International Liquidity Management:  
Sterilization Policy in Illiquid Financial Markets  

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Abstract  
During the booms that invariably precede crises in emerging economies, policy makers often struggle to limit capital flows and their expansionary consequences. The main policy tool for this task is sterilization –essentially a swap of international reserves for public bonds. However, there is an extensive debate on the effectiveness of this policy, with many arguing that it may be counterproductive once the (over-) reaction of the private sector is considered. But what forces account for the private sector’s reaction remains largely unexplained. In this paper we provide a model to discuss these issues. We first demonstrate that policies to smooth expansions in anticipation of downturns can be Pareto improving in economies where domestic financial markets are underdeveloped. We then discuss the implementation of this policy via sterilization, outlining cases in which the policy succeeds and those in which it fails. Paradoxically the greatest risk of policy arises in situations where policy is most needed – that is when financial markets are illiquid. Our mechanism is akin to the “implicit bailout” problem, despite the fact that the central bank acts non-selectively and only intervenes through open markets; illiquidity replaces corruption and ineptitude. In addition to an appreciation of the currency and the emergence of a quasi-fiscal deficit, the private sector’s reaction to sterilization may lead to an expansion rather than the wanted contraction in aggregate demand and a bias toward short term capital inflows.

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1 Introduction

During the booms that invariably precede crises in emerging economies, policy makers often struggle to limit capital flows and their expansionary consequences. They primarily rely on tight monetary policy. In particular, they attempt to sterilize capital inflows through an open market sale of domestic bonds or increased reserve requirements.\footnote{E.g., Calvo et al. (1993) p. 146 write: “Sterilized intervention has been the most popular response to the present episode of capital inflows in Latin America.” And so does the World Bank (1997), p. 181: “Sterilization was the most widely and intensively used policy response to the arrival of capital inflows among the countries in our sample.” The sample included 22 emerging economies.}

The magnitude of these sterilized interventions can be extremely large. During the early 1990s in Chile, for example, the exchange intervention meant that over three quarters of its large capital inflow —amounting to around seven percent of its GDP per year— went to international reserves accumulation at the central bank. The sterilization of this intervention increased the ratio of international reserves to monetary base from 3.5 around 1990 to over 6.0 by 1993. This pattern was repeated in many emerging economies during the early 1990’s, when capital flows to the developing world surged. In fact most of the economies involved in the crises of the second half of the 1990’s had heavily sterilized inflows. Illustratively, many of these countries’ central banks exhausted their stock of treasury securities in the process, and had to resort to alternative sterilization mechanisms (see e.g. Glick (1998)).

While sterilization is a widely used tool, both policy makers and academics have warned that it comes along with a number of difficulties and risks. Building on the Mundell-Fleming logic, many argue that sterilization is, at best, ineffective. When capital markets are integrated and there is a simultaneous attempt to stabilize the exchange rate, the central bank has no control over the money supply because the private sector can undo an open market sale of bonds for money (Mundell (1962)). The policy literature, noting the increase in capital inflows that accompany sterilizations, argues that since it is these flows that fuel what is perceived as an excessive expansion in aggregate demand and exchange rate overvaluation, sterilization is counterproductive (e.g. Calvo et al. 1993, Williamson 1995, Corbo and Hernandez 1996, Massad 1997).\footnote{In his statement on behalf of the Latin American Governors of the Fund at the joint World Bank - IMF annual meeting of the Board of Governors held in Hong Kong (1997), Massad writes: “The high rate of return on capital in a booming economy attracts large inflows of external resources. These inflows are further encouraged by the appreciation of the domestic currency, which is characteristic of economies experiencing rapid productivity growth. Capital flows stimulate domestic demand and could push up domestic interest rates if the monetary authority safeguards domestic equilibrium. This, in turn, could provide a further incentive for capital inflows. The probable outcome will be continued appreciation of the local currency, the resulting risk of widening the current account deficit, and the greater danger that these capital flows will be reversed, should some negative external shock occur.” (page 4, our emphasis).} Building on Sargent and Wallace’s (1981) unpleasant monetary arithmetics, Calvo (1991) formally shows that, by raising domestic
interest rates, the government increases its debt-service burden and creates a quasi-fiscal deficit that may jeopardize the very stabilization attempt that is supposedly being protected by the sterilization.

Uniformly, this debate has viewed the effects of sterilization as arising from changes in the composition of the government’s liabilities (money versus bonds). However, as noted above, in a typical sterilized intervention the central bank also accumulates substantial international reserves as assets and some mix of domestic currency and bonds as liabilities, while the private sector’s balance sheet changes in the opposite direction. We argue in this paper that the the impact that sterilization has on the asset side of the government’s balance sheet and on its counterpart in the private sector is central —perhaps the chief factor— to understand its consequences.

We build this view on two salient features of emerging economies vis-à-vis developed ones: First, it is crisis-prevention rather than day-to-day fine tuning that shapes their macroeconomic policy. And second, external crises are invariably associated with a country’s shortage of international liquidity.\(^3\)

To be concrete with respect to the environment and policy issues that we address in this paper, consider the following example. All domestic production in the economy requires only imported goods. In order to put up a building in downtown Bangkok, a Thai developer must import all the raw materials for the building. Suppose that this building is not acceptable collateral to a foreigner, so that loans to this developer, against the collateral of the building, will only be forthcoming from other domestics. Lacking any internationally liquid assets to exchange for the raw materials, it would appear that the real estate developer is in a dilemma. However, suppose that foreigners do accept, as internationally liquid assets, claims on export sector receivables. Then as long as the real estate developer can find a domestic with export sector revenue who will accept the building as collateral, construction can proceed. At the aggregate level, domestic investment —building construction— is constrained by the supply of internationally liquid assets. Will domestic agents assess adequately the international liquidity value and costs of the assets and liabilities they generate when making their investment decisions? Following Caballero and Krishnamurthy (1999), we show that the answer to this question is typically negative in an emerging market. That is, too much international liquidity is sacrificed in the real estate boom.

Now suppose that the economy is currently in an investment boom in which many loans

\(^3\)See Caballero and Krishnamurthy (1999) for a model of crises based on collateral shortages. For us, an internationally liquid asset is one that can be sold at a moment’s notice to an international investor. A shortage of international liquidity means that the quantity of these assets, net of any pre-existing external debt, is insufficient to meet all external financial needs. In this regard, we echo ideas in the sovereign debt literature, that tie a country’s debt capacity to its international collateral (see Eaton and Gersowitz (1981), Bulow and Rogoff (1989)).
are being made, there is much real estate development, and the economy is trading away its international liquidity. Anticipating a shortage of international liquidity in the future, the central bank tries to increase the international liquidity provisions of the economy by offering public bonds in exchange for internationally liquid assets during the boom. The sterilized intervention leaves the central bank with more international reserves while leaving the private sector with more public bonds. The direct effect of this transaction is just a reallocation of international liquidity from the private sector to the central bank. Does this financial reorganization have any real effects? In particular, does the reallocation curtail real estate investment today and have the desired effect of mitigating the future liquidity shortage? Can the action ever backfire, as practitioners warn, leading to a further loss of liquidity? We show in this paper that the answers to these questions depend on the explicit and implicit commitments of the government, and on the degree of development of domestic financial markets.

More technically, the backbone of the paper is contained in three results. First, we show that in situations where the real estate is poor international and domestic collateral—a situation which characterizes underdeveloped and illiquid financial markets—the private sector does not take adequate precautions against future shortages of international liquidity. When real estate is poor domestic collateral, lenders find that they receive a rate of return less than the marginal product of investment for the investing firm. There is a domestic spread in all lending. In a dynamic context this means that the opportunity cost of using international liquidity today (i.e. that of lending them during a future downturn) is lower than the social opportunity cost. Relative to a constrained efficient outcome, the private sector over-invests during the boom because it undervalues international liquidity.

Second, we demonstrate conditions under which sterilization can implement an objective of curtailing current investment and provisioning for the future. As in Mundell-Fleming, we find that sterilization can only succeed when the central bank is able to raise domestic interest rates. Higher interest rates however do not guarantee success. The key questions are whether the capital inflows arising from higher interest rates are purely to purchase the government bonds sold in the sterilization or whether there is a change in real domestic investment as well, and whether the latter is contractionary as desired or not. In situations where the bonds sold by the government are accepted as international liquidity by foreigners, or when this sale can be accommodated purely through a change in the portfolio of the domestic private sector, sterilization has no real effect and a Modigliani-Miller theorem applies. Sterilization does have the desired effect when some of the capital inflows substitute away from funding domestic investment and go towards purchasing these bonds. Absent other measures, we argue that success requires that foreigner’s exposure to the country’s risk is higher—relative to desired portfolios—than their exposure to that country’s government
risk.

Third, there are scenarios where sterilization backfires as capital inflows not only go toward purchasing the government bonds but also toward increasing corporate lending. If the government bonds sold in sterilization are illiquid—e.g., because they are longer term and the secondary market is illiquid—there is a sort of “liquidity” mismatch that arises in the central bank’s balance sheet. It holds international reserves that it will supply to the private sector in the event of a crisis, but the liabilities that it has issued are longer term and illiquid. Capital flows in to buy the government bonds and to take advantage of the government liquidity commitment by lending to the domestic private sector.

The basic mechanism behind backfiring is similar to a “bailout” problem, although it derives from financial markets deficiencies rather than from moral hazard or government ineptitude. For example, suppose that the government acquires international reserves today and issues completely illiquid government bonds to the market. That is, the bonds are long term and have no secondary market, so that they must be held until maturity. Now suppose that the commitment of the government to supplying the reserves in the event of the crisis is shorter in term—e.g., the government is expected to supply the reserves over the next year and the bonds do not mature for two years. Then, by its action and commitment the government has increased its support of domestic asset prices during downturns and, contrary to its goal, effectively reduced the cost of capital for real estate builders.⁴

In section 2 we lay out our basic model. We show that there when domestic financial markets are underdeveloped there will be an externality whereby the private sector draws down its international liquidity too fast (over time) relative to the constrained efficient outcome. This sets the stage for the policy discussion.

In section 3 we introduce a government/central bank and describe its rights and commitments. We demonstrate conditions under which sterilization policy is effective and leads to Pareto improvements and those under which it is completely undone by the private sector. When domestic secondary markets are illiquid, the government action can backfire leading to a net loss of international liquidity and a Pareto loss.

Section 4 adds money and a lending channel. Within the context of this expanded model we supplement our results with the standard Mundell-Fleming insight that sterilization is more likely to succeed when the central bank has no commitment to supply international

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⁴With some relabeling, this mechanism can also be illustrated via a fixed exchange rate commitment. Suppose that the government has reserves to sustain a fixed exchange rate over the next year, but has issued bonds that expire much later. Then capital inflows to purchase the bonds cannot take advantage of the fixed exchange rate unless the bonds can be sold in liquid secondary market in the next year. If this is not so, other shorter term assets will be created to take advantage of the government commitment. The domestic private sector finds that there is good demand for such assets and takes out loans (i.e. sells the asset) and increases real investment.
reserves at a pre-specified price. This advantage of a flexible exchange rate system is limited, in that the core issues discussed in the previous section remain, and that the additional success in sterilizing comes from implicit transfers rather than Pareto-improvements.

