} \langle r_i, \bar{x}_i^h \rangle, \quad i = L, H, \tag{2.16}$$

where $\bar{y}$ *is any strictly positive constant.*

Proof: See Appendix B.[16]

2.7.3 Optimality and Existence

We are now ready to explore the efficiency properties of equilibria. The following result is central to our discussion.

Lemma 2.3 *(No cross-subsidization) In equilibrium the aggregate consumption of each risk group does not exceed the corresponding aggregate endowment;*

$$\xi_i \langle r_i, \bar{x}_i^h \rangle \leq 0, \quad i = L, H. \tag{2.17}$$

Proof: The proof follows from Lemma 2.2 and the household's budget constraint.

The main result of this section has to do with the problems that the previous 'no cross-subsidy restriction' imposes on the market mechanism. On the one hand, (provided it exists) a Walrasian equilibrium is always incentive constrained efficient.

Theorem 2.4 *A Walrasian equilibrium household allocation is incentive constrained efficient.*

Proof: See Appendix B.

On the other hand, *if* there exists an incentive constrained efficient allocation which satisfies (2.17), the latter can always be supported by an equilibrium price system provided an extra assumption is introduced in the firms problem.[17]

[16] The intuition is that, if lotteries offered in the market were not priced according to the resources they use, firms would have incentives to repackage these lotteries making a profit out of such an arbitrage activity (see also Kehoe, Levine and Prescott (1998)).

[17] A relatively tedious derivation shows that this assumption is necessary for existence of such a price system (and hence, for existence of an equilibrium).

Assumption 2.1 *When one of the policies offered by the company involves a strictly negative aggregate net trade with a risk group, the firm rationally takes into account the fact that policies are sold according to the population proportions. Henceforth, if for some i* $\langle r_i, x_i^f \rangle < 0$, *then*[18]

$$\frac{||x_i^f||}{||x_j^f||} \leq \frac{\xi_i}{\xi_j}. \tag{2.18}$$

The above is nothing but a natural rationality assumption. If the firm plans to take a negative net position with one group (say t_i), it needs to finance this activity through a positive position with the other group (t_j). As default is not allowed, this position must to be large enough for the promised payments to the first group to be implementable ex post. Yet, the firm knows that contracts always end up being sold according to the population proportions. In particular, the more contracts of type t_j that are sold, the more contracts of type t_i that are sold as well. Hence, if it were to offer policies which required trading with too large a mass of t_j customers relative to the mass of t_i customers (i.e. $\frac{||x_i^f||}{||x_j^f||} > \frac{\xi_i}{\xi_j}$) it would never be able to fulfill its promises ex post. Assumption 1 states that the firm *rationally* takes this fact into account.

Theorem 2.5 *Suppose Assumption 2.1 holds. Then, an incentive constrained efficient allocation may be decentralized by a Walrasian equilibrium if an only if it satisfies condition 2.17.*

Proof: See Appendix B.

In general, however, none of the allocations in the Pareto frontier need satisfy the 'no cross-subsidy' condition (2.18). In this instances, the market will never reach a constrained efficient allocation unless a cross-subsidization scheme is introduced. It is easy to show that the planner can always find a transfer scheme according to which agents revealing a low risk type have to pay a fee, the rest of the agents receive a subsidy, and a Walrasian equilibrium (after transfers) is constrained incentive efficient. These transfers are both feasible and incentive compatible; so the scheme is implementable. In particular, low risk households are happy to pay a fee to reveal their type. Knowing that fee will be used to subsidize the (full) insurance contract of the high risk population, making misrepresentation by this group more costly, they know they will also be able to get better terms in their own insurance contract. On the other hand, high risk agents will never have incentives to pay a fee to have access to the low risk insurance market; they will be (just as) happy with their subsidized full insurance contract.

[18] $\langle r_i, x_i^f \rangle$ represents the ex post aggregate net trade of the firm with the group of t_i households.

We conclude that the relation between constrained optimal allocations and equilibria is much more subtle in the presence of adverse selection that in the standard model. Effectively, the market faces more restrictions than the social planner. Further, these restrictions make existence problematic.[19]

Corollary 2.1 *A Walrasian equilibrium exists if and only if the no cross subsidy constraint is not binding for the social planner for some choice of weights in the welfare function. Hence, for a generic set of economies, no equilibrium exists.*

3 Moral Hazard

This section presents the moral hazard model. The results extend also to the case of ex post hidden types, which is analytically equivalent. The critical feature that the hidden actions and the ex post hidden types models share is that trading takes place before asymmetries of information arise (see also Prescott and Townsend (1984a)). This is in contrast to the adverse selection model where agents privately learn their type before trading takes place.

3.1 The Economy

Consider an economy with two goods –leisure l and a single consumption good c, a continuum of ex ante identical households and a finite number of firms.

Households. Each household faces two states of nature: in state 1 it is involved in an accident and in state 2 no accident occurs. The endowment of the consumption good in each state is w_1 and w_2 respectively. Let Z be the associated net trade space. Households are also endowed with one unit of leisure, which they allocate among leisure and accident prevention activities. The amount of leisure e which is devoted to care prevention measures can either be high or low. This level, on the other hand, determines the household's probability of suffering an accident; in particular, the lower e the more likely the occurrence of an accident. Denote the probability of an accident conditional on low care (respectively, high care) by θ_L (respectively, θ_H); i.e. $0 < \theta_H < \theta_L < 1$. All agents are expected utility maximizers with Von-Neumann

[19]The idea that competitive equilibria may fail to exist in adverse selection environments of this kind is discussed in Rothschild and Stiglitz (1976). In fact, our Walrasian description of equilibrium is a reduced form of a variation of their strategic description of equilibrium. The difference is that firms are allowed to offer *pairs of contracts* – and not just a single contract. The current equilibrium concept is hence more vulnerable against arbitrage opportunities than the Rothschild-Stiglitz original construct.

Morgenstern utility function $u : \mathbf{R}^2_+ \to \mathbf{R}$; $u(c,l)$ is twice continuously differentiable and concave in c, strictly increasing (both in c and l) and satisfies $\lim_{c\to 0} \partial u/\partial c = \infty$ and $\lim_{c\to\infty} \partial u/\partial c = 0$.

Firms. Companies are large compared to their customers and may insure a continuum of households, facing (in doing so) no aggregate uncertainty.

Time and Uncertainty. The timing of the model is as follows. At time zero markets open and agents make transactions. After the trading period is over, households choose their level of care prevention, their decision being only privately observed. Uncertainty resolves and the final state of each household becomes common knowledge. Finally, all contractual obligations acquired in the trading period are enforced and consumption takes place.

Personalized Commodities. Insurance is sold (exclusively) in indivisible packages. Each package specifies both a *personal* level of care to be provided by the insured as well as (potentially random) state contingent net payments. Only contracts for which the insured agent has no interest to deviate from the specified level of care are traded.

