Multiple Traits in the Marriage Market: Does Diversity Win Sometimes?

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Abstract

A critical part of forming a long-term partnership, be it marriage, employment, co-authorship or some other commitment, is having to trade off among the various traits of one's potential partners. The nature of this trade-off depends both on the type of commitment being considered, as well as on the person making the commitment.

In this paper I focus on the impact that this trade-off has on the marriage market equilibrium. Agents differ from one another along more than one trait, and preferences over traits is not homogenous. This implies that all agents do not agree completely on the desirability of potential partners. I characterize both the core allocation as well as the equilibrium that results when there are costly search frictions.

The main finding is that in the presence of frictions, an individual who is moderately appealing to diverse tastes among the opposite sex may make a better match than someone who is found to be stunning by one group, but leaves the others cold. Assortative matching patterns emerge along more than one dimension, with the result that there is positive correlation along more than one trait in matched individuals.

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1 Introduction

A critical part of forming a long-term partnership, be it marriage, employment, co-authorship or some other commitment, is having to trade o¤ among the various traits of one's potential partners. Any person is a bundle of characteristics, and forming binding ties implies accepting the entire bundle; one cannot pick and choose among the characteristics. The nature of the trade-o¤ depends both on the type of commitment being considered, as well as on the person making the commitment.

In the labor market context, clearly, …rms in di¤erent sectors value worker skills di¤erently. When it comes to marriage, some of us are attracted by intelligence, others by wealth or power, and yet others …nd a sense of humor absolutely necessary. Are these di¤erences in preferences completely random? There is evidence that this is not so. Vandenberg (1972) talks about systematic di¤erences in the relative weights that individuals give to various personality factors and physical attractiveness in potential mates. He quotes studies that indicate that choices of dates and eventually of mates may be governed by a di¤erent combination of values for various individuals.

The existing literature on marriage does not address this issue. The usual assumption made is that all men agree upon the ranking of women as marriage partners, and all women have identical preferences over men as mates. Becker(1973) speci…es the production function for households (matched couples), and uses the properties of this function to describe preferences over mates and outcomes in the marriage market. All households have the same production function, with the result that agents have homogenous preferences. Under certain assumptions perfect assortative mating results, with the most desirable man matching with the most desirable woman, and so on down the line.

Becker's model has been extended to an environment that involves search, when one meets potential partners sequentially and utility in the future is not as valuable as utility today (see Burdett-Coles(1997), Shimer-Smith(2000), Eeckhout(2000), Bloch-Ryder(2000)). Once again, agents are assumed to be in perfect agreement on the ranking of potential partners. When utility is non-transferable, the steady state equilibrium partitions the two sides into classes according to their desirability. Women who belong to any particular class propose only to men in the same class or higher, men do the same. In equilibrium only men and women from the same class marry. Expected utility from a potential match is nondecreasing in attractiveness. The more attractive a woman is, the more likely she is to match with a highly desirable man. Matching is positive assortative even in the presence of search frictions. (Stronger assumptions than Becker's are required, see Shimer-Smith(2000)). None of these models deal with heterogenous preferences[1].

I attempt to capture the impact of systematic di¤erences in the relative weights that individuals give to various personality factors in a very simple framework. I allow agents on one side of the market to have heterogenous preferences over potential mates and model two di¤erent scenarios; in the …rst both men and women care only about a single trait in the opposite sex, but the men are split into two groups with diametrically opposed preferences. For instance, one group of men might prefer tall women, while the other prefers short; one group might prefer dark skinned women while the other prefers light skinned; one group might like blue eyes, while the other prefers dark grey.

In the second scenario, I allow women to di¤er continuously along two dimensions (or traits) and men along one. Once again men are divided into two groups by their preferences, one group of men cares more about trait 1 than about trait 2, while the second group is more concerned about trait 2 than about 1. The two groups of men therefore rank women di¤erently as potential partners. In both scenarios women have homogenous preferences, they rank men identically.

Admittedly these are special cases, but they have the advantage of providing a tractable framework and capture the essence of the trading decision. The results remain robust when the assumptions on preferences are relaxed in certain directions.

Singles of opposite sexes meet randomly. Upon meeting, each individual is perfectly informed about all the characteristics of the other individual and decides whether to propose or not. If both propose then the match is consummated. For simplicity, it is assumed that these two individuals exit the marriage market forever immediately after consummation of the match, and are replaced by two single individuals with identical characteristics (the couple has two children with the boy looking exactly like his father and the girl her mother.) This assumption ensures that the distribution of singles remains stationary through time. All agents discount the future, and prefer

[1] Burdett-Wright(1998) go to the other extreme, and assume that preferences are completely random. This would predict that on average every man and woman would receive the same number of proposals, something that we do not observe in reality.

to be matched, to remaining single. Utility is non-transferable.

The steady state equilibrium in this model also partitions the two sides according to their desirability. Since all men are ranked identically, more desirable men have a higher expected utility. The same cannot be said about women when men have diametrically opposed preferences, as assumed in the first scenario. For example, suppose women vary in their height and suppose that one group of men prefers to match with tall women while the other group prefers short women. Then in the presence of a lot of frictions, it is so difficult to meet potential partners that agents are not too choosy. Women of medium height become acceptable to both groups of men and get proposals more frequently than do very tall or very short women. As a result, their expected utility is greater than that of either the tallest or the shortest woman. As frictions go down, and it becomes easier to meet people, tall and short women start doing better than those of medium height.

Heterogenous preferences thus impacts the relative performance of women in a significant way. Those women that have the best chances of matching when there are a lot of frictions, become the very women who do worst as frictions become very small.

In the second case when women vary along more than one trait (say height and wealth), and when one group of men cares about trait 1 (prefer tall women) and the second group cares about trait 2 (prefer wealthier women), expected utility of women is non-decreasing in both traits. That is, if a woman is taller and wealthier than another, her expected utility will be at least as high as that of the other. However, the model predicts that a very tall woman could do worse than a woman of medium height with slightly more wealth, in the presence of a lot of frictions. In this sense, mediocrity in both traits sometimes wins over quality in one.

Another interesting feature of the steady state equilibrium is that self-selection sometimes emerges. Although women care only about the endowment of their partner, and not about their preferences, there sometimes exist certain classes of tall women who match only with men who prefer tall women, and classes of wealthy women who match only with men who appreciate wealth.

The paper is organized as follows: Section 2 outlines the framework; Section 3 describes the steady state equilibrium in the presence of costly search when men have diametrically opposed preferences; Section 4 characterizes steady state allocation in the presence of frictions when women vary continuously along two traits, and men have heterogenous preferences over these dimensions; Section 5 extends the analysis in Section 4 to describe the core allocation in the static assignment game; and Section 6 summarizes and discusses possible extensions. The appendix contains certain proofs.

2 Framework

There are two sides in the matching game, men and women. Women are characterized by two traits, any two from a number of traits such as height, color of eyes, color of hair, beauty, intelligence, wealth, sense of humor, etc., etc..., that matter in the matching game. In the rest of this section I will let trait 1 be the height of the woman and trait 2 be the wealth (endowment) of the woman.[2]

Let $W = [0; 1]$ £ $(0; 1]$ be the set of all women[3]. Any particular woman w is represented by the pair $(x_w^1; x_w^2)$, where x_w^1 2 $[0; 1]$ is her height, and x_w^2 2 $(0; 1]$ her endowment of wealth. The closer x_w^1 is to 1; the taller is the woman, and the closer x_w^2 is to 1; the wealthier she is.

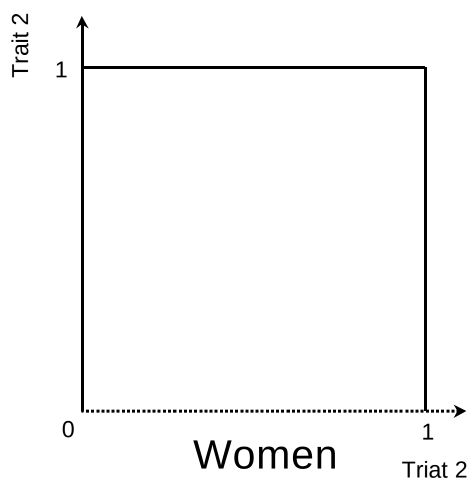

Figure 1

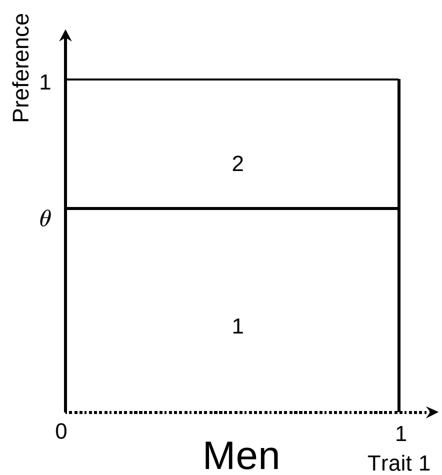

Figure 2

Let $M = [0; 1]$ £ $(0; 1]$ be the set of men. Any particular man m is characterized by the pair $(x_m; i)$; where x_m represents the man's distinguishing

[2] I pick speci…c traits simply for ease of exposition, and this is not to be interpreted as a statement of values.

[3] The reason why W is not the closed set $[0; 1]^2$ will become clear in the section on the core of the assignment game. It is a technical assumption necessary in that section.

trait in the matching game and i represents his preference over the traits of women. For ease of exposition I assume that x_m stands for his wealth. As among women, the closer x_m is to 1, the more wealthy the man.

All women prefer wealthy men. Men exhibit heterogeneity in their preferences over women. Let $M_1 = \{m_i : i \in (0, \mu]\}$ and $M_2 = \{m_i : i \in (\mu, 1]\}$ represent two different sets of men. All the men in M_1 have identical preferences over potential partners, and all the men in M_2 also agree upon how they would rank women as potential partners[4]. The preferences of the two groups are different, however. I illustrate two such differences in Section 3 and 4 of this paper.

In Section 3, I assume that height is the only trait that matters to all men and they are indifferent to the woman's wealth. Men belonging to M_1 prefer tall women, while men in M_2 prefer short women. This section examines the nature of equilibrium when the two groups of men have diametrically opposite preferences over the same trait.

In Section 4, I assume that men in M_1 still continue to care only about the height of the woman, and the taller she is the better. Men in M_2 however prefer wealthy women and are indifferent to her height. This section examines what equilibrium looks like when different groups of men do not value diverse traits in a woman identically. An implicit assumption in both sections is that the preferences of men are independent of their level of wealth.

Instantaneous utility in any period to an individual who remains single is 0. Upon matching, both the agents in the match obtain a strictly positive instantaneous utility that depends on their partner's traits, after which they immediately exit the market and are replaced by clones, one single woman and one single man, with exactly the same traits as the pair that left[5]. As a result, the distribution of traits among singles is stationary. Since staying single implies a utility of 0, all individuals strictly prefer to match at some point in time, to staying single forever.

One can see that the search models which assume that both men and women agree perfectly upon the ranking of potential mates is a special case

[4] The two groups do not have to be of equal sizes. In Figure 2 you can see that the measure of men in set $M_1 = \mu$, while the measure of men in set $M_2 = 1 - \mu$, with $\mu > \frac{1}{2}$.

[5] One may be tempted to change the story told here to one where after matching and obtaining the instantaneous utility, the pair splits up and goes back into the market as singles. The problem which that story is that any man (or woman) would then match with every person of the opposite sex that they happen to meet, obtain the instantaneous utility, and then go back to searching.

of the above set-up, with all men belonging to either the set M_1 or set M_2 (that is, all men have identical preferences).

Singles of opposite sexes meet according to a random matching function. For any individual the arrival rate of singles of the opposite sex is ®; where ® is the parameter of a Poisson process[6]. All agents discount the future at rate r:

Let the cumulative distribution of wealth among men in any period t be $G_m(:)$: Let the marginal cumulative distribution of wealth and height among women be $G_w^1(:)$ and $G_w^2(:)$; respectively. The cloning assumption made above ensures that none of these distributions depend upon t:

The individual decision problem in any period is whether to propose upon meeting someone of the opposite sex. This decision will depend upon the individual's prospects of marriage in the coming periods. Suppose proposals, which depend both upon the exogenously speci…ed arrival rate and on the strategies of members of the opposite sex, are infrequent. Then the best response for the individual facing these prospects is not to be too choosy.

I consider an equilibrium in stationary strategies. In this environment a stationary strategy for any single man is to delineate a subset of W, and to propose to any woman belonging to this set upon meeting, and reject all other women: Similarly, a stationary strategy for a single woman would be to propose to any man belonging to a particular subset of M upon meeting, and reject all other men. We assume that there is no cost to proposing, so an individual would propose to any potential mate they desire even if they knew that the other would turn the proposal down.

A stationary steady state equilibrium is de…ned as follows: given $G_m(:)$; $G_w^1(:)$ and $G_w^2(:)$; each individual agent chooses that stationary strategy that maximizes his/her utility, taking as given the strategies used by all others.

3 Diametrically opposed preferences

In this section the only trait of women that matters to men is their height. We can collapse the joint distribution of height and wealth in women into the marginal distribution of height, since men only take this into account to determine their strategies.

[6] The arrival rate does not depend upon the characteristics of the individual, it is the same for everyone.

It is possible now to represent women along a single line. We can do the same for men by taking the marginal distribution of their wealth, with the result that both sides of the market can be conveniently represented as in Figure 3. At any level of wealth, the proportion of men that prefer tall women is θ; and the proportion of men that prefer short women is $1 - \theta$

Figure 3

The instantaneous utility that a man in set M_1 receives upon matching with a woman w is $1 + x_w^1$; and the instantaneous utility that a man in set M_2 receives is $2 - x_w^1$.[7] We can see that $[1; 2]$ is the range of utility that a man can get in any match. It is also obvious that men belonging to M_1 prefer tall women (the greater x_w^1 is, the greater their utility from the match), while men belonging to M_2 prefer short women (the smaller x_w^1 is, the better off they are in the match). Any woman who matches with the man m receives instantaneous utility $1 + x_m$. This reflects her preference for wealthy men, and $[1; 2]$ is the range of instantaneous utility that a woman receives from any match.

