First Impressions in a Sequential Auction*

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Abstract

Should an informed seller lead with the best or worst good in a sequential auction? Considering the sale of two stochastically equivalent goods over two periods, we show that if second period buyers can observe the first period price, the seller has an incentive to lead with the best good so as to send a positive signal about the quality of the following good. This result holds even though the goods' values are independent because the seller's sequencing strategy endogenously generates correlation in the quality of the goods across periods. In contrast, a best for last strategy may not be as credible as the seller has an incentive to then sell his better good early. We also show that ex-ante expected profits from either of these strategies is higher than a babbling strategy of randomly sequencing the sale, even when the second period buyers do not observe the first period price.We discuss implications for the choice of sequential versus simultaneous auctions, the strategic choice of auction houses, the sequential auction of items of varying expected quality, the declining price anomaly observed in auction data, and the effects of selection bias on empirical studies of privatization auctions.

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1 Introduction

Should auctioneers sell lower value goods first, saving higher value goods for last? Or should the best goods be sold first to make the most favorable impression? We consider this question in a model where the auctioneer has imperfect private information about the values of the two goods to be soldl sequentially. If buyers of the good sold in the second period observe the auction price of the first good, their estimates of the second good's value will be affected by the seller's choice of leading with the better or worse good. For instance, a high sale price for the first good will indicate that first period buyers believed the good was of high value, raising expectations for the value of the second good. This is true even if the values of the two goods are statistically independent provided that the buyers believe that the seller's sequencing strategy is based on his information.

We find that the *impression effect* from observing the first period price favors a "best foot forward" (BFF) strategy of leading with the better good and that this effect exists even if the goods are independent. The strategy of leading with either the best or worst good, endogenously creates correlation between the two periods by shifting the conditional distribution of the second period good. If buyers believe the best good is sold first they will assume the second good is likely to be of equal or lower quality than the first. Selling the worst good first would then lead buyers to believe the second good is of even lower quality. So sellers have an incentive to stick with the best foot forward strategy when buyers believe them to be following the strategy. Conversely, if buyers believe the seller will lead with the worst good they will assume that the second good is likely to be of equal or higher quality. Deviating and selling the best good first would then lead buyers to believe the second good is of even higher quality, raising seller revenue. So sellers have an incentive to deviate from the "best for last" (BFL) strategy when buyers believe them to be following the strategy.

The following very simple example will illustrate this impression effect of leading with the better good. Suppose that buyers have common values for each good and that the value of each good is uncertain and equal to $V \in \{0, 1\}$, with $\Pr[V = 1] = \frac{1}{2}$. Suppose further that the values are drawn independently across goods and that the seller perfectly knows the value of the good and has to sell the goods sequentially[1]. Suppose, just for this illustrative example, that the buyers have no private information and that the sale price in each period is the expected value of the good given the buyers beliefs about the seller's sequencing strategy. Moreover, again for this example, suppose that the second period buyers perfectly observe the first period quality after the first period[2].

Suppose that the buyers believe that the seller is leading with his best good first. Consider the seller

[1] However, to make the problem interesting, the seller cannot credibly disclose his private information.

[2] In the model that we consider below, buyers will also have private information and as a result the first period sale price will take replace this exogenous signal for second period buyers.

with one good with value 1 and the other good with value 0. If he sells better good first then the first period price is the expected value of the good given it is the maximum of the two, which is equal to $\frac{3}{4}$. Upon observing the quality of the good sold in the first period, the second period sale price will be the expected value of the good given that it is the minimum of the two *and* given that the maximum is equal to 1. The second period price will therefore be equal to $\frac{1}{3}$. On the other hand if he deviates and sells his worse good first, then the first period price is still equal to $\frac{3}{4}$, but the second period price, equal to the expected value of the good given that it is the minimum of the two and given that the maximum is equal to 0, will now equal 0. Thus, when buyers believe that the seller is selling his better good first it is in his interest to do so: best foot forward is an equilibrium sequencing strategy.

In contrast, suppose that the buyers believe that the seller is leading with his worst good first. Consider the seller with one good with value 1 and the other good with value 0. If he sells the worse good first then the first period price is the expected value of the good given it is the minimum of the two, which is equal to $\frac{1}{4}$. Upon observing the quality of the good sold in the first period, the second period sale price will be the expected value of the good given that it is the maximum of the two *and* given that the minimum is equal to 0. The second period price will therefore be equal to $\frac{2}{3}$. On the other hand if he deviates and sells his better good first, then the first period price is still equal to $\frac{1}{4}$ but the second period price, equal to the expected value of the good given that it is the maximum of the two and given that the minimum is equal to 1, will now equal 1. Thus, when buyers believe that the seller is selling his worse good first it is *not* in his interest to do so: best for last is not an equilibrium.

We show that for a more general model, when buyers have idiosyncratic private information, and the second period buyers just observe the first period price, the impression effect illustrated above implies that the seller's incentive compatibility constraint for the best foot forward strategy is weaker than for the best for last strategy, implying best foot forward is an equilibrium of the sequencing game whenever best for last is, but not the converse. Best foot forward is therefore the unique pure strategy equilibrium for open sets of parameter values[3] whereas best for last is never the unique equilibrium. These results hold even if the seller is less precisely informed than the buyers. Of course, in all models, the buyers can always form beliefs that the seller ignores his private information, so that there always exists an equilibrium where the seller randomly chooses the good to be sold in either period. We call this the *babbling* equilibrium[4].

In the simple example above, the buyers have no private information. As a result different sequencing strategies have no effect on the seller's ex-ante expected profits[5]. However, when the buyers have idiosyncratic private information and earn informational rents, the two pure strategies BFF and BFL yield the seller higher ex-ante expected profits than the babbling strategy where the seller randomly chooses the good to be sold in either period. This is true *even when the second period buyers do not*

[3] In *any* parameterized model.

[4] To suggest an analogy with cheap talk games.

[5] Which is equal to twice the expected value of the good, in all cases.

observe the first period price. When the seller follows either pure strategy, the period in which any good is being sold is itself an informative signal to the buyers of that good, implying higher revenues by the linkage principle. The result suggests that a privately informed seller of ex-ante identical goods in ex-ante identical markets (e.g. the morning auction and the afternoon auction or the New York market and the Boston market) will nevertheless like to treat one market preferentially by always selling his better good there. We find this result interesting because of its implications for market segmentation, even if the markets operate simultaneously and without any informational spillovers from buyers in one market to those in the other.

When second period buyers observe the first period price they get an additional signal of the quality of the second period good, further increasing the revenue gain relative to the babbling strategy. Note that the informativeness of these public signals is endogenous as they depend on the seller's strategy, implying these results hold even though the qualities of the two goods sold in the two periods are independent.

Seller sequencing strategy may offer insight into the "declining price anomaly" or "afternoon effect" in which the prices for seemingly identical goods fall during the course of a sequential auction. This phenomenon has been identified in auctions for goods ranging from wine (Ashenfelter,1989), stamps (Thiel and Petry, 1990), condominiums (Ashenfelter and Geneshove, 1992, and Vanderporten, 1992) to real estate (Lusht, 1994). The empirical evidence is inconsistent with the "law of one price" that arbitrage should ensure uniform prices, and is also counter to the result from auction theory that prices should rise over time unless buyer values are unaffiliated (Milgrom and Weber, 1982c). A number of competing theoretical models try to explain this anomaly. McAfee and Vincent (1993) fail to confirm Ashenfelter's (1989) intuition that prices decline because of risk averse bidders, finding that unrealistic assumptions need to be made about behavior under risk. Menezes and Montiero (1999) assume that two identical objects are worth more as a bundle than they are separately. They show that prices decline since only buyers of the first good have the option to make use of this complementarity in the second period, an argument related to that of Black and de Meza (1992). Other explanations of the anomaly include endogenous budget constraints (Krishna and Benoit, 1998), declining number of buyers in the second auction due to participation costs (von der Fehr, 1994), a special form of auction where the winner chooses her preferred item (Gale and Hausch, 1992), and moral hazard by agents bidding on behalf of clients (Milgrom and Weber, 1982).

Our approach suggests a simpler explanation might suffice in some cases - prices fall but the law of one price is not violated because the goods are not identical across periods. The best foot forward strategy implies the quality of the second good is on average lower than the first, indicating the second period price will also be lower. For independently distributed goods we find that the price will necessarily fall.

Sequencing strategies are of particular interest in the context of privatization via auctions, one of the most important uses of auctions in recent years. When the government sells state-owned enterprises, are the more profitable firms sold first? Empirical evidence from Gupta et al.(1999) suggests that in the mass privatization programs undertaken in the Czech Republic more profitable firms appear to have been sold early. Studies which evaluate the effect of privatization or change in ownership on firm profitability and efficiency would be misleading if this potential selection issue is not accounted for. In particular, conclusions favoring rapid privatization might be affected if firms which are sold early are more successful not because of their early privatization but because they were of higher quality to begin with.

In the next section we set up the general model and in section 3 we discuss a version of the sequencing game where the second period buyers do not observe the first period price. In section 4 we use these results to analyze the sequencing game where the second period buyers do observe the first period price. In section 5 we discuss the declining price anomaly or afternoon effect and in section 6 we discuss another example. Section 7 concludes and the Appendix contains an alternative proof of the linkage principle result regarding the effect of public disclosure of private information on seller's profits, as well some other proofs of propositions in the paper.