In section 5 we show that our perspective naturally accommodates two additional sources of concern during sterilization episodes: The rise in the quasi-fiscal deficit and the shortening of the maturity of capital flows. We show that when the domestic financial markets are illiquid, the former symptom may arise even when the sterilization fails in raising corporate interest rates. Behind the second symptom, on the other hand, is that when domestic financial markets are illiquid, agents undervalue the insurance aspect of long term debt. This situation is aggravated by an sterilization policy that further strains the limited domestic financial markets. Section 6 concludes and is followed by an appendix.

2 A Model of Crises and Illiquid Financial Markets

In this section we develop a simple open economy model where two forms of liquidity, domestic and international, are required to fulfill investment plans. International liquidity determines a firm’s ability to borrow from international financiers. Domestic liquidity, on the other hand, determines a firm’s ability to borrow from domestic financiers. A claim on a piece of land in Patagonia is a domestically liquid asset in Argentina, whereas the dollar reserves backing the currency board or export sector receivables are counted as internationally liquid assets. Shortages of either form of liquidity may lead to difficulties. We shall focus on and describe an external crisis as a situation where, primarily, there is an aggregate shortage of international liquidity. That is, the sum of the international liquidity of each firm is less than that required to fulfill all investment plans. In this case, domestic liquidity serves to allocate international liquidity to the highest value of use.

We set up a model to explain how these two forms of liquidity interact and arrive at our main result of this section: in a dynamic context, when domestic liquidity is low agents will undervalue holding international liquidity relative to the constrained efficient outcome. Domestic liquidity is low when domestic financial markets are illiquid and underdeveloped. Thus, we demonstrate the existence of Pareto improving policies in cases when domestic financial markets are underdeveloped. In the next section we introduce a government and study the effectiveness of sterilization in implementing Pareto gains.\footnote{In most of our analysis, the government does not create liquidity beyond what the private sector can. This is not because we do not believe governments may have such power but because these direct benefits of policy are better understood. See, e.g., Woodford (1990), Holmstrom and Tirole (1998) and section 4 in this paper for models where the government can create liquidity.}

While the incidence of crises in emerging economies certainly have to do with aggregate shocks, in the model that follows we suppress them in order to highlight the essence of our
mechanism. We show that under certain conditions the decentralized economy will fully anticipate a crisis, and still will not do enough about it.\textsuperscript{6}

2.1 The Economic Environment, Assets, and Balance Sheets

\textit{Time.} The world lasts three periods. Time is indexed as $t = 0, 1, 2$. Date 0 is the fully flexible period when agents design the productive structure, ownership structure, and portfolio allocations. Date 1 is the crisis period, when agents must shift resources from the future to cope with shocks in the present. Date 2 represents the unconstrained future, when the economy is (relatively) rich in resources.

\textit{Agents and heterogeneity.} There are two types of agents: (i) a continuum of unit measure of domestic entrepreneurs-consumers (henceforth, domestics) with linear preferences over date 2 consumption of a single good, and (ii) foreign financiers (henceforth, foreigners) with large endowments at all dates and linear preferences with no discounting, thus the international gross interest rate is one.

\textit{International and Domestic Assets.} There are two types of assets in this economy, those that are domestically liquid and those that are internationally liquid. While either asset is a claim on one unit of the consumption good at date 2, the distinction between these assets lies in the identities of agents who assign positive value to them. A domestically liquid asset is only valued and traded among domestic agents, while an internationally liquid asset is value and traded among both domestic as well as foreign agents.\textsuperscript{7} While much of this asymmetry can have a microeconomic origin, there are sovereign issues that reinforce it. We return to this issue in the next section.\textsuperscript{8}

\textbf{Assumption 1 (Liquidity Bias)}

\textit{Foreigners only hold claims on internationally liquid assets. Domestics accept both domestic and internationally liquid claims.}

\textit{Endowment, Production, Investment, and Domestic Liquidity.} We assume that domestic agents are endowed at date 0 with $w$ units of an internationally liquid asset – e.g., the present value of export sector receivables – and access to a production technology.

\textsuperscript{6}See Caballero and Krishnamurthy (1999) for a similar model with aggregate shocks.

\textsuperscript{7}The stark distinction between domestics and foreigners in their valuation of assets is only made for simplicity. In reality, many residents behave like our foreigners at time of distress (e.g., households may be behind capital flights), and many foreigners behave like our domestics (e.g., institutional specialists well informed and connected with the domestic establishment).

\textsuperscript{8}In aggregate, the international collateral of the country is the sum of its dollar assets and a share of the present value of its export sector receivables. This determines the country’s international debt capacity. This identification is the microeconomic equivalent of that made in the sovereign debt literature (see Eaton and Gersowitz (1981) and Bulow and Rogoff (1989)).
On net, domestic agents must import materials from the rest of the world in order to produce. They do this by pledging their international liquidity to foreigners and take on foreign debt of $d_{0,f}$. Production has a time-to-build aspect. Investments are made at dates 0 and 1, and output is realized at date 2. Let $k$ denote the total amount of capital devoted to production at the beginning of date 1, inherited from date 0. Then creating capital of $k$ requires a date 0 investment of $c(k)$ units of imported goods. $c(k)$ is assumed to be strictly increasing, convex and positive.

We capture the normal churn of the economy, with its implied domestic heterogeneity, with a simple Bernoulli process. At date 1, half of the firms are spared of further investment and go on to produce $Rk$ units of goods at date 2. The rest experiences a productivity fall, $\Delta \equiv R - r > 0$, which can be offset by reinvesting a fraction $\theta \leq 1$ of $k$, in units of the imported good, in order to realize output at date 2 of:

$$\hat{R}(\theta)k = (r + \theta \Delta)k \leq Rk.$$ 

The timeline is given below.

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Date 1</th>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrowing</td>
<td>Crisis</td>
<td>Repayment</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td>Consumption</td>
</tr>
</tbody>
</table>

**Date 0 Budget Constraint:** $d_{0,f} \leq w$

**Investment Constraint:** $c(k) \leq d_{0,f}$

**Production: Time-to-build**

$$k \rightarrow^{(INTACT)} R \rightarrow R \cdot \theta \cdot k$$

Prob = 1/2

| DISTRESSED |

Need additional investment of $r \cdot k + \Delta \cdot \theta \cdot k$

$\theta \cdot k$ import goods $= R \cdot (\theta) \cdot k$

Figure 1: Time Line

This time-to-build structure underlines a critical link between financing and production during a crisis. Firms in any economy have ongoing capital needs (working capital, etc.). Starving firms of capital has the effect of shutting down production units in a potentially wasteful manner – to anticipate results, this is our crisis. The dual of this shutting down of production units is that the marginal value of capital during a crisis is very high.

We assume that the production shock at date 1 is idiosyncratic and that the identity of firms receiving the shock is private information and unverifiable. Moreover, physical
reinvestment is non-verifiable as well. As a result, an outsider can only be sure that a firm will produce $rk$ units of goods at date 2. Firms can issue claims on this quantity of output. However, these claims are only domestically liquid as international investors have no capacity to repossess the collateral of $rk$ goods.\footnote{An issue that must be addressed in the model is why foreigners do not hold domestic claims for a short period of time, and then swap them for international claims with another domestic agent when they wish. This question will be answered in the next section. The crux of the matter is that if the domestic market is illiquid (i.e. the swap encounters frictions), then having short horizons with respect to domestic claims will make foreigners averse to holding these claims. For now we proceed under the \textit{assumption} the foreigners hold no domestic claims.}

\textit{Financial structure and Balance Sheets.} Firms can raise finance at date 0 and date 1 from either domestic or foreign investors. We assume that all finance must be default free and fully secured debt – either by domestic liquidity in the case of domestics, or international liquidity in the case of foreign investors.

Date 0 decisions result in firms arriving at date 1 with installed capital of $k$ and foreign debt of $d_{0,f}$. At date 1, a firm that receives a shock is distressed (S), while a firm that escapes the shock is intact (I). The balance sheet of a domestic firm has assets of $rk$ units of domestic liquidity and $w$ units of international liquidity, and foreign debt of $d_{0,f}$. The simplest way to think of the asymmetric treatment of collateral by foreigners and domestics, is to think of foreigners studying a balance sheet of the firm as perceiving only $w$ as assets. On the other hand, a domestic sees both this quantity as well as $rk$ as assets. Thus the debt constraint with respect to foreigners at date 0 is,

$$d_{0,f} \leq w.$$ 

At date 1, if a firm takes on additional debt with foreigners, the date 1 debt constraint is:

$$d_{0,f} + d_{1,f} \leq w.$$ 

\subsection{The Microeconomic Problem}

Domestics have two sets of decisions. At date 1, given the date 0 choices of other firms (through prices) and the realization of the idiosyncratic shock, a domestic firm must decide how much to borrow (lend) and reinvest. At date 0, a firm must decide how much to invest and how much international liquidity to retain. We solve this problem by backward induction, starting from date 1.

\textit{Date 1 problem.} Consider the problem of a distressed firm in raising funds to alleviate its production shock. A choice of $\theta k$ will result in output at date 2 of $\hat{R}(\theta)k$ goods. In order to save a fraction $\theta$ of the distressed unit, the firm must raise finance and reinvest $\theta k$ import
goods. It can do this in two ways. First, the firm may have some international liquidity at date 1 that it can use to borrow directly from foreigners. That is the firm can always raise directly,

\[ d_{1,f} \leq w - d_{0,f}. \]

The latter quantity is always positive, and represents the minimum that a firm can raise at date 1. The rest must come from intact firms, which also have access to foreign investors since their capacity to borrow abroad at date 1 is also \( w - d_{0,f} \).

