Nonparametric Tests for Common Values*

Philip A. Haile
Dept. of Economics
University of Wisconsin-Madison

Han Hong
Dept. of Economics
Princeton University

Matthew Shum
Dept. of Economics
University of Toronto

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Abstract

We develop tests for the presence of common value components in bidders' valuations at symmetric first-price sealed bid auctions. These tests are fully nonparametric and require observations only of the bids submitted at each auction. The main principle of the test relies on the observation that the winner's curse is only present in common value auctions. We evaluate the performance of the test in a variety of Monte Carlo experiments and apply it to bidding data from United States Forest Service (USFS) timber auctions.

1 Introduction

Since the seminal work of Hendricks and Porter (1988), empirical studies of auctions have played an important role in demonstrating the empirical relevance of economic models of strategic interaction between agents with asymmetric information. However, a fundamental empirical issue remains unresolved: how to test between private and common value specifications of bidders' preferences. The theoretical importance of this distinction is well-known[1], and empirical strategies are needed to make these theoretical predictions practically useful

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[1]See Milgrom and Weber (1982) or the surveys of McAfee and McMillan (1988) or Wilson (1992) for detailed discussions of the distinctions between these models.

for policy purposes (for example, in the design of selling mechanisms).

In this paper, we develop an empirical strategy for testing between the common and private value paradigms using data on observed bids. The most notable feature of our approach is that it is *nonparametric*, in the sense of not requiring strong parametric assumptions about the distribution functions of bidders' private information. Previous approaches to testing between CV and PV have been largely parametric in nature (cf. Paarsch (1992), Sareen (1998)).[2] Our non-parametric approach employs methodology recently developed by Guerre, Perrigne, and Vuong (1999) (hereafter GPV) and extended by Hendricks, Pinkse, and Porter (1999) (hereafter HPP) for nonparametric structural estimation of symmetric first-price auction models.[3]

Our test relies on detecting the effects of the *winner's curse* on equilibrium bidding. The winner's curse is a distinctive adverse selection problem which only arises in common value, but not private value settings. Winning a common values auction reveals to the winner that he was more optimistic about the object's value than were any of his opponents. This is "bad news" (cf. Milgrom (1982)) in any common value auction, but particularly bad news for a bidder with many opponents, since it implies that his information was particularly over-optimistic. A rational bidder accounts for this by adjusting his expectation of the value of winning the auction accordingly. This causes a bidder with a given signal of the object's value to have a lower expected value for the object when he faces more competitors. In a private values auction, by contrast, the value a bidder places on the object does not depend on his opponents' information, so the number of bidders does not affect his expectation of the value of the object conditional on winning.

We are not the first to use the equilibrium bidding restrictions implied by the winner's curse to distinguish between the CV and PV hypotheses in first-price auction data: HPP have also proposed several tests. However, our approach differs markedly from that of HPP: while HPP test for differences in the equilibrium bid distributions in the PV and CV model near a binding reservation price, our test is based, loosely speaking, on the implication that

[2] Furthermore, there has also been reduced-form work addressing this question (eg. Gilley and Karels (1981)). Our approach is structural, in the sense that our test statistic is derived explicitly from equilibrium optimality conditions in the Milgrom and Weber (1982) affiliated values model.

[3] In general, until recently, relatively little attention has been focused on common value models. A result of Laffont and Vuong (1996) argued that with observed bid data, a common value model was observationally equivalent to a (correlated) private values model, which raises concerns about the fundamental empirical applicability of common value models. However, in obtaining this result, Laffont-Vuong do not consider data variation in reserve prices, numbers of bidders, and *ex-post* profits, all of which could potentially aid in distinguishing between the private and common value paradigms.

each bidder's expectation of the object's value conditional on winning is decreasing in the number of bidders in CV auctions.

The remainder of the paper is organized as follows. The following section summarizes the underlying affiliated values auction model, the method for estimating bidders' expected values from the observed bids, and the test statistics we consider. Section 3 then discusses several potential sources of bias in the tests that dictate key details of the estimation procedure. Section 4 provides derivation of the asymptotic distribution of our fully nonparametric test. In section 5 we present a bootstrap method for constructing the test, which can also be applied to a semi-parametric test that allows for unobserved heterogeneity in tracts that is correlated with the number of bidders. Section 6 provides Monte Carlo evidence regarding the performance of the tests. In section 7, we consider an application to two sets of auctions held by the U.S. Forest Service. We conclude in section 8.

2 Description of the Test

2.1 Equilibrium Bidding in a Symmetric First-Price Auction

The underlying theoretical framework is Milgrom and Weber's (1982) symmetric affiliated values model, which nests pure private and pure common values models as special cases. An auction has n risk-neutral bidders (indexed $i = 1, \ldots, n$), each of whom has an unknown valuation v_i for the object and receives a private signal x_i about v_i. Bidders' valuations and signals have the joint distribution $\tilde{F}(v_1, \ldots, v_n, x_1, \ldots, x_n)$. Bidders are *symmetric*, in the sense that $\tilde{F}$ is exchangeable with respect to the indices $1, \ldots, n$. The random variables

$$v_1, \ldots, v_n, x_1, \ldots, x_n$$

are affiliated.[4] Let $F(x_1, \ldots, x_n)$ denote the marginal distribution of bidders' signals. The data consist of $b_1, \ldots, b_n$, the n bidders' observed bids from a first-price sealed bid auction.

In symmetric models, it is natural to focus on symmetric Bayesian Nash equilibria in which bidders use identical strategies in equilibrium: $s_j(\cdot) \equiv s(\cdot)$. Furthermore, the affiliation assumption leads to equilibrium bidding strategies which are increasing in a bidder's signal. Symmetry and monotonicity imply that, in equilibrium, the winner is the bidder with the highest signal: $b_i > b_j \Leftrightarrow s(x_i) > s(x_j) \Leftrightarrow x_i > x_j$.

[4] Affiliation, which roughly implies that large values for some of the variables make the other variables more likely to be large than small, is similar in spirit but weaker than the condition of mutual positive correlation.

Consider a bidder i at a given first-price sealed bid auction. Given her signal x_i, she chooses a bid b_i to maximize her expected payoff, given the other bidders' equilibrium behavior:

$$\begin{aligned} b_i =& \text{argmax}_b E\left[(v_i - b)1\{x_j \leq s^{-1}(b), j \neq i\}|x_i\right] \\ =& \text{argmax}_b E\left[(v_i - b)1\{y_i \leq s^{-1}(b)\}|x_i\right] \quad \text{for} \quad y_i = \max_{j \neq i} x_j \end{aligned}$$

The first-order condition characterizing the equilibrium bid function $s(\cdot)$ is a differential equation:

$$v(x_i, x_i, n) = s(x_i) + \frac{s'(x_i)F_n(x_i|x_i)}{f_n(x_i|x_i)} \tag{1}$$

where

$$\begin{aligned} v(x, y, n) &= E\left[v_i|x_i = x, \max_{j \neq i} x_j = y\right] \\ &= E\left[v_1|x_1 = x, x_2 = y, x_3 \leq y, \ldots, x_n \leq y\right] \end{aligned}$$

and $F_n(\cdot|x_i)$ is the conditional distribution of the maximum signal among i's opponents (with $f_n(\cdot|x_i)$ the corresponding density). The conditional expectation $v(x_i, x_i, n)$ above is decreasing in the number of bidders n whenever the expectation $E[v_i|x_1, \ldots, x_n]$ depends on $x_j, j \neq i$; i.e., whenever bidders' valuations contain a common value element. This is one important manifestation of the winner's curse, and this monotonicity is the key to our nonparametric test.

Theorem 1 $E[v_i|x_1, \ldots, x_n]$ *depends on* $x_j, j \neq i$ *if and only if* $v(x, x, n)$ *is nondecreasing in* n *for all* x *and strictly increasing in* n *for some* x. □

Proof: to be written.

