Incentives, informational economies of scale, and benchmarking[1]

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1 Introduction

In a world of complete markets with well-functioning financial markets, there is no room for a theory of incentives. Competition and bankruptcy mechanisms ensure that agents have the incentives of performing to their best of knowledge and abilities. There are, however, many instances where markets are absent. For example, production by the public sector is often not sold or sold monopolistically. Long-term relationships between a supplier and a firm often involve the production of a relation-specific good, and competition cannot discipline the supplier. Within a firm, transactions are usually not exposed to external competition. In all these cases, other types of incentive mechanisms must be designed to motivate agents.

A good part of the contract theory literature deals with the provision of incentives in these cases. This literature, however, often looks at incentives in a nutshell, ignoring that some form of endogenous competition can be created when there are more than one project involved. For example, Toyota is renowned for using dual-sourcing when dealing with suppliers to generate some form of competition between them. It then seems important to better understand how and when such endogenous competition can be used to provide incentives.

Endogenous competition may have benefits in providing incentives: agents are highly motivated in performing better than their "rivals." There are however, costs to generating such competition. When using two or more agents, there may be costs that are duplicated, thus creating an inefficiency that must be supported by the principal. Higher-powered incentives based on competition must then be weighted against duplication costs.

In this paper, we analyze the problem of providing incentives when there are more than one project. A principal has access to two (possibly correlated) projects which are managed by a single agent. Before undertaking a project, the agent-manager can spend some resources to investigate its quality, namely its probability of success. Only projects with a high probability of success are profitable, and therefore should be invested in. There are two classes of strategies. First, the manager may investigate only one project to save on investigation costs, but still use the acquired information to make an investment decision on the noninvestigated project. Second, the manager could investigate both projects, and make the investment decision based on the acquired information on each project. Comparing these two strategies, it would seem that the more correlated are the projects, the better it is to investigate only one project and use the acquired information to learn about the other project. And, when correlation is low, both projects should be investigated.

There is, however, a third strategy that the principal could use. He could hire two agents, each managing one project. By making each agent's compensation dependent on the outcome of the project of the other agent, the principal creates some form of competition between them. It would seem that such competition would be more incentive when projects are highly correlated. So, intuitively, it is not obvious to know when endogenous competition should be used. When projects are highly correlated, endogenous competition provides incentives but duplicate investigation costs, while having one manager investigating only one project exploits informational economies of scale by economizing on investigation costs, but does not always yield the best investment decision since information on the noninvestigated project is not perfect.

We examine the issue of the optimal organizational structure for two projects under the assumption of asymmetric information. We assume that the investigation decision is private to the manager (moral hazard), as well as the information obtained doing so (adverse selection). We then show that the optimal structure depends, among other things, on the degree of correlation between the returns of the two projects. In general, delegating to one manager and investigating both projects is optimal when the projects are weakly correlated; delegating to two managers is optimal for intermediate values of the correlation coefficient, while delegating to one manager and investigating only one project may be optimal when projects are strongly correlated, depending on parameter values. We show that, for some parameter values, it may never be optimal to delegate to one manager and investigate only one project, and this even when projects are perfectly correlated. Endogenous competition is then optimal as it minimizes the cost of providing incentives to the managers, even though investigation costs are duplicated. In that case, delegating to two managers becomes optimal for intermediate and high values of the correlation coefficient.

2 The model

A principal has access to two indivisible risky projects whose investment cost is I. Each project can either be successful and earn R, or fail and earn 0. There are two types of projects. A valuable project is successful with probability $\overline{\pi}$, while a bad project is successful with probability $\underline{\pi} < \overline{\pi}$. We assume that $\overline{\pi}R > I$ and $\underline{\pi}R < I$, so that it is only profitable to invest in valuable projects. The return from not investing in a project is I. Before investing in a project or not, it is possible to investigate a project at cost $c > 0$. Such investigation yields a perfect signal on probability of success of the project, that is, the signal is $\overline{\pi}$ if the project is valuable, and the signal is $\underline{\pi}$ if the project is bad. The probability that a project is valuable is P. We assume that

$$P\overline{\pi}R + (1-P)I - c > \max\{I, P\overline{\pi}R + (1-P)\underline{\pi}R\},$$

which implies that it is efficient to investigate a project for making the efficient investment decision, rather than not investigate and not invest or invest blindly.[1]

The types of the two projects are correlated. The joint distribution for the types of the two projects is given in the following table.

$1 \setminus 2$	$\overline{\pi}$	$\underline{\pi}$	
$\overline{\pi}$	$P^2 + k$	$P(1-P) - k$	$[P]$
$\underline{\pi}$	$(1-P)P - k$	$(1-P)^2 + k$	$[1-P]$
	$[P]$	$[1-P]$	

The parameter k represents an index of correlation. When $k = 0$, the types of the two projects are distributed independently. When $k = P(1-P)$, the types of the two projects are perfectly correlated. We assume that $0 \le k \le P(1-P)$, that is, we assume that projects are positively correlated, which seems to be the natural assumption. (The case of negative correlation could be analyzed using the approach adopted below.) Marginal probabilities are given in square brackets in the table.

The principal pays for the investment I in each project and delegates its management to an agent. The agent is then responsible for investigating the project or not, and deciding whether to invest in it or not. We assume that the principal and the agent(s) are risk neutral, that the reservation value of the agent is 0, and that the agent has limited liability in that she cannot be paid a negative wage.

We consider two different organizational structures for managing the two projects. First, the principal can hire the same manager for the two projects. In this case, the timing is as follows.

1. The principal proposes a contract to the manager. If it is rejected, both earn 0.

[1] The basic set-up is an adaptation of a model proposed by Nier (1997) to which we add a correlation factor between projects.

2. If it is accepted, the principal gives $2I$ to the manager.

3. The manager then decides which projects to investigate to learn their type. She bears a personal cost of c per investigated project.

4. Given the signals obtained following her investigation decision, the manager makes an investment decision for each project.

5. The returns are realized and accrue to the principal. The manager is remunerated according to the contract.

The form of the contract proposed by the principal depends on informational assumptions which we introduce later. Note that the manager does not have to make the same investigation and investment decisions on the two projects.

In a second organizational structure, the principal can hire two different managers, one for each project. In that case, the timing is as follows.

1. The principal proposes a contract to each manager. If it is rejected, both earn 0.

2. If it is accepted, the principal gives I to each manager.

3. Each manager then decides whether to investigate her project or not at a personal cost of c.

4. Given the signal obtained following her investigation decision, each manager makes an investment decision for her project.

5. The returns are realized and accrue to the principal. Each manager is remunerated according to the contract.

We first assess the relative efficiency of these two structures under the assumption that the investigation decision and the signal it provides are observable to the principal. This yields a benchmark for the more interesting case of asymmetric information, that is, when the investigation decision and the signal it provides are private information to the agent(s).

3 Symmetric information

First, we assume that the principal hires only one manager for the two projects. Suppose that two projects are investigated. Given our assumptions, it is then efficient to invest in a project if and only if its signal is $\overline{\pi}$. Call such strategy d^*. The expected profit from d^* is

$$\Pi(d^*, k) - 2c \equiv 2(P\overline{\pi}R + (1-P)I - c).$$

Note that this profit is independent of the correlation factor k.