Let us now consider the decision problem that a single woman w faces at a particular point in time. Let $U_\omega(w)$ denote the expected discounted lifetime

[7] To obtain a finite number of classes in equilibrium it is necessary that the utility from being matched is strictly greater than 0; so the $1 + x$ and $2 - x$. Any pair of utility functions of the nature $c + x$ and $1 + c - x$, where c is a strictly positive constant, would do the job.

utility of this woman at the end of any period, $G_m(\cdot \mid w)$ the cumulative distribution of wealth among single men who propose to w upon meeting, and $\alpha_\omega(w)$ the arrival rate of proposals to the woman w. Both $G_m(\cdot \mid w)$ and $\alpha_\omega(w)$ are well defined given the strategies of other agents. Then $U_\omega(w)$ can be defined as

$$U_\omega(w) = \frac{1}{1+r}\left((1-\alpha_\omega(w))\,U_\omega(w) + \alpha_\omega(w)\,E\max\left[1+x_m, U_\omega(w)\right]\right)$$

where the expectation is taken over $dG_m(\cdot \mid w)$. A woman who is single this period could meet a man who proposes to her with probability $\alpha_\omega(w)$ next period, and she would accept him if she gets more from the match than she does by remaining single in the marriage market. The woman's optimal strategy is to accept any man as long as his wealth $x_m \geq U_\omega(w) - 1$. Letting the reservation wealth level that this woman accepts in a potential partner be $R_\omega(w) = U_\omega(w) - 1$, and simplifying gives us

$$R_\omega(w) = \frac{\alpha_\omega(w)}{r}\int_{R_\omega(w)}^{\infty}\left(x_m - R_\omega(w)\right)dG_m(\cdot \mid w) - 1. \qquad (1)$$

Similarly, the optimal strategy for any man $m \in M_1$ is to accept all women taller than his reservation height level of $R_\mu^1(m)$, and reject all the others, where $R_\mu^1(m)$ is given by

$$R_\mu^1(m) = \frac{\alpha_\mu(m)}{r}\int_{R_\mu^1(m)}^{\infty}\left(x_w^1 - R_\mu^1(m)\right)dG_w^1(\cdot \mid m) - 1, \qquad (2)$$

The optimal strategy of $m \in M_2$, on the other hand, is to accept all women shorter than his reservation height level of $R_\mu^2(m)$, and reject all others, where $R_\mu^2(m)$ is given by

$$R_\mu^2(m) = 2 - \frac{\alpha_\mu(m)}{r}\int_{0}^{R_\mu^2(m)}\left(R_\mu^2(m) - x_w^1\right)dG_w^1(\cdot \mid w) \qquad (3)$$

Upon manipulation and simplification of (1), (2) and (3) we get:

$$R_\omega(w) = \frac{\alpha_\omega(w)}{r}\int_{R_\omega(w)}^{1}[1 - G_m(x|w)]\,dx - 1 \qquad (4)$$

$$R_\mu^1(m) = \frac{\alpha_\mu(m)}{r}\int_{R_\mu^1(x)}^{1}\left[1 - G_w^1(x|m)\right]dx - 1 \qquad (5)$$

$$R_\mu^2(m) = 2 - \frac{\alpha_\mu(m)}{r}\int_{0}^{R_\mu^2(x)} G_w^1(x|m)\,dx \qquad (6)$$

3.1 Structure of Equilibrium Allocation.

In equilibrium, there exist two partitions on $[0;1]$ that delineate men into classes along the wealth dimension. Similarly there exist two other partitions on $[0;1]$ that divide women into different segments along the height dimension. Let $\mu^1 = [\mu^1(n_1); \ldots; \mu^1(0)]$ and $\mu^2 = [\mu^2(n_2); \ldots; \mu^2(0)]$ with $\mu^1(0) = \mu^2(0) = 1$; be the two partitions on men's wealth: The elements of these partitions correspond to the reservation wealth levels of women in equilibrium.

Classes are formed as follows:

- There are $n_1 + n_2 - 1$ classes of men.
- Men in set M_1 with $x_m \in [\mu^1(1); \mu^1(0)]$ belong to the first M_1 class of men.
- Men in set M_2 with $x_m \in [\mu^2(1); \mu^2(0)]$ belong to the first M_2 class of men.
- For $k \in \{2; \ldots; n_1\}$; men in set M_1 with $x_m \in [\mu^1(k); \mu^1(k-1))$ belong to the kth M_1 class of men.
- For $k \in \{0; \ldots; n_2\}$ men in set M_2 with $x_m \in [\mu^2(k); \mu^2(k-1)]$ belong to the kth M_2 class of men.[8]

[8] Notice that the first classes of men are delineated by closed intervals, while the other classes are half open intervals.

- There is a positive measure of men in every class.

All men belonging to the same class have the same reservation height. Let $\omega^1(k)$ be the reservation height of any man in the kth M_1 class, and $\omega^2(k)$ be the reservation height of any man in the kth M_2 class. Remember that men in set M_2 propose to all women whose height is greater than their reservation height, and men in set M_1 propose to all women whose height is lesser than their reservation height.

In a steady state equilibrium the reservation heights of men in set M_1 is nondecreasing in their wealth, and the reservation heights of men in set M_2 is nonincreasing in their wealth The intuition for this is very simple. Take two men m and m' belonging to set M_1, with $x'_{m'} > x_m$. Then any woman who is willing to propose to m will also propose to m'. This implies that m' will receive at least as many proposals as m and so cannot have a reservation strategy that is less than that of m'. The same logic works for men in set M_2

Therefore we have $\omega^1(1) \geq \omega^1(2) \geq \dots \geq \omega^1(n_1)$, and $\omega^2(n_2) \geq \dots \omega^2(2) \geq \omega^2(1)$. Let $\omega^1(0) = 1$ and $\omega^2(0) = 0$. Then $\omega^1 = [\omega^1(n_1), \dots, \omega^1(0)]$ and $\omega^2 = [\omega^2(0), \dots, \omega^2(n_2)]$ are the two partitions on the height dimension of women. All women belonging to the same class also have the same reservation wealth, and as stated before, these reservation wealth levels form the partitions π^1 and π^2 in equilibrium. But defining the class boundaries for women is a little tricky.

3.2 Construction of the equilibrium

For simplicity of exposition I will construct the steady state equilibrium after making a few simplifying assumptions. I also compare what the steady state looks like as frictions get smaller and smaller.

I begin by assuming that

- the height of women is distributed uniformly on $[0, 1]$, so $G^1_w(x) = x$ for $x \in [0, 1]$;
- the wealth of men is also distributed uniformly on $[0, 1]$, making $G_m(x) = x$ for $x \in [0, 1]$;
- there are more men who prefer tall women than men who prefer short women $\left(\theta > \frac{1}{2}\right)$.

Let m be one of the wealthiest men in set M_1. All women propose to him, and so the arrival rate of proposals that he faces is α; and $G^1_w(\cdot \mid m) = G^1_w(\cdot)$. He belongs to the 1st M_1 class and so his reservation utility is $\omega^1(1)$. Substituting this into equation 5 gives us a characterization of $\omega^1(1)$:

$$\omega^1(1) = \frac{\alpha}{r}\int_{\omega^1(1)}^{\omega^1(0)} \left[\omega^1(0) - x\right] dx - 1. \tag{7}$$

Doing the same for one of the wealthiest men in set M_2, and using equation 6 we get

$$\omega^2(1) = 2 - \frac{\alpha}{r}\int_{\omega^2(0)}^{\omega^2(1)} \left[x - \omega^2(0)\right] dx. \tag{8}$$

Straightforward inspection tells us that for all $\frac{\alpha}{r} < \infty$; $\omega^1(1) < 1$; $\omega^2(1) > 0$.

Now we could either have (1) $\omega^1(1) > \omega^2(1)$ (2) $\omega^1(1) = \omega^2(1)$ or (3) $\omega^1(1) < \omega^2(1)$. The relative performance of women is dramatically different in each of these cases, as the following figures show.

Figure 3A portrays equilibrium when $\frac{\alpha}{r}$ is small (there are a lot of frictions). The boxes in the figure indicate the matches that could form in equilibrium. No match will form in any region that is outside all the boxes. The shaded regions are the ones where women match with men in both sets M_1 and M_2. The boxes that are lettered are the regions where women match exclusively with men in set M_1 or in set M_2.

In this case the frictions are large enough for us to have $\omega^1(1) < \omega^2(1)$. What does this mean for the relative performance of women? Consider any woman with $x^1_w \in [\omega^1(1); \omega^2(1)]$. The wealthiest man in M_1 is willing to propose to her, and so all men in M_1 will propose to her. The wealthiest men in M_2 is also willing to propose to her, and so all men in M_2 propose to her. Every woman in this class gets proposals from all men, and so have the highest expected payoff possible. In other words, women with $x^1_w \in [\omega^1(1); \omega^2(1)]$ form the first class of women. I will call this class of women both the first M_1 class as well as the first M_2 class, and their reservation wealth is $\mu^1(1) = \mu^2(1)$. The reason why I do this is because men in the first M_1 class propose to women in the first M_1 class, and vice versa, and men in the first M_2 class propose to women in the first M_2 class, and vice versa.

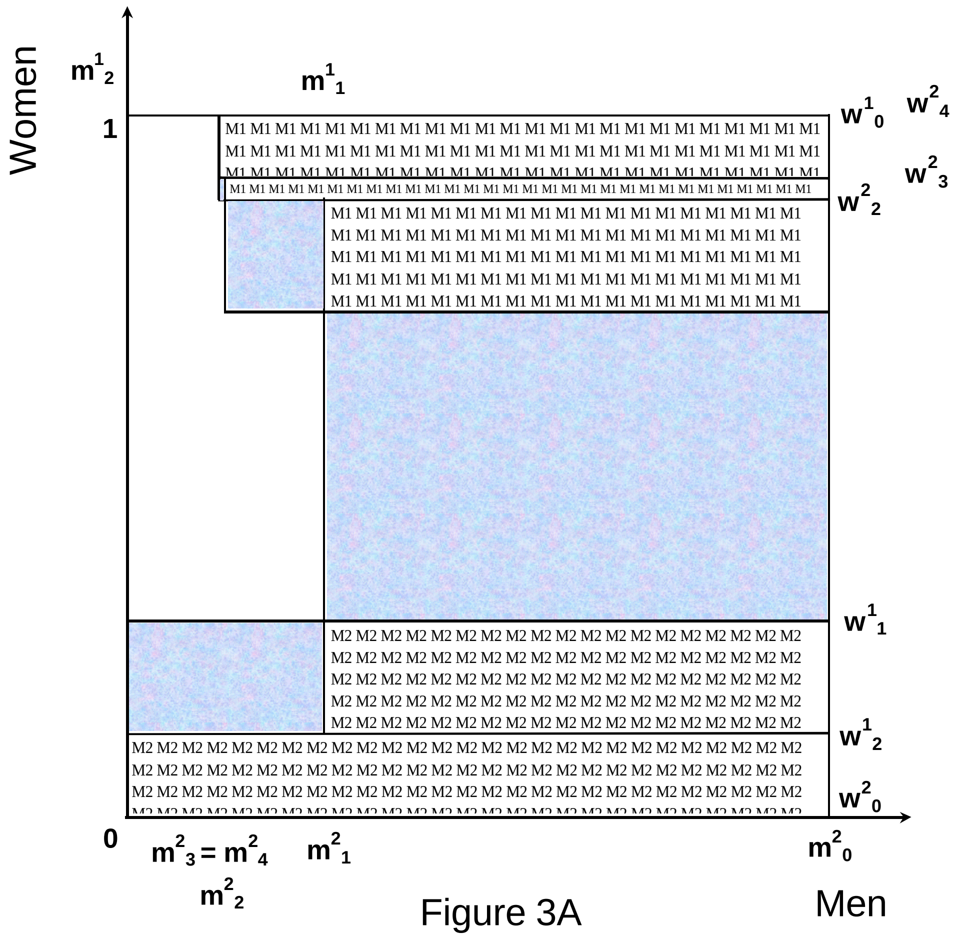

Figure 3A

As shown here it is possible that neither the tallest nor the shortest woman belongs to the …rst class of women. Outside this class, taller women do better than shorter ones. This is because there is a larger measure of men who prefer tall women. In this model, when the measure of men in set M_1 gets larger, and size of set M_2 gets smaller, taller women do better on average than shorter women (simply because they get more proposals). There is no direct impact on the expected payo¤ of men, μ does not enter their reservation height function (as you will see later). There is an indirect e¤ect from the fact that tall women get choosier, what I call the insider-outsider e¤ect. The size of the 1st M_1 class is decreasing in μ: There is no change in the expected payo¤ of men who continue to stay inside the 1st class, these are the insiders. But those that fall from the 1st M_1 class to the 2nd M_1 class have a strictly worse payo¤.

Gale and Sotomayor (1990) define stable matches in a static framework with a finite number of men and women. They find that the entry of an additional man leaves the existing men no better off (and in some cases strictly worse off). The above result seems to have the same flavor.

Women with $x^1_w \in [\omega^2(1);\omega^2(2)]$ form the 2nd M_1 class of women (The lower the M_1 class that the women belongs to, the shorter she is) . Women with $x^1_w \in [\omega^2(1);\omega^2(2)]$ form the 2nd M_2 class of women.

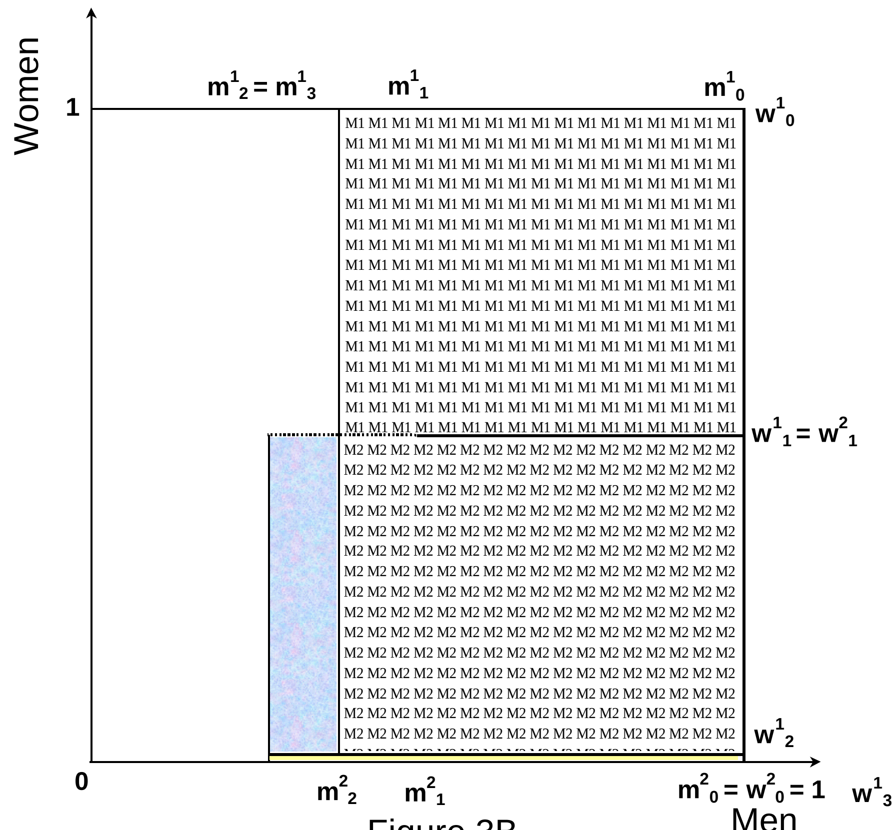

Figure 3B

As frictions grow smaller, the tallest and shortest women start doing better. Figure 3B shows the case when $\omega^1(1) = \omega^2(1)$: The 1st M_1 and the 1st M_2 classes contain women of a single height, $x^1_w = \omega^1(1) = \omega^2(1)$, in this case: Take men that belong to the 2nd M_1 class (the men with $x_m \in [\omega^1(2);\omega^1(1))$: Women in the first M_1 class do not propose to them, but they are of measure 0: Men in the 2nd M_1 class therefore have the same reservation utility as the men in the 1st M_1 class, and equilibrium partitions are as you see in the …gure.

