An Equivalence Theorem for the Anonymous Core

Chiaki Hara *

Faculty of Economics and Politics, University of Cambridge, United Kingdom

chiaki.hara@econ.cam.ac.uk

January 2000

Abstract: The purpose of this paper is to establish the equivalence between the anonymous core and the set of the Walrasian equilibrium allocations in an atomless exchange economy. The anonymous (or, synonymously, incentive-compatible or envy-free) core is the set of those consumption allocations that are anonymous and cannot be blocked by any coalition via an allocation satisfying the following dual anonymity conditions. First, every member of the coalition prefers most the consumption bundle given to him among those arising in the blocking allocation. Second, any non-member (a consumer who does not belong to the coalition) does not prefer any consumption bundle arising in the blocking allocation to the bundle he receives at the blocked allocation. We also discuss implications of our equivalence theorem on the second-best insurance problem and the relationship with the literature on the incentive-compatible core under asymmetric information.

---

* My first debt goes to Akira Yamazaki. My interest in the core theory with informational constraints arose in conversations with him. He kindly pointed out to me its relationship with the bargaining set. The conversations and correspondences with Simon Grant, Jun Iritani, Atsushi Kajii, Enrico Minelli, Rohit Rahi, Koji Shimomura, and Lin Zhou were all very helpful. Financial supports from Economic and Social Research Council of the United Kingdom is gratefully acknowledged.

1. Introduction

The purpose of this paper is to establish the equivalence between the anonymous core and the set of the Walrasian equilibrium allocations in an atomless exchange economy. The anonymous (or, synonymously, incentive-compatible or envy-free) core is the set of those consumption allocations that are anonymous and cannot be blocked by any coalition via an allocation satisfying the following dual anonymity conditions. First, every member of the coalition prefers most the consumption bundle given to him among those arising in the blocking allocation. Second, any non-member (a consumer who does not belong to the coalition) does not prefer any consumption bundle arising in the blocking allocation to the bundle he receives at the blocked allocation.

Since this notion of a blocking allocation is more stringent than in the standard definition of the core, the anonymous core includes the standard core; and hence the equivalence theorem of this paper implies the standard core equivalence theorem.

The proof method of our equivalence theorem relies much on that of the equivalence theorem for the bargaining set by Mas-Colell (1989), in that we also make full use of his "Walrasian" blocking. The interests in the anonymous core and in the bargaining set, however, are quite different. The former stems from the question of whether the presence of asymmetric information impedes the power of the Edgeworth bargaining process to bring about Walrasian equilibrium allocations, while the latter stems from the question of whether the possibility of subsequent blockings weakens the credibility of a blocking allocation so much so that some allocations other than Walrasian equilibrium allocations are left unblocked even in an atomless economy.

Indeed, after presenting and proving our equivalence theorem, we discuss some implications to a problem of optimal insurance provision. We then see that if the social planner is faced with the threat of coalitional deviations, then he cannot provide any insurance at all. To be more specific, the Walrasian equilibrium allocations at the ex post stage (when losses have already been incurred) are the only implementable ones. This is true even if every coalition is subject to the same informational constraints as the planner. This may be true even when he could attain the first-best allocation were he to need to be concerned only with individual deviations.

There is also a growing literature on the core with asymmetric information, which model the consumers' type spaces and mechanism much more explicitly.

We shall compare our result with them in the final section.

2. The Model

2.1. An Economy and Walrasian Allocations

Let L be a positive integer. Let $\mathcal{P}$ be the set of all complete, transitive, continuous, and strongly monotone preferences defined on the nonnegative orthant R^L_+. The set $\mathcal{P}$ is endowed with the closed convergence topology induced from the set of all closed subsets of $R^L_+ \times R^L_+$. It is also endowed with the Borel σ-field induced from this topology. An economy is defined to consist of an *atomless* probability measure space $(A, \mathcal{A}, \mu)$, an integrable mapping $e : A \to R^L_{++}$, and a measurable mapping $\succsim : A \to \mathcal{P}$. The set A is the set of (the names of) consumers, $\mathcal{A}$ is the set of coalitions, μ determines a population distribution over A, L is the number of goods, R^L_+ is the consumption set for every consumer, $e(a) \in R^L_+$ is the initial endowment of consumer a, and $\succsim (a) \in \mathcal{P}$ is his preference relation on the consumption set. In the rest of this paper, we write $\succsim_a$ for $\succsim (a)$. For each $C \in \mathcal{A}$, any integrable mapping $f : C \to R^L_+$ satisfying $\int_C f d\mu = \int_C e d\mu$ can be interpreted as a resource-feasible allocation among the consumers in C. Denote by $F(C)$ the set of all such mappings.