Suppose now that only one project is investigated. On the investigated project, the efficient investment decision is to invest if and only if the signal is $\overline{\pi}$. On the noninvestigated project, the efficient investment decision depends on its updated value when using the signal obtained on the investigated project. This update depends, among other things on the degree of correlation k. There are three possible cases: first, make the same investment decision on both projects (strategy $\widetilde{d}_1$); second, always invest in the noninvestigated project (strategy $\widetilde{d}_2$); and third, never invest in the noninvestigated project (strategy $\widetilde{d}_3$). The expected profit from these strategies is, respectively:

$$\Pi(\widetilde{d}_1, k) - c \equiv \left(P\overline{\pi} + (P^2 + k)\overline{\pi} + (P(1-P) - k)\underline{\pi}\right) R + (1-P)2I - c,$$

$$\Pi(\widetilde{d}_2, k) - c \equiv P\overline{\pi}R + (1-P)I + P\overline{\pi}R + (1-P)\underline{\pi}R - c,$$

$$\Pi(\widetilde{d}_3, k) - c \equiv P\overline{\pi}R + (1-P)I + I - c.$$

It is easy to show that

$$\Pi(\widetilde{d}_1, 0) < \max\left\{\Pi(\widetilde{d}_2, 0), \Pi(\widetilde{d}_3, 0)\right\}$$

and that

$$\Pi(\widetilde{d}_1, P(1-P)) > \max\left\{\Pi(\widetilde{d}_2, P(1-P)), \Pi(\widetilde{d}_3, P(1-P))\right\}.$$

When projects are independent, having the same investment strategy for both projects is never optimal. When projects are perfectly correlated, having the same investment strategy is always optimal. Note that $\Pi(\widetilde{d}_2, k)$ and $\Pi(\widetilde{d}_3, k)$ are independent of the correlation factor

k. The strategy $\widetilde{d}_2$ is preferred to strategy $\widetilde{d}_3$ if and only if blindly investing is better than not investing, that is, if and only if $P\overline{\pi}R + (1-P)\underline{\pi}R > I$.

We can now assess whether it is better to investigate one project or two. It is easy to see that $\Pi(d^*, k) > \max\left\{\Pi(\widetilde{d}_2, k), \Pi(\widetilde{d}_3, k)\right\}$ because of the assumption that investigating and investing efficiently is better than not investigating and investing blindly or not investing. It then remains to compare strategy d^* to strategy $\widetilde{d}_1$. A convenient way to rewrite $\Pi(\widetilde{d}_1, k)$ is

$$\Pi(\widetilde{d}_1, k) - c = 2(P\overline{\pi}R + (1-P)I) - c - (P(1-P) - k)(\overline{\pi} - \underline{\pi})R,$$

where the last term represents the loss of only investigating one project and investing in both. Note that this loss vanishes as correlation becomes perfect, that is, when $k = P(1-P)$. It is easy to show that

$$\Pi(\widetilde{d}_1, k^*) - c = \Pi(d^*, k^*) - 2c \quad \text{where} \quad k^* = P(1-P) - \frac{c}{R(\overline{\pi} - \underline{\pi})}.$$

This implies that strategy $\widetilde{d}_1$ is optimal if and only if $k \geq k^*$, and strategy d^* is optimal for $k \leq k^*$.

Second, suppose that the principal hires two managers, one per project. Given our assumptions, it is efficient for each manager to investigate her project and invest if and only if the signal is $\overline{\pi}$. Call such strategy $\widehat{d}$. The expected profit from $\widehat{d}$ is

$$\Pi(\widehat{d}, k) - 2c \equiv 2(P\overline{\pi}R + (1-P)I - c).$$

Note that this profit is independent of the correlation factor k, and is equivalent to $\Pi(d^*, k) - 2c$.

In a world of perfect information, the principal can easily implement the first-best strategy for all values of k. First, the principal hires a manager. If $k \leq k^*$, the principal pays her a constant wage $2c$ if she follows the strategy d^*, and 0 otherwise. If $k > k^*$, the principal pays her a constant wage c if she follows the strategy $\widetilde{d}$, and 0 otherwise. Note that it is necessary to pay the manager a positive wage to cover for her personal investigation cost c per investigated project. In both cases, the manager has incentives to follow the first-best strategy. Such contract is feasible since all decisions and signals are assumed to be observable and verifiable. Furthermore, it is not necessary to hire two managers. The first-best allocation can be attained with only one manager and an appropriate contract. We now move to the more interesting case where information is incomplete.

4 Asymmetric information

We assume that the principal cannot observe whether the manager has investigated a project or not, and if she has, what is the value of the signal. Output is, however, assumed to be observable and verifiable. Since $0 < I < R$, the principal can infer whether the manager has invested in the risky project or not. Our model is one of moral hazard (the investigation decision) and hidden information (the value of the signal). We first assume that one manager is in charge of both projects, and we evaluate the efficiency of the strategies d^*, $\widetilde{d}_1$, $\widetilde{d}_2$, and $\widetilde{d}_3$. We know when each of these strategies is optimal under symmetric information. With asymmetric information, however, informational rents may alter the ranking of these strategies. It is, therefore, necessary to re-evaluate each strategy under asymmetric information.[2]

4.1 Hiring one manager

In this environment, a remuneration contract is a wage schedule that is conditional on the observed output. There are six different outcomes depending on investment decisions and whether an invested project is successful or not. These outcomes are denoted z_s with $z_s \in \{2R, R, R+I, I, 2I, 0\}$. A wage profile is then a vector $\{w(z_s)\}$.[3]

4.1.1 Implementation of the strategy d^*

The principal wants to minimize the expected wage of the manager subject to her participation constraint and incentive constraints designed to induce the manager in implementing strategy d^*. Remember that under strategy d^*, the manager investigates the two projects and invests only when the signal is favorable ($\overline{\pi}$). The expected wage is

$$\mathrm{E}_{d^*}(w(z_s)) \equiv \left(P^2 + k\right)\left(\overline{\pi}^2 w(2R) + 2\overline{\pi}(1-\overline{\pi})w(R) + (1-\overline{\pi})^2 w(0)\right) +$$

[2] We assume here that it is optimal for the principal to implement either one of the first-best strategies. We later compare them to simpler strategies and provide conditions for these first-best strategies to be second-best optimal.

[3] Since the two projects are ex ante identical and the principal cannot observe investigation decisions, there is no loss of generality in assuming that wages are function of only the sum of the returns of the two projects.

$$2\left((1-P)P-k\right)\left(\overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\right)+\left((1-P)^2+k\right)w(2I).$$

We now describe all constraints that must be satisfied for a contract to implement strategy d^*. First, the manager's participation constraint must be satisfied.

$$\text{(1)} \quad \mathrm{E}_{d^*}(w(z_s))-2c\geq 0$$

Second, the efficient investment decisions are implemented by satisfying a set of incentive constraints. There are four possible combinations of signals that the manager can obtain, that is, she can be either one of four types: $(\overline{\pi},\overline{\pi}),(\overline{\pi},\underline{\pi}),(\underline{\pi},\overline{\pi}),(\underline{\pi},\underline{\pi})$. We consider in turn incentive constraints for these four types.

• THE SIGNALS ON THE INVESTIGATED PROJECTS ARE $(\overline{\pi},\overline{\pi})$.

When both signals are favorable, the manager should invest in both projects. Her expected wage for doing so must be at least as large as that from not investing in any project or investing in only one. This translates into the following constraints.

$$\text{(2)} \quad \overline{\pi}^2w(2R)+2\overline{\pi}(1-\overline{\pi})w(R)+(1-\overline{\pi})^2w(0)\geq w(2I)$$

$$\text{(3)} \quad \overline{\pi}^2w(2R)+2\overline{\pi}(1-\overline{\pi})w(R)+(1-\overline{\pi})^2w(0)\geq \overline{\pi}w(R+I)+(1-\overline{\pi})w(I)$$

• THE SIGNALS ON THE INVESTIGATED PROJECTS ARE $(\overline{\pi},\underline{\pi})$ or $(\underline{\pi},\overline{\pi})$.

When one signal is favorable and the other one is not, the manager must be induced in investing only in the project with the good signal, rather than not investing at all, or investing in the bad project, or investing in both projects. We then have the following incentive constraints.

$$\text{(4)} \quad \overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\geq w(2I)$$

$$\text{(5)} \quad \overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\geq \underline{\pi}w(I+R)+(1-\underline{\pi})w(I)$$

$$\text{(6)} \quad \overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\geq \overline{\pi}\underline{\pi}w(2R)+(\overline{\pi}(1-\underline{\pi})+(1-\overline{\pi})\underline{\pi})w(R)+(1-\overline{\pi})(1-\underline{\pi})w(0)$$

• THE SIGNALS ON THE INVESTIGATED PROJECTS ARE $(\underline{\pi},\underline{\pi})$.

When both signals are unfavorable, the manager should not invest, rather than invest in one project, or invest in both.

$$(7) \quad w(2I) \geq \underline{\pi} w(R+I) + (1-\underline{\pi}) w(I)$$

$$(8) \quad w(2I) \geq \underline{\pi}^2 w(2R) + 2\underline{\pi}(1-\underline{\pi}) w(R) + (1-\underline{\pi})^2 w(0)$$

We note that equation (2) is satisfied when equations (3) and (4) are, and that equation (5) is satisfied when equations (4) and (7) are.