A distressed firm can use its domestic liquidity to access the international liquidity of the intact firms. In equilibrium, the latter discount the domestic liquidity at a rate of \( L_1 \geq 1 \) in providing international liquidity. \( L_1 \) is the date 1 interest rate. It is not an interest rate that is driven by expectations of default or currency depreciation. Rather, it is driven by liquidity considerations. \( L_1 \) is strictly greater than one only when the country is "illiquid," in the sense that its aggregate availability of international liquidity is less than its needs (see below).\(^{10}\) For this reason, we refer to \( L_1 - 1 \) as the international liquidity premium.\(^{11}\)

Through this "credit chain"—which represents the domestic financial markets in our framework—the distressed firm is able to aggregate the international liquidity of the economy and pledge this to foreigners to raise resources for date 1 reinvestment. Let \( d_{1,f} \) and \( d_{1,d} \) represent the foreign and domestic debt contracted by a distressed firm at date 1, respectively. We can then write the problem of a distressed firm as,

\[
\begin{align*}
\text{(P1)} \quad V_s & \equiv \max_{\theta,d_{1,f},d_{1,d}} \quad w + \hat{R}(\theta)k - d_{0,f} - d_{1,f} - d_{1,d} \\
\text{s.t.} \quad (i) & \quad d_{1,f} + d_{0,f} \leq w \\
& \quad d_{1,d} + d_{1,f} + d_{0,f} \leq w + \frac{rk}{L_1} \\
& \quad \theta k = d_{1,f} + \frac{d_{1,d}}{L_1} \\
& \quad \theta \leq 1.
\end{align*}
\]

Constraints (i) and (ii) are balance sheet constraints (net marketable assets greater than liabilities), while constraint (iii) reflects that new investment must be fully paid with the resources received by the firm at date 1 in taking on debts of \( d_{1,f} \) and \( d_{1,d} \). Constraint (iv) is purely technological.

An intact firm at date 1 has only one decision: how much finance will it extend to the distressed firm. Suppose that the firm accepts claims at date 1 of \( x_{1,d} \) (face value of date 2

\(^{10}\) Allen and Gale (1994) also develop a liquidity based model of asset prices, in which prices may be based on "cash in the market" rather than fundamentals. Similarly, in our model, when the country is illiquid interest rates are partly driven by the supply of international liquidity.

\(^{11}\) While it can be represented as a shift in the interest parity condition, it is important to realize that it is domestic suppliers of international liquidity, rather than foreigners, who earn the liquidity premium.
goods) in return for making a date 1 contribution of $x_{1,d}/L_1$, then,

\[ (P2) \quad V_i \equiv \max_{x_{1,d}} \quad w + Rk + x_{1,d} - \frac{x_{1,d}}{L_1} \]
\[ s.t. \quad w - d_{0,f} - \frac{x_{1,d}}{L_1} \geq 0 \]

**Date 0 problem.** At date 0, a firm looking forward to date 1 can expect to find itself as either distressed or intact. Thus the decision at date 0 is,

\[ (P3) \quad \max_{k,d_{0,f}} \quad \frac{(V_n + V_i)}{2} \]
\[ s.t. \quad d_{0,f} \leq w \]
\[ c(k) = d_{0,f}. \]

### 2.3 Equilibrium and Crises

**Equilibrium.** Market clearing in the domestic debt market at date 1 (capital letters denote aggregate quantities) requires that the aggregate amount of domestic debt taken on by distressed firms is fully funded by intact firms:

\[ D_{1,d} = \frac{1}{2}d_{1,d} \]
\[ X_{1,d} = \frac{1}{2}x_{1,d}. \]

Therefore, market clearing,

\[ D_{1,d} = X_{1,d}, \tag{1} \]
determines the gross interest rate, $L_1$.

An *equilibrium* of this economy consists of date 0 and date 1 decisions, $(k,d_{0,f})$ and $(\theta,d_{1,f},d_{1,d},x_{1,d})$, respectively, and prices $L_1$. Decisions are solutions to the firms’ problems (P1), (P2), and (P3) given prices. At these prices, the market clearing condition (1) holds.

Let us now study equilibrium in more detail. Starting from date 1, consider equilibrium in the domestic debt market. Since $\Delta$ is large, a distressed firm would choose to save as many of its production units as it can. It will first borrow fully up to its international debt capacity,

\[ d_{1,f} = w - d_{0,f}. \tag{2} \]

If the amount raised from international investors, $w - d_{0,f}$, is less than the funds needed for restructuring, $k$, the firm will have to access the domestic debt market to make up the shortfall. If the firm borrows fully up to its domestic debt capacity, it will raise,

\[ d_{1,d} = \frac{rk}{L_1}. \tag{3} \]
As long as the sum of the right hand side of (2) and (3) is more than the borrowing need, the firm is unconstrained in its reinvestment at date 1 and all production units will be saved. In this case, the firm will borrow less than its domestic debt capacity.

Intact firms can tender at most their excess international debt capacity of \( w - d_{0,f} \) in return for this domestic debt. Thus a necessary condition for all production units to be saved is that,

\[
\frac{k}{2} \leq w - d_{0,f}. \tag{4}
\]

We shall refer to this constraint as the international liquidity constraint. When neither (3) nor (4) binds, all production units are saved. Since there is excess supply of funds from intact firms relative to domestic demand for funds, there is no international liquidity premium, and \( L_1 \) is equal to the international interest rate (one).

The other extreme case is when both (3) and (4) bind. Equilibrium in the domestic debt market requires that,

\[
\frac{r k}{L_1} = w - d_{0,f}.
\]

Since (3) binds, distressed firms borrow fully up to their debt capacity. As (4) binds, intact firms purchase this debt with all of their excess funds. Solving for \( L_1 \), yields

\[
L_1 = \frac{r k}{w - d_{0,f}} > 1. \tag{5}
\]

That is, in this case the international liquidity premium is positive. \( L_1 \) is above the (gross) international interest rate in order to clear the domestic market for scarce international liquidity. One half times the numerator in (5) corresponds to the transferable domestic resources owned by distressed firms. The international liquidity premium is positive when these resources are greater than one half times the denominator, which corresponds to the excess international liquidity owned by intact firms.

Define the index of domestic illiquidity as the difference between the marginal profit of saving a distressed production unit and the domestic interest rate of \( L_1 \). When (4) binds, this is simply,

\[
s_d = \Delta - L_1,
\]

Equilibrium at date 1 can place the economy in one of four regions, classified according to which of the two (domestic and international) liquidity constraints are binding. These regions are summarized in the appendix.\footnote{Depending on the date 0 choices of \( k \) and \( w - d_{0,f} \) (see the appendix) any of four regions are possible. Date 0 decisions then pin down these choices and tell us which region will prevail at date 1. A property of the model is that, depending on parameter values, any of the four regions are equilibrium outcomes of the model. This is largely due to the fact that model is one of certainty - agents at date 0 know exactly} In the main text we focus on a crisis scenario, where both liquidity constraints are binding. At the aggregate level, the economy is liquidity
constrained with respect to foreigners; at the individual level, firms are liquidity constrained with respect to other domestics since they are selling all of their domestic liquidity in aggregation; real investment is constrained; domestic spreads are positive; and the interest rate of $L_1$ is above the international interest rate. This is the most interesting configuration for the prevention-policy questions we intend to address in the main section of the paper.

**Assumption 2 (Conditions for Crisis)**

*Assume that:*

$$c^{-1} \left( \frac{\Delta + R}{1 + \Delta} \right) + c \left( c^{-1} \left( \frac{\Delta + R}{1 + \Delta} \right) \right) \leq r < \Delta \leq w < c^{-1} \left( \frac{1 + R}{2\Delta} \right) + c \left( c^{-1} \left( \frac{1 + R}{2\Delta} \right) \right).$$

**Proposition 1 (Crisis Region)**

Under assumption 2, date 0 decisions of $k$ and $d_{0,f}$ are such that both the international constraint and the domestic constraint are binding. The international liquidity premium and the domestic illiquidity index are positive, and some projects are downsized. $L_1 > 1, s_d > 0, \theta < 1.$

![Figure 2: (P3)](image)

which of the four regions will prevail at date 1. In Caballero and Krishnamurthy (1999), the model includes aggregate uncertainty (essentially in $w$), so that the occurrence of any particular region is a surprise and at date 0 agents face a date 1 world where these regions are blended.
Graphically, the solution to (P3) is represented in figure 2. The inner curve represents the budget set. This is the set of points of \((w - d_{0,f}, k)\) such that the date 0 budget constraint is satisfied with equality. The tangent line reflects the tradeoff between holding domestic and international liquidity. Its slope is given as \(-\frac{r + \frac{\Delta}{L_1}}{L_1 + \Delta}\), so that higher values of \(L_1\) make the line flatter.

In order to arrive at the crisis region, point A needs to lie between the two rays. This is the area where, \(k > w - d_{0,f}\), but \(\frac{r}{\Delta}k < w - d_{0,f}\).

2.4 Constrained Inefficiency

When the economy is in the crisis-region it is international liquidity that presents the dominant bottleneck. Domestic liquidity, on the other hand, determines the allocation of surplus during these crises. We show next that when there is insufficient domestic liquidity, the market surplus allocation will not correspond to that of the central planner. In particular, the central planner will desire to allocate all the surplus to the liquidity suppliers. This will generally not happen when domestic liquidity is low as those in need have insufficient commitment to compensate the international liquidity providers adequately.

**Proposition 2 (Underprovision of International Liquidity)** When both liquidity constraints are binding at date 1, the decentralized equilibrium is constrained inefficient. A central planner can effect a Pareto improvement by forcing the private sector to decrease \((k, d_{0,f})\), thereby increasing the international liquidity of the economy. The welfare gain from this intervention rises with the index of domestic illiquidity, \(s_d\).

Proof: First let us rewrite (P3), substituting in the value function from (P1) and (P2). A date 0 choice of \((k, d_{0,f})\) result in date 2 resources (net of any contracted debt) of,

\[(w - d_{0,f})\Delta + \frac{r k}{L_1}\Delta\]

if the firm is distressed. This is because \((w - d_{0,f})\) is directly pledged to foreigners, and the proceeds invested at the project return of \(\Delta\). The \(rk\) of domestic liquidity is sold at the interest rate of \(L_1\), and the proceeds invested at \(\Delta\). If the firm is intact, date 2 resources are,

\[(w - d_{0,f})L_1 + Rk.\]

Thus the date 0 program is,

\[
(P4) \quad \max_{k,d_{0,f}} \quad (R + r \frac{\Delta}{L_1})k + (\Delta + L_1)(w - d_{0,f})
\]

s.t. \(w \geq D_{0,f}\)

\(c(k) = d_{0,f}\)
Consider the program for a central planner who directly chooses \((k, d_{0, f})\) to maximize the equally weighted sum of utilities of agents in this economy. To do this we simply substitute the expression for \(L_1\), \((5)\), into the objective of \((P4)\), arriving at an expression that is free of prices. The program for a central planner is,

\[
(P5) \quad \max_{K, D_{0, f}} \quad (R + r)K + 2\Delta(W - D_{0, f})
\]

\[
s.t. \quad w \geq D_{0, f} \quad c(K) = D_{0, f}
\]

The solutions to \((P5)\) are the constrained efficient decisions of the economy. The only difference between the programs \((P4)\) and \((P5)\) is in the objective. Subtracting the objective in \((P5)\) from that of \((P4)\) we arrive at,

\[
s_d \left( \frac{r}{L_1} K - (W - D_{0, f}) \right).
\]

At a given equilibrium, this term must be zero. But it is apparent that individuals and the central planner value a marginal unit of international liquidity and domestic liquidity quite differently. Moreover, this misvaluation is directly proportional to \(s_d\), the domestic illiquidity index.