2 The Model

Goods, Signals & Values One seller has to sell two (indivisible) goods (or two indivisible units of the same good) to sell. The two goods are indexed by $k \in \{a, b\}$[6]. The value of each good is unknown to the buyers and the seller.

For each good k there are $n \geq 2$ buyers with private signals (or value estimates) X_{i_k}, in some set $X \subset \mathbb{R}$, $i_k \in \{1, ..., n\}$, $k \in \{a, b\}$. Denote by $X_k = (X_{1_k}, ..., X_{n_k})$ the vector of buyer signals for good k. For each i_k, let $Y_{i_k} = \max_{j_k \neq i_k}\{X_{j_k}\}$ be the maximum of the other bidders signals. Further, let Z_k be the second highest buyer signal for good k.

We assume that, for each good k the seller has a private signal (or value estimate[7]), $\theta_k \in \Theta \subset \mathbb{R}$. In the next subsection, we provide a discussion of our interpretation of, and restrictions on, the seller's information and the actions he can take given that information.

We denote by $f_k(x, \theta)$ the joint density of the random variables (X_k, θ_k) associated with good k. We suppose that f_k is symmetric in its first n arguments. Further, we suppose that the random variables (X_a, θ_a) are distributed independent of the random variables (X_b, θ_b)[8]. We also assume that the two

[6] We use the indexing k for convenience. We suppose that, for the seller and the buyers in the model, the goods have no names so that the seller's sequencing strategy cannot be conditioned on the indices k and k' of the goods.

[7] Or a summary statistic for buyer signals X_k for good k.

[8] Given our interpretation of the two goods as two units of the same good, the assumption of independence might seem strange. However, we make this assumption to illustrate that all the intertemporal correlation in this model of sequential selling will be generated endogenously. Moreover, all our results on the effects of different sequencing strategies on revenues and their incentive compatibility properties, will go through when the signals are not independent across goods.

sets of random variables are identically distributed so that $f_a(x,\theta) = f_b(x,\theta) = f(x,\theta)$ for all x, θ.

We suppose that for each good k the joint density f for the random variables (X_k, θ_k) displays affiliation. For $x, y \in \mathbb{R}^m$ let $x \vee y$ be the component-wise maximum or the join of x and y and let $x \wedge y$ be the component-wise minimum or meet of x and y. The following is a definition of affiliation for random variables in $\mathbb{R}^m$, which admit a density.

Definition 1 *If $f : \mathbb{R}^m \to \mathbb{R}$ is strictly positive density function of the random variable X in $\mathbb{R}^m$ then f is affiliated if and only if for all $x, x' \in \mathbb{R}^m$*

$$f(x \vee x')f(x \wedge x') \geq f(x)f(x')$$

Note that affiliation is equivalent to log-supermodularity. Roughly speaking, affiliation says that higher values of some of the random variables make higher values of the other random variables more likely. The interested reader is referred to MW for a more extensive discussion of affiliation and its implications.

For each good k we suppose that the buyers are symmetric. In particular we assume that there exists a (measurable) function $V : \Theta \times X^n \to \mathbb{R}$, non-decreasing in each argument, such that for each good k, for each buyer i_k, for each θ_k and X_k, the value of good k to buyer i_k is given by

$$V_{i_k} = V(\theta_k, X_{i_k}, \{X_{j_k}\}_{j_k \neq i_k})$$

That is, the valuations of all n buyers for good k depends on the seller's signal in the same way, and the valuation of each buyer depends on the signals of the other buyer's in the same way and does not depend on the identity of the other buyers[9]. Let $V_k = (V_{1_k}, ..., V_n)$ be the vector of buyer valuations for good k. Under our assumptions of affiliation and the monotonicity of the function $V(.)$, the random variables (V_k, X_k, θ_k) are affiliated[10]. We abuse notation slightly and denote by $f(v, x, \theta)$ the joint density of (V_k, X_k, θ_k) which is distributed iid across $k \in \{a, b\}$.

In general, for any random variable Z we will denote its joint density as $f_Z(z)$ and for any two random variables Y and Z we will denote the density of Y conditional on the Z having taken a value z as $f_{Y|Z}(y|z)$. We will use analogous notation for distribution functions, which will be denoted by $F_{Y|Z}(y|z)$.

The Selling Game: Seller's Information and Strategies We suppose that the seller's information on each good takes one of two possible values, so that

$$\Theta = \{H, L\}, \text{with } H > L$$

[9]This is formulation is the "general symmetric model" of MW. It allows for the possibility of pure common values, where $V_{i_k} = V$ for all i_k. It also allows for a private value component for each buyer, so that all buyers may not agree on value, even if all private signals are made public. All our examples consider the case of pure common values.

[10]As are, in addition $(\{Y_{i_k}\}, Z_k\}$ etc. See MW.

In other words, we assume that the seller can tell if good k is likely to be above average ($\theta_k = H$) or likely to be below average ($\theta_k = L$) but his information is not rich enough to enable him to say anything more about the expected quality of each good.

This implies, that the seller with one good for which $\theta_k = H$ and the other good for which[11] $\theta_{k'} = L$, the possible sequencing strategies are either to sell his better good first (best foot forward or BFF strategy) or to sell his worse good first (best for last or BFL strategy) or he can randomize. We use the symbol σ to denote a sequencing strategy for the seller and denote the BFF strategy of selling his better good first by σ^{bff} and the BFL strategy of selling his better good last by σ^{bfl}. We will also consider the mixed strategy of ignoring his information by tossing a fair coin in his sequencing decision and denote this strategy by σ^m.

We assume that the set Θ has two elements because it eliminates the need for considering more complex sequencing strategies. When the seller's information is less crude and Θ has more than two elements (for example, when Θ is a continuum) then the seller has many more pure sequencing strategies[12]. Considering all such possible strategies is technically difficult. Further, we find that many possible sequencing strategies that arise from considering a richer specification of the set Θ are somewhat unnatural[13] and do not contribute much to the economic intuition underlying seller sequencing strategies. One way out of this is to directly restrict the set of possible sequencing strategies by assuming that the seller's ability to condition his actions on his information is crude, so that the seller can either sell his better good first or sell his worse good first (or randomize), but he is not able to condition on the amount by which his better good is better than his worse good. The specification of $\Theta = \{H, L\}$, as above, is equivalent to this formulation where the seller's possible actions (*including* the act of forming *expectations*) conditional on his information is restricted in this way.

Further, for the sequencing game to be interesting, we also suppose that the seller's private information is soft information, in the sense that he cannot credibly reveal it, even though he would like to. Instead, the only way he can reveal his private information is through his sequencing strategy.

We also suppose that the seller's private information arrives after he has already proposed a selling mechanism, e.g., a particular auction form. We are thinking here of information which arrives late. With a wine auction, for example, we can think of the seller's information as arising out of tasting the samples which arrive when the goods are delivered at a date after his announcement of the date of the auction[14]. In the next subsection we set out the timing structure of this selling game.

[11] When both the seller's signals have the same value the seller will be indifferent about his sequencing strategy, and in equilibrium, and will sell either good first with probability $\frac{1}{2}$.

[12] For example, when Θ is the unit interval on the real line, the seller may employ the BFF strategy when the maximum of his two signals is less than some number $\widehat{\theta}$ and the BFL strategy otherwise, etc.

[13] As we will see below, a sequencing strategy for the seller raises the seller's revenues by revealing some of his information to the buyers. We conjecture that sequencing strategies which depend on his private information in a complicated manner may not be optimal for the seller because they might less effective than simpler strategies like BFF or BFL in revealing his information. Investigating this conjecture would be of interest.

[14] We can pretend that the seller, at the ex-ante date when he announces his selling mechanism, can also make a "speech"

The Timing Structure The timing structure of the sequential selling game is as follows:

(0) Seller proposes a selling mechanism. Buyers form beliefs about the seller's strategy.

(1) Nature chooses (V_k, θ_k, X_k), $k \in \{a, b\}$.

(2) Seller observes (θ_a, θ_b) and decides on which good to sell first.

(3) The n buyers of the good the seller sells first note that their good is being sold first and also observe their private signal X_{i_k}. They bid for the good.

(4) The n buyers of the good the seller sells second note that their good is being sold second and also observe their private signal $X_{i_{k'}}$ and *also the first period price*. They bid for the good.

We call the above game the "price observed game" and denote it as , ^{po}.Sometimes to help our analysis of the model we also consider the game where the second period buyers do not observe the first period price, which we call the "no price observed game", , ^{npo}, where (4) above is replaced by

$(4)^{npo}$ The n buyers of the good the seller sells second note that their good is being sold second and also observe their private signal $X_{i_{k'}}$ but *not the first period price*. They bid for the good.

Equilibrium We will assume that the identical buyers for each good play a symmetric BNE of the auction given their correct beliefs about the seller's strategies and their information. And that the seller's strategy is sequentially rational given the buyers' beliefs.

Public Signals Because of the symmetry in the model it will be more useful in what follows to look at the auction for each *good k* and the public and private signals that the buyers of that good receive, rather than the auction in each *period*. The auctions in the two periods are different because the buyers believe that the seller is treating the two periods differently via his sequencing strategy. However, if we think of the auction for each good (which may either be held in the first period or the second period) then, by symmetry, this auction is identical across goods. Further, we can treat the period in which a given good is sold as a public signal which the buyers of that good observe and thereby incorporate the differences in buyer beliefs in the two periods in terms of this public signal. We will therefore look at the auction for each good and the public signals that the buyers of each good receives in some detail now.

regarding his intended selling strategy, once he gets his private information. Even if buyers believe his speech, he faces incentive compatibility problems with respect to following the content of his speech or deviating from it, once he receives his private information. The purpose of this paper is to compare different speeches with respect to their interim incentive compatibility problems and to compare the speeches by the profits they earn for the seller.