The following definitions of a Walrasian allocation, a blocking, and the core are standard.

Definition 2.1. Let $f \in F(A)$. We say that f is *Walrasian* if there exists a $p \in R^L_{++}$ such that, for almost every $a \in A$, if $x \in R^L_+$ and $p \cdot x \leq p \cdot e(a)$, then $f(a) \succsim_a x$.

Definition 2.2. Let $f \in F(A)$, $C \in \mathcal{A}$, and $g \in F(C)$. We say that (C, g) is a *blocking* to f if $g(a) \succsim_a f(a)$ for almost every $a \in C$ and there exists a $B \in \mathcal{A}$ such that $B \subseteq C$, $\mu(C) > 0$, and $g(a) \succ (a) f(a)$ for almost every $a \in B$.

Definition 2.3. The *core* is the set of all allocations in $F(A)$ to which there is no anonymous blocking.

The classical core equivalence theorem states that the core coincides with the set of all Walrasian allocations.

2.2. Anonymous Allocations and Blockings

We first give the notion of anonymity for an allocation defined over the entire set A of consumers.

Definition 2.4. We say that an $f \in F(A)$ is *anonymous* if there exists a $B \in \mathcal{A}$ such that $\mu(B) = 1$ and, for every $a \in B$ and $b \in B$, if $e(a) + (f(b) - e(b)) \in R_+^L$, then $f(a) \succsim_a e(a) + (f(b) - e(b))$.

The above definition of incentive-compatibility refers to the net demands $f(a) - e(a)$. To be more specific, note that each consumer $a \in A$ receives the net demand $f(a) - e(a)$ (from, say, the markets or the social planner) to consume $f(a)$. If he pretends to be consumer $b \in A$, then he receives the net demand $f(b) - e(b)$, which is to be added to his initial endowment $e(a)$. If $e(a) + (f(b) - e(b)) \notin R_+^L$, then consumer a cannot survive with the consumption arising from this pretense; so we can imagine that he would not choose to pretend to be consumer b. If, however, $e(a) + (f(b) - e(b)) \in R_+^L$, then he can survive with this consumption vector and, in order for f to be anonymous, we require $e(a) + (f(b) - e(b))$ to be at most as good as $f(a)$ relative to his preference $\succsim_a$. Note that every Walrasian allocation is anonymous.

We next give the definition of anonymity for a blocking.

Definition 2.5. Let $f \in F(A)$ be anonymous. We say that a blocking (C, g) to f is *anonymous* if there exist a $B \in \mathcal{A}$ and a $D \in \mathcal{A}$ such that $B \subseteq C$, $D \subseteq A \backslash C$, $\mu(B) = \mu(C)$, $\mu(D) = \mu(A \backslash C)$, and:

1. For every $a \in B$ and every $b \in D$, if $e(a) + (g(b) - e(b)) \in R_+^L$, then $g(a) \succsim_a e(a) + (g(b) - e(b))$.
2. For almost every $a \in D$ and every $b \in B$, if $e(a) + (g(b) - e(b)) \in R_+^L$, then $f(a) \succsim_a e(a) + (g(b) - e(b))$.

The two conditions embody the idea of dual anonymity when a blocking coalition C is formed and the associated allocation g is implemented in ignorance of the identity of consumers. The first condition is the internal anonymity condition, in the sense that the allocation g is anonymous among the members of C. The second condition is the anonymity of the membership of the coalition C. It requires that no consumer outside the coalition C be able to receive a preferable consumption vector by pretending to be any member of the coalition.