Third, the manager must be induced in investigating both projects. We therefore have to specify incentive constraints for the investigation decision.

• Discouraging no investigation.

The manager has to prefer to investigate the two projects rather than none and not invest, invest in one of the projects, or invest in both. This translates into the following incentive constraints.

$$(9) \quad \mathrm{E}_{d^*}(w(z_s)) - 2c \geq w(2I)$$

$$(10) \quad \mathrm{E}_{d^*}(w(z_s)) - 2c \geq (P\overline{\pi} + (1-P)\underline{\pi})\, w(R+I) + (P(1-\overline{\pi}) + (1-P)(1-\underline{\pi}))\, w(I)$$

$$(11) \quad \mathrm{E}_{d^*}(w(z_s)) - 2c \geq \left(P^2 + k\right)\left(\overline{\pi}^2 w(2R) + 2\overline{\pi}(1-\overline{\pi}) w(R) + (1-\overline{\pi})^2 w(0)\right) +$$
$$2\left((1-P)P - k\right)\left(\overline{\pi}\underline{\pi} w(2R) + ((1-\overline{\pi})\underline{\pi} + \overline{\pi}(1-\underline{\pi})) w(R) + (1-\overline{\pi})(1-\underline{\pi}) w(0)\right) +$$
$$\left((1-P)^2 + k\right)\left(\underline{\pi}^2 w(2R) + 2\underline{\pi}(1-\underline{\pi}) w(R) + (1-\underline{\pi})^2 w(0)\right)$$

The manager's participation constraint (1) is satisfied when equation (9) and the limited-liability constraint on $w(2I)$ are satisfied.

• Discouraging investigating only one project.

When the manager investigates only one project, she gets only one signal. Conditional on that signal, she always invests optimally in the investigated project if the incentive constraints on the investment decisions are satisfied. On the noninvestigated project, however, she can either not invest regardless of the signal on the investigated project, invest only when she gets an unfavorable signal on the investigated project, invest only when she gets a favorable signal on the investigated projet, or always invest regardless of the signal. This yields the following four incentive constraints.

$$\text{(12)}\quad \mathrm{E}_{d^*}(w(z_s)) - 2c \geq P\left(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)\right) + (1-P)w(2I) - c$$

$$\text{(13)}\quad \mathrm{E}_{d^*}(w(z_s)) - 2c \geq \Big(P^2 + k + 2\left(P(1-P) - k\right)\Big)\left(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)\right) +$$
$$\Big((1-P)^2 + k\Big)\left(\underline{\pi}w(R+I) + (1-\underline{\pi})w(I)\right) - c$$

$$\text{(14)}\quad \mathrm{E}_{d^*}(w(z_s)) - 2c \geq \Big(P^2 + k\Big)\Big(\overline{\pi}^2 w(2R) + 2\overline{\pi}(1-\overline{\pi})w(R) + (1-\overline{\pi})^2 w(0)\Big) +$$
$$\left(P(1-P) - k\right)\left(\overline{\pi}\underline{\pi}w(2R) + (\overline{\pi}(1-\underline{\pi}) + (1-\overline{\pi})\underline{\pi})w(R) + (1-\overline{\pi})(1-\underline{\pi})w(0)\right) +$$
$$(1-P)w(2I) - c$$

$$\text{(15)}\quad \mathrm{E}_{d^*}(w(z_s)) - 2c \geq \Big(P^2 + k\Big)\Big(\overline{\pi}^2 w(2R) + 2(1-\overline{\pi})\overline{\pi}w(R) + (1-\overline{\pi})^2 w(0)\Big) +$$
$$\left(P(1-P) - k\right)\left(\overline{\pi}\underline{\pi}w(2R) + ((1-\overline{\pi})\underline{\pi} + (1-\underline{\pi})\overline{\pi})w(R) + (1-\overline{\pi})(1-\underline{\pi})w(0)\right) +$$
$$\left((1-P)P - k\right)\left(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)\right) + \Big((1-P)^2 + k\Big)\left(\underline{\pi}w(I+R) + (1-\underline{\pi})w(I)\right) - c$$

Finally, we assume that the manager has limited liability, that is, $w(z_s) \geq 0$ for all z_s.

The principal's problem is to minimize $\mathrm{E}_{d^*}(w(z_s))$ subject to constraints (1) to (15), and to limited-liability constraints. A first lemma shows that $w_{d^*}(0) = w_{d^*}(I) = w_{d^*}(R) = 0$, that is, the principal "punishes" the manager every time one of the project fails.

Lemma 1 *The manager is punished each time a project fails, that is,*

$$w_{d*}(0) = w_{d*}(I) = w_{d*}(R) = 0.$$

The intuition for the proof is straightforward: if one of these wages was positive, it would always be possible to reduce it to zero and to increase $w(2R)$ or $w(R+I)$, while keeping the expected wage payment constant and relaxing some of the constraints. This is possible because a valuable project has less probability of failing than a bad one. It is hard to provide a complete characterization of the wages necessary to implement strategy d^*. We can, however, give some limiting results.

Proposition 1 *(i) For k sufficiently near zero, the optimal wage schedule is characterized by*

$$w_{d*}(2R) = \frac{2c(P\overline{\pi} + (1-P)\underline{\pi})}{(\overline{\pi} - \underline{\pi})(\overline{\pi}^2 A(2B + PC) + \overline{\pi}\underline{\pi}(AC + 2B(B + PC)) + \underline{\pi}^2 C(B + PC))},$$

$$w_{d*}(R + I) = \frac{2c(A\overline{\pi}^2 + 2B\overline{\pi}\underline{\pi} + C\underline{\pi}^2)}{(\overline{\pi} - \underline{\pi})(\overline{\pi}^2 A(2B + PC) + \overline{\pi}\underline{\pi}(AC + 2B(B + PC)) + \underline{\pi}^2 C(B + PC))},$$

$$w_{d*}(2I) = \frac{2c(A\overline{\pi}^2 + 2B\overline{\pi}\underline{\pi} + C\underline{\pi}^2)(P\overline{\pi} + (1-P)\underline{\pi})}{(\overline{\pi} - \underline{\pi})(\overline{\pi}^2 A(2B + PC) + \overline{\pi}\underline{\pi}(AC + 2B(B + PC)) + \underline{\pi}^2 C(B + PC))};$$

the expected profit is

$$\Pi(d^*, k) - \frac{2c(A\overline{\pi}^2(P\overline{\pi} + (1-P)\underline{\pi}) + (A\overline{\pi}^2 + 2B\overline{\pi}\underline{\pi} + C\underline{\pi}^2)(2B\overline{\pi} + P\overline{\pi} + (1-P)\underline{\pi}))}{(\overline{\pi} - \underline{\pi})(\overline{\pi}^2 A(2B + PC) + \overline{\pi}\underline{\pi}(AC + 2B(B + PC)) + \underline{\pi}^2 C(B + PC))},$$

where $A = P^2 + k$, $B = P(1-P) - k$, and $C = (1-P)^2 + k$.
(ii) The cost of implementing strategy d^* tends to infinity as k tends to $P(1-P)$.

When k is near zero, only constraints (9), (10), and (11) are binding. Solving for the three remaining wages yields the characterization in the proposition. The expected profit is given by the expected revenue of the principal minus the expected wage. Note that the coefficient of $2c$ is greater than one, which implies that the manager earns some informational rents. When k is near $P(1-P)$, the expected wage gets very large as it becomes increasingly costly to force the manager to investigate both projects rather than only one, i.e., constraint (14) becomes increasingly costly to satisfy. Unfortunately, for intermediate values of k, it is not possible to provide a more complete characterization. Numerical simulations show that there are many different combinations of constraints that can bind depending on parameter values.

