APPENDIX: Derivation of The Compensation Components

In this appendix we formally derive the compensation components. We consider the labor supply problem of a worker who maximizes a well-behaved twice differentiable utility function defined on annual consumption and leisure, $V(c,l)$. We pick the composite consumption good as the numeraire, set total time equal to 1 and express the minimum (full) expenditure function for an unconstrained worker facing the wage rate $w$ as:

\[(A1) \quad e(w,V^*) = \min_{c,h} \{c + w(1-h) ; V(c,1-h) \geq V^* \}.\]

By construction, when $w = w^*$ the solution to this minimization problem yields the unconstrained equilibrium labor supply of $h = h^* < 1$, consumption level $c = w^* h^*$, and utility $V(c^*,1-h^*) = V^*$.

Next, consider the restricted expenditure function for a worker facing the labor supply constraint $h = \overline{h}$, whose reservation utility level remains at $V^*$:

\[(A2) \quad \bar{e}((1-\overline{h}),w,V^*) = \min_{c} \{c + w(1-\overline{h}) ; V(c,1-\overline{h}) \geq V^* \}.\]

Let $w_1 > w^*$ denote the wage rate which yields utility $V^*$ for $h = \overline{h} < h^*$. The increase in the minimum full expenditure that restores the worker to the reservation utility level is the sum of the change in actual consumption, plus the change in the value of leisure:

\[(A3) \quad \bar{e}((1-\overline{h}),w_1,V^*) - e(w^*,V^*) = w_1 \overline{h} - w^* h^* + w_1 (1-\overline{h}) - w^* (1-h^*) = w_1 - w^*.\]

A second equation that links the quantities of interest can be obtained by defining the virtual (real) wage $\omega(\overline{h},V^*)$ as the relative price at which the unconstrained worker would choose to supply $\overline{h}$ units of labor. Following the

\[1 \text{ See Neary and Roberts (1980: 27) for the conditions this imposes on the preference ordering.}\]
derivation in Neary and Roberts (1980: 30), or Deaton and Muelbauer (1981: 1527), we get:

\[(A4) \quad \bar{\epsilon}((1-\bar{h}), w_1, V^*) = e(\omega(\bar{h}, V^*), V^*) + (w_1 - \omega(\bar{h}, V^*))(1-\bar{h}).\]

A Taylor series expansion of the right hand side of (A4) around the point \(\bar{h} = h^*\) yields:

\[(A5) \quad \bar{\epsilon}((1-\bar{h}), w_1, V^*) \equiv e(w^*, V^*) + (w_1 - w^*)(1-\bar{h}) + \frac{1}{2\eta} \frac{w^*}{h^*}(\bar{h} - h^*)^2,\]

where \(\eta \equiv \frac{\partial h}{\partial w^*}\bigg|_{h=h^*} \frac{w^*}{h^*} > 0\) is the compensated labor supply elasticity for the unconstrained individual evaluated at the equilibrium number of hours.

To obtain (A5) we also relied on the following:

(i) \(w^* = \omega(h^*, V^*);\)

(ii) \(\partial \omega(\bar{h}, V^*)/\partial \bar{h}\bigg|_{\bar{h}=h^*} \equiv 1/(\partial h^*/\partial w^*);\)

(iii) From Shephard’s lemma, \(\partial e(\omega(\bar{h}, V^*), V^*)/\partial \omega = (1-\bar{h});\)

(iv) The ration \(\bar{h}\) and the unconstrained labor supply \(h^*\) do not depend on \(w_1\).

Combining (A3) and (A5), our second order approximation to the first compensation component for anticipated risk is obtained as:

\[(A6) \quad \frac{w_1 - w^*}{w^*} \approx \frac{1}{2\eta} \frac{(\bar{h} - h^*)^2}{h^*}.\]

Given the two argument utility function \(u(c, l)\), where \(c\) is subject to random shocks, Killingsworth (1983: 258) suggests that a measure of relative risk-aversion along the lines of Arrow and Pratt may be constructed as

\[(A7) \quad r = -c\frac{u_{cc}(c, l)}{u_c(c, l)}.\]
We follow Killingsworth and focus on the variation in consumption, ignoring the
induced variation in leisure. In our case consumption is a function of two random
variables, \( y_0 \) and \( y_1 \). We modify (A7) as

\[
(A8) \quad r_e = -c \frac{u(c(y_0),1)}{u(c(y_0),1)}, e = 0,1.
\]

That is, to characterize aversion towards unemployment and employment risk, we rely
on two sets of derivatives with respect to consumption, viewed in turn as a function of
the random \( y_0 \) and \( y_1 \). We evaluate the derivatives in question at the means of
\( y_0 \) and \( y_1 \) to get:

\[
(A9) \quad \rho_0 = c \varphi \left( \frac{V_{11}}{V_1}c + 2 \right),
\]

\[
(A10) \quad \rho_1 = c \varphi \left( \frac{V_{11}}{V_1} (c - w_3) + 2 \right) = \rho_0 - c \left( \frac{V_{11}}{V_1} w_3 \right),
\]

where \( c = w_3 (\pi_1 - \alpha \varphi) \), and \( V_1, V_{11} \) denote the first and second derivatives of \( V(.) \) with
respect to consumption, evaluated at the fully compensated rationed equilibrium
\{\( w_3 \pi_1 - b \varphi, \pi_0 \)\}. It follows that \( \rho_1 > 0 \) and \( \rho_1 > \rho_0 \).

To derive the compensation components associated with unanticipated risk,
we ignore the variation in leisure, rely on a Taylor series approximation to random
utility \( V \left( \frac{w_2 y_1 - b}{y_0 + y_1}, \frac{y_0}{y_0 + y_1} \right) \) at the point \( y_0 = \mu_0 \) and \( y_1 = \mu_1 \), and take expectations
to obtain:

\[
(A11) \quad EV \left( \frac{w_2 y_1 - b}{y_0 + y_1}, \frac{y_0}{y_0 + y_1} \right) \equiv V \left( w_3 \mu_1 - b \varphi, \mu_0 \right) + \frac{1}{2} \varphi^2 V_{11} c \left( \frac{V_{11}}{V_1} c + 2 \right)\sigma_0^2 + \frac{1}{2} \varphi^2 \left( V_{11} (c - w_3) + 2 \right)\sigma_1^2.
\]

The equilibrium condition that relates \( w_3 \) and \( w_2 \) is:
Before exploiting the equality of (A11) and (A12), we express the first term on the right hand side of (A11) as:

\[(A13)\quad V(w_3\pi_1-b\varphi, \pi_0) = V(w_2\pi_1-\alpha w_2 \varphi + (w_3-w_2)(\pi_1-\alpha \varphi), \pi_0) \equiv \overline{V}^+,\]

and note the fact that for small \((w_3-w_2)(\pi_1-\alpha \varphi),\)

\[(A14)\quad (\overline{V}^+ - \overline{V})/(w_3-w_2)(\pi_1-\alpha \varphi) \equiv \overline{V}_1.\]

With this simplification in hand, manipulation yields:

\[(A15)\quad \frac{w_3-w_2}{w^*} \equiv \frac{1}{2} \frac{\rho_0}{w^*} \left( \frac{1}{\sigma_0^2} + \frac{1}{2} \frac{\rho_1}{w^*} \frac{\mu_0 + \alpha}{\mu_1 - \alpha^2} \sigma_1^2 \right).\]