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Exchange Rates, Monetary Policy Regimes, and Beliefs

Keith Sill
Federal Reserve
Bank of Philadelphia

Je® Wrase
Federal Reserve
Bank of Philadelphia

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Abstract

We investigate an international monetary business-cycle model in which agents face monetary policy processes that incorporate regime shifts. In any given period agents cannot directly observe the policy regime, but instead form beliefs that are updated via Bayesian learning. As a result, expectation adjustment displays inertia that adds persistence to the e®ects of monetary shocks. Monetary policy process for the U.S. and an aggregate of OECD countries are estimated using Hamilton's Markov-switching model. We then solve and calibrate a version of the model and examine its quantitative properties.

1 Introduction

This paper examines an international monetary business-cycle model in which agents face monetary policy processes that incorporate regime shifts. Monetary policy regimes switch over time according to a Markov transition law. Actual money growth in any period depends both on the regime of a country's monetary authority and a monetary control error. In any given period agents cannot directly observe the regime from which money growth rates are drawn. Instead, beliefs are formed about the regime. These beliefs are rational expectations of the money growth process. They are formed from observed money growth rates and are updated using Bayesian learning. Our use of regime switching and Bayesian learning closely follows Andolfatto and Gomme [1], who analyzed a closed economy model of the Canadian economy. Our model is also closely related to recent work by Moran [9, 10], who traces out welfare costs of disin°ations in models with Bayesian learning.

We consider monetary regime switching because it appears that in many industrialized countries central bankers do shift periodically between distinct episodes of high and low average rates of money growth. Parameters of the regime switching process are tied down using maximum likelihood estimation of Hamilton's [7] Markov-switching model applied to U.S. and an aggregate of OECD country money growth rates. Our estimates provide evidence that regime switches do occur in the money growth data.

We employ a Bayesian learning mechanism to allow for inertia in the updating of expectations about money growth rates and in°ation rates. This inertia may be helpful in explaining observed persistence in the e®ects of monetary innovations on, among other variables, interest and exchange rates, and in explaining exchange rate variability (see Schlagenhauf and Wrase [13, 14] or Eichenbaum and Evans [6]). Our model has rational expectations that display inertia. The rigidity arises endogenously as Bayesian learners update their beliefs about monetary policy regimes. The learning mechanism provides an avenue for investigating exchange rate variability that stands in contrast to models of sticky prices, where unexplained price rigidities are imposed exogenously (see Obstfeld and Rogo® [11, 12], Chari, Kehoe, and McGrattan [3], and Betts and Devereaux [2]). Learning about monetary regimes has also been used to address features of exchange rate behavior other than persistence, as the survey paper by Lewis [8], and references therein, clearly articulates.

This paper proceeds as follows. Section 2 presents some statistical properties of international data, against which quantitative implications of the model we construct can be compared. Section 3 presents an open economy liquidity model in which agents form beliefs about monetary policy. The quantitative properties of a parameterized version of the model are examined in section 4. Section 5 concludes.

2 Statistical Properties of International Data

Table 1 presents international data features against which we will evaluate the models developed in subsequent sections. Moments in the table are Hodrick-Prescott-filtered quarterly observations of exchange rates and outputs for six major industrialized countries for various sample periods of flexible exchange rates. The nominal exchange rate for each country is the bilateral exchange rate vis-à-vis the U.S. dollar. Real exchange rates are measured using nominal exchange rates and the consumer price indexes of individual countries. Foreign data are from the OECD's Quarterly National Accounts and the International Monetary Fund's International Financial Statistics.

The empirical regularities evident in Table 1 are that exchange rates, nominal and real, are highly volatile with persistent movements. High persistence is indicated by first-order autocorrelation coefficients of, on average, about 0.85 across most countries and sample periods. On the quantity side, averaging across countries for the full sample period 1974:1-1994:2, the first-order autocorrelations are .79 for GDP.

Averaging across countries in the full sample period, the standard deviations of nominal and real exchange rates are close to five times higher than the standard deviation of output. Variabilities in exchange rate are highest in the 1980:1-1987:4 period, during which the U.S. dollar experienced a large appreciation and subsequent depreciation. However, even when that period is removed from consideration, standard deviations of exchange rates are high relative to standard deviations of output. Exchange rates are clearly much more variable than the output measures, except for Canada.

One quantitative issue to be addressed is whether movements in exchange rates drawn from simulations of our model are as persistent and highly volatile as in the actual data. A second issue is whether the dynamic responses of exchange rates, interest rates, and real variables to monetary shocks implied by our model correspond to impulse responses in actual data. A number of recent studies, such as Schlagenhauf and Wrase [13, 14] and Eichenbaum and Evans [6], have looked at the effects of monetary policy shocks on international variables using estimated vector autoregression (VAR) data representations. There are four key features of the empirical findings: (i) A negative shock to U.S. monetary policy (positive shock to the federal funds rate) is associated with persistent nominal and real appreciations of the U.S. dollar vis-à-vis currencies of OECD countries, including Canada, France, Germany, Japan, and the U.K.; (ii) Responses of U.S. and foreign interest rates, outputs, and nominal and real exchange rates to a U.S. monetary policy shock all persist for many quarters beyond the period of the shock; (iii) A negative U.S. monetary shock is associated with impact increases in foreign interest rates, except for Japan, which implies a widening of the U.S.-foreign interest rate differentials; and (iv) A negative U.S. monetary shock is associated with an impact increase in U.S. output, increased U.S. output for a few quarters after the shock, and subsequent output decreases. Foreign output responses to U.S. money shocks are much smaller than the U.S. output response.

The above features of the responses of international variables to a negative shock to U.S. monetary policy (positive federal funds rate shock) are the main empirical regularities against which the model in this paper is evaluated. We consider whether a parameterized version of the model can account for contemporaneous nominal and real U.S. currency appreciations, contemporaneous foreign interest rate increases, and contemporaneous increases in U.S.-foreign interest rate differentials in response to a negative U.S. money shock. We also consider whether agents' beliefs about monetary policy can help account for effects of U.S. money shocks on exchange rates, interest rates, and output across countries that persist well beyond the period of the shock.[1]

3 An Open Economy Monetary Model with Learning

This section presents an open economy monetary model in which money is nonneutral. The nonneutrality arises from a limited participation feature: households make portfolio decisions prior to observing money shocks while firms make investment and hiring decisions after observing money shocks. The model economy has two countries, domestic and foreign, linked by trade in goods and currencies. A multi-member household inhabits each country. Households consist of shoppers, firm managers, workers, and financial intermediaries. Each household member has distinct tasks to perform during a period in markets for goods, labor, and financial services. At the end of each period, all household members reunite to pool resources. Thus, all of a country's per household wealth resides with a representative household.

3.1 Trading Opportunities

The trading opportunities, objectives, and constraints of households are assumed to be isomorphic across countries. For brevity, we provide details for the representative domestic household's decisions and opportunities only. The foreign analogs are straightforward and involve obvious notational alterations. The representative domestic household begins period t with K_t^D units of capital and A_t^D units of domestic currency carried forward from the previous period.[2] At the beginning of period t, the domestic household divides nominal wealth A_t^D by sending a deposit of N_t^D currency units with its financial intermediary member to the domestic financial market. The remaining $A_t^D - N_t^D$ is allocated to trade in the currency exchange market. In the exchange market, domestic and foreign households trade currencies to arrange balances for use in purchasing consumption goods.

Domestic currency available to the domestic household in the exchange market consists of $A_t^D - N_t^D$, from the initial wealth allocation, along with the household worker's wages. The worker supplies $\bar{H}_t^D$ labor units in the domestic labor market at nominal wage W_t^D. In the foreign exchange market $A_t^D - N_t^D + \$_0 W_t^D \bar{H}_t^D$ units of domestic currency are divided into a domestic currency balance, $M_{D,t}^D$, and a foreign currency balance, $M_{F,t}^D$, at nominal exchange rate e_t (expressed in domestic per foreign currency units). Note that we allow for some of the household worker's wage receipts, $\$_0 W_t^D \bar{H}_t^D$, with $0 \leq \$_0 \leq 1$, to be used in currency trades in the foreign exchange market. The household's nominal allocation in the foreign exchange market is:

$$A_t^D - N_t^D + \$_0 W_t^D \bar{H}_t^D = M_{D,t}^D + e_t M_{F,t}^D \quad (1)$$

The household shopper purchases $C_{D,t}^D$ units of home-produced goods at price P_t^D, and $C_{F,t}^D$ units of foreign goods at price P_t^F, subject to the cash constraints:

$$P_t^D C_{D,t}^D \leq M_{D,t}^D \quad (2)$$

$$P_t^F C_{F,t}^D \leq M_{F,t}^D \qquad (3)$$

When the constraints bind as equalities, the shopper and worker combine to return home at the end of the period with goods, but no cash.[3]

The financial intermediary receives a monetary injection X_t^D in the financial market, which is deposited on behalf of the household. The intermediary then holds $N_t^D + X_t^D$ units of cash, which it lends to domestic firms. Loanable cash supplied by the intermediary is[4]:

$$\tilde{L}_t^D = N_t^D + X_t^D \qquad (4)$$

The firm manager hires workers, undertakes investment, and holds the household's capital stock K_t^D. Prior to producing output, the firm borrows L_t^D domestic currency units from an intermediary to finance acquisition of H_t^D units of labor at wage W_t^D per unit and to potentially finance capital accumulation, in the face of a cash constraint:

$$W_t^D H_t^D + \beta_1 P_t^D I_t^D \leq L_t^D \qquad (5)$$

The firm purchases $I_t^D = K_{t+1}^D - (1 - \delta) K_t^D$ units of home-produced goods to add to the household's capital stock and finances a fraction $\beta_1 P_t^D I_t^D$, with $0 \leq \beta_1 \leq 1$, representing the fraction of investment financed by loans. Capital and consumption goods are indistinguishable in the domestic goods market and sell at common price P_t^D.

Determining the household's nominal wealth evolution requires accounting for currency brought home at the end of the period by household members. From (1) - (3), the shopper and worker bring home goods but no cash when the constraints bind as equalities. The firm manager, after the close of trading in goods markets, pays loan obligation $R_{L,t}^D L_t^D$, where $R_{L,t}^D$ is the gross domestic loan rate. The manager brings home capital and cash profits of:

$$P_t^D Y_t^D - R_{L,t}^D L_t^D - (1 - \beta_1) P_t^D I_t^D \qquad (6)$$

where Y_t^D is real output per domestic household.