4.1.2 Implementation of the strategy $\widetilde{d}_1$

The principal wants to minimize expected wage payments to the manager subject to her participation constraint and incentive constraints designed to induce the manager in choosing strategy $\widetilde{d}_1$. Under strategy $\widetilde{d}_1$, the manager investigates only one project and invests in both when the signal is favorable ($\overline{\pi}$). The expected wage is

$$\mathrm{E}_{\widetilde{d}_1}(w(z_s)) \equiv \left(P^2+k\right)\left(\overline{\pi}^2 w(2R) + 2\overline{\pi}(1-\overline{\pi})w(R) + (1-\overline{\pi})^2 w(0)\right) + (1-P)w(2I) +$$

$$(P(1-P)-k)\left(\overline{\pi}\underline{\pi}w(2R) + (\overline{\pi}(1-\underline{\pi}) + \underline{\pi}(1-\overline{\pi}))w(R) + (1-\overline{\pi})(1-\underline{\pi})w(0)\right).$$

The first thing to note is that the expected wage is independent of $w(R+I)$ and $w(I)$. This is because the strategy $\widetilde{d}_1$ never calls for only one project to be invested in. It is therefore optimal to set them to zero. Taking this into account, we now describe all constraints that must be imposed on the optimal wage schedule for a contract to implement strategy $\widetilde{d}_1$. First, the manager's participation constraint must be satisfied.

$$(16)\quad \mathrm{E}_{\widetilde{d}_1}(w(z_s)) - c \geq 0$$

Second the manager must make the efficient investment decision conditional on the signal she obtains from her investigation. There are two possible signals: $\overline{\pi}$ and $\underline{\pi}$.

- The signal on the investigated project is $\overline{\pi}$.

When the signal is favorable, the manager should invest in both projects. This should be preferred to not invest at all, invest only in the investigated project, or invest only in the noninvestigated project. These last two constraints are not binding given that $w(R+I) = w(I) = 0$. There remains only one incentive constraint.

$$(17)\quad \left(\frac{P^2+k}{P}\overline{\pi} + \frac{P(1-P)-k}{P}\underline{\pi}\right)(\overline{\pi}w(2R) + (1-\overline{\pi})w(R)) +$$

$$\left(\frac{P^2+k}{P}(1-\overline{\pi}) + \frac{P(1-P)-k}{P}(1-\underline{\pi})\right)(\overline{\pi}w(R) + (1-\overline{\pi})w(0)) \geq w(2I)$$

- The signal on the investigated project is $\underline{\pi}$.

When the signal is not favorable, the optimal strategy calls for the manager not to invest at all. This should be preferred to only invest in the investigated project, to invest only in the noninvestigated one, or to invest in both projects. The only possibly binding incentive constraint is the last one.

$$(18)\quad w(2I) \geq \underline{\pi}\left(\frac{P(1-P)-k}{1-P}\overline{\pi} + \frac{(1-P)^2+k}{1-P}\underline{\pi}\right)w(2R) +$$

$$\left(\frac{P(1-P)-k}{1-P}((1-\overline{\pi})\underline{\pi}+(1-\underline{\pi})\overline{\pi})+2\frac{(1-P)^2+k}{1-P}((1-\underline{\pi})\,\underline{\pi})\right)w(R)+$$
$$(1-\underline{\pi})\left(\frac{P(1-P)-k}{1-P}(1-\overline{\pi})+\frac{(1-P)^2+k}{1-P}\,(1-\underline{\pi})\right)w(0)$$

Third, the manager must be induced in investigating only one project. We have the following incentive constraints for the implementation of the optimal investigation decision.

• Discouraging no investigation.

The manager has to prefer investigating one project rather than none and not investing, or investing in both. (Investing in only one is not binding.) This translates into the following incentive constraints.

(19) $\mathrm{E}_{\widetilde{d_1}}(w(z_s))-c\geq w(2I)$

(20) $\mathrm{E}_{\widetilde{d_1}}(w(z_s))-c\geq\left(P^2+k\right)\left(\overline{\pi}^2w(2R)+2\overline{\pi}(1-\overline{\pi})w(R)+(1-\overline{\pi})^2w(0)\right)+$

$$2\left((1-P)P-k\right)\left(\overline{\pi}\underline{\pi}w(2R)+((1-\overline{\pi})\underline{\pi}+\overline{\pi}(1-\underline{\pi}))w(R)+(1-\overline{\pi})(1-\underline{\pi})w(0)\right)+$$
$$\left((1-P)^2+k\right)\left(\underline{\pi}^2w(2R)+2\underline{\pi}(1-\underline{\pi})w(R)+(1-\underline{\pi})^2w(0)\right)$$

• Discouraging investigating two projects.

The manager must be discouraged of investigating the two projects and investing in none, investing only in the projects with a valuable signal, investing only in the projects with an unfavorable signal, investing in both projects regardless of the signals. It is obvious that, since the investigation is costly, the manager never investigates a project to then make her investment decision without using the signal. This rules out investigating the two projects and not investing, or investing in both regardless of the signal. We are therefore left with the following two incentive constraints.

(21) $\mathrm{E}_{\widetilde{d_1}}(w(z_s))-c\geq\left(P^2+k\right)\left(\overline{\pi}^2w(2R)+2\overline{\pi}(1-\overline{\pi})w(R)+(1-\overline{\pi})^2\,w(0)\right)+$

$$\left((1-P)^2+k\right)w(2I)-2c$$

(22) $\mathrm{E}_{\widetilde{d_1}}(w(z_s))-c\geq\left(P^2+k\right)w(2I)+$

$$\left((1-P)^2+k\right)\left(\underline{\pi}^2w(2R)+2\underline{\pi}(1-\underline{\pi})w(R)+(1-\underline{\pi})^2w(0)\right)-2c$$

Again, we assume that the manager has limited liability, that is, $w(z_s) \geq 0$ for all z_s.

The principal's problem is to minimize $\mathrm{E}_{\widetilde{d}_1}(w(z_s))$ subject to constraints (16) to (22), and to limited-liability constraints.

Proposition 2 *(i) The optimal wage schedule is characterized by*

$$w_{\widetilde{d}_1}(R+I) = w_{\widetilde{d}_1}(I) = w_{\widetilde{d}_1}(R) = w_{\widetilde{d}_1}(0) = 0,$$

$$w_{\widetilde{d}_1}(2I) = \frac{c\left(((1-P)^2+k)\underline{\pi}^2 + 2(P(1-P)-k)\underline{\pi}\overline{\pi} + (P^2+k)\overline{\pi}^2\right)}{(\overline{\pi}-\underline{\pi})\left((P(1-P)-k)((\overline{\pi}-\underline{\pi})P+\underline{\pi}) + k(\overline{\pi}+\underline{\pi})\right)},$$

$$w_{\widetilde{d}_1}(2R) = \frac{c}{(\overline{\pi}-\underline{\pi})\left((P(1-P)-k)((\overline{\pi}-\underline{\pi})P+\underline{\pi}) + k(\overline{\pi}+\underline{\pi})\right)}.$$

(ii) The expected wage is

$$\mathrm{E}_{\widetilde{d}_1}(w_{\widetilde{d}_1}(z_s)) = \frac{c\left((2-P)(P^2+k)\overline{\pi}^2 + (3-2P)(P(1-P)-k)\overline{\pi}\underline{\pi} + (1-P)((1-P)^2+k)\underline{\pi}^2)\right)}{(\overline{\pi}-\underline{\pi})\left((P(1-P)-k)((\overline{\pi}-\underline{\pi})P+\underline{\pi}) + k(\overline{\pi}+\underline{\pi})\right)}.$$

(iii) The principal's expected profit is

$$\Pi(\widetilde{d}_1, k) - \mathrm{E}_{\widetilde{d}_1}(w_{\widetilde{d}_1}(z_s)).$$

The optimal wage contract punishes project failures by setting $w_{\widetilde{d}_1}(R) = w_{\widetilde{d}_1}(0) = 0$. There are only two positive wages: $w_{\widetilde{d}_1}(2I)$ and $w_{\widetilde{d}_1}(2R)$. The intuition for the proof is to identify the two binding constraints, namely constraints (19) and (20), and then show that all other constraints are satisfied when these two are. After solving for the optimal wage, we compute the expected wage payment and the principal's payoff. It is easy to show that the coefficient of c in $\mathrm{E}_{\widetilde{d}_1}(w_{\widetilde{d}_1}(z_s))$ is greater than one, therefore implying that the manager earns some informational rents.