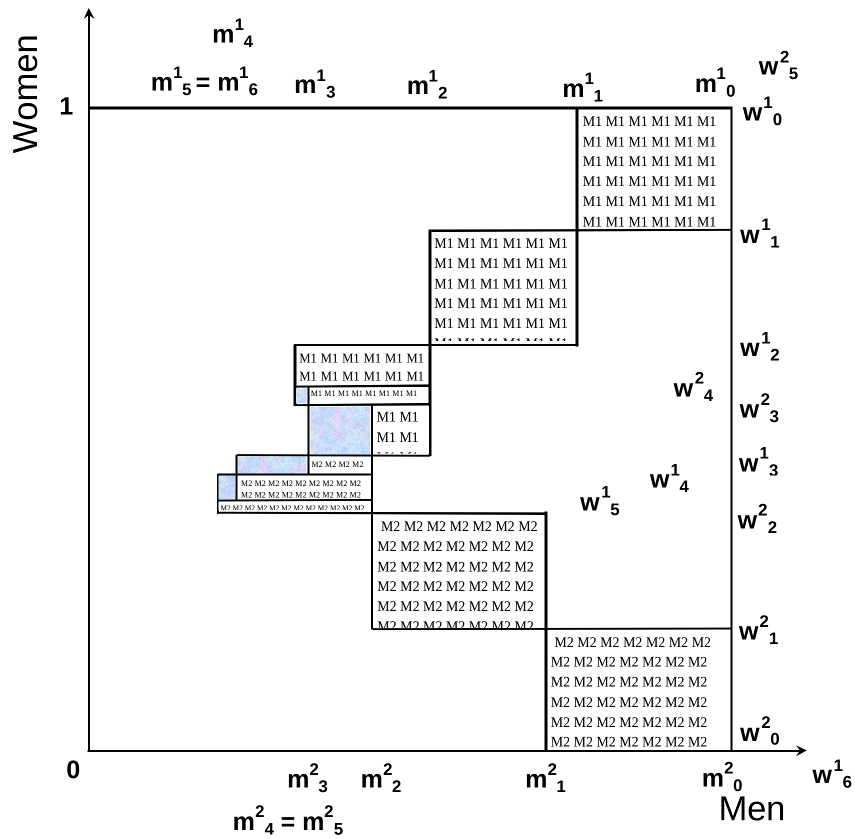

Figure 3C

Figure 3C illustrates equilibrium when $\frac{®}{r}$ increases some more. Suddenly we see that the tallest and shortest women are doing a lot better than the women of medium height. Still, at some point, the reservation height of men in set M_1 and those of men in set M_2 overlap (that is for some i and j; $!^2(j+1)$ ¸ $!^1(i+1)$ for the …rst time). Once this happens, the women in the middle do better than those slightly taller or slightly shorter than themselves. As can be seen, in Figure 3C, the reservation wealth level of women is decreases over $[!^2(0); !^2(2)]$; increases over $(!^2(2); !^1(3))$; decreases over $[!^1(3); !^1(2)]$ and increases over $[!^1(2); !^1(0)]$: So very tall and very short women perform relatively better than other women, but women in the middle still have some advantage because they appeal to diverse tastes.

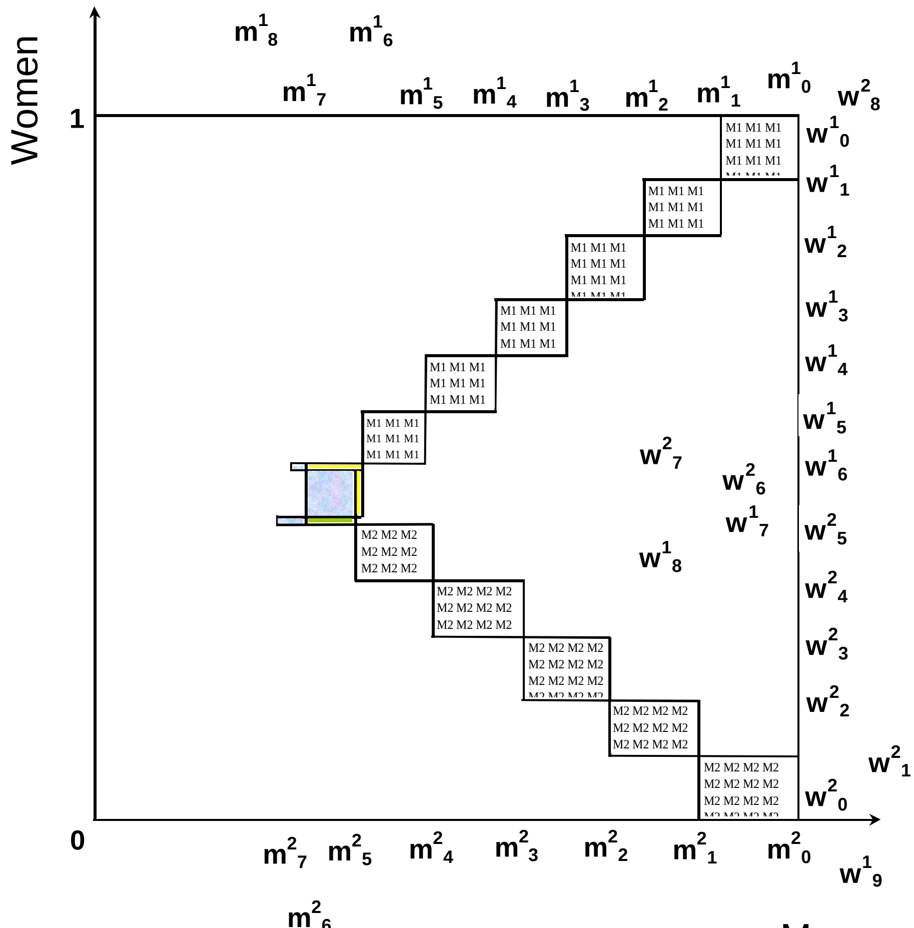

Figure 3D

As $\frac{\alpha}{r}$ goes on increasing, we can see that the women in the middle continue losing the advantage of appealing to diverse tastes. As $\frac{\alpha}{r}$ goes to infinity, we see assortative mating. The wealthiest man in set M_1 matches with the tallest woman, and the wealthiest man in set M_2 matches with the shortest woman, and so on down the line. The woman of medium height does the worst. Some men remain unmatched (For different parameter values, and distribution of traits, it could be some women who remain single).

Clearly the greater the extent of the market (the smaller the frictions, or the more efficient the matching mechanism) the greater the returns to someone who is very appealing to a niche, and the lower the returns to someone who appeals to a larger segment, albeit moderately.

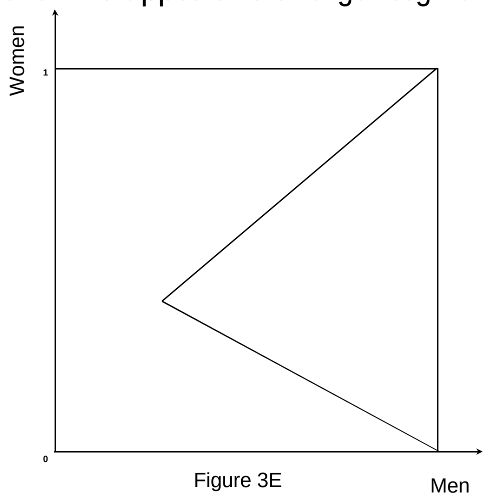

Figure 3E

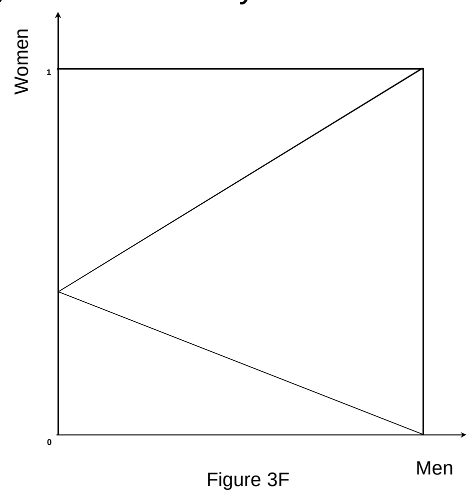

Figure 3F

Figure 3F illustrates the core of the static game. The core is defined as a one-to-one and onto assignment of men to women such that it is individually rational, and no coalition can separate away and secure greater utility (than they obtain in the core) for each of its members.

What is the effect of heterogenous preferences on the payoff of agents in the core? If all the men belonged to set M_1; then the core allocation would be one that matched every man to a woman whose height is equal to his wealth (the allocation would be along the 45 degree line). One can see that in the heterogenous case, the payoff to men in the core is greater than their level of wealth (in some cases strictly so), while the payoff to women who match with men in M_1 is less than their height (sometimes strictly), and to women who match with men in M_2 it is greater than their height.

As you can see from figures 3E and 3F, the steady state stationary equilibrium of the dynamic game does not approach the core of the static game when frictions disappear. Some men remain single in the equilibrium of the dynamic game, while this is not true in the core. The main reason for this is the cloning assumption. In the dynamic game, if a positive measure of very wealthy men find matches and depart that does not reduce the measure of very wealthy single men. Women can therefore afford to remain choosy.

This is not so in the core of the static game. In that case, there is no entry or exit of singles. Since every man and woman strictly prefers to be matched to somebody rather than be single, and since the measure of men and women is the same, nobody is single in the core allocation.

We have looked at how frictions impact the relative performance of women. What about the absolute performance? Do all women have a higher expected payo¤ as frictions go down?The e¤ect on men is interesting, and I will call it the insider-outsider e¤ect.

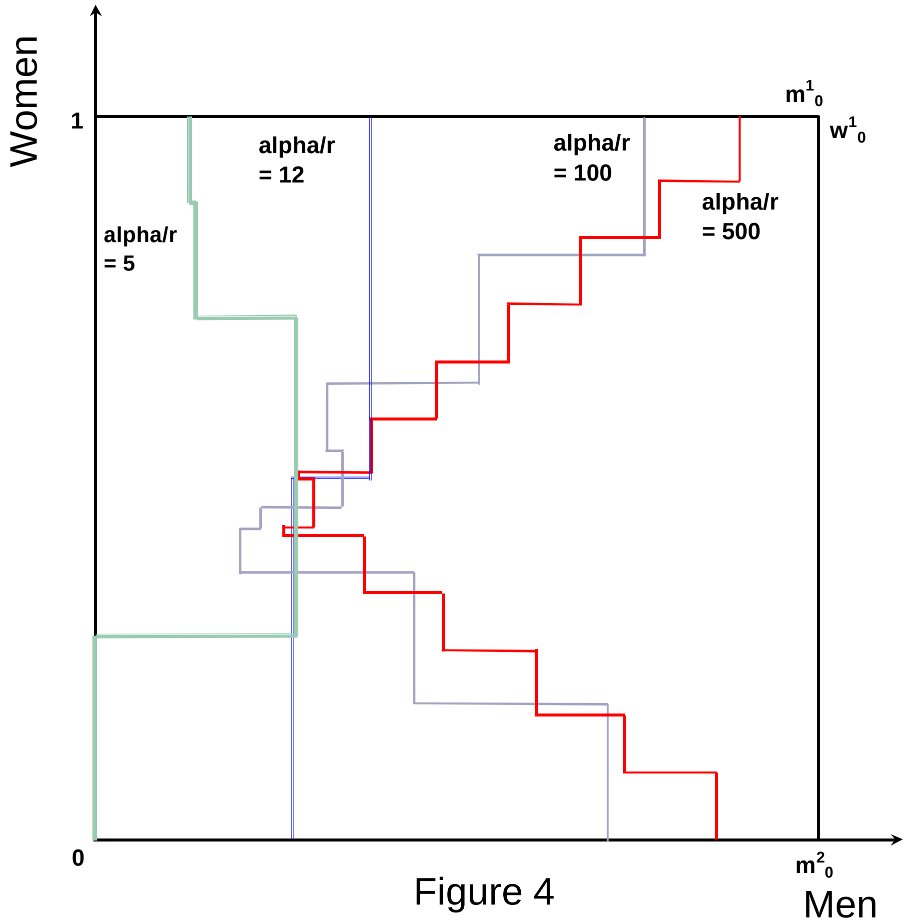

Figure 4

Figure 4 allows us to compare the change in the absolute payo¤s to women as frictions become smaller. As can be seen almost every woman does better as $\frac{®}{r}$ goes from 5 to 500:

What is the e¤ect on men when frictions go to zero? Once again, there is the insider-outsider e¤ect. Men who are insiders, that is those that continue to …nd matches as friction become smaller are better o¤, their expected utility goes up on average. At the same time, women are getting choosier, which doesn't portend well for the outsiders. So some men who are not so wealthy may be able to match with a woman in the presence of a lot of frictions, but are forced to remain single as frictions go down, since no woman will accept them.

3.3 Proposition 1

Given $(G_m; G_w^1)$, a steady state equilibrium exists and is unique. It is characterized by four partitions, $\mu^1; \mu^2; \omega^1; \omega^2$ on $[0;1]$; with μ^1 and μ^2 partitioning men on the basis of their wealth and ω^1 and ω^2 partitioning women on the basis of their height.

Proof: We will prove Proposition 1 by construction. I continue to maintain that $G_m(x) = x$ (wealth is uniformly distributed) and that $G_w^1(x) = x$ (height is uniformly distributed). The proofs do not need the uniform distribution assumption in any way, but it makes notation simpler.

- let $\omega^1(0) = \omega^2(0) = 1$; $\mu^1(0) = 1$ and $\mu^2(0) = 0$:
- let n_1 be the smallest integer such that either $0 \geq \mu^1(n_1)$ or $0 \geq \omega^1(n_1)$:
- let n_2 the smallest integer such that either $\mu^2(n_2) \geq 1$ or $0 \geq \omega^2(n_2)$:

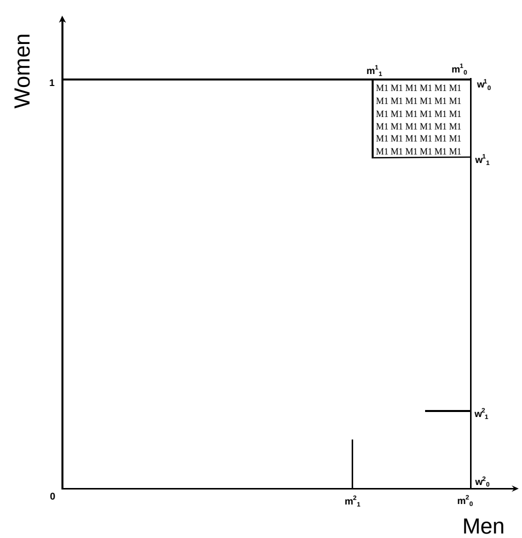

Figure 4A

Equations 7 and 8 give us $\omega^1(1)$ and $\omega^2(1)$: If $n_1 \neq 1$; and if $\omega^1(1) > \omega^2(1)$; let w_1 be a woman with height $\omega^1(1)$ and let w_2 be a woman with height $\omega^2(1)$: Denote $R_\omega(w_1)$ as $\pi^1(1)$ and $R_\omega(w_2)$ as $\pi^2(1)$: If $\pi^1(1) > \pi^2(1)$ then

Claim 1: In equilibrium, men in set M_1 with $x_m \in [\pi^1(1); 1]$ marry only women with $x_w^1 \in [\omega^1(1); 1]$; and vice versa. This set of men and women form an exclusive class and match only with members of the opposite sex as shown in figure 4A, above.

In addition, we have

$$\pi^1(1) = \frac{\alpha}{r}\int_{\pi^1(1)}^{1}(1 - x)\,dx - 1 \quad (9)$$

Proof of Claim 1: See Appendix.

The reservation values of women in what I call the first M_1 class corresponds to $\omega^1(1)$; and those of the men in this class is $\pi^1(1)$: Further elements of partitions ω^1 and π^1 will be determined in an identical fashion. We now start building partitions ω^2 and π^2:

Consider a man m_3 in set $M_1$with x_m such that $\pi^2(1) < x_m < \pi^1(1)$: Let $\omega^1(2)$ be the shortest woman that he is willing to propose to (that is, $\omega^1(2)$ is his reservation utility). If we have $\omega^1(2) > \omega^2(1)$; then let w_3 be a woman with height $\omega^1(2)$;and let $\pi^1(2)$ be her reservation utility. Again if we have $\pi^2(1) > \pi^1(2)$ then.

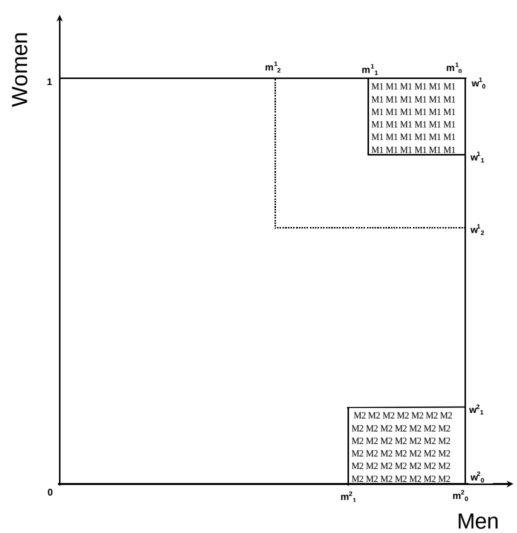

Figure 4B

Claim 2: In equilibrium, men in set M_2 with $x_m \in [\mu^2(1), 1]$ marry only women with $x_w^1 \in [0, \omega^2(1)]$, and vice versa. This set of men and women form another exclusive class whose members match only with each other, as shown in Figure 4B. Further $\mu^2(1)$ can be characterized as

$$\mu^2(1) = \frac{\alpha}{r}(1-\theta)\int_{\mu^2(1)}^{1}(1-x)\,dx - 1$$

Proof of Claim 2: See Appendix.