The intermediary receives loan repayments $R_{L,t}^D \tilde{L}_t^D = R_{L,t}^D (N_t^D + X_t^D)$ and pays a gross deposit return $R_{D,t}^D (N_t^D + X_t^D)$. The intermediary returns home at the end of the period with its household's own deposit return, $R_{D,t}^D (N_t^D + X_t^D)$; plus cash derived from intermediation $R_{L,t}^D (N_t^D + X_t^D) - R_{D,t}^D (N_t^D + X_t^D)$: Thus, the intermediary brings home a cash balance of:

$$R_{L,t}^D (N_t^D + X_t^D) \qquad (7)$$

Combining cash brought home by the household firm manager in (6), intermediary in (7), and cash that the household worker did not send to the foreign exchange market gives the household's end-of-period nominal wealth:

$$
\begin{aligned}
A^D_{t+1} \;=\;& P_t^D Y_t^D - R^D_{L,t} L^D_t - (1-\varpi_1) P_t^D I_t^D + \\
& R^D_{L,t}(N_t^D + X_t^D) + (1-\varpi_0) W^D \tilde{H}^D_t
\end{aligned} \tag{8}
$$

3.2 Preferences, Technology, and Shocks

The household maximizes utility measure:

$$
U = E_t \sum_{j=0}^{\infty} (\beta_D)^{t+j}\, \mu(C^D_{t+j}, l^D_{t+j}) \tag{9}
$$

with $0, \beta_D < 1$. Domestic consumption of home-produced goods, $C^D_{D,t}$, and foreign-produced goods, $C^D_{F,t}$, is aggregated according to a simple Cobb-Douglas aggregator:

$$
C^D_t = (C^D_{D,t})^{\upsilon}(C^D_{F,t})^{1-\nu} \tag{10}
$$

and momentary utility takes the form:

$$
\mu(C^D_{Dt}, C^D_{F,t}, 1-\tilde{H}^D_t) = \frac{1}{\rho^D}\left\{(C^D_t)^{\gamma}(l^D_t)^{1-\gamma}\right\}^{\rho^D} \tag{11}
$$

Leisure is $l^D_t = 1 - \tilde{H}^D_t$, with the time endowment normalized to unity. Foreign utility is the same as (9)-(11) except for obvious notational alterations.

For output production, each domestic firm possesses the technology:

$$
Y_t^D = f^D(K^D_t, H^D_t) = (K^D_t)^{\alpha^D}(H^D_t)^{1-\alpha^D} \tag{12}
$$

with $0 < \alpha^D < 1$. Foreign firms' technologies are the same as above except for notation.

Monetary injections in the model are $X^D_t = M^D_{s,t+1} - M^D_{s,t}$ and $X^F_t = M^F_{s,t+1} - M^F_{s,t}$, where $M^D_{s,t}$ and $M^F_{s,t}$ are per own-country-household stocks of domestic and foreign currencies. The exogenous money growth rates $\chi^D_t = \frac{X_t^D}{M^D_{s,t}}$ and $\chi^F_t = \frac{X_t^F}{M^F_{s,t}}$ depend on the monetary regimes generating monetary policy. We now turn to the nature of monetary policy.

3.3 Monetary Policy and Beliefs

The money growth rates are assumed to be independent across the two countries. The autoregressive process followed by the domestic and foreign countries are then given by:

$$(X_t^D - \bar{X}_t^D) = \rho^D (X_{t-1}^D - \bar{X}_{t-1}^D) + \epsilon_t^D \quad (13)$$
$$(X_t^F - \bar{X}_t^F) = \rho^F (X_{t-1}^F - \bar{X}_{t-1}^F) + \epsilon_t^F \quad (14)$$

with

$$\bar{X}^D \in \{\bar{X}_L^D, \bar{X}_H^D\}$$
$$\bar{X}^F \in \{\bar{X}_L^F, \bar{X}_H^F\}$$

and

$$\bar{X}_L^j < \bar{X}_H^j$$

where $\bar{X}^D$ and $\bar{X}^F$ represent the long-term rate of money expansion in the domestic and foreign country, respectively. The long-term money expansion rate depends on which monetary policy regime is in place in a country.

Monetary policy regimes switch over time in the two countries according to the transition law:

$$p_{ij}^D = Pr[\bar{X}_t^D = \bar{X}_j^D | \bar{X}_{t-1}^D = \bar{X}_i^D] \quad (15)$$
$$p_{ij}^F = Pr[\bar{X}_t^F = \bar{X}_j^F | \bar{X}_{t-1}^F = \bar{X}_i^F]$$

Agents in the economy know the parameters of the transition laws. The sense in which X_t^j is a long-term growth rate is captured by transition probabilities $p_{L,L}^j$ and $p_{H,H}^j$ being close to one. Innovations e_t^D and e_t^F represent serially independent monetary control errors. These control errors are independent of labor productivity innovations and are drawn from normal distribution functions $N(0, \sigma_a^j)$ for countries $j = D, F$ and regimes $a = L, H$.

We assume that agents cannot directly observe the current and past monetary policy regimes. Instead, they form beliefs based on the known parameters of the process governing monetary growth rates and on current and past observed actual money growth rates $X_t^j, X_{t-1}^j, X_{t-2}^j, \ldots$ For purposes of exposition, take the case of a single country and let b_t denote the belief that the current regime is characterized by low money growth. Thus, $b_t = Prob(\bar{X}_t = \bar{X}_L)$. The money supply process is given by:

$$(X_t - \bar{X}_t) = \rho (X_{t-1} - \bar{X}_{t-1}) + \epsilon_t$$

Beliefs are updated rationally using a Bayes Rule recursion:

$$b_t = \frac{g_l(b_{t-1}; X_t)}{g_l(\cdot) + g_h(\cdot)}$$

with

$$g_l = b_{t-1} \cdot p_{ll} \cdot f_{ll}(\epsilon_t^{ll}) + (1 - b_{t-1}) \cdot p_{hl} \cdot f_{hl}(\epsilon_t^{hl})$$
$$g_h = b_{t-1} \cdot p_{lh} \cdot f_{lh}(\epsilon_t^{lh}) + (1 - b_{t-1}) \cdot p_{hh} \cdot f_{hh}(\epsilon_t^{hh})$$

where ϵ_t^{ij} is the innovation implied by the money growth process under the assumption that the regime was i last period and is j this period and $f_{ij}(\cdot)$ is the normal pdf for ϵ_t^{ij}.

Given this belief structure, money growth expectations and inflation expectations could adjust sluggishly, depending on parameter values. For example, if agents have been operating in a high money growth regime in a given country and there is suddenly a switch to the low growth regime, it may take a long string of relatively low money growth observations to appreciably alter the probability agents assign to actually being in the low growth regime. Such sluggishness in expectations in the face of changes in policy regimes will influence the economic outcomes of policies such as a disinflation planned by a less-than-fully-credible monetary authority. With sluggish expectations, it may take a prolonged period for nominal interest rates to fully adjust to a planned disinflation. Nominal rates will remain high until the expected inflation premium adjusts with beliefs. The extent to which beliefs influence the economic outcomes of changes in money growth rates is considered in our quantitative evaluation of the model. In that investigation, we simulate the model using parameter values drawn from data on actual outcomes in the U.S. and other major industrial countries.

While the illustration above was for the case of a single country, the extension to a two-country case is straightforward. The two-country economy has four states: $\{(H; H); (H; L); (L; H); (L; L)\}$ where $(H; H)$ denotes domestic-High, foreign-High, etc. Since we assume that the money supply processes are independent across countries, transition probabilities between states can be calculated as simple products of individual country transition probabilities. The belief recursion is then an obvious extension of the two-state case to a four-state case.

3.4 The Economy's State and Equilibrium

The state of the world economy in period t is characterized by values for $M_{s;t}^D; M_{s;t}^F; \kappa_t^D; \kappa_t^F; A_t^D; A_t^F; K_t^D; K_t^F; b_t;$ and, S_t: $M_{s;t}^D (M_{s;t}^F)$ and $\kappa_t^D (\kappa_t^F)$ are per domestic (foreign) household money and capital stocks. $A_t^D (A_t^F)$ and $K_t^D (K_t^F)$ are the domestic (foreign) representative household's beginning currency and capital stocks. S_t denotes the vector of innovations to money growth in the home and foreign country while b_t denotes a vector of belief probabilities over the four states of global monetary policy.

An equilibrium involves state-contingent prices, wages, interest and exchange rates, and optimal household decision rules satisfying market clearing and aggregate consistency conditions. Market clearing conditions are: $\bar{H}_t^D = H_t^D$; $\bar{H}_t^F = H_t^F$ for labor; $Y_t^D = C_{D;t}^D + C_{D;t}^F + I_t^D$; $Y_t^F = C_{F;t}^F + C_{F;t}^D + I_t^F$ for goods; $\bar{L}_t^D = L_t^D$; $\bar{L}_t^F = L_t^F$ for loans; and $A_t^D + X_t^D = M_{D;t}^D + M_{D;t}^F$; $A_t^F + X_t^F = M_{F;t}^F + M_{F;t}^D$ for foreign exchange. Aggregate consistency requires that $A_t^D = M_{S;t}^D$; $A_t^F = M_{S;t}^F$ for money stocks, and $K_t^D = \kappa_t^D$; $K_t^F = \kappa_t^F$ for capital stocks.

3.5 Household Decisions and Qualitative Results

Since the choice problem facing domestic and foreign households is of the same form, we focus on the domestic household's problem. The household maximizes utility measure (9) subject to trading opportunities and constraints in (1)-(8), technology (12), and money-shock process (13)-(14).