Consider the auction for good k. The relevant random variables in which buyers of good k are interested are (V_k, X_k, θ_k) with a joint density f. Given the structure of the game, and the seller's strategy, the buyers of good k will also receive a public signal about the seller's signal θ_k of good k.

It will be convenient to consider different possible public signals of θ_k in some detail. Let ϕ be a random variable independent of the random variables related to good k. We will use the symbol ψ_k to denote in general the possible public signals of θ_k.

Consider the game Γ^{npo} above where the second period buyers do not observe the first period price. The n buyers of each good k however observe in which period each good is sold. Since the choice of the period in which to sell good k is dependent on the seller's private information, the choice of a period will (publicly)signal some information to the buyers about the seller's signal for good k and so will be a signal about the value of good k. We will denote this signal by $\tau_k \in \{\tau_H, \tau_L\}, \tau_k \in \mathbb{R}, \tau_H > \tau_L$. The joint distribution of τ_k and $\tau_{k'}$ will be correlated given the seller's strategy and, conditional on τ_k, will induce correlation in the random variables associated with the two goods a and b. We will interpret τ_H as the signal for which the buyers conclude that θ_k is likely to be high, in particular $\theta_k \geq \theta_{k'}$ and τ_L is the signal for which $\theta_k \leq \theta_{k'}$. For example, if the seller is selling his good with the better signal first (Best Foot Forward (BFF) strategy) then the buyers of good k will observe the signal τ_H if good k is being sold in the first period which happens if $\theta_k \geq \theta_{k'}$. In contrast, if the seller his selling his good with the worse signal first (Best For Last (BFL) strategy), then the buyers of good k will observe the signal τ_H if good k is being sold in the second period which also happens when $\theta_k \leq \theta_{k'}$. The distribution of τ_k conditional on θ_k will depend on the strategy of the seller.

Therefore, when the second period buyers do not observe the first period price, the only public signal received by the buyers of good k is

$$\psi_k^{npo} = \tau_k$$

In the case Γ^{po}, when good k is sold in the second period, the buyers of good k will also observe, in addition to τ_k, the first period price which is a signal of the seller's signal for the good sold in the first period[15], $\theta_{k'}$, and so, in conjunction with τ_k, is a signal of the seller's signal for good k, θ_k, which is sold in the second period. Note that the first period price is a signal of second period quality only because the seller bases his sequencing decision on both his signals. In fact, by independence of good k with good k', unconditional on τ_k, $\theta_{k'}$ contains no information about θ_k. The first period winning bid is an informative signal for the second period buyers only conditional on τ_k.

If the seller is following a best foot forward strategy, the buyers of good k observe the first period price iff good k is being sold in the second (i.e., "low quality") period, i.e. $\tau_k = \tau_L$. The corresponding public signal is

$$\psi_k^{bff} = \begin{cases} (Z_{k'}, \tau_k) & \text{if } \tau_k = \tau_L \\ (\phi, \tau_k) & \text{if } \tau_k = \tau_H \end{cases}$$

[15] Since the first period price is a function of the buyers' signals in the first period which are affiliated with the seller's signal for the good sold in the first period.

In contrast, if the seller follows a best for last strategy, the buyers of good k observe the first period price iff the good is being sold in the second (i.e., "high quality") period, i.e., $\tau_k = \tau_H$. The corresponding public signal is

$$\psi_k^{bfl} = \begin{cases} (Z_{k'}, \tau_k) & \text{if } \tau_k = \tau_H \\ (\phi, \tau_k) & \text{if } \tau_k = \tau_L \end{cases}$$

For future reference we will also note here three other possible public signals that buyers of good k may find informative. The first is the case where the buyer's of good k observe the seller's signal for the other good k' in addition to observing the signal τ_k :

$$\psi_k^{**} = (\theta_{k'}, \tau_k)$$

A less informative signal than the last one is where the buyer's of good k observe $Z_{k'}$, which is a signal of $\theta_{k'}$, in addition to τ_k :

$$\psi_k^{*} = (Z_{k'}, \tau_k)$$

Finally, denote by

$$\psi_k^{NR} = \phi$$

the null, or uninformative public signal for good k.

The effect of observing the public signals of the seller's information θ_k, will depend on the buyers' beliefs, about the seller's strategy σ. In particular, if the buyers believe that the seller is playing according to σ^m, then they will ignore any public signal whose informativeness depends on the seller's strategy. We will denote by $\widehat{\sigma}$ the (common) beliefs of the buyers' about the seller's as a sequencing strategy, σ.

The Auction, Prices and Bids Since in our model the buyers also have private information, the price for any good k will depend on the vector of the buyers private information X_k and also on any public information, denoted by ψ_k (like the first period price) which is revealed through the trades. The exact nature of this dependence depends on the nature of information in the market and the selling mechanism employed by the seller.

We will concentrate on a situation where the buyers have idiosyncratic (and statistically relevant) pieces of private information. As a result, it is natural to assume that the seller employs an auction to sell his goods in each period. The analysis below is done assuming that the seller employs a second price auction (with no reserve prices) to sell his good. However, all the results generalize to other auction forms like the English (Japanese) auction, the first price auction[16,17].

[16] We do not consider the introduction of reserve prices and entry fees, even if that choice is made ex-ante unconditioned on the seller's private information, as this may alter the set of buyers in the first period auction differently from the set of buyers in the second period auction, depending on the seller's strategy. We leave this for future research.

[17] Our results also carry over to the case where all buyers have the same information so that they earn no informational rents. In this case, the different sequencing strategies still have incentive compatibility properties similar to the auctions case, but they leave the seller's ex ante expected profits unchanged, as there are no informational rents earned by the buyers.

We will denote the price of good k when the public signal is ψ_k^l and the buyer beliefs are $\widehat{\sigma}$ by $P_k^l(X_k, \psi_k^l; \widehat{\sigma})$, where l is an index of the different public signals considered above. Accordingly, if the buyers believe that the seller is playing the BFF strategy the price for good k is denoted by $P_k^{bff}(X_k, \psi_k^{bff}; \sigma^{bff})$ and when the buyers believe that the seller the BFL strategy the price is denoted by $P_k^{bfl}(X_k, \psi_k^{bfl}; \sigma^{bfl})$ where the public signals ψ_k^{bff} and ψ_k^{bfl} have been defined above.

Notice that the price of good k is a function of the buyers signal and the public signal. The precise nature of this functional dependence depends on the public signal(s) which the buyers observe, and this has been made explicit above. Notice also that the price as a function of private and public signals also depends on buyer beliefs about the seller's sequential strategy. This has also been made explicit above but the extra notation will be dropped when it causes no confusion.

Since we are looking at the symmetric (bidding) equilibrium for the second price auction of each good, the winning price will be equal to the second highest bid in the auction. Denote the (symmetric) bidding function as $b_k^l(x, \psi; \widehat{\sigma})$, when the buyer i_k's signal X_{i_k} has value x, the public signal ψ_k^l has value ψ and the buyers beliefs about that the seller's strategy is given by $\widehat{\sigma}$.

It is well-known[18] that in a second price auction each buyer bids the expected value of the good given the public signals *and given that the realized value of his signal is tied with the highest signal for the other bidders.* Define the function

$$v_k^l(x, y, \psi; \widehat{\sigma}) = E[V_{i_k} | X_{i_k} = x, Y_{i_k} = y, \psi_k^l = \psi; \widehat{\sigma}]$$

Then the bidding function of the buyers is

$$b_k^l(x, \psi; \widehat{\sigma}) \equiv v_k^l(x, x, \psi; \widehat{\sigma})$$

It is straightforward to check that given affiliation such a bidding function is monotonic in its first argument. As a result the price in a second price auction can be written as

$$\begin{aligned} P_k^l(X_k, \psi_k^l; \widehat{\sigma}) &= 2^{nd} \max_{i_k} \{ b_k^l(X_{i_k}, \psi_k^l; \widehat{\sigma}) \} \\ &= v_k^l(Z_k, Z_k, \psi_k^l; \widehat{\sigma}) \end{aligned}$$

where $2^{nd} \max\{.\}$ is the function which chooses the second highest value from n real numbers, and we have made use of our symmetry assumptions. We will use this formulation for the price of good k in what follows.

3 No Price Observed

We first compare the ex-ante expected profits from the three strategies BFF, BFL and the mixed or babbling strategy, in this NPO case. Next we look at the question of existence of equilibria.

[18] See, for example, MW.

3.1 Profits: NPO

Let $U^{npo}(\sigma, \widehat{\sigma})$ be the profits from strategy σ in the NPO case when buyer beliefs about the seller's strategy are given by $\widehat{\sigma}$. Note that for all $\sigma \in \{\sigma^{bff}, \sigma^{bfl}, \sigma^m\}$, by symmetry, we must have

$$\begin{aligned} U^{npo}(\sigma, \sigma) &= E[P_k^{npo}(X_k, \psi_k^{npo}; \sigma)|\sigma] + E[P_{k'}^{npo}(X_{k'}, \psi_{k'}^{npo}; \sigma)|\sigma] \\ &= 2E[P_k^{npo}(X_k, \psi_k^{npo}; \sigma)|\sigma] \end{aligned}$$

Consider first the BFF strategy. Then for any good k, we identify the public signal as $\tau_k = \tau_H$ if and only if good k is being sold in period 1 which happens if $\theta_k \geq \theta_{k'}$. Similarly, $\tau_k = \tau_L$ if $\theta_k \leq \theta_{k'}$. Therefore, τ_k is an informative signal of θ_k and so of V_{i_k} for each i_k.