Notice that the reservation values of women in what I call the first M_2 class corresponds to $\mu^2(1)$, and those of the men in this class is $\omega^2(1)$. Now we proceed with determining the other elements of the partition.

Claim 3: Let $i \in \{0, ..., n_1\}$ and $j \in \{0, ..., n_2\}$ be the smallest integers such that $\omega^2(j+1) \geq \omega^1(i+1)$[9]. Then we obtain the partitions as follows:

1. For all k_1 s.t. $i \geq k_1$and k_2 s.t $j \geq k_2$,

$$\mu^1(k_1) = \frac{\alpha}{r}\theta\int_{\mu^1(k_1)}^{\mu^1(k_1-1)}\left(\mu^1(k_1-1) - x\right)dx - 1 \tag{10}$$

$$\mu^2(k_2) = \frac{\alpha}{r}(1-\theta)\int_{\mu^2(k_2)}^{\mu^2(k_2-1)}\left(\mu^2(k_2-1) - x\right)dx - 1 \tag{11}$$

2. For all k_1 s.t. $i+1 \geq k_1$and k_2 s.t $j+1 \geq k_2$,

$$\omega^1(k_3) = \frac{\alpha}{r}\int_{\omega^1(k_3)}^{\omega^1(k_3-1)}\left(\omega^1(k_3-1) - x\right)dx - 1, \tag{12}$$

$$\omega^2(k_4) = 2 - \frac{\alpha}{r}\int_{\omega^2(k_4-1)}^{\omega^2(k_4)}\left(x - \omega^2(k_4-1)\right)dx. \tag{13}$$

In equilibrium, for all k such that $i \geq k$ men in set M_1 with $x_m \in [\mu^1(k), \mu^1(k-1)]$ marry only women with $x_w^1 \in [\omega^1(k), \omega^1(k-1)]$, and

[9] Therefore $\omega^1(k_1) > \omega^2(k_2)$ for all k_1 such that $i \geq k_1$ and k_2 such that $j \geq k_2$, and $\omega^2(k_4) \geq \omega^1(k_3)$ for all $k_3 > i$ and $k_4 > j$.

vice versa, and for all k such that $j \geq k$; men in set M_2 with $x_m \in [\mu^2(k); \mu^2(k-1)]$ marry only women with $x_w^1 \in [\omega^2(k-1); \omega^2(k)]$; and vice versa,

The figure below demonstrates how these partitions form.

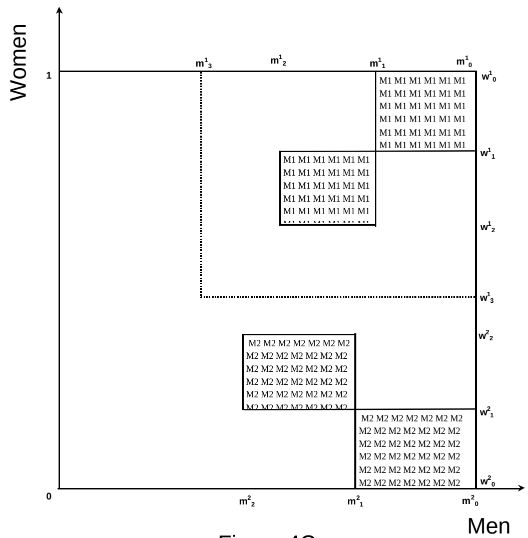

Figure 4C

Proof of Claim 3: See Appendix.

From this claim it is evident that all women in classes $k \geq i$ (tall women) and l such that $j \geq l$ (short women) are women whose height works to their advantage. They have higher reservation values (and hence higher expected payoffs) than women in classes h; $i > h > j$: Also all classes $k \geq i$ and l such that $j \geq l$ have women matching exclusively with men in either set M_1 or set M_2, but not both.

Claim 4: Let $i \in \{0, ::::, n_1\}$ and $j \in \{0, ::::, n_2\}$ be the smallest integers such that $\omega^2(j+1) \geq \omega^1(i+1)$: Then we obtain the partitions as follows:

1. For all k_1 s.t. $i < k_1$ and k_2 s.t $j < k_2$;

$$\mu^1(k_1) = \frac{\alpha}{r}\left[\int_{\mu^1(k_1)}^{\mu^1(k_1-1)} \theta\left(\mu^1(k_1-1) - x\right)dx + \int_{\mu^1(k_1)}^{\mu^2(j)} (1-\theta)\left(\mu^2(j) - x\right)dx\right] - 1; \tag{14}$$

$$\mu^2(k_2) = \frac{\alpha}{r}\left[\int_{\mu^2(k_2)}^{\mu^2(k_2-1)} (1-\theta)\left(\mu^2(k_2-1)-x\right)dx + \int_{\mu^2(k_2)}^{\mu^1(i)} \theta\left(\mu^1(i)-x\right)dx\right] - 1; \tag{15}$$

2. For all k_1 s.t. $i+1 < k_1$ and k_2 s.t $j+1 < k_2$;

$$\omega^1(k_1) = \frac{\alpha}{r}\left[\int_{\omega^1(k_1)}^{\omega^1(k_1-1)} \left(\omega^1(k_1-1)-x\right)dx + \int_{\omega^2(b_{k_1})}^{\omega^1(i)} \left(\omega^1(i)-x\right)dx + \int_{\omega^1(k_1)}^{\omega^2(b_{k_1})} \left[\omega^1(i)-\omega^2(b_{k_1})\right]dx - 1\right]; \tag{16}$$

where b_{k_1} is the smallest integer $b_{k_1} \in \{j+1, \ldots, n_2\}$ such that $\mu^1(k_1) \geq \mu^2(b_{k_1})$.

$$\omega^2(k_2) = 2 - \frac{\alpha}{r}\left[\int_{\omega^2(k_2-1)}^{\omega^2(k_2)} \left(x-\omega^2(k_2-1)\right)dx + \int_{\omega^2(j)}^{\omega^1(b_{k_2})} \left(x-\omega^2(j)\right)dx + \int_{\omega^1(b_{k_2})}^{\omega^2(k_2)} \left[\omega^1(b_{k_2})-\omega^2(k_2)\right]dx\right]. \tag{17}$$

In equilibrium, women with $x_w^1 \in [\omega^1(i+1), \omega^2(i+1)]$ which I call the $i+1$st M_1 class and the $j+1$st M_2 class, will match only with men in set M_1 with $x_m \in [\mu^1(i+1), \mu^1(i))$ and men in set M_2 with $x_m \in [\mu^2(j+1), \mu^2(j))$. The men in the $i+1$st M_1 class however match with all women in the $i+1$st M_1 class, and all women in the $j+1$st to n_2 M_2 class, that is women with $x_w^1 \in [\omega^1(i+1), \omega^1(i))$; and men in the $j+1$st M_2 class match with all women in the $i+1$st to n_1th M_1 class, and all women in the $j+1$st M_2 class, that is women with $x_w^1 \in [\omega^2(j), \omega^2(j+1))$.

For all $k_1 > i+1$ women in the type k_1st M_1 class (those with $x_w^1 \in [\omega^1(k_1), \omega^1(k_1-1))$ match with men of type M_1 belonging to the k_1st class, that is men with $x_m \in [\mu^1(k_1), \mu^1(k_1-1))$and with men in set M_2 belonging to classes j to k_1; that is men with $x_m \in [\mu^1(k_1), \mu^2(j))$ and vice versa.

For all $k_2 > j + 1$ women in the type k_2nd M_2 class (those with $x_w^1 \in [\omega^2(k_2 - 1); \omega^2(k_2))$ match with men of type M_2 belonging to the k_2nd class, that is men with $x_m \in [\mu^2(k_2); \mu^2(k_2 - 1))$and with men in set M_1 belonging to classes i to k_2; that is men with $x_m \in [\mu^2(k_2); \mu^1(i))$

Proof of Claim 4: See Appendix.

n_1 and n_2 are set in such a way that either no man stays single forever in equilibrium, or no woman stays single forever in equilibrium or both. Then equations 10 to 17 characterize the four partitions ω^1; ω^2; μ^1 and μ^2 completing the proof of Proposition 1.■

- The reservation strategy of women is decreasing for $x_w^1 \in [0; \omega^2(j)]$, is increasing for $x_w^1 \in (\omega^2(j); \omega^2(j+1)]$, is decreasing for $x_w^1 \in (\omega^2(j+1); \omega^1(i))$, and is increasing for $x_w^1 \in [\omega^1(i); 1]$.
- There may be either men or women who stay single forever in steady state equilibrium, but not both.
- Straightforward inspection shows that there are a positive measure of men in all classes and women in all classes.

4 Preferences over two Traits.

In this section I consider the case when the men in set M_1 care only about how tall the woman they match with is, since their instantaneous utility from matching with w is $1 + x_w^1$; while the men in set M_2 care only about how wealthy the woman they match with is, since their instantaneous utility from matching with w is $1 + x_w^2$. We can write out the reservation values of women, men in set M_1 and men in set M_2 as follows.

$$R_\omega(w) = \frac{\alpha_\omega(w)}{r} \int_{R_\omega(w)}^{1} [1 - G_m(x|w)]\, dx - 1;$$

$$R_\mu^1(m) = \frac{\alpha_\mu(m)}{r} \int_{R_\mu^1(x)}^{1} \left[1 - G_w^1(x|m)\right] dx - 1;$$

$$R_\mu^2(m) = \frac{\alpha_\mu(m)}{r} \int_{R_\mu^2(x)}^{1} \left[1 - G_w^2(x|m)\right] dx - 1.$$

where $G_m(:nw)$ is the distribution of wealth among men who propose to w; $G_w^1(:nm)$ is the marginal distribution of height among women who propose to m and $G_w^2(:nm)$ is the marginal distribution of wealth among women who propose to m:

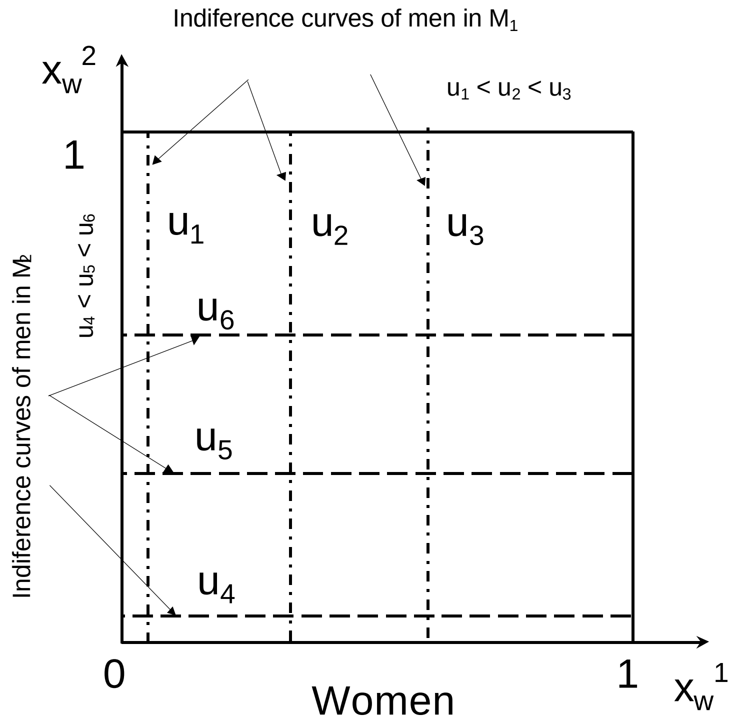

FIGURE 5

In this section I restrict attention to the case when $\mu = \frac{1}{2}$; although the generalization to other values of μ is not di¢cult. The indi¤erence curves of men in sets M_1 and M_2 are shown in …gure 5.

A stationary steady state equilibrium is de…ned as follows: given $G_m(:)$; $G_w^1(:)$ and $G_w^2(:)$; each individual agent chooses that stationary strategy that maximizes his/her utility, taking as given the strategies used by all others.

In equilibrium, there exists a partition on $[0;1]$ that delineate men into classes along the wealth dimension. There exist two other partitions on $[0;1]$, one of which divides women into di¤erent segments along the height dimension, and the other divides them into segments along the wealth dimension..

Let $\mu = [\mu(n), \ldots, \mu(0)]$ with $\mu(0) = 1$, be the partition on men's wealth. The elements of these partitions correspond to the reservation wealth levels of women in equilibrium.

All men with $x_m \in [\mu(k), \mu(k-1))$ belong to the kth class of men, for all $k > 1$. Men with $x_m \in [\mu(1), \mu(0)]$ belong to the 1st class among men. All men of type M_1 belonging to class k have the same reservation height level, which we denote as $\omega^1(k)$, and all men of type M_2 belonging to class k have the same reservation wealth level, which we denote as $\omega^2(k)$. $\omega^1 = [\omega^1(n_1), \ldots, \omega^1(0)]$, with $\omega^1(0) = 1$ forms the partition of women on the height dimension and $\omega^2 = [\omega^2(n_1), \ldots, \omega^2(0)]$ forms the partition of women on the wealth dimension.

In a steady state equilibrium the reservation values of men in both M_1 and M_2 is nondecreasing in their wealth. So we have $\omega^1(0) \geq \ldots \geq \omega^1(n_1)$, and $\omega^2(0) \geq \ldots \geq \omega^2(n_1)$. What about the reservation values of women?

Reservation values for women are monotonic in both traits, if $x^s_w > x^s_{w'}$ and $x^b_w > x^b_{w'}$, then $R_\omega(w) > R_\omega\left(w'\right)$. Moreover, holding x^2_w constant, the reservation value of women is increasing in x^1_w and holding x^1_w constant, their reservation value is increasing in x^1_w. However, we cannot say which of the two women w_1 and w_2 with $x^1_{w_1} > x^1_{w_2}$ and $x^2_{w_1} < x^2_{w^2}$ has a higher expected payoff in equilibrium. Let w_1 be along the diagonal, that is $x^1_{w_1} = x^2_{w_2}$. Also let $x^1_{w_1} = x^1_{w_2} + \varepsilon$ for some small ε, and let $x^2_{w_1} = x^2_{w_2} + M$, for some large M. That is w_2 is slightly shorter than w_1, but much wealthier. In equilibrium and in the presence of frictions, however, w_1 may get a higher expected payoff than w_2.

As in the perfectly negatively correlated case, in the presence of frictions, women like w_1 with mediocre values for both traits do better than other women like w_2 who have a much higher value in one trait, but a slightly lower one in the other. This is in sharp contrast to what happens as frictions disappear. As frictions decrease however, women along the diagonal (having the same values for both traits) do much worse that women at the north-west and south-east corners, that is women with $(1, \varepsilon)$ for $\varepsilon > 0$, small or with $(\varepsilon, 1)$. Figures 5A-D serve as visual aids in understanding what assignments look like in this case, and the effect of a reduction in frictions. Figure 5A-B represents equilibrium partitions when there are a lot of frictions (small $\frac{\alpha}{r}$).

Figure 5A is to be interpreted as follows. There are four classes of men (as you see in Figure 5B, below). The vertical line to the right represents the reservation value of the men of type M_1 in class 1, and the vertical line to the left represents the reservation value of the men of type M_1 in class 2.