4.1.3 Implementation of the strategy $\widetilde{d}_2$

The principal wants to minimize expected wage payments to the manager subject to her participation constraint and incentive constraints designed to induce the manager in choosing strategy $\widetilde{d}_2$. Under strategy $\widetilde{d}_2$, the manager investigates only one project, makes an optimal investment decision in that project, and always invests in the noninvestigated project. The expected wage is

$$\mathrm{E}_{\widetilde{d_2}}(w(z_s)) \equiv \left(P^2+k\right)\left(\overline{\pi}^2 w(2R) + 2\overline{\pi}(1-\overline{\pi})w(R) + (1-\overline{\pi})^2 w(0)\right) +$$

$$(P(1-P)-k)\left(\overline{\pi}\underline{\pi}w(2R) + (\overline{\pi}(1-\underline{\pi}) + \underline{\pi}(1-\overline{\pi}))w(R) + (1-\overline{\pi})(1-\underline{\pi})w(0)\right) +$$

$$(P(1-P)-k)\left(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)\right) + ((1-P)^2+k)\left(\underline{\pi}w(R+I) + (1-\underline{\pi})w(I)\right).$$

Note that the expected wage does not depend on $w(2I)$ as strategy $\widetilde{d}_2$ never prescribes to abstain from investing. Setting it to zero is therefore optimal. We now describe all constraints that must be imposed on the optimal wage schedule for a contract to implement strategy $\widetilde{d}_2$. First, the manager's participation constraint must be satisfied.

$$\text{(23)} \quad \mathrm{E}_{\widetilde{d_2}}(w(z_s)) - c \geq 0$$

Second the manager must make the efficient investment decision conditional on the signal she obtains from her investigation. There are two possible signals: $\overline{\pi}$ and $\underline{\pi}$.

- The signal on the investigated project is $\overline{\pi}$.

When the signal is favorable, the manager should invest in both projects. This should be preferred to not invest at all, invest only in the investigated project, or invest only in the noninvestigated project. Since $w(2I) = 0$, the first constraint is not binding. This translates into the following two incentive constraints.

$$\text{(24)} \quad \left(\frac{P^2+k}{P}\overline{\pi} + \frac{P(1-P)-k}{P}\underline{\pi}\right)\left(\overline{\pi}w(2R) + (1-\overline{\pi})w(R)\right) +$$

$$\left(\frac{P^2+k}{P}(1-\overline{\pi}) + \frac{P(1-P)-k}{P}(1-\underline{\pi})\right)\left(\overline{\pi}w(R) + (1-\overline{\pi})w(0)\right) \geq \overline{\pi}w(R+I) + (1-\overline{\pi})w(I)$$

$$\text{(25)} \quad \left(\frac{P^2+k}{P}\overline{\pi} + \frac{P(1-P)-k}{P}\underline{\pi}\right)\left(\overline{\pi}w(2R) + (1-\overline{\pi})w(R)\right) +$$

$$\left(\frac{P^2+k}{P}(1-\overline{\pi}) + \frac{P(1-P)-k}{P}(1-\underline{\pi})\right)\left(\overline{\pi}w(R) + (1-\overline{\pi})w(0)\right) \geq$$

$$\frac{P^2+k}{P}\left(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)\right) + \frac{P(1-P)-k}{P}\left(\underline{\pi}w(R+I) + (1-\underline{\pi})w(I)\right)$$

• The signal on the investigated project is $\underline{\pi}$.

When the signal is not favorable, the optimal strategy calls for the manager to invest only in the noninvestigated project. This should be preferred to not invest at all, to only invest in the bad project, or to invest in both projects. Again, the first constraint is not binding. We then have the following incentive constraints.

$$\text{(26)}\quad \frac{P(1-P)-k}{1-P}\left(\overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\right)+\frac{(1-P)^2+k}{1-P}\left(\underline{\pi}w(R+I)+(1-\underline{\pi})w(I)\right)\geq$$

$$\left(\frac{P(1-P)-k}{1-P}+\frac{(1-P)^2+k}{1-P}\right)\left(\underline{\pi}w(R+I)+(1-\underline{\pi})w(I)\right)$$

$$\text{(27)}\quad \frac{P(1-P)-k}{1-P}\left(\overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\right)+\frac{(1-P)^2+k}{1-P}\left(\underline{\pi}w(R+I)+(1-\underline{\pi})w(I)\right)\geq$$

$$\underline{\pi}\left(\frac{P(1-P)-k}{1-P}\overline{\pi}+\frac{(1-P)^2+k}{1-P}\underline{\pi}\right)w(2R)+$$

$$\left(\frac{P(1-P)-k}{1-P}((1-\overline{\pi})\underline{\pi}+(1-\underline{\pi})\overline{\pi})+2\frac{(1-P)^2+k}{1-P}((1-\underline{\pi})\,\underline{\pi})\right)w(R)+$$

$$(1-\underline{\pi})\left(\frac{P(1-P)-k}{1-P}(1-\overline{\pi})+\frac{(1-P)^2+k}{1-P}\left(1-\underline{\pi}\right)\right)w(0)$$

Third, the manager must be induced in investigating only one project. We have the following incentive constraints for the implementation of the optimal investigation decision.

• Discouraging no investigation.

The manager has to prefer investigating one project rather than none and investing in one of the projects, or investing in both. (Not investing is not binding.) This translates into the following incentive constraints.

$$\text{(28)}\quad \mathrm{E}_{\widetilde{d_2}}(w(z_s))-c\geq\left(P\overline{\pi}+(1-P)\underline{\pi}\right)w(R+I)+\left(P(1-\overline{\pi})+(1-P)(1-\underline{\pi})\right)w(I)$$

$$\text{(29)}\quad \mathrm{E}_{\widetilde{d_2}}(w(z_s))-c\geq\left(P^2+k\right)\left(\overline{\pi}^2w(2R)+2\overline{\pi}(1-\overline{\pi})w(R)+(1-\overline{\pi})^2w(0)\right)+$$

$$2\left((1-P)P-k\right)\left(\overline{\pi}\underline{\pi}w(2R)+((1-\overline{\pi})\underline{\pi}+\overline{\pi}(1-\underline{\pi}))w(R)+(1-\overline{\pi})(1-\underline{\pi})w(0)\right)+$$

$$\left((1-P)^2+k\right)\left(\underline{\pi}^2w(2R)+2\underline{\pi}(1-\underline{\pi})w(R)+(1-\underline{\pi})^2w(0)\right)$$

• Discouraging investigating two projects.

The manager must be discouraged of investigating the two projects and investing in none, investing only in the projects with a valuable signal, investing only in the projects with an unfavorable signal, investing in both projects regardless of the signals. It is obvious that, since the investigation is costly, the manager never investigates a project to then make her investment decision without using the signal. This rules out investigating the two projects and not investing, or investing in both regardless of the signal. We are therefore left with the following two incentive constraints.

$$\text{(30)}\quad \mathrm{E}_{\widetilde{d_2}}(w(z_s)) - c \geq \left(P^2 + k\right)\left(\overline{\pi}^2 w(2R) + 2\overline{\pi}(1-\overline{\pi})w(R) + (1-\overline{\pi})^2 w(0)\right) + 2\left(P(1-P) - k\right)\left(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)\right) - 2c$$

$$\text{(31)}\quad \mathrm{E}_{\widetilde{d_2}}(w(z_s)) - c \geq 2\left((1-P)P - k\right)\left(\underline{\pi}w(R+I) + (1-\underline{\pi})w(I)\right) + \left((1-P)^2 + k\right)\left(\underline{\pi}^2 w(2R) + 2\underline{\pi}(1-\underline{\pi})w(R) + (1-\underline{\pi})^2 w(0)\right) - 2c$$

Again, we assume that the manager has limited liability, that is, $w(z_s) \geq 0$ for all z_s.

The principal's problem is to minimize $\mathrm{E}_{\widetilde{d_2}}(w(z_s))$ subject to constraints (23) to (31), and to limited-liability constraints.