In contrast for the mixed strategy σ^m, where the seller ignores his information, the period in which the seller sells a given good contains no information for the buyers about the value of the seller's signal for that good and so information about the value of that good: τ_k is an uninformative signal.

Hence we have our first result, as a direct application of the linkage principle.

Proposition 1 *Using his information helps the seller in the NPO case compared to not using his information:*

$$U^{npo}(\sigma^{bff}, \sigma^{bff}) = U^{npo}(\sigma^{bfl}, \sigma^{bfl}) \geq U^{npo}(\sigma^m, \sigma^m)$$

Proof. The first equality follows from the fact that in the NPO case there is no difference between the two strategies BFF and BFL.

The inequality follows from the linkage principle from MW : in both the BFF and the BFL case the buyers of each good k observe a public signal $\psi_k^{npo} = \tau_k$ of the seller's private information θ_k for that good. From Theorem 9 in MW, the result follows[19]. □

The result states that a seller, when faced with selling two ex-ante identical and independent goods in two ex-ante identical markets which have names (e.g., period 1 & period 2, New York & Boston) would prefer to preferentially treat one market in the sense of always selling his better good there, even when the markets operate without any informational spillover from the auction in one market to the other. In other words, a privately informed seller would like to segment his markets and informationally link them by his sequencing strategy. Clearly, this result does not depend on our independence assumption but only on the fact that the seller reveals some of his private information by choosing a sequencing strategy which depends on his information.

3.2 Equilibrium: NPO

We now look at the question of the existence of an equilibrium sequencing strategy for the seller, i.e., at the incentive compatibility of the seller's ex-ante speech regarding his sequencing strategy.

[19] We provide an alternative proof of the public disclosure result or Theorem 8 from MW in the appendix.

One equilibrium which always exists is that where buyers refuse to believe that the seller conditions his sequencing decision on his private information. This is the same as supposing that the buyers believe that the seller's strategy is σ^m. Given these beliefs, the seller is indifferent between the different sequencing strategies (as the buyers of good k ignore the signal τ_k, for all k). As a result, the seller can ignore his private information and σ^m is always an equilibrium

Suppose now that the buyers believe that the seller will follow the BFF strategy, σ^{bff}, (equivalently, subject to renaming the periods, σ^{bfl}, in this NPO case). Then, for the seller with one high signal and one low signal, the expected profits when he plays according to σ^{bff} is

$$\pi^*_{HL} \equiv E[P_k^{npo}(X_k, \tau_H)|\theta_k = H] + E[P_k^{npo}(X_k, \tau_L)|\theta_k = L]$$

where we have made use of symmetry between goods k and k' and also dropped the dependence of prices on the buyer beliefs $\widehat{\sigma} = \sigma^{bff}$.

On the other hand, if the seller deviates then his expected profits are given by

$$\pi^d_{HL} \equiv E[P_k^{npo}(X_k, \tau_L)|\theta_k = H] + E[P_k^{npo}(X_k, \tau_H)|\theta_k = L]$$

Therefore the strategy σ^{bff} is an equilibrium of , ^{npo} (the speech σ^{bff} is credible) if and only if

$$E[P_k^{npo}(X_k, \tau_H) - P_k^{npo}(X_k, \tau_L)|\theta_k = H] \geq E[P_k^{npo}(X_k, \tau_H) - P_k^{npo}(X_k, \tau_L)|\theta_k = L]$$

Note that the equilibrium condition for σ^{bfl} is identical as the two strategies are identical for the game , ^{npo} subject to renaming the periods.

For the second price auction recall that

$$P_k^{npo}(X_k, \tau) = b_k^{npo}(Z_k, \tau) \text{ for all } \tau \in \{\tau_H, \tau_L\}$$

where

$$b_k^{npo}(z, \tau) = v_k^{npo}(z, z, \tau) = E[V_k|X_{i_k} = z, Y_{i_k} = z, \tau_k = \tau]$$

Whether the inequality above holds depends on the conditional distribution of Z_k given θ_k and the conditional distribution of V_k given X_{i_k}, Y_{i_k} and τ_k. In particular, note that we have an equilibrium if the jump in any buyers bid from observing the public signal τ_H over when he observes τ_L is non-decreasing in the realization of his private signal z:

$$b_k^{npo}(z, \tau_H) - b_k^{npo}(z, \tau_L) \text{ is non-decreasing in } z$$

Then, by affiliation, the result follows. However this condition is a strong sufficient condition and is not in general guaranteed by affiliation. Of particular interest is the case where $b_k^{npo}(z, \tau_H) - b_k^{npo}(z, \tau_L)$ is additively separable in z and τ, implying sellers are indifferent between strategies. The two examples in section 6 characterize the equilibrium set in terms of parameter values.

4 Price Observed

In this section we look at the game , ^{po} where the goods are sold sequentially so that the buyers in the second period observe the first period price. As above we look at the effect on profits and then look at characterizing the equilibrium set.

4.1 Profits: PO

Let $U^{po}(\sigma, \widehat{\sigma})$ be the ex-ante expected profits from strategy σ and buyer beliefs $\widehat{\sigma}$. Once again, by symmetry, we must have

$$\begin{aligned} U^{po}(\sigma^{bff}, \sigma^{bff}) &= 2E[P_k^{bff}(X_k, \psi^{bff}; \sigma^{bff})|\sigma^{bff}] \\ U^{po}(\sigma^{bfl}, \sigma^{bfl}) &= 2E[P_k^{bfl}(X_k, \psi^{bfl}; \sigma^{bfl})|\sigma^{bfl}] \\ U^{po}(\sigma^{m}, \sigma^{m}) &= 2E[P_k^{NR}(X_k)|\sigma^{m}] \end{aligned}$$

Recall that $P_k^{NR}(X_k)$ is the price of good k when the buyers observe only their private signal and no public signal.

Recall that

$$\psi_k^{bff} = \begin{cases} (Z_{k'}, \tau_k) & \text{if } \tau_k = \tau_L \\ (\phi, \tau_k) & \text{if } \tau_k = \tau_H \end{cases}$$

and

$$\psi_k^{bfl} = \begin{cases} (Z_{k'}, \tau_k) & \text{if } \tau_k = \tau_H \\ (\phi, \tau_k) & \text{if } \tau_k = \tau_L \end{cases}$$

Notice that for both the BFF case and the BFL case, the good k buyers know when the first component signal ψ_k^l is informative and when it is noise, $l \in \{bff, bfl\}$. Furthermore, the signal ψ_k^l is more informative about θ_k than the signal $\psi_k^{npo} = \tau_k$.

In contrast, for the mixed strategy, both τ_k and ψ_k are uninformative signals of θ_k. Therefore we have the following result, as a direct consequence of Theorem 9 in MW. However, we provide an alternative proof in the appendix.

Proposition 2 *When second period buyers observe the first period price, using his information further benefits the seller, compared to the NPO case:*

$$\min[U^{po}(\sigma^{bff}, \sigma^{bff}), U^{po}(\sigma^{bfl}, \sigma^{bfl})] \geq U^{npo}(\sigma^{bff}, \sigma^{bff}) = U^{npo}(\sigma^{bfl}, \sigma^{bfl}) \geq U^{npo}(\sigma^{m}, \sigma^{m}) = U^{po}(\sigma^{m}, \sigma^{m}).$$

Proof. Follows from the discussion above and Theorem 9 in MW. Also see the appendix for an alternative proof. □

A natural question is whether the ex-ante profits from σ^{bfl} and σ^{bff} can be unambiguously ranked. The relevant condition can easily be seen to be the following[20]:

[20]See the discussion of public signals in Section 2.1. for a defintion of P_k^* corresponding to the public signal ψ_k^*.

$$\begin{aligned} E[P_k^*(X_k, \tau_H, Z_{k'}) - P_k^{npo}(X_k, \tau_H)|\tau_k &= \tau_H] \\ &\geq E[P_k^*(X_k, \tau_L, Z_{k'}) - P_k^{npo}(X_k, \tau_L)|\tau_k = \tau_L] \end{aligned}$$

where we have dropped the dependence on σ^{bff} for notational brevity. In words, the BFL strategy yields higher profits to the seller than the BFF strategy, if and only if the expected gain conditional on τ_k, due to the linkage principle, of observing both signals $\{\tau_k, Z_{k'}\}$ over just observing τ_k, is greater, the higher is τ_k. Neither this condition nor its converse are guaranteed by affiliation. We provide an example (example 1) where the condition is satisfied so that BFL generates higher revenues. Roughly, the condition is satisfied when the information rents for the higher-valued good are larger than for the lower-value good, allowing the seller to recapture more of these rents via the linkage principle effect of the first period price.

4.2 Equilibria: PO

We now state our main result.

Proposition 3 *1. An equilibrium exists: σ^m is always an equilibrium in $,^{npo}$ and $,^{po}$.*

2. BFF is an equilibrium in the no price observed game if and only if BFL is an equilibrium: σ^{bff} is an equilibrium of $,^{npo}$ iff σ^{bfl} is an equilibrium of $,^{npo}$

3. The impression effect of observing the first period price favors the BFF strategy: if σ^{bff} is an equilibrium of $,^{npo}$, it is an equilibrium of $,^{po}$.