Roughly speaking, in a common values auction, when a bidder i with signal x conditions on the event that $\max_{j \neq i} x_j = x$, this tends to reduce his expectation of the object's value v_i relative to his expectation when conditioning just on his own signal x. Even if each signal x is an unbiased estimate of v_i, the highest of n signals is an *upwardly biased* estimate of v_i. This bias is increasing in n, so the conditional expectation $v(x, x, n)$ will be decreasing in n.

2.2 Structural Interpretation of Observed Bids

The observed data do not directly reveal the distribution of signals F, but rather the distribution of bids. The key insight of Guerre, Perrigne, and Vuong (1999) relies on the observation that the conditional distribution of signals F is characterized by the conditional distribution of bids G via the following relations:

$$F_n\left(y|x\right) = G_n\left(s(y)|s(x)\right) \tag{2}$$

and

$$f_n\left(y|x\right) \times \frac{1}{s'\left(y\right)} = g_n\left(s(y)|s(x)\right) \tag{3}$$

where $G_n(\cdot|s(x))$ is the conditional distribution (assuming all bidders follow $s(\cdot)$) of the highest bid submitted by i's competitors, which, given symmetry, will be the bidder with the highest signal among these $(n-1)$ competitors.

Since in equilibrium $b_i = s(x_i)$, the differential equation (1) can be rewritten

$$v(x_i, x_i, n) = b_i + \frac{G_n(b_i|b_i)}{g_n(b_i|b_i)} \equiv \xi(b_i; n). \tag{4}$$

The left side of this equation gives bidder i expected value of the object. These values cannot be observed directly. However, both $G_n(\cdot|\cdot)$ and $g_n(\cdot|\cdot)$ can be nonparametrically identified and estimated from a random sample of T_n auctions, each involving n bidders.[5] We will refer to the nonparametric estimates of these functions as, respectively, $\hat{G_n}$ and $\hat{g_n}$, (where the subscript n emphasizes the fact that different estimates are generated for auctions with different number of bidders).[6]

By evaluating $\hat{G_n}$ and $\hat{g_n}$ at each of the observed bids, we can obtain estimates of the corresponding pseudo-values using (4):

$$\mathrm{ps}_{it} \equiv b_{it} + \frac{\hat{G_n}(b_{it}|b_{it})}{\hat{g_n}(b_{it}|b_{it})} = \hat{\xi}(b_{it}; n).$$

This latter insight was first articulated in Hendricks, Pinkse, and Porter (1999), in their extension of the Guerre, Perrigne, and Vuong (1999) methodology to common value models.

[5] The nonparametric estimates are given by the following formulas:

$$\hat{G_n}(b|b) = \frac{1}{T_n \times h \times n} \sum_{t=1}^{T_n} \sum_{i=1}^{n} K\left(\frac{b - b_{it}}{h}\right) \mathbf{1}\left(b_{it}^* > b\right) \tag{5}$$

and

$$\hat{g_n}(b|b) = \frac{1}{T_n \times h^2 \times n} \sum_{t=1}^{T_n} \sum_{i=1}^{n} K\left(\frac{b - b_{it}}{h}\right) K\left(\frac{b - b_{it}^*}{h}\right). \tag{6}$$

Here h is the bandwidth, $K(\cdot)$ is a kernel, t indexes all n-bidder auctions, and i indexes each of the bidders. In addition, b_{it} is the observed bid of bidder i from auction t, and b_{it}^* is the highest observed bid among bidder i's rival bidders in auction t.

[6] Estimation of G and g is described immediately below.

2.3 Main Principle of the Test

ps_{it} is thus an estimate of $v_n(x_{it}, x_{it})$; from Theorem 1, this function is decreasing in n keeping x_{it} fixed in a CV setting, but is constant across all n in a PV setting. However, this reasoning cannot be the basis of a test; we cannot do the experiment of varying n while holding x fixed because the above procedure does not allow us to recover the signal x which generated the observed bid[7].

However, given bid data, with enough variation across n, we *can* estimate nonparametrically the distribution of pseudovalues across different n: let $F_{v,n}$ denote the distribution of pseudovalues in n-bidder auctions. Under the private value hypothesis, the distributions $F_{v,n}$, $n = 1, \ldots, N$ (where N denotes the maximum number of bidders observed in any auction in our dataset) should be identical, while under the common value hypothesis, these distributions should be "increasing" (in some appropriately defined manner) in n. This is formalized in the following corollary to theorem 1:

Corollary 1 *Let $F_{v,n}$ denote the distribution function of $v(x, x, n)$, as induced by the distribution laws on the signals x. Under the private value hypothesis:*

$$F_{v,1} = F_{v,2} = \ldots = F_{v,N}.$$

Under the common value hypothesis:

$$F_{v,1} < F_{v,2} < \ldots < F_{v,N}$$

in the sense of strict first-order stochastic dominance (FOSD).[8]

Clearly, given corollary 1, a test for common values can be restated in terms of a test for stochastic dominance of the distributions of the estimated pseudovalues. There are a wide variety of ways to test either the stochastic dominance hypothesis directly or its implications indirectly. Anderson (1996) has prescribed nonparametric tests for stochastic dominance, although these must be extended to account for the sampling error in the nonparametric first-step employed in computing the pseudovalues. In what follows, we propose two tests based on comparing the moments of $F_{v,n}$, across different n.

[7] Except in private value auctions, which Li, Perrigne, and Vuong (1999) focus on.

[8] Recall the definition of FOSD: the distribution function G_1 first-order stochastic dominates another distribution function G_2 if, for every increasing function $U(\cdot)$,

$$\int U(x)dG_1(x) > \int U(x)dG_2(x).$$

2.4 A Quantile Approach

First, we consider comparing pseudovalues for different number of bidders at the same quantile of the observed bid distribution, instead of at the same bid level. Let x_τ denote the τth quantile of the signals, and $v_{\tau,n} \equiv v(x_\tau, x_\tau, n)$. Then the private and common value hypotheses can be written as:

$$H_0(\text{private values}): \; v_{\tau,1} = v_{\tau,2} = \ldots = v_{\tau,N}$$
$$H_1(\text{common values}): \; v_{\tau,1} > v_{\tau,2} > \ldots > v_{\tau,N}.$$

In order to estimate $v_{\tau,n}$, we exploit the monotonicity properties of our equilibrium bidding strategy. Given this property, the τth quantile of the bid distribution is submitted by the bidder with the τth most optimal signal. The assumed homogeneity of the underlying joint distribution allows us to compare the pseudovalues at these equivalent quantile levels.[9] To be precise, let $\hat{b}_{\tau,n}$ be the τth quantile of the observed bid function for n bidders, i.e.:

$$\hat{b}_{\tau,n} = F_n^{-1}(\tau) = \inf\left(x : F_n(x) \geq \tau\right),$$

where $F_n(\cdot)$ is the empirical distribution of the all the $n \times T_n$ observed bid in auctions with n bidders. The pseudovalue for the τth bidder in n bidder auctions is estimated by

$$\hat{v}_{\tau,n} = \hat{b}_{\tau,n} + \frac{\hat{G_n}(\hat{b}_{\tau,n}|\hat{b}_{\tau,n})}{\hat{g_n}(\hat{b}_{\tau,n}|b_{\tau,n})}$$

Under the null private value hypothesis, $\hat{v}_{\tau,n}$ is approximately equal across n, while under the common value alternative it is decreasing over n.