Consider a case of full information, that is, a case in which households and firms have full knowledge of all current-period shocks prior to making consumption and investment decisions. Let $V^D(A_t^D; K_t^D; S_t)$ be the value function corresponding to the domestic household's problem. $V(\cdot)$ satisfies the functional equation:

$$
\begin{aligned}
V^D(A_t^D; K_t^D; S_t) \quad = \quad & \max_{(N_t^D; K_{t+1}^D; M_{F;t}^D; \bar{H}_t^D; L_t^D)} \{\mu(C_t^D; 1 - \bar{H}_t^D) + \\
& \beta_D \int V^D(A_{t+1}^D; K_{t+1}^D; S_{t+1}) \Phi(S_{t+1} \mid S_t)\}
\end{aligned}
$$

A_{t+1}^D is given by wealth evolution (8). Binding cash constraints in (2)-(3), and (5) are used to eliminate $C_{D;t}^D$; $C_{F;t}^D$; and H_t^D as separate decisions. Also, from the foreign exchange market allocation (1) we have $A_t^D - N_t^D + \varpi W_t^D \bar{H}_t^D = M_{D;t}^D + e_t M_{F;t}^D$: Consequently, choice of $M_{D;t}^D$ is implied by choices of N_t^D,

$\bar{H}^D_t$, and $M^D_{F,t}$, since A^D_t is predetermined and e_t and W^D_t are taken by the household. Optimality conditions for $N^D_t, K^D_{t+1}, M^D_{F,t}, \bar{H}^D_t, L^D_t$ are:

$$-\mu_{C^D_{D,t}}\frac{1}{P^D_t} + \beta_D \int \mu_{C^D_{D,t+1}}\frac{R^D_{L,t}}{P^D_{t+1}}\Phi(S_{t+1} \mid S_t) = 0 \tag{16}$$

$$-\int \Big[\mu_{C^D_{D,t+1}}\frac{P^D_t}{P^D_{t+1}} + \beta_D \mu_{C^D_{D,t+2}}\frac{P^D_{t+1}}{P^D_{t+2}}\left\{f^D_{K^D_{t+1}} + 1 - \delta_D\right\}\Big]\Phi(S_{t+1} \mid S_t) = 0 \tag{17}$$

$$-\mu_{C^D_{D,t}}\frac{1}{P^D_t} + \mu_{C^D_{F,t}}\frac{1}{e_t P^F_t} = 0 \tag{18}$$

$$-\mu_{l^D_t} + \mu_{C^D_{D,t}}\frac{W^D_t}{P^D_t} = 0 \tag{19}$$

$$f^D_{H^D_t} - \frac{W^D_t}{P^D_t}R^D_{L,t} = 0 \tag{20}$$

where $\mu_{l^D_t}$ is the period t marginal utility of leisure, and the period t marginal products of domestic labor and capital are denoted respectively by $f^D_{H^D_t}$ and $f^D_{K^D_t}$. Condition (16), derived from the deposit choice, relates the nominal interest rate, anticipated inflation, and the household's intertemporal marginal rate of substitution. Equation (17) governs the capital investment decision. Equation (18) is derived from decisions about consumption and the domestic and foreign currency balances to use in acquiring consumption goods. Equation (19), derived from the work effort supply choice, equates the real wage and intratemporal marginal rate of substitution between a consumption quantity and leisure. Equation (20), from the firm's loan demand decision, equates labor's marginal product and the real cost of an additional labor unit (real wage and interest cost of borrowing currency to hire labor). Beliefs enter into the Euler equations by way of the integration over the transition function $\Phi(S_{t+1} \mid S_t)$, i.e., by way of the expectations.

Note that if agents made their cash allocation decisions after observing money shocks, they will adjust amounts they send to the financial and goods markets. As a result, nominal interest rates depend only on Fisherian fundamentals–the real rate and expected inflation. However, a positive shock to money growth in a country will increase expected in°ation, and with a relatively small e®ect on the real rate, the nominal interest rate will rise. If the nominal interest rate rises, borrowing costs of ¯rms will rise, leading to reduced employment and output. These responses of interest rates and output to a positive money shock run counter to conventional wisdom and to evidence from VAR impulse responses functions. Consequently, we consider an environment in which agents choose their allocation of currency between shopping balances and balances to exchange in the foreign exchange market prior to observing contemporaneous money shocks.

When cash allocation decisions are made before observing money shocks, a liquidity e®ect, along with Fisherian fundamentals, helps determine nominal interest rates. Recall that ¯nancial intermediaries are the recipients of monetary injections in each country. Since households cannot adjust their portfolios after a money shock hits, intermediaries, °ush with cash, will induce ¯rms to borrow and disproportionately absorb any money injection through lower nominal interest rates. If nominal rates end up lower in equilibrium, employment will increase since the cost of borrowing to hire labor has fallen. The equilibrium outcome for interest rates and real activities depends on the relative strengths of the anticipated in°ation e®ect and liquidity e®ect. As Christiano [4, 5] shows in a closed economy, and Shlagenhauf and Wrase [13, 14] show in an open economy, the liquidity e®ect can dominate in a version of the model with empirically plausible parameter values. However, the liquidity e®ect lacks persistence. The cash allocation rigidity that gives rise to the liquidity e®ect vanishes in the period following a money shock and consequently, so, too, do most of the e®ects of the shock. Our interest here is to examine how sluggishness of expectations that arise from monetary regime switching and Bayesian updating of beliefs about monetary regimes a®ects the dynamics of the liquidity e®ect and the dynamics of nominal and real exchange rate responses to monetary innovations.

4 Quantitative Results

The model is solved, parameterized, and simulated to evaluate its quantitative implications. Solving the model involves combining domestic and foreign Euler equations with equilibrium and aggregate-consistency conditions. Since closed-form solutions cannot be obtained, given the nonlinear nature of the model, we solve the model using the method of undetermined coe±cients in Christiano [4]. The procedure involves: (i) transforming variables to induce stationarity; (ii) linearizing optimality conditions by taking a first-order Taylor approximation about the nonstochastic steady state and imposing equilibrium and aggregate consistency conditions; (iii) conjecturing recursive laws of motion for choice variables that are linear in the state variables; and (iv) determining coefficient values for the linear decision rules using the method of undetermined coefficients.

Values of parameters $\rho^j; v^j; \beta^j; \gamma^j; \alpha^j; \delta^j$; for $j = D; F$, and parameters of the shock process that we use in simulations are summarized in Table 2. ρ^D and ρ^F, which determine curvatures of period utility functions, are each set to -1. The share v^D (v^F) of domestic (foreign) goods in the domestic (foreign) household's Cobb-Douglas consumption composite is set to 0.5, a value used in a number of recent studies, including Stockman and Tesar [15] and Schlagenhauf and Wrase [13, 14]. Leisure shares in momentary utility for both countries are set to $\gamma^D = \gamma^F = 0.76$, which, together with the model's other parameter values, implies a steady-state allocation in each country of roughly 26 percent of nonsleep time to market activity. Discount rates β^D and β^F are set to 0.99, which implies a nonstochastic steady-state real interest rate of 1 percent per quarter in each country, close to the average return on capital over the last century in the U.S.

The production technologies we use have Cobb-Douglas capital-labor substitution. Labor's share for both countries is set to $1 - \alpha^D = (1 - \alpha^F) = 0.64$, standard values in closed-economy real-business-cycle models, and values consistent with postwar U.S. data. Capital depreciation rates δ^D and δ^F are each set to 0.025, implying annual depreciation of 10 percent, close to the average depreciation rate for the U.S. over the period 1972:1-1997:4.

The stochastic processes governing money growth shocks in the two countries are in (13) and (14). Values for the parameters in the money growth processes are maximum likelihood estimates obtained using Hamilton's [7] regime-switching model and data on growth rates of the per capita monetary base for the U.S. and nine major OECD countries (Austria, Canada, Finland, France, Germany, Italy, Japan, Switzerland, and the United Kingdom).[5] The foreign countries were aggregated to represent the foreign country of the model. Specifically, a monetary base variable was constructed using monetary data from each country converted into U.K. pounds using average 1980 exchange rates. Exchange rates are held constant so that the foreign money measure reflects changes in the monetary base only. From our estimates over the period 1972:1-1997:4, estimated values for the money growth regime processes are given in Table 2.

4.1 Transitory Money Shocks

The ¯rst quantitative exercise performed is an analysis of the responses of model variables to a temporary (one period) one-standard-deviation increase in domestic (U.S.) money growth. Both countries are assumed to have been in a low money growth regime for a long time (long enough so that beliefs settle down) when the shock occurs. Figure 1 shows the evolution of beliefs for the home country remaining in a low growth regime. While in the period of the shock there is a signi¯cant assignment of probability to the possibility that the home country has shifted to high growth, the belief quickly falls back as subsequent money growth realizations are consistent with a continuation of the low growth environment.

Figure 2 plots the response of some domestic variables and exchange rates to the transitory shock. The model is parameterized so that workers send none of their wage receipts to the foreign exchange market ($\$_0 = 0$) and all of the ¯rm's investment is funded by borrowing ($\$_1 = 1$). Impulse responses are generated under two di®erent information assumptions. The plot labeled full info is for the case where monetary policy is completely credible so that agents have no uncertainty about the true state of the monetary regime. The plot labeled uncertainty is for the case where agents are uncertain about the true monetary regime and thus form beliefs about it. For both cases it is assumed that household portfolio decisions are made prior to observing the period's money shock.

As Figure 2 shows, the model generates dominant liquidity e®ects with the domestic nominal interest rate declining and output growth rising in the face of an easing of U.S. monetary policy. The model also generates depreciations in real and nominal exchange rates. Note, though, that because of the sluggish household portfolio decision assumption, the exchange rate does not depreciate immediately (in the period of the shock) but instead responds one period after the monetary shock.

Following the period of the temporary shock, anticipated in°ation e®ects dominate in determining nominal interest rates. Of particular interest are the responses of nominal interest rates and exchange rates following the period of the shock. Note that the full information responses are quantitatively smaller after the shock relative to the uncertainty responses. This re°ects the informational assumption that fully informed agents know there has simply been a transitory domestic monetary easing and there will be no lingering inflation, so that their inflation expectations are muted relative to the uncertain agents. If households are uncertain about the true state of monetary policy when a positive monetary shock hits, they will attach increased probability to the possibility that the regime is high growth. As a result, in the next period, they send less cash to financial intermediaries than if they knew the regime remained low growth (they expect somewhat higher money growth). When the expected monetary injection is not realized, there is a liquidity "shortage," resulting in nominal interest rates being bid up and downward pressure on employment and output. The lingering anticipated inflation effect essentially accounts for the differences between the two plots in each frame of the figure. It is clear then that in models of this sort, persistent liquidity effects will not be generated in response to transitory monetary shocks. We will see though that persistence can be generated when shocks are of a more permanent nature (i.e., regime shifts).