4. The impression effect of observing the price acts against the BFL strategy: if σ^{bfl} is an equilibrium of $,^{po}$, it is an equilibrium of $,^{npo}$.

Proof. The proof is in the Appendix. □

These results follow from the best foot forward and best for last strategies being equivalent when the first period price is not observed, and the impression effect favoring the best foot forward strategy when the price is observed. The intuition for the impression effect favoring the BFF strategy and acting against the BFL strategy can be best understood by considering the simple example discussed in the introduction.

In general, whether we have pure strategy equilibria BFF or BFL depends on the information structure. The result above shows that whenever we have BFL as an equilibrium BFF must also be an equilibrium, but the converse is not true. In particular, if the seller is indifferent between all strategies in the when the first price is not observed, as occurs when $b_k^{npo}(z, \tau_H) - b_k^{npo}(z, \tau_L)$ is independent of z, then the impression effect implies that best foot forward is the unique pure strategy equilibrium when the price is observed. The parameterized examples in section 6 characterize the equilibrium set in terms of the parameters.

5 The Afternoon Effect

Let $P_t(\widehat{\sigma})$ be the price (which is a random variable) in period $t \in \{1, 2\}$ given buyer beliefs $\widehat{\sigma}$. We will be interested in the expected values of the price of good k conditional on various seller strategies. Where it is understood what the seller's strategy is (as in the proof below) we will drop the conditioning of the expectation of the price on the seller strategy. The next result shows that the expected price in the period where the seller sells his better good is higher than the expected price where he sells his worse good.

Proposition 4 *Fix the PO model where the second period buyers observe the first period price.*

1. *In the BFF case the expected second period price is lower than the expected first period price:*

$$E[P_1(\sigma^{bff})|\sigma^{bff}] \geq E[P_2(\sigma^{bff})|\sigma^{bff}]$$

2. *In the BFL case the expected second period price is higher than the expected first period price:*

$$E[P_1(\sigma^{bfl})|\sigma^{bfl}] \leq E[P_2(\sigma^{bfl})|\sigma^{bfl}]$$

3. *In the mixed strategy case the expected second period price is equal to the expected first period price:*

$$E[P_1(\sigma^{m})|\sigma^{m}] = E[P_2(\sigma^{bff})|\sigma^{bff}]$$

Proof. The Proof is in the Appendix. □

This proposition shows that when the seller plays the BFF strategy, the unconditional expected second period price will be lower than the unconditional expected first period price. Therefore we have an alternative demonstration of the afternoon effect, even when there is no exogenous correlation in the qualities of the two goods, arising endogenously out of the seller's choice of an incentive compatible sequencing strategy.

Notice that in this model the presence of the "afternoon effect" does not imply the violation of the "law of one price". The two stochastically identical goods will still sell at the same expected price, by symmetry. Sometimes, when good k is better (given the seller's information) than good k' it will sell in the first period at a higher average price, and at other times, when it is worse, it will sell in the second period at a lower average price. But unconditional on the period, both goods will sell at the same expected price.

6 Examples

6.1 Example 1

For this example let the possible values of the two goods a and b be $V = \{0, 1\}$ where $\Pr[V_k = 1] = \lambda \in (0, 1)$ for $k \in \{a, b\}$. Regarding the seller's signal, let

$$\Pr[\theta_k = H | V_k = 1] = \Pr[\theta_k = L | V_k = 0] = \alpha \in (\frac{1}{2}, 1]$$

for $k \in \{a, b\}$. In each period there are $n = 2$ buyers who each get a noisy signal of the quality of the good being sold in that period $X_{i_k} \in \{L, H\}$, where

$$\Pr[X_{i_k} = H | V_k = 1] = \Pr[X_{i_k} = L | V_k = 0] = \beta \in (\frac{1}{2}, 1),$$

for $i \in \{1, 2\},\ k \in \{a, b\}$. We assume the buyers' and the seller's signals are independent conditional on the quality of the good[21].

In a second price auction, with two bidders each bidder will bid the expected value of the good conditional on both the buyers having the same signal. Thus, when the bidders also observe a public signal ψ_k, the bidding functions will be

$$\begin{aligned} b_k(H, \psi_k) &= \frac{\beta^2 \lambda(\psi_k)}{\beta^2 \lambda(\psi_k) + (1-\beta)^2 (1 - \lambda(\psi_k))} \\ b_k(L, \psi_k) &= \frac{(1-\beta)^2 \lambda(\psi_k)}{(1-\beta)^2 \lambda(\psi_k) + (1 - (1-\beta)^2)(1 - \lambda(\psi_k))} \end{aligned}$$

where $\lambda(\psi_k)$ is the expected value of the good k conditional on the public signal ψ_k that the bidders observe (and given their beliefs about the seller's strategy).

$$\lambda(\psi_k) = E[V_k | \psi_k]$$

For example, for the case where the buyers believe that the seller is employing the BFF strategy, then in the first period the public signal that the buyers observe is that their good is being sold in the first period, i.e., $\theta_k \geq \theta_{k'}$. Thus, in this case $\lambda(\psi_k)$ or the expected value of the good given this public signal is equal to $E[V_k | \theta_k \geq \theta_{k'}]$. In the special case where the seller is perfectly informed so that $\alpha = 1$ (equivalently $\theta_k = V_k$) this is equal to $\lambda^2 + 2\lambda(1-\lambda)$. Similarly, when good k is sold in the second period the public signal that the buyers observe consist of the observation that their good is sold in the second period and also the first period price. Then $\lambda(\psi_k)$ is equal to $E[V_k | \theta_k \leq \theta_{k'}, P_{k'}]$.

Further, given a signal for the seller θ_k, in a second price auction the probability that the seller receives the high bid as a price is equal to the probability that both the buyers have a high signal; otherwise the seller receives the low bid as the price. Thus

$$\begin{aligned} \Pr[P_k &= b_k(H, \lambda(\psi_k)) | \theta_k] = \Pr[X_{1_k} = X_{2_k} = H | \theta_k] \\ \Pr[P_k &= b_k(L, \lambda(\psi_k)) | \theta_k] = 1 - \Pr[P_k = b_k(H, \lambda(\psi_k)) | \theta_k] \end{aligned}$$

First consider whether a seller benefits from a mixed "babbling" strategy or a pure strategy like BFF or BFL. Suppose first that we are in the NPO case where the second period buyers do not observe

[21]Note that we did not need to assume this for the general model.

the first period price. Even in this case, as a direct implication of the linkage principle, revenues from either the best foot forward strategy or best for last strategy should be higher, than the babbling case. The intuition for this is easy to see if we think in terms of the auction for each good rather than the auction in each period. The buyers for each good sometimes see that their good is sold in the first period and sometimes see that the good is sold in the second period. Since this is a function of the seller's private information for that good, the period in which a good is sold is an imperfect signal of the seller's private information. By the linkage principle, the expected price for each good is higher than in the babbling case and so the total revenue for the seller is higher than the babbling case.

Concentrating on this NPO case, figure 1 shows the revenue gains from either pure strategy as a proportion of the revenue from the babbling strategy when α is fixed at 0.95, meaning the seller's information is quite accurate, and β varies from 0.5 to 1, spanning the full range of accuracy for the buyers' signals, and λ is fixed at 0.5. Notice that this is non-monotonic in β. When β is very low or very high buyers either have no information or very precise information. As a result, informational rents are low. For intermediate values of β information rents are high and this is when the gain from either pure strategy is highest, as they capture more of the information rent.

In the case where the second period buyers observe the first period price and additional informative signal of the seller's private information is available to the buyers. This, again by the linkage principle, further raises the revenues of the seller. Note that both the "period" and the price signal are informative only because of the seller's strategy (or more precisely, buyer beliefs about the seller's strategy).

Figure 2 shows the gain from the BFF strategy in the PO case (where buyers observe an additional price signal) over the BFF strategy in the NPO case (as proportion of the revenue in the NPO case) as a function of β where α is again fixed at 0.95 and λ at 0.5. Once again the gain is highest for intermediate values of β when informational rents earned by the buyers is the highest.

What about the existence of equilibrium in the NPO case? Computational results show that a pure strategy is an equilibrium if $\lambda \leq \frac{1}{2}$. Intuitively, when $\lambda \leq \frac{1}{2}$, the responsiveness[22] of the price to buyer's signals is higher when buyers get a good public signal, $\tau_k = \tau_H$ compared to when the buyers get a bad public signal $\tau_k = \tau_L$. As a result, the seller has an incentive to sell his good with the higher signal in the period where buyers observe τ_H, as the good with a higher signal is the one for which buyers are more likely to have better private signals, and when $\lambda \leq \frac{1}{2}$, this increase the prices more in the period when $\tau_k = \tau_H$ than in the period where $\tau_k = \tau_L$.