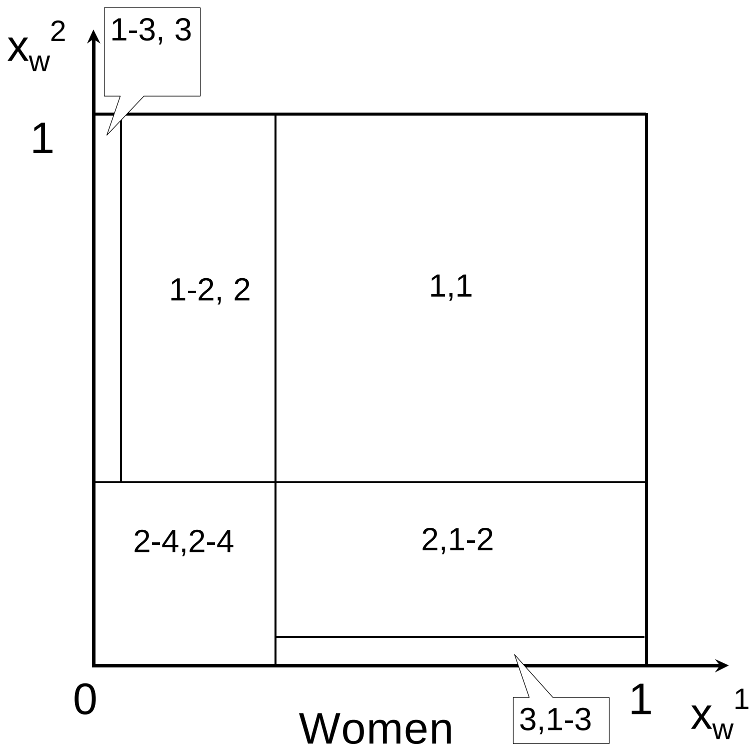

FIGURE 5A

M_1 men in classes 3 and 4 have a reservation value below 0: The horizontal lines represent the reservation values of men of type M_2 in class 1 and class 2; with the line towards the top being that of class 1: Once again, the reservation values of M_2 men in classes 3 and 4 is below 0: The reservation values of the men divide the women into classes. Each box in figure 5A represents a class of women, and all women in these classes have the same reservation value (the minimum level of wealth of the man that she will match with)[10]. The numbers within the boxes represents the classes of men with whom these woman may match in equilibrium. For instance, the box in the

[10] The reason why the boxes in 5A are symmetric around the 45 degree line is because $\mu = \frac{1}{2}$: For other values of μ that would not be the case.

top right corner has $(1;1)$ written in it. So in equilibrium, women in this box will match only with men of type M_2 in class 1, and with men of type M_1 in class 2. The box to its left has $(1-2;2)$ which says that women in that box match only with men of type M_2 in classes 1 and 2, and men of type M_1 in class 1. Notice that the wealth of women in this box is greater than the reservation value of men of type M_2 in class 1; but their height is lower than the reservation value of men of type M_1 in class 1. So when a box has $(i_{n_2} - i_{n_1}; j_{l_2} - j_{l_1})$ written in it, then in equilibrium, women in that box match with men of type M_2 in classes i_{n_2} to i_{n_1} and with men of type M_1 in classes j_{l_2} to j_{l_1}.

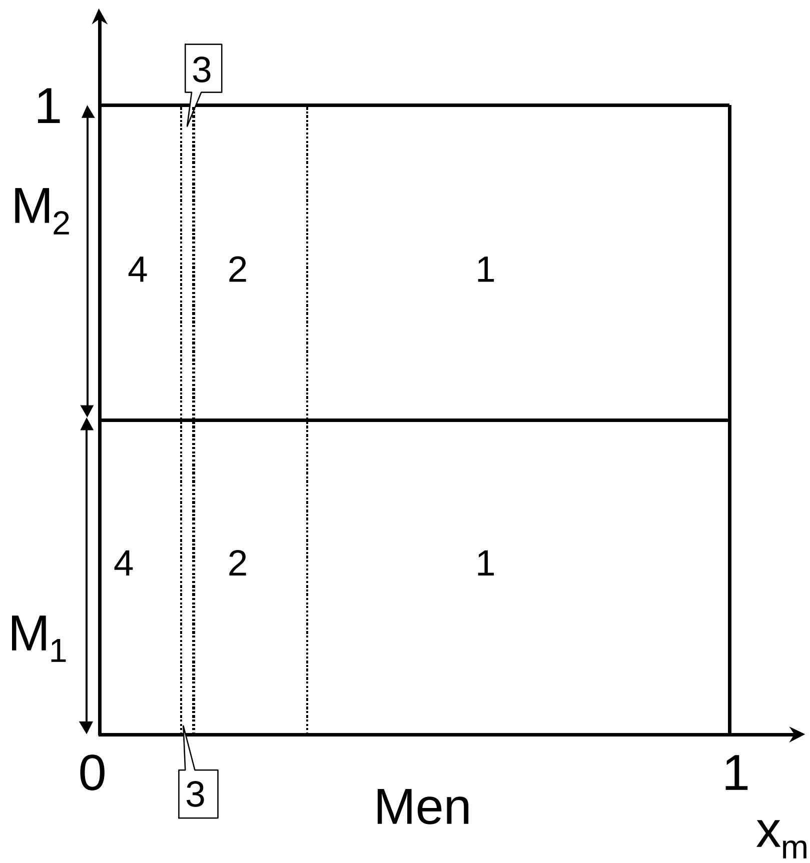

FIGURE 5B

The vertical lines in figure 5B indicate the reservation values of different classes of women. Now let us look at what the equilibrium looks like when frictions go down, or when $\frac{\circledR}{r}$ becomes larger.

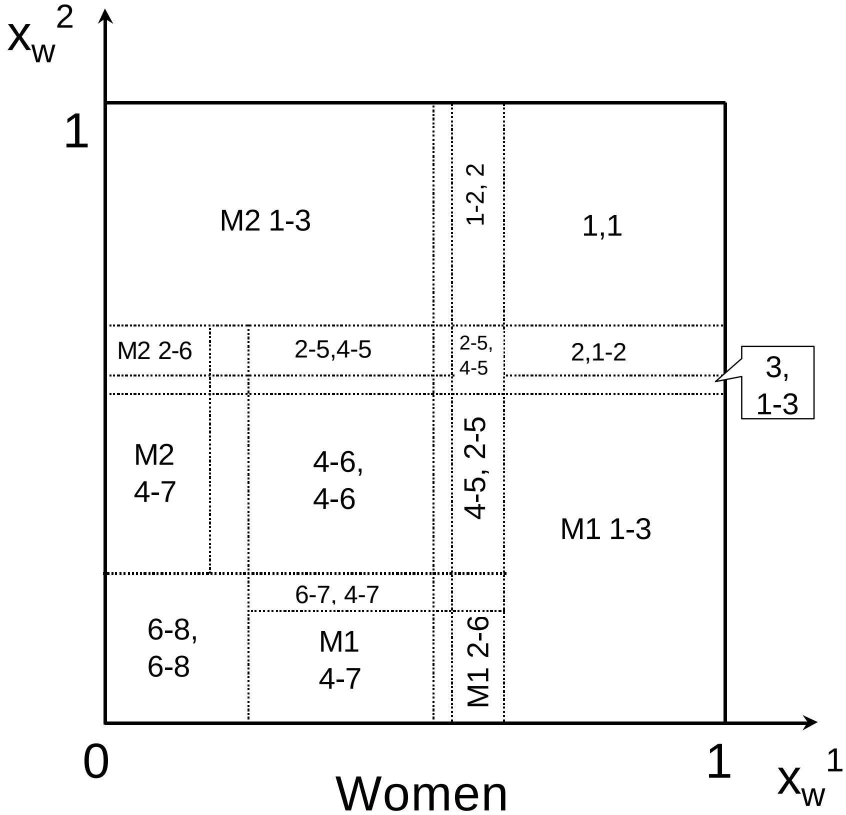

FIGURE 5C

Figure 5C describes equilibrium for an environment when frictions are a lot smaller than the previous environment. You can see that the same patterns reveal themselves as before, except that there are a lot more classes. This is because the reservation values of men went up (with frictions going down), as did the reservation values of women. The new notation M2 1-3 in one of the boxes simply implies that the women in the box match only with men of type M_2 who are in classes 1, 2 and 3:

As frictions goes up one can see, similar to the case when men had diametrically opposite preferences that women with high values in one trait (whatever the value of the other trait) begin doing better than women of medium height and wealth. There still continues to be some advantage to being moderately appealing to both groups of men. As frictions go down to zero however, appealing to diverse tastes brings no return.

Another similarity to the case with diametrically opposed preferences is that there are classes of wealthy women who match exclusively with men in set M_2, and tall women who match exclusively with men in set M_1. These classes arise endogenously, women do not care whether the men they match with are in set M_1 or in set M_2, they only care about his wealth.

I provide an algorithm for obtaining the steady state of this game in the appendix. I characterize the core of the static game when women vary along two traits and different groups of men value different traits in the next section.

5 The Core of the Assignment Game.

In this section I will define the core of the assignment game, when women differ from each other along two dimensions. I make the following assumptions in this section

- Wealth of men is distributed uniformly over $[0,1]$.
- The marginal distribution of height in women is uniform over $[0,1]$.
- The marginal distribution of wealth in women is uniform over $[0,1]$.
- The distributions of height and of wealth in women are independent of one another.
- $\theta = \frac{1}{2}$.

We need some additional notation as well.

Define $\mu_m \in \Delta(M)$, $\mu_m = \lambda^2$ (where λ^2 is the Lebesgue measure on $[0,1] \times (0,1]$), the population measure on the Borel sigma algebra of M, and $\mu_w \in \Delta(W)$, $\mu_w = \lambda^2$, the population measure on the Borel sigma algebra of W. By the definition of μ_m the measure of all men in set M_1 with $a \geq x_m$ for any $a \in [0,1]$, is $\mu_m(m : m \in \mathcal{S}, a \geq x_m) = \frac{a}{2}$.

Denote the disjoint union of individuals as $W \cup M$. We define the combined population measure on $W \cup M$ as μ. By definition $\mu(E \cup F) = \mu_w(E) + \mu_m(F)$, for all $E \subseteq W$ and $F \subseteq M$. We would like to assign women to men in such a way that (1) No man or woman prefers to remain single to being matched (2) There does not exist any pair, one man and one woman such that they would prefer to be matched with each other over their current matches.

5.1 Feasible Assignment

A feasible assignment is defined as follows:

$\pi : W \cup M \to W \cup M$, π is a measure preserving isomorphism from $W \cup M$ onto itself of order two (that is, $\pi^2(x) = x$) such that if $\pi(w) \neq w$ then $\pi(w) \in M$, and if $\pi(m) \neq m$ then $\pi(m) \in W$). By definition we have (i) π being one-to-one, onto, and measurable in both directions and (ii) $\mu(C) = \mu(\pi(C))$ for all $C \subseteq W \cup M$. If $\pi(x) = x$ then the person x remains single and is not matched with anyone under allocation π. The set of all such maps will be denoted by Π. This is also the set of all feasible allocations.

5.2 Payoff

The utility of men and women from a matching π is defined by $u : (\Pi \times W \cup M) \to [0,1]$ as follows:

$u(\pi, m) = x^i_{\pi(m)}$ if $\pi(m) \in W$ (where $i = 1$ if $m \in M_1$ and $i = 2$ if $m \in M_2$),

$u(\pi, m) = 0$ if $\pi(m) = m$ (The utility of staying single is 0),

$u(\pi, w) = x_{\pi(w)}$ if $\pi(w) \in M$ and $u(\pi, w) = 0$ if $\pi(w) = w$.

5.3 The Core

A feasible allocation belongs to the core if it satisfies the condition that

(1) there does not exist any m or any w such that $0 > u(\pi, m)$ or $0 > u(\pi, w)$ and

(2) there does not exist any pair (w, m) such that $x_m > u(\pi, w)$ and $x^i_w > u(\pi, m)$.

That is, no single person blocks the core allocation, nor is it blocked by any pair of agents.[11]

To help characterize the core of the assignment game, I will first define a few objects.

Given any feasible allocation π:

- Define $\varphi_w : \Pi \times M \rightarrow \{0;1\}$ for every $w \in W$ as

$$\varphi_w(\pi; m) = \begin{cases} 1 \text{ if } x_m > u(\pi; w); \\ 0 \text{ otherwise.} \end{cases} :$$

The set $\{m : \varphi_w(\pi; m) = 1\}$ then represents the men that the woman w would prefer to be matched with to her partner under π.

- Define for every $m \in M$; $\varphi_m : \Pi \times W \rightarrow \{0;1\}$, with

$$\varphi_m(\pi; w) = \begin{cases} 1 \text{ if } x_w^j > u(\pi; m); \\ 0 \text{ otherwise.} \end{cases}$$

The set $\{w : \varphi_m(\pi; w) = 1\}$ then represents the women that man m would prefer to be matched with to his partner under π.

- Define

$$z_\pi = \{w : \exists m \in M \text{ with } \varphi_w(\pi; m) = 1 \text{ and } \varphi_m(\pi; w) = 1.\}$$

Claim 5: A feasible allocation π is in the core if and only if $z_\pi = \emptyset$.

[11] I use the concept of the f-core in this definition. Although there are a continuum of both men and women, each point in the continuum represents an individual player and finite coalitions can block allocations. In this sense, in my model the continuum of men and women are simply approximations of very large finite sets of men and women. Since the individual is a non-negligible coalition member in finite sets, however large, the f-core works on the assumption that the individual is a non-negligible member of coalitions in the continuum as well.

Proof of Claim 5: If $z_{\pi} \neq \emptyset$ then there exists a woman $w \in W$ and a man $m \in M$ with $\varphi_w(\pi; m) = 1$ and $\varphi_m(\pi; w) = 1$. $\{w; m\}$ form a blocking coalition since each of them strictly prefers to be matched to the other, rather than to their partners under π.

If π belongs to the core, then π is not blocked by any single individual, or by any pair of agents. In particular, $\nexists \{w; m\}$, $w \in W$, $m \in M$ with $\varphi_w(\pi; m) = 1$ and $\varphi_m(\pi; w) = 1$. Therefore $z_{\pi} = \emptyset$. ■

5.4 Proposition 2

The allocation π_1 defined as:

For all $w \in W$

$$\pi_1((x_w^1; x_w^2)) = \begin{cases} \left(m : x_m = (x_w^1)^2; i = \frac{x_w^2}{2x_w^1}\right) & \text{if } x_w^1 \geq x_w^2, \\ \left(m : x_m = (x_w^2)^2; i = \frac{x_w^2 - x_w^1}{2x_w^2} + \frac{1}{2}\right) & \text{if } x_w^2 > x_w^1, \end{cases}$$

For all $m \in M \setminus (0; i); i \in (0; 1]$

$$\pi_1((x_m; i)) = \begin{cases} w : x_w^1 = \sqrt{x_m}; x_w^1 \geq x_w^2; \frac{x_w^1}{x_w^2} = 2i & \text{if } i \in \left(0; \frac{1}{2}\right]; \\ w : x_w^2 = \sqrt{x_m}; x_w^2 > x_w^1; \frac{x_w^1}{x_w^2} = 2(1 - i) & \text{if } i \in \left(\frac{1}{2}; 1\right]; \end{cases}$$

$\pi_1(m : (0; i)) = m$ for all $i \in (0; 1]$

is in the core[12]

Figure 6 gives a pictorial representation of what the core allocation looks like.

Proof of Proposition 2:

Step 1: Allocation π_1 is feasible.

Inspection of Proposition 2 shows us that $\pi_1 : W \cup M \to W \cup M$, is of order two and $\mu(C) = \mu(\pi(C))$ for all $C \subseteq W \cup M$.

Step 2: $z_{\pi_1} = \emptyset$.