3.2 Allocations

Define E to be the set of effort levels; $E = \{e_L, e_H\}$. An allocation for the households is a probability measure on the $E \times Z$. Yet, allocations may be represented in a different and yet equivalent fashion. Namely, as a pair (x_L, x_H) of measures on Z satisfying

(3.19) $$\int_Z d(x_L + x_H) = 1, \quad x_L, x_H \geq 0$$

Further, without loss of generality we restrict to measures with finite support (c.f. Appendix A).

The interpretation is as follows. When allocated the bundle (x_L, x_H), each household will be recommended low care prevention with probability π_L and high care prevention with probability π_H where

$$\pi_i = \int_Z dx_i, \quad i = L, H.$$

Once the recommendation is made, the household is delivered (potentially random) net payments. Conditional on a low care recommendation, the set of potential deliveries is given by the support of x_L and the likelihood of each delivery is given by (normalized) the mass of x_L at the corresponding point. Similarly for a high care recommendation. Notice that this description highlights two possible types of randomization: a) randomization on the level of care and, b) randomization on the net trade assignment conditional on a given recommendation. Bennardo and Chiappori (1998) have recently stressed this difference between (using their terminology) "ex ante randomization" and "ex post randomization" respectively. In our model, the former will take place whenever both x_L and x_H are strictly positive measures, while the latter will occur when x_i is a non-degenerate measure for some i.

An allocation is feasible in terms of resources if aggregate ex post consumption does not exceed aggregate endowment.

$$\langle r_L, x_L\rangle + \langle r_H, x_H\rangle \;\; \leq \;\; 0, \tag{3.20}$$

where $\langle r_i, x_i\rangle$ stands for the aggregate net trade of the group of agents who are recommended e_i;

$$\langle r_i, x_i\rangle = \int_Z \Big(\theta_i z_{i1} + (1-\theta_i)z_{i2}\Big) dx_i(z_{i1}, z_{i2}).$$

For simplicity, define $U_i(c) = u(c, 1-e_i)$ for $i = L, H$. If a level of care e_i is recommended and the e_j is actual level of care provided, the household's conditional expected utility is $\frac{1}{\pi_i}\langle EU_i, x_j$ where

$$\langle EU_i, x_j\rangle \;\; = \;\; \int_{(z_1,z_2)\in Z} \Big(\theta_j U_i(w_1+z_1) + (1-\theta_j)U_i(w_2+z_2)\Big) dx_i(z_1, z_2).$$

An allocation is *incentive compatible* if, for any level of care recommended with positive probability, the household finds it optimal to conform to such a recommendation;

$$\langle EU_i, x_i\rangle \; \geq \; \langle EU_j, x_i\rangle, \quad j \neq i,\, i = L, H. \tag{3.21}$$

Finally, an allocation is said to be feasible if it is both feasible in terms of resources and incentive compatible.

3.3 Incentive Constrained Efficiency

A Linear Semi-Infinite Program. The planner's problem corresponds to the optimization problem (D) which chooses a feasible allocation to maximize the household's expected utility.

$\max\ \langle EU_L, x_L\rangle \; + \; \langle EU_H, x_H\rangle$
subject to

$$\langle 1, x_L + x_H\rangle \;\; = \;\; 1 \tag{3.22}$$

$$
\begin{aligned}
(3.23) \qquad & -\langle EU_L, x_L\rangle + \langle EU_H, x_L\rangle \leq 0 \\
(3.24) \qquad & \langle EU_L, x_H\rangle - \langle EU_H, x_H\rangle \leq 0 \\
(3.25) \qquad & \langle r_L, x_L\rangle + \langle r_H, x_H\rangle \leq 0 \\
(3.26) \qquad & x_L, x_H \geq 0
\end{aligned}
$$

The Primal Program. Let α, β_H, β_L and q be the primal variables associated with the adding-up constraint, the incentive compatibility constraints for high and low effort and the resource constraint in program (D) respectively. The primal (P) is

min α

subject to

$$
\begin{aligned}
(3.27) \quad & \alpha \geq EU_L(z_L) - \beta_L[EU_H(z_L) - EU_L(z_L)] - qr_L(z_L), \quad \forall z_L \in Z \\
(3.28) \quad & \alpha \geq EU_H(z_H) - \beta_H[EU_L(z_H) - EU_H(z_H)] - qr_H(z_H), \quad \forall z_H \in Z \\
(3.29) \quad & \beta_H, \beta_L, q \geq 0
\end{aligned}
$$

As in Section 2, the LSIP model is well-behaved. In particular, both the primal and dual problems are solvable and there is no duality gap.

3.4 Incentive-Related External Effects

We consider an environment in which, when e is fully observable, high care prevention is optimal. In this set-up it is efficient that all households exert high care and consume their expected endowment regardless of the state. This allocation, however, fails to be incentive compatible in a world of private information (given the opportunity cost of care prevention activities) and cannot be implemented.

Given the above we let β_L=0. Consider the system of primal constraints associated to high care restated as

$$
\alpha \geq \alpha^*_H(\beta_H, q) = \sup_{z_H \in Z} EU_H(z_H) - qr_H(z_H) - \beta_H[EU_L(z_H) - EU_H(z_H)]
$$

According to this system, feasible values of α are upper bounds for a continuous real-valued function on Z. This function has three main terms. The first one gives the contribution to welfare when the household is recommended high care prevention and assigned net payments z_H, provided it conforms to the specification. The second term is the associated cost in terms of resources. (Note that, if the level of care were observable, these would be the only components of the function.)

The net contribution with high care. In the presence of incentive constraints, however, a *third* term arises which represents the contribution of any net trade assignment z_H in terms of incentives. This way, whenever it is in the household's interest to defect to low care prevention, the term gives the cost of z_H in terms of incentives measured by the utility gain of that deviation. On the other hand, for assignments providing the right incentives, the term gives the utility loss implied by a deviation to low care. The direct net contribution of any assignment –calculated as the difference between the first and second term– is then adjusted upward (respectively, downward) when it gives the right (wrong) incentives in order to determine the actual net social contribution.

The net contribution with low care. A similar interpretation holds for the constraint system associated to low care,

$$\alpha \;\geq\; \alpha_L^*(q) \;=\; \sup_{z_L \in Z} EU_L(z_L) - qr_L(z_L)$$

Yet, conditional on a low care recommendation, *no* incentive effects arise. And so, the net social contribution is just the difference between the direct contribution to welfare and the cost in terms of resources.

The Modified Primal. Feasibility in the primal requires that α should be at least as large as the maximal net contribution conditional on e_i being recommended, for any $i = L, H$. As a result, the primal may be redefined as the unconstrained convex program which chooses the price q of the consumption good and that associated to the incentive effects, β_H, to *minimize* whichever of the two maximal net contributions is *higher* at those prices.

$$\min_{\beta_H, q \geq 0} \quad \max\{\alpha_L^*(q), \alpha_H^*(\beta_H, q)\} \qquad (P')$$

Let the optimal prices be β_H^* and q^*.