The first order condition of \((P5)\) gives,

\[
c'(K) = \frac{R + r}{2\Delta},
\]

while that of \((P4)\) yields,

\[
c'(k) = \frac{R + r \Delta}{\Delta + L_1}.
\]

Graphically, we can represent the solutions to \((P4)\) and \((P5)\) in figure 3.

The two tangent lines reflect the tradeoff between retaining liquidity and making physical investments. The steeper of the two (the solid line) represents the objective in \((P4)\), the decentralized case. On the other hand, the dashed line is the objective in the central planner’s problem. From the point of view of retaining liquidity, the central planner is only concerned with protecting international liquidity. This is because international liquidity is all that can be used to attract foreign investment. However at the microeconomic (decentralized) level, both domestic and international liquidity can be used to secure financing. This is the basic tension between the individual and the central planner’s problem.\(^{13}\)

\(^{13}\)It is important to realize that the externality we have described is not of the traditional Bardhan-Harberger type, where individual borrowers do not internalize the fact that the country faces an upward slopping supply of foreign loan. Indeed, in our setup the economy does face a very steep international funds supply, for the country is rationed after some point, but the externality arises only when domestic spreads are positive.
The solutions to the programs are points A and B in figure 3. It is clear that the central planner will prefer to retain more international liquidity than individual firms when \( s_d > 0 \). In other words, a central planner must find a mechanism to induce the private sector to reduce \( k \).

![Figure 3: (P4) and (P5)](image)

3 The Government, Asset Liquidity, and Sterilization Policy

Having set the stage for policy intervention, we can now consider whether the policies typically utilized in emerging markets are indeed likely to be successful. In practice, the chief instrument to offset the expansionary effects of sustained capital inflows is sterilization. That is, the central bank sells public bonds to domestics in order to reabsorb the monetary expansion brought about by capital inflows. A capital inflow plus sterilization leaves the central bank with more international reserves as assets, and more domestic government bonds as liabilities. Thus, in reduced form, the central bank sells bonds for international reserves. In our language, this corresponds to a swap of domestic liquidity for some of the international liquidity held by the private sector (which needs to borrow abroad to buy the public bonds).

Widespread as it may be, sterilization is perceived as a “risky” strategy, hampered by the possible overreaction of the private sector. But the mechanisms behind this “risk” are not well understood, let alone modeled. In this section we offer a methodic analysis of sterilization in an economy with underdeveloped financial markets. We shall begin by noting conditions under which the private sector completely undoes the central bank’s action. In
cases where this does not occur, we demonstrate conditions under which the policy can be Pareto improving, and those under which it results in a Pareto loss. Before doing so, however, we must introduce the (consolidated) government, its instruments and constraints, and the implications of its action for asset liquidity.

3.1 Preliminaries

3.1.1 The Government and Sovereign Risk

The minimum number of ingredients we need in order to address our policy question is one public financial instrument and a tax to ensure that the government’s budget constraint is balanced. We start with this minimum and enrich the set of public financial instruments to include money in section 4, when we discuss exchange rate systems. For now, the economy is still real – or, as an equivalent interpretation, dollarized. We also consolidate the central bank and the treasury.

Public bonds and taxation. The government issues two period public bonds, with face value $B$. They are issued at date 0, but can be bought and sold in secondary markets at date 1. Our main results can also be obtained with shorter maturities, but they require additional assumptions on government specific risk – as opposed to country-wide risk. We return to this issue in the conclusion. The government has the power to tax agents, and does so exclusively to balance its budget.$^{14}$

Assumption 3 (Sovereign Risk)

The government suspends convertibility at date 1 and prevents foreigners from transporting any date 2 goods from within the economy across the border.

At date 1, when the international liquidity constraint binds, the government will suspend convertibility and repudiate any date 2 claims directly owed to foreigners. Foreigners holding claims directly on export sector revenues ($w$) are unaffected by suspension, since they can seize these receivables directly. However a foreigner holding a claim on a domestic firm or a domestic government bond has only one choice. He must sell these claims to a domestic agent who has some international liquidity and exit. That is domestic claims are

$^{14}$Importantly, taxes are always paid in units of liquidity (either domestic or international). That is, suppose that a firm has $rk$ units of domestic liquidity at date 1, and that the government levies a tax of $T$ on the firm. Then, after the transaction the firm will be left with $rk - T$ units of domestic liquidity, and the government will be left with $T$ units of domestic liquidity. Thus, if the firm has a tax liability of $T$ to the government, this simply reduces its liquidity with respect to other firms. That is, unlike Woodford(1990) and Holmstrom and Tirole (1998), our government cannot create liquidity. The simplest way to think of taxes is that they alter the balance sheet of a firm by introducing an additional liability. This then affects the firm’s ability to raise finance from other agents.
internationally liquid only to the extent that they can be exchanged for the private sector’s international liquidity at date 1.

**Lemma 1 (Foreigner’s Short Horizon)**

*Foreigners have short horizons with respect to domestic claims. Any domestic claim acquired at date 0 will always be sold at date 1. Claims on international liquidity are free of this short horizon.*

In assumption 1 we simply assumed that foreigners had a bias away from domestic claims. Sovereign risk justifies this assumption. If the sovereign may suspend convertibility at date 1, foreigner’s will shy away from holding domestic claims.

### 3.1.2 Sterilization Policy

**Definition:**

*Sterilization is an open market sale at date 0 of government bonds with face value $B$ for $\frac{B}{L_0}$ units of international liquidity.*

Suppose that sterilization takes place. To purchase these bonds the private sector borrows against its international liquidity of $w$ and acquires bonds. Let $L_0$ denote the gross interest rate on this bond, so that the government collects in total $\frac{B}{L_0}$ international reserves. At date 1, the government offers these reserves to purchase domestic assets, receiving $L_1 \frac{B}{L_0}$ units of domestic assets in return. Then the budget constraint of the government is given by:

$$T + L_1 \frac{B}{L_0} = B,$$

where any shortfall is made up with a lump-sum tax on firms (in units of domestic liquidity).

The private sector’s balance sheet is altered by this transaction. Suppose that a firm purchases $b$ units of bonds. At date 1, the firm will have an extra $b - T$ units of domestic liquidity, while its international liquidity would have shrunk by $\frac{B}{L_0}$ (or $d_{0,f}$ has risen). Thus, if the firm is intact, its date 2 output is,

$$V_i = (w - d_{0,f})L_1 + Rk + b - T,$$

while if distressed, it is

$$V_s = (w - d_{0,f})\Delta + \Delta \frac{rk + b - T}{L_1}.$$ 

But at date 0, the firm has a choice as to how much of these bonds to purchase at the interest rate $L_0$. Thus its date 0 problem can be written as,

$$(P6) \quad \max_{k,d_{0,f}} \quad (w - d_{0,f})(\Delta + L_1) + k(R + r\frac{\Delta}{L_1}) + (b - T)(1 + \frac{\Delta}{L_1})$$

s.t. \quad d_{0,f} \leq w

$$c(k) + \frac{B}{L_0} = d_{0,f}.$$
Since the government purchases domestic assets at date 1, the market clearing condition gives us that,

$$L_1 = \frac{rK + B - T}{W - D_{0,f} + 2\frac{P}{E_0}}$$

(7)

**Lemma 2 (Date 0 Interest Rates)**

If the private sector holds government bonds in equilibrium, the date 0 interest rate must satisfy,

$$L_0 \geq L_1$$

In order for domestics to hold government bonds, they must be compensated for losing their international liquidity. Taking on an extra unit of debt costs $\Delta + L_1$. However purchasing one bond yields an extra $L_0\frac{\Delta + L_1}{L_1}$. This gives us the inequality in the lemma.

With these preliminaries behind us, we now turn to the mechanisms underlying the success and failure of sterilization.

### 3.2 The Effectiveness of Policy

Sterilization is naturally associated with a capital inflow, as either foreigners or domestic investors will attempt to buy the high yield government bonds. Foreigners have short horizons, however. Thus, if foreigners see a scenario where they can purchase these bonds at date 0 and sell them back to the domestic private sector at date 1, they will step in to purchase the bonds. They will do this only to the extent that the private sector has some international liquidity to offer at date 1. Thus the liquidity of the public bonds are closely tied to the international liquidity of the private sector. The other potential buyer of the bonds is the domestic private sector itself. Since this sector does not have short horizons, it can always take advantage of the high return on the bonds by borrowing abroad (a capital inflow) to purchase the public bonds. However, once again, the capacity of the private sector to do so is limited by its international liquidity. In both cases it is the liquidity of the private sector that determines the outcome of sterilization.

#### 3.2.1 Liquidity Conservation

**Lemma 3 (Liquidity Bias and Aggregate International Illiquidity)**

If the government sterilizes so that the private sector is internationally illiquid, $d_{0,f} = w$, then foreigners will hold no domestic claims and restrict their holdings to international claims.

Proof: see appendix.
This is the date 0 effect of future suspension of convertibility and market illiquidity. If the domestic private sector has no international liquidity to offer a foreigner when he sells domestic claims at date 1, then the foreigner’s date 1 liquidity bias extends back to date 0. The foreigner anticipates that there will be a suspension of convertibility at date 1 and that there will be no buyers of his domestic claims. This means he restricts his holdings of domestic claims at date 0. The foreigner only hold direct claims on international liquidity that are free of suspension of convertibility.

**Proposition 3 (Liquidity Conservation)**
Take the case where \( d_{0,f} < w \). Let \((k', d'_{0,f}, L'_1)\) be equilibrium choices and prices to (P6) and (7) when \( B = 0 \) and let \((k, d_{0,f}, b, L_1)\) be equilibrium choices and prices to (P6) and (7) when \( B > 0 \) and \( L_0 = L_1 \). Then,

\[
\begin{align*}
L_1 &= L'_1 \\
k &= k' \\
d_{0,f} &= d'_{0,f} + \frac{B}{L_0}
\end{align*}
\]

Proof: see appendix.

The proposition is fairly intuitive. When \( d_{0,f} < w \), foreigners are willing to hold the bonds at date 0. Moreover since the private sector is not internationally liquidity constrained, it can always borrow from foreigners to purchase the bonds that the government offers. Thus the gain in reserves of the government are offset one-for-one with a loss in international liquidity of the private sector —either at date 0 if the private sector purchases the bonds, or at date 1 if foreigners purchase the bonds— and the sterilization is completely undone.

3.2.2 Pareto Improving Policy

**Proposition 4 (Liquidity Contraction)** There exists a sterilization policy of \( B > 0 \) such that the resulting equilibrium of \((k, d_{0,f}, b, L_1)\) is Pareto superior to \((k', d'_{0,f}, L'_1)\), as long as, in the resulting equilibrium, \( d_{0,f} = w \). Optimal intervention raises the date 0 interest rate relative to date 1, \( L_0 > L_1 \), while lowering the date 1 interest rate relative to the case of no-intervention, \( L_1 < L'_1 \).