Given this notion of an anonymous blocking, the definition of the anonymous core is straightforward.

Definition 2.6. The *anonymous core* is the set of all anonymous allocations in $F(A)$ to which there is no anonymous blocking.

The anonymous core includes the standard core.

3. The Result and its Proof

Theorem 3.1. *The anonymous core coincides with the set of all Walrasian allocations.*

All Walrasian allocations belong to the anonymous core because they together coincides with the core. The difficulty in proving the theorem therefore lies in establishing that if an allocation belongs to the anonymous core, then it must be Walrasian. This amounts to finding an anonymous blocking for each anonymous, but not Walrasian, allocation. It turns out that Mas-Colell's "Walrasian blockings," used to prove his equivalence theorem for the bargaining set, do the job for the anonymous core as well.

We start with giving a formal definition of a Walrasian blocking.

Definition 3.2. Let $f \in F(A)$ and (C, g) be a blocking to f. We say that (C, g) is *Walrasian* if there exists a $p \in R^L_{++}$ such that:

1. For almost every $a \in C$, $p \cdot g(a) \leq p \cdot e(a)$ and if $x \in R^L_+$ and $p \cdot x \leq p \cdot e(a)$, then $g(a) \succsim_a x$.
2. For almost every $a \in A \backslash C$ if $x \in R^L_+$ and $p \cdot x \leq p \cdot e(a)$, then $f(a) \succsim_a x$.

A Walrasian blocking can be considered as a "price-induced" blocking. The first condition states that any member a of the blocking coalition C cannot get worse off by rejecting the consumption vector $f(a)$ at the blocked allocation f and participate in market transactions under the price vector p; and the consumption vector he receives at the blocking allocation g is a most preferable one in his budget set $\{x \in R^L_+ | \ p \cdot x \leq p \cdot e(a)\}$. By strong monotonicity and $\int_C f d\mu = \int_C e d\mu$, it is possible to prove that the second part of this condition alone, that is, if $x \in R^L_+$ and $p \cdot x \leq p \cdot e(a)$, then $g(a) \succsim_a x$ for almost every $a \in C$, implies that $p \cdot g(a) = p \cdot e(a)$, that is, the weak inequality of the first part is satisfied with equality.

The second condition states that any non-member $a \in A \backslash C$ cannot get better off by rejecting the consumption vector $f(a)$ at the blocked allocation f and participate in market transactions under the price vector p; and he would thus choose to retain the consumption vector $f(a)$ at the blocked allocation f.

The following lemma by Mas-Colell is the crucial step for the equivalence theorem for the bargaining set as well as for our anonymous core.

Lemma 3.3. *Let $f \in F(A)$ and suppose that f is not a Walrasian allocation, then there exists a Walrasian blocking to f.*

It is worthwhile to recall that the proof of the above lemma involves a fixed-point argument to show that the resource feasibility constraint $\int_C f d\mu = \int_C e d\mu$ is satisfied with an appropriately chosen price vector p. Given this lemma, Theorem 3.1 follows immediately from the following lemma.

Lemma 3.4. *Let $f \in F(A)$ and suppose that f is anonymous. If (C, g) is a Walrasian blocking to f, then it is also anonymous.*

Proof of Lemma 3.4. Let $p \in R^L_{++}$ be a price vector appearing in the definition of a Walrasian blocking (Definition 3.2) together with (C, g). Then, $p\cdot(g(b)-e(b)) \leq 0$ for almost every $b \in C$ and hence $p\cdot(e(a)+(g(b)-e(b))) \leq p\cdot e(a)$ for almost every $a \in A$ as well. By Condition 1 of Definition 3.2, therefore, if $a \in C$ and $e(a) + (g(b) - e(b)) \in R^L_+$, then $g(a) \succsim_a g(a) + (g(b) - e(b))$. This shows that (C, g) satisfies Condition 1 of Definition 2.5. On the other hand, if $a \in A \backslash C$ and $e(a) + (g(b) - e(b)) \in R^L_+$, then Condition 2 of Definition 3.2 implies that $f(a) \succsim_a e(a) + (g(b) - e(b))$. Condition 2 of Definition 2.5 is thus satisfied by (C, g). ■

4. Implications to an Optimal Insurance Problem

In this section, we draw implications of our equivalence theorem to an optimal insurance provision by the social planner who does not know the consumers' identities. The central message of this section is that the threat of coalitional blockings makes it impossible for the social planner to insure consumers against their idiosyncratic risks, even if the coalitions are subject to the same informational constraint as the social planner.