Now consider the price observed case where the seller has an incentive to make a favorable first impression. The question is whether a seller with two opposing signals should sell the good with the higher or lower signal first. Due to the impression effect (see Proposition 4) favoring the BFF strategy and not favoring the BFL strategy, we know from the previous last paragraph we know that BFF is an equilibrium at least for $\lambda \leq \frac{1}{2}$ and BFL is not an equilibrium for $\lambda \geq \frac{1}{2}$. Computational results for this model indicate that there exists a cutoff value $\lambda^{bff}(\alpha, \beta) > \frac{1}{2}$ such that the best foot forward strategy

[22] That is, the difference between the prices for high buyer signals and low buyer signals.

is always an equilibrium for $\lambda \in (0, \lambda^{bff})$ while there exists another cutoff value $\lambda^{bfl} < \frac{1}{2}$ such that best for last strategy is an equilibrium for $\lambda \in (0, \lambda^{bfl})$. Therefore, for "low" lambda we have two pure strategy equilibria, BFF and BFL. For intermediate values we have a unique pure strategy equilibrium in BFF. And for "high" lambda we have no pure strategy equilibrium of the sequencing game. Figure 3 shows the net gains from following the BFF strategy and the BFL strategy in both the PO and the NPO case as a function of λ with α fixed at 0.95 and β fixed at 0.75. The curve which intersects the x-axis at the lowest point corresponds to the BFL strategy and the curve which intersects it at the highest point corresponds to the BFF strategy. The curve in between, intersecting the x-axis at 0.5 corresponds to the NPO case.

Figure 4 shows the net gains from following the BFF strategy and the BFL strategy as a function of the informativeness of the buyers' signal. For figure 4 α is again fixed at 0.95, λ is fixed at 0.4 and β varies from 0.5 to 1. Since $\lambda < 0.5$ a BFF equilibrium exists for all β (and α). A BFL equilibrium exists only when the buyers' signal is sufficiently imprecise (β less than about 0.775). As the buyers' signal becomes informative, the price signal becomes more precise and as a result BFL becomes less likely to be an equilibrium.

Now consider what happens to average prices in the first and second periods. From Proposition 6 the second period price should always be lower than the first period price when the seller follows the best foot forward strategy. Figure 5 shows the expected prices for the two periods, with the first period price on top, when α and β have the same values, varying together from 0.5 to 1 (and $\lambda = 0.5$). Note that the average price is 0.5 for $\alpha = \beta = 0.5$. In this case all signals are uninformative so the buyers both bid the unconditional expected value. As the accuracy of the signals increases, a gap between the prices emerges because the seller is now able to sell the better good in the first period with some accuracy. This gap reaches a maximum when both signals are perfectly accurate. Note that the average of the two prices is lower when the signals are partly accurate rather than completely uninformative or completely informative. With partly accurate signals the buyers can have opposing signals, allowing one buyer to collect information rents at the expense of the seller.

A final concern is whether the ex ante profits from committing to a strategy of best foot forward or best for last are higher. Computational results show that best for last revenues are always higher than best foot forward revenues, regardless of whether BFL or BFF is an equilibrium. As the next example shows this result is dependent on the information structure of this example an does not generalize to all information structures satisfying affiliation. Figure 6 shows the revenue gain from the BFL strategy over the BFL strategy as a proportion of the revenue from the BFF strategy as a function of β when λ is fixed at 0.5 and α is fixed at 0.95.

6.2 Example 2

For this example assume again that the possible values of the two goods a and b are $V \in \{0,1\}$ where $\Pr[V_k = 1] = \lambda \in (0,1)$ for $k \in \{a,b\}$. Suppose that the seller perfectly knows the value of the good.

In each period there are n buyers. Each buyer, with probability $\beta \in (0,1)$ may be informed (type I) who perfectly knows the value of the good, or may be uninformed (type U) with probability $(1-\beta)$ in which case he knows nothing apart from public signals about the quality of the good. We assume the buyers' signals are independent conditional on the quality of the good[23].

In this model, in the NPO case, the seller is always (for all λ, β) strictly prefers playing BFF to deviating from it: BFF is always a equilibrium. As a result, in the PO case, from Proposition 4, a BFF equilibrium always exists. Computational results indicate that a BFL equilibrium also exists for certain parameter values. As is true in general, the BFF equilibrium yields higher revenues than the mixed strategy babbling equilibrium

However, the result from the previous example where BFL always yields higher revenue than BFF, does not hold in this example. Either strategy may yield higher revenues depending on the values of the parameters.

7 Conclusion

This model assumed the two goods were independently drawn from the same distribution, and then showed that correlation across periods was endogenously generated by the seller's strategy. If instead the two goods were ex ante positively correlated, this should strengthen the impression effect favoring a best foot forward strategy since second period buyers would learn even more from the first period price. For this same reason, positive correlation increases revenues when sellers can credibly commit to a strategy. Since more accurate public information is revealed about the expected value of the second period good, sellers recover even more information rents from buyers. Under what conditions this favors best foot forward or best for last is unclear. Positive ex ante correlation clearly does not strengthen the declining price result. Since the second period buyers observe a stronger signal of the quality of the second period good through the first period price, they will bid more aggressively in the second period, raising the second period price. With sufficient correlation, this linkage effect from observing the first period price may outweigh the opposing best foot forward effect of the second period good being lower quality.

The potential conflict between revenue maximizing and equilibrium strategies highlight consideration

[23] Our results depend on the value of any good, the seller's signal for that good and the buyers' signals for that good to be affiliated. To see that this signal structure satisfies the condtions of affiliation, suppose that buyers' signals take values x_H,x_M,x_L with $x_H > x_M > x_L$, and the probability that it takes value x_H equal to β , value x_M equal to $(1-\beta)$ and value x_L equal to 0 conditional on the value of the good being equal to 1; and, conditional on the value of the good being equal to 0, the probability that it takes value x_H equal to 0 , value x_M equal to $(1-\beta)$ and value x_L equal to β.

of reputation. However, since we have assumed that the seller's information is ex-post unverifiable[24] it is not clear that reputational considerations will allow the seller to solve his incentive compatibility problems with regard to his sequencing decision. Further, even when the seller can disclose his private information, sequencing strategies will be important as long as the their is correlation across goods, because that will allow the seller to link the information of the buyers across goods.

Relaxing the assumption that both goods come from the same distribution would make the model applicable to a broader range of auctions. For instance, if two paintings which are known to have different expected values are to be sold, which painting should be auctioned first? Since the buyers and sellers share the same knowledge of which good is likely to be better, buyers can observe directly which strategy the seller is following, allowing sellers to credibly commit to the best for last strategy. In this case the seller's strategy would not endogenously generate correlation in the values of the goods, but any ex ante positive correlation would still favor a best for last strategy.

8 Appendix

Proof of the Profitable Public Disclosure Result We provide here a proof of the profitable public disclosure result for the second price auction. See MW for a more general discussion.

Let ψ_k be the public signal of the quality of the good. Let $P_k(X_k)$ be the price when no public information is disclosed and $P_k^{\psi}(X_k, \psi_k)$ when all buyers observe the public signal.

Proposition 5 *Disclosure of the public signal raises the expected price:*

$$E[P_k^{\psi}(X_k, \psi_k)] \geq E[P_k(X_k)]$$

Proof. By definition,

$$\begin{aligned} E[P_k^{\psi}(X_k, \psi_k)] &= \int_{Z_k} \int_{\psi_k} v_k^{\psi}(z, z, \psi) f_{Z_k \psi_k}(z, \psi) dz d\psi \\ &= \int_{Z_k} \{ \int_{\psi_k} v_k^{\psi}(z, z, \psi) f_{\psi_k | Z_k}(\psi | z) d\psi \} f_{Z_k}(z) dz \\ &= E[\overline{b}_k^{\psi}(Z_k)] \end{aligned}$$

where

$$\begin{aligned} \overline{b}_k^{\psi}(z) &\equiv \int_{\psi_k} v_k^{\psi}(z, z, \psi) f_{\psi_k | Z_k}(\psi | z) d\psi \\ &= E[v_k^{\psi}(z, z, \psi_k) | Z_k = z] \\ &= E[v_k^{\psi}(z, z, \psi_k) | X_{1_k} \geq z, X_{2_k} = z, X_{i_k} \leq z \forall i_k > 2] \end{aligned}$$

[24] Otherwise he can credibly disclose it.

where the last equality follows from symmetry. Notice that $\overline{b}_k^{\psi}(z)$ is the expected winning bid in the case where the public information is disclosed, the expectation being taken over values of the public signal, *conditional on the second highest signal being* z.

On the other hand

$$\begin{aligned} E[P_k(X_k)] &= \int_{Z_k} v_k(z,z) f_{Z_k}(z) dz \\ &= E[v_k(Z_k, Z_k)] \end{aligned}$$

Now, by symmetry,

$$\begin{aligned} v_k(z,z) &= E[V_{1_k} | X_{1_k} = z, Y_{1_k} = z] \\ &= E[E[V_{1_k} | X_{1_k} = z, Y_{1_k} = z, \psi_k] | X_{1_k} = z, Y_{1_k} = z] \\ &= E[v_k^{\psi}(z,z,\psi_k) | X_{1_k} = z, Y_{1_k} = z] \\ &= E[v_k^{\psi}(z,z,\psi_k) | X_{1_k} = z, X_{2_k} = z, X_{i_k} \leq z \forall i_k > 2] \end{aligned}$$

where we have used symmetry. Notice that $v_k(z,z)$ is the bid of any buyer in the no disclosure case given that the realization of his signal equals z. This is equal to the expected value of the bid for that buyer in the case where the public information is disclosed, the expectation being taken over values of the public signal, *conditional on the highest and the second highest signal being* z, i.e., conditional on his signal and the fact that he wins the auction.