Take any particular $w \in W$. Either $u(\pi_1; w) = (x_w^1)^2$ (when $x_w^1 \geq x_w^2$); or $u(\pi_1; w) = (x_w^2)^2$ (when $x_w^1 \geq x_w^2$). Therefore $u(\pi_1; w) = (\max\{x_w^1; x_w^2\})^2$. Take any man $m \in M$ such that $\varphi_w(\pi; m) = 1$. Then $x_m > u(\pi_1; w) = (\max\{x_w^1; x_w^2\})^2$. So $\sqrt{x_m} > x_w^1$ and $\sqrt{x_m} > x_w^1$; which makes $\varphi_m(\pi_1; w) = 0$. So every man in the set $\{m : \varphi_w(\pi; m) = 1\}$ strictly prefers his partner under

[12] The core maps any woman with $x_w^2 \leq x_w^1 = a$ for some $a \in [0; 1]$ to the man with wealth a^2 in set M_1; and any woman with $x_w^1 < x_w^2 = a$ to the man with wealth a^2 in set M_2.

π_1 to w; and $w \notin z_{\pi_1}$: Since this is true for every woman $z_{\pi_1} = \emptyset$; and from claim 5, π_1 belongs to the core of the assignment game. ■

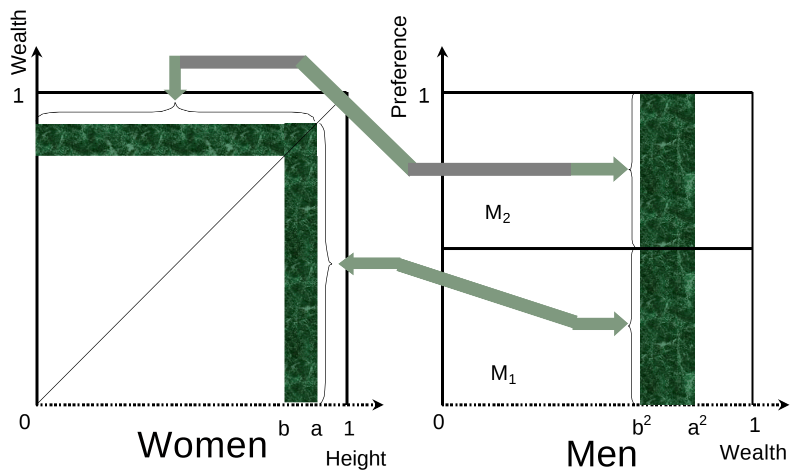

Figure 6

In the appendix I provide a sequence of games with finite numbers of players, the set of women converging to W; the set of men converging to M; and a sequence of allocations that belong in the core of each finite game in the sequence, that converges (in measure) to π_1:

5.5 Proposition 3

In any core allocation, the payoff to man m is $\sqrt{x_m}$; and the payoff to the woman is $\left(\max\left\{x_w^s; x_w^b\right\}\right)^2$:

Proof of Proposition 3: See appendix.

In any core allocation,

- All women whose height is greater than their level of wealth ($x_w^1 \ge x_w^2$) match with men in set M_1; all women whose level of wealth is greater than their height ($x_w^2 > x_w^1$) match with men in set M_2;
- Each woman receives a payoff which equals $(\max\{x_w^1; x_w^2\})^2$; and every man receives a payoff $\sqrt{x_m}$:

What is the effect of having heterogenous preferences on the payoffs of agents in the market?

If all men belonged to set M_1, (when only one trait of a woman is relevant), then the payoff to the men in any core allocation would be x_m, and the payoff to the women would be x^1_w[13]. Every man does better in the heterogenous preferences case. Among women, all those who have $x^1_w \geq x^2_w$ do worse, since there are fewer men who prefer tall women, and all those with $x^1_w < x^2_w$ do better, since now there are some men who value the trait in which these women have a comparative advantage.

The other important difference between the core of this game and the dynamic equilibrium defined in the previous section is that in the core, women along the diagonal (those with $x^1_w = x^2_w$) do strictly worse than any woman w' with $\max\left(x^1_{w'}, x^2_{w'}\right) > x^1_w$. So in the case of the two women w_1 and w_2 (that we discussed in Section 4) with w_1 on the diagonal and $x^1_{w_1} = x^1_{w_2} + \varepsilon$ for some small ε, and $x^2_{w_1} = x^2_{w_2} + M$ for some large M, w_1 gets a payoff of $\left(\max\left(x^1_{w_1}, x^2_{w_1}\right)\right)^2$, and w_2 gets $\left(\max\left(x^1_{w_1} - \varepsilon, x^2_{w_1} + M\right)\right)^2$. It is very likely that $x^2_{w_1} + M > x^1_{w_1}$ and so w_2 does a lot better than w_1 in this case, the exact opposite of the relative performance of these two women in the dynamic equilibrium with frictions.

6 Conclusion

One of the main differences is that in the presence of frictions, an individual who is moderately appealing to diverse tastes among the opposite sex may make a better match than someone who is found to be stunning by one group, but leaves the others cold. A second difference is that the perfect assortion prediction by Becker (1973) does not arise. Instead one sees assortative patterns along more than one dimension. This seems more in tune with empirical evidence of correlation between traits being around 60% along more than one dimension.

In this paper I attempt to analyze the equilibrium that arises in the

[13] If all men belonged to set M_1, each man would match with the woman whose intelligence is exactly the same as his level of wealth, and each woman would match with the man whose wealth is exactly the same as her level of intelligence. This can be established trivially. In the core of such a game (when only one trait is valued) the payoff to each woman has to be nondecreasing in her intelligence (else a blocking coalition can be formed). This leads to the assignment outlined above.

marriage market when one side of the market has heterogenous preferences. When women vary from one another along one trait only (as in Becker (1973)), but when men have diametrically opposed preferences for that one trait, then in the presence of search frictions, women with mediocrity in both traits do better than women that excel in one. This continues to be true in the case when women differ along more than one trait (I do the analysis with 2 traits), and some men care only about one of the traits and the others care only about the other trait. Some women who are mediocre in both traits do better than other women who are a lot better in one of the characteristics, and just slightly less good in the other trait. Which women would prefer to trade places with which other women depends entirely upon the extent of frictions and the specific parameters of the matching technology.

The findings in this paper extend to the case when men are divided into two sets, with one set getting instantaneous utility $\tau x_w^1 + (1 - \tau) x_w^2$ upon matching with w; and the other set getting instantaneous utility $(1 - \tau) x_w^1 + \tau x_w^2$ upon matching with w; for some $\tau \in [0; 1]$: That is, one set of men care more about trait 1, while the second cares more about trait 2. Once again in the presence of frictions women who are mediocre do better than those that excel in one and are less than mediocre in the other. However, as frictions become smaller and smaller, mediocrity loses its attraction to women, and finally, mediocre women (those along the diagonal) do the worst when there are no frictions at all.

It is possible to extend this research in many directions. The first obvious direction is: what do the patterns of matching predict if agents on both sides of the market differ in more than one trait? The second is to question what would happen if the preferences of men differed continuously, instead of discreetly as assumed in this paper. A third direction is to consider this set-up with transferable utility. The results of such a model would be more easily applicable to the labor market. It would also be interesting to consider different institutions, for instance, suppose agents could direct search by forming discos or book clubs, would all agents benefit from such segregation? Would agents willingly join such clubs, and if not, would the clubs vanish, or would they continue with some agents in them and some not?

Another direction in which this research could head is whether the results obtained in the static two-sided matching literature, with no search, can be extended to the dynamic two-sided matching literature with search and without the cloning assumption.

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7 Appendix

7.1 Proofs from Section 3

Claim 1:

Assumptions: If $n_1 \neq 1$, and if $\omega^1(1) > \omega^2(1)$, let w_1 be a woman with height $\omega^1(1)$ and let w_2 be a woman with height $\omega^2(1)$. Denote $R_\omega(w_1)$ as $\mu^1(1)$ and $R_\omega(w_2)$ as $\mu^2(1)$, and if $\mu^1(1) \geq \mu^2(1)$ then

Statement of Claim 1: In equilibrium, men in set M_1 with $x_m \in [\mu^1(1), 1]$ marry only women with $x^1_w \in [\omega^1(1), 1]$, and vice versa. This set of men and women form an exclusive class and match only with members of the opposite sex from the same class.

In addition, we have

$$\mu^1(1) = \frac{\alpha}{r}\left(\int_{\mu^1(1)}^{1} (1 - x)\, dx\right) - 1$$

Proof of Claim 1:

Step 1: All women taller than w_1 have reservation values lower than that of w_1. That is $\mu^1(1) \geq R_\omega(w')$ for all w' such that $x^1_{w'} \in [\omega^1(1), 1]$.

All men in set M_1 propose to both w' and w_1, and any man in set M_2 who proposes to w' also proposes to w_1. Therefore $R_\omega(w_1) = \mu^1(1) \geq R_\omega(w')$.

Step 2: All the men in set M_1 with $x_m \in [\mu^1(1), 1]$ have the same reservation value as the wealthiest man in M_1. That is $R^1_\mu(m') = \omega^1(1)$ for all $m' \in [\mu^1(1), 1] \times (0, a]$.

Take any $m' \in [\mu^1(1), 1] \times (0, a]$. All women with $x^1_{w'} \in [\omega^1(1), 1]$ propose to him (since their reservation value is smaller than $\mu^1(1)$). Let's suppose that no other woman would propose to m'. We can then compute $R = \min(R^1_\mu(m'))$ as follows

$$R = \frac{\alpha}{r}\left[(1 - \omega^1(1)) \int_R^{\omega^1(1)} (1 - 0)\, dx + \int_{\omega^1(1)}^{1} \left(1 - \frac{x - \omega^1(1)}{1 - \omega^1(1)}\right) dx\right] - 1$$

Simplifying and substituting from 7, we have:

$$R = \frac{\alpha}{r}\left[\int_{R}^{\omega^1(1)} \left(1 - \omega^1(1)\right) dx\right] + \omega^1(1)$$

$R^1_1(1) = \omega^1(1) \geq R^1_1\left(m'\right) \geq R$; since the reservation values of men is increasing in their level of wealth by claim 1. The first term in the above equation is therefore positive. But given that $\omega^1(1) \geq R$; the first term must be 0. Therefore $R = R^1_1\left(m'\right) = \omega^1(1)$. ∎

Step 3: All women shorter than w_2 have reservation values lower than that of w_2. That is $\mu^2(1) \geq R_\omega\left(w'\right)$ for all w' such that $x^2_{w'} \in [0, \omega^2(1)]$.

All men in set M_2 propose to both w' and w_2, and any man in set M_1 who proposes to w' also proposes to w_2. Therefore $R_\omega(w_2) = \mu^2(1) \geq R_\omega\left(w'\right)$. ∎

Step 4: All the men in set M_2 with $x_m \in [\mu^2(1), 1]$ have the same reservation value as the wealthiest man in M_1. That is $R^2_1\left(m'\right) = \omega^2(1)$ for all $m' \in [\mu^2(1), 1] \times (a, 1]$.

Take any $m' \in [\mu^1(1), 1] \times (a, 1]$. All women with $x^1_{w'} \in [0, \omega^2(1)]$ propose to him (since their reservation value is smaller than $\mu^2(1)$). Let's suppose that no other woman would propose to m'. Similar to what we did in step 2, we will set $R = \min\left(R^2_1\left(m'\right)\right)$, and after substitution from 8,

$$R = 2 - \frac{\alpha}{r}\omega^2(1)\left[\int_0^{\omega^2(1)} \frac{x}{\omega^2(1)} dx + \int_{\omega^2(1)}^{1} 0 dx\right] = \omega^2(1).$$

∎

Step 5: The woman w_1 (with height $x^1_w = \omega^1(1)$) matches only with men in set M_1 in equilibrium.

We know that $R_\omega(w_1) = \mu^1(1)$, by definition. No man in set M_2 with wealth $x_m \geq \mu^1(1)$ ever proposes to w_1 (since he doesn't propose to anyone taller than $\omega^2(1)$). Since she does not accept any proposals from men with wealth $\mu^1(1) \geq x_m$, she matches in equilibrium only with men in set M_1 ∎

Notice that w_1 does not reject men in M_2 with $x_m \geq \omega^1(1)$. It is they who do not accept her proposal.

We can now characterize $\mu^1(1)$ as

$$\mu^1(1) = \frac{\alpha}{r}\theta \int_{\mu^1(1)}^{1} (1 - x)\,dx - 1$$

Step 6: All women taller than w_1 also have a reservation value $\mu^1(1)$ and match only with men in set M_1 in equilibrium.

Take any woman w' taller than w_1: All the men in M_1 propose to her. Let's suppose that no man in set M_2 proposes to her, and let $R = \min\left(R_\omega\left(w'\right)\right)$: Then

$$R = \frac{\alpha}{r}\theta \int_{R}^{1} (1 - x)\,dx - 1 = \mu^1(1)$$

Therefore $R_\omega\left(w'\right) \geq R = \mu^1(1)$: From step 1, we have $\mu^1(1) \geq R_\omega\left(w'\right)$: Combining the two we get $R_\omega\left(w'\right) = \mu^1(1)$

This establishes that in equilibrium, men in set M_1 with $x_m \in [\mu^1(1); 1]$ marry only women with $x_w^1 \in [\omega^1(1); 1]$; and vice versa. $\blacksquare$

Claim 2:

Assumptions: $m_3 \in M_1$; with x_m such that $\mu^2(1) < x_m < \mu^1(1)$: $\omega^1(2)$ is the reservation utility of m_3, and $\omega^1(2) > \omega^2(1)$: w_3 is a woman with height $\omega^1(2)$;and her reservation utility is $\mu^1(2)$; we have $\mu^2(1) > \mu^1(2)$:

Statement of Claim 2: In equilibrium, men in set M_2 with $x_m \in [\mu^2(1); 1]$ marry only women with $x_w^1 \in [0; \omega^2(1)]$; and vice versa. This set of men and women form another exclusive class whose members match only with each other.

Further $\mu^2(1)$ can be characterized as

$$\mu^2(1) = \frac{\alpha}{r}(1 - \theta) \int_{\mu^2(1)}^{1} (1 - x)\,dx - 1$$

Proof of Claim 2:

Step 1: The woman w_2 (with height $x_w^1 = \omega^2(1)$) matches only with men in set M_2 in equilibrium.

We know that $R_\omega(w_2) = \mu^2(1)$; by definition. No man in set M_1 with wealth $x_m \geq \mu^2(1)$ ever proposes to w_2 (since he doesn't propose to anyone shorter than $\omega^1(2)$): Since w_2 does not accept any proposals from men with wealth $\mu^2(1) \geq x_m$; she matches in equilibrium only with men in set M_2

Notice that w_2 has no preferences over whether the man belongs to M_1 or M_2, provided that he is rich enough. But a wealthy man who prefers short women find her too tall to propose to.

We can now characterize $\mu^2(1)$ as

$$\mu^2(1) = \frac{\alpha}{r}(1-\theta)\int_{\mu^2(1)}^{1}(1-x)\,dx - 1$$

Step 2: All women shorter than w_2 also have a reservation value $\mu^2(1)$ and match only with men in set M_2 in equilibrium.