Proposition 3 *(i) If $0 \leq k < P(1-P)$, the optimal wage schedule is*

$$w_{\widetilde{d_2}}(R) = w_{\widetilde{d_2}}(0) = w_{\widetilde{d_2}}(2I) = 0,$$

$$w_{\widetilde{d_2}}(I) = w_{\widetilde{d_2}}(R+I) = \frac{c(A\overline{\pi}^2 + 2B\overline{\pi}\underline{\pi} + C\underline{\pi}^2)}{(\overline{\pi} - \underline{\pi})(A(1-P)\overline{\pi} + CP\underline{\pi})},$$

$$w_{\widetilde{d_2}}(2R) = \frac{c}{(\overline{\pi} - \underline{\pi})(A(1-P)\overline{\pi} + CP\underline{\pi})};$$

the expected wage is

$$\mathrm{E}_{\widetilde{d_2}}(w_{\widetilde{d_2}}(z_s)) = \frac{c\left(A\overline{\pi}^2 + B\overline{\pi}\underline{\pi} + (1-P)(A\overline{\pi}^2 + 2B\overline{\pi}\underline{\pi} + C\underline{\pi}^2)\right)}{(\overline{\pi} - \underline{\pi})(A(1-P)\overline{\pi} + CP\underline{\pi})},$$

where $A = P^2 + k$, $B = P(1-P)$, *and* $C = (1-P)^2 + k$.
(ii) If $k = P(1-P)$, *the optimal wage schedule is*

$$w_{\widetilde{d}_2}(R) = w_{\widetilde{d}_2}(0) = w_{\widetilde{d}_2}(2I) = w_{\widetilde{d}_2}(R+I) = 0,$$

$$w_{\widetilde{d}_2}(I) = \frac{c(P\overline{\pi}^2 + (1-P)\underline{\pi}^2)}{P(1-P)(\overline{\pi}^2(1-\underline{\pi}) - \underline{\pi}^2(1-\overline{\pi}))},$$

$$w_{\widetilde{d}_2}(2R) = \frac{c(P(1-\overline{\pi}) + (1-P)(1-\underline{\pi}))}{P(1-P)(\overline{\pi}^2(1-\underline{\pi}) - \underline{\pi}^2(1-\overline{\pi}))};$$

the expected wage is

$$\mathrm{E}_{\widetilde{d}_2}(w_{\widetilde{d}_2}(z_s)) = \frac{c\left(P^2\overline{\pi}^2(1-\overline{\pi}) + 2P(1-P)\overline{\pi}^2(1-\underline{\pi}) + (1-P)^2\underline{\pi}^2(1-\underline{\pi})\right)}{P(1-P)(\overline{\pi}^2(1-\underline{\pi}) - \underline{\pi}^2(1-\overline{\pi}))},$$

(iii) The principal's expected profit is

$$\Pi(\widetilde{d}_2, k) - \mathrm{E}_{\widetilde{d}_2}(w_{\widetilde{d}_2}(z_s)).$$

It turns out that the solution differs depending on the value of k. The constraint set is not continuous at $k = P(1-P)$. Constraint (26) vanishes as correlation becomes perfect. Since this constraint is binding for lower values of k, the solution jumps discontinuously at $k = P(1-P)$. When $k < P(1-P)$, only constraints (26), (28), and (29) bind. The optimal wage contract punishes project failures when two projects have been invested in by setting $w_{\widetilde{d}_2}(R) = w_{\widetilde{d}_2}(0) = 0$. This discourages the manager from investing blindly in both projects rather than investigating one. There remain three positive wages: $w_{\widetilde{d}_2}(I)$, $w_{\widetilde{d}_2}(R+I)$, and $w_{\widetilde{d}_2}(2R)$. A first result is that $w_{\widetilde{d}_2}(I) = w_{\widetilde{d}_2}(R+I)$. This can be understood by looking at constraint (26). This constraint simplifies to

$$B(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)) \geq B(\underline{\pi}w(R+I) + (1-\underline{\pi})w(I)).$$

When $k < P(1-P)$, $B > 0$. The constraint then implies that $w_{\widetilde{d}_2}(I) = w_{\widetilde{d}_2}(R+I)$. Intuitively, constraint (26) says that, when the signal is $\underline{\pi}$, the manager should prefer investing in the unknown noninvestigated project, rather than investing in the bad investigated one. This implies that success must be rewarded, i.e., $w_{\widetilde{d}_2}(R+I) \geq w_{\widetilde{d}_2}(I)$. But this is incompatible with providing incentives for investigating (constraints (28) and (29)). These constraints are relaxed if $w(R+I)$ is low. The tradeoff between these constraints is then to set the two wages equal. Another interpretation is that, since the manager is forced to invest in the noninvestigated project, it is not optimal to punish her for failure when she invests only in that project, hence, $w_{\widetilde{d}_2}(I) = w_{\widetilde{d}_2}(R+I)$. When $k = P(1-P)$, constraint (26) vanishes, and only constraints (28) and (29) bind. The solution prescribes to set $w_{\widetilde{d}_2}(R+I) = 0$.

After solving for the optimal wage, we compute the expected wage payment and the principal's payoff. It is easy to show that the coefficient of c in $\mathrm{E}_{\widetilde{d}_2}(w_{\widetilde{d}_2}(z_s))$ is greater than one, therefore implying that the manager earns some informational rents.

4.1.4 Implementation of the strategy $\widetilde{d}_3$

The principal wants to minimize expected wage payments to the manager subject to her participation constraint and incentive constraints designed to induce the manager in choosing strategy $\widetilde{d}_3$. Under strategy $\widetilde{d}_3$, the manager investigates only one project, makes an optimal investment decision in that project, and never invests in the noninvestigated project. The expected wage is

$$\mathrm{E}_{\widetilde{d}_3}(w(z_s)) \equiv P(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)) + (1-P)w(2I).$$

The expected wage is independent of $w(2R)$, $w(R)$, and $w(0)$. This implies that setting them to zero relaxes the incentive constraints. In the following, we can therefore set them to zero. We now describe all constraints that must be imposed on the optimal wage schedule for a contract to implement strategy $\widetilde{d}_3$. First, the manager's participation constraint must be satisfied.

(32) $\mathrm{E}_{\widetilde{d}_3}(w(z_s)) - c \geq 0$

Second the manager must make the efficient investment decision conditional on the signal she obtains from her investigation. There are two possible signals: $\overline{\pi}$ and $\underline{\pi}$.

• THE SIGNAL ON THE INVESTIGATED PROJECT IS $\overline{\pi}$.

When the signal is favorable, the manager should invest only in the investigated project. This should be preferred to not invest at all, invest only in the noninvestigated project, or invest in both. Since investing in both yields zero, this constraint is never binding, and is therefore omitted. We have the following two incentive constraints.

$$(33)\quad \overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\geq w(2I)$$

$$(34)\quad \overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\geq$$

$$\frac{P^2+k}{P}\left(\overline{\pi}w(R+I)+(1-\overline{\pi})w(I)\right)+\frac{P(1-P)-k}{P}\left(\underline{\pi}w(R+I)+(1-\underline{\pi})w(I)\right)$$

• The signal on the investigated project is $\underline{\pi}$.

When the signal is not favorable, the optimal strategy calls for the manager not to invest at all. This should be preferred to only invest in the investigated project, to invest only in the noninvestigated one, or to invest in both projects. Again this last possibility yields zero, and its associated incentive constraint is omitted. We then have the following incentive constraints.

$$(35)\quad w(2I)\geq\underline{\pi}w(R+I)+(1-\underline{\pi})w(I)$$

$$(36)\quad w(2I)\geq\left(\frac{P(1-P)-k}{1-P}\overline{\pi}+\frac{(1-P)^2+k}{1-P}\underline{\pi}\right)w(R+I)+$$

$$\left(\frac{P\,(1-P)-k}{1-P}(1-\overline{\pi})+\frac{(1-P)^2+k}{1-P}\,(1-\underline{\pi})\right)w(I)$$

Third, the manager must be induced in investigating only one project. We have the following incentive constraints for the implementation of the optimal investigation decision.

• Discouraging no investigation.

The manager has to prefer investigating one project rather than none and not investing, investing in one of the projects, or investing in both. Again this last constraint is omitted. This translates into the following incentive constraints.

$$(37)\quad \mathrm{E}_{\widetilde{d_3}}(w(z_s))-c\geq w(2I)$$

$$(38)\quad \mathrm{E}_{\widetilde{d_3}}(w(z_s))-c\geq\left(P\overline{\pi}+(1-P)\underline{\pi}\right)w(R+I)+\left(P(1-\overline{\pi})+(1-P)(1-\underline{\pi})\right)w(I)$$

• Discouraging investigating two projects.