Proof: As we showed in proposition 2, the decentralized equilibrium is constrained inefficient. Optimal policy should move the economy from point A to point B on figure 3. That is to say, optimal policy should reduce date 0 investment so that \( k < k' \).
Suppose that the central bank offers $B$ bonds at $L_0$. The program for a firm choosing to purchase $b$ bonds is,

$$\max_{k, d_{0,f}} \quad (w - d_{0,f})(\Delta + L_1) + k(R + r\frac{\Delta}{L_1}) + (b - T)(1 + \frac{\Delta}{L_1})$$

s.t. \quad \begin{align*}
    d_{0,f} &\leq w \\
    c(k) + \frac{b}{L_0} & = d_{0,f}.
\end{align*}

From the previous proposition, we know that sterilization is ineffective if $d_{0,f} < w$. Taking the other case ($d_{0,f} = w$), we can rewrite the program as,

$$\max_{k,b} \quad k(R + r\frac{\Delta}{L_1}) + (b - T)(1 + \frac{\Delta}{L_1})$$

s.t. \quad \begin{align*}
    c(k) + \frac{b}{L_0} & = w.
\end{align*}

The first order conditions for this program yield,

$$L_0 c'(k) = \frac{R + r\frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}}$$

Now optimal policy will be such that $k < k'$. For this to be the case, we must have that $c'(k) < c'(k')$. $L_0$ satisfies,

$$1 - \frac{\Delta L_0}{R + r\frac{\Delta}{L_1}} < c'(k') = \frac{R + r\frac{\Delta}{L_1}}{\Delta + L_1'}$$

Rewriting this yields

$$L_0 > L_1 \quad \frac{R + r\frac{\Delta}{L_1}}{R + r\frac{\Delta}{L_1}} \frac{\Delta + L_1'}{\Delta + L_1}.$$  \hfill (8)

Consider the market clearing condition for $L_1$.

$$L_1 = L_0 \frac{rK + B - T}{2B}$$

which we can rewrite as,

$$L_1 \frac{B}{L_0} + L_1 \frac{B}{L_0} = rK + B - T.$$

Substituting in the government’s budget constraint and rewriting,\footnote{The budget constraint for the government is that,

$$T + L_1 \frac{1}{L_0} B = B.$$}

$$L_1 = L_0 \frac{rK}{B}$$

Whereas,

$$L_1' = \frac{rK'}{W - D_{0,f}}.$$
If $k < k'$ then $\frac{B}{L_0} = W - c(K) > W - D'_{0,f}$. Therefore, $L_1 < L'_1$ and $B > 0$. Combining this with (8), we can conclude that $L_0 > L_1$.\(^{16}\)

When the central bank offers bonds at a high interest rate ($L_0 > L_1$), there is clearly a capital inflow to purchase the bonds. However, when the private sector is internationally illiquid ($d_{0,f} = w$), foreigners are unwilling to purchase the bonds since they recognize that there will be no buyers for the bonds at date 1. Hence the only buyers of the bonds are domestics. As long as domestics have reached their international debt capacity, government bonds cannot be purchased purely by borrowing abroad and saving. Expenditure must be reduced as well, which is why $k$ falls. This is the heart of intervention, by issuing domestic bonds, the government crowds out private investment, while at the same time acquiring all of the international liquidity of the private sector. It can then transfer this back to the private sector at date 1. The government smooths shocks between date 0 and date 1. It removes liquidity from the private sector at date 0 by raising interest rates, then it injects this liquidity back into the markets at date 1, thereby lowering the cost of liquidity.

The main obstacle faced by the government in its attempt to reduce aggregate demand (investment in $k$) at date 0 is the reaction of the private sector. In a successful sterilization $L_1$ falls both below $L_0$ and below $L'_1$. Remember that the private sector's incentives for international liquidity provision are linked to $L_1$. When $L_1$ is low the private sector has very little incentive to do so. Thus, when the government intervenes, it actually reduces the private sector’s incentives for international liquidity provisioning. So much so that the private sector chooses to sell all of its liquidity provisions and hold domestic government bonds returning $L_0 > L_1$. As long as $d_{0,f} = w$ the job of liquidity provisioning is left fully in the hands of the central bank.

The private sector anticipates a “bailout” and takes action. If the central bank is committed to supplying liquidity at date 1—in practice, via support of an exchange rate, or via a government guarantee—the private sector will try to take full advantage of this support. First, the private sector will have no incentive to hedge (independently hold its own international liquidity provisions.) Second, the private sector will attempt to undo the liquidity provisioning of the government. They will borrow abroad paying the opportunity cost of $L_1$ and invest locally at the higher return of $L_0$. When $d_{0,f} < w$ this action by the private sector fully offsets the liquidity provisioning by the central bank. Effectively, the private sector appropriates any international reserves held in the central bank. When

\(^{16}\)We also need to make sure that $\Delta > L_1$ after the intervention so that we are still in the region where both liquidity constraints are binding.

$$rK < \frac{\Delta}{L_0}B = \Delta(W - c(K))$$

Since $K < K'$, if the condition is satisfied at the decentralized solution, it must also be satisfied at the central planner's solution.
\( d_{0,f} = w \) the private sector reaches its international debt capacity and it must reduce
domestic expenditure in order to purchase government bonds.

### 3.3 Policy “Mistakes”

Is liquidity provisioning by the central bank a policy mistake? The response of the private
sector, when viewed in isolation, would certainly suggest so. However, to appropriately
judge the policy one must view the private sector response in conjunction with the central
bank action at both date 0 and date 1.

When sterilization is succesful, the private sector is not the marginal international liq-
uidity provider. The central bank takes over this job. Indeed, since \( L_1 \) falls, the private
sector has little incentive to do so. Is it possible to arrive at scenario where both, \( L_1 \) falls
so that the private sector has little incentive to liquidity provision, and the private sec-
tor remains the marginal liquidity provider? If so, the private sector would free-ride by
cutting its liquidity provisioning (relative to the case of no-intervention) and on net the
economy would lose international liquidity. In this section we show that this scenario is a
very real possibility when domestic markets are illiquid. We shall demonstrate conditions
under which the capital inflow accompanying sterilization leads not just to a purchase of
government bonds but also to increased lending to the domestic private sector.

#### 3.3.1 Illiquid Secondary Markets

At date 1, government bonds are exchanged for international liquidity by both domestic
distressed firms and any potential foreign investors. Distressed firms sell in order to receive
funds for investment, while foreign holders sell in order to exit the market. We assume that
this transaction suffers frictions.

**Assumption 4 (Secondary Market Illiquidity)**

A sale at date 1 of one unit of a date 2 government bond suffers a real cost of \( \alpha < 1 \). Selling
\( X \) units of bonds only recovers \( \frac{X(1-\alpha)}{L_1} \) units of international liquidity.

The illiquid secondary market makes it harder to liquidate the two period government
bond and exchange it for international liquidity. Transactions costs must be paid, there are
search costs involved in making the exchange, and potentially even rents must be paid to
market makers. This will further raise the required return for holding bonds at date 0, and
will increase the domestic illiquidity index, \( s_d \), at date 1.\(^{17}\)

\(^{17}\)We have assumed this illiquidity to only affect the secondary market for government bonds, but in
practice one would expect that it extends to the corporate lending market as well. That is, since distressed
firms sell \( rk \) domestic claims at date 1, one may expect that this transaction too would suffer frictions. This
extension does not alter any of the results.
3.3.2 Backfiring Policy

Suppose that the government sells bonds at date 0 in an attempt to sterilize, but does not intervene sufficiently so that the private sector is still internationally liquid at date 0. B government bonds are sold at interest rate of \( L_0 \). Consider the program for a firm choosing to hold \( b \) bonds at date 0,

\[
(P7) \quad \max_{k, d_0, f, b} \quad (w - d_{0,f})(\Delta + L) + k(\rho + (1 - \alpha)r \frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha)\frac{\Delta}{L_1}) \\
\text{s.t.} \quad d_{0,f} \leq w \\
\quad c(k) + \frac{b}{L_0} = d_{0,f}.
\]

Lemma 4 (Government Interest Rates)
The date 0 interest rate on government bonds will always exceed the date 1 interest rate. \( L_0 > L_1 \).

This lemma is easy to verify. Purchasing one government bond costs \( \frac{1}{L_0} \) units of international liquidity. At date 1, if the firm is distressed the bond can be sold for \( \frac{1 - \alpha}{L_1} \) and the proceeds invested to yield output at date 2 of \( \Delta \frac{1 - \alpha}{L_1} \). If the firm is not distressed, the bond is held till maturity to return one.\(^{18}\) The opportunity cost of using one unit of international liquidity is to use it at date 1. If the firm is distressed, this yields \( \Delta \) at date 2, while if the firm is intact lending yields \( L_1 \) at date 2. Thus firms purchase bonds as long as,

\[
\Delta \frac{1 - \alpha}{L_1} + 1 \geq \frac{1}{L_0}(\Delta + L_1).
\]

Rewriting,

\[
L_0 \geq L_1 + \frac{\alpha \Delta L_0}{\Delta + L_1} > L_1.
\]

Since government bonds are illiquid at date 1, they must pay a high interest rate in order to be held. Corporate borrowing rates on the other hand are determined by \( L_1 \), not \( L_0 \). If a firm was to sell some its domestic liquidity at date 0 and borrow (say, from another domestic), it would pay the interest rate of \( L_1 \). This is because another domestic would value the lost international liquidity at the opportunity cost – the return to lending at date 1, which is \( L_1 \).

Policy can backfire because sterilization leads to a fall in \( L_1 \). When the government sterilizes and holds international liquidity provisions, it commits to supplying these to the

\(^{18}\)It is also easy to check that foreigners will never purchase government bonds. The cost \( \alpha \) is only suffered if claims are liquidated at date 1. A foreigner has short horizons with respect to domestic claims. A domestic has short horizons only if his firm is distressed. With probability one-half, the domestic holds the claim to maturity and does not liquidate. Thus domestics have longer horizons and bear less of the illiquidity cost. Which means that as long as domestic are holding government bonds, it must be that foreigners shy away from holding them.
market at date 1. If the bonds that the government issues are longer term and illiquid, these liquidity provisions go toward supporting domestic private sector assets. The corporate borrowing rate, \( L_1 \), falls and corporates find it attractive to borrow and increase investment at date 0.