Since the sample quantile converges at rate $\sqrt{T_n}$ to the true quantile, the convergence rate of $v_{\tau,n}$ to v_{x_τ,x_τ}, where x_τ is the τth quantile of the marginal distribution of bidder signal, is governed by the slow pointwise nonparametric convergence rate of $\hat{g}_n(\cdot)$. As shown in Guerre, Perrigne, and Vuong (1999), for fixed b, $\hat{g}(b|b)$ converges at $\sqrt{T_n h^2}$ to $g(b|b)$ due to the estimation of a univariate conditional density. Theorem 2 below describes the limiting behavior of each $\hat{v}_{\tau,n}$:

[9] Generally, this invariance property of quantiles to monotonic transformations has been previously exploited in the literature of semiparametric estimation of limited dependent variable models (Powell (1994)), but we believe this is the first attempt to use it in other settings. Previously, this monotonicity was exploited in Hong and Shum (1999) to facilitate estimation of a structural auction model. As we noted there, these monotonicity properties are potentially very useful in other incomplete information settings where the equilibrium strategies (or the "policy functions") are monotonic transformations of the unobserved types: for example, in nonlinear pricing (and, more generally, mechanism design) models, the policy function $p(x)$ is often constrained to be monotonic in the type x in order to be implementable (i.e., satisfy incentive compatibility). See (Fudenberg and Tirole, 1991. 257ff.).

Theorem 2 *Assuming (i) $K(\mu)$ and $|\mu K(\mu)|$ are bounded. (ii) $\int \mu K(\mu)\, d\mu = 0$. (iii) $\int \mu^2 K(\mu)\, d\mu < \infty$. (iv) $\lim T_n h^2 = \infty$ and $\lim T_n h^6 = 0$. Then for each fixed b where $g_n(b|b) f(b) > 0$:*

$$\sqrt{T_n h^2}\left(\hat{v}\left(s^{-1}(b), s^{-1}(b), n\right) - v\left(s^{-1}(b), s^{-1}(b)\right), n\right)$$
$$= \sqrt{T_n h^2}\left(\frac{\hat{G}_n(b|b)}{\hat{g}_n(b|b)} - \frac{G_n(b|b)}{g_n(b|b)}\right) + o_p(1)$$
$$\xrightarrow{d} N\left(0, \frac{1}{n}\frac{G_n^2(b|b)}{g_n^3(b|b) f(b)}\left[\int\int K^2(x) K^2(y)\, dx dy\right]\right)$$

Also let $F_x(\cdot)$ be the marginal distribution function of a bidder's signal, then

$$\hat{b}_{\tau,n} - s\left(F_x^{-1}(\tau)\right) = O_p\left(\frac{1}{\sqrt{T_n}}\right)$$
$$= \frac{1}{T_n n}\frac{1}{f_b\left(s\left(F_x^{-1}(\tau)\right)\right)}\sum_{t=1}^{T}\sum_{i=1}^{n}\left[1\left(b_{it} \le s\left(F_x^{-1}(\tau)\right)\right) - \tau\right] + o_p\left(\frac{1}{\sqrt{T_n}}\right)$$

In addition, for distinct values of $\tau_1, \ldots, \tau_L$, assuming

$$g_n\left(s\left(F_x^{-1}(\tau_l)\right)|s\left(F_x^{-1}(\tau_l)\right)\right) f\left(s\left(F_x^{-1}(\tau_l)\right)\right) > 0$$

for each $l = 1, \ldots, L$, the L-dimensional vector of

$$\sqrt{T_n h^2}\left(\hat{v}\left(s^{-1}\left(\hat{b}_{\tau_l,n}\right), s^{-1}\left(\hat{b}_{\tau,n}\right), n\right) - v\left(F_x^{-1}(\tau_l), F_x^{-1}(\tau_l)\right), n\right)$$

converges weakly to the vector Z which is jointly normally distributed with diagonal variance-covariance matrix whose lth diagonal element is given by

$$\Omega_l = \frac{1}{n}\frac{G_n^2\left(s\left(F_x^{-1}(\tau)\right)|s\left(F_x^{-1}(\tau)\right)\right)}{g_n^3\left(s\left(F_x^{-1}(\tau)\right)|s\left(F_x^{-1}(\tau)\right)\right) f(b)}\left[\int\int K^2(x) K^2(y)\, dx dy\right]$$

□

In short, test statistics based on comparisons of pseudovalues at a fixed number of quantiles converge at the slow rate $\sqrt{T_n h^2}$, as a result of the slow pointwise convergence rate of the nonparametric estimate of the conditional density $g_n(b|b)$. The second test we propose is based on comparing centered moments of the pseudovalues across n, which converge at a faster rate due to the averaging involved in calculating the sample moments.

2.5 A First Moment Approach

Theorem 1 also implies that $\mathcal{E}_x v(x, x, n)$ is decreasing in n in symmetric CV auctions, but is constant across n for PV auctions, where the expectation is taken over the marginal distribution $F(x)$ (which by symmetry is identical across all bidders). This forms the basis of our next test. Precisely, the test is based on the same assumption that the joint marginal distribution for any $n-1$ bidders in a n-bidder auction is the same as the joint distribution for a $n-1$-bidder auction. This assumption, of course, can be made conditional on all observed heterogeneity across objects for sale. It is obvious from the monotonicity of $\mathcal{E}_x v\,(x,x,n)$ that it is well defined for all n as long as it is defined for $n=2$, which will be assumed through the rest of the paper. For example, for a log-normal joint distribution where (v_1,x_1,x_2) are jointly normally distributed,

$$\mathcal{E}_x v\,(x,x,2) = \mathcal{E}_x \exp\left(\beta x + \alpha\right)$$

for β and α determined by the mean μ and covariances Σ of (v_1,x_1,x_2) and is well defined for all μ and Σ. On the other hand, for procurement auctions, the pseudovalue $v\,(x,y,n) = \mathcal{E}\,(c_1|x_1=x,x_2=y,x_3\geq y,\ldots,x_n\geq y)$ is increasing in n, but since

$$\mathcal{E}\,(c_1|x_1=x,x_2=x,x_3\geq x,\ldots,x_n\geq x) \sim E\,(c_1|x_1=\ldots,=x_n=x)$$

as $x\longrightarrow\infty$ whenever the later is integrable, it suffices to have the later integrable over the marginal distribution of x, which is true, for example, for (c,x) jointly log-normally distributed.

If we assume that signals are drawn randomly across all the observed auctions, we can estimate $\mathcal{E}_x[v(x,x,n)]$ for the set of n-bidder auctions using the sample average of the pseudovalues calculated across all the auctions with n bidders:

$$\mathcal{E}_x[v(x,x,n)] \doteq \frac{1}{n*T_n}\sum_{t=1}^{T_n}\sum_{i=1}^{n}\mathrm{ps}_{it} \equiv \hat{\mu}_n \tag{7}$$

where ps_{it} is the pseudovalue for the ith bidder in auction t, and T_n is the number of n-bidder auctions observed in the data set.

Under the private value assumption, $v\,(x,x,n)=x$, so that $\mathcal{E}_x v(x,x,n)$ should be constant across n in a series of auctions where identical objects are being sold.[10] The null hypothesis of private values, then, can be stated as

$$H_0: \mathcal{E}_x v(x,x,1) = E_x v(x,x,2) = \cdots = E_x v(x,x,N) = E_x x. \tag{8}$$

We are particularly interested in the common value alternative hypothesis which, as the above discussion indicates, implies:

$$H_1: E_x v(x,x,1) > E_x v(x,x,2) > \cdots > E_x v(x,x,n). \tag{9}$$

[10]Or when object heterogeneity is controlled for by regressors in the nonparametric regression, although for clarity of presentation we do not explicitly condition on regressors in describing the kernel regression procedure here.