3.5 Ex post Randomization

In this section we apply the Complementary Slackness theorem to characterize incentive constrained efficient allocations.

Theorem 3.1 *(Complementary Slackness) Given feasible primal and dual variables* (β_H^*, q^*) *and* (x_L^*, x_H^*)*, the latter are optimal if and only if*

$$q^*\Big(\langle r_L, x_L^*\rangle + \langle r_H, x_H^*\rangle\Big) \;=\; 0 \qquad (3.30)$$

$$\beta_H^*\langle EU_L - EU_H, x_H^*\rangle \;=\; 0 \qquad (3.31)$$

$$
\begin{aligned}
(3.32) \qquad \alpha_L^*(q^*) &= EU_L(z_L^*) - q^* r_L(z_L^*) \quad \forall z_L^* \in \text{supp} x_L^* \\
(3.33) \qquad \alpha_H^*(q^*, \beta_H^*) &= EU_H(z_H^*) - q^* r_H(z_H^*) - \beta_H^*[EU_L(z_H^*) - EU_H(z_H^*)] \\
& \quad \forall z_L^* \in \text{supp} x_L^*
\end{aligned}
$$

A series of results follow from the theorem. In particular, (3.32) has the following direct implication.

Corollary 3.1 *Constraint efficient allocations satisfy that, conditional on a low-care recommendation, households are fully insured. Hence, randomization is always suboptimal conditional on this type of recommendation.*[20]

Note first that feasibility of optimal primal solutions implies (for unbounded utilities) that $q^* > 0$. When agents are risk averse the net social contribution conditional on low care is always strictly concave, having a unique global maximum. The full insurance result follows directly from the first order conditions of the maximization. Intuitively, recommending low care prevention does not give rise to incentive-effects, and so it is always optimal to provide the households with full insurance.

The case of high care prevention is rather different. Despite the fact that preferences are convex, the incentive effects identified in the previous section may give rise to *non-convexities* in the net contribution function associated to a high care recommendation. As a result, there may be benefits from assigning random payments conditional on this type of recommendation. Similarly to the adverse selection model, there is a special situation in which (ex post) lotteries are always suboptimal. (The proof is essentially that of Proposition 2.2 in Section 2.6).

Proposition 3.1 *If utility is separable in consumption and effort, assigning a lottery conditional on a high care recommendations is suboptimal.*[21]

In more general instances, however, randomization might be beneficial. On the fashion of the example in Section 2.6 consider the case in which t households exerting high care are risk neutral and households with a low level of care prevention are risk averse (i.e. U_L is linear and U_H strictly concave). Then, it is easy to devise an allocation which is in both incentive compatible and first best efficient. Such an allocation recommends high care with probability one and then assigns the household a lottery with expectation equal to the full information expected net trade. At the same time the lottery involves enough risk to preclude the agent from shirking its level of care.

[20]The second part of this proposition is also stated by Bennardo and Chiappori (1998).

[21]This result was first established by Holmström (1979).

Sufficient conditions for ex post randomization to be suboptimal are established below. (The proof is essentially that of Proposition 2.3 in Section 2.6)[22]

Proposition 3.2 *If absolute risk aversion increases with effort, assigning random payments conditional on a high care recommendation is suboptimal.*

3.6 Ex Ante Randomization

Bennardo and Chiappori (1998) study a similar moral hazard model in which absolute risk aversion increases with the level of care, and so, ex post randomization is always suboptimal. They argue, however, that there may be benefits to yet another type of randomization. (Note that, after all, moral hazard economies with discrete effort levels are non-convex economies.) The underlying idea is that when leisure and consumption are complementary commodities –effort and consumption are substitutes– there may be a limit to the amount of the good that the household can consume while still willing to provide high care prevention.

In terms of the LSIP model, restricting the planner's choice to deterministic allocations (recommending high care) may lead to a non-binding resource constraint. This is clearly suboptimal given the strict monotonicity of preferences and the incentive-free effect of consumption with low care prevention. In other words, at any such allocation the maximal net contribution conditional on a low care recommendation is higher than that of a high care (i.e. $\alpha_H(0, \beta_H) < \alpha_L(0)$) and yet the households are not being recommended a low care prevention at all. This highlights that, in these instances, recommending low care is an optimal way to transfer resources to the household without perversely affecting their incentives. At the incentive constrained efficient allocation the two maximal net contributions ought to be equated, $\alpha_H(q^*, \beta_H^*) = \alpha_L(q^*)$.

On the other hand, it is easy to see from (3.31)–() that (when ex ante randomization is optimal[23]) the expected utility of households exerting high care is strictly lower than that of households with low care prevention activities. Furthermore, for high risk agents, the marginal utility is not even equated across states.[24] While this may seem odd at first sight, it really is not, as the *marginal utility net of incentive external effects (the social marginal utility)* is equated both across states and effort levels. So the result indeed conforms to the general notion of an efficient allocation with external effects.

[22]Arnott and Stiglitz (1988) derive this result through a different argument.

[23]See Bennardo and Chiappori (1998) for sufficient conditions.

[24]For a related discussion see Bennardo (1998).

Let MU_{is}^S stand for the marginal social utility of consumption in state s conditional on e_i being recommended; i.e.

$$\begin{aligned} MU_{H1}^S(c_{H1}) &= U_H'(c_{H1}) + \beta_H[U_H'(c_{H1}) - \tfrac{\theta_L}{\theta_H}U_L'(c_{H1})] & MU_{L1}^S(c_{L1}) &= U_L'(c_{L1}) \\ MU_{H2}^S(c_{H2}) &= U_H'(c_{H2}) + \beta_H[U_H'(c_{H2}) - \tfrac{(1-\theta_L)}{(1-\theta_H)}U_L'(c_{H2})] & MU_{L2}^S(c_{L2}) &= U_L'(c_{L2}) \end{aligned}$$

Proposition 3.3 *In the Bennardo-Chiappori model* $q^* = MU_{is}^S(w_s + z_{is}^*)$, $\forall i, \forall s$.

Because no external effects arise when low care prevention is recommended, in that case the social marginal utility coincides with the private marginal utility (driving the full insurance result). For the case of high care, however, the marginal social utility of consumption is strictly lower in the event of an accident. Hence it is optimal to have this agents consume less in that state.

3.7 The Insurance Market

Consider an insurance market in which the households buy insurance from the companies introduced in Section 3.1. We shall focus on a single firm with constant returns to scale technology.

Let $P = C(Z) \times C(Z)$. We shall consider the weak topology $\sigma(X, P)$ on X, which makes P the natural price space. Given a price system $p \in P$, the cost of a bundle $x \in X$ is given by the linear functional

$$\langle p, x\rangle = \sum_{i=L,H} \langle p_i, x_i\rangle.$$

3.7.1 Market Equilibrium

We assume traders take prices as given and define an equilibrium in the usual way.