By affiliation and the monotonicity of $v_k^{\psi}(.,.,.)$ in ψ, therefore (see MW theorem 5), for all z,

$$\begin{aligned} \overline{b}_k^{\psi}(z) &= E[v_k^{\psi}(z,z,\psi_k) | X_{1_k} \geq z, X_{2_k} = z, X_{i_k} \leq z \forall i_k > 2] \\ &\geq E[v_k^{\psi}(z,z,\psi_k) | X_{1_k} = z, X_{2_k} = z, X_{i_k} \leq z \forall i_k > 2] \\ &= v_k(z,z) \end{aligned}$$

Therefore,

$$\begin{aligned} E[P_k^{\psi}(X_k, \psi_k)] &= E[\overline{b}_k^{\psi}(Z_k)] \\ &\geq E[v_k(Z_k, Z_k)] \\ &= E[P_k(X_k)] \end{aligned}$$

This concludes the proof. □

Proof of Proposition 3: Equilibrium in PO Case The first two parts are immediate.

For 3 and 4 note first that the equilibrium condition, in the NPO case, for the identical pure strategies σ_{bfl} and σ_{bff} can be written as

$$\begin{aligned} \text{(NP0)} \quad E[P_k^{npo}(X_k, \tau_H) | \theta_k &= H] - E[P_k^{npo}(X_k, \tau_H) | \theta_k = L] \\ &\geq E[P_k^{npo}(X_k, \tau_L) | \theta_k = H] - E[P_k^{npo}(X_k, \tau_L) | \theta_k = L] \end{aligned}$$

Similarly the equilibrium condition for the strategy σ_{bff} can be written as

$$\text{(BFF)}\quad \begin{aligned} E[P_k^{npo}(X_k,\tau_H)|\theta_k &= H] - E[P_k^{npo}(X_k,\tau_H)|\theta_k = L] \\ &\geq E[P_k^*(X_k,\tau_L,Z_{k'})|\theta_k = H, \theta_{k'} = L] - E[P_k^*(X_k,\tau_L,Z_{k'})|\theta_k = L, \theta_{k'} = H] \end{aligned}$$

and that for strategy σ_{bfl} can be written as

$$\text{(BFL)}\quad \begin{aligned} E[P_k^*(X_k,\tau_H,Z_{k'})|\theta_k &= H, \theta_{k'} = L] - E[P_k^*(X_k,\tau_H,Z_{k'})|\theta_k = L, \theta_{k'} = H] \\ &\geq E[P_k^{npo}(X_k,\tau_L)|\theta_k = H] - E[P_k^{npo}(X_k,\tau_L)|\theta_k = L] \end{aligned}$$

where we have made use of symmetry and the fact that

$$\psi_k^{bff} = \begin{cases} \psi_k^{npo} & \text{when } \tau_k = \tau_H \\ \psi_k^* & \text{when } \tau_k = \tau_L \end{cases}$$

and

$$\psi_k^{bfl} = \begin{cases} \psi_k^{npo} & \text{when } \tau_k = \tau_L \\ \psi_k^* & \text{when } \tau_k = \tau_H \end{cases}$$

where $\psi_k^* = (Z_{k'}, \tau_k)$ and $\psi_k^{npo} = \tau_k$.

To prove 3 it suffices to demonstrate that

$$\begin{aligned} E[P_k^*(X_k,\tau_L,Z_{k'})|\theta_k &= H, \theta_{k'} = L] - E[P_k^*(X_k,\tau_L,Z_{k'})|\theta_k = L, \theta_{k'} = H] \\ &\leq E[P_k^{npo}(X_k,\tau_L)|\theta_k = H] - E[P_k^{npo}(X_k,\tau_L)|\theta_k = L] \end{aligned}$$

and for 4 that

$$\begin{aligned} E[P_k^*(X_k,\tau_H,Z_{k'})|\theta_k &= H, \theta_{k'} = L] - E[P_k^*(X_k,\tau_H,Z_{k'})|\theta_k = L, \theta_{k'} = H] \\ &\leq E[P_k^{npo}(X_k,\tau_H)|\theta_k = H] - E[P_k^{npo}(X_k,\tau_H)|\theta_k = L] \end{aligned}$$

We prove a stronger version of these inequalities in the lemma below.

Lemma 6 *Fix $\tau \in \{\tau_H, \tau_L\}$ and $\theta \in \{H, L\}$. Then*

$$\begin{aligned} E[P_k^*(X_k,\tau,Z_{k'})|\theta_k = \theta, \theta_{k'} = L] \\ \leq E[P_k^{npo}(X_k,\tau)|\theta_k = \theta] \\ \leq E[P_k^*(X_k,\tau,Z_{k'})|\theta_k = \theta, \theta_{k'} = H] \end{aligned}$$

Proof of lemma. Note first that

$$E[P_k^{npo}(X_k,\tau)|\theta_k = \theta] = \int_{Z_k} v_k^{npo}(z,z,\tau) f_{Z_k|\theta_k}(z|\theta)dz$$

where

$$\begin{aligned} v_k^{npo}(z,z,\tau) &= E[V_{i_k}|X_{i_k}=z, Y_{i_k}=z, \tau_k=\tau] \\ &= E[v_k^*(z,z,\tau,Z_{k'})|X_{i_k}=z, Y_{i_k}=z, \tau_k=\tau] \\ &= \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|X_{i_k}Y_{i_k}\tau_k}(z'|z,z,\tau)dz' \end{aligned}$$

Now, for any x, y, τ, z'

$$f_{Z_{k'}|X_{i_k}Y_{i_k}\tau_k}(z'|x,y,\tau) = \frac{\sum_{\theta'} f_{Z_{k'}|X_{i_k}Y_{i_k}\tau_k\theta_{k'}}(z'|x,y,\tau,\theta') f_{X_{i_k}Y_{i_k}\tau_k\theta_{k'}}(x,y,\tau,\theta')}{f_{X_{i_k}Y_{i_k}\tau_k}(x,y,\tau)}$$

where the sum is over values of $\theta_{k'}$ in $\{H, L\}$. By independence of the random variables related to good k from good k' the density of $Z_{k'}$ conditional on X_{i_k}, Y_{i_k}, τ_k and $\theta_{k'}$ depends only on $\theta_{k'}$:

$$f_{Z_{k'}|X_{i_k}Y_{i_k}\tau_k\theta_{k'}}(z'|x,y,\tau,\theta') = f_{Z_{k'}|\theta_{k'}}(z'|\theta')$$

Using this we obtain

$$v_k^{npo}(z,z,\tau) = \sum_{\theta'} \frac{f_{X_{i_k}Y_{i_k}\tau_k\theta_{k'}}(z,z,\tau,\theta')}{f_{X_{i_k}Y_{i_k}\tau_k}(z,z,\tau)} \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|\theta')dz'$$

Note that for fixed z, τ, by affiliation of $\theta_{k'}$ with $Z_{k'}$ we have

$$\begin{aligned} E[v_k^*(z,z,\tau,Z_{k'})|\theta_{k'} &= H] = \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|H)dz' \\ &\geq \sum_{\theta'} \frac{f_{X_{i_k}Y_{i_k}\tau_k\theta_{k'}}(z,z,\tau,\theta')}{f_{X_{i_k}Y_{i_k}\tau_k}(z,z,\tau)} \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|\theta')dz' \\ &\geq \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|L)dz' = E[v_k^*(z,z,\tau,Z_{k'})|\theta_{k'}=L] \end{aligned}$$

Therefore

$$\begin{aligned} E[P_k^{npo}(X_k,\tau)|\theta_k &= \theta] \\ &= \int_{Z_k} \{\sum_{\theta'} \frac{f_{X_{i_k}Y_{i_k}\tau_k\theta_{k'}}(z,z,\tau,\theta')}{f_{X_{i_k}Y_{i_k}\tau_k}(z,z,\tau)} \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|\theta')dz'\} f_{Z_k|\theta_k}(z|\theta)dz \\ &\geq \int_{Z_k} \{\int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|L)dz'\} f_{Z_k|\theta_k}(z|\theta)dz \\ &= E[P_k^*(X_k,\tau,Z_{k'})|\theta_k=\theta, \theta_{k'}=L] \end{aligned}$$

where in the last line we have again made use of the independence of the random variables related to goods k and k'. Similarly

$$E[P_k^{npo}(X_k,\tau)|\theta_k = \theta]$$

$$
\begin{aligned}
&= \int_{Z_k} \{ \sum_{\theta'} \frac{f_{X_{i_k} Y_{i_k} \tau_k \theta_{k'}}(z,z,\tau,\theta')}{f_{X_{i_k} Y_{i_k} \tau_k}(z,z,\tau)} \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|\theta') dz' \} f_{Z_k|\theta_k}(z|\theta) dz \\
&\leq \int_{Z_k} \{ \int_{Z_{k'}} v_k^*(z,z,\tau,z') f_{Z_{k'}|\theta_{k'}}(z'|H) dz' \} f_{Z_k|\theta_k}(z|\theta) dz \\
&= E[P_k^*(X_k, \tau, Z_{k'}) | \theta_k = \theta, \theta_{k'} = H]
\end{aligned}
$$

This concludes the proof of the lemma. □

Proof of Proposition (continued). The proof of 4 follows by setting $\tau = \tau_H$ and setting $\theta = H$ and then $\theta = L$. The proof of 3 follows by setting $\tau = \tau_L$ and setting $\theta = H$ and then $\theta = L$.□

Proof of Proposition 4: The Afternoon Effect 1. We start by proving the afternoon effect for the BFF strategy. Fix the buyer beliefs $\widehat{\sigma} = \sigma^{bff}$. Note that then the random variable τ_k is affiliated with the signals related to good k, in particular with V_k from the buyers' perspective. We proceed in steps.