In the appendix we derive the consistency and asymptotic distribution of $\hat{\mu}_n$. We show that the convergence rate for our estimate of each $\hat{\mu}_n$ is $\frac{1}{\sqrt{T_n h}}$, which is slower than the parametric rate $\frac{1}{\sqrt{T_n}}$, but faster than the $\frac{1}{\sqrt{T_n h^2}}$ rate of the quantiles-based test described above. Intuitively, the intermediate $\frac{1}{\sqrt{T_n h}}$ rate of convergence arises because $\hat{g}_n(b)$ is an estimated bivariate density function, but in estimating each $\tilde{V}_n$ we average only along the one-dimensional 45^o line $(b_{ij},\, b^*_{ij} = b_{ij})$ (cf. Newey (1994)). Furthermore, the asymptotic distribution of $\hat{\mu}_n$ depends on the kernel function used in the nonparametric estimation. More precisely:

Lemma 1 *Under suitable conditions on the choice of the bandwidth parameter h and the kernel function $K(\cdot)$, and under smoothness conditions on the joint distribution of bids,*

$$\sqrt{T_n h}\left(\hat{\mu}_n - Ev(x,x,n)\right) \xrightarrow{d} N(0,V)$$

where

$$V = \left[\int \left(\int K(v)\, K(u+v)\, dv\right)^2 du\right] \left[\frac{1}{n}\int \frac{G_n^2(b;b)}{g_n^3(b;b)} db\right]. \qquad (10)$$

and the double integral is over the support of the kernel function.

While the test based on averaged pseudovalues converges faster than that based on fixed number of quantiles, but the better rate comes at the cost of more stringent smoothness conditions at the boundary of the bid distribution.

3 Implementing the test

For the remainder of this paper, we focused on tests based on the first moment approach. We consider a test based on regressions of pseudo-values on the number of bidders. The primary advantage of this approach is that it allows the use of instrumental variable techniques. This can be important in many applications where one may be concerned that observed heterogeneity in auctions is correlated with the number of bidders. [11] This could arise if, for example, participation is higher in auctions involving more valuable objects, thus introducing unobserved factors which are correlated with both n_t and ps_{it}.

Denote the sample of pseudo-values by $\{\mathrm{ps}_{it}\}$ for auctions $t = 1, \ldots, T$ and bids $i = 1, \ldots, n_t$ for auction t, where n_t is the number of bidders in auction t. One can then estimate the

[11] See, for example, Haile (1999) and Hendricks and Porter (1999).

regression model:

$$\mathrm{ps}_{it} = \phi(n_t) + \epsilon_{it} \tag{11}$$

where $\phi(n_t)$ is a known function of n_t. Letting

$$\phi(n_t) = \sum_n \beta_n 1\{n_t = n\} \tag{12}$$

and testing the hypothesis $\beta_n = \beta \ \forall n$ is obviously equivalent to the test proposed in the preceding section. However, more parsimonious specifications (linear or polynomial functions, for example) are also possible. The private values hypothesis restricts $\phi'(n) = 0$ for all n while $\phi'(n) < 0$ for all n if common values are present. Note that in the affiliated values framework, which is a maintained assumption throughout the paper, pseudo-values can only be constant in n or strictly decreasing in n. Hence even when $\phi(n_t) = \alpha + n_t\beta$ this test has power to detect all alternatives to the pure private values models. More flexible specifications like (12), however, will have greater power to detect nonmonotonicities or other violations of the affiliated values assumption as well as greater power against the common values alternative.

3.1 Details of the implementation

Li, Perrigne, and Vuong (1997) focus on consistent estimation of pseudo-values given a sample of auctions with a constant number of bidders. We rely heavily on their work. However, a potentially significant source of bias can arise when applying their estimation method to make comparisons across auctions with different numbers of bidders. This is due to the effects of "trimming" bids near the boundary of the support of the observed data. This trimming is necessary to obtain consistent kernel estimates of pseudo-values, but care must be taken to avoid introducing biases that are systematically related to the number of bidders. We explain below how this can be done. We then point out problems that arise if one pools data from auctions with different numbers of bidders.

3.1.1 Trimming

Li, Perrigne, and Vuong (1997) describe bandwidth selection for a given sample of observed bids. As usual one must trim observations near the boundaries of the support of the observed data in order to obtain consistent kernel estimates. Trimming a symmetric distribution leaves the mean (and, therefore, the consistency of our tests) in tact as long as the same interval is trimmed off both boundaries. However, this is not the case with an asymmetric distribution. Furthermore, the bias introduced by trimming depends on the amount trimmed, which typically varies with n if one trims a fixed number of bandwidths for each n. As a result, spurious relationships between pseudo-values and n may be detected.

To see this, suppose that private values are *i.i.d.* with distribution $F(v) = v^2$ and that we have auctions with $n = 2$ and $n = 3$. When $n = 2$, bids are distributed on the interval $\left[0, \frac{2}{3}\right]$. Imagining that we observe bids over this full range, we then trim one bandwidth[12] to use bids on the interval $\left[h, \frac{2}{3} - h\right]$. When bids are used to estimate pseudo-values, bids are scaled up by a factor of $\frac{3}{2}$ (since $b(x) = \frac{2}{3}x$), giving pseudo-values distributed (ignoring sampling error) on $\left[\frac{3}{2}h, 1 - \frac{3}{2}h\right]$ according to $\frac{F(v)-F(\frac{3}{2}h)}{F(1-\frac{3}{2}h)-F(\frac{3}{2}h)}$. Similarly, when $n = 3$, pseudo-values have a truncated distribution on $\left[\frac{5}{4}h', 1 - \frac{5}{4}h'\right]$, where h' is the bandwidth used. Our test for common values examines $E\left[\text{ps}_{it}|n_t = 2, \text{not trimmed}\right] - E\left[\text{ps}_{it}|n_t = 3, \text{not trimmed}\right]$. The following table shows this difference for some hypothetical values of h and h'.

	h'	.10	.15	.20	.25	.30
h	.1	-.012	.017	.040	.058	.071
	.15	-.043	-.015	.009	.027	.040
	.20	-.067	-.038	-.015	.003	.016
	.25	-.083	-.055	-.031	-.013	.000
	.30	-.092	-.063	.040	.022	.009

This example demonstrates that there is bias that depends on the bandwidths used, and that this bias is not eliminated by equalizing the range of bids trimmed from each subsample of auctions, defined by the number of bidders. The bias can, however, be eliminated if the trimming is equalized across subsamples, not in the space of *bids*, but in the space of *pseudo-values.* In this example, where we know the true distribution, if

$$\frac{3}{2}h = \frac{5}{4}h'$$

the bias will be gone. This can be seen in the table above, where $h = .25$ and $h' = .30$. The problem is that we of course don't know $F(\cdot)$ in practice. However, this problem can be overcome using the following procedure:

- estimate $\hat{G_n}$ and $\hat{g_n}$ using the bandwidths h_n specified by Li, Perrigne, and Vuong (1997)

[12] Li, Perrigne, and Vuong suggest trimming two bandwidths. This does not affect the argument, however.

- construct $\hat{\xi}(b;n)$ for each n as described above
- choose a "trimming width" h'_n for each n so that $\hat{\xi}(h'_n;n)$ is the same for all n, with $h'_n \geq \max_m h_m \forall n$. Specifically, letting $\overline{\xi}$ denote $\max_n \hat{\xi}(h_n;n)$, choose h'_n for each n so that $\hat{\xi}(h'_n;n) = \overline{\xi}$.
- re-calculate the sample of pseudo-values, using the original estimates $\hat{G_n}$ and $\hat{g_n}$, but trimming based on h'_n.

Note that this procedure leaves the consistency of the pseudo-value estimates (shown in Guerre, Perrigne, and Vuong (1999) and Li, Perrigne, and Vuong (1997)) intact.

3.1.2 Estimation with pooled data

In many cases one will have data from a fairly large number of auctions but a relatively small number of observations for each value of n. This may suggest pooling data across auctions with similar numbers of bidders for estimation. However, the derivation of pseudo-values from the distribution of bids described above assumes a constant number of bidders. Pooling data from auctions with different numbers of bidders can lead to biased estimates that render our tests invalid.