Figures 3 and 4 examine the case of a foreign transitory money growth shock (one-standard-deviation increase) and its effect on U.S. nominal and real variables. Both countries are assumed to have been in the low growth regime for a sufficient period of time for beliefs to settle down when the shock hits. Figure 3 is a plot of beliefs for a foreign transitory money shock. Given our parameter estimates, the belief revision is much smaller in magnitude than for the case of a domestic monetary shock. As before, beliefs show little persistence, returning to their initial state after two to three periods.

Figure 4 plots the U.S. response to a foreign transitory shock. The foreign money growth increase generates impact nominal and real appreciations of the domestic currency, an increase in the domestic nominal interest rate, and an impact decrease in domestic real output growth. Note that there is virtually no difference in the paths for the full information case and for the case with uncertainty about the monetary regime. This arises from the very small effect that a foreign transitory shock has on beliefs about the state of monetary policy (Figure 3). Note also that the overall effect of a foreign shock on U.S. variables is very small and arises primarily from effects of changes in expectations about foreign inflation on domestic agents' choices.

4.2 Changes in Regime

To analyze a transition in monetary regimes, we perform the following experiment. The economy begins in a global monetary state that has each country in a high money growth regime. We then transit one of the countries to a low money growth regime and analyze the e®ects on model variables. The same parameterization is used as in the transitory shock experiments ($\$_0 = 0$ and $\$_1 = 1$).

Figure 5 shows the e®ect on beliefs when the home country (U.S.) undertakes a permanent disin°ation. It takes approximately six quarters for beliefs to fully adjust to the new monetary regime. Figure 6 plots the e®ect of the disin°ation policy on domestic variables. There is a dominant liquidity e®ect on the nominal interest rate and output growth increases on impact. Note that there is increased persistence in the uncertainty case relative to the certainty case. In the case of output growth, it takes about ¯ve quarters for output growth to settle down when agents are uncertain about the monetary regime. This compares to about three quarters for the certainty case. However, the absolute di®erence between the two paths is rather small. Similarly for the nominal exchange rate. Regime uncertainty adds three to four quarters of persistence to the interest rate response compared to the certainty case, but the di®erence between the two paths is somewhat less than 1 percent (at a quarterly rate) at its maximum. The nominal exchange rate appreciates (with a one-period lag) in response to the disin°ation. Some persistence is generated by regime uncertainty relative to the certainty case. The real exchange rate appreciates on impact, then depreciates as output growth and in°ation settle down.

Why does uncertainty about the monetary regime generate persistence in the case of a regime switch? Suppose the economy has been in a high growth regime for some time. Households send an amount to the ¯nancial intermediary consistent with their expectation that money growth will be high. If the regime switches to low growth, intermediaries initially ¯nd they are short on liquidity, which puts upward pressure on the nominal interest rate. Next period, households revise their beliefs, putting increased weight on the possibility that the regime has switched. However, they are not yet fully convinced and so place less funds with the ¯nancial intermediary than they would if they knew with certainty that the regime was low growth. Again, a low money growth realization comes in, banks are short on liquidity, there is upward pressure on nominal rates relative to the full information case. This upward pressure on nominal rates in a period continues until households attach the appropriate probability to the low growth regime outcome and send the appropriate amount of funds to the ¯nancial intermediary.

Figure 7 plots beliefs for an experiment in which the foreign country undertakes a disin°ation policy while the home country stays in a high growth regime. Given our estimates on the money supply process, the adjustment of beliefs for this case is much more rapid than in the case of a U.S. disin°ation policy. Here, beliefs have fully adjusted after about three quarters.

Figure 8 shows domestic variable responses to a foreign monetary regime switch from a high to a low growth regime. There do not appear to be significant di®erences in domestic in°ation, interest rate, or output e®ects across the full information and the uncertainty economies. The signi¯cant di®erences show up in exchange rate responses. For the nominal exchange rate, the fully informed agents ratchet nominal exchange rates down almost immediately to the new stationary level, as displayed by the almost immediate and permanent domestic nominal appreciation. The real exchange rate initially appreciates, followed by some protracted real depreciation. For both the nominal and real exchange rates, there are clear and signi¯cant di®erences in the adjustments across the full information and uncertainty economies, re°ecting di®erent expected in°ation paths across the two economies.

5 Conclusion

We have constructed, solved, and simulated an open economy monetary model in which agents do not know with certainty the true state of monetary policy. Agents form beliefs based on observed money growth rates, but they cannot disentangle monetary control errors from mean shifts. Our interest is in whether a calibrated version of the model can help account for the observed persistent e®ects of monetary innovations on key economic variables. Calibrating the money supply processes required us to estimate Hamilton's regime-switching models for money growth processes for the U.S. and an aggregate of nine major OECD countries. The model provides evidence for additional persistence in e®ects on key variables of monetary innovations when regime switching and learning about regimes are included. However, the overall e®ect on persistence is small under our calibration.

The model suggests that some of the observed persistence the data display in response to monetary policy shocks can be accounted for by sticky expectations generated by learning. For the parameterization considered in this paper, learning introduces quantitatively signi¯cant e®ects: the di®erences in magnitude of response between the full information and uncertainty cases were noticeable as was the amount of persistence generated in response to monetary shocks. The next step we are pursuing is to determine how sensitive our results are to alternative configurations of parameters in the model.

ENDNOTES

1. In the models there is no distinction between the terms of trade and the real exchange rate. Moments for the two international relative price measures are provided in Table 1 to illustrate that, in general, using alternative measures, international relative prices possess high volatilities.

2. The notational conventions are: A subscript denotes the country of origin of a good or money balance. A superscript denotes the residence of the household choosing the variable. A tilde "~" denotes a quantity supplied; household choice variables without tildes are quantities demanded.

3. In simulations, parameter values are used for which agents drive cash constraints to bind as strict equalities. That is, the gross nominal interest rates exceed unity.

4. As long as the gross loan rate exceeds unity, intermediaries lend all available cash to ¯rms.

5. Ideally, monetary base data would be an appropriate proxy to use as a correspondence to the monetary process controlled by a monetary authority in the model. The closest analog in the IFS database is "reserve money," which we found to contain too many data entry errors to be reliable. The monetary series for foreign countries that we use is the series "money," line 34 in the IFS database, which is composed of transferable deposits and currency outside banks. This is the closest reliable data analog to what the foreign monetary authorities actually control that we have found.

References

[1] David Andolfatto and Paul Gomme. Monetary policy regimes and beliefs. Universite du Quebec a Montreal, Working Paper 48, 1997.

[2] Caroline Betts and Michael B. Devereaux. Exchange rate dynamics in a model of pricing-to-market. University of British Columbia Working Paper, 1998.

[3] V. V. Chari, Patrick Kehoe, and Ellen McGrattan. Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem? Federal Reserve Bank of Minneapolis Sta® Report 217, 1996.

[4] Lawrence J. Christiano. Computational algorithms for solving variants of Fuerst's model. Federal Reserve Bank of Minneapolis Working Paper 467, 1990.

[5] Lawrence J. Christiano. Modeling the liquidity e®ect of a money shock. Federal Reserve Bank of Minneapolis Quarterly Review, Winter, 1990.

[6] Martin Eichenbaum and Charles Evans. Some empirical evidence on the e®ects of monetary policy shocks on exchange rates. Northwestern University Working Paper, 1992.

[7] James D. Hamilton. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica, 57:357{384, 1989.

[8] Karen K. Lewis. Puzzles in international ¯nancial markets. In Gene Grossman and Kenneth Rogo®, editors, Handbook of International Economics, volume III. Elsevier Science B.V., 1995.

[9] Kevin Moran. Bayesian updating and welfare costs of the transition to a lower in°ation rate. Unpublished, University of Rochester, 1997.

[10] Kevin Moran. Accounting for the transition in the computation of the welfare bene¯ts of lower in°ation. Unpublished, University of Rochester, 1998.

[11] Maurice Obstfeld and Kenneth Rogo®. Exchange rate dynamics redux. Journal of Political Economy, 103:621{660, 1995.

[12] Maurice Obstfeld and Kenneth Rogo®. Risk and exchange rates. NBER Working Paper 6694, 1998.

[13] Don E. Schlagenhauf and Je®rey M. Wrase. Exchange rate dynamics and international e®ects of monetary shocks in monetary equilibrium models. Journal of International Money and Finance, 14:155{177, 1995.

[14] Don E. Schlagenhauf and Je®rey M. Wrase. Liquidity and real activity in a simple open economy model. Journal of Monetary Economics, 35:431{461, 1995.

[15] Allan C. Stockman and L.L. Tesar. Tastes and technology in a two-country model of the business cycle: Explaining international comovements. Rochester Center for Economic Research Working Paper 255, 1990.

Table 1: Exchange Rate Volatility and Persistence

Country	Sample Period	Standard Deviation			Autocorrelation	
		Nom FX	Real FX	Real output	Nom FX	Real FX
Canada	1973:1-1992:1	2.693	2.876	2.227	0.832	0.837
	1973:1-1979:4	2.863	2.745	1.658	0.838	0.817
	1980:1-1987:4	2.712	3.292	3.281	0.880	0.883
	1988:1-1992:1	1.942	2.071	1.954	0.457	0.422
France	1973:1-1992:1	9.587	8.867	0.940	0.870	0.851
	1973:1-1979:4	7.213	6.645	1.092	0.723	0.705
	1980:1-1987:4	12.395	11.349	0.738	0.872	0.852
	1988:1-1992:1	7.085	6.984	0.914	0.568	0.563
Germany	1973:1-1992:1	9.327	8.921	1.433	0.841	0.826
	1973:1-1979:4	7.415	7.191	1.827	0.663	0.649
	1980:1-1987:4	11.694	11.072	1.329	0.844	0.831
	1988:1-1992:1	7.318	6.995	0.796	0.548	0.525
Japan	1973:1-1992:1	9.372	9.333	1.073	0.859	0.851
	1973:1-1979:4	9.286	8.954	1.472	0.866	0.843
	1980:1-1987:4	9.7315	9.815	0.787	0.822	0.807
	1988:1-1992:1	9.029	8.913	0.701	0.745	0.738
UK	1973:1-1992:1	9.179	8.867	1.699	0.852	0.850
	1973:1-1979:4	8.241	6.645	1.960	0.836	0.706
	1980:1-1987:4	10.871	11.348	1.018	0.846	0.852
	1988:1-1992:1	7.020	6.984	1.945	0.606	0.563
USA	1973:1-1992:1	{	{	1.879	{	{
	1973:1-1979:4	{	{	2.171	{	{
	1980:1-1987:4	{	{	1.711	{	{
	1988:1-1992:1	{	{	1.522	{	{

These statistics are for Hodrick-Prescott filtered quarterly data over the sample periods listed. The nominal exchange rate for each country is the bilateral exchange rate vis-à-vis the US dollar. Real exchange rates are measured using the nominal exchange rates and individual country consumer price indexes. Data are from the International Monetary Fund's International Financial Statistics.