A Walrasian equilibrium is an allocation for the economy $(\bar{x}^h, \bar{x}^f)$ *and a price system* $\bar{p} \in P$ *such that the following conditions hold.*

(i) *Optimality for the households:*

$$\begin{aligned} \bar{x}^h &= \arg\max_{x^h \in X^h} \quad \langle EU, x^h\rangle \\ & \qquad s.t. \quad \langle \bar{p}, x^h\rangle \leq 0, \end{aligned}$$

where $X^h = \{(x_L^h, x_H^h) \in X : \sum_{i=L,H}\langle 1, x_i^h\rangle = 1,\ x_i^h \geq 0,\ i = L, H\}$ *is the household's trading possibilities set.*

(ii) *Optimality for the firm:*

$$\bar{x}^f = \arg\min_{x^f \in X^f} \langle \bar{p}, x^f\rangle$$

where X^f is the set of allocations for the firm which are technologically feasible and incentive compatible. That is, $x^f = (x^f_L, x^f_H) \in X$ belongs to X^f if and only if

$$\begin{aligned} -\langle EU_L, x^f_L\rangle + \langle EU_H, x^f_L\rangle &\geq 0 \\ \langle EU_L, x^f_H\rangle - \langle EU_H, x^f_H\rangle &\geq 0 \\ \langle r_L, x^f_L\rangle + \langle r_H, x^f_H\rangle &\geq 0 \\ x^f_L, x^f_H &\leq 0. \end{aligned}$$

(iii) *Market clearing:*

$$\bar{x}^f + \bar{x}^h = 0.$$

Since the firm's trading possibilities set X^f is a pointed cone, **(ii)** implies the following.

Lemma 3.1 *The firm makes zero profits in equilibrium.*

In addition, the following is a critical *no-arbitrage* equilibrium property.

Lemma 3.2 *Equilibrium allocations are priced according to the amount resources which are used conditional on any given care recommendation.*

$$\langle \bar{p}_i, \bar{x}^h_i\rangle = \bar{y}\langle r_i, \bar{x}^h_i\rangle.$$

where $\bar{y}$ is any strictly positive constant.

Proof: The proof is identical to that of lemma 2.2.

3.7.2 Optimality

As the main result of this section we establish the existence of a one-to-one correspondence between Walrasian equilibria and incentive constrained efficient allocations.

Theorem 3.2 *A Walrasian equilibrium household allocation is incentive constrained efficient. Conversely, an optimal solution to the planner's problem can be decentralized as a Walrasian equilibrium.*

Proof: See Appendix B

In the light of the previous theorem, the existence of optimal solutions to the planner's problem (c.f. Appendix A) guarantees also the existence of an equilibrium.

Theorem 3.3 *A Walrasian equilibrium always exists.*

Appendix A

A.1 The Primal Program

Let $\mathbf{R}^n$ be equipped with the Euclidean norm and partially ordered by means of the cone

$$K_m^n = \{\, y = (y_1, ..., y_n) \in \mathbf{R}^n : \, y_j \geq 0, \, j = 1, ..., m \,\}, \qquad 0 \leq m \leq n.$$

Given $w \in \mathbf{R}_+^2$, define the set $Z = \{\, z \in \mathbf{R}^2 \; : \; z \geq -w \,\}$. Let the vector space $C(Z)$ of continuous real-valued functions on Z, endowed with the topology of uniform convergence on compact sets, be partially ordered by means of the cone

$$C(Z)_+ = \{\, f \in C(Z) : \, f(z) \geq 0 \quad \forall z \in Z \,\}.$$

Let a vector $c \in \mathbf{R}^n$, a continuous linear mapping $A : \mathbf{R}^n \to C(Z) \times C(Z)$, and a fixed element $b \in C(Z) \times C(Z)$ be given.

Problem (P). The primal LSIP program, with value $\nu(P)$, is

$$\begin{array}{ll} \inf & c \cdot y \\ s.t. & Ay \geq b \\ & y \in K_m^n. \end{array}$$

A.2 The Standard Dual

Let $C(Z) \times C(Z)$ be paired in duality with its topological dual space, $M_C(Z) \times M_C(Z)$; i.e. $M_C(Z)$ is the space of compactly supported signed Borel measures on Z which are finite on compact sets (c.f. Hewitt (1959)). The reflexive space $\mathbf{R}^n$ shall be paired with itself. The two pairings are endowed with their natural bilinear forms. [The notation below highlights the dimensionality of the spaces in the pairing: whereas the dot product notation applies to finite dimensions, $\langle\, .\, , .\, \rangle$ is used for infinite dimensional spaces.]

$$\text{(A.1)} \qquad \langle f, x \rangle = \int_Z f_L dx_L + \int_Z f_H dx_H, \qquad f = (f_L, f_H) \in C(Z) \times C(Z), \quad x = (x_L, x_H) \in M_C(Z) \times M_C(Z)$$

$$\text{(A.2)} \qquad y \cdot z = \sum_{j=1}^{n} y_j z_j, \qquad y \in \mathbf{R}^n, \, z \in \mathbf{R}^n.$$

The mapping $A^* : M_C(Z) \times M_C(Z) \to \mathbf{R}^n$ which is *adjoint* to A is defined by

$$\text{(A.3)} \qquad y \cdot (A^* x) = \langle Ay, x \rangle \qquad \forall y \in K_m^n, \, \forall x \in M_C(Z)_+ \times M_C(Z)_+.$$

Program (D_S). The dual of (P), with value $\nu(D_S)$, is posed in $M_C(Z) \times M_C(Z)$ as

$$\begin{array}{ll} \inf & \langle b, x\rangle \\ s.t. & A^* x \leq c \\ & x \geq 0. \end{array}$$

We may write $Ay = \sum_{j=1}^{n} y_j f^j$, where $f^j \in C(Z) \times C(Z),\ j = 1, \cdots, n$; so

$$y \cdot (A^* x) = \sum_{j=1}^{n} y_j \langle f^j, x\rangle \qquad \forall y \in K_m^n, \forall x \in M_C(Z)_+ \times M_C(Z)_+$$

The statement $A^* x \leq c$ is then equivalent to

$$\sum_{i=1}^{n} \qquad y_j(\langle f^j, x\rangle - c_j) \leq 0 \qquad \forall y = (y_1, ..., y_n) \in K_m^n$$

and (D_S) can be expressed as

$$\begin{array}{lll} \sup & \langle b, x\rangle & \\ s.t. & \langle f^j, x\rangle \leq c_j, & j = 1, ..., m \\ & \langle f^j, x\rangle = c_j, & j = m+1, ..., n \\ & x \geq 0. & \end{array}$$