Take the case of independent private values distributed uniform $[0,1]$ and assume the same number of auctions of each size n. In this case

$$\xi(b;n) = \frac{n}{n-1}b$$

for given n; so for a fixed $b, \xi(b;n)$ decreases in n. Since in any private values model and many/most other affiliated values models raising the number of bidders shifts the distribution of bids to the right, this monotonicity is likely to hold in many cases. So if one combines auctions with different n to estimate G and g, this will lead to pseudo-values *within this group* that are larger than the true $v(x,x)$ generating the bids when n is low and pseudo-values that are smaller than the true $v(x,x)$ when n is large. So over the sample one would estimate a spurious negative relation between pseudo-values and n.

Since there are more observations from auctions with n large in this example, this also results in bias in the estimated average pseudo-value within the group. If one divided the sample into sets of, say "small n" and "large n" auctions, the size of the bias could vary *across the groups*, leading to spurious variation in average pseudo-values across groups. More generally, the fact that the distribution of n within group may vary across groupings in real data—in some cases there may be fewer observations from larger auctions, for example—this could create unpredictable differences in the bias across groups.

3.2 Bootstrapping

Given the computational difficulties in deriving an approximation of the asymptotic variance of our regression coefficients (which will be linear combinations of the elements in the asymptotic covariance matrix (10)), we consider the alternative of using the bootstrap principle to construct confidence intervals for our regression coefficients.[13] The procedure is straightforward, although we must account for the possibility that bids (pseudo-values) from each auction are correlated. Hence, we use a simple type of "block bootstrap" procedure. For simplicity we describe the approach for the regression-based test using the simple linear specification

$$\phi(n_t) = \alpha + n_t \beta.$$

After obtaining the pseudo-value estimates and an estimate of $\hat{\beta}$ of β, we perform a large number (R) bootstrap iterations. For each bootstrap iteration $r = 1, \ldots, R$ we:

1. Draw a bootstrap "sub-sample" separately for each value of n, where each subsample replicates the number of bids from n-bidder auctions in the data. This is done by drawing with replacement one of the T_n auctions and including all bids from that auction, thereby incorporating any correlation of bids within auction in the bootstrap data generating process.

2. Construct the full bootstrap sample of bids by combining the bootstrap sub-samples across all n.

3. Estimate the pseudo-values associated with the rth bootstrap sample of bids: call these $\vec{\text{ps}}^r$.

4. Run an OLS regression for the rth sample of pseudo-values $\vec{\text{ps}}^r$ on n, and call the estimate $\hat{\beta}_r$

5. Finally, calculate confidence bands around the original estimate $\hat{\beta}$ based on percentiles of the empirical distribution of the β_r's across all bootstrap iterations r.[14]

[13]For a discussion of the bootstrap, see Hall (1994) or Efron and Tibshirani (1993).

[14]Two-sided tests can easily be constructed. Given our focus on a private values null and a common values alternative, we use a one-sided test.

4 Monte Carlo Evidence

To assess the performance of our tests we perform several Monte Carlo experiments. We focus first on the regression-based test. To address both the size and power of the test, we consider several samples of artificial bidding data, some generated according to a private values model; others generated by a common values model. We consider three private values specifications which differ only in the underlying joint distribution of values $v(x,x) = x$:

(1) $F(x_1, \ldots, x_n) = \prod_{i=1}^{n} x_i^2, x_i \in [0,1]$;

(2) $F(x_1, \ldots, x_n) = \prod_{i=1}^{n} x_i, x_i \in [0,1]$;

(3) $F(x_1, \ldots, x_n) = \int_0^1 F(x_1, \ldots, x_n|c)\, dc = \int_0^1 \prod_{i=1}^{n} \frac{x_i}{c}\, dc, x_i \in [0,1]$.

The first two are examples of independent private values. The two cases $F(x) = x$ and $F(x) = x^2$ are considered because the effects of trimming on the bias of the test can vary with the convexity of the distribution. The third case is an example of a correlated private values environment. Here bidders' values are *i.i.d.* conditional on an unobserved random variable C. Conditional on $C = c$, each x_i is distributed uniformly on $[0,c]$. We include this example to illustrate that our tests address not the the correlation of bidders' private information—often a feature of common values models, although this is neither necessary nor sufficient for common values—but the winner's curse that arises from the fact that each bidder's expected value for the object depends on opponents' private information.

This point is emphasized further by the comparison to a pure common value model in which the information structure is identical to that in the third example above, but now the (common) value of the object to each bidder is the unknown random variable C. As above, each bidder i observes a signal X_i of this value, where each X_i is independently distributed (conditional on c) with distribution

$$F(x_i|c) = \frac{x_i}{c}, x_i \in [0,c]$$

while C itself is uniform on $[0,1]$. The symmetric equilibrium bid function (see Matthews (1984)) is

$$b(x) = \int_0^x \hat{v}(t,n) \left(\frac{n-1}{x}\right) \left(\frac{t}{x}\right)^{n-2} dt \tag{13}$$

where

$$\hat{v}(t,n) = \int_x^1 c\, g(c|x,n)\, dc$$

and[15]

$$g(c|x,n) = \frac{\frac{n}{c}\left(\frac{x}{c}\right)^{n-1}}{\int_x^1 \frac{n}{w}\left(\frac{x}{w}\right)^{n-1} dw}.$$

Finally, we consider an affiliated values model with both private and common value components. Each bidder i has valuation v_i where

$$\ln v_i = \ln a_i + \ln v$$

and signal x_i where

$$\ln x_i = \ln a_i + \ln v + \epsilon_i.$$

The random variables $\ln v$, $\ln a_i$, and ϵ_i are drawn from standard normal distributions. The equilibrium bid function for this model (see Hong and Shum (1999)) is ******

With each of these models we consider a sample of auctions with the number of bidders varying between 2 and 9. To allow for varying precision in the pseudo-value estimates that is likely to arise in many applications, we consider cases in which the number of auctions of each size is held fixed (at 100) and in which the number of bids from each auction is held approximately fixed (at as close to 200 as possible given the divisibility constraints). These choices reflect typical sample sizes for auction data. To illustrate the importance of estimating pseudo-values separately for each n, we compare results obtained following this approach to those obtained when auctions are only divided into "low participation" ($n \leq 5$) and "high participation" auctions for estimation. Finally, we illustrate the importance of equalizing trimming in the space of pseudo-values by comparing the results obtained this way with those obtained when we simply trim two bandwidths for each subsample of pseudo-values (one for each n).

This yields a total of 48 combinations of model and estimation method. For each of these we perform 1000 Monte Carlo simulations (calculating bootstrap tests using 500 bootstrap samples at each iteration) and report the mean and standard deviation of the estimated derivative $\frac{dps_{it}}{dn_t}$. We also report the frequency of rejecting the null hypothesis that this derivative is zero—an indication of the test size in the case of the private values models and of the power in the case of the common value models. Tables 1 and 2 report the results.

[15]To understand the bid function (13), note that each player's valuation $\hat{v}(t,n)$ is his expectation of C conditional on his own signal and the inference that when he wins the auction, this signal was the highest among those of all n bidders. This conditioning is due to the winner's curse. For strategic reasons, each bidder then optimally shades his bid below his valuation, bidding his expectation of the second highest valuation conditional on his own being the highest.

5 An Application to USFS Timber Auctions

In process

6 Conclusions

We have provided a simple method to test for common values in symmetric first-price sealed bid auctions. We have also pointed out the applicability of tools developed in Guerre, Perrigne, and Vuong (1999): not just for structural estimation of auction models, but also for direct tests of the PV hypothesis. Compared to previous parametric approaches, our test can be more powerful, since we are not testing against a specific parametric CV alternative, but rather against the range of all symmetric models where $\mathcal{E}_x v(x, x, n)$ is not constant across n.