The manager must be discouraged of investigating the two projects and investing in none, investing only in the projects with a valuable signal, investing only in the projects with an unfavorable signal, investing in both projects regardless of the signals. It is obvious that, since the investigation is costly, the manager never investigates a project to then make her investment decision without using the signal. This rules out investigating the two projects and not investing, or investing in both regardless of the signal. We are therefore left with the following two incentive constraints.

$$\text{(39)}\quad \mathrm{E}_{\widetilde{d}_3}(w(z_s)) - c \geq 2\,(P(1-P)-k)\,(\overline{\pi}w(R+I) + (1-\overline{\pi})w(I)) + \left((1-P)^2+k\right) w(2I) - 2c$$

$$\text{(40)}\quad \mathrm{E}_{\widetilde{d}_3}(w(z_s)) - c \geq \left(P^2 + k\right) w(2I) + 2\,((1-P)P - k)\,(\underline{\pi}w(R+I) + (1-\underline{\pi})w(I)) - 2c$$

Again, we assume that the manager has limited liability, that is, $w(z_s) \geq 0$ for all z_s.

The principal's problem is to minimize $\mathrm{E}_{\widetilde{d}_3}(w(z_s))$ subject to constraints (32) to (40), and to limited-liability constraints.

Proposition 4 *(i) The optimal wage schedule is*

$$w_{\widetilde{d}_3}(2R) = w_{\widetilde{d}_3}(R) = w_{\widetilde{d}_3}(0) = w_{\widetilde{d}_3}(I) = 0,$$

$$w_{\widetilde{d}_3}(R+I) = \frac{c}{P(1-P)(\overline{\pi}-\underline{\pi})},$$

$$w_{\widetilde{d}_3}(2I) = \frac{c(P\overline{\pi} + (1-P)\underline{\pi})}{P(1-P)(\overline{\pi}-\underline{\pi})};$$

the expected wage is

$$\mathrm{E}_{\widetilde{d}_3}(w_{\widetilde{d}_3}(z_s)) = \frac{c\,(P\overline{\pi} + P(1-P)\overline{\pi} + (1-P)^2\underline{\pi})}{P(1-P)(\overline{\pi}-\underline{\pi})}.$$

(ii) The principal's expected profit is

$$\Pi(\widetilde{d}_3, k) - \mathrm{E}_{\widetilde{d}_3}(w_{\widetilde{d}_3}(z_s)).$$

Investing in both projects is punished, and failure on the undertaken project is also punished. This leaves only two positive wages, $w_{\widetilde{d_3}}(2I)$ and $w_{\widetilde{d_3}}(R+I)$. Only constraints (37) and (38) bind, and optimal wages are computed solving for these two binding constraints. Note that the solution is completely independent of k since none of the constraints where k appears is binding. After solving for the optimal wage, we compute the expected wage payment and the principal's payoff. It is easy to show that the coefficient of c in $\mathrm{E}_{\widetilde{d_2}}(w_{\widetilde{d_2}}(z_s))$ is greater than one, therefore implying that the manager earns some informational rents.

4.2 Hiring two managers

We now introduce a different form of organizational structure where each project is under the responsibility of a single manager. We assume that the returns on both projects are observable so that the compensation of each manager can depend on the returns of both projects. This amounts to using relative-performance compensation to stimulate a form of competition between the two managers. For example, manager 1's compensation is denoted by $w_1(r_1; r_2)$, where r_i is the return on project i, and $r_i \in \{0, R, I\}$.

Each manager must make investigation and investment decisions, without knowing what the other manager does or observes, namely, manager i cannot observe the investigation decision of manager j, the signal manager j obtains, or her investment decision. We also assume that the two managers cannot communicate prior to investigation or investment, therefore ruling out any possibility for collusion.

The first-best strategy from the point of view of the principal is to investigate each project and only invest in those with a favorable signal, that is strategy $\widehat{d}$. Under such strategy, the expected wage is

$$\mathrm{E}_{\widehat{d}}(w_i(r_i; r_j)) \equiv (P(1-P)-k)\left(\overline{\pi} w_i(I_i; R_j) + (1-\overline{\pi}) w_i(I_i; 0)\right) +$$

$$((1-P)P-k)\left(\overline{\pi} w_i(R_i; I_j) + (1-\overline{\pi}) w_i(0; I_j)\right) + \left((1-P)^2+k\right) w_i(I_i; I_j) +$$

$$\left(P^2+k\right)\left(\overline{\pi}^2 w_i(R_i; R_j) + \overline{\pi}(1-\overline{\pi}) w_i(R_i; 0) + \overline{\pi}(1-\overline{\pi}) w_i(0; R_j) + (1-\overline{\pi})^2 w_i(0; 0)\right).$$

The principal must design wage schedules for each manager so that they have incentives to follow such strategy. Wages must then satisfy the following constraints.

First, each manager must find her contract acceptable. For $i, j = 1, 2$, we have the following participation constraint.

$$(41) \quad \mathrm{E}_{\widehat{d}}(w_i(r_i; r_j)) - c \geq 0$$

Second, each manager must invest efficiently conditional on the signal she receives.

- The signal on the investigated project is $\overline{\pi}$.

When the signal is favorable, the manager should invest in the project rather than abstaining from it. This translates into the following incentive constraint.

$$(42) \quad \overline{\pi}\frac{P^2+k}{P}\left(\overline{\pi}w_i(R_i; R_j) + (1-\overline{\pi})w_i(R_i; 0)\right) + \overline{\pi}\frac{P(1-P)-k}{P}w_i(R_i; I_j) +$$

$$(1-\overline{\pi})\frac{P^2+k}{P}\left(\overline{\pi}w_i(0; R_j) + (1-\overline{\pi})w_i(0; 0)\right) + (1-\overline{\pi})\frac{P(1-P)-k}{P}w_i(0; I_j) \geq$$

$$\frac{P^2+k}{P}\left(\overline{\pi}w_i(I_i; R_j) + (1-\overline{\pi})w_i(I_i; 0)\right) + \frac{P(1-P)-k}{P}w_i(I_i; I_j)$$

Note that manager i uses her signal to update the distribution of signals of the other manager. Manager i also assumes that manager j follows the prescribed strategy.

- The signal on the investigated project is $\underline{\pi}$.

When the signal is not favorable, the manager must be induced in not investing.

$$(43) \quad \frac{P(1-P)-k}{1-P}\left(\overline{\pi}w_i(I_i; R_j) + (1-\overline{\pi})w_i(I_i; 0)\right) + \frac{(1-P)^2+k}{1-P}w_i(I_i; I_j) \geq$$

$$\frac{P(1-P)-k}{1-P}\left(\overline{\pi}(\underline{\pi}w_i(R_i; R_j) + (1-\underline{\pi})w_i(0; R_j)) + (1-\overline{\pi})(\underline{\pi}w_i(R_i; 0) + (1-\underline{\pi})w_i(0; 0))\right) +$$

$$\frac{(1-P)^2+k}{1-P}\left(\underline{\pi}w_i(R_i; I_j) + (1-\underline{\pi})w_i(0; I_j)\right)$$

Third, the manager must be induced in investigating one project and investing optimally rather than not investigating and not investing, or investing blindly. This translates into the following incentive constraints.

$$\mathrm{E}_{\widehat{d}}(w_i(r_i; r_j)) - c \geq \left(P^2+k\right)\left(\overline{\pi}w_i(I_i; R_j) + (1-\overline{\pi})w_i(I_i; 0)\right) + \left((1-P)^2+k\right)w_i(I_i; I_j) +$$

$$\left(P(1-P)-k\right)\left(\overline{\pi}w_i(I_i; R_j) + (1-\overline{\pi})w_i(I_i; 0) + w_i(I_i; I_j)\right)$$

This constraint can be simplified to:

$$(44)\quad \left(P^2+k\right)\left(\overline{\pi}^2 w_i(R_i;R_j)+\overline{\pi}(1-\overline{\pi})\left(w_i(R_i;0)+w_i(0;R_j)\right)+(1-\overline{\pi})^2 w_i(0;0)\right)+$$

$$((1-P)P-k)\left(\overline{\pi}w_i(R_i;I_j)+(1-\overline{\pi})w_i(0;I_j)\right)-c\geq$$

$$\left(P^2+k\right)\left(\overline{\pi}w_i(I_i;R_j)+(1-\overline{\pi})w_i(I_i;0)\right)+((1-P)P-k)\,w_i(I_i;I_j).$$