Table 2: Calibration Parameters
Preference and Technology Parameters

ν	home good share in consumption	0.5
ρ	utility curvature	-1.0
β	discount factor	0.99
γ	leisure share in utility	0.76
α	capital share in technology	0.36
δ	depreciation	0.025

Money Supply Process Parameters

$a^D; a^F$	AR coefficients on money growth	.3365	.0282
		(.114)	(.114)
$\bar{\mu}^D_H; \bar{\mu}^F_H$	High-growth regime means	.0182	.0333
		(.0073)	(.0019)
$\bar{\mu}^D_L; \bar{\mu}^F_L$	Low-growth regime means	.0094	.0097
		(.0022)	(.0024)
$p^D_{hh}; p^F_{hh}$	high-to-high transition probability	.952	.969
		(.032)	(.024)
$p^D_{ll}; p^F_{ll}$	low-to-low transition probability	.786	.965
		(.119)	(.028)
$\sigma^D_H; \sigma^F_H$	high-growth regime standard deviation	.00386	.1332
$\sigma^D_L; \sigma^F_L$	low-growth regime standard deviation	.00358	.00838

Figure 1: Change in beliefs in response to 1 standard deviation transitory increase in domestic money growth.

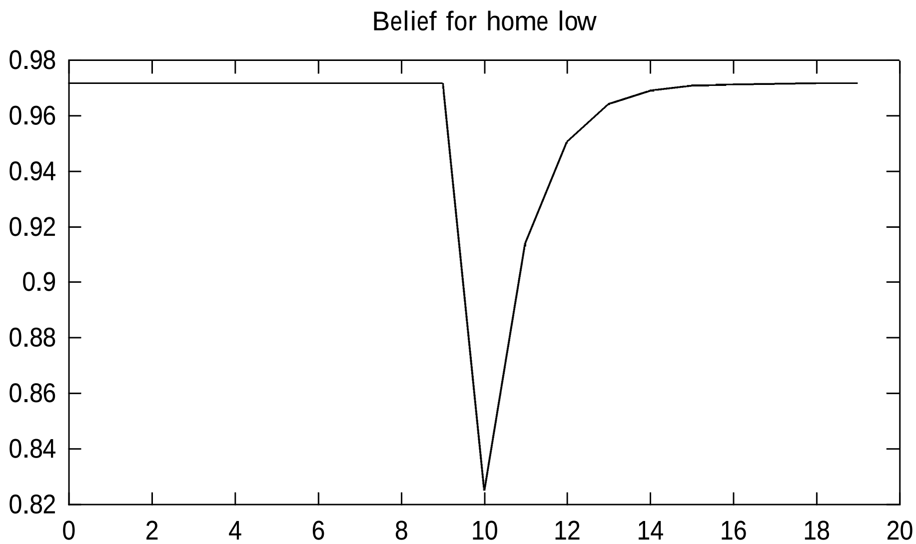
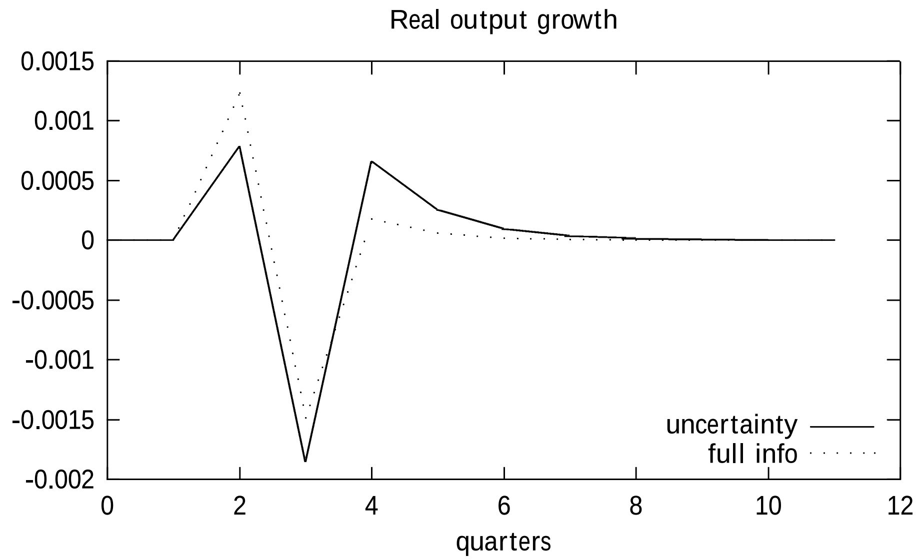
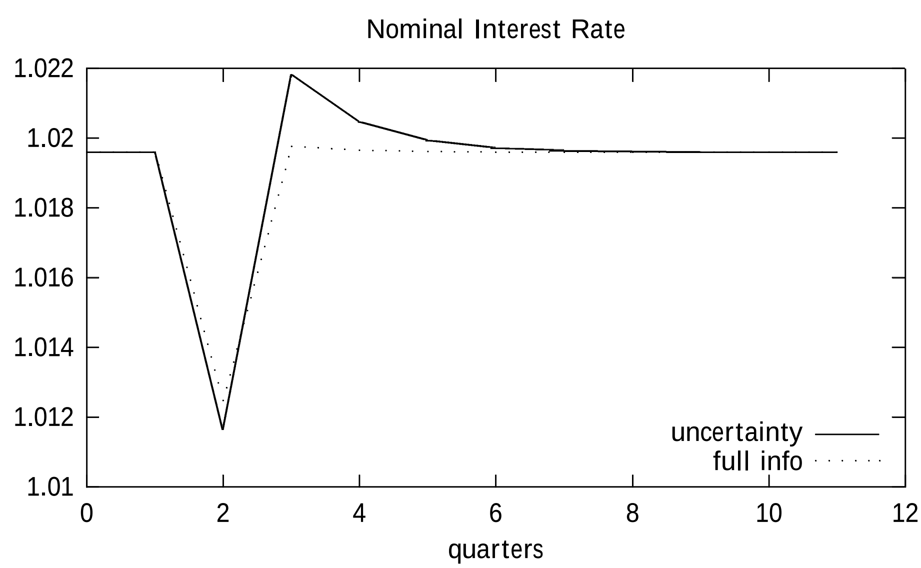


Figure 2: Domestic response to 1 standard deviation transitory increase in domestic money growth.





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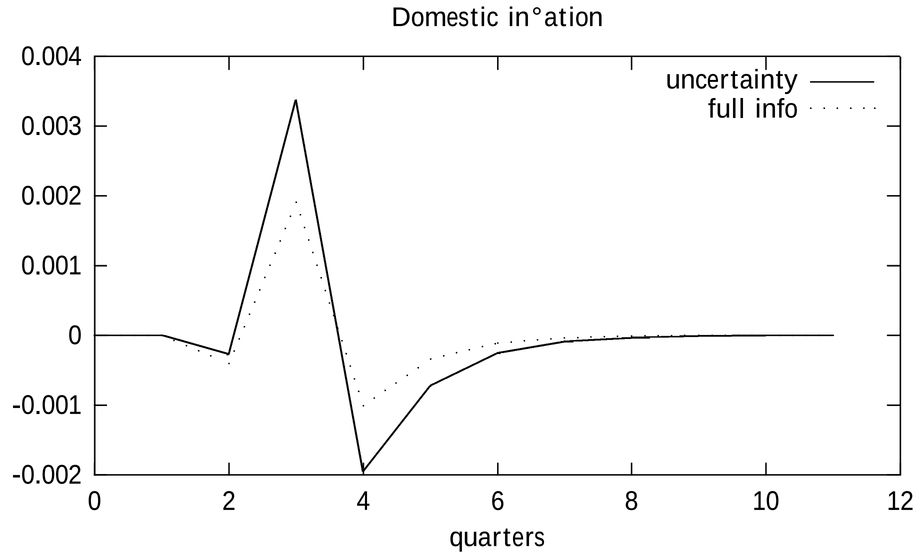