We can extend our testing procedure to detect common value elements with *ex ante* asymmetric bidders, as long as it is possible to identify a bidder who competes with two different set of rivals in two different set of auctions. The only modification of the test is that we focus on a particular bidder instead of treating them symmetrically in forming the test statistics. Consider, without loss of generality, bidder 1. Under the private value hypothesis:

$$\mathcal{E}\left(v_1 | x_1 = s_1^{-1}(b), x_j = s_j^{-1}(b), x_k \leq s_k^{-1}(b), k \neq 1, j\right) = x_1 = b_1 + \frac{\hat{G}_1(b)}{\sum_{j \neq 1} \hat{G}_{1j}(b)}$$

(The index n on the number of bidders has been omitted for clarity.) Therefore a test for the presence of common values for bidder 1 can be similarly based on the average pseudovalue:

$$\hat{\mu}_{1n} \equiv \mathcal{E} x_{1n} \approx \frac{1}{T_n} \sum_{l=1}^{T_n} \left(b_{1l} + \frac{\hat{G}_1(b_{1l})}{\hat{g}_1(b_{1l})} \right)$$

where $\hat{G}_1(b)$ and $\hat{G}_{1j}(b)$ are nonparametric estimates analogous to those in equations (5) and (6) above, and b_{l1}^* denotes the highest bid among bidder 1's rivals in auction l. Under the private value hypothesis, $\hat{\mu}_{1n}$ should be constant across n. However, under the alternative common value hypothesis, the magnitude of $\hat{\mu}_{1n}$ are not clearly ordered in the asymmetric model, since the combined effect of asymmetry and affiliation can result in either higher or lower estimated $\hat{\mu}_{1n}$ as n increases. Nevertheless, a test of equality of average pseudovalues for different n can still suggest absence or presence of common value elements.

Additional extensions

- simple multi-object environments (multi-object, unit demand, pay-your bid auctions, as in Weber (1983))?
- Major drawback is that current test requires n to be common knowledge, but test would also be valid for HPP random-participation model which augments the non-parametric LPV approach with a parametric model of participation?

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A Consistency of the nonparametric test

The proof essentially follows steps contained in Lavergne and Vuong (1996), as well as Powell, Stock, and Stoker (1989), Lewbel (1998), Horowitz(1998) **(cite?)**, among others. The steps are standard and it is only neccessary to give an outline.

Assumption 1 *There are n auctions, M bidders for each auction, $M \geq 3$. Bidders are ex ante symmetric. The joint distribution of v, x are exchangeable in $l = 1, \dots, M$.*

Define $v(x) = E(v_1 | x_1 = x, x_2 = x, x_3 \leq x, \dots, x_n \leq n)$. As shown in Guerre, Perrigne, and Vuong (1999) and Hendricks, Pinkse, and Porter (1999), $v(x) = v\left(s^{-1}(b)\right) = b + \frac{G(b|b)}{g(b|b)} = b + \frac{G(b;b)}{g(b;b)}$.

Define

- $G(b;b) \equiv G_{B_1,b_1}(b;b) = P(B_1 \leq b; b_1 = b)$
- $g(b;b) \equiv g_{B_1,b_1}(b;b) = f(B_1 = b; b_1 = b)$, where $B_1 = \max(b_j, j > 1)$.

Assumption 2 *Both $G(b;b)$ and $g(b;b)$ have mth order bounded derivatives. The support of b is bounded.*

As suggested in Guerre, Perrigne, and Vuong (1999) and Hendricks, Pinkse, and Porter (1999), we estimate G and g at each b_i using kernels:

- $\hat{G}(b_i; b_i) = \frac{1}{n-1}\frac{1}{M}\sum_{j\neq i}^{n}\sum_{l=1}^{M}\frac{1}{h}K\left(\frac{b_{jl}-b_i}{h}\right)1(B_{jl} \leq b_i)$, where $B_{jl} = \max(b_{jk}, k \neq l)$.
- $\hat{g}(b_i; b_i) = \frac{1}{n-1}\frac{1}{M}\sum_{j\neq i}\sum_{l=1}^{M}\frac{1}{h^2}K\left(\frac{b_{jl}-b_i}{h}\right)K\left(\frac{B_{jl}-b_i}{h}\right)$.

Assumption 3 *The kernel function $K(\cdot)$ is a mth order kernel with bounded support and bounded total variation: $\int K(u)\,du = 1$, $\int u^k K(u)\,du = 0$, $1 \leq k < m$. $\int u^m |K(u)|du < \infty$.*

Assumption 4 *Let h_n be the bandwidth parameter and let τ_n be a trimming scalar such that $h_n \to 0$, $\tau_n \to 0$, $\tau_n \sqrt{nh_n^2}\left(\sqrt{\log n}\right)^{-1} \to \infty$. $\tau_n^{-1} h_n^m \to 0$.*

Define the test statistic by: $\hat{T} = \frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\left[b_{il} + \frac{\hat{G}(b_{il};b_{il})}{\hat{g}(b_{il};b_{il})}\right]1\left(\hat{g}(b_{il};b_{il}) \geq \tau_n\right)$.

Theorem 3 *Under assumptions (1), (2), (3), (4), $\hat{T} \xrightarrow{p} Ev(x) = Ev\left(s^{-1}(b)\right)$.*

Proof: The following uniform nonparametric rates of convergence are standard,

$$\sup_{b\in R}\left|\hat{G}(b;b) - G(b;b)\right| = O_p\left(\sqrt{\frac{\log n}{nh_n}}\right) + O(h_n^m) \quad \sup_{b\in R}\left|\hat{g}(b;b) - g(b;b)\right| = O_p\left(\sqrt{\frac{\log n}{nh_n^2}}\right) + O(h_n^m)$$

Consequently, it follows from assumption (2), where $\tilde{G}(b;b) = \hat{G}(b;b) - G(b;b)$, and $\tilde{g}(b;b) = \hat{g}(b;b) - g(b;b)$.

$$\tau_n^{-1}\sup_b |\tilde{G}(b;b)| = o_p(1) \quad \tau_n^{-1}\sup_b |\tilde{g}(b;b)| = o_p(1)$$

Let $I_{\tau il}$ denote $1\left(g(b_{il};b_{il}) \geq \tau_n\right)$. Furthermore, define

- $\tilde{T}=\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\left[b_{il}+\frac{\hat{G}(b_{il};b_{il})}{\hat{g}(b_{il};b_{il})}\right]I_{\tau il}$.
- $T_0=\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\left(b_{il}+\frac{G(b_{il};b_{il})}{g(b_{il};b_{il})}\right)I_{\tau il}$.
- $T_1=\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\frac{\tilde{G}(b_{il};b_{il})}{g(b_{il};b_{il})}I_{\tau il}$.
- $T_2=-\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\frac{G(b_{il};b_{il})}{g^2(b_{il};b_{il})}\tilde{g}\left(b_{il};b_{il}\right)I_{\tau il}$.
- $T_3=\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}I_{\tau il}\left\{-\frac{\tilde{G}_{il}\tilde{g}_{il}}{\hat{g}_{il}g_{il}}+\frac{G_{il}}{g_{il}^2\hat{g}_{il}}\tilde{g}_{il}^2\right\}$.