A.3 The Haar Dual

Let $\mathbf{R}^{(Z)}$ be the vector space of all functions $\lambda_i : Z \to \mathbf{R}$ which vanish outside a finite subset of Z; the so-called supporting set of λ_i ($\operatorname{supp} \lambda_i = \{z_i \in Z : \lambda_i(z_i) \neq 0\}$). The elements of $\mathbf{R}^{(Z)}$ are known as *generalized finite sequences* in $\mathbf{R}$ (c.f. Goberna and López (1998)). Following Charnes et al. (1963), let $C(Z) \times C(Z)$ be paired in duality with $\mathbf{R}^{(Z)} \times \mathbf{R}^{(Z)}$, with associated bilinear form

$$\begin{aligned} \langle f, \lambda\rangle \quad &= \sum_{z_L \in \operatorname{supp} \lambda_L} f_L(z_L)\lambda_L(z_L) + \sum_{z_H \in \operatorname{supp} \lambda_H} f_H(z_H)\lambda_H(z_H) \\ & f = (f_L, f_H) \in C(Z) \times C(Z), \quad \lambda = (\lambda_L, \lambda_H) \in \mathbf{R}^{(Z)} \times \mathbf{R}^{(Z)}. \end{aligned}$$

Program (D_H). A similar derivation to that in Section A.2 gives the *dual problem in Haar's sense*, with value $\nu(D_H)$.

$$\begin{array}{lll} \sup & \langle b, \lambda\rangle & \\ s.t. & \langle f^j, \lambda\rangle \leq c_j, & j = 1, ..., m \\ & \langle f^j, \lambda\rangle = c_j, & j = m+1, ..., n \\ & \lambda \geq 0. & \end{array}$$

	Adverse Selection	Moral Hazard
(n,m)	$(5,3)$	$(4,3)$
y	$(\beta_L, \beta_H, q, \alpha_L, \alpha_H)$	$(\beta_L, \beta_H, q, \alpha)$
c	$(0,0,0,1,1)$	$(0,0,0,1)$
b	$(\gamma_L EU_L, (1-\gamma_L)EU_H)$	(EU_L, EU_H)
f^1	$(-EU_L, EU_L)$	$(-EU_L + EU_H, 0)$
f^2	$(EU_H, -EU_H)$	$(0, EU_L - EU_H)$
f^3	$(\xi_L r_L, (1-\xi_L) r_H)$	(r_L, r_H)
f^4	$(1,0)$	$(1,1)$
f^5	$(0,1)$	—

Table A.1: Adverse Selection and Moral Hazard Models

Any pair $\lambda = (\lambda_L, \lambda_H) \in \mathbf{R}^{(Z)} \times \mathbf{R}^{(Z)}$ gives rise to a pair of finitely supported measures $x = (x_L, x_H)$ where, for example, $x_L = \sum_{z_L \in \text{supp}\,\lambda_L} \lambda_L(z_L)\delta_{z_L}$. Formally, $\mathbf{R}^{(Z)} \times \mathbf{R}^{(Z)}$ is isomorphic to the space X of allocations defined in Sections 2 and 3. Finally, it can be seen from Table A.1 that (D_H) is equivalent to the planner's problem in each such section.

A.4 Existence of Optimal Solutions and No Duality Gap

Since $\mathbf{R}^{(Z)} \times \mathbf{R}^{(Z)}$ is isomorphic to a subspace of $M_C(Z) \times M_C(Z)$, $\nu(D_H) \leq \nu(D_S)$. The Weak Duality Theorem for $\{(P), (D_S)\}$ (c.f. Krabs (1979)) implies then

$$\nu(D_H) \leq \nu(D_S) \leq \nu(P);$$

so the pair $\{(P), (D_H)\}$ satisfies also the weak duality inequality. In this section we show that $\nu(D_H) = \nu(P)$, so it in fact is sufficient to consider the Haar pair. The following fact regarding the system of primal constraints is critical in the proof.

Lemma A.1 *There exists a compact subset $T \subset Z$ such that all primal constraints associated to elements in Z which lie outside T may be eliminated without altering the set of optimal solutions.*

Proof: Let Y denote the set of feasible primal solutions; a closed convex subset of $\mathbf{R}^n$. Any $y \in Y$ satisfies

$$\text{(A.4)} \qquad 0 \;\geq\; h_L(z_L; y) \;=\; b_L(z_L) - \sum_{j=1}^{n} y_j f_L^j(z_L), \quad \forall z_L \in Z$$

$$\text{(A.5)} \qquad 0 \;\geq\; h_H(z_H; y) \;=\; b_H(z_H) - \sum_{j=1}^{n} y_j f_H^j(z_H), \quad \forall z_H \in Z$$

Since preferences are convex, it is easy to see from Table A.1 that this system is consistent, so $Y \neq \emptyset$. We establish the Lemma through a sequence of claims.

Claim 1: $\exists M_j$, $j = 1, \dots, n$ such that all optimal primal solutions lie in the set

$$M = \{y \in Y : \quad y_j \leq M_j, \quad j = 1, \dots, n\}.$$

Since feasible solutions for (P) belong to K_m^n and satisfy (A.4) and (A.5), it is clear from Table A.5 that Y is bounded below. For $j = \{n-m+1, \dots, n\}$ the existence of M_j follows from the objective of (P), which chooses $y \in Y$ to minimize $\sum_{j=n-m+1}^{n} y_j$. Since any optimal solution y^* must satisfy (A.4) and (A.5) with strict equality, this implies that y_j^* is bounded above for $j = \{1, \dots, n-m\}$.

Claim 2: $\exists \epsilon > 0$ s.t. $\forall y \in M,\ y_{n-m} > \epsilon$.

Assume not. Then there is a sequence $\{y^k\}$ in M for which $0 \leq y_{n-m}^k < \frac{1}{k}$, for all $k \in \mathbf{N}$. Since one of the incentive constraint will always be redundant (this is obvious with moral hazard and was established in Section 2 for the case of adverse selection). Without loss of generality let $y_1 = 0$. Table A.1 implies then that for some $i \in \{L, H\}$ and all $y \in Y$

$$0 \ \geq \ h_i(z_i; y) \geq b_i(z_i) - y_{n-m} f_i^{n-m}(z_i) - y_n, \quad \forall z_i \in Z$$

Let $y = y^k$. Rearranging and taking limits,

$$\lim_k y_n^k \geq b_i(z_i) - \lim_k y_{n-m}^k f_i^{n-m}(z_i) \ = \ b_i(z_i), \quad \forall z_i \in Z.$$

It utility is unbounded, $\lim_k y_{n-m+1}^k = \infty$, contradicting Claim 1. If utility is bounded, $\lim_{z_i \to \infty} b_i(z_i) = B_i$, M_n can then always be found in $(0, B_i)$, leading to a similar contradiction.