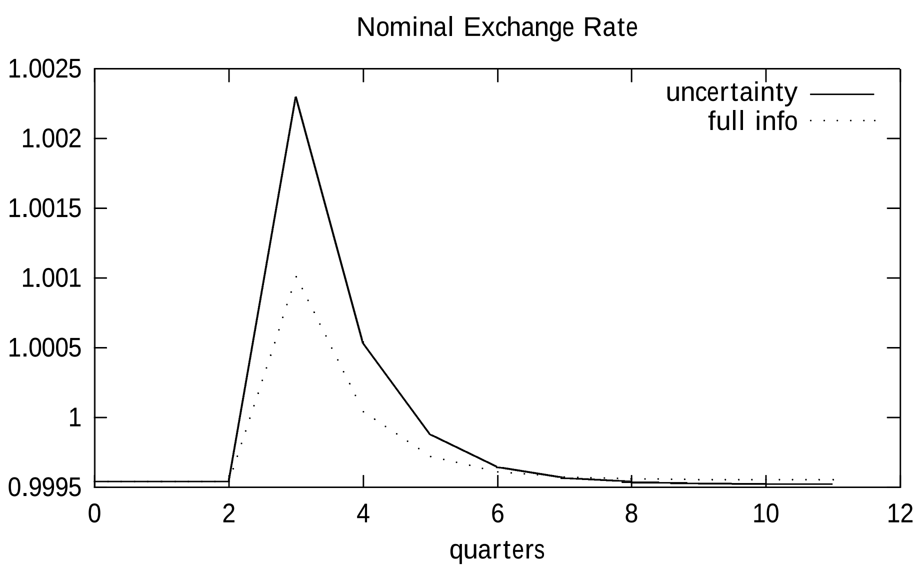

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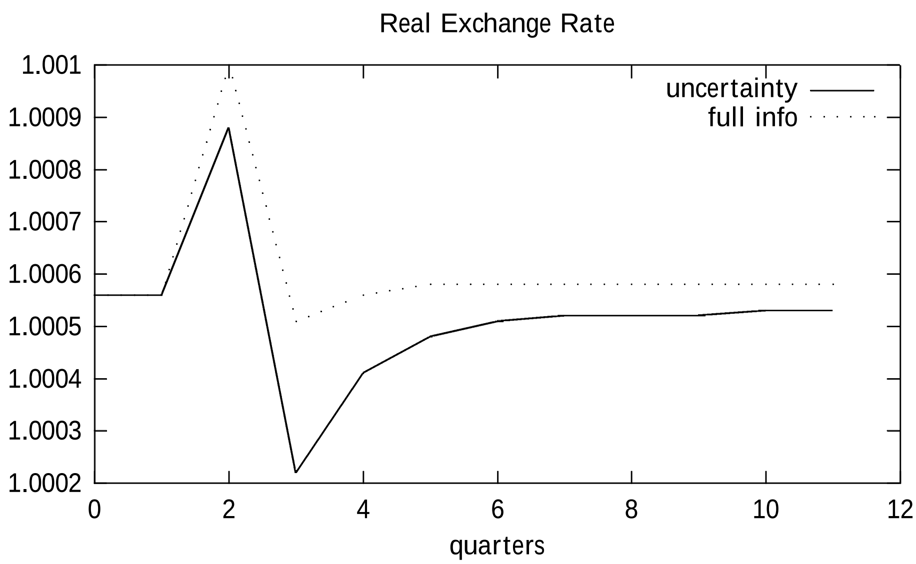


Figure 3: Change in beliefs in response to 1 standard deviation transitory increase in foreign money growth.

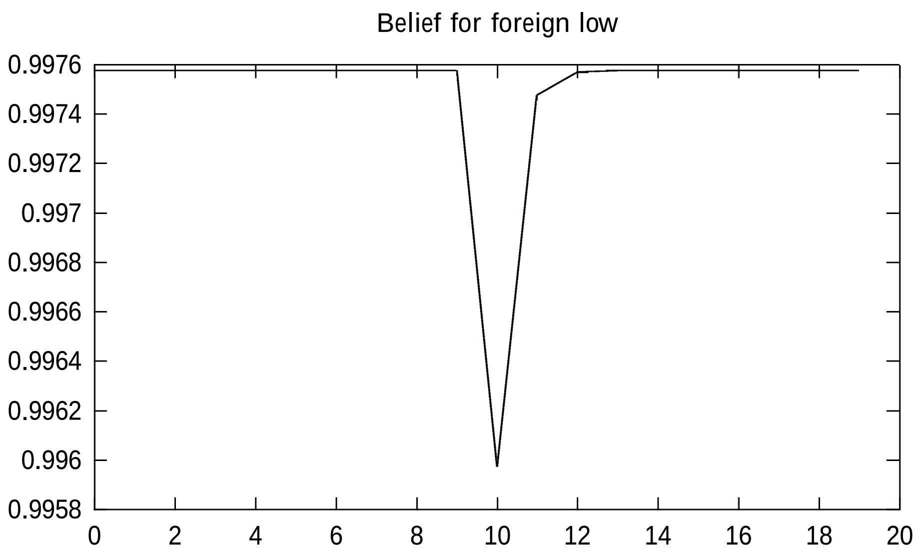
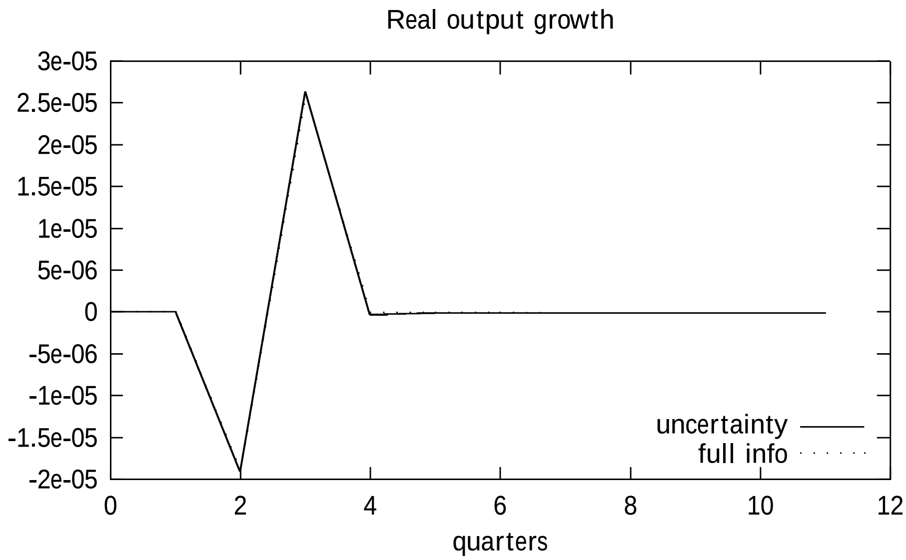
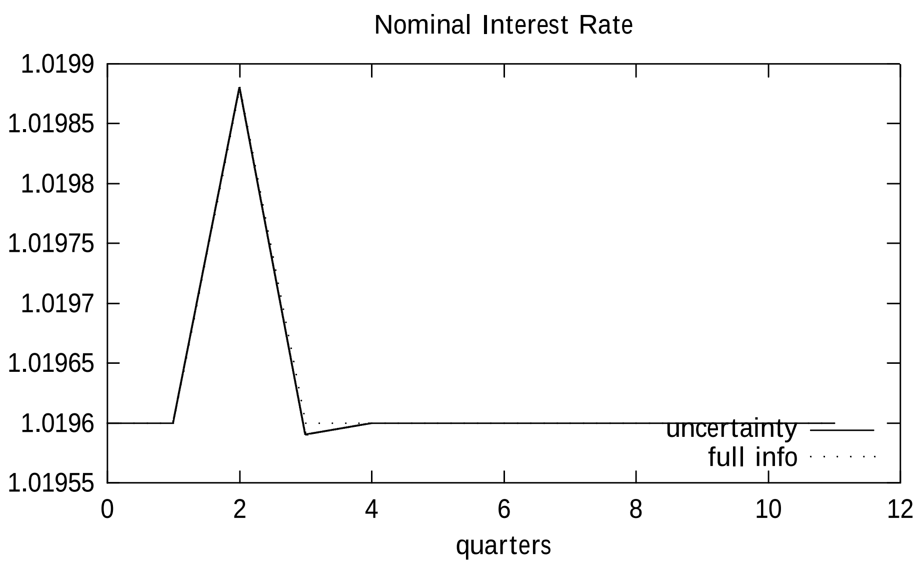


Figure 4: Domestic response to 1 standard deviation transitory increase in foreign money growth.





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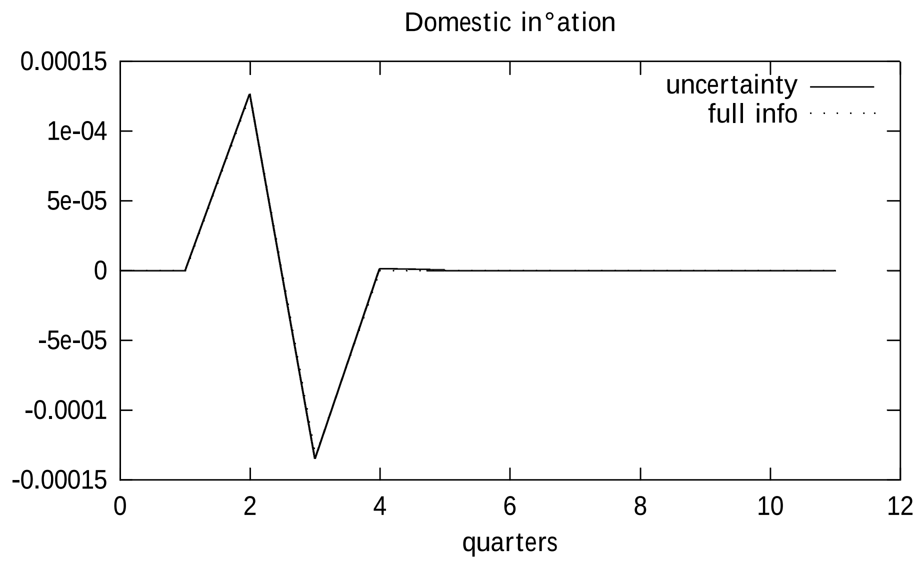