By construction, $\tilde{T}=T_0+T_1+T_2+T_3$. It can be shown, following the same arguments as in Lavergne and Vuong (1996), that $T_1=o_p(1)$, $T_2=o_p(1)$, $T_3=o_p(1)$. It also follows by dominated convergence theorem and Markov inequality that

$$T_0-\frac{1}{n}\sum_{i=1}^{n}\left[b_i+\frac{G\left(b_i;b_i\right)}{g\left(b_i;b_i\right)}\right]I_{\tau i}=\frac{1}{n}\sum_{i=1}^{n}\left[b_i+\frac{G\left(b_i;b_i\right)}{g\left(b_i;b_i\right)}\right]\left(1-I_{\tau i}\right)=o_p\left(1\right)$$

Application of strong law of large number shows $\frac{1}{n}\sum_{i=1}^{n}\left[b_i+\frac{G(b_i;b_i)}{g(b_i;b_i)}\right]\stackrel{p}{\longrightarrow}Ev\left(x\right)$. The same arguments can be repeated after replacing τ_n by $\tau_n+\epsilon_n$, such that $\tau_n^{-1}\epsilon_n\to 0$ and $\epsilon_n^{-1}\sup_b\left|\tilde{g}\left(b;b\right)\right|=o_p\left(1\right)$, to show that for $\bar{T}=\frac{1}{n}\sum_{i=1}^{n}\left[b_i+\frac{G(b_i;b_i)}{g(b_i;b_i)}\right]1\left(g\left(b_i;b_i\right)\geq\tau_n+\epsilon_n\right)=Ev\left(x\right)+o_p\left(1\right)$. The rest is only to show that

$$\begin{aligned}\hat{T}-\bar{T}=&\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\left[b_{il}+\frac{G\left(b_{il};b_{il}\right)}{g\left(b_{il};b_{il}\right)}\right]*\\&\left\{1\left(\hat{g}\left(b_{il};b_{il}\right)>\tau_n,g\left(b_{il};b_{il}\right)\leq\tau_n+\epsilon_n\right)+1\left(\hat{g}\left(b_{il};b_{il}\right)\leq\tau_n,g\left(b_{il};b_{il}\right)>\tau_n+\epsilon_n\right)\right\}\end{aligned}$$

is $o_p\left(1\right)$. Again this follows the same arguments as in Lavergne and Vuong (1996).

B Asymptotic distribution of the nonparametric test

Assumption 5 *Conditions on h_n and τ_n are changed to $\tau_n^2\sqrt{n}h_n^2\left(\log n\right)^{-1}\to\infty$, $\tau_n^{-2}\sqrt{n}h_n^m\to 0$.*

Lemma 2

$$\begin{aligned}\sqrt{n}\left(T_1'-E\frac{G\left(b_{il},b_{il}\right)}{g\left(b_{il},b_{il}\right)}\right)&=\sqrt{n}\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\frac{1}{g\left(b_{il},b_{il}\right)}\hat{G}\left(b_{il},b_{il}\right)-E\frac{G\left(b_{il},b_{il}\right)}{g\left(b_{il},b_{il}\right)}\\&=\sqrt{n}\frac{1}{nM}\sum_{i=1}^{n}\sum_{l=1}^{M}\left[\frac{G\left(b_{il},b_{il}\right)}{g\left(b_{il},b_{il}\right)}+\frac{1\left(B_{il}\leq b_{il}\right)}{g\left(b_{il},b_{il}\right)}f\left(b_{il}\right)-2E\frac{G\left(b_{il},b_{il}\right)}{g\left(b_{il},b_{il}\right)}\right]+o_p\left(1\right)\end{aligned}$$

In particular, $\left(T_1'-E\frac{G(b_{il},b_{il})}{g(b_{il},b_{il})}\right)=O_p\left(\frac{1}{\sqrt{n}}\right)$.

Proof: Consider $T_1' = \sqrt{n}\binom{n}{2}^{-1}\sum\sum_{i>j} p_n(z_i, z_j)$ for

$$p_n(z_i, z_j) = \frac{1}{2M^2}\sum_{l=1}^{M}\sum_{l'=1}^{M}\left[\frac{1}{g(b_{il}, b_{il})}\frac{1}{h}1(B_{jl'} \le b_{il}) K\left(\frac{b_{jl'} - b_{il}}{h}\right) + \frac{1}{g(b_{jl'}, b_{jl'})}\frac{1}{h}1(B_{il} \le b_{jl'}) K\left(\frac{b_{il} - b_{jl'}}{h}\right)\right]$$

First, to use lemma 3.1 in Powell, Stock, and Stoker (1989), we need to show first $E\left[p_n(z_i, z_j)^2\right] = o(n)$. For that it suffices to show

$$\frac{1}{n}E\frac{1}{g(b_{il}, b_{il})^2}\frac{1}{h_n^2}1(B_{jl'} \le b_{il}) K^2\left(\frac{b_{jl'} - b_{il}}{h_n}\right) \longrightarrow 0$$

By change of variable it is tedious to verify that the term is $O(nh) \to \infty$, since by assumption $\tau^2 nh^2 \to \infty$, $\tau \to 0$, $h_n \to 0$. By lemma 3.1 in Powell, Stock, and Stoker (1989),

$$T_1' = \frac{2}{n}\sum_{i=1}^{n} E(p_n(z_i, z_j)|z_i) - E\frac{1}{g(b_{il}, b_{il})}\frac{1}{h}1(B_{jl'} \le b_{il}) K\left(\frac{b_{jl'} - b_{il}}{h}\right) + o_p\left(\frac{1}{\sqrt{n}}\right)$$

By another change of variable in integration, together with assumption (2), it can be seen that

$$E\frac{1}{g(b_{il}, b_{il})}\frac{1}{h}1(B_{jl'} \le b_{il}) K\left(\frac{b_{jl'} - b_{il}}{h}\right) = \int \frac{G(b_i; b_i)}{g(b_i; b_i)} f(b_i)\, db_i + O(h^m)$$

Note that $h^m = o\left(\frac{1}{\sqrt{n}}\right)$ by assumption (4). It remains to calculate $E(p(z_i, z_j)|z_i)$, consider two terms separately

$$\begin{aligned}
&E\left[\frac{1}{g(b_{il}, b_{il})}\frac{1}{h}1(B_{jl'} \le b_{il}) K\left(\frac{b_{jl'} - b_{il}}{h}\right)|b_{il}\right] \\
=&\frac{1}{g(b_{il}, b_{il})}\int \frac{1}{h}G(b_{il}; b_{jl'}) K\left(\frac{b_{jl'} - b_{il}}{h}\right) db_{jl'} \\
=&\frac{1}{g(b_{il}, b_{il})}\int G(b_{il}, b_{il} + uh) K(u)\, du = \frac{1}{g(b_{il}, b_{il})}(G(b_{il}, b_{il}) + O(h_m))
\end{aligned}$$

$$\begin{aligned}
&E\left[\frac{1}{g(b_{jl'}, b_{jl'})}\frac{1}{h}1(B_{il} \le b_{jl'}) K\left(\frac{b_{il} - b_{jl'}}{h}\right)\bigg| b_{il}, B_{il}\right] \\
=&\int \frac{1}{g(b_{jl'}, b_{jl'})}\frac{1}{h}1(B_{il} \le b_{jl'}) K\left(\frac{b_{il} - b_{jl'}}{h}\right) f(b_{jl'})\, db_{jl'} \\
=&\int \frac{1}{g(b_{il} + uh, b_{il} + uh)}1(B_{il} \le b_{il} + uh) K(u) f(b_{il} + uh)\, du \\
=&\frac{1(B_{il} \le b_{il}) f(b_{il})}{g(b_{il}, b_{il})} + O(h^m)
\end{aligned}$$

The last equality is not immediately obvious due to the nonsmooth indicator function, however, it becomes obvious when comparing the integral $\int_{\frac{B_{il}-b_{il}}{h}}^{\infty} \frac{1}{g(b_{il}+uh, b_{il}+uh)} K(u) f(b_{il} + uh)\, du$ to the result, taking into account the bounded support of b_{il}. Q.E.D.