4.1. Description of the Insurance Problem

First, we impose the following assumptions on our atomless economy. Let $u : R^L_+ \to R$ be continuous, strongly monotone, and strictly concave and assume that $\succsim_a$ is represented by u for every $a \in A$. So all consumers have the same preference. We also assume that there is a measurable partition $\{A_1, \cdots, A_J\}$ such that $\mu(A_j) > 0$ for every $j \in \{1, \cdots, J\}$ and that $e(a) = e(b)$ for every $j \in \{1, \cdots, J\}$, $a \in A_j$, and $b \in A_j$. Denote this initial endowment vector by e_j and write $\mu_j = \mu(A_j)$, then this assumption means that a group of consumers with population μ_j have the common initial endowment vector e_j.

Then, we imagine that there is a stage that is not modelled in, and precedes, our model of the atomless economy. There is no consumption taking place at this "ex ante" stage. The consumers know that their preference relation will be $\succsim_a$ at the next, "ex post," stage, but they do not know what their initial endowments will be. They however know that they will be endowed with e_j with probability μ_j. The uncertainty faced with a consumer at the ex ante stage is thus with regards to his initial endowments; for example, if $e_j > e_k$, then he is richer when he is of type j and when he is of type k. Although the preference relation $\succsim_a$ is deterministic, the uncertainty in initial endowments gives rise to the uncertainty in the preference over the *net* demand, which will later become relevant for the anonymity constraints.

We assume that the shocks that determine individual consumers' initial endowment vectors are idiosyncratic. The well-known technical problems aside, this assumption implies that there is always a group of consumers with population μ_j who have the common initial endowment e_j, regardless of realizations of individual consumers' idiosyncratic risks. Hence the mean (aggregate) endowment vector of the economy, $\sum_{j=1}^{J} \mu_j e_j$, and the Walrasian equilibrium price vectors are deterministic. If a consumer consumes a consumption vector $x_j \in R_+^L$ whenever his initial endowment vector has turned out to be e_j, then the "expected" utility that he has at the ex ante stage is equal to $\sum_{j=1}^{J} \mu_j u(x_j)$. The strict concavity of u represents the consumers's risk aversion, as well as his preference over the vectors of L goods.

Finally, we can introduce a social planner in this two-stage setting. We assume that the planner does not know which consumer has which initial endowment vector, and yet distributes the aggregate endowment $\sum_{j=1}^{J} \mu_j e_j$ so that each consumer with endowment e_j receives a consumption vector x_j. Since the realized initial endowments are private information, he has to implement the allocation $(x_1, \cdots, x_J)$ in the incentive compatible, or, equivalently, anonymous manner. His objective is to maximize every consumer's expected utility $\sum_{j=1}^{J} \mu_j u(x_j)$ at the ex ante stage under the resource and incentive compatibility constraint. We shall now turn to discuss what kind of notion of incentive compatibility constraint is most appropriate.

4.2. Incentive Compatibility Conditions and an Application of the Equivalence Theorem

In our insurance problem, a typical incentive compatibility condition for the allocation $(x_1, \cdots, x_J)$, where every consumer with the endowment vector e_j receives the consumption vector x_j, would be the following:

> For every j and k, if $e_j + (x_k - e_k) \in R^L_+$, then $u(x_j) \geq u(e_j + (x_k - e_k))$.