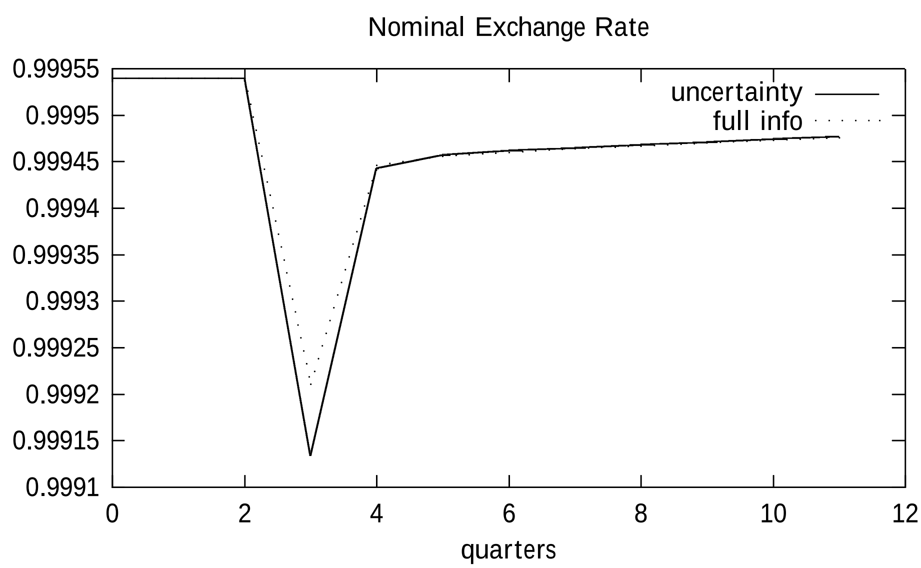

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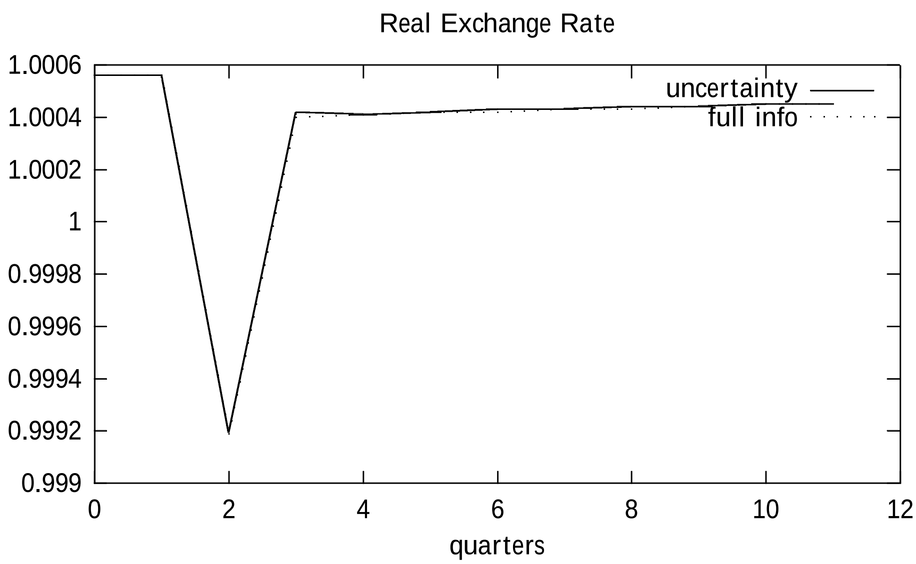


Figure 5: Change in beliefs in response to a permanent reduction in home money growth

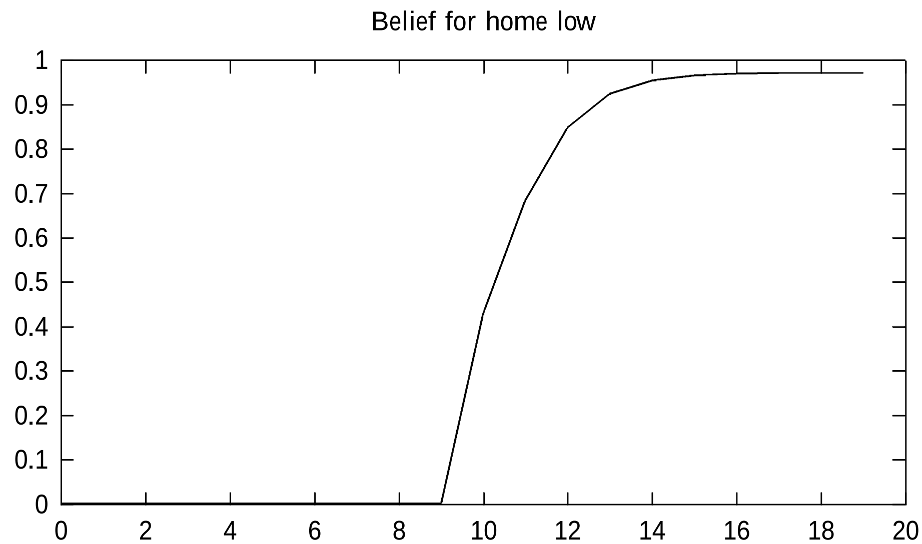
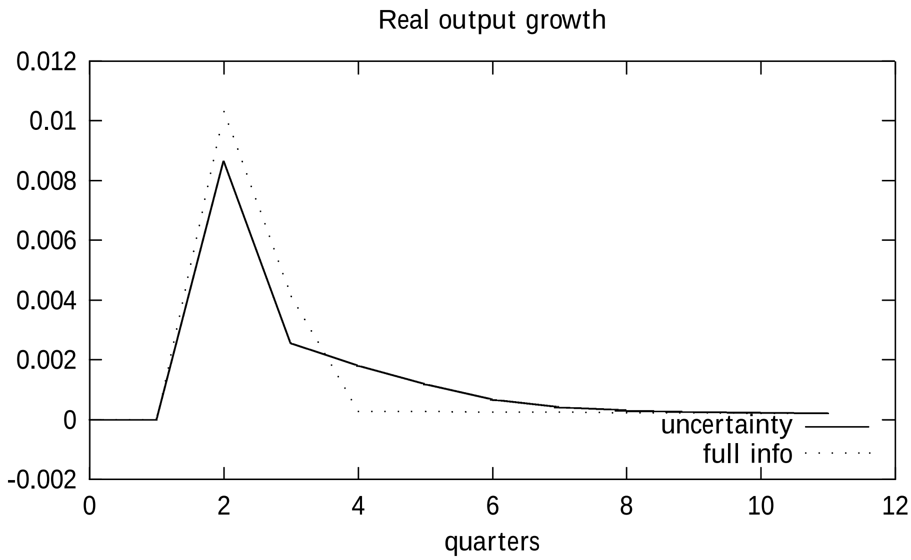
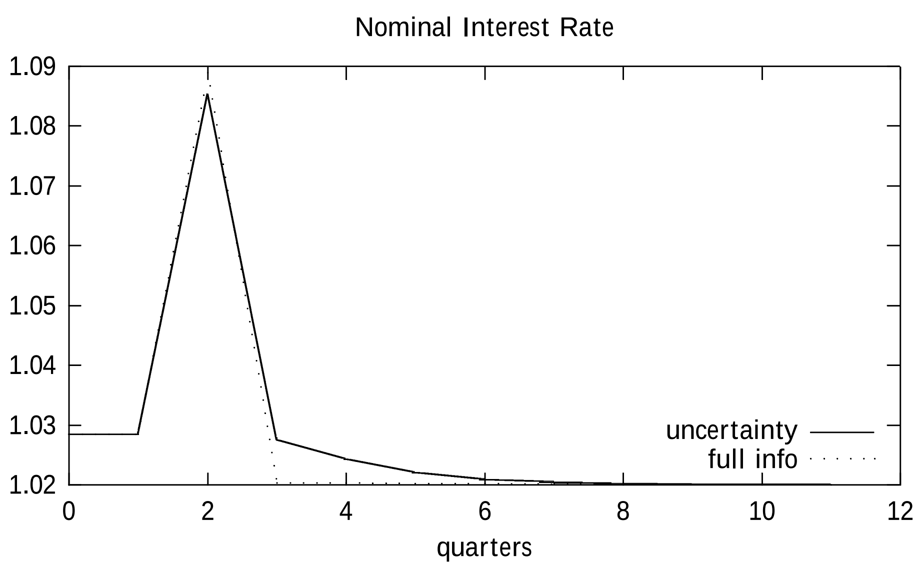


Figure 6: Domestic response to a permanent reduction in home money growth.





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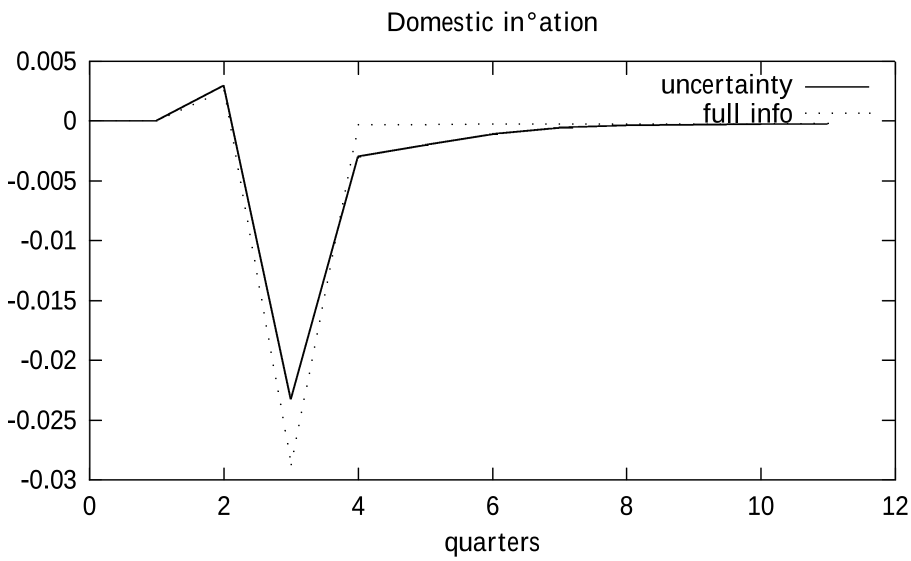