Claim 3: $\exists \bar{z}$ s.t. $\forall y \in M$, $\nabla h_i(z_i; y) << 0$, $\forall z_i \geq \bar{z}$, $i \in \{L, H\}$.

Without loss of generality, take $i = L$. First, note that $\nabla f_L^j = 0, j = n-m+1, \dots, n$. Second, $\nabla f_L^{n-m}(z_L) = \bar{g}_L \in \mathbf{R}_{++}^2$. Hence,

$$\begin{aligned} \nabla h_L(z_L; y) &= \nabla b_L(z_L) - \sum_{j=1}^{n-m} y_j \nabla f_L^j(z_L) \\ &= \nabla b_L(z_L) - \sum_{j=1}^{n-m-1} y_j(\nabla f_L^{j+}(z_L) - \nabla f_L^{j-}(z_L)) - y^{n-m} \nabla f_L^{n-m}(z_L) \end{aligned}$$

Where, $\nabla f_L^{j+}, \nabla f_L^{j-} \geq 0$ stand for the positive and negative parts of ∇f_L^j.

Because marginal utility decreases asymptotically to zero,

$$\begin{aligned} \lim_{z_L \to +\infty} \nabla b_L(z_L) &= 0 \\ \lim_{z_L \to +\infty} \nabla f_L^j(z_L) &= 0, \quad j = \{1, \dots, n-m-1\} \end{aligned}$$

Claims 1 and 2 imply then

$$\nabla h_L(z_L;y) \;\leq\; \nabla b_L(z_L) + \sum_{j=1}^{n-m-1} M_j \nabla f_L^{j^-}(z_L) - \epsilon \bar{g}_L \qquad \forall z_L \in Z$$

Taking limits,

$$\begin{aligned} \lim_{z_L \to +\infty} \nabla h_L(z_L;y) \;&\leq\; \lim_{z_L \to +\infty} \nabla b_L(z_L) + \sum_{j=1}^{n-m-1} M_j \lim_{z_L \to +\infty} \nabla f_L^{j^-}(z_L) - \epsilon \bar{g}_L \\ &=\; -\epsilon \bar{g}_L << 0 \end{aligned}$$

Since $h_L(.;y)$ is a continuously differentiable map by assumption, there exists $\bar{z}_L$ with $\nabla h_L(z_L;y) << 0$, for all $z_L > \bar{z}_L$.

A similar derivation gives $\bar{z}_H$. Let $\bar{z} = \max\{\bar{z}_L, \bar{z}_H\}$.

Finally, by Claim 3, set $T = [-w_1, \bar{z}] \times [w_2, \bar{z}]$ satisfies the lemma. □

Consider the pair $\{(P^T), (D_H^T)\}$ which arises by replacing Z by T in the primal and (Haar) dual programs. We establish the following.

Lemma A.2 *The system of constraints in* (P^T) *is canonically closed in the sense of Charnes et al. (1965).*

Proof. First, since T is compact and b and f^j, $j = 1, ...n$, correspond to pairs of continuous functions, the set

$$\{(f^1(t), f^2(t), \ldots, f^n(t), b(t)) : \, t \in T\}$$

is compact in $\mathbf{R}^{n+1}$. Second, the Slater qualification constraint is satisfied; e.g. take $y_1^0 = \cdots = y_{n-m-1}^0 = 0$. The map f^{n-m} is linear and (given the convexity of preferences) b corresponds to a pair of concave functions. Hence, there exist constants $\delta_L > 0$ and $\delta_H > 0$ and values for $y_{n-m}^0, \cdots, y_n^0$ such that,

$$\begin{aligned} \delta_L \;&\geq\; h_L(z_L;y^0) \;=\; b_L(z_L) - y_{n-m}^0 f_L^{n-m}(z_L) - y_{n-m+1}^0, \quad \forall z_L \in Z \\ \delta_H \;&\geq\; h_H(z_H;y^0) \;=\; b_H(z_H) - y_{n-m}^0 f_H^{n-m}(z_H) - y_n^0, \quad \forall z_H \in Z \end{aligned}$$

making y^0 a Slater point. □

Lemma A.3 $\nu(D_H^T)$ *is attained and* $\nu(P^T) = \nu(D_H^T)$.

Proof. Given Lemma A.2, the Inhomogeneous Haar Theorem of Charnes et al. (1965) implies that the system of constraints in (P_T) has the Farkas-Minkoswki property. Since (P^T) and (D_H^T) are consistent, the Extended Duality Theorem (Charnes et al. (1962, 1963)) implies then that (D_H^T) is solvable and $\nu(D_H^T) = \nu(P^T)$. □

Given the previous results, the proof of Theorem 2.1 is readily established.

Proof of Theorem 2.1. Since $\mathbf{R}^{(T)} \subset \mathbf{R}^{(Z)}$, $\nu(D_H^T) \leq \nu(D_H)$. By Lemma A.1, $\nu(P) = \nu(P^T)$. Finally, weak duality of the pair $\{(P), (D_H)\}$ and Lemma A.3 imply $\nu(P) = \nu(D_H)$. Further, the solvability of (D_H^T) guarantees that of (D_H) as both programs have the same value. □

We also establish existence of optimal primal solutions.

Proof of Theorem 2.2. Y is closed, and by Claim 1 in Lemma A.1, may be assumed bounded. Hence, the primal program is equivalent to a program that maximizes a continuous function on a compact set, and so, its value is attained.□

Appendix B

Proof of Lemma 2.2. It suffices to show that there is a $\bar{y} \geq 0$ such that $\langle \bar{p}_i, \bar{x}_i^f \rangle = \bar{y}\langle r_i, \bar{x}_i^f \rangle$ for $i = L, H$.

We first claim that $\langle r_i, \bar{x}_i^f \rangle = 0$ implies $\langle \bar{p}_i, \bar{x}_i^f \rangle = 0$. Without loss of generality, let $i = L$ and assume $\langle \bar{p}_L, \bar{x}_L^f \rangle < 0$ instead. Consider $\hat{x}^f = (\gamma \bar{x}_L^f, \bar{x}_H^f)$ with $\gamma > 1$. Since $\bar{x}^f \in X^f$, also $\hat{x}^f \in X^f$. Further, $\langle \bar{p}_L, \hat{x}_L^f \rangle = \gamma \langle \bar{p}_L, \bar{x}_L^f \rangle < \langle \bar{p}_L, \bar{x}_L^f \rangle$, and so $\langle \bar{p}, \hat{x}^f \rangle < \langle \bar{p}, \bar{x}^f \rangle$; contradicting (ii). A similar argument applies for $\langle \bar{p}_L, \bar{x}_L^f \rangle > 0$, letting $\gamma < 1$.