To see why, consider an extreme case of illiquidity where the government bonds have no secondary market and must be held till maturity (\( \alpha = 1 \)). Market clearing at date 1 is as follows: Distressed firms sell \( rK - T \) total of domestic liquidity at the date 1 interest rate of \( L_1 \), the \( B \) government bonds have no secondary market and must be held until maturity. Since one-half of firms are distressed, in total, firms raise for investment

\[
\frac{rK - T}{2L_1}
\]

Now at the other side of the market are intact firms and the government. The government has international liquidity of \( \frac{B}{L_0} \), while intact firms have in aggregate, \( \frac{W - D_{0,f}}{2} \). Thus market clearing is,

\[
\frac{rK - T}{2L_1} = \frac{W - D_{0,f}}{2} + \frac{B}{L_0}.
\]

(9)

Rewriting this expression, yields

\[
\frac{rK - T}{L_1} + \left( \frac{B}{L_1} - \frac{B}{L_0} \right) = W - D_{0,f} - \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}.
\]

The loss to the government of intervening by issuing bonds at date 0 at the high rate of \( L_0 \) and purchasing bonds at date 1 at the lower rate of \( L_1 \) must be made up in taxes. Thus, rewrite this expression by substituting in, \( T = B - \frac{L_1}{L_0}B \), as,

\[
\frac{rK}{L_1} = W - D_{0,f} - \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}.
\]

(10)

Now imagine that the private sector exactly offsets the government’s reserve accumulation. That is to say, suppose that for every unit that \( \frac{B}{L_0} \) rises, the private sector takes on an extra unit of date 0 debt. In this case, the term in the parentheses on the right hand side would be unchanged by sterilization. However if this is the case, and \( B > 0 \), it must be that \( L_1 \) falls, since the supply of liquidity to purchase private sector assets has risen by \( \frac{B}{L_1} \). This is why sterilization backfires. The government supports private sector assets and hence lowers corporate borrowing costs.

The case where the government bonds are fully liquid highlights the key role played by illiquid markets in the previous conclusion. Suppose that \( \alpha = 0 \), so that government bonds can be sold at date 1 without any friction. In this case, distressed firms sell \( B \) bonds at the price of \( L_1 \) at date 1. Thus append \( \frac{B}{L_1} \) to the left hand side of (10) to arrive at,

\[
\frac{rK}{L_1} + \frac{B}{L_1} = W - D_{0,f} - \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}.
\]
The government’s reserves go towards purchasing back the bonds that it issued at date 0. Sterilization does not bring additional support for corporate assets, and hence \( L_1 \) is unaffected.

Let us now state this result more formally.

**Proposition 5 (International Liquidity Loss)**
Consider the case where \( \alpha > 0 \). Let \((k', \varphi_{0,f}', L'_1)\) be equilibrium choices and prices to (P7) and (9) when \( B = 0 \) and let \((k, \varphi_{0,f}, b, L_0, L_1)\) be equilibrium choices and prices to (P7) and (9) when \( B > 0 \) but \( d_{0,f} < w \). Then, we have that,

\[
L_1 < L'_1 \\
k > k' \\
d_{0,f} > \varphi_{0,f} + \frac{B}{L_0}
\]

Proof: see appendix.

An equivalent way to think about this result is in terms of capital inflows. When the government sterilizes and commits to providing liquidity at date 1, foreigners come in to purchase short term assets that will be supported by the government. If these assets are the government bonds that are issued in sterilization, then the sterilization is offset. However, when the secondary market for government bonds is illiquid, foreigners will shy away from holding these bonds and will instead demand other short term assets. The corporate sector sees this as a borrowing opportunity on good terms and increases investment. Thus a symptom of counterproductive sterilization is that corporate borrowing rates fall relative to government borrowing rates. In other words, as \( B \) rises, the spread between the interest rate on government bonds and that on corporate lending, \( L_0 - L_1 \) rises.

Backfiring is a policy mistake, but one that is not easy to avoid in the environment of emerging markets. It occurs because the government does not sterilize enough — the private sector is left with some international debt capacity that it borrows against on favorable terms — and the domestic instruments that the government issues are illiquid. How much sterilization is enough? In the model, the government has to sell enough bonds so that the private sector reaches its international debt capacity \((\varphi_{0,f} = w)\). That is the government takes all of the private sector’s liquidity. This is clearly an abstraction. Our supply curve of international funds is kinked at \( \varphi_{0,f} = w \). A more gradual slope would cause interest rates to rise for smaller interventions. Indeed the key point is that in unsuccessful sterilizations, corporate borrowing rates fall not rise, both absolutely and relative to government rates. Nevertheless, during the booms when international capital markets are all too willing to lend to emerging economies, the amount of required sterilization is much higher and as such
the possibility that policy may backfire is very real. The second factor behind the failure of sterilization, that of illiquid secondary markets, is more structural and seems unavoidable in emerging markets.

3.4 Optimal Policy with Illiquid Secondary Markets

When there are illiquid secondary markets for government bonds, optimal policy must take this into account. Are there always Pareto improving policies? We answer this question next.

**Proposition 6 (Sterilization with Illiquid Markets)** There exists a sterilization policy of $B > 0$ such that the resulting equilibrium of $(k, d_{0,f}, b, L_0, L_1)$ is Pareto superior to $(k', d_{0,f}', L_1')$, as long as,

(i) Domestic markets are sufficiently liquid (i.e small $\alpha$), and

(ii) In the resulting equilibrium, $d_{0,f} = w$, so that the private sector is internationally illiquid.

Optimal policy requires that, in equilibrium, the government raises date 0 interest rates, $L_0 > L_1$, and the date 1 interest rate falls relative to no-intervention, $L_1 < L_1'$.

Proof: At date 1, the private sector sells the government bonds in return for the international liquidity. When $\alpha > 0$, this transaction suffers a real cost. Thus intervention can be costly because it requires the private sector to sell more bonds into an illiquid market. Policy can result in a Pareto improvement as long as this cost is not too high. In the extreme case when $\alpha = 0$, policy always leads to Pareto improvement.

First let us define $UPRIV$ as,

\[
(P8) \quad \max_{k, d_{0,f}} \quad U^{PRIV} \equiv (w - d_{0,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1})
\]

\[\text{s.t.} \quad d_{0,f} \leq w\]

\[c(k) = d_{0,f}.\]

The first order condition for this program is,

\[c'(k') = \frac{R + r \frac{\Delta}{L_1}}{\Delta + L_1}\]

The optimal choice is denoted $k'$ to refer to the no-intervention point. The market clearing condition remains that of (5).

Suppose that a central bank offered $B$ bonds for sale at the interest rate of $L_0$, but bearing the illiquidity cost of $\alpha$. This program must be altered as follows:

\[
\max_{k, d_{0,f}, b} \quad (w - d_{0,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha) \frac{\Delta}{L_1})
\]

\[\text{s.t.} \quad d_{0,f} \leq w\]

\[c(k) + \frac{b}{L_0} = d_{0,f}.\]
Since we require that the central bank sell enough bonds so that \( d_{0,f} = w \), we can rewrite as:

\[
\max_{k,b} \quad k(R + r\frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha)\frac{\Delta}{L_1}) \\
\text{s.t.} \quad c(k) + \frac{b}{r} = w.
\]

The first order condition for this program is,

\[
c'(k) = \frac{1}{L_0} \frac{R + r\frac{\Delta}{L_1}}{1 + (1 - \alpha)\frac{\Delta}{L_1}}
\]

and market clearing gives,

\[
L_1 = \frac{rK + (1 - \alpha)(B - T)}{2\frac{B}{L_0}}
\]

Let us now simplify this expression using the fact that the government’s budget constraint is \( T + B\frac{L_0}{L_b} = B \). Then, this can be rewritten as,

\[
L_1 = \frac{1}{1 + \alpha} \frac{rK}{\frac{B}{L_0}}.
\] (11)

Substituting this back into the objective in (P8) allows us to generate the welfare function for the central planner assuming that it intervenes sufficiently so that \( d_{0,f} = w \). This expression is given by,

\[
U^{CP}(K) \equiv K(R + \frac{r}{1 + \alpha}) + (W - c(K))2\Delta.
\]

The central bank can choose \( B \) to arrive at any point on this function. Thus, assuming intervention, the central bank will choose to implement,

\[
C'(K^{CP}) = \frac{R + \frac{1}{1 + \alpha}r}{2\Delta}.
\]

We can subtract the objective in \( U^{CP} \) with that in (P8) to see how the private sector’s choice differs from that of a central planner.

\[
U^{CP} - U^{PRIV} = (W - c(K))s_d - \frac{1}{1 + \alpha} \frac{rK}{L_1}(s_d + \alpha\Delta)
\] (12)

As in proposition 2, it is clear that the central planner values international liquidity higher than the private sector, and domestic liquidity less. Thus the benefit of intervention is that it moves the private sector away from a sub-optimal choice. That is, \( K^{CP} < K' \). However intervention has a cost, since firms must sell their bonds into an illiquid market. To compute this cost let us substitute market clear condition for \( L_1 \), (11), into (12):

\[
U^{CP} - U^{PRIV} = -\alpha \frac{1}{1 + \alpha} rK.
\]

Thus, it is clear that intervention always lowers the welfare function when \( \alpha > 0 \). Intervention is beneficial as long \( U^{CP}(K^{CP}) > U^{PRIV}(K^{PRIV}) \). But this depends on the size of the
externality versus the cost of intervention. The private sector always chooses an inefficient point - point C on the figure. The cost of intervention is that it lowers welfare to points on the lower curve. Thus if the cost is sufficiently high, point A lies below point C and intervention is not beneficial. On the other hand, in the extreme case where $\alpha = 0$, $U^{CP}$ rises to $U^{PRIV}$, so that point B is chosen by the central planner. It is always the case that intervention is beneficial in this case.

![Graph](image)

**Figure 4: U^{CP} and U^{PRIV}**

### 4 Intermediation, Exchange Rate Systems, and the Mundell-Fleming Mechanism

Up to now, we have removed from our analysis any source of ex-ante heterogeneity among domestics and made no distinction between domestic assets issued by the central bank (money and bonds, in particular). In this section we relax these assumptions with the goal of connecting our framework to the traditional Mundell-Fleming based discussion.

Introducing ex-ante heterogeneity gives a role to domestic asset markets at date 0, when sterilization takes place. Money is an asset with unique transaction services in these financial markets. In particular, it facilitates bank lending. While none of our basic conclusions are modified by these extensions, we find that indeed — and for more or less the standard reasons — a flexible exchange rate system facilitates the success of sterilization. However, in our lending-channel example, this additional power is gained at the cost of losing the Pareto nature of the intervention, as it involves a dynamic transfer from banks/savers to
4.1 Money and the Banking System

We introduce a minimalist structure to capture the special role of money in facilitating domestic financial transactions at date 0. Rather than starting with homogeneous domestic agents, we take the international wealth away from the corporate sector and allocate it to a blended banking/savers sector (henceforth, referred to as bankers). Bankers, like entrepreneurs, have linear preferences over date 2 consumption but have no investment opportunities aside from lending to the corporate sector.