The second incentive constraint is:

$$\mathrm{E}_{\widehat{d}}(w_i(r_i;r_j))-c\geq(P(1-P)-k)\left((1-\underline{\pi})\overline{\pi}w_i(0;R_j)+(1-\underline{\pi})(1-\overline{\pi})w_i(0;0)\right)+$$

$$((1-P)P-k)\left(\overline{\pi}w_i(R_i;I_j)+(1-\overline{\pi})w_i(0;I_j)+\underline{\pi}\overline{\pi}w_i(R_i;R_j)+\underline{\pi}(1-\overline{\pi})w_i(R_i;0)\right)+$$

$$\left(P^2+k\right)\left(\overline{\pi}^2 w_i(R_i;R_j)+\overline{\pi}(1-\overline{\pi})\left(w_i(R_i;0)+w_i(0;R_j)\right)+(1-\overline{\pi})^2 w_i(0;0)\right)+$$

$$\left((1-P)^2+k\right)\left(\underline{\pi}w_i(R_i;I_j)+(1-\underline{\pi})w_i(0;I_j)\right).$$

We can simplify it to:

$$(45)\quad (P(1-P)-k)\left(\overline{\pi}w_i(I_i;R_j)+(1-\overline{\pi})w_i(I_i;0)\right)+\left((1-P)^2+k\right)w_i(I_i;I_j)-c\geq$$

$$(P(1-P)-k)\left(\underline{\pi}(\overline{\pi}w_i(R_i;R_j)+(1-\overline{\pi})w_i(R_i;0))+(1-\underline{\pi})(\overline{\pi}w_i(0;R_j)+(1-\overline{\pi})w_i(0;0))\right)+$$

$$\left((1-P)^2+k\right)\left(\underline{\pi}w_i(R_i;I_j)+(1-\underline{\pi})w_i(0;I_j)\right).$$

Limited liability constraints are $w_i(r_i;r_j)\geq 0$ for all r_i and r_j.

For $i,j=1,2$, the principal's problem is to minimize $\mathrm{E}_{\widehat{d}}(w_i(r_i;r_j))$ subject to constraints (41) to (45). We note that the participation constraint (41) is satisfied when constraint (44) is, and that constraints (42) and (43) are satisfied when constraints (44) and (45) are. We are therefore left with only two constraints, (44) and (45).

Proposition 5 *For $i,j=1,2$, the optimal wage schedule is characterized by*

$$w_{i\widehat{d}}(0;I_j)=w_{i\widehat{d}}(0;R_j)=w_{i\widehat{d}}(0;0)=w_{i\widehat{d}}(R_i;0)=w_{i\widehat{d}}(I_i;0)=w_{i\widehat{d}}(I_i;R_j)=w_{i\widehat{d}}(R_i;I_j)=0,$$

$$w_{i\widehat{d}}(R_i;R_j)=\frac{c(1-P)}{\overline{\pi}\left(\overline{\pi}(P^2+k)((1-P)^2+k)-\underline{\pi}(P(1-P)-k)^2\right)},$$

and

$$w_{i\widehat{d}}(I_i;I_j) = \frac{c\,(\overline{\pi}(P^2+k)+\underline{\pi}(P(1-P)-k))}{\overline{\pi}(P^2+k)((1-P)^2+k)-\underline{\pi}(P(1-P)-k)^2}.$$

The expected wage payment to manager i is

$$\mathrm{E}_{\widehat{d}}(w_{i\widehat{d}}(r_i;r_j)) = \frac{c\,(\overline{\pi}(P^2+k)((1-P)(2-P)+k)+\underline{\pi}(P(1-P)-k)((1-P)^2+k))}{\overline{\pi}(P^2+k)((1-P)^2+k)-\underline{\pi}(P(1-P)-k)^2}.$$

The expected profit is

$$\Pi(\widehat{d},k) - 2\mathrm{E}_{\widehat{d}}(w_{i\widehat{d}}(r_i;r_j)).$$

Project failures are penalized to give incentives to the manager to invest optimally. This is optimal since bad projects have a higher probability of failure than good projects. The wage $w_{i\widehat{d}}(R_i;0)$ is also set to zero. This is not necessary for an optimal wage schedule but is without loss of generality. Manager i has no control over manager j's project. Her payoff only depends on $\overline{\pi}w_i(R_i;R_j) + (1-\overline{\pi})w_i(R_i;0)$. There is no loss of generality in setting $w_{i\widehat{d}}(R_i;0)$ to zero. The wages $w_{i\widehat{d}}(I_i;R_j) = w_{i\widehat{d}}(R_i;I_j) = w_{i\widehat{d}}(I_i;0) = 0$ because it is optimal to punish managers that have correlated information and that invest differently. Incentives are more powerful when only $w_{i\widehat{d}}(R_i;R_j)$ and $w_{i\widehat{d}}(I_i;I_j)$ are positive. (Note that only two wages need to be positive as only two constraints bind.) Solving for the two constraints (44) and (45) yields the closed-form solution for the two remaining wages. We can then calculate the expected wage payment to manager i. As before, the coefficient of c in $\mathrm{E}_{\widehat{d}}(w_{i\widehat{d}}(r_i;r_j))$ is greater than one, implying that the manager earns some informational rents.

5 Comparison of the different structures

It is hard to compare the different structures because the case in which one manager investigate two projects cannot be fully characterized. We do, however, have some limiting results around $k = 0$ and $k = P(1-P)$. We can also compare the class of strategies when one manager investigates one project.

Proposition 6 *(i) When the correlation between projects is weak enough (k near 0) and I is close to* $P\overline{\pi}R + (1-P)\underline{\pi}R$*, it is always optimal to use only one manager and having her*

investigate the two projects. If I is small (large) relative to $P\overline{\pi}R + (1-P)\underline{\pi}R$, it may be optimal to investigate one project and always (never) invest in the noninvestigated project.
(ii) When the correlation between projects is strong enough (k near $P(1-P)$), it is optimal to use two managers and relative-performance compensation schemes if and only if

$$P(3P-2)(\overline{\pi}^2 - \underline{\pi}^2) + \underline{\pi}^2 > 0,$$

otherwise, it is optimal to use one manager and investigate only one project.

When projects are weakly correlated and I is close to $P\overline{\pi}R+(1-P)\underline{\pi}R$, it is always optimal to use only one manager and having her investigate the two projects. The reason this dominates hiring two managers is that there are informational economies of scale. There is more room to punish a manager when she manages two projects, and therefore incentive constraints are relaxed compared to the case where there are two managers. This implies that there are informational economies of scale in the management of projects. Another implication of this is that one manager dominates two managers near $k = 0$ for any value of I and $P\overline{\pi}R + (1-P)\underline{\pi}R$. Relative-performance schemes can only be optimal if projects are sufficiently correlated.

When the correlation is strong (k near $P(1-P)$), hiring two managers is optimal if $\underline{\pi}$ is large enough. The intuition for this is the following. Near perfect correlation, hiring one manager that investigates one project and makes the same investment decision on both projects dominates all other strategies using only one manager. When this strategy is compared to hiring two managers, the parameter $\underline{\pi}$ is important. When the signal is $\underline{\pi}$, the single manager does not invest. When $\underline{\pi}$ is low, her incentive constraints are then relaxed as it is easy to provider her with incentives not to invest. When $\underline{\pi}$ is large, the two projects are similar, and it becomes more costly to provide incentives to the single manager. When there are two managers competing, it is not the difference in the quality of the projects that matter, but rather having the same investment strategy as the other manager. This implies that variations in $\underline{\pi}$ do not affect the efficiency of having two managers as much as that of only one manager.

6 Conclusion

This paper shows that any structure can be optimal depending on parameter values. Relative-performance incentives may be optimal when the correlation between projects is large enough and the difference in the quality of the projects is large enough. This implies that it is not necessarily optimal to exploit informational economies of scale: there are cases where investigating only one project is never optimal.