That is, no consumer can enjoy a preferable consumption by pretending to have an initial endowment vector different from his own. Note that he does not need to commit himself to receiving any particular consumption bundle at the ex ante stage, so that his pretence can be contingent on the realized endowment vector. If we rename the consumers (and put some technical issues aside again), by using the notation up to the previous section, we can formulate the social planner's welfare maximization problem as follows:

$$
\begin{aligned}
\max_{f} \quad & \textstyle\int_A u(f(\cdot))d\mu \\
\text{s.t.} \quad & f \in F(A); \\
& \text{for almost every } a \in A,\ u(f(a)) \geq u(e(a)); \\
& \text{there exists a } B \in \mathcal{A} \text{ such that } \mu(B) = 1 \text{ and, for every } a \in B \text{ and } b \in B, \\
& \text{if } e(a) + (f(b) - e(b)) \in R^L_+, \text{ then } f(a) \succsim_a e(a) + (f(b) - e(b)).
\end{aligned}
\tag{4.1}
$$

The first constraint is the standard resource constraint. The second one is the individual rationality constraint, saying that no consumer a gets worse off by choosing the consumption vector $f(a)$ over his own endowment vector $e(a)$. The third one is the incentive compatibility constraint, which reformulates the previous incentive compatibility constraint and coincides with our notion of anonymity (Definition 2.4). Note that we allow for the possibility that two distinct consumers with the same initial endowment vector (namely, in the same A_j) are given different consumptions vectors. The incentive compatibility constraint, however, implies that $u(f(a)) = u(f(b))$ for almost all $a \in A_j$ and $b \in A_j$.

Is this incentive compatibility condition sufficiently strong to guarantee the absence of plausible objections to f from the consumers' side? We shall argue that it is not. This can be best done by means of an example.

Example 4.1. *Let $L = 2$. Let A be the half open unit interval $[0, 1[$, $\mathcal{A}$ to be the set of Lebesgue measurable subsets of $[0, 1[$, and μ to the Lebesgue measure. Let $A_1 = [0, 1/3[$, $A_2 = [1/3, 2/3[$, and $A_3 = [2/3, 1[$. Hence $\mu_1 = \mu_2 = \mu_3 = 1$.*

Define $e : A \to R$ by

$$\begin{aligned} e(a) &= (9, 1) \text{ for every } a \in A_1, \\ e(a) &= (1, 9) \text{ for every } a \in A_2, \\ e(a) &= (2, 2) \text{ for every } a \in A_3. \end{aligned}$$

Hence $e_1 = (9, 1)$, $e_2 = (1, 9)$, and $e_3 = (2, 2)$.

To define the common utility function, for each $r > 0$, we first let $v^r : R_+ \to R$ be a continuous, strongly monotone, and strictly concave function such that $v^r(t) = -t^{-r}$ for every $t > 1/r$. Then define $u^r : R^2_+ \to R$ by $u^r(x) = v^r(x^1) + v^r(x^2)$, where $x = (x^1, x^2) \in R^2_+$

We required $v^r(t) = -t^{-r}$ only for $t > r^{-1}$ so that v^r can take a finite value at $t = 0$. The existence of such a v^r can be easily established. Note that u^r converges to the Leontief utility function $u^\infty(x) = \min\{x^1, x^2\}$ in the following sense: if $x \in R^2_+$, $y \in R^2_+$, and $u^\infty(x) > u^\infty(y)$, then there exists a $\overline{r} > 0$ such that $u^r(x) > u^r(y)$ for every $r > \overline{r}$.

In Example 4.1, the social planner can even attain the first best allocation:

Proposition 4.2. *Suppose that $r > 0$ is sufficiently large.*

1. Define $f : A \to R^2_{++}$ by $f(a) = (4, 4)$ for every $a \in A$, then f is the solutions to the first-best allocation problem:

$$\begin{array}{ll} \underset{f}{\text{Max}} & \int_A u(f(\cdot))d\mu \\ s.t. & f \in F(A). \end{array} \tag{4.2}$$

with $u = u^r$. Moreover, f satisfies the individual rationality and the incentive compatibility constraints in (4.1). Hence f is the solution to (4.1) with $u = u^r$.

2. Let $C = A_1 \cup A_2$ and define $g : C \to R^2_+$ by

$$\begin{aligned} g(a) &= (5, 5) \text{ for every } a \in A_1, \\ g(a) &= (5, 5) \text{ for every } a \in A_2, \end{aligned}$$

then (C, g) is a Walrasian, and hence anonymous, blocking to f.