In order to lend to the corporate sector at date 0, bankers require domestic collateral from corporations and need to hoard $\mu < 1$ units of domestic money between dates 0 and 1 per unit of debt’s face value. They are born with $M^0$ units of high powered money. This means that the maximum amount of loans that the banking sector can make (to the corporate sector) is also $\frac{M^0}{\mu}$.\(^{19}\)

The Central Bank fully backs up $M^0$ with international reserves, which are returned to money at date 1. Thus, the nominal exchange rate at date 1, $E_1$, is equal to one regardless of the exchange rate system prevailing at date 0. Banks are not needed at date 1, so bankers can participate directly in the financial markets without holding any money.

Taken together, these assumptions create a transmission mechanism for domestic monetary policy via the “lending channel.”\(^{20}\) A central bank that contracts or expands $M^0/\mu$ can affect the amount of lending from bankers to the corporate sector. Our assumptions are designed to isolate the impact of this mechanism on the date 0 problem of taming the boom, as it is apparent that at date 1 the Central Bank will attempt to ensure perfect domestic aggregation of international collateral.

4.2 Monetary Policy in a Flexible Exchange Rate System

We remove bonds for now and fix $M^0$, so monetary policy takes the form of a tightening in reserve requirements, $\mu$. None of our main conclusions is affected by reintroducing bonds

---

\(^{19}\) These steps can be disentangled more finely. For example, separating bankers from savers, the story goes as follows: Banks face a reserve requirement on taking deposits from savers. Let $\mu$ be the reserve requirement. Then, given $M^0$ banks can only take in deposits of $\frac{M^0}{\mu}$. We additionally assume that banks cannot raise funds from savers in any other way than by taking deposits. This means that the maximum amount of loans that the banking sector can make (to the corporate sector) is also $\frac{M^0}{\mu}$. Last, we assume that the $rK$ of domestic liquidity that firms create is only tradeable among the corporate sector and banks. That is loans to a firm with some domestic liquidity can only be made by other firms (for example, via asset sales, trade credit, or mergers), or from savers through the banking sector.

\(^{20}\) For discussion and evidence of the lending channel in the U.S., see for example, Kashyap, Stein and Wilcox (1993).
and implementing the monetary contraction via open market operations instead (see below).

At date 0, bankers lend as much as they can to firms as long as $L_0 > L_1$. That is, they supply:

$$X_0^s = \min \left[ \frac{M^0}{\mu}, W \right].$$

(13)

Let $X_0^{dh}$ denote the potential demand for loans, defined as the demand that would arise if $L_0 = L_1$. Then the amount lent, $X_0$, is:

$$X_0 = \min[X_0^s, X_0^{dh}]$$

(14)

We focus on the case where the first terms in square brackets in both (13) and (14) bind.\(^{21}\) Thus:

$$X_0 = \frac{M^0}{\mu}.$$

In equilibrium, corporate investment is effectively determined by credit availability:

$$c(K) = \frac{M^0}{\mu},$$

while the rest of bankers’ funds is supplied at date 1:

$$X_1 = W - X_0.$$

It is apparent that by controlling $\mu$ the central bank can manipulate the timing of investment and hence aggregate liquidity provisions. It is here that exchange rate flexibility plays a role, as it limits the private sector’s mechanisms to undo the monetary squeeze.

It is important to notice, nonetheless, that unlike the policies we discussed in the previous section, this mechanism relies on transfers as opposed to Pareto changes. In fact, since in this section we have allocated all the wealth to a sector which has no real investment margin (the bankers), the decentralized equilibrium is constrained efficient and $\mu$ should be set to a very low value. Reducing the fragility of the corporate sector by raising $\mu$ comes primarily at the expense of curtailing financial investment opportunities to bankers.

Furthermore, as in Mundell-Fleming, the counterpart of monetary control is that the exchange rate at date 0, $E_0$, is determined by the market. $E_0$ must be such that banks are indifferent between selling their stock of money at date 0 and collecting the return for the funds that otherwise would have been involved in a loan package, and the return on the loan itself:

$$L_1 \left( \frac{\mu}{E_0} + 1 \right) = L_0 + L_1 \mu.$$

\(^{21}\)If $X_0^{dh}$ binds instead, $L_0 = L_1$. This case is uninteresting for us since monetary policy, of the type discussed here, has obviously no effect on lending.
Solving for the exchange rate, yields:

$$E_0 = \frac{L_1 \mu}{L_1 \mu + s^0} < 1,$$

where $s^0 \equiv L_0 - L_1$. While in our simplified framework such appreciation has no impact on real allocations, in reality it may be in itself a source of concern. For example, if bankers had a choice on whether to consume an imported or a domestic good at date 0, some of their international liquidity would be diverted to the early consumption of foreign goods.

### 4.3 Monetary Policy in a Fixed Exchange Rate system

Suppose now that the exchange rate is fixed at one at date 0 as well, and the central bank stands ready to swap international reserves for domestic money at the private sector’s will.

There are two basic scenarios to consider in this fixed exchange rate system. In the first one, foreigners do not value domestic money as collateral. It is apparent that in this case bankers will offset any monetary contraction by selling their $X_1$ to the central bank in exchange for domestic money. They will do so for as long as $L_0 > L_1$ and $X_1 > 0$. Thus, as in Mundell-Flemming, monetary policy is futile. This changes once $X_1 = 0$ for then the bankers are constrained in the same sense as firms were in section 3 in the scenario where sterilization worked. The “holy trinity” of open economy macroeconomics establishes that only two out of the following three are possible: effectiveness of monetary policy, control of the exchange rate, and free capital mobility. It is the endogenous canceling of the latter that gives back its powers to monetary policy.\(^{23}\)

The other limit case to consider is when high-power money is part of international collateral. In that case, neither $X_0$ nor $X_1$ can be affected by monetary policy, thus monetary policy is useless. This case highlights an important aspect of the policy considerations we have stressed throughout the paper: In order to be successful in preventing an external crisis, the policymaker needs to be able to “hide” some of the private sector’s international liquidity at date 0. This, it will not be able to do if it attempts it be selling highly internationally liquid instruments to its private sector.

### 5 Further Costs of Failed Sterilizations

In addition to fueling capital flows, sterilization often appreciates the currency, generates quasi-fiscal deficits, and biases capital flows toward short term debt (e.g. Reinhart and

\(^{22}\)Recall that bankers need to hold the money at date 0 in order to make the loans. Hence, this assumption does not mean that foreigners wouldn’t accept money as a method of payment at date 0, but that money in banks’ hands does not count as collateral.

\(^{23}\)Reisen (1993) argues that the “holy trinity” does not apply when the central bank uses reserve requirements rather than open market operations. We do not find support for such claim in our model.
Reinhart (1998), Montiel and Reinhart (1997), and Calvo et al. (1993)). While each of these regularities may just be the equilibrium counterpart of the higher domestic interest rates associated to a successful sterilization, policymaker’s are often concerned that they are too large or too severe to correspond to the simple counterpart of a successful policy. We have already argued that failed sterilization leads to an increase in capital flows and aggregate demand. In this section, we show that the backfiring result we described in section 3 also produces the quasi-fiscal deficit and the shortening in the maturity of capital flows. Thus, these regularities may also be viewed as symptoms of failed sterilization.

5.1 The Quasi-Fiscal Deficit Problem

Returning to the simple single-public financial instrument described in section 3, consider the budget constraint of the government. This equates revenues, comprised of taxes and the proceeds from selling the international reserves acquired during the sterilization episode, to the face value of the bonds issued:

\[ T + L_1 \frac{B}{L_0} = B. \]

There are many ways in which this constraint can lead to an increase in the fiscal deficit. While when \( L_0 = L_1 \), the budget balances without having to raise taxes, when \( L_0 > L_1 \) it must be that \( T > 0 \) to pay the high interest on the government debt. As we have shown, this situation arises with a successful sterilization (hence the high interest rates) but it also does when there is an illiquid secondary market for bonds.\(^{24}\) In lemma 4, we showed that when \( \alpha > 0 \), it must be that \( L_0 > L_1 \). Hence, attempting to sterilize in an illiquid bond market can have large detrimental effects on the quasi-fiscal deficit, even when it fails to raise domestic corporate borrowing rates and slowdown aggregate demand.

Needless to say, this deterioration in the fiscal situation can be worsened if the reserves of international liquidity are not targeted back to the private sector in an efficient fashion at date 1. This will occur if the government receives a return less than \( L_1 \) on its international reserves.

\(^{24}\)Quantitatively, it is seldom the case that the quasi-fiscal costs associated with sterilization are much larger than a quarter of a percent of GDP - a small number when compared with other factors involved in crises (see e.g. Kletzer and Spiegel (1998) who compute these costs for several Latin American and Asian economies and conclude that they are not only small, but also draw little reaction from governments and central banks). It is for this reason that we think of these costs as yet another dimension of a failed sterilization, rather than as the primary source of concern.
5.2 Excessive Short Term Capital Flows

The observed shortening in the maturity of capital flows following sterilization is particularly interesting from our point of view. The step in arriving at this result from our model is in defining short and long term debt in terms of their insurance features. One can think of long term debt as short term debt plus rescheduling insurance. A simple extension of the model presented in section 2 shows that agents undervalue the insurance component of long term debt as long as \( L_1 < \Delta \), which is the case when domestic financial markets are underdeveloped. The result is akin to our previous result on the undervaluation of international liquidity.

Suppose that only a fraction \( 1 - \psi \), where \( 0 < \psi < 1 \), of \( w \) is directly pledgeable to foreigners at date 1.\(^{25}\) Debt that is taken on against this \( 1 - \psi \) of international liquidity will be viewed as short term debt. Now suppose that the rest, \( \psi w \), can be seized by foreigners at date 2, however doing so requires payment of a monitoring cost of \( 0 < \epsilon < (\Delta - 1) \) at date 2. A domestic firm has two choices. (A) It can take on one period debt up to the limit of \((1 - \psi)w\) at the international interest rate of one. Then at date 1, if it needs the funds, it can roll this over and take on additional debt of \( \psi w \). However, the interest rate on this additional debt will obviously be above one to compensate the foreign lenders for bearing the monitoring cost. (B) It can take on long term debt against the full \( w \), in which case the foreign lenders will always pay the monitoring cost to seize the additional \( \psi w \). Thus, domestics face an upward sloping term structure of borrowing.

With option A, only firms that are distressed at date 1 will take on the additional debt and draw down \( \psi w \). With option B, on the other hand, all firms will have pledged their extra-collateral ex-ante, and the intact firms will sell the corresponding international funds at date 1 to the distressed firms. The latter option is clearly socially preferable, since the social value of an extra unit of liquidity is \( \Delta - 1 > \epsilon \). The problem, as before, is that the return to intact firms is only \( L_1 - 1 \), which could well be below \( \epsilon \) if domestic financial markets are illiquid. If this last inequality holds, the equilibrium is one where no firm values the insurance service of long term debt and is willing to pay the high interest. This comes at great cost in terms of the aggregate supply of international liquidity since a central planner would prefer that firms take on the long term debt.