Second, if $\langle r_i, \bar{x}_i^f \rangle \neq 0$ for $i = L, H$,

$$\frac{\langle \bar{p}_L, \bar{x}_L^f \rangle}{\langle \bar{r}_L, \bar{x}_L^f \rangle} = \frac{\langle \bar{p}_H, \bar{x}_H^f \rangle}{\langle r_H, \bar{x}_H^f \rangle}. \tag{B.1}$$

When $\langle \bar{p}_H, \bar{x}_H^f \rangle = 0$, this follows trivially from Lemma 2.1. Assume $\langle \bar{p}_H, \bar{x}_H^f \rangle \neq 0$ and, without loss of generality, let left-hand side of (B.1) be larger than the right-hand side. Because $\bar{x}^f \in X^f$,

$$\frac{\langle \bar{p}_L, \bar{x}_L^f \rangle}{\langle \bar{p}_H, \bar{x}_H^f \rangle} > \frac{\langle r_L, \bar{x}_L^f \rangle}{\langle r_H, \bar{x}_H^f \rangle} \geq -1.$$

Hence, $\langle \bar{p}_L, \bar{x}_L^f \rangle + \langle \bar{p}_H, \bar{x}_H^f \rangle > 0$, contradicting Lemma 2.1.

Finally, for any i the sign of $\langle \bar{p}_i, \bar{x}_i^f \rangle$ must equal that of $\langle r_i, \bar{x}_i^f \rangle$. Say $i = L$. Assume that $\langle \bar{p}_L, \bar{x}_L^f \rangle < 0$ and $\langle r_L, \bar{x}_L^f \rangle > 0$. The bundle $\hat{x}^f = (\gamma \bar{x}_L^f, \bar{x}_H^f)$ with $\gamma > 1$ is in X^f. Also, $\langle \bar{p}, \hat{x}^f \rangle < \langle \bar{p}, \bar{x}^f \rangle$, contradicting (ii). A similar argument goes through when $\langle \bar{p}_L, \bar{x}_L^f \rangle > 0$ and $\langle r_L, \bar{x}_L^f \rangle < 0$, letting $\gamma < 1$. □

The next two lemmas are critical in the adverse selection model.

Lemma B.1 *Let $\bar{\nu}_i^h = \langle EU_i, \bar{x}_i^h \rangle$ and let $\bar{\lambda}_i^h$ be the equilibrium marginal utility of money for households of type t_i. Then, $\bar{\nu}_i^h \geq EU_i(z_i) - \bar{\lambda}_i^h \bar{p}_i(z_i), \forall z_i \in Z$, with strict equality iff $z_i \in \text{supp}\bar{x}_i^h$.*

Proof. The household's problem in (i) is a LSIP problem (in its dual version), and the above are just the complementary slackness conditions of the associated primal □

Lemma B.2 *There is an array $(\bar{\alpha}_L^f, \bar{\alpha}_H^f, \bar{\beta}_L^f, \bar{\beta}_H^f, \bar{q}^f) \in \mathbf{R}^5$ such that $\bar{\alpha}_L^f + \bar{\alpha}_H^f = 0$, $\bar{\beta}_L^f, \bar{\beta}_H^f, \bar{q}^f \geq 0$, which satisfies*

$$\begin{aligned} \bar{\alpha}_L^f &\geq \xi_L \bar{p}_L(z_L) + \bar{\beta}_L^f EU_L(z_L) - \bar{\beta}_H^f EU_H(z_H) - \bar{q}^f \xi_L r_L(z_L), \quad \forall z_L \in Z; \\ \bar{\alpha}_H^f &\geq (1-\xi_L)\bar{p}_H(z_H) - \bar{\beta}_L^f EU_L(z_H) + \bar{\beta}_H^f EU_H(z_H) - \bar{q}^f (1-\xi_L) r_H(z_H), \quad \forall z_H \in Z; \end{aligned}$$

with strict equality iff $z_L \in \text{supp}\bar{x}_L^f$ and $z_H \in \text{supp}\bar{x}_H^f$, respectively.

Proof. Since $(\bar{x}^f_L, \bar{x}^f_H)$ solves the firm problem, $(-\frac{\bar{x}^f_L}{\xi_L}, -\frac{\bar{x}^f_H}{1-\xi_L})$ is an optimal solution for the (dual) LSIP problem:

$$\max\ \xi_L\langle p_L, x_L\rangle + (1-\xi_L)\langle p_H, x_H\rangle \qquad \text{s.t.}$$

$$\begin{aligned} \langle 1, x_L\rangle &= 1 \\ \langle 1, x_H\rangle &= 1 \\ -\langle EU_L, x_L\rangle + \langle EU_L, x_H\rangle &\leq 0 \\ \langle EU_H, x_L\rangle - \langle EU_H, x_H\rangle &\leq 0 \\ \xi_L\langle r_L, x_L\rangle + (1-\xi_L)\langle r_H, x_H\rangle &\leq 0 \\ x_L, x_H &\geq 0 \end{aligned}$$

The Lemma states the complementary slackness conditions for the associated primal, with optimal solution $(\bar{\beta}^f_L, \bar{\beta}^f_H, \bar{q}^f, \bar{\alpha}^f_L, \bar{\alpha}^f_H)$. In the absence of a duality gap, Lemma 2.1 implies $\bar{\alpha}^f_L + \bar{\alpha}^f_L = 0$. □

Lemma B.3 *Let* $\gamma_L = \left(1 + \frac{(1-\xi_L)\bar{\lambda}^h_L}{\xi_L\bar{\lambda}^h_H}\right)^{-1}$. *Consider the array* $(\bar{\alpha}_L, \bar{\alpha}_H, \bar{\beta}_L, \bar{\beta}_H, \bar{q})$ *where*

$$\bar{\alpha}_L = \tfrac{\xi_L}{\bar{\lambda}^h_L}\bar{\nu}^h_L - \bar{\alpha}^f_L, \quad \bar{\alpha}_H = \tfrac{1-\xi_L}{\bar{\lambda}^h_H}\bar{\nu}^h_H - \bar{\alpha}^f_H, \quad \bar{\beta}_L = \bar{\beta}^f_L, \quad \bar{\beta}_H = \bar{\beta}^f_H, \quad \bar{q} = \bar{q}^f.$$

Then (a) $(\bar{\alpha}_L, \bar{\alpha}_H, \bar{\beta}_L, \bar{\beta}_H, \bar{q})$ *is feasible for* (P), *(b)* $(\bar{x}^h_L, \bar{x}^h_H)$ *is feasible for* (D) *and (c) the complementary slackness conditions for* (P) *and* (D) *are satisfied.*

Proof:

(a) By Lemma B.1, any x^h_i in X^h satisfies $\bar{v}_i \geq \langle EU_i, x^h_i\rangle - \bar{\lambda}^h_i\langle \bar{p}_i, x^h_i\rangle$. For $i = L$, Lemma B.2 then implies

$$\bar{v}_L \geq \langle EU_L, x^h_L\rangle - \bar{\lambda}^h_L\langle -\frac{\bar{\beta}^f_L}{\xi_L}EU_L + \frac{\bar{\beta}^f_H}{\xi_L}EU_H + \bar{q}^f r_L + \frac{\bar{\alpha}^f_L}{\xi_L}, x^h_L\rangle.$$

for all $x^h_L \in X^h$. In particular,

$$\bar{v}_L \geq \langle EU_L, \delta_{z_L}\rangle - \bar{\lambda}^h_L\langle -\frac{\bar{\beta}^f_L}{\xi_L}EU_L + \frac{\bar{\beta}^f_H}{\xi_L}EU_H + \bar{q}^f r_L, \delta_{z_L}\rangle - \bar{\lambda}^h_L\frac{\bar{\alpha}^f_L}{\xi_L},$$

for all $z_L \in Z$. Rearranging,

$$\text{(B.2)} \quad \frac{\xi_L\bar{v}_L}{\bar{\lambda}^h_L} + \bar{\alpha}^f_L \geq \frac{\xi_L}{\bar{\lambda}^h_L}EU_L(z_L) + \bar{\beta}^f_L EU_L(z_L) - \bar{\beta}^f_H EU_H(z_L) - \bar{q}^f r_L(z_L).$$

Similarly for $i = H$, all $z_H \in Z$ satisfy

$$\text{(B.3)} \quad \frac{(1-\xi_L)\bar{v}_H}{\bar{\lambda}^h_H} + \bar{\alpha}^f_H \geq \frac{(1-\xi_L)}{\bar{\lambda}^h_H}EU_H(z_H) - \bar{\beta}^f_L EU_L(z_H) + \bar{\beta}^f_H EU_H(z_H) - \bar{q}^f r_H(z_H).$$

Thus, $(\bar\beta_L, \bar\beta_H, \bar q, \bar\alpha_L, \bar\alpha_H)$ satisfies both systems of primal constraints when the weights of t_L and t_H in the social welfare function are given by $\frac{\xi_L}{\lambda_L^h}$ and $\frac{1-\xi_L}{\lambda_H^h}$. It remains to normalize the weights.

(b) Follows directly from the definitions of X_i^h and X^f, given **(iii)**.

(c) Complementary slackness for (P) follows from Lemmas B.1 and B.2, which imply that conditions (B.2) and (B.4) hold with strict equality for $z_L \in \bar x_L^h$ and $z_H \in \bar x_L^h$, respectively.

Lemma B.2 also implies that

$$\begin{aligned}\langle \bar p, \bar x^f\rangle &= \bar q^f(\langle r_L, \bar x_L^f\rangle + \langle r_H, \bar x_H^f\rangle) + \bar\beta_L^f\Big(\langle\frac{EU_L}{1-\xi_L}, \bar x_H^f\rangle - \langle\frac{EU_L}{\xi_L}, \bar x_L^f\rangle\Big)\\ &+ \bar\beta_H^f\Big(\langle\frac{EU_H}{\xi_L}, \bar x_L^f\rangle - \langle\frac{EU_H}{1-\xi_L}, \bar x_H^f\rangle\Big) + \bar\alpha_L^f + \bar\alpha_H^f\end{aligned}$$

Since $\bar\beta_L, \bar\beta_H, \bar q$ are non–negative and $\bar x^f \in X^f$, Lemma 2.1 and (**iii**) yield

$$\begin{aligned}0 &= \bar q^f(\xi_L\langle r_L, \bar x_L^h\rangle + (1-\xi_L)\langle r_H, \bar x_H^h\rangle) + \bar\beta_L^f(\langle EU_L, \bar x_H^h\rangle - \langle EU_L, \bar x_L^h\rangle) +\\ &+\bar\beta_H^f(\langle EU_H, \bar x_L^h\rangle - \langle EU_H, \bar x_H^h\rangle) + \bar\alpha_H^f + \bar\alpha_H^f\end{aligned}$$

$$\begin{aligned}\bar q^f(\xi_L\langle r_L, \bar x_L^h\rangle + (1-\xi_L)\langle r_H, \bar x_H^h\rangle) &\le 0\\ \bar\beta_L^f(\langle EU_L, \bar x_H^h\rangle - \langle EU_L, \bar x_L^h\rangle) &\le 0\\ \bar\beta_H^f(\langle EU_H, \bar x_L^h\rangle - \langle EU_H, \bar x_H^h\rangle) &\le 0\end{aligned}$$

Yet, since $\bar\alpha_L^f + \bar\alpha_H^f = 0$, the three inequalities are in fact strict equalities. □

Proof of Theorem 2.4. Follows from Lemma B.3 and the Complementary Slackness Theorem. □

Proof of Theorem 2.5. By Lemma 2.3, (2.17) is necesary for decentralization. We next show it is also sufficient. Suppose a constrained optimal allocation exists which satisfies (2.17). Let $p^* \in P$ be completely by the optimal solution $(\beta_L^*, \beta_H^*, q^*)$ of the associated (modified) primal program as

$$\begin{aligned}p_L(z) &= q^* r_L(z) - \frac{\beta_L^*}{\xi_L} EU_L(z) + \frac{\beta_H^*}{\xi_L} EU_H(z) + K_L^*,\\ p_H(z) &= q^* r_H(z) + \frac{\beta_L^*}{(1-\xi_L)} EU_L(z) - \frac{\beta_H^*}{(1-\xi_L)} EU_H(z) + K_H^*,\\ K_L &= \frac{\beta_L^*}{\xi_L}\langle EU_L, x_L^{h*}\rangle - \frac{\beta_H^*}{\xi_L}\langle EU_H, x_L^{h*}\rangle,\\ K_H &= -\frac{\beta_L^*}{1-\xi_L}\langle EU_L, x_H^{h*}\rangle + \frac{\beta_H^*}{1-\xi_L}\langle EU_H, x_H^{h*}\rangle.\end{aligned}$$

Clearly, (x_L^{h*}, x_H^{h*}) satisfies (2.16) (e.g. let $y^* = q^*$.) Thus, by Lemma 2.3, x_i^{h*} belongs to the type-t_i household's budget set. Finally, the complementary slackness conditions for (P) imply that x_i^{h*} is optimal for the households ($\lambda_i^* = 1$). Now, complementary slackness for (D) yields $\xi_L K_L^* + (1 - \xi_L) K_H^* = 0$ and $\langle p_L^*, \xi_L x_L^{h*} \rangle + \langle p_H^*, (1 - \xi_L) x_H^{h*} \rangle = 0$. Finally, it is easy to check that given Assumption 2.18, any $x^f \in X^f$ satisfies $\langle p^*, x^f \rangle \geq 0$. So, $x^{f^*} = (-\xi_L x_L^{h*}, -(1 - \xi_L) x_H^{h*})$ is optimal for the firm and markets clear. □

Proof of Theorem 3.2. The proof of the first statement is identical to that of Theorem 2.4. The proof of the second statement is a simplified version of that of Theorem 2.5. □

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