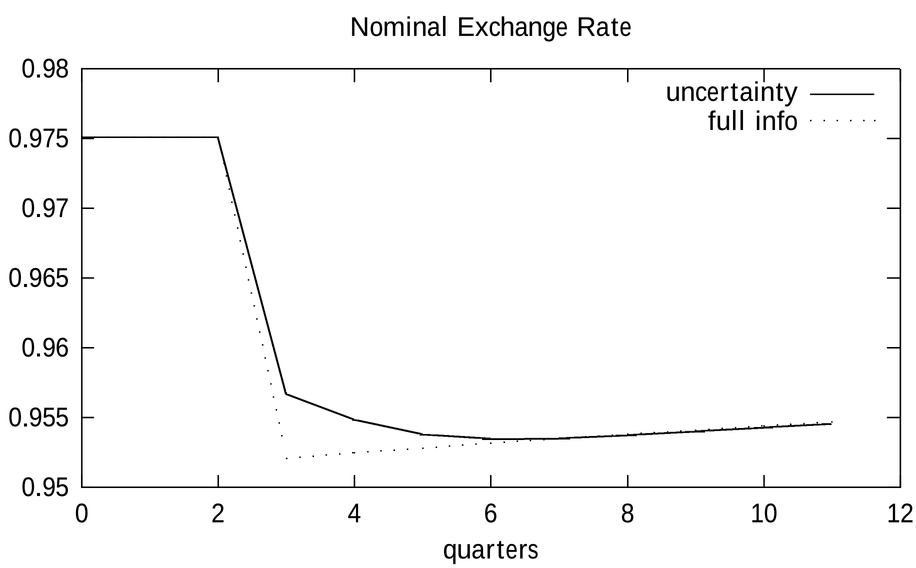

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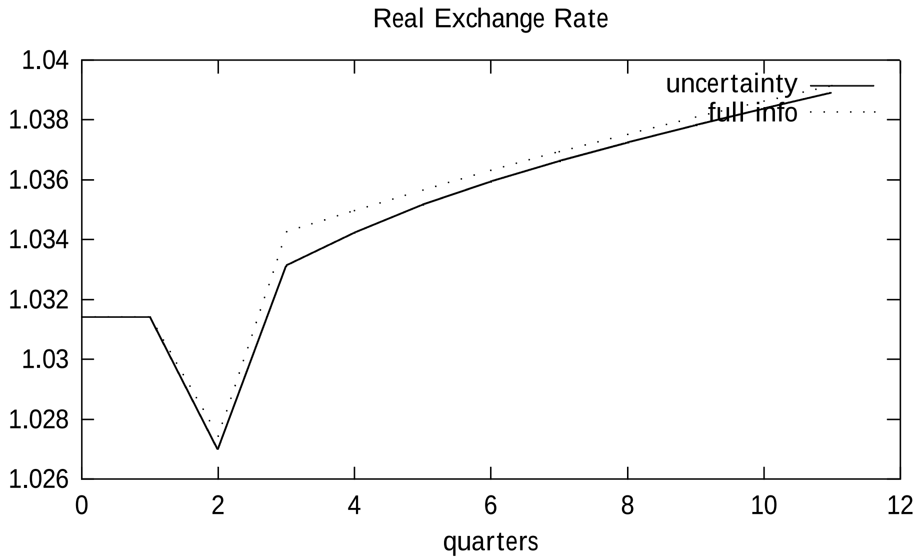


Figure 7: Change in beliefs in response to a permanent reduction in foreign money growth

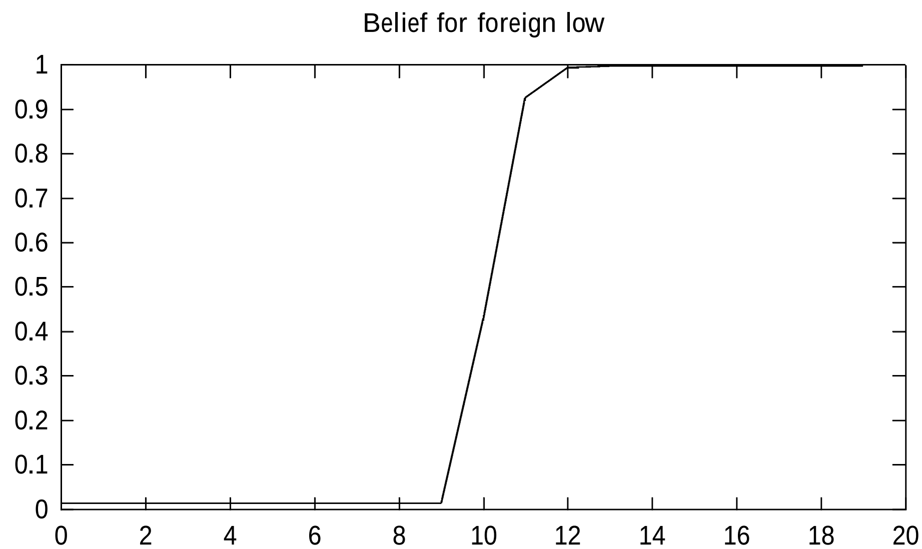
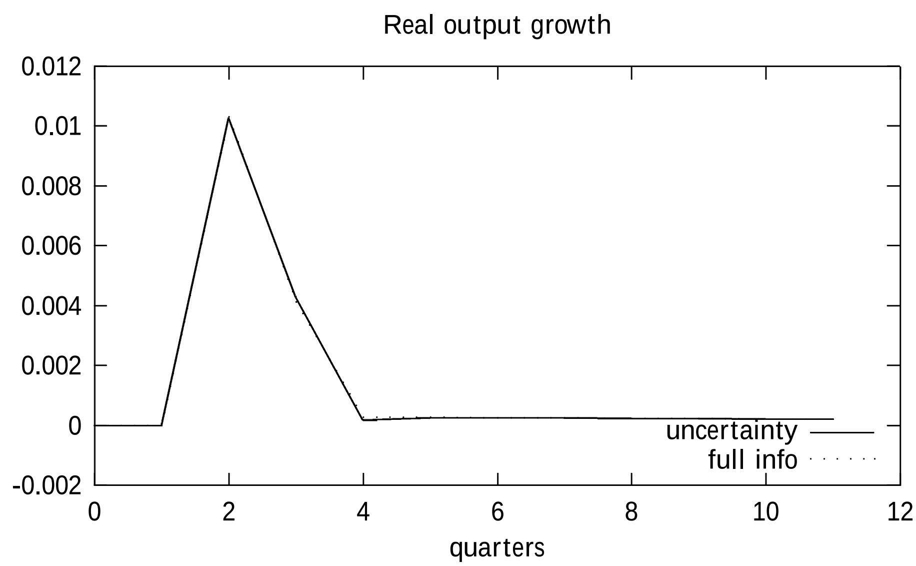
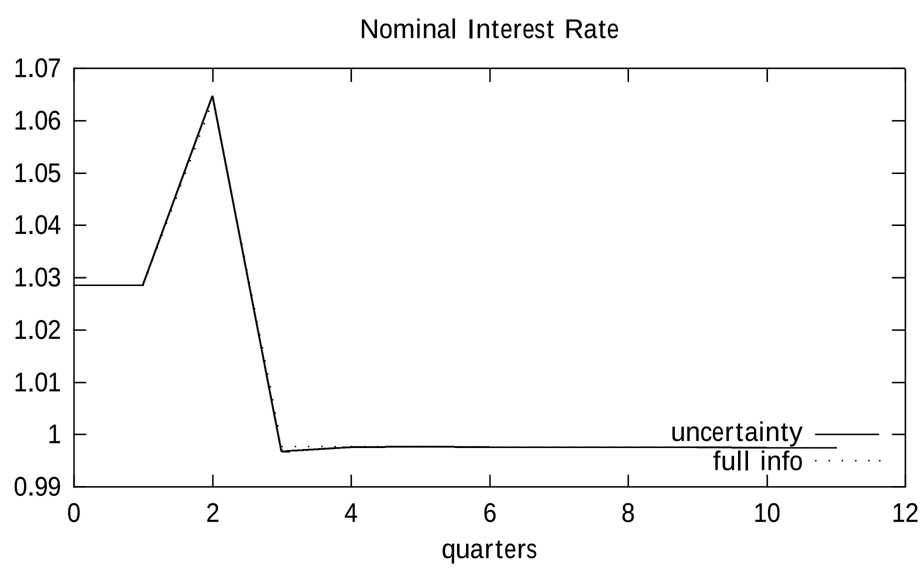


Figure 8: Domestic response to a permanent reduction in foreign money growth.





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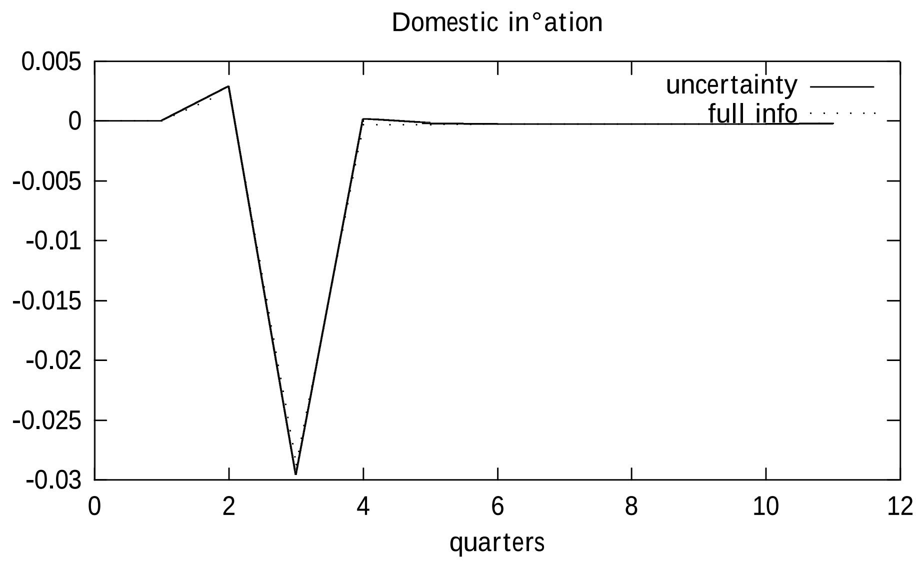

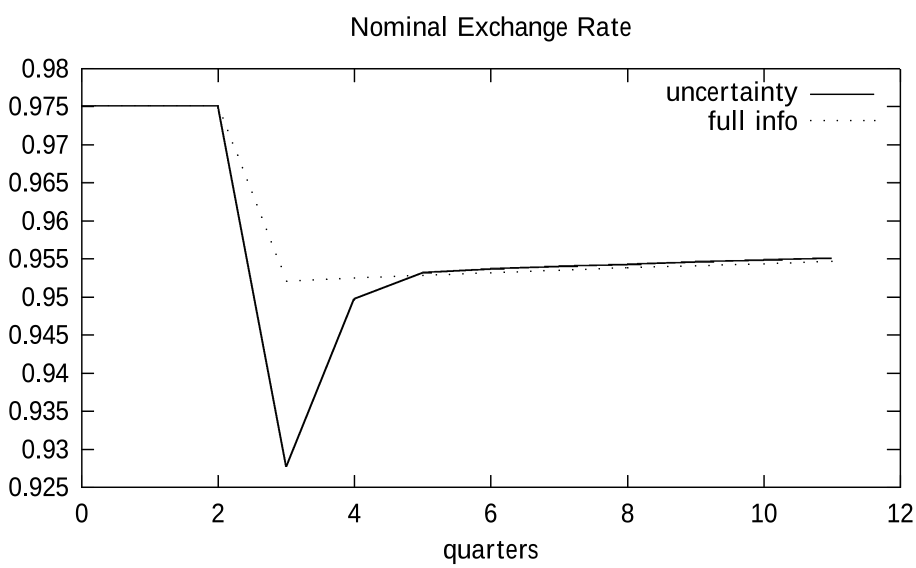

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