Since a failed sterilization lowers \( L_1 \), the incentives for short versus long term debt are adversely affected by it. Thus the capital inflow-composition problem is worsened when there is backfiring.\(^{26}\)

\(^{25}\)Diamond (1991) develops a model of debt maturity structure based on liquidity risk. The sketch of our model is related but the maturity structures depends on aggregate liquidity risk.

\(^{26}\)Since a successful sterilization also lowers \( L_1 \), the reader may wonder whether the shortening of capital flows is also an implication of a successful sterilization. The answer to this question is that it will not. Since government bonds yield \( L_0 \) which is high, firms will find it profitable to borrow as much as they can at date
6 Final Remarks

A central consideration of macroeconomic policy in emerging economies is external-crisis prevention. Public and private reserves management, aggregate budget constraints, anticipation of large changes in key asset prices, or the separation of domestic and international liquidity, are all first order considerations for the emerging market policymaker that are less central to his counterpart in developed economies.

In this paper we have offered a simple framework that tries to improve on the “conventional wisdom” model on some of these dimensions. It is devoid of much of the efficient structure of that model, and hence narrower in its goals, but in exchange it gives a central and very explicit role to the actions and reactions of the private sector and the different financial mechanisms behind these. Policymakers understand well that failure of policy is rooted in an unwelcome response by the private sector, and in that sense our structure, while only the beginning, is well aimed. It is also comforting that the motive for intervention is explicit and arises in the same financial market imperfections that complicate the policy outcome. Underdeveloped and illiquid domestic financial markets both justify intervention and represent its worst hazards. Illiquidity generates problems similar to those of the bailout/moral hazard arguments, while arguably being more pervasive and harder to deal with.

The model is highly stylized, hence its regions and effects are sharply delimited. In reality, neither the distinction between domestics and foreigners, or between domestic and international collateral, is as stark as in our model. Neither is the government as decidedly biased or as benevolent and competent as we have portrayed it.

One way to recount our main results, less constrained by the specific equations of the model, is to adopt the perspective of an international investor deciding its exposure to the country. Sterilization is successful when foreigners are near their limit on sovereign risk (international collateral limit), and hence cut down their holdings of corporate assets in order to absorb the government bonds sold in sterilization. A foreigner should view these bonds as substitutable (or better) than existing corporate paper, but he should not view these bonds as good substitutes for foreign bonds. Since it is sovereign risk that determines whether the bonds are substitutable for foreign bonds, if a country has little external debt (relative to its international collateral), then the new bonds are good substitutes for foreign bonds and sterilization is undone. As we ride up the country risk curve, substitutability breaks down and sterilization is successful. The equilibrium counterpart to this is that all interest rates within the country rise.

Sterilization fails when domestic assets are viewed as imperfect substitutes because the

0, which will include both short and (expensive) long-term debt. That is, the high $L_0$, replaces $L_1$ in the preceding cost-benefit tradeoff.
specific asset markets are illiquid. In particular, the sale of government bonds into an illiquid bond market raises government bond interest rates because it rides up the government bonds risk curve (as opposed to the sovereign risk curve). Foreigners will shift out of government bonds and start buying corporate paper which now are perceived as more liquid because the government holds international reserves that it has committed to supply to all asset markets. In this case effective corporate interest rates fall (especially relative to government rates). Government rates rise but for the “wrong” reason – foreigners are still willing to invest in the country, they just do not want more government bonds.

While sterilization may be the tool of choice in the short run, long term solutions to the problems we have highlighted are not cyclical but structural in nature. Our framework not only illustrates the second best options and policy problems, but also points at domestic financial underdevelopment as the primitive source of concern. It is important when thinking about second best solutions to also ask whether they will have any long run effects on the primitive problem. Taxing capital flows, for example, while obviously appropriate from the second best point of view, and even useful as a companion to sterilization, loses appeal once one thinks in terms of the medium and long term development of financial markets. Flexible exchange rates may have an advantage over fixed - for Mundell-Fleming reasons - but they may have long run detrimental effects on financial markets.

Regardless of the specific answers to these concerns, it appears to us that there is an increasing realization that a modern debate on issues such as the advantages and drawbacks of dollarization, capital flows taxation, liquidity requirements, and so on, ought to consider the asset markets aspects of the problem, and that the structure we have proposed here is a useful tool for such a task. We are currently exploring some of these structural problems in ongoing work.
A Appendix

A.1 Regions

Taking as given date 0 decisions of $k$ and $d_{0,f}$, there are four possible regions that the economy can be in at date 1.

- Region I occurs when both the international liquidity constraint (4) and the domestic liquidity constraint (3) are slack. In this case, both the international liquidity premium and the domestic illiquidity index are zero and there is full project completion. $L_1 = 1, s_d = 0, \theta = 1$.

- Region II occurs when the international constraint is slack but the domestic constraint is binding. The international liquidity premium remains at zero, however the domestic illiquidity index is positive as the economy fails to aggregate all of its resources resulting in some projects being downsized. $L_1 = 1, s_d > 0, \theta < 1$.

- Region III occurs when the international constraint is binding but the domestic constraint is slack. The economy aggregates all of its resources, however this is insufficient in the aggregate and some projects are downsized. $L_1 > 1, s_d = 0, \theta < 1$.

- Region IV occurs when both the international constraint and the domestic constraint are binding. The international liquidity premium and the domestic illiquidity index are positives, and some projects are downsized. $L_1 > 1, s_d > 0, \theta < 1$.

A.2 Proof of Lemma 3

Lemma 3 stated that under the assumption of suspension of convertibility and if $d_{0,f} = w$, then foreigners will not hold any domestic claims at date 0. The proof is as follows.

Suppose that the private sector holds $B$ government bonds, $rK$ domestic claims, and has \( w - d_{0,f} = 0 \). Suppose foreigners hold $B_f$ government bonds. We show that $B_f = 0$ in equilibrium.

When $w - d_{0,f} = 0$, the government controls all of the international liquidity in the economy. If the government suspends convertibility at date 1, it only releases reserves when shown an invoice for an imported good to fulfill investment plans.

Either distressed or intact firms can tender $rK + B - T$ to the government for $\frac{B + B_f}{L_0}$ of reserves. If intact firms tender, they onsell the imported goods to distressed firms in exchange for some of $rK + B - T$ of distressed firms. Suppose an intact firm tenders one domestic claim, it receives $1/L_1$ import goods, which it sells to the distressed firm for $L_1$ domestic claims. Thus it is indifferent between tendering and not. Assume that it does not.
If a foreigner tender’s its bond of $B_f$, it also receives $B_f/L_1$ import goods. It must sell this to the domestic distressed firm for $B_f$ domestic claims. However, these claims have no value to foreigners. Thus, $B_f = 0.27$

Distressed firms receive all imported goods totalling,

$$rK + B - T \over L_1$$

¿From (7), this is exactly equal to $B + B_f \over L_0$ which is all of the government’s reserves. Thus when only distressed firms tender, this is an equilibrium.

A.3 Proof of Proposition 3

Proposition 3 stated conditions under which the private sector completely undoes the central banks sterilization operation.

Take prices first. Let us rewrite (7) to give,

$$L_1(W - D_{0,f}) + 2L_1 {B \over L_0} = rK + B - T.$$ 

Now suppose that $k = k'$ and $d_{0,f} = d'_{0,f} + {B \over L_0}$, then,

$$L_1(W - D'_{0,f}) + L_1 {B \over L_0} = rK' + B - T.$$ 

However, given the government’s budget constraint, (6), we can conclude that $L_1$ does not depend on $B$.

Fixing prices at $L_1 = L_1'$, let us consider the firm’s optimization problem and verify that $k = k'$ and $d_{0,f} = d'_{0,f} + {B \over L_0}$ is a solution. Since,

$$w - d_{0,f} > 0,$$

---

We have made two very unrealistic assumptions here. First, we have said that foreigners cannot take the imported goods and liquidate them outside the country for international liquidity. If there is any liquidation cost in this transaction, it is easy to see that foreigners would prefer not to hold domestic claims, since they must always bear this cost, while domestics never bear the cost. The more interesting case is that of over-invoicing. During periods of capital-controls domestic firms routinely over-invoice their imported goods. That is they claim higher prices than actual ones for their goods, thereby getting their hands on more valuable international reserves. Suppose that a firm can get away with over-invoicing by a multiple of $M > 1$. A firm that tenders one unit of domestic claim receives $1 \over M L_1$ import goods, and $1 \over M L_1$ international reserves. The import goods can be sold to a distressed firm, for $1 \over M$ domestic claims. The international reserves can now be sold to a foreigner to redeem some of the foreigner’s bonds. Foreigners selling $B_f$ bonds can receive at maximum $B_f \over M L_1$ reserves. Thus let,

$$L_{1,f} = B_f \over M L_1 P_0 (B + B_f).$$

This is the discount that foreigners sell their bonds at. In equilibrium, foreigners will hold only enough bonds so that $L_{1,f} = L_1$. This means that a fraction $M L_1$ of international reserves can be promised away to foreigners by the private sector at date 0. Over-invoicing creates a leak in the system.
it must also be that,
\[ w - d'_{0,f} > \frac{B}{L_0} > 0. \]

Additionally, since,
\[ c(k) + \frac{B}{L_0} = d_{0,f}, \]
then substituting,
\[ c(k') + \frac{B}{L_0} = d'_{0,f} + \frac{B}{L_0}. \]
Thus the solutions satisfy the budget constraints. Since the objective of the program is linear, they also satisfy the F.O.C’s. The last part of the proposition, \( L_0 = L_1 \), follows from domestic arbitrage.

\[ \blacksquare \]

### A.4 Proof of Proposition 5

The statement is that when domestic secondary markets are illiquid and \( d_{0,f} < w \), sterilization can backfire.

The proof is by showing that there is no other possible equilibrium. Suppose there was an equilibrium with \( k \leq k' \) and \( L_1 \geq L'_1 \). The private sector purchases the bonds through increased borrowing and reduced expenditure. However if this is the case then from market clearing we have a contradiction,
\[ L_1 = \frac{rK + (1 - \alpha)(B - T)}{W - D_{0,f} + 2 \frac{B}{L_0}} < \frac{rK'}{W - D'_{0,f}} = L'_1 \]

Consider an equilibrium with \( L_1 \leq L'_1 \) and \( K \leq K' \) - where at least one of the inequalities is strict. From the firm’s first order conditions,
\[ c'(k) = \frac{R + r \frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}} < \frac{R + r \frac{\Delta}{L'_1}}{1 + \frac{\Delta}{L'_1}} = c'(k'). \]

Since \( c(k) \) is strictly convex, this is a contradiction. The case with \( L_1 \geq L'_1 \) and \( K \geq K' \), can be ruled out by the same logic. Hence the only equilibrium is, \( L_1 < L'_1, k > k' \), and \( d_{0,f} > d'_{0,f} + \frac{B}{L_0} \).

\[ \blacksquare \]
References