3. Define $h : A \to R^2_{++}$ by

$$
\begin{aligned}
h(a) &= (5,5) \text{ for every } a \in A_1, \\
h(a) &= (5,5) \text{ for every } a \in A_2, \\
h(a) &= (2,2) \text{ for every } a \in A_3.
\end{aligned}
$$

then h is a Walrasian allocation with a price vector $p = (1,1)$.

Proof of Proposition 4.2. It follows from the strict concavity of u that f is the solution to the first-best allocation problem (4.2). To prove that f satisfies the individual rationality and the incentive compatibility constraints, note that

$$
\begin{aligned}
f(a) - e(a) &= (-5,3) \text{ for every } a \in A_1, \\
f(a) - e(a) &= (3,-5) \text{ for every } a \in A_2, \\
f(a) - e(a) &= (2,2) \text{ for every } a \in A_3.
\end{aligned}
$$

It is then easy to show that f satisfies all those constraints with respect to u^∞ with strict inequalities. Hence it does so with respect to u^r with every sufficiently large r.

2. Since

$$
\begin{aligned}
g(a) - e(a) &= (-4,4) \text{ for every } a \in A_1, \\
g(a) - e(a) &= (4,-4) \text{ for every } a \in A_2,
\end{aligned}
$$

it is easy to show that g is a Walrasian blocking with a price vector $p = (1,1)$.

3. This follows from the definition of u^r. ■

Part 1 of Proposition 4.2 means that the first-best, complete-insurance allocation can be implemented by the planner if, besides the incentive compatibility constraint, he only has to be concerned with is individual deviations. If, however, he also has to face with coalitional deviations, then he can no longer attain the first-best allocation f, as claimed by Part 2. This should be taken as quite intuitive. Since the initial endowment vectors e_1 and e_2 are very biased between the two goods, the consumers endowed with e_1 and e_2 find it beneficial to trade between them by supplying a large amount of one good and demanding the other. Once such beneficial trades are possible, the utility level of the consumers endowed with, and consuming, e_3 becomes relatively low. The planner would then find it desirable to transfer some of the goods from those with e_1 and e_2 to those e_3.

Those with e_3 cannot deliver the large supply required in the net demand vector $f(a) - e(a)$ for $a \in A_1 \cup A_2$. Those with e_1 or e_2 do not wish to pretend to be with e_3, because he needs a large amount of one of the two good to increase utility. Hence f is incentive-compatible. But the consumers with e_1 and e_2 would like to form a coalition without involving those with e_3, so that they can now consume the net demand $(2, 2)$ given to e_3. Those with e_1 can receive the net demand $(-4, 4)$ and those with e_2 can receive $(-4, 4)$. What is important is that this can be implemented without identifying individual consumers' endowments. This can be seen by the same argument as for the incentive-compatibility of f.

More generally, if every coalition in $\mathcal{A}$ were able to be formed without reference to the allocation to be blocked and implement any resource-feasible allocation among its members as envisaged in the standard notion of a blocking (Definition 2.2), then, not surprisingly, the standard core equivalence theorem would tell us that the only implementable allocations are the Walrasian ones. This is, however, not a significant result in our context because each coalition is assumed to have a much better information than the planner when it comes to selecting members and implementing a blocking allocation. In other words, no coalition is faced with the incentive-compatibility constraint in the planner's problem. The relevant question here is which allocations are implementable and optimal when every coalition is subject to the same informational constraints as the planner.

Our notion of an anonymous blocking (Definition 2.5) captures these informational constraints. Since any coalition (or any member of any coalition) does not know any consumers' initial endowments, it can choose its members only in the self-selecting manner (Condition 2 of Definition 2.5) and implement an allocation that is anonymous among its members (Condition 1 of Definition 2.5). The planner's problem taking this coalitional rationality can be formulated as

$$\begin{array}{ll} \underset{f}{\text{Max}} & \int_A u(f(\cdot))d\mu \\ \text{s. t.} & f \in F(A), \\ & \text{there is no anonymous blocking to } f. \end{array} \tag{4.3}$$