Take any woman w' shorter than w_1: All the men in M_2 propose to her. Let's suppose that no man in set M_1 proposes to her, and let $R = \min\left(R_\omega\left(w'\right)\right)$: Then

$$R = \frac{\alpha}{r}(1-\theta)\int_{R}^{1}(1-x)\,dx - 1 = \mu^2(1)$$

Therefore $R_\omega\left(w'\right) \geq R = \mu^2(1)$: From step 3 of claim 1, we have $\mu^2(1) \geq R_\omega\left(w'\right)$: Combining the two we get $R_\omega\left(w'\right) = \mu^2(1)$ ■

This establishes that in equilibrium, men in set M_2 with $x_m \in [\mu^2(1); 1]$ marry only women with $x_w^1 \in [0; \omega^1(1)]$; and vice versa. ■

Claim 3: Let $i \in \{0; ::::; n_1\}$ and $j \in \{0; ::::; n_2\}$ be the smallest integers such that $\omega^2(j+1) \geq \omega^1(i+1)$: Then we obtain the partitions as follows:

1. For all k_1 such that $i \geq k_1$ and k_2 such that $j \geq k_2$ we have

$$\mu^1(k_1) = \frac{\alpha}{r}\theta\int_{\mu^1(k_1)}^{\mu^1(k_1-1)}\left(\mu^1(k_1-1)-x\right)dx - 1$$

$$\mu^2(k_2) = \frac{\alpha}{r}(1-\theta)\int_{\mu^2(k_2)}^{\mu^2(k_2-1)}\left(\mu^2(k_2-1)-x\right)dx - 1$$

2. For all k_3 such that $i+1 \geq k_3$ and k_4 such that $j+1 \geq k_4$ we have

$$\omega^1(k_3) = \frac{\alpha}{r}\int_{\omega^1(k_3)}^{\omega^1(k_3-1)} \left(\omega^1(k_3-1) - x\right) dx - 1;$$

$$\omega^2(k_4) = 2 - \frac{\alpha}{r}\int_{\omega^2(k_4-1)}^{\omega^2(k_4)} \left(x - \omega^2(k_4-1)\right) dx.$$

In equilibrium, for all k such that $i \geq k$ men in set M_1 with $x_m \in [\xi^1(k); \xi^1(k-1)]$ marry only women with $x_w^1 \in [\omega^1(k); \omega^1(k-1)]$; and vice versa, and for all j such that $j \geq k$; men in set M_2 with $x_m \in [\xi^2(k); \xi^2(k-1)]$ marry only women with $x_w^1 \in [\omega^2(k); \omega^2(k-1)]$; and vice versa.

Proof of Claim 3: We will prove this claim by induction. Let claim 3 be true for all $k_1 < l_1$ for some l_1 with $i \geq l_1$; and for all $k_2 < l_2$ for some l_2 with $j \geq l_2$. By assumption we have $\omega^1(k_1) > \omega^2(k_2)$. Let w_1 be a woman with height $\omega^1(k_1)$ and let w_2 be a woman with height $\omega^2(k_2)$. Denote $R_\omega(w_1)$ as $\xi^1(k_1)$ and $R_\omega(w_2)$ as $\xi^2(k_2)$. Either $\xi^1(k_1) < \xi^2(k_2)$ or $\xi^1(k_1) \geq \xi^2(k_2)$. If $\xi^1(k_1) < \xi^2(k_2)$; then

Step 1: $R_\omega(w) < \xi^2(k_2)$ for all women w with $x_w^1 \in (\omega^2(k_2); \omega^1(k_1))$. Suppose not: let $\exists w'$ be a woman with $x_w^1 \in (\omega^2(k_2); \omega^1(k_1))$ and $R_\omega\left(w'\right) > \xi^2(k_2)$. Such a woman would never marry, since no man in set M_1 with $x_m \in [\xi^1(k_1); \infty]$ ever proposes to her (since all M_1 men in classes k_1 and above only marry M_1 women in their own classes), and no man in set M_2 with $x_m \in [\xi^2(k_2); \infty]$ ever proposes to her. It is therefore strictly better off for her to choose a reservation strategy $R_\omega\left(w'\right) < \xi^2(k_2)$.

Step 2: Take the man m' in set M_2 with $x_m = \xi^2(k_2) - \varepsilon > \xi^1(k_1)$. Such a man receives proposals from all women with $x_w^1 \in (\omega^2(k_2); \omega^1(x))$; where $k_1 \geq x$ is such that $\xi^1(x) \geq \xi^2(k_2)$ (there exists such an x; since $\xi^1(0) = \infty \geq \xi^2(k_2)$). What is his strategy? We know that $\omega^1(x) \geq \omega^1(k_1) \geq \omega^1(i) \geq \omega^2(k_2+1)$. Therefore,

$$\omega^2(k_2+1) = 2 - \frac{\alpha}{r}\left(\omega^1(x) - \omega^2(k_2)\right)\int_{\omega^2(k_2)}^{\omega^2(k_2+1)} \left(\frac{x - \omega^2(k_2)}{\omega^1(x) - \omega^2(k_2)}\right) dx.$$

and simplifying we have

$$\omega^2(k_2+1) = 2 - \frac{\alpha}{r}\int_{\omega^2(k_2)}^{\omega^2(k_2+1)} \left(x - \omega^2(k_2)\right) dx.$$

Step 3: If $\mu^1(k_1) \geq \mu^2(k_2)$; then we could retrace the steps 1 and 2 to get

$$\omega^1(k_1+1) = \frac{\alpha}{r}\int_{\omega^1(k_1+1)}^{\omega^1(k_1)} \left(\omega^1(k_1) - x\right) dx - 1;$$

Step 4: If $k_1 < i$; and $k_2 < j$; then by assumption we have $\omega^1(k_1+1) > \omega^2(k_2+1)$. Let w_3 be a woman with height $\omega^1(k_1+1)$ and let w_4 be a woman with height $\omega^2(k_2+1)$. Denote $R_\omega(w_3)$ as $\mu^1(k_1+1)$ and $R_\omega(w_4)$ as $\mu^2(k_2+1)$. Either $\mu^1(k_1+1) < \mu^2(k_2+1)$ or $\mu^1(k_1+1) \geq \mu^2(k_2+1)$. If $\mu^1(k_1+1) < \mu^2(k_2+1)$; then

No man in set M_1 with wealth $x_m \geq \mu^2(k_2+1)$ ever proposes to w_4 (since he doesn't propose to anyone shorter than $\omega^1(k_1+1)$). Since w_4 does not accept any proposals from men with wealth $x_m < \mu^2(k_2+1)$ (that being her reservation value) she matches in equilibrium only with men in set M_2

We can now characterize $\mu^2(k_2+1)$ as

$$\mu^2(k_2+1) = \frac{\alpha}{r}(1-\theta)\int_{\mu^2(k_2+1)}^{\mu^2(k_2)} \left(\mu^2(k_2) - x\right) dx - 1$$

Step 5: If $\mu^1(k_1+1) \geq \mu^2(k_2+1)$; then no man in set M_2 with wealth $x_m \geq \mu^1(k_1+1)$ ever proposes to w_3; and since she does not accept any proposals from men with wealth $x_m < \mu^1(k_1+1)$; she matches in equilibrium only with men in set M_1

We can now characterize $\mu^1(k_1+1)$ as

$$\mu^1(k_1+1) = \frac{\alpha}{r}\theta\int_{\mu^1(k_1+1)}^{\mu^1(k_1)} \left(\mu^1(k_1) - x\right) dx - 1$$

Steps 1, 2 and 3 prove that equations 12 and 13 characterize $\omega^1(k_1)$ and $\omega^2(k_2)$ for all k_1 such that $i+1 \geq k_1$ and k_2 such that $j+1 \geq k_2$. However, at the beginning of Step 4 we assumed that $k_1 < 1$; and proved the statement for $k_1 + 1$. Therefore equations 10 and 11 characterize $\mu^1(k_1)$ and $\mu^2(k_2)$ for all k_1 such that $i \geq k_1$ and k_2 such that $j \geq k_2$.

That completes the proof of claim 3. ∎

Claim 4: Let $i \in \{0, \ldots, n_1\}$ and $j \in \{0, \ldots, n_2\}$ be the smallest integers such that $\omega^2(j+1) \geq \omega^1(i+1)$. Then we obtain the partitions as follows:

1. For all k_1 s.t. $i < k_1$ and k_2 s.t $j < k_2$;

$$\mu^1(k_1) = \frac{\alpha}{r}\left[\int_{\mu^1(k_1)}^{\mu^1(k_1-1)} \theta\left(\mu^1(k_1-1) - x\right)dx + \int_{\mu^1(k_1)}^{\mu^2(j)} (1-\theta)\left(\mu^2(j) - x\right)dx\right] - 1;$$

$$\mu^2(k_2) = \frac{\alpha}{r}\left[\int_{\mu^2(k_2)}^{\mu^2(k_2-1)} (1-\theta)\left(\mu^2(k_2-1) - x\right)dx + \int_{\mu^2(k_2)}^{\mu^1(i)} \theta\left(\mu^1(i) - x\right)dx\right] - 1;$$

2. For all k_1 s.t. $i+1 < k_1$ and k_2 s.t $j+1 < k_2$;

$$\begin{aligned}\omega^1(k_1) = \frac{\alpha}{r}\Bigg[&\int_{\omega^1(k_1)}^{\omega^1(k_1-1)} \left(\omega^1(k_1-1) - x\right)dx + \\ &\int_{\omega^2(b_{k_1})}^{\omega^1(i)} \left(\omega^1(i) - x\right)dx + \int_{\omega^1(k_1)}^{\omega^2(b_{k_1})} \left[\omega^1(i) - \omega^2(b_{k_1})\right]dx - 1\Bigg];\end{aligned}$$

where b_{k_1} is the smallest integer $b_{k_1} \in \{j+1, \ldots, n_2\}$ such that $\mu^1(k_1) \geq \mu^2(b_{k_1})$.

$$\begin{aligned}\omega^2(k_2) = 2 - \frac{\alpha}{r}\Bigg[&\int_{\omega^2(k_2-1)}^{\omega^2(k_2)} \left(x - \omega^2(k_2-1)\right)dx + \\ &\int_{\omega^2(j)}^{\omega^1(b_{k_2})} \left(x - \omega^2(j)\right)dx + \int_{\omega^1(b_{k_2})}^{\omega^2(k_2)} \left[\omega^1(b_{k_2}) - \omega^2(k_2)\right]dx\Bigg].\end{aligned}$$

Proof of Claim 4:

Step 1: Any woman w' with $x^1_{w'} \in [\omega^1(i+1), \omega^2(j+1)]$ does not receive proposals from any M_1 man in the first i classes, that is any man with $x_m \in [\mu^1(i), \infty]$; and from any M_2 man in the first j classes, that is any man with $x_m \in [\mu^2(j), \infty]$. She receives proposals from all other men, and so her reservation value is given by

$$\mu^1(i+1) = \frac{\alpha}{r}\left[\int_{\mu^1(i+1)}^{\mu^1(i)} \theta\left(\mu^1(i) - x\right)dx + \int_{\mu^1(i+1)}^{\mu^2(j)} (1-\theta)\left(\mu^2(j) - x\right)dx\right] - 1.$$

This class forms the $i+1$st M_1 class as well as the $j+1$st M_2 class as so we have $\mu^1(i+1) = \mu^2(j+1)$. In equilibrium, women in the $i+1$st M_1 class and the $j+1$st M_2 class, match only with men in the $i+1$st M_1 class and men in the $j+1$th M_2 class. Men in the $i+1$st M_1 class however match with women in the $i+1$st M_1 class and to women in the $j+1$st to n_2th M_2 class. Similarly men in the $j+1$st M_2 class match with women in the $j+1$st M_2 class and to women in the $i+1$st to n_1th M_1 class

Step 2: For all $k > i+1$ any woman w' with $x^1_{w'} \in [\omega^1(k), \omega^1(k-1))$, gets proposals from all men in the kth M_1 class, and men in the kth to $j+1$st M_2 class and so

$$\mu^1(k) = \frac{\alpha}{r}\left[\int_{\mu^1(k)}^{\mu^1(k-1)} \theta\left(\mu^1(k-1) - x\right)dx + \int_{\mu^1(k)}^{\mu^2(j)} (1-\theta)\left(\mu^2(j) - x\right)dx\right] - 1.$$

and for all $k > j+1$ any woman w' with $x^1_{w'} \in (\omega^2(k-1), \omega^2(k)]$, get proposals from all men in the kth M_2 class, and men in the kth to $i+1$st M_1 class and so

$$\mu^2(k) = \frac{\alpha}{r}\left[\int_{\mu^2(k)}^{\mu^2(k-1)} \theta\left(\mu^2(k-1) - x\right)dx + \int_{\mu^2(k)}^{\mu^1(i)} (1-\theta)\left(\mu^1(i) - x\right)dx\right] - 1.$$

Step 3: For all $k > i+1$, any m such that $x_m \in [\mu^1(k), \mu^1(k-1))$, gets proposals from all women with $x_w^1 \in [\omega^1(k), \omega^1(k-1))$ and women with $x_w^1\ (\omega^2(b_k), \omega^1(i)]$ where b_k is the smallest integer $b_k \in \{j+1, ..., n_2\}$ such that $\mu^1(k) \geq \mu^2(b_k)$. And so

$$\omega^1(k) = \frac{\alpha}{r}\left[\int_{\omega^1(k)}^{\omega^1(k-1)} \left(\omega^1(k-1) - x\right)dx + \int_{\omega^2(b_k)}^{\omega^1(i)} \left(\omega^1(i) - x\right)dx + \int_{\omega^1(k)}^{\omega^2(b_k)} \left[\omega^1(i) - \omega^2(b_k)\right]dx - 1\right];$$

similarly for all $k > j+1$ we get

$$\omega^2(k) = 2 - \frac{\alpha}{r}\left[\int_{\omega^2(k-1)}^{\omega^2(k)} \left(x - \omega^2(k-1)\right)dx + \int_{\omega^2(j)}^{\omega^1(b_k)} \left(x - \omega^2(j)\right)dx + \int_{\omega^1(b_k)}^{\omega^2(k)} \left[\omega^1(b_k) - \omega^2(k)\right]dx\right].$$

where b_k is the smallest integer $b_k \in \{i+1, ..., n_1\}$ such that $\mu^2(k) \geq \mu^1(b_k)$.

That completes the proof of claim 4□.

7.2 Section 4

Characterizing steady state equilibrium in the environment when women differ continuously along two traits, height and wealth, and when men have heterogenous preferences, with one group preferring tall women, and the other preferring wealthy women:

Given (G_m, G_w^1, G_w^2), a steady state equilibrium exists and is unique. It is characterized by three partitions, μ, ω^1, ω^2 on $[0,1]$, with μ partitioning men on the basis of their wealth, ω^1 partitioning women on the basis of their height and ω^2 partitioning women on the basis of their wealth.

These partitions can be determined as follows: Set $\omega^1(0) = \omega^2(0) = \mu(0) = 1$. Begin by computing the reservation value of the woman $(1, 1)$ who gets proposals from all men, and term this reservation value $\mu(1)$. All men with $x_m \geq \mu(1)$ belong to the first class of men.

Next take a man in set M_1 who belongs in the first class: He gets proposals from all women and we will compute and denote his reservation value as $\omega^1(1)$. This is also the reservation values of all M_1 men in class 1. Do the same for a man in set M_2 who belongs to class 1, and denote his reservation value as $\omega^2(1)$. The reservation values of all M_2 men in class 1 is $\omega^2(1)$. Now all women with $x_w^1 \geq \omega^1(1)$ and $x_w^2 \geq \omega^2(1)$ form the first class of women.

Now take a woman who assumes she gets proposals from all M_1 men, and all M_2 men who are not in class 1. Compute her reservation value and term it $a(1)$. Term the reservation value of the woman who assumes she gets proposals from all M_2 men, and all M_1 men who are not in class 1. Term this $b(1)$.

If $a(1) \geq b(1)$, then $\mu(2) = a(1)$. Men in $[\mu(2), \mu(1))$ form the second class of men and the reservation values of men of type M_1 in this class is $\omega^1(2)$, and those of men M_2 in this class is $\omega^2(2)$. All women with $x_w^1 \geq \omega^1(1)$ and $\omega^2(1) > x_w^2 \geq \omega^2(2)$, have reservation value $\mu(2)$ (If $a(1) = b(1)$, then women with $x_w^2 \geq \omega^2(1)$ and $\omega^1(1) > x_w^1 \geq \omega^1(2)$, also have reservation value $\mu(2)$).

Now take the woman who assumes she gets proposals from all M_1 men, and all M_2 men who are not in classes 1 and 2, and let her reservation value be $a(2)$. If $a(2) \geq b(1)$, then $\mu(3) = a(2)$ and men in $[\mu(3), \mu(2))$ form the thrid class of men. Let $\omega^1(3)$ be the reservation value of M_1 men in class 3 and $\omega^2(3)$ be the reservation value of M_2 men in class 3. Then all women with $x_w^1 \geq \omega^1(1)$, and $\omega^2(2) > x_w^2 \geq \omega^2(3)$ have reservation value $\mu(3)$.