Unfortunately, the variation of the second term T_2 is by an order of magnitude larger than $\sqrt{n}$ and it is the variation of this term that dominates the asymptotic distribution of our test statistics, regardless of the smoothness of the joint distribution of the bids. It is unfortunately that the asymptotic distribution depends on both the bandwidth and the Kernel function chose. However this is unavoidable due to the fact that we are estimating a two dimensional density but are averaging only along one dimension. This is a similar problem to lemma 5.3 in Newey (1994) in a different settting.

Lemma 3 *Let*

$$V = \left[\int \left(\int K(v) K(u+v)\, dv\right)^2 dv\right]\left[\frac{1}{M}\int \frac{G^2(b;b)}{g^3(b;b)} db\right]$$

then

$$\sqrt{nh}\left(T_2' - E\frac{G(b_i;b_i)}{g(b_i;b_i)}\right) \xrightarrow{d} N(0,V).$$

where $T_2' = \frac{1}{nM}\sum_{i=1}^n \sum_{l=1}^M \frac{G(b_{il};b_{il})}{g^2(b_{il};b_{il})}\tilde{g}(b_{il};b_{il})$.

Proof: By the same U-statistics projection argument as in lemma 1, it can be shown similarly that

$$T_2' = \frac{1}{n}\sum_{i=1}^n E p_n(z_i, z_j | z_i) - ET_2' + o_p\left(\frac{1}{\sqrt{n}}\right) = \frac{2}{n}\sum_{i=1}^n E p_n(z_i, z_j | z_i) - ET_2' + o_p\left(\frac{1}{\sqrt{nh}}\right)$$

for

$$\begin{aligned} p_n(z_i, z_j) = & \frac{1}{M^2}\sum_{l=1}^M \sum_{l'=1}^M \left[\frac{G(b_{il}, b_{il})}{g^2(b_{il}, b_{il})}\frac{1}{h^2}K\left(\frac{b_{jl'} - b_{il}}{h}\right) K\left(\frac{B_{jl'} - b_{il}}{h}\right) + \right. \\ & \left. \frac{G(b_{jl'}, b_{jl'})}{g^2(b_{jl'}, b_{jl'})}\frac{1}{h^2}K\left(\frac{b_{il} - b_{jl'}}{h}\right) K\left(\frac{B_{il} - b_{jl'}}{h}\right)\right] \end{aligned}$$

The two terms of $E(p_n(z_i, z_j)|z_i)$, one has variation of $\frac{1}{\sqrt{n}}$ while the other has variation $\frac{1}{\sqrt{nh}}$ which dominates the asymptotic distribution. The $O_p\left(\frac{1}{\sqrt{n}}\right)$ term is given by

$$\frac{1}{M}\sum_{l=1}^M E\left[\frac{G(b_{il}, b_{il})}{g^2(b_{il}, b_{il})}\frac{1}{h^2}K\left(\frac{b_{jl'} - b_{il}}{h}\right) K\left(\frac{B_{jl'} - b_{il}}{h}\right)|b_{il}\right] = \frac{1}{M}\sum_{l=1}^M \frac{G(b_{il}, b_{il})}{g(b_{il}, b_{il})} + o_p\left(\frac{1}{\sqrt{n}}\right)$$

The dominating term of order $O_p\left(\frac{1}{\sqrt{nh}}\right)$ is given by

$$\begin{aligned} T_{22,i}' = & \frac{1}{M}\sum_{l=1}^M E\left[\frac{G(b_{jl'}, b_{jl'})}{g^2(b_{jl'}, b_{jl'})}\frac{1}{h^2}K\left(\frac{b_{il} - b_{jl'}}{h}\right) K\left(\frac{B_{il} - b_{jl'}}{h}\right)|b_{il}\right] \\ = & \frac{1}{M}\sum_{l=1}^M \frac{1}{h}\int \frac{G(b_{il} + vh; b_{il} + vh)}{g^2(b_{il} + vh; b_{il} + vh)} K(v) K\left(\frac{B_i - b_{il}}{h} + v\right) f(b_{il} + vh)\, dv \end{aligned}$$

From the following calculation of $hVar\left(T'_{22,i}\right)$ it will be clear that the $\sqrt{nh}\frac{1}{n}\sum_{i=1}^{n}\left(T_{22,i}-ET_{22,i}\right)\stackrel{d}{\to}$ $N\left(0,V\right)$, for $V=\lim_{n\to\infty}Var\left(T_{22,i}\right)$. It is also standard that $hVar\left(T'_{22,i}\right)=hE\left(T'^{2}_{22,i}\right)+o\left(1\right)$, therefore it suffices to calculate

$$hE\left(T'_{22,i}\right)^{2}=\frac{1}{Mh}Eq\left(b,B\right)^{2}+\left(1-\frac{1}{M}\right)\frac{1}{h}Eq\left(b,B\right)q\left(b',B'\right)$$

for

$$q\left(b,B\right)=\int\frac{G\left(b+vh;b+vh\right)}{g^{2}\left(b+vh;b+vh\right)}K\left(v\right)K\left(\frac{B-b}{h}+v\right)f\left(b+vh\right)dv$$

It is tedious to verify by pointwise convergence and suitable version of dominated convergence theorem that

$$\begin{aligned}\frac{1}{h}Eq\left(b,B\right)^{2}&=\frac{1}{h}\int\int\left[\int\frac{G\left(b+vh;b+vh\right)}{g^{2}\left(b+vh;b+vh\right)}K\left(v\right)K\left(\frac{B-b}{h}+v\right)f\left(b+vh\right)dv\right]^{2}g\left(B,b\right)dBdb\\&=\int\int\left[\int\frac{G\left(b+vh;b+vh\right)}{g^{2}\left(b+vh;b+vh\right)}K\left(v\right)K\left(u+v\right)f\left(b+vh\right)dv\right]^{2}g\left(b+uh,b\right)dudb\\&\longrightarrow\left(\int\left[\int K\left(v\right)K\left(u+v\right)dv\right]^{2}du\right)\int\frac{G^{2}\left(b;b\right)}{g^{3}\left(b;b\right)}db\end{aligned}$$

Similar calculation shows that $\frac{1}{h}Eq\left(b,B\right)q\left(b',B'\right)=O\left(h\right)\longrightarrow 0$. This is similar to the asymptotic independence of nonparametric estimates at distinct points.

Theorem 4 *Under the stated conditions on the joint distribution of $\left(B_l,b_l,l=1,M\right)$ and under the stated conditions on the bandwidth and trimming parameters, $\sqrt{nh}\left(\hat{T}-Ev\left(x\right)\right)\stackrel{d}{\longrightarrow}N\left(0,V\right)$ for V the variance-covariance matrix given in the lemma above.*

Proof: Define

$$\bar{T}=\frac{1}{n}\sum_{i=1}^{n}\left[b_i+\frac{G\left(b_i;b_i\right)}{g\left(b_i;b_i\right)}\right]1\left(g\left(b_i;b_i\right)\geq\tau_n+\epsilon_n\right)$$

The desired result will follows by combining lemma 1, lemma 2 together with the collection of the following results(to be completed)

$$\begin{aligned}\bar{T}-\hat{T}&=o_p\left(\frac{1}{\sqrt{nh}}\right)\\\bar{T}-\tilde{T}&=o_p\left(\frac{1}{\sqrt{nh}}\right)\\\tilde{T}_0-\frac{1}{n}\sum_{i=1}^{n}\sum_{l=1}^{M}b_{il}+\frac{G\left(b_{il};b_{il}\right)}{g\left(b_{il};b_{il}\right)}&=o_p\left(\frac{1}{\sqrt{nh}}\right)\\\tilde{T}_1-T'_1&=o_p\left(\frac{1}{\sqrt{nh}}\right)\\\tilde{T}_2-T'_2&=o_p\left(\frac{1}{\sqrt{nh}}\right)\end{aligned}$$

Q.E.D.