Our equivalence theorem (Theorem 3.1) states that, even under these informational constraints, the Walrasian allocations are still the only ones that admit no coalitional deviation. Given that there are typically only finitely many Walrasian allocations, the task of finding a solution to the above problem boils down to merely comparing the finitely many values of the objective function $\int_A u(f(\cdot))d\mu$ for different Walrasian allocations f. Like the allocation h in Part 3 of Example 4.1, a Walrasian allocation typically involves inequality among the utility levels at the ex post stage, and hence randomness in utilities from the ex ante viewpoint. We can therefore conclude from this exercise that the threat of ex post coalition deviations eliminate insurance opportunities.

5. Relationship with the Core under Asymmetric Information

There is now a growing literature on the core with asymmetric information. They investigate what kind of allocations will or will not survive blockings when coalitions must block via allocations that are incentive compatible among the members. The underlying idea is that the individual members' private information is not shared within any coalition. The set of those allocations that survive such blockings is termed as the "incentive compatible core." We have opted for the "anonymous core," because we have not modelled the type space of individual consumers as others did. We also note that the anonymity requirement for the membership (Part 2 of Definition 2.5) of a coalition does not seem to have been explicitly dealt with in the literature. Mas-Colell (1989, Remark 8) already pointed out the significant difference between this anonymity requirement and the incentive compatibility for the blocking allocation within the coalition.

Forges, Heifetz, and Minelli (1999) used a competitive equilibrium concept of the type of Prescott and Townsend (1984a,b) and established a limit theorem for the incentive compatible core for a replica sequence of economies when the private information is of independent, private value. The nature of private information in this paper is of this type as well, because it only reveals each consumer's own endowment and preference. Their limit theorem, however, is concerned with an "ex ante" competitive equilibrium, where consumers can transfer income across different states corresponding to different realizations of uncertainty. Theorem 3.1 of this paper, on the other hand, establishes the equivalence with ex post equilibria, that is, those equilibria which can be reached when consumers are engaged in market transactions only after they know their own endowments and preferences. Note that there is no financial tool that allows them to insure themselves against any risks.

Vohra (1999) showed that the incentive compatible core, according to his definition, may be empty if the allocation must be deterministic; later Forges and Minelli (1999) showed that allowing random allocations may retrieve non-emptiness. Vohra also clarified the relationship with the preceding results, for the details of which the reader is referred to his paper. Yet, we ought to note that our equivalence theorem may not hold if we replace the definition of a blocking (Definition 2.2) by a stronger requirement that $g(a) \succ_a f(a)$ for almost every $a \in C$. This was already pointed out by Mas-Colell (1989) for the case of the bargaining set. Moreover, in the insurance model of Section 4 of this paper, when there are some consumers $a \in A_j$ in a coalition C that blocks an allocation $f \in F(A)$ via $g \in F(C)$ who are indifferent between joining the coalition and staying outside it (that is, $f(a) \sim_a g(a)$), it is quite possible that only a subset of A_j indeed joins C. This can be regarded as a randomization in allocations between $f(a)$ and $g(a)$. Hence our result appears to be consistent with Forges and Minelli (1999).

References

[1] Forges, Francoise, Aviad Heifetz, and Enrico Minelli, 1999, Incentive compatible core and competitive equilibria in differential information economies, Thema Discussion Paper Series 99-06, University of Paris X.

[2] Forges, Francoise, and Enrico Minelli, 1999, A note of the incentive compatible core, Thema Discussion Paper Series 99-02, University of Paris X.

[3] Mas-Colell, Andreu, 1989, An equivalence theorem for a bargaining set, Journal of Mathematical Economics, 18, 129-139.

[4] Prescott, Edward, and Robert Townsend, 1984a, Pareto optima and competitive equilibria with adverse selection and moral hazard," Econometrica 52, 21-45.

[5] Prescott, Edward, and Robert Townsend, 1984b, General competitve analysis in an economy with private information, International Economic Review 25, 1-20.

[6] Vohra, Rajiv, 1999, Incomplete information, incentive compatibility, and the core, Journal of Economic Theory 86, 123-147.