One continues in this manner until we get the equilibrium partition.

A word of explanation on what produces the classes when very wealthy woman match only with men in set M_2 (or when very tall women match only with men in set M_1). Let some woman be such that $x_w^2 \geq \omega^2(1)$ and $x_w^2 < \omega^1(3)$. So this woman gets proposals from all men in set M_1, but does not get proposals from M_1 men in classes 1, 2 and 3. She computes her reservation value assuming that men in M_1 below class 3 propose to her, and denote it $b(3)$: If $b(3) \geq \mu(3)$; then this woman accepts no proposals from any man in set M_1 (since the only M_1s who propose to her have $x_m < \mu(3)$); and so in equilibrium she matches only with men in set M_2:

7.3 Proofs from Section 5

7.3.1 Proposition 3

In any core allocation, the payoff to man m is $\sqrt{x_m}$; and the payoff to the woman is $\left(\max\left\{x_w^s; x_w^b\right\}\right)^2$:

Proof of Proposition 3:

Step 1: Let $T = \{w : \pi(w) \in M_1\}$; be the set of all women matched with men in set M_1: Similarly let $S = \{w : \pi(w) \in M_2\}$; be the set of women matched with men in set M_2:

Claim 6: If π is a core allocation. The utility of women $u(\pi; w)$ is nondecreasing in x_w^1 over the set T, and the utility of men $u(\pi; m)$ is non-decreasing in x_m over M_1: Also the utility of women is nondecreasing in x_w^2 over the set S.and that of men is nondecreasing in x_m over M_2:

Proof of Claim 6: Suppose $u(\pi; m)$ is not weakly increasing in x_m over the set M_1: Then $\exists m; m'$ s.t. $(m; m') \in M_1$, $x_m > x_{m'}$; but $u(\pi; m) < u(\pi; m')$: Then m and $w = \pi(m')$ form a coalition that blocks π: Now suppose $u(\pi; w)$ is not weakly increasing over T: Then $\exists w; w'$ st $(w; w') \in T$, $x_w^s > x_{w'}^s$; and $u(\pi; w) < u(\pi; w')$: Then w and $m = \pi(w')$ form a coalition that blocks π: Similarly we can prove that the utility of women is nondecreasing in x_w^2 over the set S.and that of men is nondecreasing in x_m over M_2: $\blacksquare$

Step 2: Let $W_1^c = \{w : x_w^1 \geq c; x_w^1 \geq x_w^2\}$; for some $c \in [0; 1]$ and let $W_2^c = \{w : x_w^2 \geq c; x_w^2 \geq x_w^1\}$: Let $E_1^c = \{w : x_w^1 \geq x_w^2 \geq c\}$; and $E_2^c = \{w : x_w^2 \geq x_w^1 \geq c\}$ (Figures 6A and B indicate these sets). It is obvious that $E_1^c \subset W_1^c$ and $E_2^c \subset W_1^c$:

Let $M_1^c = \left\{m : x_m > c; i \in \left(0; \frac{1}{2}\right]\right\}$; be all the man in set M_1 with wealth strictly greater than c; and $M_2^c = \left\{m : x_m > c; i \in \left(\frac{1}{2}; 1\right]\right\}$; be all the men in set M_2 with wealth strictly greater than c:

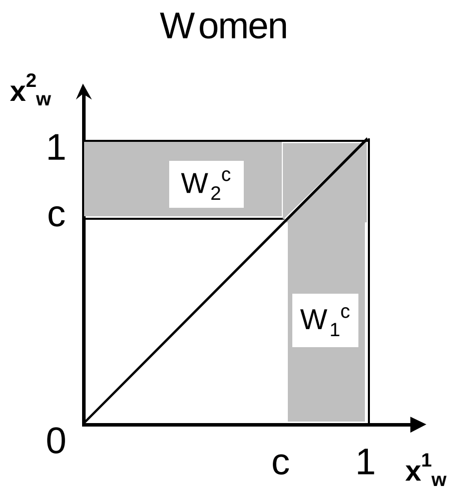

Figure 6A

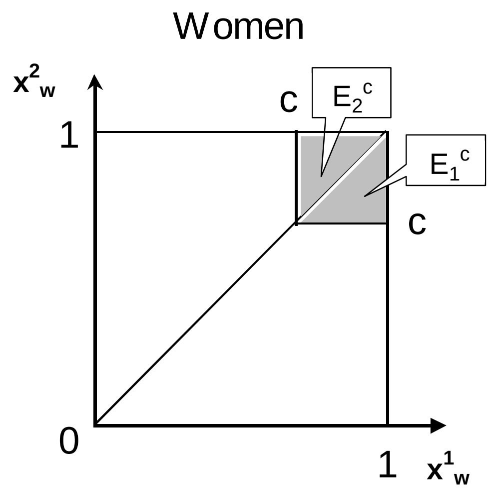

Figure 6B

Claim 7: If π is a core allocation, then it must be true that $u(\pi; w) \geq c^2$ for all women $w \in E_1^c \cup E_2^c$, for all $c \in (0, 1)$.

Proof of Claim 7: Suppose not, then $\exists w_1 \in E_1^c \cup E_2^c$ st $u(\pi; w_1) < c^2$. Let the wealth of her partner under π be $x_{\pi(w_1)} = c^2 - \varepsilon$ for some $\varepsilon > 0$. All men $m_1 \in M_1$ with $x_{m_1} > c^2 - \varepsilon$ must have $u(\pi; m) \geq x_w^1 \geq c$; else $\{w_1, m_1\}$ form a blocking coalition. That is $\pi\left(M_1^{c^2-\varepsilon}\right) \in W_1^c \cup E_2^c$. All men $m_2 \in M_2$ with $x_m > c^2 - \varepsilon$ must have $u(\pi; m) \geq x_w^2 \geq c$; else $\{w_1, m_2\}$ form a blocking coalition. That is $\pi\left(M_2^{c^2-\varepsilon}\right) \in W_2^c \cup E_1^c$. $\mu_m\left(M_1^{c^2-\varepsilon} \cup M_2^{c^2-\varepsilon}\right) = 1 - c^2 + \varepsilon$. $\mu_w\left(\pi\left(M_1^{c^2-\varepsilon} \cup M_2^{c^2-\varepsilon}\right)\right) = \mu_w\left(W_1^c \cup E_2^c \cup W_2^c \cup E_1^c\right) = 1 - c^2$. Contradiction, since a feasible allocation must have $\mu_m\left(M_1^{c^2-\varepsilon} \cup M_2^{c^2-\varepsilon}\right) = \mu_w\left(\pi\left(M_1^{c^2-\varepsilon} \cup M_2^{c^2-\varepsilon}\right)\right)$. ■

This claim establishes that $u(\pi; w) \geq \left(\min\left(x_w^1, x_w^2\right)\right)^2$ for all $w \in W$.

Claim 8: If π is a core allocation, then it must be true that $u(\pi; w) \geq c^2$ for all women $w \in W_1^c \cup W_2^c$ for all $c \in (0, 1)$.

Proof of Claim 8: Suppose not, then $\exists w_1 \in W_1^c \cup W_2^c$ st $u(\pi; w_1) < c^2$. Let $w_1 \in W_1^c$ and let the wealth of her partner under π be $x_{\pi(w_1)} = c^2 - \varepsilon$ for some $\varepsilon > 0$. All men $m_1 \in M_1$ with $x_{m_1} > c^2 - \varepsilon$ must have $u(\pi; m) \geq x_{w_1}^1 = d$; else $\{w_1, m_1\}$ form a blocking coalition. Therefore $\pi\left(M_1^{c^2-\varepsilon}\right) \subset W_1^c \cup E_2^c$. Now $\mu_w(W_1^c) = \frac{1-c^2}{2}$ and $\mu_m\left(M_1^{c^2-\varepsilon}\right) = \frac{1-c^2+\varepsilon}{2}$; therefore it must be true that the set of men $m \in M_1^{c^2-\varepsilon}$ with $\pi(m) \in E_2^c$ is a set of positive measure. That is $M_1^{c^2-\varepsilon} \cap \pi(E_2^c)$ has measure $\delta \geq \frac{\varepsilon}{2} > 0.$ Given claim 6, we know that $u(\pi; w)$ is increasing over $\pi\left(M_1^{c^2-\varepsilon}\right)$ in $x_w^1.$ Therefore there exists a set of women $w \in E_1^c \cup E_2^c$ with $u(\pi; w) < c^2.$ This constitutes a contradiction of Claim 7. $\blacksquare$

This establishes that $u(\pi; w) = (\max(x_w^1, x_w^2))^2.$ It is now trivial to see that $T = \{w : x_w^1 \geq x_w^2\}$ and $S = \{w : x_w^1 < x_w^2\}$. That is, the set of women with $x_w^1 \geq x_w^2$ is exactly the same as the set of women matched to men in set M_1, and the set of women with $x_w^2 > x_w^1$ is exactly the same as the set of women matched to men in set M_2. That ends the proof of proposition 3. $\blacksquare$

7.3.2 Proposition 4

The exists a sequence of games with finite numbers of players, with the set of women converging to W, the set of men converging to M, and a sequence of allocations that belong in the core of each finite game in the sequence, that converges (in measure) to $\pi_1.$

Proof of Proposition 4: I will prove this proposition by construction. Let $N = 2^n$ for $n = 1, 2, \ldots.$ For each N, there are a finite number of men and women $M_n = W_n = (N+1)N.$ Any man m is characterized by his level of wealth and his preference over the traits of women, (x_m, i). The level of wealth of any man could be $x_m = 0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}, 1$, and preferences could be $i = \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}, 1$. If $i \in \left(0, \frac{1}{2}\right)$, then the man belongs to set M_1; he derives utility from the height of the woman he mates with, and is indifferent to her level of wealth. If $i \in \left(\frac{1}{2}, 1\right)$, then he belongs to set M_2; he derives utility from the level of wealth of the woman and is indifferent to her height. A woman's height could be one of $x_w^1 = 0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}, 1$, and her level of wealth could be one of $x_w^2 = \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}, 1.$ We can see that as $n \to \infty$, $M_n \to M$, and $W_n \to W.$

For any n, let $s = 0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}, 1.$ In each of these finite games we use the following algorithm to define one element in the core of that finite game. These core elements converge to π_1 as $n \to \infty.$

Algorithm: By the algorithm all men of type M_1 are assigned to women with $x_w^1 \geq x_w^2.$ All men of type M_2 are assigned to women with $x_w^2 > x_w^1.$ The precise definition of the algorithm that results in a one-to-one and onto assignment of assignment of men to women would require a lot of notation. I describe it roughly here.

- Assign any man in set M_1 with $(s; i)$ to a woman as follows:

(1) Her height $x_w^1 = \frac{j+k}{N}$; where j is given by

$$N^2 s \geq j^2 + j \tag{18}$$
$$\text{and } j^2 + 3j + 2 \geq N^2 s \tag{19}$$

and $K \geq j + k$; K given by

$$N^2 \left(s + \frac{1}{N} \right) \geq K^2 + K \tag{20}$$
$$\text{and } K^2 + 3K + 2 \geq N^2 \left(s + \frac{1}{N} \right) \tag{21}$$

It can be shown that $\sqrt{N} \geq k$. Therefore s determines her height.

(2) The level of wealth x_w^2 of the woman that the man is assigned to is determined as $i = \frac{x_w^2}{2x_w^1} \pm \frac{l}{N}$; where l could be different for each man, and $\sqrt{N} \geq l$. l is decided on by taking care that each woman is assigned to only one man. Therefore i determines the woman's wealth level.

- Assign any man in set B with $(s; i)$ to a woman with wealth $x_w^2 = \frac{j+k}{N}$; where j is given by 18 and 19, and j by 20 and 21.

Notice that s determines wealth of the woman.

The woman's height is determined by i, by setting $i = \frac{x_w^2 - x_w^1}{2x_w^1} \pm \frac{l}{N} + \frac{1}{2}$; where l could be different for each man, and $\sqrt{N} \geq l$; once again making sure that each woman is assigned to one man only.

Claim A1:

The assignment described by the above algorithm belongs in the core of each finite game.

Proof of Claim A1:

Any man with wealth s in set M_1 is assigned to some woman with height $x_w^1 = \frac{j-k}{N}$; j and k defined as above. For any given s; the number of men with $x_m < s$; and belonging to set M_1 is $\frac{N^2 s}{2}$: The number of women with $\frac{j}{N} \geq x_w^1$ is $\frac{(j+1)j}{2}$. Every man with $x_m < s$ must be assigned to some woman with $\frac{j}{N} \geq x_w^1$ in the core (else a blocking pair can be formed). Therefore the j should be the smallest integer such that

$$\frac{(j+1)j}{2} \geq \frac{N^2 s}{2}$$

which gives us 18. The fact that j should be the smallest integer such that this is true gives us 19. At the same time $j+k$ should also be such that m with $x_m = s$ in set M_1 is not assigned a woman with greater intelligence than any woman assigned to men with $x_m > s$. This gives us conditions for k. Since the men are indifferent to wealth, the level of wealth of the woman they are assigned to is immaterial in determining if the allocation is in the core. The above argument can be repeated in the case when the men belong to set M_2. This completes the proof that the allocation described by the algorithm is in the core. $\blacksquare$

Notice that the core allocation is by no means unique.

Claim A2: The sequence of core allocations of the finite game converges to π_1.

Proof of Claim A2: Any man with wealth s in set M_1 matches with a woman of height $\frac{j+k}{N}$; j given in equations 18 and 19, and $\sqrt{N} \geq k$. Approximating j

$$j^2 + j = N^2 s$$

$$j = \frac{1 \pm \sqrt{1+4N^2 s}}{2}.$$

Therefore we have the intelligence of the woman who matches with this man is:

$$x_w^1 = \frac{\frac{1+\sqrt{1+4N^2 s}}{2} + k}{N};$$

simplifying which we get

$$x_w^1 = \frac{1}{2N} + \sqrt{\frac{1}{4N^2} + s + \frac{k}{N}}$$

As $n \to \infty$; $N \to \infty$; therefore $x_w^1 \to \sqrt{s}$. Therefore in the limit $u(\pi; m) = \sqrt{x_m}$ for any man with x_m that is rational. Similarly, $u(\pi; w)$ for any woman w with rational $(x_w^1; x_w^2)$ is $(\max\{x_w^1; x_w^2\})^2$ in the limit.

The level of wealth x_w^2 of the woman that the man in set M_1 with preference i is assigned to is determined as $i = \frac{x_w^2}{2x_w^1} \pm \frac{l}{N}$; where $\sqrt{N} \geq l$. When $n \to \infty$; we have $i = \frac{x_w^2}{2x_w^1}$. The height x_w^1 of the woman that the man in set M_2 with preference i is assigned to is determined as $i = \frac{x_w^2 - x_w^1}{2x_w^2} \pm \frac{l}{N} + \frac{1}{2}$; where $\sqrt{N} \geq l$. When $n \to \infty$; we have $i = \frac{x_w^1 - x_w^2}{2x_w^2} + \frac{1}{2}$.

Notice that all the men with $x_m = 0$ match with some woman in the core of each finite game. However, in the limit, the height of each of these women converges to 0 as does their wealth. But $(0; 0) \notin W$; and so in the continuum game, these men remain single. Therefore the convergence of the core of the finite games to the continuum game is only in measure.

We have shown that the core assignments in the finite game does converge to the core assignment in the continuum game for all men with levels of wealth that are rational except those with $x_m = 0$. Since π_1 is a continuous function a.e, this ensures that the core assignments in the finite game does converge to the core assignment in the continuum game for all men with levels of wealth, except for those with $x_m = 0$.