Step 1 *The expected second period price in the BFF case is the expected price for any good k conditional on it being sold in the second period:*

$$E[P_2(\sigma^{bff})|\sigma^{bff}] = E[P_k^{bff}(X_k, \psi_k^{bff}; \sigma^{bff})|\tau_k = \tau_L]$$

Follows immediately from symmetry. Notice that the second expectation above will also be conditioned on the seller's strategy σ^{bff} but we will drop that conditioning in what follows, for notational brevity.

Step 2 *The expected second period price is higher when the second period buyers directly observe the seller's first period signal compared to when they only observe the first period price*:

$$E[P_k^{bff}(X_k, \psi_k^{bff}; \sigma^{bff})|\tau_k = \tau_L] \leq E[P_k^{**}(X_k, \psi_k^{**}; \sigma^{bff})|\tau_k = \tau_L]$$

Note that

$$E[P_k^{bff}(X_k, \psi_k^{bff}; \sigma^{bff})|\tau_k = \tau_L] = E[P_k^{*}(X_k, \psi_k^{*}; \sigma^{bff})|\tau_k = \tau_L] \leq E[P_k^{**}(X_k, \psi_k^{**}; \sigma^{bff})|\tau_k = \tau_L]$$

where the first equality follows from the fact that given $\tau_k = \tau_L$, the two signals ψ_k^{bff} and ψ_k^* are identical:

$$\psi_k^{bff} = (Z_{k'}, \tau_k) = \psi_k^* \text{ when } \tau_k = \tau_L$$

and the last inequality follows from the Linkage Principle and the fact that the signal ψ_k^{**} contains more information about θ_k than the signal ψ_k^* , unconditionally and conditional on $\tau_k = \tau_L$: (see Theorem 9 in MW)

$$
\begin{aligned}
\psi_k^{**} &= (\theta_{k'}, \tau_k) \\
\psi_k^{*} &= (Z_{k'}, \tau_k)
\end{aligned}
$$

Step 3 *The expected price is higher in the no revelation case compared to the BFF case when the second period buyers observe the seller's first period signal :*

$$E[P_k^{**}(X_k, \psi_k^{**}; \sigma^{bff}) | \tau_k = \tau_L] \leq E[P_k^{NR}(X_k)]$$

Note that, by definition,

$$E[P_k^{**}(X_k, \psi_k^{**}; \sigma^{bff}) | \tau_k = \tau_L] = E[v_k^{**}(Z_k, Z_k, \theta_{k'}, \tau_L; \sigma^{bff}) | \tau_k = \tau_L]$$

where

$$\begin{aligned} v_k^{**}(z, z, \theta, \tau_L; \sigma^{bff}) &= E[V_{i_k} | X_{i_k} = z, Y_{i_k} = z, \theta_{k'} = \theta, \tau_k = \tau_L; \sigma^{bff}] \\ &\leq E[V_{i_k} | X_{i_k} = z, Y_{i_k} = z, \theta_{k'} = \theta; \sigma^{bff}] \\ &= E[V_{i_k} | X_{i_k} = z, Y_{i_k} = z; \sigma^{bff}] \\ &\equiv v_k^{NR}(z, z) \end{aligned}$$

where the first equality is definitional, the inequality follows from affiliation of τ_k with the other random variables related to good k (given buyer beliefs) (see Theorem 5 in MW), the next equality follows from the fact that $\theta_{k'}$ is independent of θ_k (and contains information about θ_k only in conjunction with τ_k), and the last equality is definitional. Therefore,

$$E[v_k^{**}(Z_k, Z_k, \theta_{k'}, \tau_L; \sigma^{bff}) | \tau_k = \tau_L] \leq E[v_k^{NR}(Z_k, Z_k) | \tau_k = \tau_L] \leq E[P_k^{NR}(X_k)]$$

and we have the desired result.

Step 4 *The expected first period price in the BFF case is the expected price for any good k conditional on it being sold in the first period:*

$$E[P_1(\sigma^{bff}) | \sigma^{bff}] = E[P_k^{bff}(X_k, \psi_k^{bff}; \sigma^{bff}) | \tau_k = \tau_H]$$

Follows immediately from symmetry.

Step 5 *The expected first period price in the BFF case is higher than the expected price in the no revelation case :*

$$E[P_k^{bff}(X_k, \psi_k^{bff}; \sigma^{bff}) | \tau_k = \tau_H] \geq E[P_k^{NR}(X_k)]$$

Note first that

$$E[P_k^{bff}(X_k, \psi_k^{bff}; \sigma^{bff}) | \tau_k = \tau_H] = E[P_k^{npo}(X_k, \psi_k^{npo}; \sigma^{bff}) | \tau_k = \tau_H]$$

as

$$\psi_k^{bff} = \tau_k = \psi_k^{npo} \text{ when } \tau_k = \tau_H$$

Moreover,

$$E[P_k^{npo}(X_k, \psi_k^{npo}; \sigma^{bff}) | \tau_k = \tau_H] = E[v_k^{npo}(Z_k, Z_k, \tau_H, \sigma^{bff}) | \tau_k = \tau_H]$$

where

$$\begin{aligned} v_k^{npo}(z,z,\tau_H;\sigma^{bff}) &= E[V_{i_k}|X_{i_k}=z,Y_{i_k}=z,\tau_k=\tau_H;\sigma^{bff}] \\ &\geq E[V_{i_k}|X_{i_k}=z,Y_{i_k}=z;\sigma^{bff}] \\ &= v_k^{NR}(z,z) \end{aligned}$$

where the two equalities are definitional and the inequality follows from affiliation.. Thus

$$\begin{aligned} E[v_k^{npo}(Z_k,Z_k,\tau_H,\sigma^{bff})|\tau_k &= \tau_H] \geq E[v_k^{NR}(Z_k,Z_k)|\tau_k=\tau_H] \\ &\geq E[P^{NR}(X_k)] \end{aligned}$$

This concludes the proof of 1.

The proof 2 is similar and that of 3 is immediate from the definition of the strategy. □

9 Bibliography

1. Ashenfelter, O. (1989), "How auctions work for wine and art," *Journal of Economic Perspectives, 3, 23–36.*

2. Ashenfelter, O. and D. Genesove (1992), "Testing for price anomalies in real-estate auctions," *American Economic Review*, 82, 501–505.

3. Avery, C. (1998), "Strategic jump bidding in English auctions," *Review of Economic Studies*, 65, 185–210.

4. Benoit, J-P. and V. Krishna (1998), "Multi-object auctions with budget constrained bidders," Working paper, New York University and Pennsylvania State University.

5. Bernhardt, D. and D. Scoones (194), "A note on sequential auctions," *American Economic Review*, 84, 653–657.

6. Black, J. and D. de Meza (1992), "Systematic price divergences between successive auctions are no anomaly", *Journal of Economics and Management Strategy, 1,* 607–628.

7. Beggs, A. W. and K. Graddy (1997), "Declining values and the afternoon effect: evidence from art auctions. *Rand Journal of Economics*, 28, 544–565.

8. Cassady, R., Jr. (1967), *Auctions and Auctioneering,* University of California Press, Berkeley and Los Angeles.

9. Engelbrecht-Wiggans, R. (1994), "Sequential auctions of stochastically equivalent goods," *Economics Letters*, 44, 87–90.

10. von der Fehr, N-H. M. (1994), "Predatory bidding in sequential auctions, *Oxford Economic Papers,* 46, 345–346.

11. Frutos, M.A. and R.W. Rosenthal (1997), "On some myths about sequenced common-value auctions," Boston University, ISP Discussion Paper #77.

12. Gale, I. L. and D. B. Hausch (1994), "Bottom-fishing and declining prices in sequential auctions," *Games and Economics Behavior, 7,* 318–331.

13. Klemperer, P. D. (1999), "Auction theory: a guide to the literature," *Journal of Economic Surveys*,13.

14. Laffont, J-J. (1997), "Game theory and empirical economics: the case of auction data," *European Economic Review*, 41, 1–35.

15. Lusht,-K.-M..(1994), "Order and price in a sequential auction," *Journal of Real Estate Finance and Economics*, 8, 259-266.

16. Maskin, E. (1992), "Auctions and privatization," in *Privatization*, edited by H. Siebert, Institut fur Weltwirtschaft an der Universitat Kiel.

17. McAfee, R. P. and D. Vincent (1993), "The declining price anomaly," *Journal of Economic Theory, 60,* 191–212.

18. Milgrom, P.R. and R. J. Weber (1982a), "A theory of auctions and competitive bidding," *Econometrica*, 50, 1089–1122.

19. Milgrom, P.R. and R. J. Weber (1982b), "The value of information in a sealed-bid auction," *Journal of Mathematical Economics*, 10, 105–114.

20. Milgrom, P.R. and R. J. Weber (1982c), "A theory of auctions and competitive bidding II," Mimeo, Northwestern University.

21. Ortega-Reichert, A. (1968), "A sequential game with information uncertainty," *Models for Competitive Bidding under Uncertainty*, Ch. 8, Stanford University Ph.D. thesis.

22. Tong, Y.L.: *Probability Inequalities for Multivariate Distributions.* New York: Academic Press, 1980

23. Topkis, D.M.: *Supermodularity and Complementarity.* Princeton University Press, 1998.

24. Vanderporten, B. (1992), "Strategic behavior in pooled condominium auctions," *Journal of Urban Economics*, 31, 123–37.

25. W. Vickrey (1961), "Counterspeculation, auctions, and competitive sealed tenders," *Journal of Finance*, 16, 15–27.